EXERCISE # 1

STRAIGHT OBJECTIVE TYPE

1. A (1, –1, –3), B (2, 1, –2) & C (–5, 2, –6) are the position vectors of the vertices of a triangle ABC. The length of the bisector of its internal angle at A is :

(A) $\sqrt{10}/4$ (B) $3\sqrt{10}/4$ (C) $\sqrt{10}$ (D) none

2. Let $\vec{p}$ is the p.v. of the orthocentre & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p} = K\,\vec{g}$, then K =

(A) 3 (B) 2 (C) 1/3 (D) 2/3

3. A vector $\vec{a}$ has components 2p & 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components p + 1 & 1 then,

(A) p = 0 (B) p = 1 or p = –1/3 (C) p = –1 or p = 1/3 (D) p = 1 or p = –1

4. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0)$ & $\vec{b}\,(0, 1, 1)$ is:

(A) 1 (B) 2 (C) 3 (D) ∞

5. Four points A(+1, –1, 1) ; B(1, 3, 1) ; C(4, 3, 1) and D(4, – 1, 1) taken in order are the vertices of

(A) a parallelogram which is neither a rectangle nor a rhombus

(B) rhombus

(C) an isosceles trapezium

(D) a cyclic quadrilateral.

6. Let α, β & γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$; $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ and $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

(A) are collinear (B) form an equilateral triangle

(C) form a scalene triangle (D) form a right angled triangle

7. If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ constitute the sides of a ΔABC, then the length of the median bisecting the vector $\vec{c}$ is

(A) $\sqrt{2}$ (B) $\sqrt{14}$ (C) $\sqrt{74}$ (D) $\sqrt{6}$

8. Let A(0, –1, 1), B(0, 0, 1), C(1, 0, 1) are the vertices of a ΔABC. If R and r denotes the circumradius and inradius of ΔABC, then $\frac{r}{R}$ has value equal to

(A) $\tan\frac{3\pi}{8}$ (B) $\cot\frac{3\pi}{8}$ (C) $\tan\frac{\pi}{12}$ (D) $\cot\frac{\pi}{12}$

9. $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a}+\vec{b}$ is collinear with $\vec{c}, \vec{b}+\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+\vec{b}+\vec{c}$ is equal to -

(A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{c}$ (D) none of these

10. If the three points with position vectors (1, a, b) ; (a, 2, b) and (a, b, 3) are collinear in space, then the value of a + b is

(A) 3 (B) 4 (C) 5 (D) none

11. Consider the following 3 lines in space

$L_1 : \vec{r} = 3\hat{i} - \hat{j} + 2\hat{k} + \lambda\,(2\hat{i} + 4\hat{j} - \hat{k})$

$L_2 : \vec{r} = \hat{i} + \hat{j} - 3\hat{k} + \mu\,(4\hat{i} + 2\hat{j} + 4\hat{k})$

$L_3 : \vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k} + t\,(2\hat{i} + \hat{j} + 2\hat{k})$

Then which one of the following pair(s) are in the same plane.

(A) only L_1L_2 (B) only L_2L_3 (C) only L_3L_1 (D) L_1L_2 and L_2L_3

12. The acute angle between the medians drawn from the acute angles of an isosceles right angled triangle is:

(A) $\cos^{-1}(2/3)$ (B) $\cos^{-1}(3/4)$ (C) $\cos^{-1}(4/5)$ (D) none

13. The vectors $3\hat{i} - 2\hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$ form the sides of a triangle. Then triangle is

(A) an acute angled triangle (B) an obtuse angled triangle

(C) an equilateral triangle (D) a right angled triangle

14. If the vectors $3\bar{p} + \bar{q}; 5\bar{p} - 3\bar{q}$ and $2\bar{p} + \bar{q}; 4\bar{p} - 2\bar{q}$ are pairs of mutually perpendicular vectors then sin $(\bar{p}\,\hat{}\,\bar{q})$ is

(A) $\sqrt{55}/4$ (B) $\sqrt{55}/8$ (C) 3/16 (D) $\sqrt{247}/16$

15. Consider the points A, B and C with position vectors $\left(-2\hat{i} + 3\hat{j} + 5\hat{k}\right)$, $\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $7\hat{i} - \hat{k}$ respectively.

Statement-1: The vector sum, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

Because

Statement-2: A, B and C form the vertices of a triangle.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

16. The set of values of c for which the angle between the vectors $cx\,\hat{i} - 6\hat{j} + 3\hat{k}$ and $x\hat{i} - 2\hat{j} + 2cx\hat{k}$ is acute for every $x \in R$ is

(A) (0, 4/3) (B) [0, 4/3] (C) (11/9, 4/3) (D) [0, 4/3)

17. Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, then $|\vec{w}.\hat{n}|$ is equal to

(A) 1 (B) 2 (C) 3 (D) 0

18. If the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ is decomposed into vectors parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ then the vectors are :

(A) $-\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $7\hat{i} - 2\hat{j} - 5\hat{k}$

(B) $-2\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $8\hat{i} - \hat{j} - 4\hat{k}$

(C) $+ 2\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $4\hat{i} - 5\hat{j} - 8\hat{k}$

(D) none

19. Let $\vec{r} = \vec{a} + \lambda\vec{\ell}$ and $\vec{r} = \vec{b} + \mu\vec{m}$ be two lines in space where $\vec{a} = 5\hat{i} + \hat{j} + 2\hat{k}, \vec{b} = -\hat{i} + 7\hat{j} + 8\hat{k}$, $\vec{\ell} = -4\hat{i} + \hat{j} - \hat{k}$ and $\vec{m} = 2\hat{i} - 5\hat{j} - 7\hat{k}$ then the p.v. of a point which lies on both of these lines, is

(A) $\hat{i} + 2\hat{j} + \hat{k}$

(B) $2\hat{i} + \hat{j} + \hat{k}$

(C) $\hat{i} + \hat{j} + 2\hat{k}$

(D) non existent as the lines are skew

20. Let A(1, 2, 3), B(0, 0, 1), C(–1, 1, 1) are the vertices of a ΔABC.

(i) The equation of internal angle bisector through A to side BC is

(A) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu\,(3\hat{i} + 2\hat{j} + 3\hat{k})$

(B) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 3\hat{k})$

(C) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu\,(3\hat{i} + 3\hat{j} + 2\hat{k})$

(D) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu\,(3\hat{i} + 3\hat{j} + 4\hat{k})$

(ii) The equation of median through C to side AB is

(A) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p\,(3\hat{i} - 2\hat{k})$

(B) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p\,(3\hat{i} + 2\hat{k})$

(C) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p\,(-3\hat{i} + 2\hat{k})$

(D) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p\,(3\hat{i} + 2\hat{j})$

(iii) The area (ΔABC) is equal to

(A) $\frac{9}{2}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\frac{17}{2}$ (D) $\frac{7}{2}$

21. If $\vec{a} + b + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}\ \&\ \vec{b}$ is :

(A) $\pi/6$ (B) $2\pi/3$ (C) $5\pi/3$ (D) $\pi/3$

22. A line passes through the point $A\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and is parallel to the vector $\vec{V}\left(\hat{i} + \hat{j} + \hat{k}\right)$. The shortest distance from the origin, of the line is -

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$

23. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b}+\vec{c}, \vec{b}$ to $\vec{c}+\vec{a}$ and $\vec{c}$ to $\vec{a}+\vec{b}$. Then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is :

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$

24. The set of values of x for which the angle between the vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ acute and the angle between the vector $\vec{b}$ and the axis of ordinates is obtuse, is

(A) $1 < x < 2$ (B) $x > 2$ (C) $x < 1$ (D) $x < 0$

25. If a vector $\vec{a}$ of magnitude 50 is collinear with vector $\vec{b} = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with positive z-axis then :

(A) $\vec{a} = 4\vec{b}$ (B) $\vec{a} = -4\vec{b}$ (C) $\vec{b} = 4\vec{a}$ (D) none

26. A, B, C & D are four points in a plane with pv's $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ respectively such that $\left(\vec{a}-\vec{d}\right)\cdot\left(\vec{b}-\vec{c}\right) = \left(\vec{b}-\vec{d}\right)\cdot\left(\vec{c}-\vec{a}\right) = 0$. Then for the triangle ABC, D is its

(A) incentre (B) circumcentre (C) orthocentre (D) centroid

27. $\vec{a}$ and $\vec{b}$ are unit vectors inclined to each other at an angle α, $\alpha \in (0, \pi)$ and $|\vec{a}+\vec{b}| < 1$. Then $\alpha \in$

(A) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ (B) $\left(\frac{2\pi}{3}, \pi\right)$ (C) $\left(0, \frac{\pi}{3}\right)$ (D) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

28. Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is, $\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda\left(\hat{i} + 3\hat{j} + 5\hat{k}\right)$ has the position vector

(A) (– 9, 5, 2) (B) (9, 5, – 2) (C) (9, – 5, – 2) (D) none

29. Let $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector. If pairwise angles between $\hat{a}+\hat{b}+\hat{c}$ are θ_1, θ_2 and θ_3 respectively then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$ equals

(A) 3 (B) – 3 (C) 1 (D) – 1

30. A tangent is drawn to the curve $y = \frac{8}{x^2}$ at a point A (x_1, y_1), where $x_1 = 2$. The tangent cuts the x-axis at point B. Then the scalar product of the vectors $\overrightarrow{AB}$ and $\overrightarrow{OB}$ is

(A) 3 (B) – 3 (C) 6 (D) – 6

31. Cosine of an angle between the vectors $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ if $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a} \wedge \vec{b} = 60°$ is

(A) $\sqrt{3/7}$ (B) $9/\sqrt{21}$ (C) $3/\sqrt{7}$ (D) none

32. An arc AC of a circle subtends a right angle at the centre O. The point B divides the arc in the ratio 1 : 2. If $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$, then the vector $\overrightarrow{OC}$ in terms of $\vec{a}$ & $\vec{b}$, is

(A) $\sqrt{3}\,\vec{a}+2\vec{b}$ (B) $-\sqrt{3}\,\vec{a}+2\vec{b}$ (C) $2\vec{a}-\sqrt{3}\,\vec{b}$ (D) $-2\vec{a}+\sqrt{3}\,\vec{b}$

33. Given three vectors $\vec{a}, \vec{b}$ & $\vec{c}$ each two of which are non collinear. Further if $\left(\vec{a}+\vec{b}\right)$ is collinear with $\vec{c}$, $\left(\vec{b}+\vec{c}\right)$ is collinear with $\vec{a}$ & $|\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{2}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$:

(A) is 3 (B) is – 3 (C) is 0 (D) cannot be evaluated

34. The vector equations of two lines L_1 and L_2 are respectively

$\vec{r} = 17\vec{i} - 9\hat{j} + 9\hat{k} + \lambda\left(3\hat{i}+\hat{j}+5\hat{k}\right)$ and $\vec{r} = 15\hat{i} - 8\hat{j} - \hat{k} + \mu\left(4\hat{i}+3\hat{j}\right)$

I L_1 and L_2 are skew lines

II (11, –11, –1) is the point of intersection of L_1 and L_2

III (–11, 11, 1) is the point of intersection of L_1 and L_2

IV $\cos^{-1}\left(3/\sqrt{35}\right)$ is the acute angle between L_1 and L_2

then , which of the following is true?

