Numerical Modeling and Optimization of Evaporated and Condensed Milk Production

Individual Project (CHN-302)

Final Report

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Contents

1 Introduction & Objectives

Evaporated and condensed milk are essential products in the dairy industry, widely used for long-term storage, transport, and culinary applications. It's production involves removing a portion of water from raw milk using thermal evaporation under controlled conditions.

The primary objectives of this study are:

- To develop a numerical model of the milk evaporation process.
- To analyze effect of temperature, pressure on evaporation rate and final concentration.
- To optimize process parameters for improved product quality and reduce energy requirement.

1.1 Scope and Assumptions

To simplify the modeling modelling process, the following assumptions are made:

- The milk is treated as a single-phase homogeneous fluid.
- Steady-state operation is assumed throughout the process.
- Heat losses to the environment and fouling are neglected. (adiabatic process)
- Physical properties (density, specific heat, latent heat) are considered constant.
- Boiling point elevation (BPE) is included using a simplified colligative approach.
- Vapor generated is assumed to be pure water vapor.
- Flow is assumed to be well mixed and uniformly distributed.

1.2 Overview of Evaporated Milk Production

The production of evaporated milk involves multiple unit processes to ensure consistency, and extended shelf life. The schematic in next section outlines the typical industrial flow.

1. Clarification and Storage : Raw milk is first clarified to remove impurities and then stored temporarily to ensure the continuity in the process. It is stored in silos.

2. Separation and Standardisation : Milk is separated by centrifuge into cream and skim milk. Desired fat content is achieved through standardisation. Additives like carrageenan, dipotassium phosphate, and vitamins are also introduced at this stage, which act as stabilizers and add nutitive content.

3. Evaporation : A portion of the water content is removed by evaporation at controlled temperature and pressure, increasing the concentration of solids.

4. Homogenization and Cooling : To prevent fat separation, the product is homogenized. The mixture is then cooled and filtered to remove any residual particles.

5. Sterilization and Filling : Milk is sterilized, filled in containers, and subjected to sample inspection to ensure quality control.

