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Optimisation of variable-length code for data compression of memoryless Laplacian source

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Abstract: In this study, the authors present an efficient technique for compression and coding of memoryless Laplacian sources, which uses variable-length code (VLC). That technique is based on the combination of two companding quantisers in the first case and three companding quantisers in the second case. These quantisers have disjoint support regions, different number of representation levels and different compressor functions. The closed-form expressions are obtained for the distortion, average bit rate and signal to quantisation noise ratio (SQNR). The presented numerical results point out the effects of rate-distortion (R-D) optimisation on the system performances. Since our model assumes the general case of Laplacian distribution, it has wide applications like the coding of speech and images. It is shown that the difference of SQNR of our model and classical companding quantiser based model is 2.8 dB for two quantisers and 4.2 dB in three quantisers model. The authors have also made a comparison between our model, combination of the optimal uniform quantiser and Huffmann lossless coder and combination of optimal companding quantiser and simple lossless coder.

1 Introduction

Quantisers play an important role in the theory and practice of modern day signal processing. They are applied for the purpose of storage and transmission of continual signals. All data-compression schemes assume a digital source of information with known statistical properties as an input. The output of the source is a set of symbols with a given probability of occurrence. Compression is achieved by assigning shorter codewords to the more frequent symbols and longer codewords to the less frequent ones. The compressed output is simply the concatenation of such codewords. This is an important application of the variable-length codes (VLCS).

Many sources that we deal with, have a distribution which is quite peaked at zero. For example, speech consists mainly of silence. Therefore samples of speech will be zero or close to zero with high probability. On the other hand, image pixels do not have any attraction to small values. But there is a high degree of correlation among pixels. Therefore a large number of the pixel-to-pixel differences will have values close to zero. In these situations, Laplacian distribution provides a close match to data. Memoryless Laplacian source is commonly used and important in many areas of telecommunications and computer science.

An efficient algorithm for the design of the optimal quantiser for the source with known distribution was developed by Lloyd and Max [1]. However, this method is time consuming for the large number of quantisation levels. One solution that overcomes these difficulties is the companding model [2, 3]. Quantisers based on the companding model have simple realisation structure and performances close to the optimal ones. Its simplicity, parameters, and many characteristics can be described in closed-form relations. Examples are: speech signal, images, video signal etc. The design of such quantisers is also more efficient than Lloyd–Max's algorithm since it does not require the iterative method. This difference is very notable for some commonly used sources including the Laplacian source.

For the purpose of transmission, processing and storing those signals, simple and fast compression algorithms are desirable. One solution is given in [4] where a uniform quantiser is considered. In paper [5] lossless compression algorithm provided only the additional compression of the digitised signal (PCM), but without providing a quality improvement. In [6] forward adaptive technique is given for Lloyd–Max's algorithm implementation in speech coding algorithm. Fixed-rate scalar quantisers for Laplacian source have already been the topic of earlier research [7, 8]. The well-known efficient algorithm for lossless coding of the information sources with known symbol probability, is Huffman algorithm [9, 10]. It requires a very complex realisation structure and also is time consuming. Hardware implementations of popular compression algorithms such as the Huffman coding [11], Lempel–Ziv coding [12], binary arithmetic coding [13] and the Rice algorithm [14] have been reported in the literature. A 12-bit A/D with a simplified Huffman encoder is presented in [15]. Compression algorithm for Laplacian source, consisting of an optimal bounded companding quantiser and simple lossless coder is given in [16]. Multi-resolution scalar quantisers are described in [17].

In this paper we give the simple solution based on two non-uniform companding quantisers, in the first case, and three non-uniform companding quantisers, in the second case. For a fixed value of average bitrate R, we provide the optimisation of region bounds as well as the number of quantisation levels of each companding quantiser.

We compare our model to the combination of the optimal uniform quantiser and Huffmann lossless coder. It is shown that our model presents better results, with much simple and more efficient realisation structure. Comparison is also made with the combination of optimal companding quantiser and simple lossless coder [16]. The model in paper [16] is performing coding and decoding the groups of three samples and transmitting the side information about number of bits (bitrates $R-1$ or R) used for coding. In our paper coding and decoding is performed sample by sample and transmitted side information is used for determining which quantiser corresponds to which sample. Advantages of our model are better results and higher flexibility in quantiser designing (changeability of number of quantisers and their bit rates), with slightly more complex realisation structure.

We also deal with an application of our model in speech signal coding. It is used for the compression of the sample speech signal and the obtained experimental results are compared with theory.

This paper is organised as follows. Section 2 recalls some basic theories of quantisers and companding model. In Section 3 we give a description of the VLC for data compression of memoryless Laplacian source, which consists of two and tree companding quantisers. Section 4 contains some numerical examples. We also performed an optimisation of the quantiser distortion for prescribed value of average bit rate. In Section 5, for the purpose of testing, we considered the adaptive variant of our three quantiser VLC model which is tested on the sample speech signal. Section 6 concludes the paper by summarising the key features of the coder design and its applications.