(A) II and IV (B) I and IV (C) IV only (D) III and IV

35. For two particular vectors $\vec{A}$ and $\vec{B}$ it is known that $\vec{A}\times\vec{B} = \vec{B}\times\vec{A}$ .What must be true about the two vectors?

(A) At least one of the two vectors must be the zero vector.

(B) $\vec{A}\times\vec{B} = \vec{B}\times\vec{A}$ is true for any two vectors.

(C) One of the two vectors is a scalar multiple of the other vector.

(D) The two vectors must be perpendicular to each other.

36. For some non zero vector $\vec{V}$, if the sum of $\vec{V}$ and the vector obtained from $\vec{V}$ by rotating it by an angle 2α equals to the vector obtained from $\vec{V}$ by rotating it by α then the value of α, is

(A) $2n\pi \pm \frac{\pi}{3}$ (B) $n\pi \pm \frac{\pi}{3}$ (C) $2n\pi \pm \frac{2\pi}{3}$ (D) $n\pi \pm \frac{2\pi}{3}$

where n is an integer.

37. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals

(A) 2 (B) $\sqrt{7}$ (C) $\sqrt{14}$ (D) 14

38. If $\vec{a}$ and $\vec{b}$ are non zero, non collinear, and the linear combination
$(2x - y)\vec{a} + 4\vec{b} = 5\vec{a} + (x - 2y)\vec{b}$ holds for real x and y then x + y has the value equal to

(A) – 3 (B) 1 (C) 17 (D) 3

39. Given an equilateral triangle ABC with side length equal to 'a'. Let M and N be two points respectively on the side AB and AC such that $\overrightarrow{AN} = K\overrightarrow{AC}$ and $\overrightarrow{AM} = \dfrac{\overrightarrow{AB}}{3}$. If $\overrightarrow{BN}$ and $\overrightarrow{CM}$ are orthogonal then the value of K is equal to

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

40. If $\vec{p}$ & $\vec{s}$ are not perpendicular to each other and $\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$ & $\vec{r}.\vec{s} = 0$, then $\vec{r}$ =

(A) $\vec{p}.\vec{s}$ (B) $\vec{q} + \left(\dfrac{\vec{q}.\vec{p}}{\vec{p}.\vec{s}}\right)\vec{p}$ (C) $\vec{q} - \left(\dfrac{\vec{q}.\vec{s}}{\vec{p}.\vec{s}}\right)\vec{p}$ (D) $\vec{q} + \mu\vec{p}$ for all scalars μ

41. If $\vec{u}$ and $\vec{v}$ are two vectors such that $|\vec{u}| = 3$; $|\vec{v}| = 2$ and $|\vec{u} \times \vec{v}| = 6$ then the correct statement is

(A) $\vec{u} \wedge \vec{v} \in (0, 90°)$ (B) $\vec{u} \wedge \vec{v} \in (90°, 180°)$ (C) $\vec{u} \wedge \vec{v} = 90°$ (D) $(\vec{u} \times \vec{v}) \times \vec{u} = 6\vec{v}$

42. Given a parallelogram OACB. The lengths of the vectors $\overrightarrow{OA}, \overrightarrow{OB}$ & $\overrightarrow{AB}$ are a, b & c respectively. The scalar product of the vectors $\overrightarrow{OC}$ & $\overrightarrow{OB}$ is :

(A) $\dfrac{a^2 - 3b^2 + c^2}{2}$ (B) $\dfrac{3a^2 + b^2 - c^2}{2}$ (C) $\dfrac{3a^2 - b^2 + c^2}{2}$ (D) $\dfrac{a^2 + 3b^2 - c^2}{2}$

43. Vectors $\vec{a}$ & $\vec{b}$ make an angle $\theta = \dfrac{2\pi}{3}$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ then $\left\{\left(\vec{a} + 3\vec{b}\right) x \left(3\vec{a} - \vec{b}\right)\right\}^2$ =

(A) 225 (B) 250 (C) 275 (D) 300

44. If the vector product of a constant vector $\overrightarrow{OA}$ with a variable vector $\overrightarrow{OB}$ in a fixed plane OAB be a constant vector, then locus of B is :

(A) a straight line perpendicular to $\overrightarrow{OA}$ (B) a circle with centre O radius equal to $|\overrightarrow{OA}|$
(C) a straight line parallel to $\overrightarrow{OA}$ (D) none of these

45. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $\left|\vec{a} \times \vec{b}.\vec{c}\right| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if ;

(A) $\vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0$ (B) $\vec{c}.\vec{a} = 0,\ \vec{a}.\vec{b} = 0$

(C) $\vec{a}.\vec{c} = 0,\ \vec{b}.\vec{c} = 0$ (D) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

46. The vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ & $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are -

(A) not coplanar

(B) coplanar but cannot form a triangle

(C) coplanar but can form a triangle

(D) coplanar & can form a right angled triangle

47. Given the vectors

$$\vec{u} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{v} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{w} = \hat{i} - \hat{k}$$

If the volume of the parallelopiped having $-c\vec{u}$, $\vec{v}$ and c $\vec{w}$ as concurrent edges, is 8 then 'c' can be equal to

(A) ± 2 (B) 4 (C) 8 (D) cannot be determined

48. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j}$; $(\vec{a} \,\hat{}\, \vec{b}) = \pi/2$, $\vec{a}\cdot\vec{c} = 4$ then

(A) $[\vec{a}\,\vec{b}\,\vec{c}]^2 = |\vec{a}|$ (B) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|$ (C) $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ (D) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|^2$

49. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$; $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ & $\vec{b}$. If the angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$$

(A) 0 (B) 1

(C) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$ (D) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

50. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three?

(A) $\vec{u}\cdot(\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w})\cdot\vec{u}$ (C) $\vec{v}\cdot(\vec{u} \times \vec{w})$ (D) $(\vec{u} \times \vec{v})\cdot\vec{w}$

51. Let $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ & $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors , $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c}$ are coplanar then $\frac{\alpha}{\beta}$ is

(A) 1 (B) 2 (C) 3 (D) – 3

52. A rigid body rotates with constant angular velocity ω about the line whose vector equation is, $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. The speed of the particle at the instant it passes through the point with p.v. $2\hat{i} + 3\hat{j} + 5\hat{k}$ is :

(A) $\omega\sqrt{2}$ (B) 2ω (C) $\omega/\sqrt{2}$ (D) none

53. Given 3 vectors $\vec{V}_1 = a\hat{i} + b\hat{j} + c\hat{k}$; $\vec{V}_2 = b\hat{i} + c\hat{j} + a\hat{k}$; $\vec{V}_3 = c\hat{i} + a\hat{j} + b\hat{k}$

In which one of the following conditions $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$ are linearly independent?

(A) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

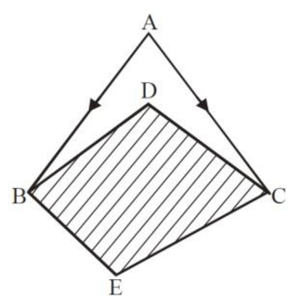
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$

(D) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

54. Given unit vectors $\vec{m}, \vec{n}$ & $\vec{p}$ such that angle between $\vec{m}$ & $\vec{n}$ = angle between $\vec{p}$ and $\left(\vec{m} \times \vec{n}\right)$ = π/6, then $\left[\vec{n}\,\vec{p}\,\vec{m}\right] =$

(A) $\sqrt{3}/4$ (B) 3/4 (C) 1/4 (D) none

55. Let $\overrightarrow{AB} = 3\hat{i} - \hat{j}, \overrightarrow{AC} = 2\hat{i} + 3\hat{j}$ and $\overrightarrow{DE} = 4\hat{i} - 2\hat{j}$. The area of the shaded region in the adjacent figure, is-



(A) 5 (B) 6 (C) 7 (D) 8

56. The altitude of a parallelopiped whose three coterminous edges are the vectors, $\vec{A} = \hat{i} + \hat{j} + \hat{k}$; $\vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 3\hat{k}$ with $\vec{A}$ and $\vec{B}$ as the sides of the base of the parallelopiped, is

(A) $2/\sqrt{19}$ (B) $4/\sqrt{19}$ (C) $2\sqrt{38}/19$ (D) none

57. Consider ΔABC with A ≡ $(\vec{a})$; B ≡ $(\vec{b})$ & C ≡ $(\vec{c})$. If $\vec{b} \cdot (\vec{a} + \vec{c}) = \vec{b} \cdot \vec{b} + \vec{a} \cdot \vec{c}$; $|\vec{b} - \vec{a}| = 3$;

$|\vec{c}-\vec{b}| = 4$ then the angle between the medians $\overrightarrow{AM}$ and $\overrightarrow{BD}$ is

(A) $\pi - \cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$ (B) $\pi - \cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$

(C) $\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$ (D) $\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$

58. If A (– 4, 0, 3) ; B (14, 2, –5) then which one of the following points lie on the bisector of the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ ('O' is the origin of reference)

(A) (2, 1, –1) (B) (2, 11, 5) (C) (10, 2, –2) (D) (1, 1, 2)

59. Position vectors of the four angular points of a tetrahedron ABCD are A(3, – 2, 1); B(3, 1, 5); C(4, 0, 3) and D(1, 0, 0). Acute angle between the plane faces ADC and ABC is

(A) $\tan^{-1}(5/2)$ (B) $\cos^{-1}(2/5)$ (C) $\text{cosec}^{-1}(5/2)$ (D) $\cot^{-1}(3/2)$

60. The volume of the tetrahedron formed by the coterminus edges $\vec{a},\vec{b},\vec{c}$ is 3. Then the volume of the parallelopiped formed by the coterminus edges $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ is

(A) 6 (B) 18 (C) 36 (D) 9

61. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$, then the vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ & $\vec{a}\times\vec{c} = \vec{b}$ is -

(A) $\frac{1}{3}\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ (B) $\frac{1}{3}\left(-\hat{i}+2\hat{j}+5\hat{k}\right)$ (C) $\frac{1}{3}\left(\hat{i}+2\hat{j}-5\hat{k}\right)$ (D) $\frac{1}{3}\left(3\hat{i}+2\hat{j}+\hat{k}\right)$

62. $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b} = 0$, then the acute angle between $\vec{a}$ & $\vec{c}$ is :

(A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $5\pi/12$

63. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = 4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c} = \hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors & $|\vec{c}| = \sqrt{3}$ then

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$ (C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

64. A vector of magnitude $5\sqrt{5}$ coplanar with vectors $\hat{i}+2\hat{j}$ and $\hat{j}+2\hat{k}$ and the perpendicular vector $2\hat{i}+\hat{j}+2\hat{k}$ is

(A) $\pm 5\left(5\hat{i}+6\hat{j}-8\hat{k}\right)$ (B) $\pm\sqrt{5}\left(5\hat{i}+6\hat{j}-8\hat{k}\right)$

(C) $\pm 5\sqrt{5}\left(5\hat{i}+6\hat{j}-8\hat{k}\right)$ (D) $\pm\left(5\hat{i}+6\hat{j}-8\hat{k}\right)$

65. Let $\vec{\alpha} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{\beta} = \hat{i}+\hat{j}$. If $\vec{\gamma}$ is a unit vector, then the maximum value of $\left[\vec{\alpha}\times\vec{\beta}\ \vec{\beta}\times\vec{\gamma}\ \vec{\gamma}\times\vec{\alpha}\right]$ is equal to

(A) 2 (B) 3 (C) 4 (D) 9

MATRIX MATCH TYPE

66. If A(0, 1, 0), B(0, 0, 0), C(1, 0, 1) are the vertices of a ΔABC. Match the entries of **column-I** with **column-II**.

