The energy equation of the snake $\gamma(s)$ is given by

$$E_{snake}(s) = \oint [\alpha\|\gamma'(s)\|^2 + \beta\|\gamma''(s)\|^2 - \delta\|\nabla(\mathrm{G} \otimes \mathrm{I})\|^2\, \gamma(s)]\, ds$$

The E-L equation is given by,

$$\frac{\partial E}{\partial \gamma} - \frac{\partial}{\partial s}\frac{\partial E}{\partial \gamma'} + \frac{\partial^2}{\partial s^2}\frac{\partial E}{\partial \gamma''} = 0$$

We convert it to time-differential equation, and $\gamma(s)$ becomes $\gamma(s,t)$.

$$\frac{\partial \gamma(s,t)}{\partial t} = \frac{\partial E(s,t)}{\partial \gamma} - \frac{\partial}{\partial s}\frac{\partial E(s,t)}{\partial \gamma'} + \frac{\partial^2}{\partial s^2}\frac{\partial E(s,t)}{\partial \gamma''}$$

$$\frac{\partial \gamma(s,t)}{\partial t} = -\delta\, \nabla\|\nabla(\mathrm{G} \otimes \mathrm{I})\|^2\, \gamma(s,t) - \alpha\frac{\partial^2 \gamma(s,t)}{\partial s^2} + \beta\frac{\partial^4 \gamma(s,t)}{\partial s^4}$$

The equation can be decomposed into x and y components.

$$\frac{\partial \gamma_x(s,t)}{\partial t} = -\delta\, \nabla_{\mathrm{x}}\|\nabla(\mathrm{G} \otimes \mathrm{I})\|^2\, \gamma_x(s,t) - \alpha\frac{\partial^2 \gamma_x(s,t)}{\partial s^2} + \beta\frac{\partial^4 \gamma_x(s,t)}{\partial s^4}$$

$$\frac{\partial \gamma_y(s,t)}{\partial t} = -\delta\, \nabla_y\|\nabla(\mathrm{G} \otimes \mathrm{I})\|^2\, \gamma_y(s,t) - \alpha\frac{\partial^2 \gamma_y(s,t)}{\partial s^2} + \beta\frac{\partial^4 \gamma_y(s,t)}{\partial s^4}$$

The second derivative is given by:

$$\frac{\partial^2 \gamma_x(s,t)}{\partial s^2} = D_2\gamma_x(s,t)$$

Where D_2 is a matrix

$$D_2 = \begin{bmatrix} -2 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 & -2 \end{bmatrix}$$

The fourth derivative is given by:

$$\frac{\partial^4 \gamma_x(s,t)}{\partial s^4} = D_4\gamma_x(s,t)$$

Where D_4 is a matrix

$$D_4 = \begin{bmatrix} -6 & -4 & -1 & 0 & \cdots & 0 & 1 & -4 \\ -4 & 6 & -4 & 1 & \cdots & 0 & 0 & 1 \\ 1 & -4 & 6 & -4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & -4 & 6 & -4 \\ -4 & 1 & 0 & 0 & \cdots & 1 & -4 & 6 \end{bmatrix}$$

From the discrete temporal derivative,

$$\frac{\partial \gamma(s,t)}{\partial t} = \frac{\gamma(s,t) - \gamma(s,t-1)}{\Delta t}$$

$$\gamma(s,t) - \Delta t \frac{\partial \gamma(s,t)}{\partial t} = \gamma(s,t-1)$$

Substituting the value of $\frac{\partial \gamma(s,t)}{\partial t}$ we get,

$$\gamma(s,t) - \Delta t \left[-\delta \, \nabla \|\nabla(\mathrm{G} \otimes \mathrm{I})\|^2 \, \gamma(s,t) \; - \alpha \frac{\partial^2 \gamma(s,t)}{\partial s^2} + \beta \frac{\partial^4 \gamma(s,t)}{\partial s^4} \right] = \gamma(s,t-1)$$

$$\gamma(s,t) - \Delta t[-\delta \, \nabla \|\nabla(\mathrm{G} \otimes \mathrm{I})\|^2 \, \gamma(s,t) \; - \alpha \, D_2 \, \gamma(s,t) + \beta \, D_4 \, \gamma(s,t)] = \gamma(s,t-1)$$

$$[eye() + \Delta t(-\alpha \, D_2 + \beta \, D_4] \, \gamma(s,t) = \; \gamma(s,t-1) + \Delta t \, \delta \, P\left(\gamma_x(s,t-1), \gamma_y(s,t-1)\right)$$

$$A \, \gamma(s,t) = b(s,t-1)$$

$$\gamma(s,t) = A^{-1} \, b(s,t-1)$$