Recall

let $S_1, S_2, \ldots, S_n$ be sets
The cartesian product is the set

$$S_1 \times S_2 \times \ldots \times S_n = \{(s_1, s_2, \ldots, s_n) \mid s \in S_i\}_{i=1..n}$$

## Theorem

let $G_1, G_2, \ldots, G_n$ be groups
(wrt binary operations $*_1, *_2, \ldots *_n$)
Then

$$G_1 \times G_2 \times \ldots \times G_n = \{(g_1, g_2, \ldots, g_n) \mid g_i \in G_i, i=1..n\}$$

is a group called the
direct product of $G_i$'s
wrt a binary operation defined componentwise

## Proof

1) $\forall (g_1, g_2, \ldots, g_n), (g_1', g_2', \ldots, g_n') \in G_1 \times \ldots \times G_n$

$$\exists! \left(g_1 \underset{*_1}{\cdot} g_1', \dots, g_n \underset{*_n}{\cdot} g_n'\right)\left(g_1 \underset{*_1}{\cdot} g_1', \dots, g_n \underset{*_n}{\cdot} g_n'\right) \in G_1 \times \dots \times G_n$$

The bin op defined on $G_1 \times \dots \times G_n$ is associative

Identity is $(e_1, \dots, e_n)$ where $e_i$ is identity of $G_i$

Inverse of $(g_1, \dots, g_n)$ is $(g_1^{-1}, \dots, g_n^{-1})$ where $g_i^{-1}$ is the inverse of $g_i \in G_i$