Column-I		**Column-II**	
(A)	Orthocentre of ΔABC.	(P)	$\frac{\sqrt{2}}{2}$
(B)	Circumcentre of ΔABC.	(Q)	$\frac{\sqrt{3}}{2}$
(C)	Area (ΔABC).	(R)	$\frac{\sqrt{3}}{3}$
(D)	Distance between orthocentre and centroid.	(S)	$\frac{\sqrt{3}}{6}$
(E)	Distance between orthocentre and circumcentre.	(T)	(0, 0, 0)
(F)	Distance between circumcentre and centroid.	(U)	$\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$
(G)	Incentre of ΔABC.	(V)	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$
(H)	Centroid of ΔABC	(W)	$\left(\frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}},\frac{\sqrt{2}}{\sqrt{1}+\sqrt{2}+\sqrt{3}},\frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}\right)$

EXERCISE # 2

1. Given the vector $\overrightarrow{PQ} = -6\hat{i} - 4\hat{j}$ and Q is the point (3, 3), find the point P.

2. Find the unit vector (in xy plane) obtained by rotating j counterclockwise $3\pi/4$ radian about the origin.

3. Show that the vector v = ai + bj is perpendicular to the line ax + by = c.

4. In ΔABC, a point P is chosen on side $\overrightarrow{AB}$ so that AP : PB = 1 : 4 and a point Q is chosen on the side $\overrightarrow{BC}$ so that CQ : QB = 1 : 3. Segment $\overrightarrow{CP}$ and $\overrightarrow{AQ}$ intersect at M. If the ratio $\frac{MC}{PC}$ is expressed as a rational numbers in the lowest term as $\frac{a}{b}$, find (a + b).

5. Let O be an interior point of ΔABC such that $2\overrightarrow{OA} + 5\overrightarrow{OB} + 10\overrightarrow{OC} = \vec{0}$. If the ratio of the area of ΔABC to the area of ΔAOC is t, where 'O' is the origin. Find [t].
(where [ ] denotes greatest integer function)

6. If the distance from the point P(1, 1, 1) to the line passing through the points Q(0, 6, 8) and R(–1, 4, 7) is expressed in the form $\sqrt{p/q}$ where p and q are coprime, then the value of $\frac{(p+q)(p+q-1)}{2}$.

7. Let S(t) be the area of the ΔOAB with O(0, 0, 0), A (2, 2, 1) and B(t, 1, t + 1).
The value of the definite integral $\int_1^e (S(t))^2 \ell n t\,dt$, is equal to $\left(\frac{e^3 + a}{b}\right)$ where a, b $\in$ N, find (a + b).

8. Given $f^2(x) + g^2(x) + h^2(x) \le 9$ and U(x) = 3f(x) + 4g(x) + 10h(x), where f(x), g(x) and h(x) are continuous $\forall$ x $\in$ R. If maximum value of U(x) is $\sqrt{N}$, then find N.

9. If $\vec{a}$ and $\vec{b}$ are non collinear vectors such that $\vec{p} = (x+4y)\vec{a} + (2x+y+1)\vec{b}$ and $\vec{q} = (y-2x+2)\vec{a} + (2x-3y-1)\vec{b}$, find x and y such that $3\vec{p} = 2\vec{q}$.

10. (a) Show that the points $\vec{a} - 2\vec{b} + 3\vec{c}$; $2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-7\vec{b} + 10\vec{c}$ are collinear.

(b) Prove that the points A(1, 2, 3), B(3, 4, 7), C(–3, –2, –5) are collinear and find the ratio in which B divides AC.

11. Find out whether the following pairs of lines are parallel, non-parallel & intersecting, or nonparallel and non-intersecting.

(a) $\vec{r}_1 = \hat{i} + \hat{j} + 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$
$\vec{r}_2 = 2\hat{i} + \hat{j} + 3\hat{k} + \mu(-6\hat{i} + 4\hat{j} - 8\hat{k})$

(b) $\vec{r}_1 = \hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$
$\vec{r}_2 = 2\hat{i} + 4\hat{j} + 6\hat{k} + \mu(2\hat{i} + \hat{j} + 3\hat{k})$

(c) $\vec{r}_1 = \hat{i} + \hat{k} + \lambda(\hat{i} + 3\hat{j} + \hat{k})$
$\vec{r}_2 = 2\hat{i} + 3\hat{j} + \mu(4\hat{i} - \hat{j} + \hat{k})$

12. If $\vec{r}$ and $\vec{s}$ are non zero constant vectors and the scalar b is chosen such that $|\vec{r} + b\vec{s}|$ is minimum, then show that the value of $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2$ is equal to $|\vec{r}|^2$.

13. In a unit cube. Find
(a) The angle between the diagonal of the cube and a diagonal of a face skew to it.
(b) The angle between the diagonals of two faces of the cube through the same vertex.
(c) The angle between a diagonal of a cube and a diagonal of a face intersecting it.

Instruction for question nos. 14 to 16 :

Suppose the three vectors $\vec{a}, \vec{b}, \vec{c}$ on a plane satisfy the condition that
$|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{a} + \vec{b}| = 1$; $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b} \cdot \vec{c} > 0$, then

14. Find the angle formed by $2\vec{a} + \vec{b}$ and $\vec{b}$.

15. If the vector $\vec{c}$ is expressed as a linear combination $\lambda\vec{a} + \mu\vec{b}$ then find the ordered pair (λ, μ).

16. For real numbers x,y the vector $\vec{p} = x\vec{a} + y\vec{c}$ satisfies the condition $0 \le \vec{p} \cdot \vec{a} \le 1$ and $0 \le \vec{p} \cdot \vec{b} \le 1$. Find the maximum value of $\vec{p} \cdot \vec{c}$.

17. (a) Find the minimum area of the triangle whose vertices are A(–1,1,2); B(1,2,3) and C(t,1,1) where t is a real number.

(b) Let $\overrightarrow{OA} = \vec{a}$; $\overrightarrow{OB} = 100\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$ where O, A and C are non collinear points. Let P denotes the area of the parallelogram with $\overrightarrow{OA}$ and $\overrightarrow{OC}$ as adjacent sides and Q denotes the area of the quadrilateral OABC. If Q = λP. Find the value of λ.

18. Given that $\vec{a}$ and $\vec{b}$ are two unit vectors such that angle between $\vec{a}$ and $\vec{b}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If $\vec{c}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$, such that $|\vec{c}| = 4$, $\vec{c} \times \vec{b} = 2\vec{a} \times \vec{b}$ and $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ then, find (a) the value of λ, (b) the sum of values of μ and (c) the product of all possible values of μ.

19. Let $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{C} = \hat{j} + \hat{k}$.

If the vector $\vec{B}\times\vec{C}$ can be expressed as a linear combination $\vec{B}\times\vec{C} = x\vec{A} + y\vec{B} + z\vec{C}$ where x, y, z are scalars, then find the value of (100x + 10y + 8z).

20. The base vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are given in terms of base vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ as $\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$; $\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$ and $\vec{a}_3 = -2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3$. If $\vec{F} = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$, then express $\vec{F}$ in terms of $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$.

21. The vector $\overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

22. The pv's of the four angular points of a tetrahedron are $A\left(\hat{j}+2\hat{k}\right)$; $B\left(3\hat{i}+\hat{k}\right)$; $C\left(4\hat{i}+3\hat{j}+6\hat{k}\right)$ & $D\left(2\hat{i}+3\hat{j}+2\hat{k}\right)$. Find :

(i) the perpendicular distance from A to the line BC.
(ii) the volume of the tetrahedron ABCD.
(iii) the perpendicular distance from D to the plane ABC.
(iv) the shortest distance between the lines AB & CD.

23. Let a 3 dimensional vector $\vec{V}$ satisfies the condition $2\vec{V} + \vec{V}\times\left(\hat{i}+2\hat{j}\right) = 2\hat{i} + \hat{k}$.

If $3|\vec{V}| = \sqrt{m}$, where m ∈ N, then find m.

24. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying
$[(a-2)\alpha^2 + (b-3)\alpha + c]\vec{x} + [(a-2)\beta^2 + (b-3)\beta + c]\vec{y} + [(a-2)\gamma^2 + (b-3)\gamma + c]\ (\vec{x} \times \vec{y}) = 0$
where α, β, γ are three distinct real numbers, then find the value of $(a^2 + b^2 + c^2)$.

25. Solve the simultaneous vector equations for the vectors $\vec{x}$ and $\vec{y}$.
$\vec{x} + \vec{c}\times\vec{y} = \vec{a}$ and $\vec{y} + \vec{c}\times\vec{x} = \vec{b}$ where $\vec{c}$ is a non zero vector.

26. Vector $\vec{V}$ is perpendicular to the plane of vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and satisfies the condition $\vec{V}.\left(\hat{i}+2\hat{j}-7\hat{k}\right) = 10$. Find $|\vec{V}|^2$.

27. Let two non-collinear vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = 4$. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given as $\overrightarrow{OP} = \left(e^t + e^{-t}\right)\vec{a} + \left(e^t - e^{-t}\right)\vec{b}$. If the least distance of P from origin is $\sqrt{2}\sqrt{\sqrt{p}-q}$ where p, q ∈ N then find the value of (p + q).

