UNIT-01

> Definition:

An equation involving derivaties of one or more dependent variable with respect one or more independent variable is called Differential equation.

Type of D.Eqn

→ ① Ordinary Differential eqn: A differential eqn is said to be ordinary the equation consisting of only one independent variable

Eg: $\left(\frac{d^2y}{dx^2}\right)^1 + \left(\frac{dy}{dx}\right)^5 + 3y = 0$

$$\left(\frac{d^3y}{dx^3}\right)^2 + 4\left(\frac{dy}{dx}\right)^5 + 5y = 0$$

→ ② Partial Differential equation:- A Differential eqn is said to be partial if the eqn consisting more than one independent variable.

Eg:- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial z} = 0$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is called two dimension Laplace eqn.

→ **Order**:- The highest derivative in the given differential is called order

→ **Degree**:- The power of highest derivative in the given differential eqn is called Degree.

$\left(\frac{\partial^2 y}{\partial x^2}\right)^1 + \left(\frac{\partial y}{\partial x}\right)^5 + 3y = 0$ order = 2, Degree - 1

$\left(\frac{\partial^3 y}{\partial x^3}\right)^2 + 4\left(\frac{\partial y}{\partial x}\right)^5 = 0$ order - 3, degree - 2

$\left(\frac{\partial x}{\partial x}\right) + \sqrt{\left(\frac{\partial^2 y}{\partial x^2}\right)^2} + 2y = 0$ order - 2, Degree - 1.

① Find the order and degree of the given differential eq^n $y = x\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Sol:- Given

$$y - x\frac{dy}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

S.O.B.S

$$\left(y - x\frac{dy}{dx}\right)^2 = 1+\left(\frac{dy}{dx}\right)^2$$

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = 1+\left(\frac{dy}{dx}\right)^2$$

$$y^2 - 2xy\frac{dy}{dx} - 1 = \left(\frac{dy}{dx}\right)^2\left[1 - x^2\right]$$

order - 1

Degree - 2

→ Solution of D.Eq^n :- Any relation between the dependent and independent variable not containing there derivatives which satisfies the given Differential equation is called solution of D.Eq^n

Eg:- $y = A\cos x + B\sin x$

$$\frac{d^2y}{dx^2} + y = 0$$

① Form the differential equation by eliminating arbitary constant $y = Ae^{x} + Be^{-x}$ where a and b are arbitary constant.

Sol:- Given $y = Ae^{x} + Be^{-x}$ — ①

$\boxed{y - Ae^{x} - Be^{-x}}$

we have 2 constante

The Differention is 2 time

Differentiating w.r.to x, we get

$$\frac{d}{dx}(y) = \frac{d}{dx}(Ae^{x} + Be^{-x})$$

$$= A\cdot e^{x} + B(-1)e^{-x}$$

$$\frac{dy}{dx} = Ae^{x} - Be^{-x}.$$

Again Diff w.r.to x, we get to eqⁿ ②

$$\frac{\partial^2 y}{\partial x^2} = \frac{d}{dx}(Ae^{x} - Be^{-x})$$

$$\frac{\partial^2 y}{\partial x^2} = Ae^{x} - (-1)Be^{-x}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = Ae^{x} + Be^{-x}} \quad \text{--- (3)}$$

put eq^n (1) in eq^n (3)

$$\boxed{\frac{\partial^2 y}{\partial x^2} = y.}$$

$$\frac{\partial^2 y}{\partial x^2} \Rightarrow \boxed{y = Ae^{x} + Be^{-x}}$$

(2) Form differential eq^n by eleminating orbitary constents $y = e^{x}[A\cos x + B\sin x]$

Sol: The given eq^n is $y = e^{x}[A\cos x + B\sin x]$ ← (1)

$$y - e^{x}[A\cos x + B\sin x] = 0$$

differenting eq^n (1) with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}[e^{x}(A\cos x + B\sin x)]$$

$$= e^{x}[-A\sin x + B\cos x] + [A\cos x + B\sin x]e^{x}$$

put eq^n (1) in above.

$$\frac{dy}{dx} = e^x[-A\sin x + B\cos x] + y$$

$$\frac{dy}{dx} - y = e^x[-A\sin x + B\cos x] \quad -②$$

diff w.r.t eqⁿ ③ to x

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x[-A\cos x + B(-\sin x)]$$
$$+ [A\sin x + B\cos x]$$

put eqⁿ ② in bove

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -e^x[A\cos x + B\sin x] +$$
$$\frac{dy}{dx} - y$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -y + \frac{dy}{dx} - y$$

$$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0,$$

which is required Differential equation

③ form the D.Eqn by eliminating orbitary Constants of the eqn $\sin^{-1}x + \sin^{-1}y = c$. where 'c' is orbitary constant.

Sol:- Given soln we have only one constant

Diff only one time.

$$\sin^{-1}x + \sin^{-1}y = c \quad \text{—} ①$$

diff w.r.to 'x' in eqn ①

$$\frac{d}{dx}\left[\sin^{-1}x + \sin^{-1}y\right] = 0$$

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\frac{1}{\sqrt{1-y^2}} + \frac{1}{\sqrt{1-x^2}} = 0$$

∴ which is required soln.

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1) Variable Separable :-

In variable separable separating the variables as x variables and y variables then apply integration

→ Solve above integration we get regular soln.

that soln consisting of consisting of constants and expaling variables

problems

(1) Solve the given differential eqn $\frac{dy}{dx} + \sqrt{\frac{1+y^2}{1+x^2}} = 0$

Sol:-
$$\frac{dy}{dx} = \frac{-\sqrt{1+y^2}}{\sqrt{1+x^2}}$$

$$dy = \frac{-\sqrt{1+y^2}}{\sqrt{1+x^2}}\, dx$$

$$\frac{dy}{\sqrt{1+y^2}} = -\frac{dx}{\sqrt{1+x^2}}$$

Applying integration on b.s

$$\int \frac{1}{\sqrt{1+y^2}}\, dy = -\int \frac{1}{\sqrt{1+x^2}}\, dx + C$$

$$\sinh^{-1} y = -\sinh^{-1}(x) + C$$

$$\therefore \boxed{\sinh^{-1}(y) + \sinh^{-1}(x) = C}$$

Required soln

② Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

Sol:- $\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$

$= \frac{e^x}{e^y} + \frac{x^2}{e^y}$

$\frac{dy}{dx} = \frac{e^x + x^2}{e^y}$.

$e^y dy = (e^x + x^2) dx$

$\Rightarrow \int e^y dy = \int (e^x + x^2) dx$.

$e^y = \int e^x dx + \int x^2 dx + C$

$e^y = e^x + \frac{x^3}{3} + C$

$\boxed{e^y - e^x - \frac{x^3}{3} = C}$

③ Solve $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$.

Sol:- Let $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$

$(e^y + 1) \cos x \, dx = -e^y \sin x \, dy$.

$\frac{e^y (e^y + 1) \cos x}{dy} = \sin x \, dx$.

$$\frac{\cos x}{\sin x}\,dx = \frac{-e^y}{(e^y+1)}\,dy.$$

Apply $\int$ on b.s.

$$\int\frac{\cos x}{\sin x}\,dx = -\int\frac{e^y}{(e^y+1)}\,dy + c$$

$$\Rightarrow \log(\sin x) = -\log(e^y+1)+c \qquad \int\frac{f'(x)}{f(x)} = \log f(x)$$

$$\Rightarrow \log(\sin x)+\log(e^y+1) = \log c$$

$$\log[\sin x(e^y+1)] = \log c$$

$$\therefore \boxed{\sin x(e^y+1) = c}$$

④ $[y-yx]\,dx+(x+xy)\,dy = 0$

Sol:- $y(1-x)\,dx + x(1+y)\,dy = 0$

$$\int\frac{(1-x)}{x}\,dx = \int-\frac{(1+y)}{y}\,dy$$

$$\log x \quad \frac{y}{1+y}\,dy = -\frac{x}{1-x}\,dx$$

$$-\log x\ \frac{x^2}{2} = -\log y\ \frac{y^2}{2}+c$$

$$\log x\ \frac{x^2}{2} = -\log\frac{y^2}{2}+c.$$

$$\int \frac{1}{x} dx - \int 1\, dx = -\int \frac{1}{y} dy - \int 1\, dy + c$$

$$\boxed{\log x - x = -\log y - y + c}$$

V.Imp

Exact Differential Equations

$$M\,dx + N\,dy = 0$$

d.m.

Process :- The given differential eq^n is of the form $\boxed{M\,dx + N\,dy = 0}$

2) Check the exactness condition of the given diff eq^n i.e $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ is satisfied.

