Sets & Relations

$\mathbb{R}$ set of real numbers

$\mathbb{C}$ complex

$\mathbb{Q}$ rational $\{n/m \mid n, m \in \mathbb{Z} \wedge m \neq 0 \wedge \gcd(n,m) = 1\}$

$\mathbb{Z}$ integers

$\mathbb{N}$ natural $\{1, 2, \ldots\}$

$$A^* = \{a \in A \mid a \neq 0\}$$

$$A^+ = \{a \in A \mid a > 0\}$$

$$A^- = \{a \in A \mid a < 0\}$$

$n\mathbb{Q} \wedge \mathbb{R} = \overline{\mathbb{Q}} = \mathbb{R} \setminus \mathbb{Q}$ is the set of irrational numbers

$$\mathbb{Q} \cap n\mathbb{Q} = \emptyset$$

$\exists x$ - there exists at least one

$\exists! x$ - " " only one

Definitions

A is a **subset** of B $\quad A \subseteq B$
iff $\forall a \in A,\ a \in B$

A is a **proper subset** of B
iff $\forall a \in A,\ a \in B \wedge \exists b \in B,\ b \notin A$
(denoted $A \subset B$)

A **relation** R on set A is a subset of $A \times A$

$$R \subseteq A \times A$$

R is an **equivalence relation** iff

1) $\forall a \in A,\ aRa \quad ((a,a) \in R)$ [reflexive]
2) $\forall a,\ a \in A \quad aRb \Rightarrow bRa$ [symmetric]
3) $\forall a,b,c \in A \quad aRb \wedge bRc \Rightarrow aRc$
[transitive]

Operations

$A \cup B = \{x \mid x \in A \vee x \in B\}$ union

$A \cap B = \{x \mid x \in A \wedge x \in B\}$ intersection

$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ complement of B in A

$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$ cartesian cross product

* Note

$A \cup B = B \cup A$
$A \cap B = B \cap A$ } commutative

$A \times B \neq B \times A$

Definitions

A fn $f: A \to B$ is **bijective** iff

f is surjective (onto B)

$f(A) = B$ iff $(\forall b \in B \ \exists a \in A) \ f(a) = b$

and

f is injective (1-1)

$\forall a_1, a_2 \in A \quad f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

* Note
A bijective fn has an inverse

Let S be a collection of sets
relation R on S, $A, B \in S$, ARB iff

$\exists f: A \to B$, f-bijective.

If ARB, A & B are **equanimous**

(of the same size b/c theres
a 1-1 correspondence)

Notice from the last definition

1) $\forall A \in S \quad ARA$ bcs [reflexive]

$\exists f = id_A : A \to A$, f-bijective

2) $\forall A, B \in S \quad ARB \Rightarrow BRA$ bcs [symmetric]

$\exists f : A \to B$, f-bij $\Rightarrow \exists g = f^{-1} : B \to A$, g-bij

3) $\forall A, B, C \in S \quad ARB \wedge BRC \Rightarrow ARC$

$\exists f : A \to B$, $\exists g : B \to C$, f, g-bij [transitive]

$\Rightarrow \exists h : A \to C$ h-bij

$\therefore$ R is an equiv relation on S
+ partitions S

☆ Note

$E_A = \{B \in S \mid ARB\}$
contain all sets of size $|A|$

Definitions

Set A is **denumerable** iff

Set A is **denumerable** iff
A is equinumerous to $\mathbb{N}$

Set A is **countable** iff
A is finite or denumerable