2 Scalar quantisers and companding technique

Assume that an input signal is characterised by continuous random variable X with probability density function (PDF) $p(x)$. The first approximation to the long-time-averaged PDF of amplitudes is provided by a two-sided exponential or Laplacian model. Waveforms are sometimes represented in terms of adjacent-sample differences. The PDF of the difference signal for an image waveform follows the Laplacian function [2, p. 33]. Laplacian source can be also used for modelling of the speech signal [18, p. 384]. In the rest of the paper, we assume that information source is Laplacian source with memoryless property and zero mean value. The PDF of that source is given by

$$p(x)=\frac{1}{\sqrt{2\sigma^2}}\mathrm{e}^{-(|x|\sqrt{2}/\sigma)} \tag{1}$$

where x is zero-mean statistically independent Laplacian random variable of variance σ^2.

The sources with exponential and Laplacian PDF are commonly encountered and the methods for designing quantisers for these sources are very similar. Without loss of generality, we can suppose that $\sigma=1$ and expression (1) becomes

$$p(x)=\frac{1}{\sqrt{2}}\mathrm{e}^{-\sqrt{2}|x|} \tag{2}$$

An N-point fixed rate scalar quantiser is characterised by the set of real numbers $t_1, t_2, \ldots, t_N$, called 'decision thresholds', which satisfy

$$-\infty=t_0<t_1<\cdots<t_{N-1}<t_N=+\infty \tag{3}$$

and set $y_1, y_2, \ldots, y_N$, called 'representation levels', which satisfy

$$y_j\in\alpha_j=(t_{j-1},t_j], \quad \text{for} \quad j=1,\ldots,N \tag{4}$$

Sets $\alpha_1, \alpha_2, \ldots, \alpha_N$ form the partition of the set of real numbers $\mathbb{R}$ and are called 'quantisation cells'. The quantiser is defined as many-to-one mapping $Q\colon \mathbb{R}\to\mathbb{R}$, $Q(x)=y_j$ where $x\in\alpha_j$. In practice, input signal value x is discretised (quantised) to the value y_j. Cells $\alpha_2, \alpha_3, \ldots, \alpha_{N-1}$ are 'inner cells' (or 'granular cells') while α_1 and α_N are 'outer cells' (or 'overload cells'). In such way, cells $\alpha_2, \alpha_3, \ldots, \alpha_{N-1}$ form granular while cells α_1 and α_N form an overload region. Since variable rate and scalar quantisers are the only types of quantisers considered in the paper, we just briefly recall their properties.

The quality of the quantiser is measured by distortion of resulting reproduction in comparison to the original one. Mostly used measure of distortion is mean-squared error. It is defined by

$$D(Q)=E(X-Q(X))^2=\sum_{i=1}^{N}\int_{t_{i-1}}^{t_i}(x-y_i)^2p(x)\,\mathrm{d}x \tag{5}$$

The N-point quantiser Q is 'optimal' for the source X if there is no other N-point quantiser Q_1 such that $D(Q_1)<D(Q)$. We also define granular distortion $D_\mathrm{g}(Q)$ and overload $D_\mathrm{ol}(Q)$ distortion by

$$D_\mathrm{g}(Q)=\sum_{j=2}^{N-1}\int_{t_{j-1}}^{t_j}(x-y_j)^2p(x)\,\mathrm{d}x \tag{6}$$

$$D_\mathrm{ol}(Q)=\int_{-\infty}^{t_1}(x-y_1)^2p(x)\,\mathrm{d}x+\int_{t_{N-1}}^{+\infty}(x-y_N)^2p(x)\,\mathrm{d}x \tag{7}$$

Obviously follows that $D(Q)=D_\mathrm{g}(Q)+D_\mathrm{ol}(Q)$.

Considerable amount of work has been focused on the design of optimal quantisers for compression sources in image, speech and other applications. Denote by D_N^* the distortion of an optimal N-point quantiser. As it was discovered by Panter and Dite [11], for large N holds $D_N^*\cong c_\infty/N$. Here c_∞ is the Panter–Dite constant

$$c_\infty=\frac{1}{12}\left(\int_{-\infty}^{+\infty}p^{1/3}(x)\,\mathrm{d}x\right)^3 \tag{8}$$

The general method for the design of an optimal N-point quantiser for the given source X is Lloyd–Max algorithm [1, 2, 19]. Owing to the computational complexity of this method, it is not suitable for the design of optimal quantisers with more than 128 levels. Hence, other methods for the construction of nearly optimal quantisers for large number of quantisation levels are developed. One of the commonly used techniques for this purpose is the companding technique [12]. It forms the core of the ITU-T G.711 PCM standard, recommended for coding speech signals. Companding technique consists of the following steps:

1. Compress the input signal x by applying the compressor function $c(x)$.
2. Apply the uniform quantiser Q_u on the compressed signal.
3. Expand the quantised version of the compressed signal using an inverse compressor function $c^{-1}(x)$.