3) Find the general sol^n

$$\boxed{\underbrace{\int M\,dx}_{y\text{-constant}} + \underbrace{\int N\,dy}_{\text{free of } x\text{-terms}} = c}$$

① Solve $(2x-y+1)\,dx+(2y-x-1)\,dy=0$

Sol:- The given differential is of the form

$$M\,dx+N\,dy=0$$

Here

$M=(2x-y+1)$ & $N=2y-x-1$

$$\boxed{\frac{\partial M}{\partial y}=-1} \qquad \boxed{\frac{\partial N}{\partial x}=-1}$$

The $\boxed{\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=-1}$

The given D.eqⁿ is Exact differe eqⁿ

G.S. ⇒

$$\int_{y\text{-const}} M\,dx+\int_{\text{free of }x\text{-terms}} N\,dy=C$$

$N=2y-\cancel{x}-1$

$\boxed{N=2y-1}$

$$\int(2x-y+1)\,dx+\int(2y-1)\,dy=C$$

$$\int 2x\,dx-\int y\,dx+\int dx+2\int y\,d\ldots$$

$$\cancel{2}\frac{x^2}{\cancel{2}}-\frac{y^2(x)}{2}+x+\cancel{2}\frac{y^2}{\cancel{2}}-y=2C$$

$$\boxed{x^2-y(x)+x+y^2)-2y=2C}$$

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(2). Solve $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$

Sol:- ~~There~~ given differential eq^n is in the form $Mdx + Ndy = 0$

Here $M = (e^y+1)\cos x$ & $N = e^y \sin x$

$\frac{\partial M}{\partial y} = -(e^y+1)\sin x.$ $\frac{\partial N}{\partial x} = e^y \cos x$

$= \cancel{-(e^y+1)\sin x + \cos x(0)}$

$= \cancel{-(e^y+1)\sin x}.\ e^y \cos x.$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = e^y \cos x}$

The given Diff eq^n is ~~not~~ exactness eq^n so, the

G. S

$\Rightarrow \underset{y\ \text{const}}{\int M dx} + \underset{\text{free of } x\text{-terms}}{\int N dy} = C$

$N = e^y \sin x$

$\Rightarrow \int (e^y+1)\cos x + \int 0\, dy = C.$ $N = 0$

$(e^y+1)\int \cos x + 0 = C. \Rightarrow \sin x.(e^y+1) = C.$

$\cancel{\sin x\,(e^y+1) = C.}$ $\boxed{(e^y+1)\sin x = C}$

$\sin x = C$

H.W

(3) solve $(x+y-1)\,dy - (x-y+2)\,dx = 0$

sol:- The given differential eqn is in the form $M\,dx + N\,dy = 0$.

Here $M = -(x-y+2)$ & $N = (x+y-1)$

$\frac{\partial M}{\partial y} = +1 \qquad \frac{\partial N}{\partial x} = 1$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 1}$$

G.S $\Rightarrow \int M\,dx + \int N\,dy = c$

y - const free of x-terms

$$\boxed{N = y-1.}$$

$\Rightarrow \int -(x-y+2)\,dx + \int (y-1)\,dy = c$

$\Rightarrow -\frac{x^2}{2} + yx - 2x + \frac{y^2}{2} - y = c$

$\Rightarrow \frac{y^2}{2} - \frac{x^2}{2} + yx - 2x - y = c.$

$y^2 - x^2 + 2yx - 4x - 2y = 2c.$

$$\boxed{y^2 - x^2 + 2x(y-2) - 2y = 2c}$$

V.Imp

(4) Solve $x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x$.

Sol^n: $x^3 \sec^2 y \frac{dy}{dx} = (-3x^2 \tan y + \cos x)$

$x^3 \sec^2 y \, dy = (\cos x - 3x^2 \tan y) dx$.

$x^3 \sec^2 y \, dy - (\cos x - 3x^2 \tan y) dx = 0$

This is in the form $M dx + N dy = 0$

Here $M = -\cos x + 3x^2 \tan y$ & $N = x^3 \sec^2 y$.

$\frac{\partial M}{\partial y} = 0 + 3x^2 \sec^2 y$. $\frac{\partial N}{\partial x} = \sec^2 y (3x^2)$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3x^2 \sec^2 y}$$

G.S $\Rightarrow \int M dx + \int N dy = 0$

y const   free of x term

$N = x^3 \sec^2 y$

$\int (-\cos x + 3x^2 \tan y) dx + \int 0 dy = c$ $N = 0$

$\Rightarrow -\int \cos x \, dx + 3 \tan y \int x^2 dx = c$

$-(\sin x) + \cancel{3} \tan y \frac{x^3}{\cancel{3}} = c$.

$\boxed{-\sin x + \tan y (x^3) = c}$

⑤ Solve $\left(xe^{xy}+2y\right)\frac{dy}{dx}+ye^{xy}=0$.

Sol:- $\left(xe^{xy}+2y\right)\frac{dy}{dx}+ye^{xy}dx=0$.

~~$xe^{xy}+2y$~~

This is in the form $M\,dx+N\,dy=0$

$M=ye^{xy}$ & $N=xe^{xy}+2y$.

$$\frac{\partial M}{\partial y}=y\,e^{xy}(x)+e^{xy},\qquad \frac{\partial N}{\partial x}=xe^{xy}y+e^{xy}.$$

$$=e^{xy}+xye^{xy}\qquad =xe^{xy}y+e^{xy}.$$

$$=e^{xy}(xy+1)\qquad =e^{xy}(xy+1)$$

$$\boxed{\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=e^{xy}(xy+1)}$$

G.S $\Rightarrow \int M\,dx+\int N\,dy=0$

$N=2y$.

$$\int(ye^{xy})+\int 2y\,dy=C$$

$$y\frac{e^{xy}}{y}+2\left(\frac{y^2}{2}\right)=C.$$

$$\boxed{e^{xy}+y^2=C}$$

NOT EXACT DIFFERENTION Eqⁿ

→ Eqⁿ redusable to exact differential eqⁿ

process :-

1) write the given differential eqⁿ is of the form $Mdx + Ndy = 0$

2) The exactness condition is not satisfied

i.e $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

3) Find Integrating factor and multiply integrating factor to the given differential eqⁿ we get exact differential eqⁿ.

4) find the general solⁿ of exact diff eqⁿ.

Method-1 :-

To find an integrating factor of $Mdx + Ndy = 0$

$$I.F = \frac{1}{Mx + Ny}$$ (Homogeneous diff. eqⁿ)

Note: The given diff eqⁿ $Mdx + Ndy = 0$ is not exact and which is homogeneous differential eqⁿ then use $I.F = \frac{1}{Mx+Ny}$

Q1 Solve the given diff eq^n $x^2 dy + (y^2 - xy) dx = 0$

Sol:- This is in the form $M dx + N dy = 0$

Here $M = (y^2 - xy)$ & $N = x^2$.

Now $\frac{\partial M}{\partial y} = 2y - x$ & $\frac{\partial N}{\partial x} = 2x$.

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Now I.F is the given diff eq^n is homogeneous then $I.F = \frac{1}{Mx + Ny}$.

$$I.F = \frac{1}{(y^2 - xy)x + (x^2)y} = \frac{1}{xy^2}$$

multiply I.F with given diff then.

$$(1) \Rightarrow \frac{1}{xy^2} \times x^2 dy + (y^2 - xy) dx = 0$$

$$\Rightarrow \frac{x}{y^2} dy + \frac{(y^2 - xy)}{xy^2} \cdot dx = 0$$

$$\Rightarrow \frac{x}{y^2} dy + \left(\frac{y}{x} - \frac{1}{y}\right) dx = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = \left(\frac{1}{x} - \frac{1}{y}\right)$$

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$$\frac{\partial M_1}{\partial y} = 0 + \left(\frac{+1}{y^2}\right) \quad \frac{\partial N_1}{\partial x} = \frac{+1}{y^2}$$

$$\boxed{\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}} \quad - (2)$$

This is exact diff eq^n

$$\underset{y\text{-const}}{\int M_1\, dx} + \underset{\text{free of } x \text{ terms}}{\int N_1\, dy} = 0$$

$N_1 = 0$.

$$\Rightarrow \int \left(\frac{1}{x} - \frac{1}{y}\right) dx + 0 = C.$$

$$\boxed{\log x - \frac{x}{y} = C}$$

H.W

(2) Solve $x^2 y\, dx - (x^3 + y^3)\, dy = 0$

Sol :- This is in the form $M dx + N dy = 0$

$$M = x^2 y, \quad N = -(x^3 + y^3)$$

$$\frac{\partial M}{\partial y} = x^2 (1), \quad \frac{\partial N}{\partial x} = -3x^2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

$$I \cdot F = \frac{1}{(x^2 y)x + x^3 y - y^5 y} = \frac{-1}{y^4}$$

The diff eq^n = I.F × given diff eq^n

$$= \frac{-1}{y^4} x^2 y dx + \frac{(x^3+y^3)}{y^4} dy$$

$$= \frac{-x^2}{y^3} dx + \frac{x^3}{y^4} dy + \frac{y^3}{y^4} dy.$$

$$= \frac{-x^3}{y^3} dx + \left(\frac{x^3}{y^4} + \frac{1}{y}\right) dy.$$

$\Rightarrow \int M_1 dx + \int N_1 dy.$ $\quad M_1 = -\frac{x^3}{y^3}$

$$\frac{\partial M_1}{\partial y} = -x^3 \frac{1}{3y^2} \qquad \frac{\partial N_1}{\partial x} = \frac{-1}{y^3}(3x^2)$$

$$= -\frac{x^3}{3y}\left(\frac{-1}{y^2}\right) \qquad \left(\frac{x^3}{y^4} + \frac{1}{y}\right)$$

$$= \frac{x^3}{3y^3} \qquad \frac{3x^2}{y^4}$$

$$\underbrace{\int M_1 dx}_{y\text{-const}} + \underbrace{\int N_1 dy}_{\text{free of } x\text{-terms}} = 0$$

$$-\frac{1}{y^3}\int x^3 dx + x^3 \int \frac{1}{y^4} + \int \frac{1}{y} dy = C$$

$$\Rightarrow -\frac{1}{y^3}\frac{x^4}{4} + x^3 \frac{y^{-3}}{-3} + \log(y) = C.$$

