Let S be a set

Relation $*$ is a **binary operation** on S iff

$$\forall (a,b) \in S \times S \quad \exists! c = a * b \quad c \in S$$

that is

$*$ is a f-n $*: S \times S \to S$

To show $*$ is a bin. op. on S:

1) $*$ is everywhere well-defined on S

$$\forall a, b \in S \quad \exists! c, \; c = a * b$$

2) S is closed under $*$

$$\forall a, b \in S, \quad a * b \in S$$