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(D.5) A) Convert to s-domain.

$m\ddot{y} + b\dot{y} + ky = F(t)$, $\boxed{ms^2 Y(s) + bsY(s) + kY(s) = F(s)}$

B) $Y(s)(ms^2 + bs + k) = F(s)$, $\boxed{\frac{Y(s)}{F(s)} = \frac{1}{ms^2 + bs + k}}$

C) $\boxed{F(s) \rightarrow \boxed{\frac{1}{ms^2+bs+k}} \rightarrow Y(s)}$

(D.6) Find the linear state space equations.

$\vec{x} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$, $\dot{\vec{x}} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix}$ $\vec{y} = [z]$, $\vec{u} = [F]$

$\ddot{z} = \frac{F(t)}{m} - \frac{kz}{m} - \frac{b\dot{z}}{m}$

$$\boxed{\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ m \end{bmatrix} F}$$

$$\boxed{y = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x} + [0] F}$$

E.5 1) Convert the linear equations to the s-domain:

$$m_1 \ddot{\tilde{z}} + m_1 g \hat{\theta} = 0 , \quad \boxed{m_1 s^2 \tilde{Z}(s) + m_1 g \tilde{\Theta}(s) = 0} ,$$

$$\boxed{\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) s^2 \tilde{\Theta}(s) + m_1 g \tilde{Z}(s) = \ell \tilde{F}(s)}$$

B) Find $\frac{\tilde{Z}(s)}{\tilde{F}(s)}$, and $\frac{\tilde{\Theta}(s)}{\tilde{F}(s)}$, and $\frac{\tilde{Z}(s)}{\tilde{\Theta}(s)}$

$$\begin{bmatrix} m_1 s^2 , & m_1 g \\ \frac{m_1}{\ell} g , & \left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) s^2/\ell \end{bmatrix} \begin{bmatrix} \tilde{Z}(s) \\ \tilde{\Theta}(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tilde{F}(s)$$

$$m_1 s^2 \tilde{Z}(s) = -m_1 g \tilde{\Theta}(s) , \quad \boxed{\frac{\tilde{Z}(s)}{\tilde{\Theta}(s)} = -\frac{g}{s^2}}$$

$$\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) s^2 + m_1 g \frac{\tilde{Z}(s)}{\tilde{\Theta}(s)} = \ell \frac{\tilde{F}(s)}{\tilde{\Theta}(s)} , \quad \left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) s^2 - \frac{m_1 g^2}{s^2} = \ell \frac{\tilde{F}(s)}{\tilde{\Theta}(s)}$$

$$\frac{\tilde{F}(s)}{\tilde{\Theta}(s)} = \left(\frac{m_2 \ell}{3} + \frac{m_1 z_e^2}{\ell}\right) s^2 - \frac{m_1 g^2}{\ell s^2} = \frac{s^4\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) - m_1 g^2}{\ell s^2}$$

$$\frac{\tilde{\Theta}(s)}{\tilde{F}(s)} = \frac{s^2 \ell}{s^4\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) - m_1 g^2} \qquad \boxed{\frac{\tilde{\Theta}(s)}{\tilde{F}(s)} = \frac{\ell}{s^2\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) - \frac{m_1 g^2}{s^2}}}$$

$$\frac{\tilde{Z}}{\tilde{F}} = \frac{\tilde{Z}}{\tilde{\Theta}} \frac{\tilde{\Theta}}{\tilde{F}} = \boxed{\frac{-g \ell}{s^4\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) - m_1 g^2} = \frac{\tilde{Z}(s)}{\tilde{F}(s)}}$$

C) Let $m_1 g \tilde{z} = 0$, then $\boxed{\frac{\tilde{\Theta}(s)}{\tilde{F}(s)} = \frac{\ell}{s^2\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right)}}$

$$\boxed{\frac{\tilde{Z}(s)}{\tilde{F}(s)} = \frac{-g \ell}{s^4\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right)}}$$

D)

$$\tilde{F}(s) \rightarrow \boxed{\frac{\ell}{s^2\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right)}} \xrightarrow{\tilde{\Theta}(s)} \boxed{\frac{-g}{s^2}} \longrightarrow \tilde{Z}(s)$$

(5.6)

$$\tilde{x} = \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{bmatrix} \qquad \tilde{u} = \tilde{F} \qquad \tilde{y} = \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \end{bmatrix} \qquad \dot{\tilde{x}} = \begin{bmatrix} \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \\ \ddot{\tilde{z}} \\ \ddot{\tilde{\theta}} \end{bmatrix}$$

$$m_1 \ddot{\tilde{z}} + m_1 g \tilde{\theta} = 0$$

$$\ddot{\tilde{z}} = -g \tilde{\theta}$$

$$\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right) \ddot{\tilde{\theta}} + m_1 g \tilde{z} = \ell \tilde{F}$$

$$\ddot{\tilde{\theta}} = \frac{-m_1 g}{\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right)} \tilde{z} + \frac{\ell}{\left(\frac{m_2 \ell^2}{3} + m_1 z_e^2\right)} \tilde{F}$$