The corresponding non-uniform quantiser consisting of a compressor, a uniform quantiser, and an expander in cascade is called 'companding quantiser' (compandor). Hence, the companding quantiser can be represented as $Q(x) = c^{-1}(Q_{\mathrm{u}}(c(x)))$, where $Q_{\mathrm{u}}(x)$ is uniform quantiser in the interval $[-1, 1]$. Denote by $t_{\mathrm{u},i}$ and $y_{\mathrm{u},i}$ decision thresholds and representation levels of the uniform quantiser $Q_{\mathrm{u}}(x)$. Corresponding values t_i and y_i of the companding quantiser $Q(x)$ can be determined as the solutions of the following equations

$$c(t_i) = t_{\mathrm{u},i} = -1 + \frac{2i}{N}, \quad c(y_i) = y_{\mathrm{u},i} = -1 + \frac{2i-1}{N} \tag{9}$$

There are several ways how to choose the compressor function $c(x)$ for compression law. Originally, in [12] and also in [8], compressor function $c_0\colon \mathbb{R} \to [-1, 1]$ is defined as

$$c_0(x) = \frac{\int_{-\infty}^{x} p^{1/3}(x)\,\mathrm{d}x}{\int_{-\infty}^{+\infty} p^{1/3}(x)\,\mathrm{d}x} \tag{10}$$

In this paper, we use the similar definition of the compressor function which will be described in the following section.

3 Description and construction of VLC coder and decoder

In this section we describe our model consisting of two and three companding quantisers, in the first and the second cases respectively. Optimisation of the bounds of support regions and numbers of representation levels, is performed for a fixed average bitrate R.

3.1 Two companding quantisers VLC model

The coder consists of two companding quantisers with different number of representation levels and different compressor functions. First quantiser Q_1 is applied on the inner segment $I = [-t_1, t_1]$, whereas the second quantiser Q_2 is applied on the outer segment $O = (-\infty, -t_1] \cup [t_1, +\infty)$. Value t_1 is called the threshold value. We denote by $N_i = 2^{k_i}$ the number of the quantisation levels of quantiser Q_i, where k_i is number of bits per sample and $i = 1, 2$. Let $c_1\colon I \to [-1, 1]$ and $c_2\colon O \to [-1, 1]$ be the corresponding compressor functions. An optimal compressor function c_1 is given by the following expression [20]

$$c_1(x) = -1 + 2\frac{\int_{-t_1}^{x} p^{1/3}(x)\,\mathrm{d}x}{\int_{-t_1}^{t_1} p^{1/3}(x)\,\mathrm{d}x}, \quad -t_1 < x < t_1 \tag{11}$$

By analogy, the optimal compressor function $c_2(x)$ is given by (see (12))

$$c_2(x) = \begin{cases} -1 + 2\dfrac{\int_{-\infty}^{x} p^{1/3}(u)\,\mathrm{d}u}{\int_{-\infty}^{-t_1} p^{1/3}(u)\,\mathrm{d}u + \int_{t_1}^{+\infty} p^{1/3}(u)\,\mathrm{d}u}, & -\infty < x < -t_1 \\[2ex] -1 + 2\dfrac{\int_{-\infty}^{-t_1} p^{1/3}(u)\,\mathrm{d}u + \int_{t_1}^{x} p^{1/3}(u)\,\mathrm{d}u}{\int_{-\infty}^{-t_1} p^{1/3}(u)\,\mathrm{d}u + \int_{t_1}^{+\infty} p^{1/3}(u)\,\mathrm{d}u}, & t_1 < x < \infty \end{cases} \tag{12}$$

Since the function $p(x)$ is symmetric, by direct evaluation we obtain, for every $t_1 > 0$, that

$$\int_{-\infty}^{-t_1} p^{1/3}(x)\,\mathrm{d}x = \int_{t_1}^{+\infty} p^{1/3}(x)\,\mathrm{d}x = 3\sqrt[3]{\frac{\sigma}{4}}\exp\left(-\frac{\sqrt{2}t_1}{3}\right)$$

Hence for $t_1 > 0$, compressor functions c_1 and c_2 can be expressed as

$$c_1(x) = \begin{cases} -\dfrac{1 - \exp\left(\sqrt{2}x/3\sigma\right)}{1 - \exp\left(-\sqrt{2}t_1/3\sigma\right)}, & -t_1 < x < 0 \\[2ex] \dfrac{1 - \exp\left(-\sqrt{2}x/3\sigma\right)}{1 - \exp\left(-\sqrt{2}t_1/3\sigma\right)}, & 0 < x < t_1 \end{cases}$$

$$c_2(x) = \begin{cases} -1 + \exp\left(\dfrac{\sqrt{2}}{3\sigma}(x + t_1)\right), & -\infty < x < -t_1 \\[2ex] 1 - \exp\left(-\dfrac{\sqrt{2}}{3\sigma}(x - t_1)\right), & t_1 < x < +\infty \end{cases} \tag{13}$$

