Available online at www.sciencedirect.com

ScienceDirect

Procedia - Social and Behavioral Sciences 195 (2015) 1860 – 1864

Procedia
Social and Behavioral Sciences

World Conference on Technology, Innovation and Entrepreneurship

# Performance Analysis of Karatsuba Multiplication Algorithm for Different Bit Lengths

Can Eyupoglu*

*Istanbul Commerce University, Faculty of Engineering and Design, Department of Computer Engineering, Istanbul, Turkey*

**Abstract**

In computer arithmetic, multiplication is one of the most significant operations. Multiplication is used in many operations such as division, squaring and computing reciprocal. In addition, the efficiency of multiplication is crucial due to the use of digital signal processing applications such as correlation, filtering, frequency analysis and image processing. Karatsuba algorithm is one of the algorithms developed for increasing the efficiency and reducing the cost in order to simplify multiplication. In this study, the performance of Karatsuba algorithm is analyzed in terms of the number of multiplication and the total process time for different bit lengths.


Peer-review under responsibility of Istanbul Univeristy.

*Keywords:* Computer Arithmetic, Multiplication, Karatsuba Algorithm

## 1. Introduction

Multiplication is a very significant arithmetic operation for many signal processing applications which are correlation, convolution, frequency analysis, image processing etc. The speed of multiplication operation is important for the process time of these applications (Patil et al., 2014). Moreover, multiplication is an arithmetic operation usually used in hardware level in digital filtering (Madke and Zafar, 2014). In addition, the efficiency of multiplication operation is a base for implementation of modulators, cryptosystems, ALU (Arithmetic Logic Unit) and many other devices like these (Mishra and Pradhan, 2012).

* Corresponding author. Tel.: 444-0413/3235; fax: +90-216-489-0269.
*E-mail address: ceyupoglu@ticaret.edu.tr*

1877-0428 
Peer-review under responsibility of Istanbul Univeristy.
doi:10.1016/j.sbspro.2015.06.420

In classical multiplication method, adding two integers of $n$-bits takes $O(n)$ bit operations and multiplying two integers of $n$-bits takes $O(n^2)$ bit operations. Complexity of addition operation is optimum in terms of the number of bit operations. However, complexity of multiplication operation is not optimal for integers (Dwivedi, 2013). There are various algorithms explored to enhance $O(n^2)$ multiplication bound. The simplest algorithm is Karatsuba multiplication algorithm (Aho et al., 1974). In order to multiply two integers of $n$-bits, Karatsuba algorithm uses divide and conquer technique and takes $O(n^{\log 3})$ bit operations. Karatsuba algorithm performs multiplication operation by replacing some multiplications with subtraction and addition operations which are less costly (Karatsuba and Ofman, 1963). Karatsuba algorithm runs slower than classical multiplication method for small inputs due to recursion overhead and it is more efficient for large numbers (Dwivedi, 2013). In this study, it is thought that multiplication is performed by multiplicand and multiplier having equal length.

The rest of the paper is organized as follows. Section 2 describes Karatsuba multiplication algorithm. Section 3 analyses the performance of Karatsuba algorithm for different bit lengths. Finally, conclusions being under study are summarized in section 4.

## 2. Method

The simplest technique in order for multiplying two $n$-digit integers is classical method or long multiplication method and requires $O(n^2)$ multiplication operations. In order to improve this requirement, there are many algorithms. The simplest one is Karatsuba algorithm which utilizes divide and conquer technique. Classical method needs four multiplications for performing 2-digit multiplication operation. In Karatsuba algorithm, 2-digit multiplication operation can be performed with three multiplications (Dwivedi, 2013).

Karatsuba algorithm is used by Intel and other companies to perform faster multiplication, because it requires less number of steps (Madke and Zafar, 2014). Besides, Karatsuba algorithm can be implemented as recursive and iterative (Mishra and Pradhan, 2012). The multiplication of two 2-digit decimal numbers is calculated as shown in Fig. 1.

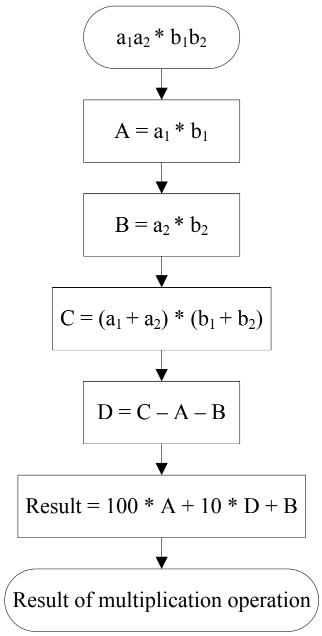


Fig. 1. Flowchart of Karatsuba algorithm (Multiplication of two 2-digit numbers).