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$$M_1 = -\frac{x^2}{y^3}, \quad N_1 = \frac{x^3}{y^4} + \frac{1}{y}.$$

$$\frac{\partial M_1}{\partial y} = -x^2\left(\frac{\partial}{\partial y} y^{-3}\right) \qquad \frac{\partial N}{\partial x} = \frac{1}{y^4} 3x^2$$

$$= 3x^2 y^{-4} \qquad \frac{3x^2}{y^4}$$

$$\boxed{\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}}$$

G.S $\Rightarrow \int M dx + \int N dy = c$

$$-\int \frac{x^2}{y^3} dx + \int \left(\frac{1}{y}\right) dy = c$$

$$-\frac{1}{y^3}(x^2) + \log y = c$$

$$\boxed{-\frac{1}{y^3}\left(\frac{x^3}{3}\right) + \log y = c}$$

H.W

(3) $(x^2+y^2)dx - 2xy\, dy = 0$

(4) $xy\,dx - (x^2+2y^2)dy = 0$

(5)

Method-2 :-

01/02/2020

To find integrating factor of $Mdx+Ndy=0$.

$$I.F = \frac{1}{Mx-Ny}$$

Note:- The given diff eq^n is not exact and that is non Homogoneous then find integrating factor of $mdx+Ndy=0$

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(1). Solve $y(x^2y^2+2).dx + x(2-2x^2y^2)dy=0$. — (1)

Sol:- It is in the form $mdx+Ndy=0$.

Here $M = x^2y^3+2y$ & $N = 2x-2x^3y^2$.

$$\frac{\partial M}{\partial y} = x^2 3y^2+2. \qquad \frac{\partial N}{\partial x} = 2-6x^2y^2.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$I.F = \frac{1}{x^3y^3+\cancel{2xy}-\cancel{2xy}+2x^3y^3}$$

$$\boxed{I.F = \frac{1}{3x^3y^3}}$$

multiply I.f to eq^n (1)

$$(1) \Rightarrow \frac{\cancel{y}(x^2y^2+2)}{3x^3y^{\cancel{3}2}}\,dx + \frac{\cancel{x}(2-2x^2y^2)}{3x^{\cancel{3}2}y^3}\,dy = 0$$

$$\Rightarrow \left(\frac{\cancel{x^2y^2}}{3x^{\cancel{3}}\cancel{y^2}} + \frac{2}{3x^3y^2}\right)dx + \left(\frac{2}{3x^2y^3} - \frac{2x^2y^2}{3x^2y^3}\right)dy = 0$$

$$\left(\frac{1}{3x} + \frac{2}{3x^3y^2}\right)dx + \left(\frac{2}{3x^2y^3} - \frac{2}{3y}\right)dy = 0$$

$$\boxed{M_1\,dx + N_1\,dy = 0}$$

$$M_1 = \frac{1}{3x} + \frac{2}{3x^3y^2}, \quad N_1 = \frac{2}{3x^2y^3} - \frac{2}{3y}.$$

$$\frac{\partial M_1}{\partial y} = 0 + \frac{2}{3x^3}\left(\frac{-2}{y^3}\right), \quad \frac{\partial N_1}{\partial x} = \frac{2}{3y^3}\left(\frac{-1}{x^2}\right) - 0$$

$$= \frac{-2}{3}$$

$$\boxed{\frac{\partial M_1}{\partial y} = \frac{-4}{3x^2y^3}}, \quad \boxed{\frac{\partial N_1}{\partial x} = \frac{-4}{3x^2y^3}}$$

Eqn (2) is.

$$G.S \Rightarrow \underset{y\text{-const}}{\int M_1\,dx} + \underset{\text{free of } x\text{-term}}{\int N_1\,dy} = C. \qquad \boxed{N_1 = \frac{-2}{3y}}$$

$$\int\left(\frac{1}{3x} + \frac{2}{3x^3y^2}\right)dx + \int\left(\frac{-2}{3y}\right)dy = c$$

$$\frac{1}{3}\int\left(\frac{1}{x}\right)dx + \frac{2}{3y^2}\int x^{-3}\,dx - \frac{2}{3}\int\frac{1}{y}\,dy = C$$

$$\frac{\log x}{3} \quad \frac{\cancel{\log x}}{3\,\cancel{\log x}} + \frac{2}{3y^2}\left(\frac{x^{-2}}{-2}\right) - \frac{2}{3}\log y = C.$$

v. IMP
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(2) Solve diff eq^n $y(1+xy)dx + x(1-xy)dy = 0$ → (1)

Sol:- It is in the form $Mdx + Ndy = 0$.

Here $M = y + xy^2$ & $N = x - x^2y$

$$\boxed{\frac{\partial M}{\partial y} = 1 + 2xy} \neq \boxed{\frac{\partial N}{\partial x} = 1 - 2xy.}$$

x^n
nx^{n-1}

$-2(\frac{-1}{y^2})$ Now eq^n (1) is not exact diff. eq^n.

y^{-2}
$-2 \cdot y^{-1}$
$\frac{-2 \cdot y}{y}$ Now I.F is The given is non-homogeneous

$$\therefore I \cdot F = \frac{1}{Mx - Ny} = \frac{1}{xy + x^2y^2 - xy + x^2y^2}$$

$$= \frac{1}{2x^2y^2}.$$

Multiply I.F to the given diff eq^n.

$$(1) \Rightarrow \frac{\cancel{y}(1+xy)}{2x^2y^{\cancel{2}}}dx + \frac{\cancel{x}(1-xy)}{2x^{\cancel{2}}y^2}dy = 0$$

$$\left(\frac{1}{2x^2y} + \frac{xy}{2x^2y}\right)dx + \left(\frac{1}{2xy^2} - \frac{xy}{2xy^2}\right)dy = 0$$

$$\left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \left(\frac{1}{2xy^2} - \frac{1}{2y}\right)dy = 0$$

$M_1 dx + N_1 dy = 0$ → (2)

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$M_1 = \frac{1}{2x^2y} + \frac{1}{2x}$ $\quad N_1 = \frac{1}{2xy^2} - \frac{1}{2y}$.

$\frac{\partial M_1}{\partial y} = \frac{1}{2x^2}\left(\frac{-1}{y^2}\right)$ $\quad \frac{\partial N_1}{\partial x} = \frac{1}{2y^2}\left(\frac{-1}{x^2}\right)$

$$\boxed{\frac{\partial M_1}{\partial y} = \frac{-1}{2x^2y^2}} = \boxed{\frac{\partial N_1}{\partial x} = \frac{-1}{2x^2y^2}}$$

$\therefore$ Eq (2) is Exact diff eq^n.

G.S $\rightarrow$ $\int M_1 dx + \int N_1 dy = 0$ $\quad N_1 = \frac{-1}{2y}$.

y-const $\quad$ free of x

$$\int \left(\frac{1}{2x^2y} + \frac{1}{2x}\right) dx + \int \left(\cancel{\frac{1}{2xy^2}} - \frac{1}{2y}\right) dy = 0$$

N

$$\frac{1}{2y}\int x^{-2} dx + \frac{1}{2}\int \frac{1}{x} dx - \frac{1}{2}\int \frac{1}{y} dy = 0$$

$$\boxed{\frac{1}{2y}\left(\frac{x^{-1}}{-1}\right) + \frac{1}{2}(\log x) - \frac{1}{2}(\log y) = C}$$

H.W

(3) $(y - xy^2)dx - (x + x^2y)dy = 0$

(4) $y(1 - xy)dx - x(1 + xy)dy = 0$

(5) $(2xy + 1)y\,dx + (1 + 2xy - x^3y^3)x\,dy = 0$

Method - III

03/02/2020

To find integrating factor of $Mdx+Ndy=0$

$\Rightarrow$ $I.F = e^{\int f(x)dx}$

where $f(x) = \frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]$

Method - IV

To find an integrating factor of $Mdx+Ndy=0$

$\Rightarrow$ $I.F = e^{\int g(y)dy}$

where $g(y) = \frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right]$

M-III

1. Solve $2xy\,dy - (x^2+y^2+1)dx = 0$. —(1)

Sol:- It is in the form $Mdx+Ndy=0$

Here $M = -(x^2+y^2+1)$ & $N = 2xy$.