6. Distribution : The final product enters the supply chain for commercial distribution.

1.3 Schematic Diagram of Evaporated Milk Production

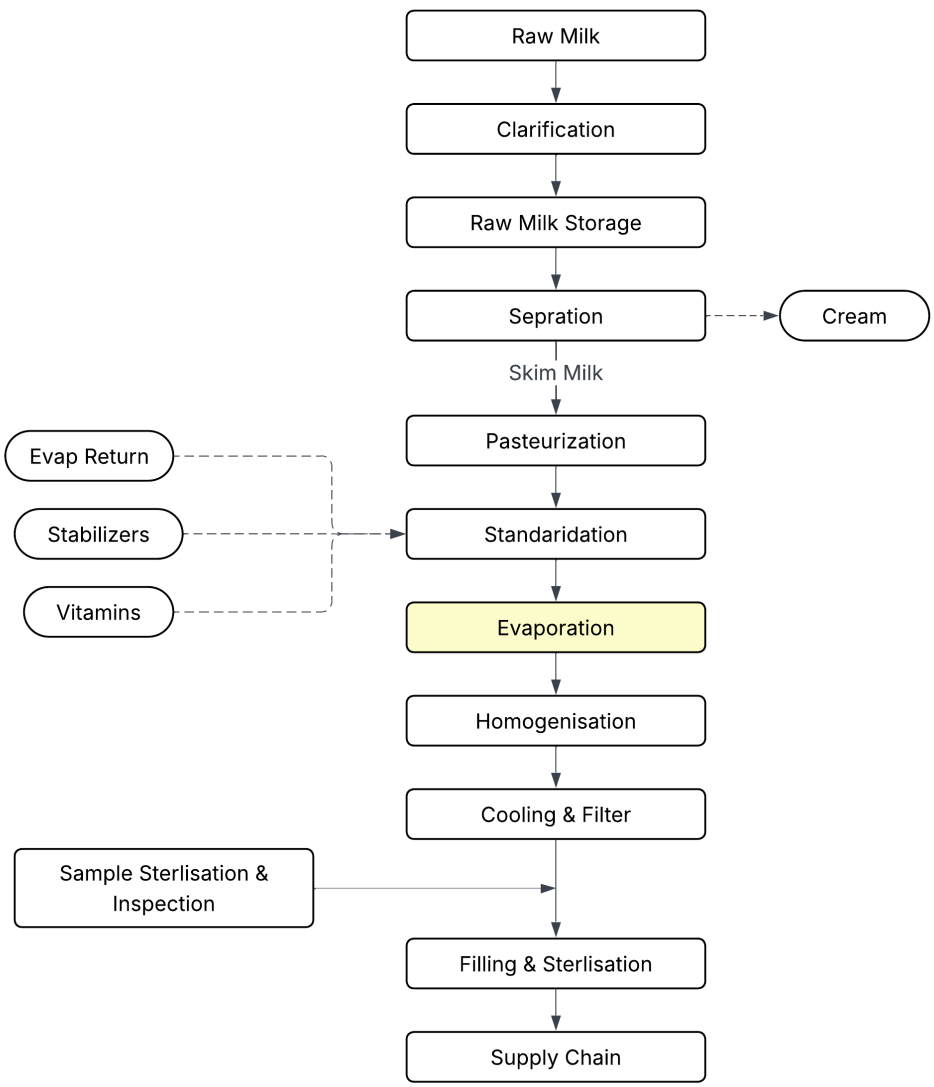


Figure 1: Schematic diagram of Evaporated Milk Production

2 Methodology and Model Formulation

This section explains the mass and energy balances used to model the evaporator unit. The empirical correlations derived from experimental data are mentioned.

2.1 Mass Balance

The overall mass balance considering solids content:

$$\dot{m}_{\text{feed}} \cdot X_0 = \dot{m}_{\text{concentrate}} \cdot X_{\text{final}} \tag{1}$$

$$\dot{m}_{\text{feed}} = \dot{m}_{\text{concentrate}} + \dot{m}_{\text{vapor}} \tag{2}$$

Where:

- X_0 = initial mass fraction of solids
- X_{final} = final mass fraction of solids
- $\dot{m}_{\text{feed}}, \dot{m}_{\text{concentrate}}, \dot{m}_{\text{vapor}}$ = mass flow rates (kg/s)

2.2 Energy Balance

The energy balance, accounting for sensible and latent heat and boiling point elevation, is:

$$Q = \dot{m}_{\text{steam}} \cdot \lambda_{\text{steam}} = \dot{m}_{\text{vapor}} \cdot \lambda_{\text{solution}} + \dot{m}_{\text{feed}} \cdot c_p \cdot (T_{\text{solution}} - T_{\text{inlet}}) \tag{3}$$

In simplified models, the sensible heat term ($\dot{m}_{\text{feed}} \cdot c_p \cdot (T_{\text{solution}} - T_{\text{inlet}})$) is often neglected when latent heat dominates.

The heat transfer formulation is:

$$Q = U \cdot A \cdot \Delta T_{\text{LM}} \quad \text{(kW)} \tag{4}$$

The log mean temperature difference is:

$$\Delta T_{\text{LM}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

In simplified models, ΔT_{LM} may be approximated as $T_{\text{steam}} - T_{\text{solution}}$.

Parameters:

- $\lambda_{\text{solution}}$: Latent heat at solution temperature (kJ/kg)
- ΔT_{LM}: Log mean temperature difference (K)
- U: Overall heat transfer coefficient (kW/m^2·K)
- A: Heat transfer area (m^2)
- c_p: Specific heat capacity of milk (kJ/kg·K)

2.3 Evaporation Rate Modeling

Latent heat based vapor mass flow rate is:

$$\dot{m}_{\text{vapor}} = \frac{U \cdot A \cdot \Delta T_{\text{LM}}}{\lambda_{\text{solution}}} \tag{5}$$

2.4 Final Concentration Calculation

The final solids concentration is found using the equation :

$$X_{\text{final}} = \frac{X_0}{1 - \left(\frac{\dot{m}_{\text{vapor}}}{\dot{m}_{\text{feed}}}\right)} \tag{6}$$

