

Kernel Matrix Calculation

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The Kernel Matrix

Consider a data matrix X consisting of n vectors x_1, x_2, \dots, x_n each of size m . The kernel matrix \mathbf{K} is symmetric and calculated from kernel functions of the form $K(x_i, x_j)$ which forms element $K_{i,j}$ of the kernel matrix [1].

Kernel Functions

The following kernel functions are used in the article [2, 3, 4]:

- Linear Kernel (dot product)

$$K(x_i, x_j) = x_i^T x_j \quad (1)$$

- Gaussian

$$K(x_i, x_j) = \exp \left(\frac{-\|x_i - x_j\|^2}{\theta} \right) \quad (2)$$

- Polynomial

$$K(x_i, x_j) = (x_i^T x_j + \text{Offset})^d \quad (3)$$

- Exponential

$$K(x_i, x_j) = \exp \left(\frac{-\|x_i - x_j\|}{\theta} \right) \quad (4)$$

- Log Kernel

$$K(x_i, x_j) = -\log(1 + \|x_i - x_j\|^\beta) \quad (5)$$

- Cauchy

$$K(x_i, x_j) = \frac{1}{1 + \frac{\|x_i - x_j\|^2}{\sigma}} \quad (6)$$

- Power

$$K(x_i, x_j) = -\|x_i - x_j\|^\beta \quad (7)$$

- Wave

$$K(x_i, x_j) = \frac{\theta}{\|x_i - x_j\|} \sin\left(\frac{\|x_i - x_j\|}{\theta}\right) \quad (8)$$

- Multiquadric

$$K(x_i, x_j) = \sqrt{\|x_i - x_j\|^2 + \sigma^2} \quad (9)$$

The kernel matrix calculation scales $O(mn^2)$ with the dimensions of the data matrix X .

References

- [1] An introduction to support vector machines and other kernel-based learning methods, Nello Cristianini & John Shawe-Taylor
- [2] Data Variant Kernel Analysis, Yuichi Motai
- [3] Classes of Kernels for Machine Learning: A Statistics Perspective, Marc G. Genton.
- [4] Conditionally Positive Definite Kernels for SVM Based Image Recognition, S. Boughorbel, J. -. Tarel and N. Boujemaa, 2005 IEEE International Conference on Multimedia and Expo, Amsterdam, 2005, pp. 113-116