Karatsuba algorithm can be recursively applied with dividing multiplicand and multiplier into two parts for large-digit multiplication operation. The cost of multiplication operation is higher than the cost of addition and subtraction operations (Dwivedi, 2013). For example, the calculation of 97*98 multiplication operation using Karatsuba algorithm is shown in Fig. 2. In classical multiplication method, this multiplication operation is calculated with four 1-digit multiplication in addition to some addition and shift operations. According to the flowchart, Karatsuba algorithm requires three multiplications along with some addition, subtraction and shift operations in order to perform this multiplication operation. In step C, the result of 16*17 operation can be calculated recursively by applying the method once again.

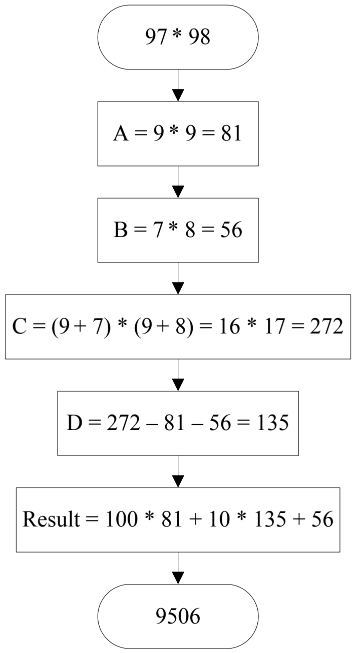


Fig. 2. Calculation of 97*98 multiplication operation using Karatsuba algorithm.

## 3. Results and Discussion

In this paper, the performance of Karatsuba algorithm is investigated for multiplicand and multiplier having 4, 8, 16 and 32 bit length. Moreover, the performance of Karatsuba algorithm is analyzed in terms of the number of multiplication and the total process time. The applications used for performance analysis are implemented using MATLAB R2014a and the computer used for testing has these features: Windows 7 64 bit Operating System, Intel Core i5-3317U CPU @ 1.70 GHz Processor and 4 GB RAM. The performance analysis of Karatsuba algorithm in terms of the number of multiplication for different bit lengths is given in Fig. 3.

The bit length increases along with the number of multiplication due to the processing of Karatsuba algorithm. In addition, the more the number of multiplication raises, the more the amount of hardware increases. Therefore, the cost required for performing multiplication operation rises. When compared to each other, the number of multiplication of Karatsuba algorithm is less than classical multiplication method. The performance of Karatsuba algorithm in terms of the total process time for different bit lengths is analyzed as shown in Fig. 4.

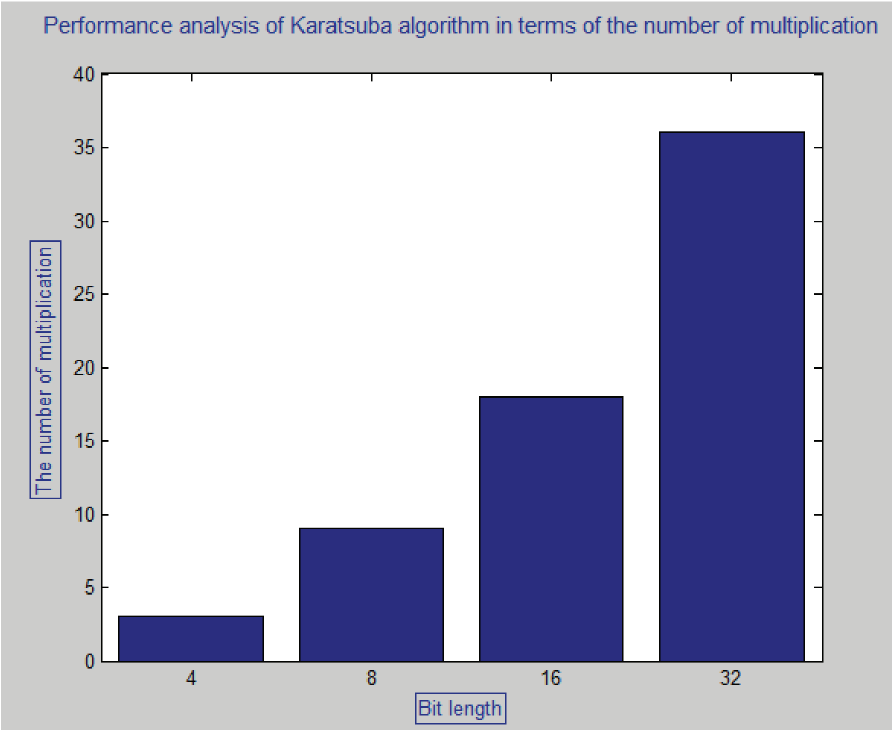


Fig. 3. Performance analysis of Karatsuba algorithm in terms of the number of multiplication for different bit lengths.