The total signal distortion D is given by $D = D_{\mathrm{i}} + D_{\mathrm{o}}$ where D_{i} and D_{o} are distortions for the inner and outer regions, respectively. They can be approximated using Bennet integral as follows

$$\begin{aligned} D_{\mathrm{i}} &= \frac{2}{3N_1^2}\left(\int_0^{t_1} p^{1/3}(u)\,\mathrm{d}u\right)^3 = \frac{9\sigma^2\left(1 - \exp\left(-\sqrt{2}t_1/3\sigma\right)\right)^3}{2N_1^2} \\ D_{\mathrm{o}} &= \frac{2}{3N_2^2}\left(\int_{t_1}^{+\infty} p^{1/3}(u)\,\mathrm{d}u\right)^3 = \frac{9\sigma^2\exp\left(-\sqrt{2}t_1/\sigma\right)}{2N_2^2} \end{aligned} \tag{14}$$

According to the last expression we see that total distortion D is the function of the parameters N_1, N_2 and t_1, that is we may write

$$\begin{aligned} D = D(N_1, N_2, t_1) = &\frac{9\sigma^2(1 - \exp(-\sqrt{2}t_1/3\sigma))^3}{2N_1^2} \\ &+ \frac{9\sigma^2\exp(-\sqrt{2}t_1/\sigma)}{2N_2^2} \end{aligned} \tag{15}$$

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Similarly the average number of bits per sample R is given by $R = p_1 \log_2 N_1 + p_2 \log_2 N_2$, where p_1 and p_2 are probabilities that one signal sample will belong to I and O, respectively. Since

$$p_1 = \int_{-t_1}^{t_1} p(u)\,\mathrm{d}u = 1 - \exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right) \tag{16}$$

and $p_2 = 1 - p_1$ we see that R can be also expressed as a function of N_1, N_2 and t_1 in the following way

$$\begin{aligned} R &= R(N_1, N_2, t_1) \\ &= \left(1 - \exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)\right)(\log_2 N_1 + 1) \\ &\quad + \exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)(\log_2 N_2 + 1) \end{aligned} \tag{17}$$

The additional bit in expression (17) determines which quantiser is used in the coding process. This information is necessary for decoding. Note that threshold t_1 can be computed directly from the value R using

$$t_1 = -\frac{\sigma}{\sqrt{2}} \ln\left(\frac{R - \log_2 N_1 - 1}{\log_2 N_2 - \log_2 N_1}\right) \tag{18}$$

3.2 Three companding quantisers VLC model

The coder consists of three companding quantisers with different number of representation levels and different compressor functions. First quantiser Q_1 is applied on the inner segment, $I_1 = [-t_1, t_1]$, second quantiser Q_2 is applied on the second inner segment $I_2 = [-t_2, -t_1] \cup [t_1, t_2]$, while the third quantiser Q_3 is applied on the outer segment $O = (-\infty, t_2] \cup [t_2, +\infty)$. We say that values t_1, t_2 and t_3 are threshold values. We also denote by $N_i = 2^{k_i}$ the number of the quantisation levels of quantiser Q_i, where k_i is number of bits per sample and $i = 1, 2, 3$. Let $c_1 : I_1 \to [-1, 1]$, $c_2 : I_2 \to [-1, 1]$ and $c_3 : O \to (-1, 1)$ be corresponding optimal compressor functions. These functions are given similarly as in the case of two quantisers (relations (11) and (12))

$$c_1(x) = \begin{cases} -\dfrac{1 - \exp(\sqrt{2}x/3\sigma)}{1 - \exp(-\sqrt{2}t_1/3\sigma)}, & -t_1 < x < 0 \\[2ex] \dfrac{1 - \exp(-\sqrt{2}x/3\sigma)}{1 - \exp(-\sqrt{2}t_1/3\sigma)}, & 0 < x < t_1 \end{cases}$$

$$c_2(x) = \begin{cases} -\dfrac{1 - \exp(\sqrt{2}(x + t_1)/3\sigma)}{1 - \exp(-\sqrt{2}(t_2 - t_1)/3\sigma)}, & -t_2 < x < -t_1, \\[2ex] \dfrac{1 - \exp(-\sqrt{2}(x - t_1)/3\sigma)}{1 - \exp(-\sqrt{2}(t_2 - t_1)/3\sigma)}, & t_1 < x < t_2 \end{cases}$$

$$c_3(x) = \begin{cases} -1 + \exp\left(\dfrac{\sqrt{2}}{3\sigma}(x + t_2)\right), & -\infty < x < -t_2 \\[2ex] 1 - \exp\left(-\dfrac{\sqrt{2}}{3\sigma}(x - t_2)\right), & t_2 < x < +\infty \end{cases}$$

The total signal distortion D is given by $D = D_1 + D_2 + D_3$, where D_i is distortion for the inner or outer regions, respectively. The average number of bits per sample R is given by $R = p_1 \log_2 N_1 + p_2 \log_2 N_2 + p_3 \log_2 N_3$, where p_i are probabilities that one signal sample belongs to I_i or O, respectively. After some basic calculations, similarly to the previous case, we see that D and R can be expressed in the following way

$$\begin{aligned} &D(N_1, N_2, N_3, t_1, t_2) \\ &\quad = \frac{9\sigma^2(1 - \exp(-\sqrt{2}t_1/3\sigma))^3}{2N_1^2} \\ &\qquad + \frac{9\sigma^2(\exp(-\sqrt{2}t_1/3\sigma) - \exp(-\sqrt{2}t_2/3\sigma))^3}{2N_2^2} \\ &\qquad + \frac{9\sigma^2(\exp(-\sqrt{2}t_2/3\sigma))^3}{2N_3^2} \end{aligned} \tag{19}$$

