A solutions manual for Topology by James Munkres

1. Fundamental Concepts

Exercises

1. Check the distributive laws for $\cup$ and $\cap$ and DeMorgan's laws.

2. Determine which of the following statements are true for all sets A, B, C, and D. If a double implication fails, determine whether one or the other of the possible implications holds. If an equality fails, determine whether the statement becomes true if the "equals" symbol is replaced by one or the other of the inclusion symbols $\subset$ or $\supset$.

(a) $A \subset B$ and $A \subset C \Leftrightarrow A \subset (B \cup C)$.
(b) $A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cup C)$.
(c) $A \subset B$ and $A \subset C \Leftrightarrow A \subset (B \cap C)$.
(d) $A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cap C)$.
(e) $A - (A - B) = B$.
(f) $A - (B - A) = A - B$.
(g) $A \cap (B - C) = (A \cap B) - (A \cap C)$.
(h) $A \cup (B - C) = (A \cup B) - (A \cup C)$.
(i) $(A \cap B) \cup (A - B) = A$.
(j) $A \subset C$ and $B \subset D \Rightarrow (A \times B) \subset (C \times D)$.
(k) The converse of (j).
(l) The converse of (j), assuming that A and B are nonempty.
(m) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.
(n) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
(o) $A \times (B - C) = (A \times B) - (A \times C)$.
(p) $(A - B) \times (C - D) = (A \times C - B \times C) - A \times D$.
(q) $(A \times B) - (C \times D) = (A - C) \times (B - D)$.

3. (a) Write the contrapositive and converse of the following statement: "If $x < 0$, then $x^2 - x > 0$," and determine which (if any) of the three statements are true.
(b) Do the same for the statement "If $x > 0$, then $x^2 - x > 0$."

4. Let A and B be sets of real numbers. Write the negation of each of the following statements:
(a) For every $a \in A$, it is true that $a^2 \in B$.
(b) For at least one $a \in A$, it is true that $a^2 \in B$.
(c) For every $a \in A$, it is true that $a^2 \notin B$.
(d) For at least one $a \notin A$, it is true that $a^2 \in B$.

5. Let $\mathcal{A}$ be a nonempty collection of sets. Determine the truth of each of the following statements and of their converses:
(a) $x \in \bigcup_{A \in \mathcal{A}} A \Rightarrow x \in A$ for at least one $A \in \mathcal{A}$.
(b) $x \in \bigcup_{A \in \mathcal{A}} A \Rightarrow x \in A$ for every $A \in \mathcal{A}$.

(c) $x \in \bigcap_{A \in \mathcal{A}} A \Rightarrow x \in A$ for at least one $A \in \mathcal{A}$.
(d) $x \in \bigcap_{A \in \mathcal{A}} A \Rightarrow x \in A$ for every $A \in \mathcal{A}$.

6. Write the contrapositive of each of the statements of Exercise 5.

7. Given sets A, B, and C, express each of the following sets in terms of A, B, and C, using the symbols $\cup, \cap$, and $-$.

$$\begin{aligned} D &= \{x \mid x \in A \text{ and } (x \in B \text{ or } x \in C)\}, \\ E &= \{x \mid (x \in A \text{ and } x \in B) \text{ or } x \in C\}, \\ F &= \{x \mid x \in A \text{ and } (x \in B \Rightarrow x \in C)\}. \end{aligned}$$

8. If a set A has two elements, show that $\mathcal{P}(A)$ has four elements. How many elements does $\mathcal{P}(A)$ have if A has one element? Three elements? No elements? Why is $\mathcal{P}(A)$ called the powerset of A?

9. Formulate and prove DeMorgan's laws for arbitrary unions and intersections.

10. Let $\mathbb{R}$ denote the set of real numbers. For each of the following subsets of $\mathbb{R} \times \mathbb{R}$, determine whether it is equal to the cartesian product of two subsets of $\mathbb{R}$.
(a) $\{(x, y) \mid x \text{ is an integer}\}$.
(b) $\{(x, y) \mid 0 < y \leq 1\}$.
(c) $\{(x, y) \mid y > x\}$.
(d) $\{(x, y) \mid x \text{ is not an integer and } y \text{ is an integer}\}$.
(e) $\{(x, y) \mid x^2 + y^2 < 1\}$.