$$aH = Ha = \phi^{-1}(2\phi(a))$$

# Definition

Let  $H \leq G$  w/ kernel HThen H is a normal subgroup of G, denoted

#### Theorem

Let  $\phi:G \to G'$  be a group homomorphism w | Kerkell HThen the cosets of H in Gform a group, called the

coset/botor group, denoted H wrt the operation defined by aH.bH = a.bH

Moreover, the map  $\mu: G_H \to \Phi(G)$   $\text{Eath} \mid a \in G_S^2$ where  $\Phi(G) \leq G'$ ,  $\text{Sefined by } \mu(aH) = \Phi(a)$ , is isomorphic

Proof

1) Yatl, bH & G/H, 3'aH.bH = a.bH (1)

a.bH & G/H

bes Marb EG J'arb, arb EG

2)  $\forall aH, bH, cH \in {}^{G}_{H},$   $(aH \cdot bH) \cdot cH = a \cdot bH \cdot cH = (a \cdot b) \cdot cH$   $= (a \cdot b \cdot c) H$   $= aH \cdot bcH$ 

= aH. (bH.cH)

3) 3'H=eHEGH WatteGH eHall=eaH=atleH identity

4) Watle GH = J'a'HEGH a-1 H.aH = H = aH.a-1+1 inverses /

.. GH has goop structure

Notice the map  $\mu: GH \to \Phi(G)$   $\mu(aH) = \Phi(a)$ ,  $a \in G$  is a homomorphism bus  $(\forall aH, b \notin GH)$   $\mu(aH \cdot bH) = \mu(a \cdot bH)$   $= \Phi(ab)$  $= \mu(aH) \cdot \mu(bH)$  the Kernel of M is = 2 aH C GH | acH = Ker \$3 - 2 H3 hence m is toivial 4
m is injective m is also surjective :- u is a bijective homomorphism n is an isomorphism GH ~ 4(0) where  $\phi(G) \leq G'$ 

Example G G'=Q(G)Let  $\Phi: \mathbb{Z} \to \mathbb{Z}_n$   $\Phi(m) = m \mod n = 0$ where m = nk + r

KEZ USES N

Then \$ is a group homomorphism w/ kernel H = kn = nZ

By thm, Zn is a factor group whose elements are right/left cosets

nZ+0, nZ+1, ---, nZ+(n-1)

w/ identity being

nZ+0 = nZ = <n>
the inverse of nZ+k is

nZ+ (n-k)

Moreover  $\mu: \mathbb{Z}_{n\mathbb{Z}} \to \mathbb{Z}_{n}$   $\mathcal{Z}_{n} \to \mathbb{Z}_{n}$   $\mathcal{Z}_{n} \to \mathbb{Z}_{n}$ Actined by  $\mathcal{Z}_{n} \to \mathbb{Z}_{n}$ is an isomorphism  $\mathbb{Z}_{n} \to \mathbb{Z}_{n}$ 

## Theorem

Let H ≤ G

The left (right) coset "multiplication" is well-defined iff

H 46

# Proof

=> Assume of on cosets defined by aH.bH = a.bH for a,b E6 is well-defined

GH is a group, show aH=Ha aEG

Let x Eatl then x H= a H

(by def. of left coset)

Since alt & GH = a + H & GH

Then  $xH \cdot a^{-1}H = aH \cdot a^{-1}H = H$ So  $xH \cdot a^{-1}H = x \cdot a^{-1}H = H$ 

 $xa^{-1}EH$ ,  $xa^{-1}=h$ , hEHthen  $x = ha \in Ha$ 

we can show Ha = aH

(is similar)

### Theorem

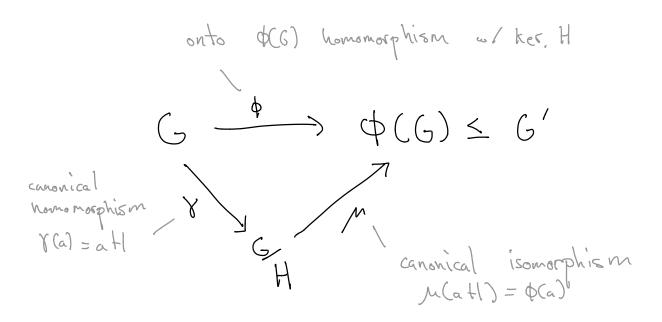
Let  $H \not= G$ Then  $y:G \rightarrow GrH$  defined y(a) = aHis a homomorphism w/ kernel H

Proot

$$\forall a_1 b \in G$$
  $\chi(a \cdot b) = \alpha \cdot b \cdot H = \alpha \cdot H \cdot b \cdot H$   
=  $\chi(\alpha) \cdot \chi(b)$ 

is a homomorphism  $\ker \gamma = \underbrace{\text{deg}}_{a} + \underbrace{\text{deg}}_{a} = \underbrace{\text{Hdg}}_{a}$   $= \underbrace{\text{deg}}_{a} + \underbrace{\text{deg}}_{a} + \underbrace{\text{Hdg}}_{a}$   $= \underbrace{\text{Hdg}}_{a} + \underbrace{\text{Hdg}}_{a} + \underbrace{\text{Hdg}}_{a}$ 

Overview



O. 
$$G \rightarrow H$$

def  $\gamma(\alpha) = \alpha H$ 

is a homomorphism w/ kernel  $H$ 
 $\forall a \in G$ 
 $\forall (\alpha) = M(\gamma(\alpha)) = (M \circ \gamma)(\alpha)$ 
 $\phi = M \circ \gamma$