$$\begin{aligned} &R(N_1, N_2, N_3, t_1, t_2) \\ &\quad = \left(1 - \exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right)\right)(\log_2 N_1 + 1) \\ &\qquad + \left(\exp\left(-\frac{\sqrt{2}t_1}{\sigma}\right) - \exp\left(-\frac{\sqrt{2}t_2}{\sigma}\right)\right)(\log_2 N_2 + 2) \\ &\qquad + \exp\left(-\frac{\sqrt{2}t_2}{\sigma}\right)(\log_2 N_3 + 2) \end{aligned} \tag{20}$$

Extra one and two bits, respectively, are added in the rates of every used quantisers in expression (20). They determine which quantiser is used in the coding process. This is the side information necessary for decoding. If the source sample belongs to region I_1, the first bit of the codeword is set to zero and the following bit rate $R_1 = \log_2 N_1 + 1$ is used. Otherwise, if source sample belongs to second or third segment, the first two bits of codeword are set to 10 or 11, respectively. Since other $\log_2 N_2$ and $\log_2 N_3$ are used for information, corresponding bit rates are $R_2 = \log_2 N_1 + 2$ and $R_3 = \log_2 N_1 + 2$. Such coding enables the simple decoder structure. Since the coder depends on two parameters t_1 and t_2, we perform an optimisation for the fixed value of average bit rate. In other words, we solve the following optimisation problem

$$\begin{aligned} \min \quad & D \\ \text{s.t.} \quad & R = R_0 \\ & 0 \le t_1 \le t_2 \end{aligned}$$

The optimisation procedure can be described as follows. First we express t_2 from (20), considering that $R = R_0$, and then replace it in (19). Hence we obtain the distortion D only as a function of t_1, that is, $D(t_1)$. Then it can be minimised using one of the well-known unconstrained optimisation methods (e.g., simplex method, variant I) [21].

The above given optimisation is valid if thresholds follow relationship $0 \le t_1 \le t_2$, that is

$$\exp\left(-\sqrt{2}t_2/\sigma\right) \le \exp\left(-\sqrt{2}t_1/\sigma\right) \le 1 \tag{21}$$

Combining (20) and (21) we can derive the conditions for the cases represented in Table 1. It can be seen that there exist four cases with three conditions, respectively. Our method of optimisation is correct for any of these cases. Following these conditions in the terms of t_1, k_1, k_2, k_3 and R_0, we have obtained a solution that provides a unique minimum value of D, which means that our method of optimisation is

Table 1 Four cases with three range conditions in each case as a function of threshold t_1, sample bit rates k_1, k_2, k_3 and R_0 for which our method of optimisation is correct, respectively

	1st condition	2nd condition	3rd condition
1.	$t_d \le t_1 \le t_u$	$k_1 - 1 \le k_2 \le k_3$	$k_1 + 1 - R_0 \le 0$
2.	$t_d \le t_1 \le t_u$	$k_1 - 1 \ge k_3 \ge k_2$	$k_1 + 1 - R_0 \ge 0$
3.	$t_d \le t_1 \le t_u$	$k_1 - 1 \ge k_2 \ge k_3$	$k_1 + 1 - R_0 \ge 0$
4.	$t_d \ge t_1 \ge t_u$	$k_1 - 1 \le k_3 \le k_2$	$k_1 + 1 - R_0 \le 0$

correct. In Table 1, t_d and t_u are defined as

$$t_d = -\frac{1}{\sqrt{2}} \ln \frac{k_1 + 1 - R_0}{k_1 - k_2 - 1} \tag{22}$$

$$t_u = -\frac{1}{\sqrt{2}} \ln \frac{R_0 - k_1 - 1}{k_3 - k_1 + 1} \tag{23}$$

and they denote lower and upper borders of range in which interval threshold t_1 can take its values to satisfy the constraints in optimisation model.

3.3 Block diagrams of coder and decoder

Block schemes of our VLC model are shown in Fig. 1.

On the transmission side, input signal I first goes through quantiser selector where it is compared with the threshold values t_1 in the case of two ($i = 2$) and t_1, t_2 in the case of three quantisers ($i = 3$). Quantiser selector sends the signal to the corresponding companding quantiser (consisting of compressor and uniform quantiser). Output consists of the quantiser output O and the information about the selected quantiser (O_Q).

On the reception side, both information go to the decoder selector which sends the quantiser output O into the corresponding decoder (consisting of the uniform quantiser decoder and expandor), according to the information O_Q.

