S_n - the symmetric group on n-lefts
(group of permutations of set I_n)

Let A_n be a subset of S_n of all even permutations of I_n

Let B_n be " " odd permutations of I_n

Then $|A_n| = |B_n| = n!/2$

bcs $A_n \cap B_n = \emptyset$

&

$A_n \cup B_n = S_n$

<u>Fact</u>

The subset A_n of all even permutations of S_n is a subgroup,
called the alternating group

Proof

1) $\forall\, \delta, \sigma \in A_n$, $\delta \cdot \sigma \in A_n$

$\underset{2k}{\delta}, \underset{2k'}{\sigma}$ $\quad 2(k+k')$

(bcs the product of 2 even permutations has an even number of transpositions)

2) Identity for S_n $n \geq 2$ is

$$(1,2)(2,1) \quad (1,2)^2 \in A_n$$

bcs it is even

3) let $\delta \in A_n$, then $\delta^{-1} \in A_n$

bcs it has same # of transpositions

$$\delta = (a_1, a_2) \cdot \ldots \cdot (a_k, a_{k+1})$$

$$\delta^{-1} = (a_k, a_{k+1}) \cdot \ldots \cdot (a_1, a_2)$$

$\therefore$ $A_n \leq S_n$ of order $n!/2$

Note B_n does not have group structure wrt perm. mult bcs B_n is not closed under op (product of two odds is even).

Definition

let G be a group + let $a \in G$

a is of order $n \in \mathbb{Z}^+$ iff n is the least pos. int. st $a^n = e$ (where e is id of G)

that is,

$$a^n = e \wedge \forall k < n \quad a^k \neq e$$

Ex

$$(3\ 1\ 5\ 7) \Rightarrow n=4$$

so $(3\ 1\ 5\ 7)^4 = id$

Fact 1 The order of a cycle in S_n is the length of the cycle

Fact 2 The order of a permutation in S_n is the least common multiple of the lengths of cycles in the disjoint cycle decomposition for this permutation