$\boxed{\frac{\partial M}{\partial y} = -2y} \neq \boxed{\frac{\partial N}{\partial x} = 2y}$

eqn (1) is not exact diff eqn.

Now I.F :- Choosing $I.F = e^{\int f(x)dx}$

where $f(x) = \frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]$

$$f(x) = \frac{1}{2xy}\left[-2y-2y\right]$$

$$f(x) = \frac{-\cancel{4}^{2}\cancel{y}}{\cancel{2}x\cancel{y}} = -\frac{2}{x}$$

Now $I.f = e^{-2\int \frac{1}{x}dx}$

$$= e^{-2\log_e x}$$

$$= e^{\log_e x^{-2}} \Rightarrow x^{-2}$$

$$I.F. = \frac{1}{x^2}$$

Multiply I.F to eqn ①.

$$① \Rightarrow \frac{2xy}{\cancel{x^2}}\,dy - \frac{(x^2+y^2+1)}{x^2}dx = 0$$

$$\frac{2y}{x}dy - \left(1+\frac{y^2}{x^2}+\frac{1}{x^2}\right)dx = 0$$

$$\boxed{\frac{2y}{x}dy - \left(1+\frac{y^2}{x^2}+\frac{1}{x^2}\right)dx = 0}$$

This is in the form $M_1dx + N_1dy = 0$

$$M_1 = -\left(1+\frac{y^2}{x^2}+\frac{1}{x^2}\right)$$

$$N_1 = \frac{2y}{x}$$

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$$\boxed{\frac{\partial M_1}{\partial y} = \frac{-2y}{x^2}} = \boxed{\frac{\partial N_1}{\partial x} = \frac{-2y}{x^2}}$$

$\therefore$ Eqn (2) is exact diff eqn

G.S $\Rightarrow$ $\int M_1 dx + \int N_1 dy = 0$

y - const   free of x-terms

$N_1 = \cancel{\frac{2y}{x^2}} = 0$

$$\int \left(-1 - \frac{y^2}{x^2} - \frac{1}{x^2}\right) dx + \int 0 \cdot dy = c.$$

$$-\int dx - y^2 \int \frac{1}{x^2} dx - \int \frac{1}{x^2} dx + 0 = c.$$

$$\boxed{-x - y^2\left(\frac{x^{-1}}{-1}\right) - \left(\frac{x^{-1}}{-1}\right) = c}$$ required soln

M-4

(1) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ — (1)

Sol: It is in the form $Mdx + Ndy = 0$

$M = xy^3 + y$ $N = 2x^2y^2 + 2x + 2y^4$.

$$\boxed{\frac{\partial M}{\partial y} = 3xy^2 + 1} \neq \boxed{\frac{\partial N}{\partial x} = 4xy^2 + 2}$$

eqn (1) is not exact diff eqn -

Now I.F $= e^{\int g(y) dy}$

Choosing

$$\cancel{g(y) = y(xy^2+1)}$$

$$g(y) = \frac{1}{M}\left[\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\right]$$

$$g(y) = \frac{1}{y(xy^2+1)}\left[4xy^2+2-3xy^2-1\right]$$

$$= \frac{1}{y\cancel{(xy^2+1)}}\left[\cancel{xy^2+1}\right)$$

$$g(y) = \frac{1}{y}$$

$$\text{Now I.F} = e^{\int g(y)\,dy}$$

$$= e^{\int\left(\frac{1}{y}\right)dy}$$

$$= \cancel{e}^{\log_e y} = y$$

Multiply I.F to eq^n (1)

$$(1) \Rightarrow y\frac{(xy^3+y)}{y}\,dx + y\frac{2(x^2y^2+x+y^4)}{y}\,dy = 0$$

$$y(xy^3+y)\,dx + 2x^2y^3+xy+y^5\,dy = 0 \quad (2)$$

$$\Rightarrow (xy^4+y^2)\,dx + (2x^2y^3+2xy+2y^5)\,dy = 0$$

$$\boxed{\frac{\partial M}{\partial y} = 4xy^3+2y} = \boxed{\frac{\partial N}{\partial x} = 4xy^3+2y}$$

eq^n (2) is exact diff eq^n

Now. I.t is in the form

$$\int M_1\,dx + N_1\,dy = 0$$

Now $\Rightarrow \int M_1\,dx + \int N_1\,dy = 0$. $N_1 = 2y^5$

$$\Rightarrow \int (x y^4 + y^2)\,dx + \int 2y^5\,dy = 0$$

$$\Rightarrow y^4\int x\,dx + y^2\int dx + 2\int y^5\,dy = C$$

$$\boxed{y^4\frac{x^2}{2} + y^2(x) + \cancel{2}\frac{y^6}{\cancel{6}\,3} = C}$$

$$\boxed{y^4\frac{x^2}{2} + xy^2 + \frac{y^6}{3} = C}$$

$y.(1+y)$

H.M

(1) $(y+y^2)\,dx + xy\,dy = 0$.

$\frac{1}{N}$

Sol:- $M = y+y^2$, $N = xy$

$1+2y$.

$$I.F = \frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]$$

$$= \frac{1}{xy}\left[1+2y-x\right]$$

$$= \frac{1}{1+2y}\left[\frac{x-1-2y}{x-(1+2y)}\right]$$

$1+2y$

N. Imp **Linear Differential Equation.** 05/03/2020

An eq^n of the form $\frac{dy}{dx} + p(x) \cdot y = Q(x)$

where $p(x)$ and $Q(x)$ are functions of x

is called linear Differential eq^n.

working rule:-

→ Solve linear diff eq^n.

1) write the given linear diff eq^n $\frac{dy}{dx} + P(x) \cdot y = Q$

2) $I.F = e^{\int p(x) dx}$

3) General sol^n of linear diff eq^n is.

$$y(I.F) = \int Q(x)(I.F) dx + C.$$

Another form of linear diff eq^n and solving

1) $\frac{dx}{dy} + P(y) \cdot x = Q(y)$

2) $I.F = e^{\int P(y) dy}$

3) $x(I.F) = \int Q(y)(I.F) dy + C$

$$\boxed{\int \log x = x \log x - x + C}$$

① Solve $x\frac{dy}{dx}+y=\log x$.

Sol:- ~~Thus~~ Given that $x\frac{dy}{dx}+y=\log x$

divide with 'x' O.B.S.

$$\boxed{\frac{dy}{dx}+\frac{1}{x}\cdot y=\frac{\log x}{x}} \quad \text{--- } ①$$

This is in the form of $\frac{dy}{dx}+P(x)\cdot y=Q(x)$

where $P(x)=\frac{1}{x}$ & $Q(x)=\frac{\log x}{x}$.

I.F :- $\text{I.F}=e^{\int P(x)\,dx}$

$=e^{\int\frac{1}{x}dx} \Rightarrow e^{\log x}$.

$=x$.

Now G.S is

$\Rightarrow y(\text{I.F})=\int Q(x)(\text{I.F})\,dx+C$.

$yx=\int\frac{\log x}{x}(x)\,dx+C$

$$\therefore \boxed{xy=x\log x-x+C}$$

② Solve: $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$

Sol:- Let $\frac{dy}{dx}-\frac{y}{x+1}=e^{3x}(x+1)$

Now $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{-1}{x+1}$ $\quad Q(x) = e^{3x}(x+1)$

$I.F = e^{-\int\left(\frac{-1}{x+1}\right)dx}$ $\qquad \frac{f'(x)}{f(x)} =$

$= e^{-\int\frac{-1}{(x+1)}dx}$

$= e^{+\log(x+1)}$

$= \frac{1}{(x+1)} \Rightarrow$

Now $G.S \Rightarrow y(I.F) = \int Q(x)\, I.F\, dx + C$

$y.\frac{1}{(x+1)} = \int e^{3x}\cancel{(x+1)}\frac{1}{\cancel{x+1}}dx + C$

$\frac{y}{x+1} = e^{3x} + C$

$\boxed{y = (x+1)\frac{e^{3x}}{3} + C}$

$\Rightarrow \boxed{\frac{y}{x+1} = \frac{e^{3x}}{3} + C}$

13

③ Solve $\frac{dy}{dx} + y\sec x = \tan x$

Sol:- $P(\cancel{\sec x}) = \sec x$ & $Q(x) = \tan x$.