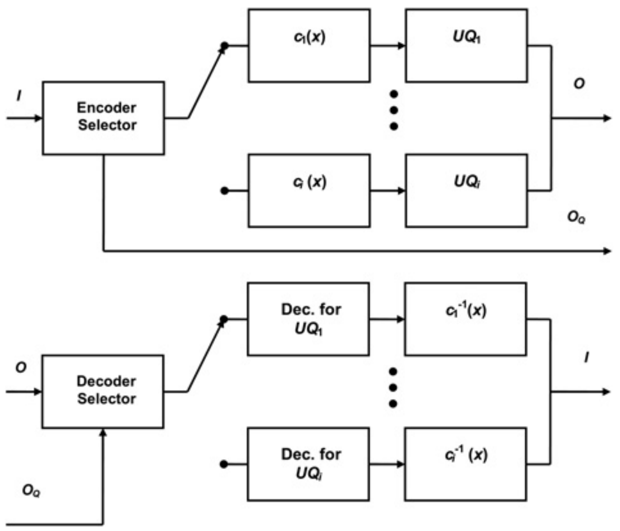

Fig. 1 *Block schemes of coder (upper scheme) and decoder (lower scheme), corresponding to two ($i = 2$) and three ($i = 3$) quantiser VLC model*

4 Numerical examples and optimisation

The value often used for description of the quality of quantiser is 'signal to quantiser noise ratio' (SQNR), defined by

$$\text{SQNR} = 10 \log\left(\frac{\sigma^2}{D}\right) \tag{24}$$

In this section we use value of SQNR to measure the performance of the quantisers, instead of the distortion D.

Numerical results corresponding to the first model (two quantisers) are shown in Fig. 2. We plotted the value of SQNR in relation to the average bit rate R for different values of N_1 and N_2. Every line is obtained as a parametric curve $[R(t_1), \text{SQNR}(t_1)]$. Range of parameter t_1 is chosen in the way that $R(t_1)$ goes from its minimum to its maximum value. Optimisation is performed by varying different values of k_1 and k_2 and computing SQNR where $R = R_0$ is fixed.

Fig. 3 shows the numerical results corresponding to the second model (three quantisers). Value of SQNR is plotted in relation to the average bit rate R for different numbers of representation levels N_1, N_2 and N_3. Optimal dependency is practically linear with slope value equal to six, that is, SQNR increase 6 dB per bit.

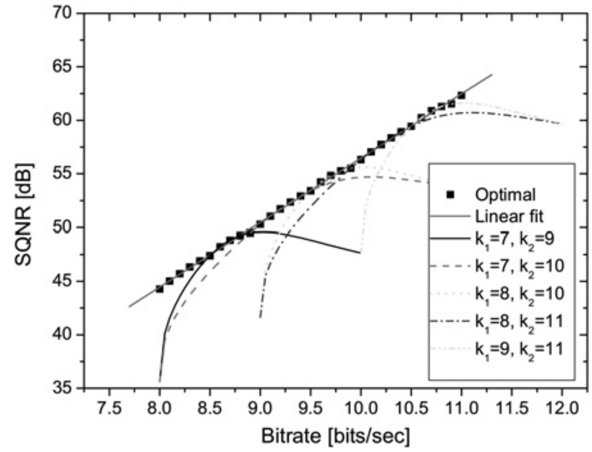

Fig. 2 *SQNR in relation to the average bit rate R for different numbers of quantisation points in each of two quantisers*

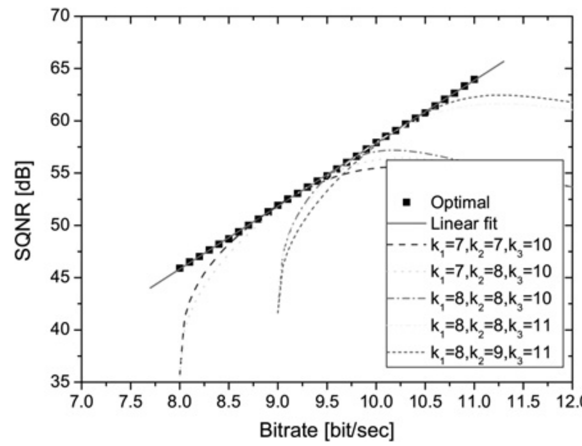

Fig. 3 *SQNR in relation to the average bit rate R for different numbers of quantisation points in each of three quantisers*

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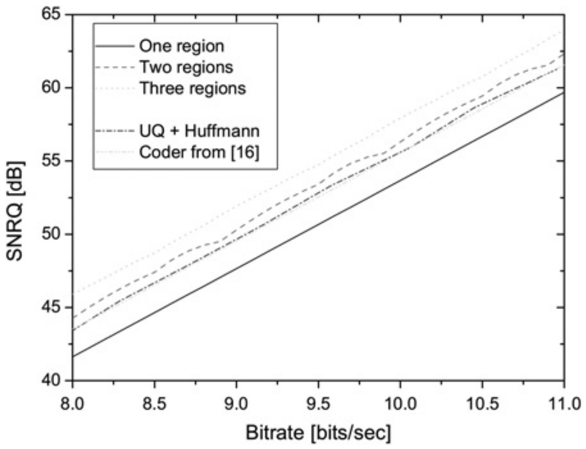

Fig. 4 *SQNR in relation to the average bit rate R for quantisation models with one, two, three regions, combination of the optimal uniform quantiser and Huffmann lossless coder and combination of optimal bounded companding quantiser and simple lossless coder [16]*