EXERCISE # 3 (JM)

1. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}$, $\overrightarrow{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A onto BC. the magnitude of the resultant is- **[AIEEE-2006]**

(1) $\frac{(AB)(AC)}{AB+AC}$ (2) $\frac{1}{AB}+\frac{1}{AC}$ (3) $\frac{1}{AD}$ (4) $\frac{AB^2+AC^2}{(AB)^2(AC)^2}$

2. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\,\hat{u} \times 3\,\hat{v}$ is a unit vector for- **[AIEEE-2007]**

(1) Exactly two values of θ (2) More than two values of θ

(3) No value of θ (4) Exactly one value of θ

3. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals - **[AIEEE-2007]**

(1) 0 (2) 1 (3) –4 (4) –2

4. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$, lies in the plane the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisect the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ? **[AIEEE-2008]**

(1) α= 2, β = 2 (2) α = 1, β = 2 (3) α = 2, β = –1 (4) α = 1, β = 1

5. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality **[AIEEE-2009]** $\left[3\vec{u}\ p\vec{v}\ p\vec{w}\right] - \left[p\vec{v}\ \vec{w}\ q\vec{u}\right] - \left[2\vec{w}\ q\vec{v}\ q\vec{u}\right] = 0$ holds for :-

(1) More than two but not all values of (p,q)

(2) All values of (p, q)

(3) Exactly one value of (p, q)

(4) Exactly two values of (p, q)

6. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is : **[AIEEE-2010]**

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$ (3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

7. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$ then the vector $\vec{d}$ is equal to :- **[AIEEE-2011]**

(1) $\vec{b}+\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (2) $\vec{c}-\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ (3) $\vec{b}-\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (4) $\vec{c}+\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

8. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\vec{b} = \frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\vec{a}-\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is :

[AIEEE-2011]

(1) 5 (2) 3 (3) – 5 (4) – 3

9. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is : **[AIEEE-2011]**

(1) $\vec{a}+\vec{c}$ (2) $\vec{a}$ (3) $\vec{c}$ (4) $\vec{0}$

10. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[AIEEE-2012]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

11. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q},\overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[AIEEE-2012]**

(1) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (2) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (4) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

12. If the vectors $\overrightarrow{AB} = 3\hat{i}+4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of median through A is :

(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{45}$

13. Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k},\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vectors of the type $\vec{b}+\lambda\vec{c}$ for some scalar λ, whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$, is :

[JEE-MAINS Online 2013]

(1) $-2\hat{i}-\hat{j}+5\hat{k}$ (2) $2\hat{i}+\hat{j}+5\hat{k}$ (3) $2\hat{i}-\hat{j}+5\hat{k}$ (4) $2\hat{i}+3\hat{j}+3\hat{k}$

14. Let $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}$, $\vec{b} = \hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\bullet\vec{c} = |\vec{c}|$, $|\vec{c}-\vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30º, then $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ equals : **[JEE-MAINS Online 2013]**

(1) $\frac{3}{2}$ (2) 3 (3) $\frac{1}{2}$ (4) $\frac{3\sqrt{3}}{2}$

15. If $\left[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right] = \lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to : **[JEE(Main)-2014]**

(1) 2 (2) 3 (3) 0 (4) 1

16. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non – zero vectors such that no two of them are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

[JEE(Main)-2015]

(1) $\frac{2}{3}$ (2) $\frac{-2\sqrt{3}}{3}$ (3) $\frac{2\sqrt{2}}{3}$ (4) $\frac{-\sqrt{2}}{3}$

17. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :- **[JEE(Main)-2016]**

(1) $\frac{5\pi}{6}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{2}$ (4) $\frac{2\pi}{3}$

18. Let $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}| = 3$, $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to : **[JEE (Main)-2017]**

(1) $\frac{1}{8}$ (2) $\frac{25}{8}$ (3) 2 (4) 5

19. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b} = \hat{j}+\hat{k}$, If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to **[JEE (Main)-2018]**

(1) 84 (2) 336 (3) 315 (4) 265

20. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ is ; **[JEE (Main)-2019]**

(1) $\sqrt{6}$ (2) $3\sqrt{6}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

21 If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then the distance of R from the origin is : **[JEE (Main)-2019]**

(1) 6 (2) $\sqrt{53}$ (3) $2\sqrt{14}$ (4) $2\sqrt{21}$

22. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $\left|\vec{a} \times \vec{b}\right| = r$ is possible if : **[JEE (Main)-2019]**

(1) $r \ge 5\sqrt{\frac{3}{2}}$ (2) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$ (3) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (4) $0 < r \le \sqrt{\frac{3}{2}}$

23. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to: **[JEE (Main)-2019]**

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $3\hat{i} - 9\hat{j} - 5\hat{k}$ (3) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$ (4) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

24. If a unit vector $\vec{a}$ makes angle $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is : **[JEE (Main)-2019]**

(1) $\frac{5\pi}{6}$ (2) $\frac{5\pi}{12}$ (3) $\frac{\pi}{4}$ (4) $\frac{2\pi}{3}$

25. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is : **[JEE (Main)-2019]**

(1) 6 (2) $2\sqrt{13}$ (3) 7 (4) $4\sqrt{3}$

26 If the volume of parallelopiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to **[JEE (Main)-2019]**

(1) $-\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) $-\frac{1}{\sqrt{3}}$ (4) $\sqrt{3}$

27. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is: **[JEE (Main)-2019]**

(1) $4(-2\hat{i} - 2\hat{j} + \hat{k})$ (2) $4(2\hat{i} - 2\hat{j} - \hat{k})$ (3) $4(2\hat{i} + 2\hat{j} - \hat{k})$ (4) $4(2\hat{i} + 2\hat{j} + \hat{k})$

28. Let $\alpha \in R$ and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set S = {$\alpha$: $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar} **[JEE (Main)-2019]**

(1) is singleton
(2) contains exactly two numbers only one of which is positive
(3) is empty
(4) contains exactly two positive numbers

29. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $\left|\vec{c}\right|^2$ is equal to : **[JEE (Main)-2019]**

(1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

30. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}k$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}k$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}k$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular of $\vec{c}$, then $|\vec{b}|$ is equal to :

(1) $\sqrt{22}$ (2) 4 (3) 6 (4) $\sqrt{32}$

[JEE (Main)-2019]

31. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is: **[JEE (Main)-2019]**

(1) $\left(\frac{1}{2}, 4, -2\right)$ (2) (1, 5, 1) (3) $\left(-\frac{1}{2}, 4, 0\right)$ (4) (1, 3, 1)

32. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is: **[JEE (Main)-2019]**

(1) –3 (2) 4 (3) –4 (4) 3

33. Let $\vec{a} = \hat{i} + 2\hat{j} + 4k$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4k$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)k$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is : **[JEE (Main)-2019]**

(1) $-10\hat{i} + 5\hat{j}$ (2) $-14\hat{i} + 5\hat{j}$ (3) $-10\hat{i} - 5\hat{j}$ (4) $-14\hat{i} - 5\hat{j}$

34. Let $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1 - \beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is: **[JEE (Main)-2019]**

(1) 3 (2) 4 (3) 1 (4) 2

35. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i} + \hat{j} + k$, $\hat{i} + \mu\hat{j} + k$, $\hat{i} + \hat{j} + \mu k$ are co–planar, is : **[JEE (Main)-2019]**

(1) 2 (2) 0 (3) –1 (4) 1

36. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to: **[JEE (Main)-2019]**

(1) 30º (2) 45º (3) 90º (4) 60º

37. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ $(\alpha, \beta \in R)$ lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then : **[JEE (Main)-2020]**

(1) $\vec{a} \cdot \hat{k} + 4 = 0$ (2) $\vec{a} \cdot \hat{i} + 1 = 0$ (3) $\vec{a} \cdot \hat{k} + 2 = 0$ (4) $\vec{a} \cdot \hat{i} + 3 = 0$

38. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vector such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ if $\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ then the ordered pair, $(\lambda, \vec{d})$ is equal to : **[JEE (Main)-2020]**

(1) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (2) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$ (3) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$ (4) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$

39. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to .... **[JEE (Main)-2020]**

40. Let the volume of a parallelepiped whose coterminous edges are given by $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}$ $\vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{w}=2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. unit. If θ be the angle between the edge $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be : **[JEE (Main)-2020]**

(1) $\frac{7}{6\sqrt{6}}$ (2) $\frac{5}{3\sqrt{3}}$ (3) $\frac{7}{6\sqrt{3}}$ (4) $\frac{5}{7}$

41. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c} = \vec{b}\times\vec{a}$ and $\vec{c}.\vec{a}=0$ then $\vec{c}.\vec{b}$ is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{2}$ (2) -1 (3) $-\frac{3}{2}$ (4) $-\frac{1}{2}$

42. If the vectors,

$\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$,

$\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and

$\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ ($a \in R$) are coplanar and $3(\vec{p}\cdot\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$ then value of λ is .... **[JEE (Main)-2020]**

43. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=5$, $\vec{b}\cdot\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to_______.

[JEE (Main)-2020]

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$. Then $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$ is equal to______. **[JEE (Main)-2020]**

45. The lines $\vec{r}=(\hat{i}-\hat{j})+l(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}+\hat{j}-\hat{k})$ **[JEE (Main)-2020]**

(1) intersect when l = 1 and m = 2 (2) do not intersect for any values of l and m

(3) intersect for all values of l and m (4) intersect when $l = 2$ and m = $\frac{1}{2}$

46. Let a, b, c $\in$ R be such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = \cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is : **[JEE (Main)-2020]**

(1) $\frac{\pi}{2}$ (2) 0 (3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

47. Let x_0 be the point of local maxima of $f(x) = \vec{a}\cdot(\vec{b}\times\vec{c})$, where $\vec{a} = x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b} = -2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c} = 7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x = x_0$ is : **[JEE (Main)-2020]**

(1) –30 (2) –22 (3) 14 (4) –4

48. If $\vec{a} = 2\hat{i}+\hat{j}+2\hat{k}$, then the value $\left|\hat{i}\times\left(\vec{a}\times\hat{i}\right)\right|^2+\left|\hat{j}\times\left(\vec{a}\times\hat{j}\right)\right|^2+\left|\hat{k}\times\left(\vec{a}\times\hat{k}\right)\right|^2$ of is equal to _______.

[JEE (Main)-2020]

49. If the volume of a parallopiped, whose coterminos edges are given by the vectors $\vec{a} = \hat{i}+\hat{j}+n\hat{k}$, $\vec{b} = 2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c} = \hat{i}+n\hat{j}+3\hat{k}$ ($n \geq 0$), is 158 cu. units, Then : **[JEE (Main)-2020]**

(1) $\vec{a}\cdot\vec{c} = 17$ (2) $\vec{b}\cdot\vec{c} = 10$ (3) n = 9 (4) n = 7

50. Let the vectors $\vec{a},\vec{b},\vec{c}$ be such that $|\vec{a}| = 2$, $|\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $\left|\vec{a}+\vec{b}-\vec{c}\right|$ is ______. **[JEE (Main)-2020]**

51. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\,|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is-

[JEE (Main)-2020]

52. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}| = \vec{x}$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is _____. **[JEE (Main)-2020]**

53. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+k) = 1$ and $\vec{r}.(\hat{i}-2\hat{j}) = -2$, and the point (1, 0, 2) is **[JEE (Main)-2021]**

(A) $\vec{r}.(\hat{i}-2\hat{j}+3k) = \frac{7}{3}$ (B) $\vec{r}.(3\hat{i}+7\hat{j}+3k) = 7$

(C) $\vec{r}.(\hat{i}+7\hat{j}+3k) = 7$ (D) $\vec{r}.(\hat{i}-7\hat{j}+3k) = \frac{7}{3}$

54. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2k$ and $\hat{i}+2\hat{j}-4k$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, .2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of the vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is **[JEE (Main)-2021]**

(A) $\sqrt{482}$ (B) $\sqrt{171}$ (C) $\sqrt{5}$ (D) $\sqrt{227}$

55. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to

(A) 1 (B) $-\frac{5}{4}$ (C) $\frac{4}{5}$ (D) –1

56. Let $\vec{a}$ and $\vec{b}$ be two-zero vectors perpendicular to each other and $|\vec{a}|=|\vec{b}|$. If $|\vec{a}\times\vec{b}|=|\vec{a}|$, then the angle between the vectors $(\vec{a}+\vec{b}+(\vec{a}\times\vec{b}))$ and $\vec{a}$ is equal to **[JEE (Main)-2021]**

(A) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (C) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (D) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

57. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c},\ \vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following not true? **[JEE (Main)-2021]**

(A) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2 (B) $|3\vec{a}+\vec{b}-2\vec{c}|^2=51$

(C) $[\vec{a}\vec{b}\vec{c}]+[\vec{c}\vec{a}\vec{b}]=8$ (D) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))=\vec{0}$

58. Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies.