$$I.F = e^{\int P(x)\cdot dx}$$

$$= e^{\int \sec x\, dx}$$ $\int \sec x \tan x = \int \sec x$

$$= e^{\log(\sec x + \tan x)}$$

$$= \sec x + \tan x$$

G.S is $\Rightarrow y(I.F) = \int Q(x)(I.F)\,dx + C.$

$$\Rightarrow y(\sec x + \tan x) = \int \tan x(\sec x + \tan x) + C$$

$$\Rightarrow y(\sec x + \tan x) = \int \tan x \sec x\, dx + \int \tan^2 x\, dx + C$$

$$\Rightarrow y(\sec x + \tan x) = \sec x + \int (\sec^2 x - 1)\,dx + C$$

$$= \sec x + \int \sec^2 x\, dx - \int dx + C$$

$$\Rightarrow \boxed{y(\sec x + \tan x) = \sec x + \tan x - x + C}$$

④ Solve $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}.$

Sol:- $P(x) = \frac{1}{x\log x}$, $Q(x) = \frac{\sin 2x}{\log x}$

$\Rightarrow$ I.F $= e^{\int P(x)\,dx}$

$= e^{\int \frac{1}{x\log x}}$

$= e^{\log(\log x)}$

$\Rightarrow$ $\boxed{I.F = \log(\cancel{\log}\, x)}$

G.S $\Rightarrow$ $y(I.F) = \int Q(x)\cdot(I.F)\,dx + C$

$y\log x = \int \frac{\sin 2x}{\cancel{\log x}}\,\cancel{\log x}\,dx + C$

$y\log x = \int \sin 2x\,dx + C$

$\therefore$ $\boxed{y\log x = \frac{-\cos 2x}{2} + C}$

H.W

5. Solve $(2y - x^3)\,dx + x\,dy = 0.$

$(2y - x^3)\,dx = -x\,dy$

$\cancel{\frac{dx}{dy} + 2y - x^3}$

$-x\,dy = (2y - x^3)\,dx$

$\frac{dy}{dx} = \frac{-(2y - x^3)}{x}$

$\not{x} \frac{dy}{dx} = \not{x}(x^3 - 2y).$

divid with x on both sides

$$\frac{dy}{dx} = \frac{x^3}{x} - \frac{2y}{x}$$

$$\frac{dy}{dx} + \frac{2}{x} \cdot y = x^2$$

$$I \cdot F = e^{\int P(x)\,dx}$$

$$= e^{2\log x}$$

$$= x^2.$$

G.S

$$\Rightarrow y(I \cdot F) = \int Q(x)(I \cdot F)\,dx + C$$

$$y(x^2) = \int x^2 \cdot x^2\,dx + C$$

$$\Rightarrow yx^2 = \int x^4\,dx + C$$

$$\Rightarrow yx^2 = \frac{x^5}{5} + C.$$

⑥ Solve $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy.$

$$\frac{dx}{dy} = \frac{1+y^2}{\quad}$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$$

$\Rightarrow I \cdot F = e^{\int P(x)\,dx}$

$= e^{\int \frac{1}{x\log x}}$

$= e^{\log(\log x)}$

$\Rightarrow \boxed{I \cdot F = \log(\cancel{\log}\, x)}$

$G \cdot S \Rightarrow y(I \cdot F) = \int Q(x) \cdot (I \cdot F)\, dx + C$

$y \log x = \int \frac{\sin 2x}{\cancel{\log x}} \cancel{\log x}\, dx + C$

$y \log x = \int \sin 2x\, dx + C$

$\therefore \boxed{y \log x = \frac{-\cos 2x}{2} + C}$

H.W

(5). Solve $(2y - x^3)\,dx + x\,dy = 0.$

$(2y - x^3)\,dx = -x\,dy$

$\frac{d\cancel{x}}{dy} + \cancel{2y - x^3}$

$-x\,dy = (2y - x^3)\,dx$

$\frac{dy}{dx} = -\frac{(2y - x^3)}{x}$

$$f x \frac{dy}{dx} = f(x^3 - 2y).$$

divid with x on both sidy

$$\frac{dy}{dx} = \frac{x^3}{x} - \frac{2y}{x}$$

$$\frac{dy}{dx} + \frac{2}{x} \cdot y = x^2.$$

$$I \cdot F = e^{\int P(x)\, dx}$$

$$= e^{2\int og x}$$

$$= x^2$$

G.S

$$\Rightarrow y(I \cdot F) = \int Q(x)(I \cdot F)\, dx + C.$$

$$y(x^2) = \int x^2 \cdot x^2\, dx + C$$

$$\Rightarrow y x^2 = \int x^4\, dx + C$$

$$\Rightarrow y x^2 = \frac{x^5}{5} + C.$$

⑥ Solve $(1+y^2)\, dx = (\tan^{-1} y - x)\, dy.$

$$= \frac{1+y^2}{}$$

$$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}.$$

$\Rightarrow$ It is in the form $\frac{dx}{dy} + P(y)\cdot x = Q(y)$

Here $P(y) = \frac{1}{1+y^2}$ & $Q(y) = \frac{\tan^{-1}y}{1+y^2}$.

Now $I.F = e^{\int P(y)dy}$

$$\boxed{I.F = e^{\tan^{-1}y}}$$

G.S $\Rightarrow$ $I.F(x) = \int Q(y)(I.F)\,dy + C$.

$$x e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} e^{\tan^{-1}y}\,dy + C$$

$$x e^{\tan^{-1}y} = \int \frac{1}{1+y^2} \tan^{-1}y \, e^{\tan^{-1}y}\,dy + C$$

Now put $\tan^{-1}y = t$.

$$\frac{1}{1+y^2}dy = dt$$

$$x e^t = \int \underset{u}{t}\, \underset{v}{e^t}\, dt + C$$

$$= t(e^t - 1)e^t + C.$$

$$x e^{\tan^{-1}y} = \tan^{-1}y\,(e^{\tan^{-1}y} - e^{\tan^{-1}y} + C.$$

$*$ (7) Solve $(x+2y^3)\frac{dy}{dx} = y$.

$\boxed{\frac{x}{a+b} = \frac{a+b}{x}}$

Sol

$$\frac{dy}{dx} = \frac{y}{x+2y^3}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x+2y^3}{y}$$

$$\frac{dx}{dy} - \frac{1}{y}x = 2y^2$$

It is in the form $\frac{dx}{dy} + P(y)\cdot x = Q(y)$

$$P(y) = -\frac{1}{y} \quad \& \quad Q(y) = 2y^2$$

$$I\cdot F = e^{\int P(y)\,dy}$$

$$= e^{-\log y}$$

$$= y^{-1} \Rightarrow \frac{1}{y}.$$

$$\boxed{I\cdot F = \frac{1}{y}}$$

$$G\cdot S \Rightarrow x(I\cdot F) = \int Q(y)(I\cdot F)\,dy + C.$$

$$x\frac{1}{y} = \int 2y^2\cdot\frac{1}{y}\,dy + C$$

$$\frac{x}{y} = \cancel{2}\frac{y^2}{\cancel{2}} + C$$

$$\boxed{\frac{x}{y} = y^2 + C}$$

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⑧ Solve $(1+y^2) + (x - e^{\tan^{-1}y})\dfrac{dy}{dx} = 0$

Sol:-

$$(1+y^2) = -(x - e^{\tan^{-1}y})\frac{dy}{dx}$$

$$\frac{dx}{dy} = \frac{-x + e^{\tan^{-1}y}}{1+y^2}$$

$$\frac{dx}{dy} + \frac{1}{1+y^2}\cdot x = \frac{e^{\tan^{-1}y}}{1+y^2}$$

This is in the form $\frac{dx}{dy} + P(y)\cdot x = Q(y)$

$P(y) = \frac{1}{1+y^2}$ & $Q(y) = \frac{e^{\tan^{-1}y}}{1+y^2}$

$$I.F = e^{\int P(y)dy}$$

$$\boxed{I.F = e^{\tan^{-1}y}}$$

G.S $\Rightarrow$ $x(I.F) = \int Q(y)\cdot(I.F)\,dy + c$

$$x\cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2}\cdot e^{\tan^{-1}y}\,dy + c$$

$e^a \cdot e^a$

$$t = e^{\tan^{-1}y}.$$

$$= e^{\tan^{-1}y}\frac{1}{1+y^2}dy.$$

$$x\cdot e^{\tan^{-1}y} = \int e^{2t}\,dt + C$$

$$\boxed{x\cdot e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + C}$$

H.W

(9) $\frac{dy}{dx}+2y=e^{x}+x$

$P(x)=$

$\frac{dy}{dx}-e^{x}-x=-2y.$

Sol:- $\frac{dy}{dx}+2y-x=e^{x}$

→ Bernoulli's Differential equations:-

→ An diff eqⁿ is of the form

$$\frac{dy}{dx}+P(x)\cdot y=Q(x)\cdot y^{n}$$

→ where $P(x)$ and $Q(x)$ are functions of 'x'
is called Bernoullies Diff. eqⁿs

Solving Bernoulli's diff eqⁿs :- process)

1. write the B. Diff eqⁿ that is

$$\boxed{\frac{dy}{dx}+P(x)\cdot y=Q(x)\cdot y^{n}} \quad (1)$$

O.B.S divide with y^{n}

$$\frac{1}{y^{n}}\frac{dy}{dx}+P(x)\cdot\frac{y}{y^{n}}=Q(x)\cdot 1$$

$$\boxed{y^{-n}\frac{dy}{dx}+P(x)\cdot y^{1-n}=Q(x)} \quad (2)$$

2) By substituting method change eqⁿ no.2 into linear differⁿ eqⁿ in "v -variable

that is $\boxed{\frac{dv}{dx} + p(x) \cdot v = Q(x)}$

$\rightarrow I \cdot f = e^{\int p(x) dx}$

G.S $\Rightarrow$ $\boxed{v(I \cdot f) = \int Q(x)(I \cdot f) dx + c}$.

and follow substitution method.

