Boolean Operations

Truth Tables

Truth Tables are a tool we use to compute the values of logical expressions over each combination of input value. For example, a truth table for the logical AND of two logic variables, p and q, would look something like this:

p	q	p ^ q
true	true	true
true	false	false
false	true	false
false	false	false

not

the *NOT* operation (\sim) is the logical negation of p. \sim p = false if p = true and \sim p = true if f = false.

Truth Table for ~p:

p	~p
true	false
false	true

or

the OR operation (v) holds true when either p or q are true.

Truth table for p v q:

p	q	p v q	
true	true	true	
true	false	true	
false	true	true	
false	false	false	

and

the AND operation (\wedge) holds true only when both p and q are true.

Truth table for $p \wedge q$:

p	\mathbf{q}	p v d	
true	true	true	
true	false	false	

false	true	false
false	false	false

xor

The *EXCLUSIVE-OR* operation (\oplus) holds true when either *p* or *q* are true, but not both.

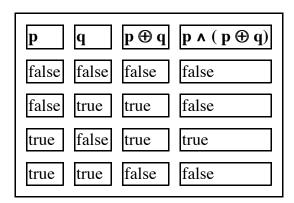
Truth table for $p \oplus q$:

p	q	p
true	true	false
true	false	true
false	true	true
false	false	false

Boolean Algebra

Example: Evaluate the boolean function: $F(p,q) = p \land (p \oplus q)$

Let us construct a truth table:



$$F(0,0) = 0$$

$$F(0,1) = 0$$

$$F(1,0) = 1$$

$$F(1,1) = 0$$

Boolean Algebra Identities

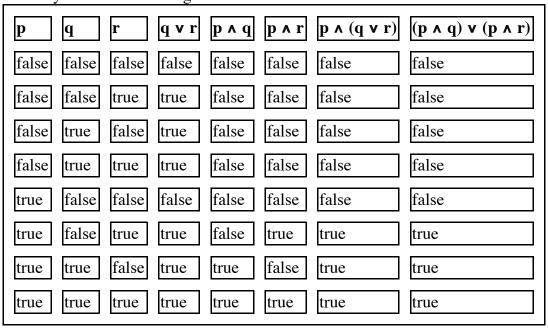
Identity	AND form	OR form
Identity Laws	p ∧ 1 = p	$p \vee 0 = p$
Dominance Laws	$p \wedge 0 = 0$	p v 1 = 1

Idempotent Laws	$p \wedge p = p$	$p \vee p = p$
Complement Laws	$p \land \sim p = 0$	p v ~p = 1
Double Complement Laws	\sim (\sim p) = p	
Commutative Laws	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Associative Laws	$p \wedge (q \wedge r) = (p \wedge q) \wedge r$	$p \lor (q \lor r) = (p \lor q) \lor r$
Distributive Laws	$p \land (q \lor r) = (p \land q) \lor (p \land r)$	$p \lor (q \land r) = (p \lor q) \land (p \lor r)$
Absorption Laws	$p \land (p \lor q) = p$	$p \lor (p \land q) = p$
DeMorgan's Laws	\sim (p \land q) = \sim p \lor \sim q	\sim (p v q) = \sim p \wedge \sim q

We can use the identiies to help us evaluate boolean functions. Each law can be proved by either a truth table or by using other laws.

Example: Use a truth table to prove the distributive law over the AND operation:

Proposition: $p \land (q \lor r) = (p \land q) \lor (p \land r)$ Proof by construction using a truth table:



Example: Use the laws of Boolean Algebra to prove the Absorption Law over the AND operation:

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Proposition: p \land (p \lor q) = p
p \land (p \lor q) = (p \land p) \lor (p \land q) \qquad \text{by Distributive Law}
= p \lor (p \land q) \qquad \text{by Idempotent Law}
= p \land (1 \lor q) \qquad \text{by Distributive Law}
= p \land 1 \qquad \text{by Dominance Law}
= p \qquad \text{by Identity Law}
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