Comparison of quantisation models with one, two and three regions is given in Fig. 4. The increase in SQNR between methods with one and two regions is 2.8 dB, and between methods with one and three regions is 4.2 dB approximately, in favour of our multi-resolution scalar quantiser. We also made a comparison between our model and coder consisting of the optimal uniform quantiser and Huffmann lossless coder. We assume that the uniform quantiser $Q_u(x)$ is applied in the support region $[-t_{max}, t_{max}]$ where the bound t_{max} is optimised. From Fig. 4 we can observe that our method outperforms the well-known mentioned methods. Comparison is also made with the combination of optimal companding quantiser and simple lossless coder [16]. Our model has more complex quntiser, but simpler coder for two quantiser model reaches gain of 1.3 dB. Considering model with three regions we reach gain of 2.7 dB, with slightly more complex realisation structure.

5 Application in speech coding

We have tested our coding scheme on the speech coding. The sample signal is taken from the base which is derived from the TIMIT corpus [22]. The TIMIT corpus of speech has been designed to provide speech data for the acquisition of acoustic-phonetic knowledge and for the development and evaluation of automatic speech recognition systems.

For the purpose of testing, we consider the adaptive variant of our three quantiser VLC model. The block scheme is given in Fig. 5.

The original VLC model assumes that the input signal is Laplacian source with variance (power of the signal) equal to $\sigma = 1$. In general, speech signal can be approximated by Laplacian source with variable variance. Hence, we divide the input signal into the frames and for each frame we estimate the signal variance and normalise all samples before coding.

Consider the n samples of the input signal $x_1, x_2, \ldots, x_n$ and assume that signal samples are divided in F frames and each frame consist of M samples. Furthermore, denote by $x_{i,j}$ the jth sample of the ith frame ($i = 0, \ldots, F-1$ and $j = 0, \ldots, M-1$), that is, $x_{i,j} = x_{iM+j}$. In the ith frame signal variance is estimated using $\sigma_i^2 = (1/M)\sum_{j=0}^{M-1} x_{i,j}^2$. The source samples are then normalised and sent to the

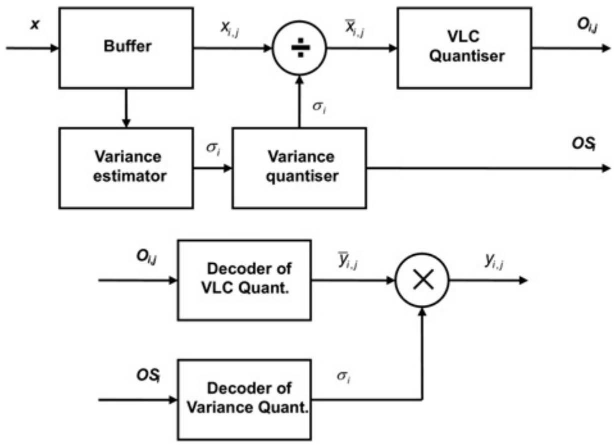

Fig. 5 *Block scheme of adaptive VLC quantiser: transmitter (upper scheme) and receiver (lower scheme)*

quantiser. When received, the signal has to be denormalised. For that purpose, we also need to transmit the signal variance σ_i. It is quantised using log-uniform quantiser with N_g levels and sent to the beginning of each frame. Other signal samples are normalised to $\bar{x}_{i,j} = x_{i,j}/\tilde{\sigma}_i$, where $\tilde{\sigma}_i$ is quantised signal variance, and then sent to the quantiser.

The representation levels and decision thresholds of log-uniform quantiser $Q_{lu}(\sigma)$ are defined as

$$\log(y_{lu,i}) = \log(\sigma_{min}) + \frac{2i-1}{2N_g}\log\frac{\sigma_{max}}{\sigma_{min}}, \quad i = 1, 2, \ldots, N_g$$

$$\log(t_{lu,i}) = \log(\sigma_{min}) + \frac{i}{N_g}\log\frac{\sigma_{max}}{\sigma_{min}}, \quad i = 0, 1, 2, \ldots, N_g$$

where σ_{min} and σ_{max} are, respectively, maximum and minimum possible values of the signal variance. In other words, log-uniform quantiser is the uniform quantiser in decibels scale. We used the dynamic range of the variance $(20\log(\sigma_{max}/\sigma_{min})\,\text{dB})$ of 40 dB and $N_g = 32$ levels of log-uniform quantiser. In Fig. 6, we show the SQNR value as the function of σ for adaptive variant of three quantiser VLC model and different values of N_g.