Problems:-

① Solve diff eqn $x \frac{dy}{dx} + y = x^3 y^6$

Sol:- $x \frac{dy}{dx} + y = x^3 y^6$

divide with 'x' on B.S

$\boxed{\frac{dy}{dx} + \frac{1}{x} y = x^2 \cdot y^6}$ —— ① linear diff eqn

This is in the form

$\boxed{\frac{dy}{dx} + p(x) \cdot y = Q(x) \cdot y^n}$

Now divide 'y^6' O.B.S

$\frac{1}{y^6} \frac{dy}{dx} + \frac{1}{x} \cdot y^{-5} = x^2$ —— ②

$\frac{dy}{dx} = \frac{dy}{dv}$

put $y^{-5} = v$

$-5 y^{-6} \frac{dy}{dx} = \frac{dv}{dx}$

y^{-5}

$-5 y^{-5-1}$

$-5y^{-6} \frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow y^{-6} \frac{dy}{dx} = \frac{-1}{5} \frac{dv}{dx}$

② $\Rightarrow$ $\frac{-1}{5} \frac{dv}{dx} + \frac{1}{x} \cdot v = x^2$

multiply (-5) O.B.S

$$\frac{dv}{dx} + \left(\frac{-5}{x}\right) \cdot v = (-5x^2)$$

$$\frac{dv}{dx} + P(x) \cdot v = Q(x)$$

$$P(x) = \frac{-5}{x} \quad \& \quad Q(x) = -5x^2.$$

I.F $\Rightarrow$ $I.F = e^{\int P(x)\,dx}$

$$= e^{-5\log x} \Rightarrow e^{\log x^{-5}}$$

$$= \frac{1}{x^5}$$

G.S $\Rightarrow$ $v\,(I.F) = \int Q(x)\,(I.F)\,dx + C$

$$y^{-5}\left(\frac{1}{x^5}\right) = \int -5x^2 \cdot \frac{1}{x^5}\,dx + C.$$

$$\frac{1}{x^5y^5} = -5\left[\frac{x^{-3+1}}{-3+1}\right] + C$$

$$\boxed{\frac{1}{x^5y^5} = \frac{5}{2}x^{-2} + C}$$ req^d sol^n.

(2) Solve B.D.eq^n $\frac{dy}{dx} + y \cdot \tan x = y^2 \sec x.$

Sol:- $\frac{dy}{dx} + \tan x \cdot y = \sec x \cdot y^2$ —(1)

Divide with y^2 O.B.S

$$\boxed{\frac{1}{y^2}\frac{dy}{dx} + \tan x \cdot y^{-1} = \sec(x)} \quad (2)$$

Change this differ^n into Linear D.eq^n in v

put $\boxed{y^{-1} = v}$

$$y^{-2}\frac{dy}{dx} = (-1)\frac{dv}{dx}$$

③ ⇒ $(-1)\frac{dv}{dx} + \tan x . v = \sec x.$

multiply '-1' O.B.S

$$\boxed{\frac{dv}{dx} + (\tan x) . v = (-\sec x)}$$

$P(x) = \tan x$ & $Q(x) = -\sec x.$

$$I.F = e^{\int P(x)\,dx}$$

$$= e^{-\int \tan x\,dx}$$

$$= e^{-\log(\sec x)}$$

$$I.F = \left(\frac{1}{\sec x}\right) = \cos x.$$

G.S ⇒ $v(I.F) = \int Q(x)\,(I.F)\,dx + C$

$$y^{-1}(\cos x) = \int -\sec x \cdot \cos x\,dx + C$$

$$= -\int \frac{1}{\cos x} \cdot \cos x\,dx + C$$

$$y^{-1}(\cos x) = -x + C$$

$$\boxed{\frac{\cos x}{y} = -x + C}$$

required sol^{n}.

(B) Solve diff eqⁿ $\frac{dy}{dx} + y\cos x = y^3 \sin 2x$

Sol:- $\frac{dy}{dx} + \cos x \cdot y = \sin 2x \cdot y^3$ — (1)

O·B·S divide with y^3

$$\frac{1}{y^3}\frac{dy}{dx} + \cos x \cdot \frac{y}{y^3} = \sin 2x \cdot \frac{y^3}{y^3}$$

$$\Rightarrow \boxed{y^{-3}\frac{dy}{dx} + \cos x \cdot y^{-2} = \sin 2x} \text{ — (2)}$$

Change this eqⁿ (2) into linear diff eqⁿ in 'v' variable.

put $\boxed{y^{-2} = v}$

$$-2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\boxed{y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}}$$

(2) $\Rightarrow$ $-\frac{1}{2}\frac{dv}{dx} + \cos x \cdot v = \sin 2x$.

multiply (-2) O.B.S

$$\boxed{\frac{dv}{dx} + (-2\cos x)\cdot v = (-2\sin 2x)} \text{ — (3)}$$

$\frac{dv}{dx} + P(x)\cdot v = Q(x)$ liner diff eqⁿ

$P(x) = -2\cos x$ & $Q(x) = -2\sin 2x$

$I.F = e^{\int P(x)dx}$

$= e^{-2\int \cos x\, dx}$

$\boxed{I.F = e^{-2\sin x}}$

$G.S \Rightarrow v.(I.F) = \int Q(x)(I.F)\,dx + c$

$y^{-2}(e^{-2\sin x}) = \int -2\sin 2x \cdot e^{-2\sin x}\,dx + c$

$-2\sin x = t$

$-2\cos x\,dx = dt$

$= -1\int -2(-2\sin x\cos x)\, e^{-2\sin x}\,dx + c$

$= -\int t.e^t dt + c$

$= -[t e^t - e^t] + c$

$\boxed{y^{-2}(e^{-2\sin x}) = -[2\sin x . e^{-2\sin x} - e^{-2\sin x}] + c}$

(5) Solve $e^x \frac{dy}{dx} = 2xy^2 + ye^x$

Sol:

$e^x \frac{dy}{dx} - ye^x = 2xy^2$

divide with e^x on B.S

$\boxed{\frac{dy}{dx} - (1)y = \frac{2x}{e^x} . y^2} \quad \text{--- (1)}$

Divide y^2 O.B.S

$$\boxed{y^{-2}\frac{dy}{dx}+(-1)y^{-1}=\frac{2x}{e^{x}}} \quad -(2)$$

Change the above into L.D.eq

put $y^{-1}=v$

$$-1\,y^{-2}\frac{dy}{dx}=\frac{dv}{dt}$$

$$\boxed{y^{-2}\frac{dy}{dx}=(-1)\frac{dv}{dx}}$$

$$(-1)\frac{dv}{dx}+(-1)v=\frac{2x}{e^{x}}$$

$(2) \Rightarrow$ $(-1)\frac{dv}{dx}+(-1)\cdot v=\frac{2x}{e^{x}}$

multiply (-1) O.B.S

$$\boxed{\frac{dv}{dx}+v=\frac{-2x}{e^{x}}} \quad -(3)$$

This is in the form $\frac{dv}{dx}+P(x)\cdot v=Q(x)$

$P(x)=1$ $\quad Q(x)=\frac{-2x}{e^{x}}$

I.F $\quad$ $I.F=e^{\int P(x)dx}$

$$\boxed{I.F=e^{x}}$$

$$G.S \Rightarrow V(I.F) = \int Q(x)(I.F)\,dx + C$$

$$y^{-1}(e^{x}) = \int \frac{-2x}{\cancel{e^{x}}} \cdot e^{x}\,dx + C.$$

$$\frac{e^{x}}{y} = -2\int x\,dx + C$$

$$= -\cancel{2}\frac{x^2}{\cancel{2}} + C$$

$$\therefore \boxed{\frac{e^{x}}{y} = -x^2 + C}$$

H.W

1) $\frac{dy}{dx} + \frac{y}{x} = y^2 x$

2) $(1-x^2)\frac{dy}{dx} + xy = xy^2$

3) $\frac{dy}{dx} = x^3y^3 - xy$

4) $x^3\frac{dy}{dx} - x^2y + y^4\cos x = 0.$

5) $\cos x\,dy = y(\sin x - y)\,dx.$

H.W

① Let $\frac{dy}{dx} + \frac{1}{x} \cdot y = y^2 x$ — ①

Sol:

divide with y^2 O.B.S.