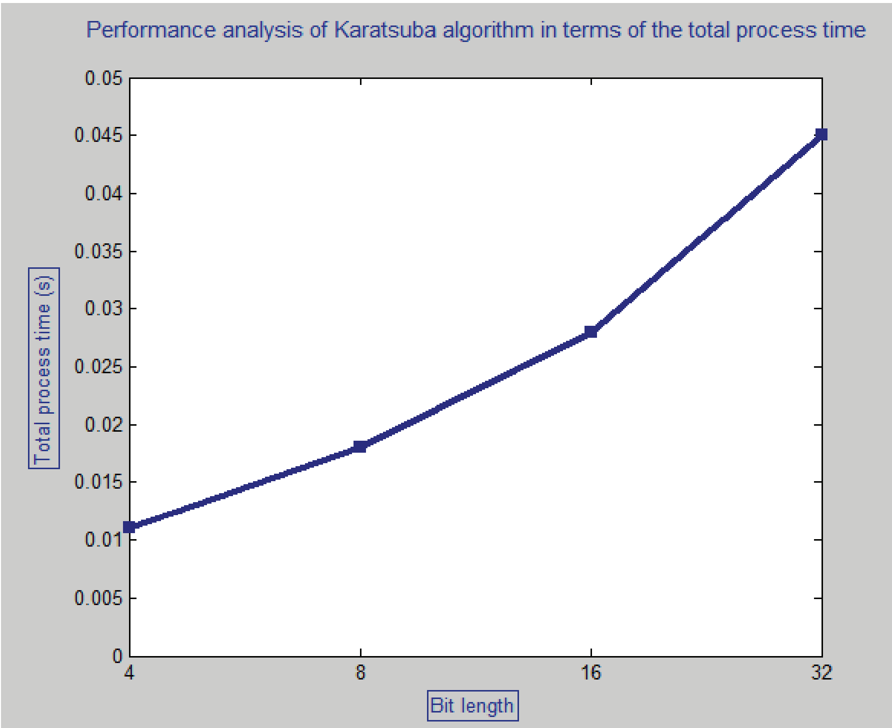


Fig. 4. Performance analysis of Karatsuba algorithm in terms of the total process time for different bit lengths.

As shown in the graph, the more the bit length increases, the more the total process time raises. The reason is that the number of required multiplications increases along with the bit length. Moreover, the total process time is inversely proportional to the processing speed. The increase of the total process time decreases the processing speed of multiplication. When compared to classical multiplication method, Karatsuba algorithm gives better results in terms of the total process time.

## 4. Conclusion

In this study, the performance of Karatsuba multiplication algorithm is analysed. The performance analysis is carried out for different bit lengths. The number of multiplication and the total process time are used as analysis parameters. According to the study results, the bit length raises along with the number of multiplication owing to the processing of Karatsuba algorithm. The more the bit length increases, the more the total process time rises. Besides, Karatsuba algorithm has better results than classical multiplication method in terms of the number of multiplication and the total process time, because the number of multiplication and the cost required for performing multiplication operation are less than classical multiplication method.

As a future work, the performance of Karatsuba algorithm can be compared with the performance of the other algorithms developed for performing multiplication operation. In addition, an algorithm can be developed for increasing the efficiency of multiplication operation and reducing the cost of process.

## References

Aho, A. V., Hopcroft, J. E., & Ullman, J. D. (1974). *The design and analysis of computer algorithms*, Addison-Wesley.

Dwivedi, S. P. (2013). An efficient multiplication algorithm using Nikhilam method, Fifth International Conference on Advances in Recent Technologies in Communication and Computing (ARTCom), 20-21 Sept, Bangalore, IET, ISBN: 978-1-84919-842-4, 223-228.

Karatsuba, A., & Ofman, Y. (1963). *Multiplication of multidigit numbers on automata*, English Translation in Soviet Physics Doklady.

Madke, D., & Zafar, S. (2014). Polynomial Multiplication Using Karatsuba and Nikhilam Sutra, *International Journal of Advanced Research in Computer Science and Software Engineering*, *4 (6),* 1423-1428.

Mishra, S., & Pradhan, M. (2012). Synthesis Comparison of Karatsuba Multiplierusing Polynomial Multiplication, Vedic Multiplier and Classical Multiplier, *International Journal of Computer Applications*, *41 (9),* 13-17.

Patil, S., Manjunatha, D. V., & Kiran, D. (2014). Design of Speed and Power Efficient Multipliers Using Vedic Mathematics with VLSI Implementation, International Conference on Advances in Electronics, Computers and Communications (ICAECC), 10-11 Oct., Bangalore, IEEE, DOI: 10.1109/ICAECC.2014. 7002484, 1-6.