$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$, then $\vec{r}$ is equal to **[JEE (Main)-2021]**

(A) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$ (B) $\frac{1}{3}(2\vec{a}+\vec{b}-\vec{c})$ (C) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$ (D) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$

59. $\vec{c}$ is coplanar with $\vec{a}=-\hat{i}+\hat{j}+k\ \&\ \vec{b}=2\hat{i}+k$ $\vec{a}.\vec{c}=7\ \&\ \vec{c}\perp\vec{b}$. then the value of $2|\vec{a}+\vec{b}+\vec{c}|^2$ is __________.

60. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3k,\vec{b}=\beta\hat{i}-\alpha\hat{j}-k$ and $\vec{c}=\hat{i}-2\hat{j}-k$ such that $\vec{a}.\vec{b}=1$ and $\vec{b}.\vec{c}=-3$, then $\frac{1}{3}((\vec{a}\times\vec{b}).\vec{c})$ is equal to ____________. **[JEE (Main)-2021]**

61. Let $\vec{a}=\hat{i}+5\hat{j}+\alpha k,\vec{b}=\hat{i}+3\hat{j}+\beta k$ and $\vec{c}=-\hat{i}+2\hat{j}+3k$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _________.

[JEE (Main)-2021]

62. Let $\vec{a} = 2\hat{i} - \hat{j} + 2k$ and $\vec{b} = \hat{i} + 2\hat{j} - k$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - k$ and its projection on $\vec{a}$ is 19 units, then $\left|2\vec{v}\right|^2$ is equal to ___________. **[JEE (Main)-2021]**

63. Let the vectors $\hat{a} = (1+t)\hat{i} + (1-t)\hat{j} + k$, $\vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2k$ and $\vec{c} = t\hat{i} + t\hat{j} + k, t \in R$ such that for abg $\alpha, \beta, \gamma \in R$, $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is: **[JEE (Main)-2021]**

64. Let a vector $\vec{a}$ has magnitude 9, Let a vector $\vec{b}$ be such that for every (x, y) $\in$ R × R – {(0, 0)}, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$ then the value of $\left|\vec{a} \times \vec{b}\right|$ is equal to **[JEE (Main)-2022]**

(A) $9\sqrt{3}$ (B) $27\sqrt{3}$ (C) 9 (D) 81

65. Let P(–2, –1, 1) and $Q\left(\frac{56}{11}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus PRQS. If the direction ratios of the diagonal RS are α, –1, β where both α and β are integers orf minimum absolute values, then $\alpha^2 + \beta^2$ is equal to : **[JEE (Main)-2022]**

66. If $\vec{a} = 2\hat{i} + \hat{j} + 3k, \vec{b} = 3\hat{i} + 3\hat{j} + k$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\vec{k}$ are complanar vectors and $\vec{a}.\vec{c} = 5, \vec{b} \perp \vec{c}$, then 122 $(c_1 + c_2 + c_3)$ is equal to ________. **[JEE (Main)-2022]**

67. Let the position vectors of the points A, B, C and D be $5\hat{i} + 5\hat{j} + 2\lambda k, \hat{i} + 2\hat{j} + 3k, -2\hat{i} + \lambda\hat{j} + 4k$ and $-\hat{i} + 5\hat{j} + 6k$. Let the set S = {$\lambda \in R$: the points A, B, C and D are coplanar}. Then $\sum_{\lambda \in S} (\lambda + 2)^2$ is equal to : **[JEE (Main)-2023]**

(A) $\frac{37}{2}$ (B) 13 (C) 25 (D) 41

68. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4k, b = \hat{i} - 2\hat{j} - 2k$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3k$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a}.\vec{d} = 18$, then $[\vec{a} \times \vec{d}]^2$ is equal to: **[JEE (Main)-2023]**

(A) 760 (B) 640 (C) 25 (D) 41

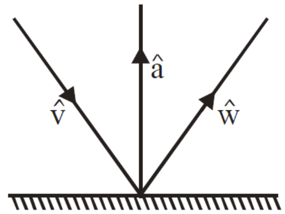
69. Let $\vec{u} = \hat{i} - \hat{j} - 2k, \vec{v} = 2\hat{i} + \hat{j} - k, \vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to **[JEE (Main)-2023]**

(A) 2 (B) $\frac{3}{2}$ (C) 1 (D) $-\frac{2}{3}$

70. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to **[JEE (Main)-2023]**

EXERCISE # 4 (JA)

1. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector .The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. **[JEE 05 (Mains)4]**



2. **(a)** Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ has the magnitude equal to $\frac{1}{\sqrt{3}}$ is -

(A) $4\hat{i} - \hat{j} + 4\hat{k}$ (B) $3\hat{i} + \hat{j} - 3\hat{k}$ (C) $2\hat{i} + \hat{j} - 2\hat{k}$ (D) $4\hat{i} + \hat{j} - 4\hat{k}$

(b) Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is – **[JEE 2006, 3+5]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

3. **(a)** The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}, \hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is -

(A) zero (B) one (C) two (D) three

(b) Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct?

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$ (B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

(C) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$ (D) $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are mutually perpendicular

(c) Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement-1 : $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$.

because

Statement-2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True. **[JEE 2007, 3+3+3]**

4. **(a)** The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = \frac{1}{2}$. Then, the volume of the parallelopiped is :

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

(b) Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}$ cost + $\hat{b}$ sin t. when P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then-

(A) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$ (B) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$

(C) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{1/2}$ (D) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{1/2}$

[JEE 2008, 3+3]

5. **(a)** If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$, then –

(A) $\vec{a}, \vec{b}, \vec{c}$ are non - coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non - coplanar

(C) $\vec{b}, \vec{d}$ are non - parallel (D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

(b) Match the statements / expression given in Column I with the value given in Column II.

	Column – I		**Column - II**
(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(P)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$ where [y] denotes the largest integer less than or equal to y	(Q)	$\frac{\pi}{4}$
(C)	Volume of the parallelepiped with its edges	(R)	$\frac{\pi}{3}$
	represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(S)	$\frac{\pi}{2}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$	(T)	π

[JEE 2009, 3+8]

6. **(a)** Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD} = -\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB then the cosine of the angle α is given by –

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

(b) If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is **[JEE 2010, 5 + 3]**

7. **(a)** Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$ (C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

(b) The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

(c) Let $\vec{a} = -\hat{i}-\hat{k}, \vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is **[JEE 2011, 3+4+4]**

8. **(a)** If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

(b) If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is **[JEE 2012, 4+3]**

(A) 0 (B) 3 (C) 4 (D) 8

9. Let $\overrightarrow{PR} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ} = \hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}$, $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **[JEE-Advanced 2013, 2M]**

(A) 5 (B) 20 (C) 10 (D) 30

10. Consider the set of eight vectors V = $\left\{a\hat{i}+b\hat{j}+c\hat{k} : a, b, c \in \{-1,1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE-Advanced 2013, 4, (–1)]**

11. Match List – I with List – II and select the correct answer using the code given below the lists.

	List – I		**List – II**
P.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2\left(\vec{a}\times\vec{b}\right), 3\left(\vec{b}\times\vec{c}\right)$ and $\left(\vec{c}\times\vec{a}\right)$ is	1.	100
Q.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3\left(\vec{a}+\vec{b}\right)$, $\left(\vec{b}+\vec{c}\right)$ and $2\left(\vec{c}+\vec{a}\right)$ is	2.	30
R.	Area of triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $\left(2\vec{a}+3\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ is	3.	24
S.	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $\left(\vec{a}+\vec{b}\right)$ and $\vec{a}$ is	4.	60

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

[JEE-Advanced 2013, 3, (–1)]

12. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then **[JEE (Advanced)-2014, 3]**

(A) $\vec{b}=(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$ (B) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$ (C) $\vec{a}\cdot\vec{b}=-(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$ (D) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is **[JEE(Advanced)-2014, 3]**

14. Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 24$, then which of the following is (are) true ?

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ (B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$ (C) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$ (D) $\vec{a}.\vec{b} = -72$

15. **(a)** Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p} + \vec{q} + \vec{r})$, $(\vec{p} - \vec{q} + \vec{r})$ and $(-\vec{p} - \vec{q} + \vec{r})$ are x, y and z, respectively, then the value of $2x + y + z$ is **[JEE 2015, 4,, –0M]**

(b)

	Column-I		Column-II
(A)	In a triangle ΔXYZ, let a, b and c be the length of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)	(P)	1
(B)	In a triangle ΔXYZ, let a, b and c be the length of the sides opposite to the angles, X Y and Z, respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)	(Q)	2
(C)	In R^2, Let $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ and $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	(R)	3
(D)	Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = \lvert\alpha x - 1\rvert + \lvert\alpha x - 2\rvert + ax$, where $\alpha \in \{0,1\}$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$, is (are)	(S)	5
		(T)	6

[JEE (Advanced) 2015]

16. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $\mathbb{R}^2$ and $\hat{w} = \frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}+2\hat{k}\right)$. Given that there exist a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u}\times\vec{v}| = 1$ and $\hat{w}.(\hat{u}\times\vec{v}) = 1$. Which of the following statement(s) is (are) correct ?

(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choice for such $\vec{v}$

(C) if $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(D) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$ **[JEE(Advanced)-2016, 4(–2)]**

17. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS} = \overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS} = \overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its **[JEE (Advanced)-2017]**

(A) circumcentre (B) incentre (C) centroid (D) orthocenter

PARAGRAPH-1

Let O be origin, and $\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ}$ be three unit vectors in the direction of the sides $\overrightarrow{QR},\overrightarrow{RP},\overrightarrow{PQ}$, respectively of a triangle PQR. **[JEE (Advanced)-2017]**

18. $\left|\overrightarrow{OX}\times\overrightarrow{OY}\right| =$

(A) sin (Q + R) (B) sin 2R (C) sin (P + R) (D) sin (P + Q)

19. If the triangle PQR varies. then the minimum value of

cos (P + Q) + cos (Q + R) + cos (R + P) is

(A) $-\frac{5}{3}$ (B) $-\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $\frac{3}{2}$

20. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b}=0$. For some x, y $\in$ R, let $\vec{c} = x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inlined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ____. **[JEE (Advanced)-2018]**

21. Consider the cube in the first with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0,0,0) is the origin, Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagranal OT. If $\vec{p} = \overrightarrow{SP}$, $\vec{q} = \overrightarrow{SQ}$, $\vec{r} = \overrightarrow{SR}$, and $\vec{t} = \overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is ___.