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{y}{x \cdot y^2} = x$$

$$\boxed{y^{-2}\frac{dy}{dx} + \frac{1}{x} y^{-1} = x} \quad \text{— ②}$$

put $y^{-1} = v$

$$(-1) y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$y^{-2}\frac{dy}{dx} = (-1)\frac{dv}{dx}$$

$$-1\frac{dv}{dx} + \left(\frac{1}{x}\right) \cdot v = x$$

$$\boxed{\frac{dv}{dx} - \frac{1}{x} \cdot v = -x} \quad \text{— ③}$$

$$\frac{dv}{dx} + P(x) v = Q(x)$$

$$P(x) = \frac{-1}{x} \quad \text{\&} \quad Q(x) = -x$$

I.F $\Rightarrow$ $I.F = e^{\int P(x) dx}$

$$= e^{-\int \frac{1}{x} dx}$$

$$= e^{-\log x} = \frac{1}{x}$$

$G.S \Rightarrow v(I.F) = \int Q(x)(I.F)\,dx + C$

$y^{-1}\left(\frac{1}{x}\right) = \int (-x)\left(\frac{-1}{x}\right) dx + C$

$\therefore \boxed{\frac{1}{xy} = -x + C}$ required sol^n.

H.W

(3)

Sol: The diff eq^n $\frac{dy}{dx} = x^3y^3 - xy$.

$\boxed{\frac{dy}{dx} + x \cdot y = x^3 y^3}$ — (1)

$\Rightarrow \boxed{y^{-3}\frac{dy}{dx} + xy^{-2} = x^3}$ — (2)

$\boxed{y^{-2} = v}$

$-2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$

$\boxed{y^{-3}\frac{dy}{dx} = \frac{-1}{2}\frac{dv}{dx}}$

$\therefore \boxed{\frac{-1}{2}\frac{dv}{dx} + xv = x^3}$

e^{-x^2}

Multiply -2 on B.S

$\boxed{\frac{dv}{dx} + (-2x) \cdot v = -2x^3}$. $P(x) = -2x$, $Q(x) = -2x^3$

$I.F = e^{\int P(x)\,dx} \Rightarrow e^{-2\int x\,dx}$

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$$I.F = e^{-2\frac{x^2}{2}}$$

$$\boxed{I.F = e^{-x^2}}$$

$$G.S \Rightarrow (I.F)\, v = \int Q(x)\,(I.F)\, dx + C$$

$$y^{-2} e^{-x^2} = \int -2x^3 (e^{-x^2})\, dx + C$$

put $-x^2 = t$

$-2x\, dx = dt$

$$y^{-2} e^{t} = \int -2x \cdot x^2 (e^{-x^2})\, dx + C$$

$-x^2 = t$, $-2x\,dx = dt$

$$= -\int dt\,(-x^2)\,(e^{t}) + C$$

$-x^2 = t$

$$= -\int t e^{t}\, dt + C$$

$$= -[t e^{t} - 1 \cdot e^{t}] + C$$

$$= -[e^{t}(t-1)] + C$$

$$\underline{y^{-2} e^{t}} = -[e^{-x^2}(-x^2-1)] + C$$

MID - I

Imp *) Newton's Law of Cooling

The rate of change of the temperature of a body is proportional to the difference the temp of body and that of the surrounding medium.

Let $\theta \to$ be the body temp

$\theta_0 \to$ surroundings temp

By the Newton's law of cooling

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

variable separable

$$\frac{d\theta}{\theta - \theta_0} = -k\,dt$$

Apply I.O.B's

$$\int \frac{1}{\theta - \theta_0}\, d\theta = -k \int dt + c$$

$$\therefore \left[\int \frac{1}{x-1}\, dx = \log(x-1)\right]$$

$$\therefore \boxed{\log(\theta - \theta_0) = -kt + c}$$

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$$\log(\theta-\theta_0) = -Kt + \log c$$

$$\log(\theta-\theta_0) - \log c = -Kt$$

$$\log_e \frac{(\theta-\theta_0)}{C} = -Kt$$

Apply exp (e) O.B.S

$$e^{\log_e \left(\frac{\theta-\theta_0}{C}\right)} = e^{-Kt}$$

$$\therefore \boxed{\frac{\theta-\theta_0}{C} = e^{-Kt}}$$

① A body is originally at 80°C and cools down 60°C in 20 min. If the temp of the air is 40°C. Find the temp of body after 40 min.

Sol :- From the given data

the body cools from 80°c to 60°c in 20min

$\Rightarrow \theta_0 = 40°C$

Find θ at $t = 40$ min

θ = body temp

$\theta_0 \rightarrow$ Surroundings temp

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\frac{d\theta}{\theta - \theta_0} = -k\,dt$$

$\Rightarrow$ Apply $\int$ O.B.S

$$\int \frac{1}{\theta - \theta_0} d\theta = -\int k\,dt$$

$$\left(\because \int \frac{1}{x-1} dx = \log(x-1)\right)$$

$$\Rightarrow \boxed{\log(\theta - \theta_0) = -kt + c} \quad \text{— (I)}$$

$\theta_0 = 40°C$

Case(i):- At $t = 0 \Rightarrow \theta = 80°C$

$$(I) \Rightarrow \log(80 - 40) = -k(0) + c$$

$$\boxed{\log 40 = c} \quad \text{— (a)}$$

Case(ii):- At $t = 20$ min $\Rightarrow \theta = 60°C$

$$(I) \Rightarrow \log(60 - 40) = -k(20) + \log 40$$

$$\log 20 = -20k + \log 40$$

$$\log 20 - \log 40 = -20k$$

$$\log\left(\frac{20}{40}\right) = -20k$$

$\log(\frac{1}{2}) = -20K$

$$\boxed{K = \frac{-1}{20}\log(\frac{1}{2})} \quad \text{---} \; B$$

put K, C & θ_0 values in eqⁿ I

I $\Rightarrow \log(\theta - \theta_0) = -Kt + C$

$\log(\theta - 40) - \log 40 = -Kt$

$$\boxed{\log\left(\frac{\theta - 40}{40}\right) = \frac{t}{20}\log(\frac{1}{2})} \text{ --- } B$$

Now $\theta = ?$ $t = 40$.

$$\log\left(\frac{\theta - 40}{40}\right) = \frac{\cancel{40}^{\,2}}{\cancel{20}}\log(\frac{1}{2})$$

$$\Rightarrow \frac{\theta - 40}{40} = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \frac{\theta - 40}{\cancel{40}10} = \frac{1}{\cancel{4}}$$

$$\Rightarrow \theta - 40 = 10$$

$\therefore \boxed{\theta = 50^\circ C}$ At $t = 40$ min

H.W

1) If the temp of the body is changing from 100°c to 70°c in 15 min. Find when the temp will be 40°c. The surroundings temp is 30°c

2) A body cools from 140°c to 80°c in 20 min with the air temp is 25°c then find when the body cools down 35°c

3) a) If the temp air is 20°c and temp of the body drops from 100°c to 80°c in 10 min. what will be its temp after 20 min.

b) when will be the temp 40°c

1

Sol:- From the given data

The body temp changing from 100°c to 70°c in 15 min

$\theta_0 = 30°C$

$\frac{d\theta}{dt} \propto (\theta - \theta_0)$

$$\frac{d\theta}{dt} = -K(\theta-\theta_0)$$

$$\frac{d\theta}{\theta-\theta_0} = -K\,dt.$$

Apply S.O.B.S

$$\int \frac{1}{\theta-\theta_0}\,d\theta = -\int K\,dt$$

$$\boxed{\log(\theta-\theta_0) = -Kt + C} \quad \text{--- (1)}$$

$\theta_0 = 30^\circ C.$

Case(i): At $t = 0 \Rightarrow \theta = 100^\circ C$

$(1) \Rightarrow \log(100 - 30^\circ C) = -K(0) + C$

$$\boxed{\log 70 = C} \quad \text{--- (a)}$$

Case(ii): At $t = 15$ min $\Rightarrow \theta = 70^\circ C$

$(1) \Rightarrow \log(70-30) = -K(15) + \log 70$

$\log(40) - \log 70 = -15K$

$\log\left(\frac{40}{70}\right) = -15K.$

$\log\left(\frac{4}{7}\right) = -15K.$

$$\boxed{K = -\frac{1}{15}\log\left(\frac{40}{70}\right)} \quad \text{--- (b)}$$

put K, C & θ_0 in eqⁿ (1)

(1) ⇒ $\log(\theta - \theta_0) = -Kt + C$

$$\log(\theta - 30) = -Kt + \log 70$$

$$\log(\theta - 30) - \log 70 = -Kt$$

$$\log\left(\frac{\theta - 30}{70}\right) = -Kt$$

$$\log\left(\frac{\theta - 30}{70}\right) = \frac{t}{15}\log\left(\frac{40}{70}\right)$$