$$\boxed{\dot{\tilde{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -g & 0 & 0 \\ \frac{-m_1 g}{\frac{m_2 \ell^2}{3} + m_1 z_e^2} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\ell}{\frac{m_2 \ell^2}{3} + m_1 z_e^2} \end{bmatrix} \tilde{F}}$$

$$\boxed{\tilde{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tilde{F}}$$

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F.S A)

$$(m_c+2m_r)\ddot{\check{Z}} + \mu\dot{\check{Z}} = -F_c\check{\Theta}$$
$$(m_c+2m_r)\ddot{\check{h}} = \check{F}$$
$$(J_c+2m_r d^2)\ddot{\check{\theta}} = \check{\tau}$$

$$(m_c+2m_r)s^2\check{Z}(s) + \mu s\check{Z}(s) = -F_c\check{\Theta}(s)$$
$$(m_c+2m_r)s^2\check{H}(s) = \check{F}(s)$$
$$(J_c+2m_r d^2)s^2\check{\Theta}(s) = \check{T}(s)$$

B) Find

$$\frac{\check{H}(s)}{\check{F}(s)} = \frac{1}{s^2(m_c+2m_r)}$$

C) Find $\frac{\check{\Theta}(s)}{\check{T}(s)}$ and $\frac{\check{Z}(s)}{\check{T}(s)}$ and $\frac{\check{Z}(s)}{\check{\Theta}(s)}$

$$\frac{\check{\Theta}(s)}{\check{T}(s)} = \frac{1}{s^2(J_c+2m_r d^2)}$$

$$\frac{\check{Z}(s)}{\check{\Theta}(s)} = \frac{-F_c}{s^2(m_c+2m_r)+s\mu}$$

$$\frac{\check{Z}(s)}{\check{T}(s)} = \left(\frac{\check{Z}(s)}{\check{\Theta}(s)}\right)\left(\frac{\check{\Theta}(s)}{\check{T}(s)}\right) = \frac{-F_c}{s^4(J_c+2m_r d^2)(m_c+2m_r)+s^3\mu(J_c+2m_r d^2)}$$

D) Draw a block diagram of the 2 systems

$$\check{F}(s) \rightarrow \boxed{\frac{1}{s^2(m_c+2m_r)}} \rightarrow \check{H}(s)$$

$$\check{T}(s) \rightarrow \boxed{\frac{1}{s^2(J_c+2m_r d^2)}} \xrightarrow{\check{\Theta}(s)} \boxed{\frac{-F_c}{s^2(m_c+2m_r)+s\mu}} \rightarrow \check{Z}(s)$$

or

$$\check{T}(s) \rightarrow \boxed{\frac{-F_c}{s^4(J_c+2m_r d^2)(m_c+2m_r)+s^3\mu(J_c+2m_r d^2)}} \rightarrow \check{Z}(s)$$

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F.6 A) $\tilde{x}_{lon} = \begin{bmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{bmatrix}$, $\tilde{u}_{lon} = \tilde{F}$, $\tilde{y}_{lon} = \tilde{h}$, $\dot{\tilde{x}}_{lon} = \begin{bmatrix} \dot{\tilde{h}} \\ \ddot{\tilde{h}} \end{bmatrix}$

$\ddot{\tilde{h}} = \frac{\tilde{F}}{(m_c + 2m_r)}$, $\dot{\tilde{h}} = \dot{\tilde{h}} = \tilde{x}_{lon}(2) = \dot{\tilde{x}}_{lon}(1)$

$$\boxed{\dot{\tilde{x}}_{lon} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_c + 2m_r} \end{bmatrix} \tilde{F}} \quad \boxed{\tilde{y}_{lon} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{bmatrix} + [0] \tilde{F}}$$

B) $\tilde{x}_{lat} = \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{bmatrix}$ $\tilde{u}_{lat} = \tilde{\tau}$ $\tilde{y}_{lat} = \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \end{bmatrix}$ $\dot{\tilde{x}}_{lat} = \begin{bmatrix} \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \\ \ddot{\tilde{z}} \\ \ddot{\tilde{\theta}} \end{bmatrix}$

$(m_c + 2m_r)\ddot{\tilde{z}} + \mu \dot{\tilde{z}} = -F_e \tilde{\theta}$ $\quad (J_c + 2m_r d^2)\ddot{\tilde{\theta}} = \tilde{\tau}$

$\ddot{\tilde{z}} = \frac{-F_e}{m_c + 2m_r}\tilde{\theta} - \frac{\mu}{m_c + 2m_r}\dot{\tilde{z}}$ $\quad \ddot{\tilde{\theta}} = \frac{\tilde{\tau}}{J_c + 2m_r d^2}$

$$\boxed{\dot{\tilde{x}}_{lat} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-F_e}{m_c + 2m_r} & \frac{-\mu}{m_c + 2m_r} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_c + 2m_r d^2} \end{bmatrix} \tilde{\tau}}$$

$$\boxed{\tilde{y}_{lat} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{bmatrix} + [0] \tilde{\tau}}$$