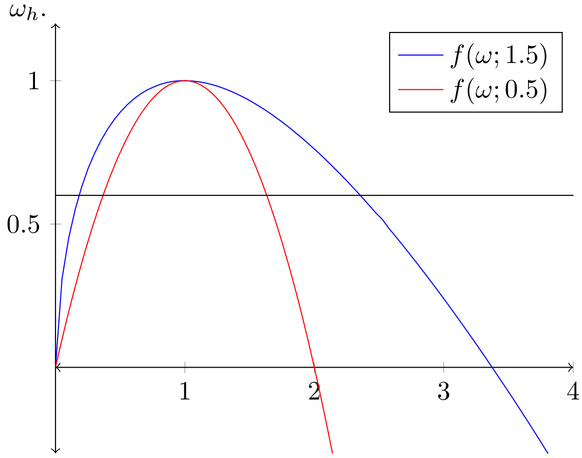
Utility
$$U = \frac{\theta x^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}} - px + b(E) + \delta + u$$
FOC
$$\theta x^{*-\frac{1}{\varepsilon}} = p \text{ or } x^* = p^{-\varepsilon}\theta^{\varepsilon} \text{ or } \ln x^* = -\varepsilon \ln p + \varepsilon \ln \theta$$
Value
$$U^* = \frac{p^{1-\varepsilon}\theta^{\varepsilon}}{\varepsilon - 1} + \delta + u$$
Expenditure $E = px$
$$U(E) = \frac{\theta (E/p)^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}} - E + b(E) + \delta + u$$
$$U(E) > U^* \Leftrightarrow \frac{\theta (E/p)^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}} - E + b(E) > \frac{p^{1-\varepsilon}\theta^{\varepsilon}}{\varepsilon - 1}$$
Let $\omega = \theta^{\varepsilon} p^{1-\varepsilon}/E$, then
$$U(E) > U^* \Leftrightarrow \frac{\omega^{\frac{1}{\varepsilon}} E}{1-\frac{1}{\varepsilon}} - E + b(E) > \frac{\omega E}{\varepsilon - 1}$$
or
$$U(E) > U^* \Leftrightarrow \frac{\varepsilon \omega^{\frac{1}{\varepsilon}} - \omega}{\varepsilon - 1} > 1 - \frac{b(E)}{E}$$
$f(\omega;\varepsilon) = \frac{\varepsilon \omega^{\frac{1}{\varepsilon}} - \omega}{\varepsilon - 1}$ is concave with maximum at $\omega = 1$ and 2 roots at $\omega = 0$ and $\omega = \varepsilon^{\frac{\varepsilon}{\varepsilon - 1}}$. The equation $f(\omega;\varepsilon) = 1 - b$ thus have exactly 2 solutions ω_l and ω_h.



To solve for $f(\omega;\varepsilon) - (1-b) = 0$, can use Newton's method
$$\omega' = \omega - \frac{\varepsilon \omega^{\frac{1}{\varepsilon}} - \omega + (1-\varepsilon)(1-b)}{\omega^{\frac{1}{\varepsilon}-1} - 1} = \frac{(1-\varepsilon)(\omega^{\frac{1}{\varepsilon}} - 1 + b)}{\omega^{\frac{1}{\varepsilon}-1} - 1}$$
Choice between two different expenditures:

$$U(E) > U(E') \Leftrightarrow \frac{\theta(E/p)^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}} - E + b(E) > \frac{\theta(E'/p)^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}} - E' + b(E')$$

or

$$\frac{\theta p^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}}\left(E^{1-\frac{1}{\varepsilon}} - E'^{1-\frac{1}{\varepsilon}}\right) > E - E' - b(E) + b(E')$$

or

$$\frac{\varepsilon\omega^{\frac{1}{\varepsilon}}}{\varepsilon-1}\left(1-\left(\frac{E'}{E}\right)^{1-\frac{1}{\varepsilon}}\right) > 1 - \frac{E'}{E} - b + b'\frac{E'}{E}$$

When consumers choose a "non-optimal" quantity of choice j, the likelihood is

$$\begin{aligned} l &= Pr\left(U_j(E) > U_j^* \wedge U_j(E) > \max(U_k(E), U_k^*)\right) \\ &= Pr(U_j(E) > U_j^*) \times Pr\left(U_j(E) > \max(U_k(E), U_k^*) \mid U_j(E) > U_j^*\right) \end{aligned}$$

The condition $U_j(E) > U_j^*$ is equivalent to $\omega \in [\omega_l, \omega_h]$. Thus, the probability $Pr\left(U_j(E) > \max(U_k(E), U_k^*) \mid U_j(E) > U_j^*\right)$ can be estimated with simulation by drawing from the truncated distribution of ω.

When consumers choose the optimal quantity instead:

$$\begin{aligned} l &= Pr\left(E_j^* \wedge U^* > U_j(E) \wedge U^* > \max(U_k(E), U_k^*)\right) \\ &= Pr(E_j^* \wedge U^* > U_j(E)) \times Pr\left(U_j(E) > \max(U_k(E), U_k^*) \mid E_j^* \wedge U^* > U_j(E)\right) \end{aligned}$$

Draw r: (p_{jr}, θ_{jr})