Definition

Let G, G' be groups

Then $\phi : G \to G'$ is a

group homomorphism iff

$$\forall g_1, g_2 \in G \quad \phi(g_1 \cdot g_2) = \phi(g_1) \cdot \phi(g_2)$$

Note: There is at least one homomorphism btwn any 2 groups, namely the <u>trivial homomorphism</u>

$\phi : G \to G'$ defined $\phi(g) = e'$ $g \in G$

since

$\forall g_1, g_2 \in G \quad g_1 \cdot g_2 \in G$

$$\phi(g_1 \cdot g_2) = e' = e' \cdot e' = \phi(g_1) \cdot \phi(g_2)$$

Example

1) $\phi : S_n \to \mathbb{Z}_2$ $\quad \phi(\sigma) = \begin{cases} 0 & \text{if } \sigma \text{ even} \\ 1 & \text{if } \sigma \text{ odd} \end{cases}$

group homomorphism bcs

$\forall \sigma, \delta \in S_n, \quad \phi(\sigma \cdot \delta) = \phi(\sigma) +_2 \phi(\delta)$

2) $\phi : \mathbb{Z} \to \mathbb{Z}_n \quad \phi(m) = m \bmod n = r$

where $m = nk + r$

$$\forall \underset{\overset{\parallel}{k_1 n + r_1}}{m_1}, \underset{\overset{\parallel}{k_2 n + r_2}}{m_2} \quad \phi(\underset{\overset{\parallel}{(k_1+k_2)\cdot n + (r_1+r_2)}}{m_1 + m_2}) = r_1 +_n r_2 = \phi(m_1) +_n \phi(m_1)$$

3) Let F be additive group of all real valued f-n's of a real variable

let $c \in D$, $D \subseteq \mathbb{R}$ wher $f \in F$

$f : D \to \mathbb{R}$

Define $\phi : F \to \mathbb{R}$ by $\phi(f) = f(c)$

ϕ is group homomorphism bcs

$\forall f, g \in F$

$$\phi(f+g) = (f+g)(c) = f(c) + g(c) = \phi(f) + \phi(g)$$