[JEE (Advanced)-2018]

22. Let L_1 and L_2 denote the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}),\ \lambda \in \mathbb{R} \quad \text{and}$$

$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? **[JEE (Advanced)-2019]**

(A) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(B) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(C) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(D) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

23. Three lines

$L_1 :\ \vec{r} = \lambda\hat{i},\ \lambda \in \mathbb{R}$

$L_2 :\ \vec{r} = \hat{k} + \mu\hat{j}, \mu \in \mathbb{R}$ and

$L_3 :\ \vec{r} = \hat{i} + \hat{j} + v\hat{k},\ v \in \mathbb{R}$

are given. For which point(s) Q and L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear ? **[JEE (Advanced)-2019]**

(1) $\hat{k} + \frac{1}{2}\hat{j}$ (2) $\hat{k} + \hat{j}$ (3) $\hat{k}$ (4) $\hat{k} - \frac{1}{2}\hat{j}$

24. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, α, $\beta \in$ R. If the projections of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equals _____ **[JEE (Advanced)-2019]**

25. In a triangle PQR, let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\frac{\vec{a}.(\vec{c} - \vec{b})}{\vec{c}(\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$, then the value of $\left|\vec{a} \times \vec{b}\right|^2$ is ______ **[JEE (Advanced)-2020]**

26. Let a and b be positive real numbers. Suppose $\overrightarrow{PQ} = a\hat{i} + b\hat{j}$ and $\overrightarrow{PS} = a\hat{i} - b\hat{j}$ are adjacent sides of a parallelogram PQRS. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w} = \hat{i} + \hat{j}$ along $\overrightarrow{PQ}$ and

$\overrightarrow{PS}$, respectively. If $|\vec{u}|+|\vec{v}|=|\vec{w}|$ and if the area of the parallelogram PQRS is 8, then which of the following statements is/are TRUE ? **[JEE (Advanced)-2020]**

(A) a + b = 4

(B) a – b = 2

(C) The length of the diagonal PR of the parallelogram PQRS is 4

(D) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

27. Let 0 be the origin and $\overrightarrow{OA}=2\hat{i}+2\hat{j}+\hat{k}\overrightarrow{OB}=\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC}\frac{1}{2}\left(\overrightarrow{OB}-\lambda\overrightarrow{OA}\right)$ for some $\lambda>0$. If $\left|\overrightarrow{OB}\times\overrightarrow{OC}\right|=\frac{9}{2}$, then which of the following statements is (are) TRUE ?

[JEE (Advanced)-2021]

(A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(B) Area of the triangle OAB is $\frac{9}{2}$

(C) Area of the triangle ABC is $\frac{9}{2}$

(D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

28. Let $\vec{u},\vec{v}$ and $\vec{w}$, be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u},.\vec{w}=1,\vec{v}.\vec{w}=1,\vec{w}.\vec{w}=4$. If the volume of the paralleloipiped, whose adjacent sides are represented by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ is $\sqrt{2}$ then the value of $\left|3\vec{u}+5\vec{v}\right|$ is ____________. **[JEE (Advanced)-2021]**

29. Let S be the reflection of a point Q with respect to the plane given by $\vec{r}=-(t+p)\hat{i}+t\hat{j}+(1+p)\hat{k}$ where t, p are real parameters and $\hat{i},\hat{j},\hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE ? **[JEE (Advanced)-2022]**

(A) $3(\alpha+\beta)=-101$ (B) $3(\beta+\gamma)=-71$

(C) $3(\gamma+\alpha)=-86$ (D) $3(\alpha+\beta+\gamma)=-121$

30. Let $\hat{i},\hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$\vec{a}=3\vec{i}+\hat{j}-\hat{k}$

$\vec{b}=\hat{i}+b_2\hat{j}+b_3\hat{k}$ $\quad b_2b_3\in R$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ $\quad c_1,c_2,c_3\in R$

Be three vectors such that $b_1b_3>0,\vec{a}.\vec{b}=0$ and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix}\begin{pmatrix} 3 & -c_1 \\ 1 & -c_2 \\ -1 & -c_3 \end{pmatrix}$$

Then, which of the following is/are True ? **[JEE (Advanced)-2022]**

(A) $\vec{a}.\vec{c}=0$ (B) $\vec{b}.\vec{c}=0$ (C) $|\vec{b}|>\sqrt{10}$ (D) $|\vec{c}|\ge\sqrt{11}$

31. Let the position vectors of the points P, Q, R and S be $\vec{a}=\hat{i}+2\hat{j}-5k, \vec{b}=3\hat{i}+6\hat{j}+3k, \vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively. Then which of the following statements is true ? **[JEE (Advanced)-2023]**

(A) The points P, Q, R and S are NOT coplanar

(B) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b}\times\vec{d}$ is 95

EXERCISE # 5

STRAIGHT OBJECTIVE TYPE

1. Given a parallelogram ABCD. If $|\overrightarrow{AB}| = a$, $|\overrightarrow{AD}| = b$ and $|\overrightarrow{AC}| = c$, then $\overrightarrow{DB}\cdot\overrightarrow{AB}$ has the value

(A) $\frac{3a^2+b^2-c^2}{2}$ (B) $\frac{a^2+3b^2-c^2}{2}$ (C) $\frac{a^2-b^2+3c^2}{2}$ (D) none

2. L_1 and L_2 are two lines whose vector equations are

$$L_1 : \vec{r} = \lambda\left[\left(\cos\theta+\sqrt{3}\right)\hat{i}+\left(\sqrt{2}\sin\theta\right)\hat{j}+\left(\cos\theta-\sqrt{3}\right)\hat{k}\right]$$

$$L_2 : \vec{r} = \mu\left(a\hat{i}+b\hat{j}+c\hat{k}\right),$$

where λ and μ are scalars and α is the acute angle between L_1 and L_2.
If the angle 'α' is independent of θ then the value of 'α' is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

3. In the isosceles triangle ABC, $|\overrightarrow{AB}| = |\overrightarrow{BC}| = 8$, a point E divides AB internally in the ratio 1 : 3, then the cosine of the angle between $\overrightarrow{CE}$ and $\overrightarrow{CA}$ is (where $|\overrightarrow{CA}| = 12$)

(A) $-\frac{3\sqrt{7}}{8}$ (B) $\frac{3\sqrt{8}}{17}$ (C) $\frac{3\sqrt{7}}{8}$ (D) $\frac{-3\sqrt{8}}{17}$

4. If $\vec{p} = 3\vec{a}-5\vec{b};\ \vec{q} = 2\vec{a}+\vec{b};\ \vec{r} = \vec{a}+4\vec{b};\ \vec{s} = -\vec{a}+\vec{b}$ are four vectors such that $\sin\left(\vec{p}\wedge\vec{q}\right) = 1$ and $\sin\left(\vec{r}\wedge\vec{s}\right) = 1$ then $\cos\left(\vec{a}\wedge\vec{b}\right)$ is :

(A) $-\frac{19}{5\sqrt{43}}$ (B) 0 (C) 1 (D) $\frac{19}{5\sqrt{43}}$

5. In a quadrilateral ABCD, $\overrightarrow{AC}$ is the bisector of the $\left(\overrightarrow{AB} \wedge \overrightarrow{AD}\right)$ which is $\frac{2\pi}{3}$,

$15\,|\overrightarrow{AC}| = 3|\overrightarrow{AB}| = 5|\overrightarrow{AD}|$ then cos $\left(\overrightarrow{BA} \wedge \overrightarrow{CD}\right)$ is

(A) $-\frac{\sqrt{14}}{7\sqrt{2}}$ (B) $-\frac{\sqrt{21}}{7\sqrt{3}}$ (C) $\frac{2}{\sqrt{7}}$ (D) $\frac{2\sqrt{7}}{14}$

6. If the two adjacent sides of two rectangles are represented by the vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} + \vec{b}$ respectively, then the angle between the vectors $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$

(A) is $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ (B) is $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

(C) is $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ (D) cannot be evaluated

7. A rigid body rotates about an axis through the origin with an angular velocity $10\sqrt{3}$ radians/sec. If $\vec{\omega}$ points in the direction of $\hat{i} + \hat{j} + \hat{k}$ then the equation to the locus of the points having tangential speed 20 m/sec. is

(A) $x^2 + y^2 + z^2 - xy - yz - zx - 1 = 0$
(B) $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx - 1 = 0$
(C) $x^2 + y^2 + z^2 - xy - yz - zx - 2 = 0$
(D) $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx - 2 = 0$

MULTIPLE OBJECTIVE TYPE

8. If $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$ then which of the following always hold(s) good?

(A) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs (B) $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = |\vec{b}|$

(C) $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = |\vec{c}|^2$ (D) $|\vec{b}| = |\vec{c}|$

9. Given the following information about the non zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$

(i) $\left(\vec{A} \times \vec{B}\right) \times \vec{A} = \vec{0}$ (ii) $\vec{B} \cdot \vec{B} = 4$ (iii) $\vec{A} \cdot \vec{B} = -6$ (iv) $\vec{B} \cdot \vec{C} = 6$

Which one of the following holds good?

(A) $\vec{A} \times \vec{B} = \vec{0}$ (B) $\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = 0$ (C) $\vec{A} \cdot \vec{A} = 8$ (D) $\vec{A} \cdot \vec{C} = -9$

10. If $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are four non zero vectors in the same plane no two of which are collinear then which of the following hold(s) good?

(A) $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = 0$

(B) $(\vec{A}\times\vec{C})\cdot(\vec{B}\times\vec{D}) \neq 0$

(C) $(\vec{A}\times\vec{B}) \times (\vec{C}\times\vec{D}) = \vec{0}$

(D) $(\vec{A}\times\vec{C}) \times (\vec{B}\times\vec{D}) \neq \vec{0}$

11. If $\vec{a}, \vec{b}, \vec{c} \,\&\, \vec{d}$ are the pv's of the points A, B, C & D respectively in three dimensional space & satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, then :

(A) A, B, C & D are coplanar

(B) the line joining the points B & D divides the line joining the point A & C in the ratio 2 : 1.

(C) the line joining the points A & C divides the line joining the points B & D in the ratio 1 : 1

(D) the four vectors $\vec{a}, \vec{b}, \vec{c} \,\&\, \vec{d}$ are linearly dependent.

12. The vectors $\vec{u} = \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix}$; $\vec{v} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}$; $\vec{w} = \begin{bmatrix} 3 \\ 2 \\ -6 \end{bmatrix}$

(A) form a left handed system

(B) form a right handed system

(C) are linearly independent

(D) are such that each is perpendicular to the plane containing the other two.

13. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non-collinear vectors such that a vector

$\vec{p} = ab\cos\left(2\pi - \left(\vec{a} \wedge \vec{b}\right)\right)\vec{c}$ and a vector $\vec{q} = ac\cos\left(\pi - \left(\vec{a} \wedge \vec{c}\right)\right)\vec{b}$ then $\vec{p}+\vec{q}$ is

(A) parallel to $\vec{a}$

(B) perpendicular to $\vec{a}$

(C) coplanar with $\vec{b} \,\&\, \vec{c}$

(D) coplanar with $\vec{a}$ and $\vec{c}$

14. Which of the following statement(s) hold good ?

(A) if $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{b} = \vec{c} (\vec{a} \neq 0)$

(B) if $\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{b} = \vec{c} (\vec{a} \neq 0)$

(C) if $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{b} = \vec{c}$ $(\vec{a} \neq 0)$

(D) if $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are non coplanar vectors and $\vec{k}_1 = \dfrac{\vec{v}_2 \times \vec{v}_3}{\vec{v}_1\cdot(\vec{v}_2\times\vec{v}_3)}$; $\vec{k}_2 = \dfrac{\vec{v}_3 \times \vec{v}_1}{\vec{v}_1\cdot(\vec{v}_2\times\vec{v}_3)}$

and $\vec{k}_3 = \dfrac{\vec{v}_1 \times \vec{v}_2}{\vec{v}_1\cdot(\vec{v}_2\times\vec{v}_3)}$ then $\vec{k}_1.(\vec{k}_2\times\vec{k}_3) = \dfrac{1}{\vec{v}_1.(\vec{v}_2\times\vec{v}_3)}$

15. If the line $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \sqrt{2}\,\hat{k})$ makes angles α, β, γ with xy, yz and zx planes respectively then which of the following are not possible?

(A) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ & $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\tan^2\alpha + \tan^2\beta + \tan^2\gamma = 7$ & $\cot^2\alpha + \cot^2\beta + \cot^2\gamma = 5/3$

(C) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$ & $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

(D) $\sec^2\alpha + \sec^2\beta + \sec^2\gamma = 10$ & $\operatorname{cosec}^2\alpha + \operatorname{cosec}^2\beta + \operatorname{cosec}^2\gamma = 14/3$

16. If a, b, c are different real numbers and $a\hat{i} + b\hat{j} + c\hat{k}; b\hat{i} + c\hat{j} + a\hat{k}$ & $c\hat{i} + a\hat{j} + b\hat{k}$ are position vectors of three non-collinear points A, B & C then :

(A) centroid of triangle ABC is $\frac{a+b+c}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

(B) $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

17. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point P(1, 0) can be

(A) $6\hat{i} + 8\hat{j}$ (B) $-6\hat{i} + 8\hat{j}$ (C) $6\hat{i} - 8\hat{j}$ (D) $-6\hat{i} - 8\hat{j}$

18. Let OAB be a regular triangle with side length unity (O being the origin). Also M,N are the points of trisection of AB,M being closer to A and N closer to B. Position vectors of A,B,M and N are $\vec{a}, \vec{b}, \vec{m}$ and $\vec{n}$ respectively. Which of the following hold(s) good ?

(A) $\vec{m} = x\vec{a} + y\vec{b} \Rightarrow \frac{2}{3}$ and y = $\frac{1}{3}$

(B) $\vec{m} = x\vec{a} + y\vec{b} \Rightarrow \frac{5}{6}$ and y = $\frac{1}{6}$

(C) $\vec{m}.\vec{n}$ equals $\frac{13}{18}$

(D) $\vec{m}.\vec{n}$ equals $\frac{15}{18}$

19. If A($\bar{a}$); B($\bar{b}$); C($\bar{c}$) and D($\bar{d}$) are four points such that

$\bar{a} = -2\hat{i} + 4\hat{j} + 3\hat{k};\ \bar{b} = 2\hat{i} - 8\hat{j};\ \bar{c} = \hat{i} - 3\hat{j} + 5\hat{k};\ \bar{d} = 4\hat{i} + \hat{j} - 7\hat{k}$

d is the shortest distance between the lines AB and CD, then which of the following is True?

(A) d = 0, hence AB and CD intersect

(B) d = $\frac{\left[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{BD}\right]}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$

(C) AB and CD are skew lines and d = $\frac{23}{13}$

(D) d = $\frac{\left[\overrightarrow{AB}\ \overrightarrow{CD}\ \overrightarrow{AC}\right]}{|\overrightarrow{AB} \times \overrightarrow{CD}|}$

20. Which of the following statement(s) is(are) incorrect ?

(A) The relation $|(\vec{u} \times \vec{v})| = |\vec{u} . \vec{v}|$ is only possible if atleast one of the vectors $\vec{u}$ and $\vec{v}$ is null vector.

(B) Every vector contained in the line $\vec{r}(t) = \langle 1+2t, 1+3t, 1+4t \rangle$ is parallel to the vector $\langle 1,1,1 \rangle$.

(C) If scalar triple product of three vectors, $\vec{u}, \vec{v}, \vec{w}$ is larger than $|\vec{u} \times \vec{v}|$ then $|\vec{w}| > 1$.

(D) The distance between the x-axis and the line x = y = 1 is $\sqrt{2}$.

21. Given three vectors $\vec{U} = 2\hat{i} + 3\hat{j} - 6\hat{k}$; $\vec{V} = 6\hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{W} = 3\hat{i} - 6\hat{j} - 2\hat{k}$

Which of the following hold good for the vectors $\vec{U}$, $\vec{V}$ and $\vec{W}$?

(A) $\vec{U}$, $\vec{V}$ and $\vec{W}$ are linearly dependent

(B) $(\vec{U} \times \vec{V}) \times \vec{W} = \vec{0}$

(C) $\vec{U}$, $\vec{V}$ and $\vec{W}$ form a triplet of mutually perpendicular vectors

(D) $\vec{U} \times (\vec{V} \times \vec{W}) = \vec{0}$

22. Which of the following statement(s) is/are true in respect of the lines

$\vec{r} = \vec{a} + \lambda\vec{b}; \vec{r} = \vec{c} + \mu d$ where $\vec{b} \times \vec{d} \neq 0$

(A) acute angle between the lines is $\cos^{-1}\left(\dfrac{|\vec{b} \cdot \vec{d}|}{|\vec{b}||\vec{d}|}\right)$

(B) The lines would intersect if $[\vec{c}\ \vec{b}\ \vec{d}] = [\vec{a}\ \vec{b}\ \vec{d}]$

(C) The lines will be skew if $[\vec{c} - \vec{a}\ \vec{b}\ \vec{d}] \neq 0$

(D) If the lines intersect at $\vec{r} = \vec{r}_0$, then the equation of the plane containing the lines is $[\vec{r} - \vec{r}_0\ \vec{b}\ \vec{d}] = 0$

23. Let $\vec{a}$ and $\vec{b}$ be two non-zero and non-collinear vectors then which of the following is/are always correct?

(A) $\vec{a} \times \vec{b} = [\vec{a}\,\vec{b}\,\hat{i}]\hat{i} + [\vec{a}\,\vec{b}\,\hat{j}] + [\vec{a}\,\vec{b}\,\hat{k}]\hat{k}$

(B) $\vec{a}.\vec{b} = (\vec{a}.\hat{i})(\vec{b}.\hat{i}) + (\vec{a}.\hat{j})(\vec{b}.\hat{j}) + (\vec{a}.\hat{k})(\vec{b}.\hat{k})$

(C) if $\vec{u} = \hat{a} - (\hat{a} \cdot \hat{b})\hat{b}$ and $\vec{v} = \hat{a} \times \hat{b}$ then $|\vec{u}| = |\vec{v}|$

(D) if $\vec{c} = \vec{a} \times (\vec{a} \times \vec{b})$ and $\vec{d} = \vec{b} \times (\vec{a} \times \vec{b})$ then $\vec{c} + \vec{d} = \vec{0}$

COMPREHENSION TYPE

Paragraph for questions nos. 24 to 26

Consider three vectors $\vec{p} = \hat{i} + \hat{j} + \hat{k}, \vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$ and let $\vec{s}$ be a unit vector, then

24. $\vec{p}, \vec{q}$ and $\vec{r}$ are

(A) linearly dependent

(B) can form the sides of a possible triangle

(C) such that the vectors $(\vec{q}-\vec{r})$ is orthogonal to $\vec{p}$

(D) such that each one of these can be expressed as a linear combination of the other two

25. If $(\vec{p}\times\vec{q})\times\vec{r} = u\vec{p}+v\vec{q}+w\vec{r}$, then (u + v + w) equals to

(A) 8 (B) 2 (C) – 2 (D) 4

26. The magnitude of the vector $(\vec{p}\cdot\vec{s})(\vec{q}\times\vec{r})+(\vec{q}\cdot\vec{s})(\vec{r}\times\vec{p})+(\vec{r}\cdot\vec{s})(\vec{p}\times\vec{q})$ is

(A) 4 (B) 8 (C) 18 (D) 2

MATRIX MATCH TYPE

27.

	Column-I		**Column-II**
(A)	P is point in the plane of the triangle ABC. The pv's of A,B and C are $\vec{a},\vec{b}$ and $\vec{c}$ respectively with respect to P as the origin. If $\left(\vec{b}+\vec{c}\right)\cdot\left(\vec{b}-\vec{c}\right)$ = 0 and $\left(\vec{c}+\vec{a}\right)\cdot\left(\vec{c}-\vec{a}\right)$ = 0, then w.r.t. the triangle ABC,P is its	(P)	centroid
(B)	If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the three non collinear points A,B and C respectively such that the vector $\vec{V}=\overline{PA}+\overline{PB}+\overline{PC}$ is a null vector then w.r.t. the ΔABC, P is its	(Q)	orthocentre
(C)	If P is a point inside the ΔABC such that the vector $\vec{R}=(BC)\overrightarrow{PA}+(CA)\left(\overrightarrow{PB}\right)+(AB)\left(\overrightarrow{PC}\right)$ is a null vector then w.r.t. the ΔABC, P is its	(R)	Incentre
(D)	If P is a point in the plane of the triangle ABC such that the scalar product $\overrightarrow{PA}.\overrightarrow{CB}$ and $\overrightarrow{PB}.\overrightarrow{AC}$ vanishes, then w.r.t. the ΔABC, P is its	(S)	circumcentre

EXERCISE # 6

1. Given a tetrahedron D-ABC with AB = 12 , CD = 6. If the shortest distance between the skew lines AB and CD is 8 and the angle between them is $\frac{\pi}{6}$, then find the volume of tetrahedron.

2. A vector $\vec{V} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ satisfies the following conditions :

(i) magnitude of $\vec{V}$ is $7\sqrt{2}$

(ii) $\vec{V}$ is parallel to the plane x – 2y + z = 6

(iii) $\vec{V}$ is orthogonal to the vector $2\hat{i} - 3\hat{j} + 6\hat{j}$ and (iv) $\vec{V} \cdot \hat{i} > 0$

Find the value of $(v_1 + v_2 + v_3)$.

3. Let $(\vec{p} \times \vec{q}) \times \vec{r} + (\vec{q} \cdot \vec{r})\, \vec{q} = (x^2 + y^2)\, \vec{q} + (14 - 4x - 6y)\, \vec{p}$ and $(\vec{r} \cdot \vec{r})\, \vec{p} = \vec{r}$ where $\vec{p}$ and $\vec{q}$ are two non-zero non-collinear vectors and x and y are scalars. Find the value of (x + y).

