Tutorial #2: Mapping

1 Exercise 1:

$$f(p/q) = \frac{p+1}{p-2} \tag{1}$$

When $p = 2, \forall q \in \mathbb{Q}^*$, we have: $f(2/q) = \frac{3}{0}$.
There exist (p/q) in domain such that $f(p/q)$ in codomain doesn't exist.
Thus, (1) is not mapping.

$$f(p/q) = \frac{p}{p+q} \tag{2}$$

When $p = -q$, we have: $f(-1) = \frac{p}{0}$.
There exist (p/q) in domain such that $f(p/q)$ in codomain doesn't exist.
Thus, (2) is not mapping.

2 Exercise 2:

a)

$$f : \mathbb{R} \to \mathbb{R} \text{ defined by } f(x) = e^x$$

As $f'(x) = e^x > 0 \ \forall x \in \mathbb{R}$, the function f only increases in $\mathbb{R}$. Thus, the function is injective. However, for negative elements in codomain ($f(x) < 0$), there exists no $x \in \mathbb{R}$ maps to $f(x)$. Hence, the function injective but not subjective.

b)

$$f : \mathbb{Z} \to \mathbb{Z} \text{ defined by } f(n) = n^2 + 5$$

As $f(-n) = f(n) \ \forall n \in \mathbb{Z}$, the function is not injective. In addition, when $f(n) = 3$, there no such $x \in \mathbb{Z}$ maps to $f(x)$. Thus, the function is neither subjective nor injective.

c)

$$f : \mathbb{R} \to \mathbb{R} \text{ defined by } f(x) = sin(x)$$

As $f(2\pi) = f(0) = 1$, the function is not injective. In addition, when $f(n) = 2$, there no such $x \in \mathbb{Z}$ maps to $f(x)$. Thus, the function is neither subjective nor injective.

3 Exercise 3:

a)

$$f(x) = \frac{1}{2}x + 7$$

Let $y = f(x)$, we re-write the equation as

$$y = \frac{1}{2}x + 7 \iff x = 2y - 14$$

Thus, the inverse function is $f^{-1}(x) = 2x - 14$

b)

$$f(x) = (x-2)^3 + 1$$

let $y = f(x) - 1$, we re-write the equation as

$$y = (x-2)^3 \iff x = \sqrt[3]{y} + 2$$

Thus, the inverse function is $f^{-1}(x) = \sqrt[3]{x-1} + 2$

c)

$$f(x) = \frac{1+2x}{7+x}$$

Let $y = f(x)$, we re-write the equation as

$$y = \frac{1+2x}{7+x} \iff x = \frac{1-7y}{y-2}$$

Thus, the inverse function is $f^{-1}(x) = \frac{1-7x}{x-2}$

4 Exercise 4:

The plot leaves for reader ;)
The domain of f: $x \in \mathbb{R} \setminus \{0\}$. The range of f: $f \in \mathbb{R} \setminus \{0\}$
Let $y = f(x)$, we re-write the equation as

$$y = \frac{x+1}{x-1} \iff x = \frac{y+1}{y-1}$$

Thus, the inverse function of f is $f^{-1} = \frac{x+1}{x-1}$.
Compare:

$$f \circ f^{-1} = f(f^{-1}(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = x$$

$$f^{-1} \circ f = f^{-1}(f(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = x$$

Conclusion: $f \circ f^{-1} = f^{-1} \circ f$