2.5 Boiling Point Elevation (BPE)

Solid component in milk raise the boiling point of the solution. The elevation is:

$$\Delta T_b = K_b \cdot m \cdot i \tag{7}$$

Where:

- $K_b = 0.51$ °C·kg/mol (water's ebullioscopic constant)
- m = molality of dissolved solids (mol/kg solvent)
- $i = 1.8$ (van't Hoff factor for milk solids)

2.6 Pressure–Temperature Relationship (Antoine Equation)

To estimate the boiling temperature of water using Pressure, we use Antoine Equation:

$$T_{\text{boil}} = \frac{B}{A - \log_{10}(P_{\text{mmHg}})} - C \tag{8}$$

Where:

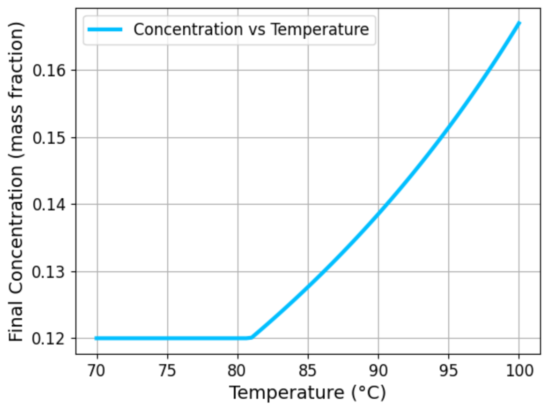
- P_{kPa} is converted to mmHg: $P_{\text{mmHg}} = P_{\text{kPa}} \cdot 7.50062$
- $A = 8.07131$, $B = 1730.63$, $C = 233.426$ (Antoine constants for water)

3 Python code for Simulation

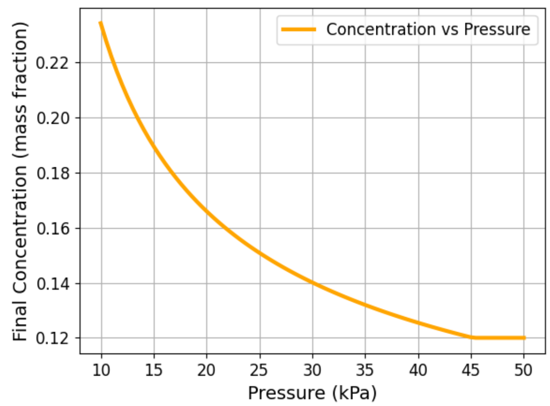
```python
import numpy as np
import matplotlib.pyplot as plt

# Constants and parameters
A = 8.07131; B = 1730.63; C = 233.426 # Antoine constant for water
Kb = 0.51; i = 1.8 # Boiling point elevation
lambda_water = 2257; cp = 3.9; U = 1.5; A_heat = 10; X0 = 0.12

temps = np.linspace(70, 100, 80)
pressures = np.linspace(10, 50, 80)

def boiling_point_elevation(X):
    m = X * 10 # molality
    delta_Tb = Kb * m * i
    return delta_Tb

def antoine_boiling_temp(P_kPa):
    P_mmHg = P_kPa * 7.50062
    return B / (A - np.log10(P_mmHg)) - C

def final_concentration(T_steam, P, F):
    Tb_water = antoine_boiling_temp(P) # base boiling temp at pressure
    delta_Tb = boiling_point_elevation(X0) # solute-induced
    T_solution = Tb_water + delta_Tb
    delta_T_LM = T_steam - T_solution

    if delta_T_LM <= 0: # evaporation won't take place in this case
        return X0

    lambda_solution = 0.9 * lambda_water
    m_vapor = (U * A_heat * delta_T_LM) / lambda_solution # kg/s

    if m_vapor >= F: # physically not possible
        return np.nan

    X_final = X0 / (1 - (m_vapor / F))
    return X_final


# Data for plots
conc_vs_T = [final_concentration(T, 47, 0.5) for T in temps]
conc_vs_P = [final_concentration(80, P, 0.5) for P in pressures]

# Contour data
T_grid, P_grid = np.meshgrid(temps, pressures)
conc_grid = np.zeros_like(T_grid)
for i in range(T_grid.shape[0]):
    for j in range(T_grid.shape[1]):
        conc_grid[i, j] = final_concentration(T_grid[i, j], P_grid[i, j], 1)
```