4. In a ΔABC, points E and F divide sides AC and AB respectively so that $\frac{AE}{EC} = 4$ and $\frac{AF}{FB} = 1$. Suppose D is a point on side BC. Let G be the intersection of EF and AD and suppose D is situated so that $\frac{AG}{GD} = \frac{3}{2}$. If the ratio $\frac{BD}{DC} = \frac{a}{b}$, where a and b are in their lowest form, find the value of (a + b).

5. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $|\vec{u} - \hat{i}|$ is geometric mean of $|\vec{u}|$ and $|\vec{u} - 2\hat{i}|$ where $\hat{i}$ is the unit vector along x-axis then find the value of $|\vec{u}|$.

6. $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points A ≡ (x, y, z);B ≡ (y, –2z, 3x) ;C ≡ (2z, 3x,–y) and D ≡ (1, –1, 2) respectively. If $|\vec{a}| = 2\sqrt{3}$; $(\vec{a} \wedge \vec{b}) = (\vec{a} \wedge \vec{c})$; $(\vec{a} \wedge \vec{d}) = \frac{\pi}{2}$ and $(\vec{a} \wedge \hat{j})$ is obtuse, then find x, y, z.

7. The length of the edge of the regular tetrahedron D–ABC is 'a'. Point E and F are taken on the edges AD and BD respectively such that E divides $\overrightarrow{DA}$ and F divides $\overrightarrow{BD}$ in the ratio 2 : 1 each. Then find the area of triangle CEF.

8. The position vectors of the points A, B, C are respectively (1, 1, 1) ; (1, –1, 2) ; (0, 2, –1). Find a unit vector parallel to the plane determined by ABC &perpendicular to the vector(1,0,1).

9. The position vectors of the vertices A,B and C of a tetrahedron are (1,1,1), (1,0,0) and (3,0,0) respectively. The altitude from the vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of side AD is 4 and volume of the tetrahedron is $2\sqrt{2}/3$ then find the all possible position vectors of the point E.

10. Given non zero number x_1, x_2, x_3 ; y_1, y_2, y_3 and z_1, z_2 and z_3

(i) Can the given numbers satisfy

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0 \text{ and } \begin{cases} x_1x_2 + y_1y_2 + z_1z_2 = 0 \\ x_2x_3 + y_2y_3 + z_2z_3 = 0 \\ x_3x_1 + y_3y_1 + z_3z_1 = 0 \end{cases}$$

(ii) If $x_i > 0$ and $y_i < 0$ for all i = 1, 2, 3 and P (x_1, x_2, x_3); Q(y_1, y_2, y_3) and O(0, 0, 0) can the triangle POQ be a right angled triangle ?

11. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}$, $\vec{b}\cdot\vec{q} = 0$ and $(\vec{b})^2 = 1$, where μ is a scalar then prove that $|(\vec{a}\cdot\vec{q})\vec{p} - (\vec{p}\cdot\vec{q})\vec{a}| = |\vec{p}\cdot\vec{q}|$.

12. Let $g(\theta) = \int_{-(\hat{a}.\hat{b})^2}^{|\hat{a}\times\hat{b}|^2} (2t+1)\,dt$, where θ is the angle between $\hat{a}$ and $\hat{b}$. If volume of the parallelopiped whose coterminous edges are represented by vectors $\hat{a}, \hat{a}\times\hat{b}$ and $\hat{a}\times\left(\hat{a}\times\hat{b}\right)$ (where angle between $\hat{a}$ and $\hat{b}$ is taken from the equation $2g(\theta) - 1 = 0$), is $\frac{p}{q}$ then find the least value of (p + q).

13. **(a)** Find a unit vector $\hat{a}$ which makes an angle $(\pi/4)$ with axis of z & is such that $\hat{a} + \hat{i} + \hat{j}$ is a unit vector.

(b) If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the range of $\frac{3|\vec{a}+\vec{b}|}{2} + 2|\vec{a}-\vec{b}|$.

14. Given four non zero vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$. The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and

$\left(\vec{a}\wedge\vec{b}\right) = \left(\vec{b}\wedge\vec{c}\right) = \frac{\pi}{3}$, $\left(\vec{d}\wedge\vec{a}\right) = \alpha$, $\left(\vec{d}\wedge\vec{b}\right) = \beta$, then prove that $\left(\vec{d}\wedge\vec{c}\right) = \cos^{-1}(\text{cso}\beta - \cos\alpha)$

15. Given three points on the xy plane on O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $\left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right) + 3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 0$. If the maximum and minimum values of $|\overrightarrow{PA}||\overrightarrow{PB}|$ are M and m respectively then find the values of $M^2 + m^2$.

16. Let $\vec{a}, \vec{b}, \vec{c}$ are unit vectors where $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}+\vec{a}|^2 = 3$, then $|\vec{a}+2\vec{b}+3\vec{c}|^2$ is equal to

ANSWER KEY

EXERCISE # 1

1.	B	**2.**	A	**3.**	B	**4.**	B	**5.**	D	**6.**	B	**7.**	D
8.	B	**9.**	D	**10.**	B	**11.**	D	**12.**	C	**13.**	D	**14.**	B
15.	C	**16.**	D	**17.**	C	**18.**	A	**19.**	A	**20.**	(i) D, (ii) B, (iii) B		
21.	D	**22.**	A	**23.**	D	**24.**	D	**25.**	B	**26.**	C	**27.**	B
28.	B	**29.**	D	**30.**	A	**31.**	A	**32.**	B	**33.**	B	**34.**	A
35.	C	**36.**	A	**37.**	C	**38.**	B	**39.**	A	**40.**	C	**41.**	C
42.	D	**43.**	D	**44.**	C	**45.**	D	**46.**	B	**47.**	A	**48.**	D
49.	C	**50.**	C	**51.**	D	**52.**	A	**53.**	D	**54.**	A	**55.**	C
56.	C	**57.**	A	**58.**	D	**59.**	A	**60.**	C	**61.**	B	**62.**	A
63.	D	**64.**	D	**65.**	B								

66. (A) T; (B) U ; (C) P; (D) R ; (E) Q; (F) S; (G) W; (H) V

EXERCISE # 2

1. (9, 7) **2.** $-\frac{1}{2}(\hat{i}+\hat{j})$ **4.** 13 **5.** 3 **6.** 4950 **7.** 7

8. 1125 **9.** x = 2, y = –1 **10.** **(b)** externally in the ratio 1 : 3

11. **(i)** parallel **(ii)** the lines intersect at the point p. v. $-2\hat{i} + 2\hat{j}$

(iii) lines are skew

13. **(a)** $\cot^{-1}(0)$; **(b)** $\cot^{-1}\frac{1}{\sqrt{3}}$; **(c)** $\cot^{-1}\sqrt{2}$ **14.** $\frac{\pi}{2}$

15. $\left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ **16.** $\sqrt{3}$ **17.** **(a)** $\frac{\sqrt{3}}{2}$, **(b)** 51

18. **(a)** 2, **(b)** –1, **(c)** –12 **19.** 101 **20.** F= $2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$

21. $\frac{4}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$ **22.** **(i)** $\frac{6}{7}\sqrt{14}$ **(ii)** 6 **(iii)** $\frac{3}{5}\sqrt{10}$ **(iv)** $\sqrt{6}$

23. 6 **24.** 13 **25.** $\vec{x} = \frac{\vec{a}+(\vec{c}.\vec{a})\vec{c}+\vec{b}\times\vec{c}}{1+\vec{c}^2}$, $y = \frac{\vec{b}+(\vec{c}.\vec{b})\vec{c}+\vec{a}\times\vec{c}}{1+\vec{c}^2}$

26. 75 **27.** 488

EXERCISE # 3 (JM)

1.	3	**2.**	4	**3.**	4	**4.**	4	**5.**	3	**6.**	1	**7.**	2
8.	3	**9.**	4	**10.**	4	**11.**	3	**12.**	3	**13.**	1	**14.**	1
15.	4	**16.**	3	**17.**	1	**18.**	3	**19.**	2	**20.**	3	**21.**	3
22.	1	**23.**	3	**24.**	4	**25.**	3	**26.**	2	**27.**	2	**28.**	3
29.	1	**30.**	3	**31.**	3	**32.**	3	**33.**	1	**34.**	3	**35.**	3
36	1	**37.**	Bonus	**38.**	1	**39.**	4.00	**40.**	3	**41.**	4	**42.**	1.00
43.	30.00	**44.**	2.00	**45.**	2	**46.**	1	**47.**	2	**48.**	18.00	**49.**	2
50.	6.00	**51.**	4.00	**52.**	1.00	**53.**	C	**54.**	B	**55.**	D	**56.**	B
57.	B	**58.**	C	**59.**	75	**60.**	2	**61.**	90	**62.**	1494	**63.**	C
64.	B	**65.**	450	**66.**	150	**67.**	D	**68.**	C	**69.**	C	**70.**	B

EXERCISE # 4 (JA)

1. $\hat{w} = \hat{v} - 2(\hat{a}.\hat{v})\hat{a}$ **2.** **(a)** A **(b)** B

3. **(a)** C **(b)** B **(c)** C **4.** **(a)** A **(b)** A

5. **(a)** C **(b)** (A) Q,S; (B) P,R,S,T; (C) T, (D) R

6. **(a)** B **(b)** 5

7. **(a)** C **(b)** A,D **(c)** 9 **8.** **(a)** 3 **(b)** C

9. C **10.** 5 **11.** C **12.** A,B,C **13.** 4 **14.** A,C,D

15. **(a)** Bonus, **(b)** (A$\rightarrow$P,R,S); (B $\rightarrow$P); (C $\rightarrow$P,Q); (D $\rightarrow$S; T)

16. B,C **17.** D **18.** D **19.** B **20.** 3.00 **21.** 0.50

22. A,C,D **23.** A,D **24.** 18.00 **25.** 108.00 **26.** A,C **27.** A,B,C

28. 7 **29.** A,B,C **30.** B,C,D **31.** B

EXERCISEss # 5

1. A **2.** A **3.** C **4.** D **5.** C **6.** B **7.** C

8. A,C **9.** A,B,D **10**. B,C **11.** A,C,D **12.** A,C,D **13.** B,C **14.** C,D

15. A,B,D **16.** A,B,C,D **17.** A,D **18.** A,C **19.** B,C,D **20.** A,B,D

21. B,C,D **22.** A,B,C,D **23.** A,B,C **24.** C **25.** B **26.** A

27. (A) S; (B) P; (C) R; (D) Q

EXERCISE # 6

1. 48 **2.** 12 **3.** 5 **4.** 9 **5.** $\sqrt{2}-1$ **6.** x = 2, y = –2, z = –2

7. $\dfrac{5a^2}{12\sqrt{3}}$ sq. units **8.** $\pm\dfrac{1}{3\sqrt{3}}\left(\hat{i}+5\hat{j}-\hat{k}\right)$ **9.** (–1, 3, 3) and (3, –1, –1)

10. NO, NO **12**. 5 **13.** **(a)** $\frac{-1}{2}\hat{i}-\frac{1}{2}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$, **(c)** Range : [3, 5]

15. 34 **16.** 19