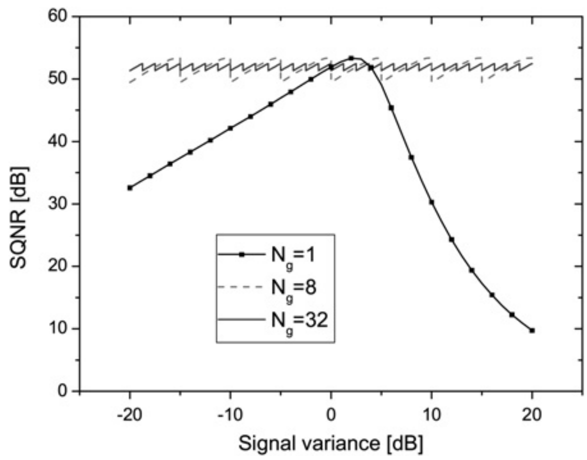

Fig. 6 *Theoretical dependence of the SQNR value as a function of signal variance σ (in decibels) for non-adaptive ($N_g = 1$) and adaptive ($N_g = 8, 32$) three quantiser VLC model*

Parameters of VLC model are equal to $k_1 = k_2 = 7$, $k_3 = 10$, $t_1 = 0.574$ and $t_2 = 1.192$

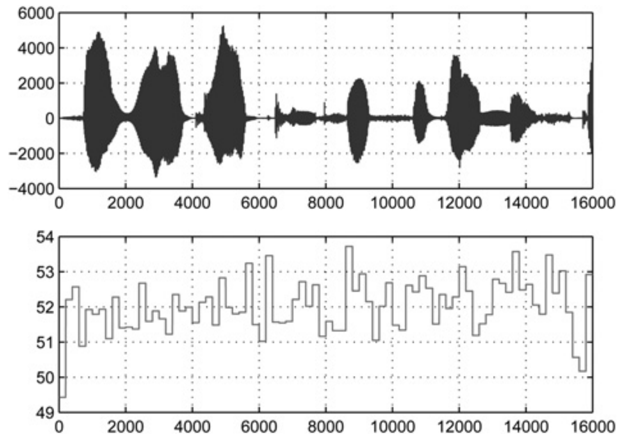

Fig. 7 *Input signal (upper graph) and experimental value of SQNR (lower graph) for the optimal three quantiser VLC model with R = 9*

Table 2 Comparison between theoretical and experimental value of SQNR of optimal three quantiser VLC model, for different values of bitrate *R*

R	k_1	k_2	k_3	t_1	t_2	SQNR	SQNR$^{\text{ex}}$
11	9	10	12	0.636	1.677	63.7116	63.7187
10.5	8	9	11	0.419	1.148	60.754	61.1961
10	8	8	10	0.467	1.004	57.0683	57.3965
9.5	7	8	10	0.419	1.148	54.7334	55.1332
9	7	7	10	0.574	1.192	51.9223	52.0192

Value $N_g = 1$ corresponds to the non-adaptive case, that is, when input signal goes directly to quantiser. Note that SQNR value is not attending its maximum at point $\sigma = 0$ dB (it is attending at $\sigma^* = 2.45$ dB). However, varying σ violates the condition $R = R_0$, that is, bit rate R is also changing. Therefore we have to adapt variance to initial value $\sigma = 0$ dB ($\sigma = 1$). As it can be seen in Fig. 6, SQNR value is almost constant for $N_g = 32$. Hence $N_g = 32$ is good choice for the number of levels of variance quantiser.

For the purpose of the experiment, we choose the frame size $M = 200$ and total $F = 800$ frames. We determine the experimental value $\text{SQNR}_i^{\text{ex}}$ for each frame $i = 0, 1, \ldots, F-1$. In Fig. 7 we show the input signal (upper graph) and $\text{SQNR}_i^{\text{ex}}$ values (lower graph). Experiment is done for $R = 9$ and the corresponding optimal three quantiser VLC model (parameters are $k_1 = k_2 = 7$, $k_3 = 10$, $t_1 = 0.574$ and $t_2 = 1.192$).

Note that the x scale on lower graph still represents the index of the sample (not the frame). The average SQNR value of all frames is equal to $\text{SQNR}^{\text{ex}} = (1/F)\sum_{i=0}^{F-1}\text{SQNR}_i^{\text{ex}}$. In our case it follows that $\text{SQNR}^{\text{ex}} = 52.0192$ and theoretical value is $\text{SQNR} = 51.9223$ (from Figs. 3 and 4). Hence, we obtain good agreement between theory and experiment in this case. We compared theoretical and experimental values of SQNR for the several different values of the bitrate R. As it can be seen from Table 2, there is a good agreement between theory and experiment in all cases.

6 Conclusion

This paper provides the simple structure coder for memoryless Laplacian source. We have used the companding model based on the two companding quantisers in the first case, and then the three companding quantisers in the second case, with the different number of representation levels and different compressor functions. There are analytical estimates of the distortion, average bit rate and signal to quantisation noise ratio derived. We have also performed the R–D optimisation for both two and three quantisers cases. Generally our method gives a very simple realisation structure and performances close to optimal ones and hence it is very useful in practical applications, such as speech signals, images etc. That is the main advantage of our model. We have also made a comparison between our model and coder consisting of the uniform quantiser (with optimal support range) and Huffmann lossless coder second comparison has bean made between our model and combination of optimal bounded companding quantiser and simple lossless coder. It is shown that our coder presents better results, with much simple and efficient realisation structure. Theoretical results are verified by the experiment on the sample speech signal. The above discussion points to the fact that our method outperforms well-known mentioned methods.

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