4 Results

4.1 Plots



(a) Concentration vs Temperature



(b) Concentration vs Pressure

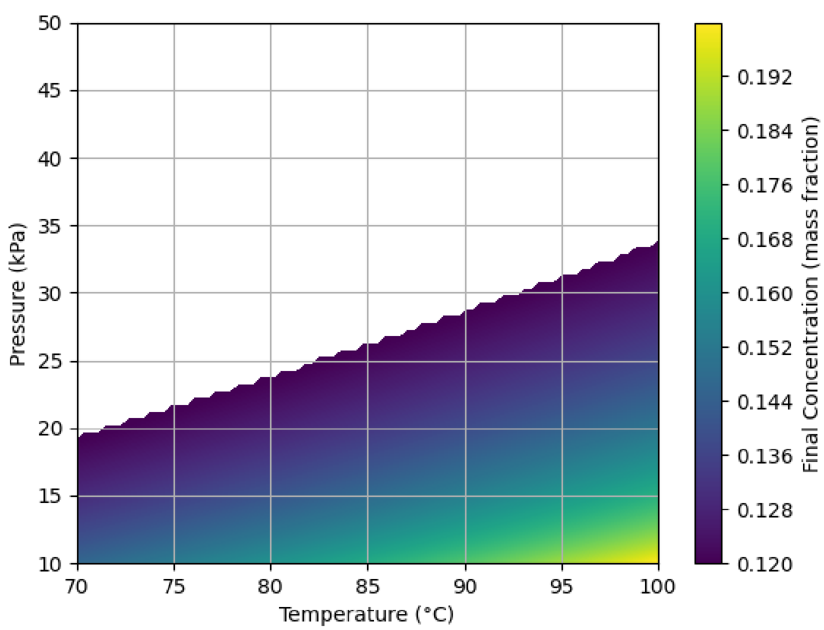


Figure 3: Contour Plot: Concentration vs Temperature and Pressure

4.2 Numerical Results

The concentration ranged from **0.12** (which is the assumed initial solid concentration based on the data in the current literature) to approximately **0.199**. Data suggests it is optimal solid content for condensed milk available in markets. Reduction in pressure below a certain limit decreases concentration, while increasing temperature steadily improves it.

5 Discussion

This model depicts how temperature and pressure affect the evaporation process in condensed milk production. The results match well with basic thermodynamic principles and what is observed in real industrial evaporators.

5.1 Temperature Effects

Temperature strongly affects evaporation. A threshold near 80°C is observed, which aligns with standard industrial steam conditions. Below this, concentration change is minor, but above it, even small temperature increases significantly boost efficiency. Operating around 80°C is both effective and cost-efficient.

5.2 Pressure Effects

Lowering pressure usually improves concentration, as expected in vacuum evaporation. But when pressure goes below 15 kPa, the improvement slows down. At that point, the extra energy needed to maintain such low pressure isn't worth it, it is what's seen in real industrial setups.

5.3 Process Optimization Implications

Based on the contour plot and simulations, we can summarize a few key points:

- Best results come from using the highest safe temperature and the lowest practical pressure.
- There's a sensitive zone in the temperature-pressure space where even small changes in settings lead to big changes in output concentration. This is a crucial area for fine-tuning.

6 Limitations and Recommendations

The current model simplifies several aspects of the real process. Some of the main limitations and areas for future improvement include:

1. **Fouling Effects:** As concentration increases, fouling can reduce heat transfer efficiency. Future models should include variable heat transfer coefficients to reflect this.
2. **More Accurate BPE Modeling:** The current boiling point elevation estimate is simplified. A better approach would account for milk as a complex, multi-component solution.

3. **Energy Considerations:** Right now, the model focuses mainly on achieving high concentration. Including energy usage in the analysis would help find a better balance between product quality and process efficiency.

In summary, this model helps in understanding and optimizing condensed milk production. It shows how temperature, pressure, and feed rate are connected, and provides a strong starting point for better evaporator design and control.

References

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