Now $\theta = 40$ $t = ?$

$$\log\frac{\theta - 30}{70} = \frac{t}{15}\log\left(\frac{40}{70}\right)$$

$$\Rightarrow \cancel{\log}\frac{\theta - 30}{70} = \cancel{\log}\left(\frac{40}{70}\right)^{\frac{t}{15}}$$

$$\log\left(\frac{40 - 30}{70}\right) = \left(\frac{40}{70}\right)^{\frac{t}{15}}$$

$$\log\frac{1}{7} = \left(\frac{40}{70}\right)^{\frac{t}{15}}$$

$$\frac{1}{7} = \left(\frac{4}{7}\right)^{\frac{t}{15}}$$

$$= (0.571)^{t/15}$$

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$$\log\left(\frac{10}{70}\right) = \frac{t}{15}\log\left(\frac{40}{70}\right)$$

$$\Rightarrow \log\left(\frac{1}{7}\right) = \frac{t}{15}\log\left(\frac{4}{7}\right)$$

$$\Rightarrow +0.8450 = \frac{+t}{15}(0.2430)$$

$$0.8450 \times 15 = t(0.2430)$$

$$\Rightarrow \frac{12.675}{0.2430} = t$$

$$\boxed{t = 52.16}$$

(2) from the given data

Sol:- The body cools from $140^\circ c$ to $80^\circ c$ in 20 min

$$\theta_0 = 25^\circ C$$

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

$$\frac{d\theta}{\theta - \theta_0} = -K\,dt$$

Apply S.O.BS

$$\boxed{\log(\theta - \theta_0) = -Kt + c} \quad (1)$$

At case (i):-

$$\theta_0 = 25^\circ c$$

$$\log(140 - 25) = c \Rightarrow \boxed{c = \log 115}$$

Case (ii) At $t = 20$ min $\Rightarrow \theta = 80°C$

$$\log(80 - 25) = -20K + \log 115°$$

$$\log 55 = -20K + \log 115°$$

$$\log\left(\frac{55}{115}\right) = -20K$$

$$\boxed{K = \frac{-1}{20}\log\left(\frac{55}{115}\right)} \quad \text{— b}$$

Put K, C & θ_0 in eqn (1)

$$\log(\theta - \theta_0) = -Kt + C$$

$$\log(35 - 25) = \frac{t}{20}\log\left(\frac{55}{115}\right) + \log 115$$

$$\log\left(\frac{10}{115}\right) = \frac{t}{20}\log\left(\frac{55}{115}\right)$$

$$-2.4423 = -\frac{t}{20}(0.7375)$$

$$48.846 = t(0.7375)$$

$$t = \frac{48.846}{0.7375}$$

$$\boxed{t = 66.23 \text{ min}}$$

(3) Sol:- From the given data.

The body temp drops from 100° to 80

in 10 min.

$\theta_0 = 20°C$

$\frac{d\theta}{dt} \propto (\theta - \theta_0)$

$\frac{d\theta}{dt} = -K(\theta - \theta_0)$

$\frac{d\theta}{\theta - \theta_0} = -K\,dt$

Apply $\int$ O.B.S

$\int \frac{1}{\theta - \theta_0} d\theta = -K \int dt + C$

$\therefore \boxed{\log(\theta - \theta_0) = -Kt + C}$ — (I)

$\theta_0 = 20°C$

Case(i) :- At $t = 0 \Rightarrow \theta = 100°C$

(I) $\Rightarrow \log(100 - 20) = C$

$\boxed{C = \log 80}$ — (a)

Case(ii) :- At $t = 10\,min \Rightarrow \theta = 80°C$

$\log(80 - 20) = -10K + \log 80$

$$\log\left(\frac{60}{80}\right) = -10K$$

$$\boxed{K = \frac{-1}{10}\log\left(\frac{60}{80}\right)} \text{ — } (B)$$

Sub C, K & θ_0 values in eq^n (I)

$$(I) \Rightarrow \log(\theta - 20°) = \frac{t}{10}\log\left(\frac{60}{80}\right) + \log 80$$

$$= \frac{\cancel{20}^{2}}{\cancel{10}}\log\left(\frac{60}{80}\right) + \log 80$$

$$\log\left(\frac{\theta - 20}{80}\right) = \log\left(\frac{\cancel{60}^{3}}{\cancel{80}_{4}}\right)^2$$

$$\left(\frac{\theta - 20}{80}\right) = \left(\frac{3}{4}\right)^2$$

$$\left(\frac{\theta - 20}{80}\right) = \frac{9}{16}$$

$$\theta - 20 = (0.75)^2 \times 80.$$

$$\theta - 20 = 0.5625 \times 80$$

$$\theta = 45 + 20$$

$$\boxed{\theta = 65°C}$$

If $\theta = 40°C$

$$\log\left(\frac{40 - 20}{80}\right) = \frac{t}{10}\log\left(\frac{60}{80}\right)$$

$$\Rightarrow \log\left(\frac{20}{80}\right) = \frac{t}{10} \log\left(\frac{60}{80}\right)$$

$$\Rightarrow 0.6020 = \frac{t}{10}(0.1249)$$

$$\frac{6.02}{0.1249} = t$$

$$\Rightarrow \boxed{t = 48.1985 \text{ min}}$$

Law of Natural growth or Decay

growth $\uparrow$ $+K$; Decay $\downarrow$ $-K$

Let N be the amount of substance at time 't' and let the substance be getting converted chemically. a law of chemical conversion that the rate of change of amount N of chemically changing substance is proportional to the amount of the substance available at that time.

$$\frac{dN}{dt} \propto N \quad \text{--- (1)}$$

$$\uparrow \frac{dN}{dt} = +KN \qquad \frac{dN}{dt} = -KN \downarrow$$

Consider $\frac{dN}{dt} = KN$

use variable separable

$$\frac{dN}{dt} = KN$$

$$\frac{dN}{N} = K\,dt$$

Apply $\int$ O.B.S

$$\int \frac{1}{N}\,dN = K\int dt + c$$

$$\Rightarrow \log N = Kt + \log c$$

$$\log N - \log c = Kt$$

Apply e O.B.S

$$e^{\log\left(\frac{N}{c}\right)} = e^{Kt}$$

$$\frac{N}{c} = e^{Kt}$$

$\uparrow$ $\boxed{N = c\cdot e^{Kt}}$ — (I) for growth

& $\boxed{N = c\cdot e^{-Kt}}$ $\downarrow$ — (J).

for Decay.

① The no. 'N' of bacteria in a culture growing up at a rate proportional to N. The value of N was initially 100 and increased to 332 in 1 hour. What was the value of N after one and half ($1\frac{1}{2}$) hour

$1\frac{1}{2} = \frac{3}{2}$

Sol:- let $\frac{dN}{dt} \propto N$ — ①

$\frac{dN}{dt} = kN$

66.5

$\frac{dN}{N} = k\,dt$

Apply $\int$ o. B. S

$\cancel{100}\ \int \frac{1}{N} dN = k\int dt + c$

$\log N = kt + \log c$

$\Rightarrow \log N - \log c = kt$

$\log\left(\frac{N}{c}\right) = kt$

Apply 'e' O. B. S

$e^{\log\left(\frac{N}{c}\right)} = e^{kt}$

$\frac{N}{c} = e^{kt}$

$\boxed{N = c \cdot e^{kt}}$ — (I)

use Initial conditions

Case (i) :

(I) $\Rightarrow$ At $t=0 \Rightarrow N=100$

$100 = C \cdot e^{0K}$

$$\boxed{C=100} \quad -\text{ⓐ}$$

Case (ii) :

At $t=1$ hour $\Rightarrow N=332$

(I) $\Rightarrow \quad 332 = 100 \cdot e^{K}$

$$e^{K} = \frac{332}{100}$$

$$\boxed{e^{K}=3.32} \quad -\text{ⓑ}$$

put K & C values in eqⁿ (1)

$$N = 100\,[e^{K}]^{t}$$

$$\boxed{N = 100\,(3.32)^{t}}$$

Now $t = \frac{3}{2}$ hour $N = ?$

$$N = 100\,(3.32)^{3/2}$$

$$= 100 \times (6.0493)$$

$$N = 604.93 \cong 605$$

$\boxed{N=605}$ at $t = \frac{3}{2}$ hours.

(2) A bacterial culture decreases exponensially from 400 gm to 100 gm in 10 hours how much was present after 3 hours from the initial state.

Sol: From the given data the bacteria decreases 400 gm to 100 gm in 10 hours

To Find N at t = 3 hours

Natural Decay $\frac{dN}{dt} \propto N$

$$\frac{dN}{dt} = -KN$$

$$\Rightarrow \frac{dN}{N} = -Kdt$$

Apply $\int$ O.B.S

$$\int \frac{1}{N} dN = -K\int dt + c.$$

$$\Rightarrow \log N = -Kt + \log c$$

$$\log\left(\frac{N}{c}\right) = -Kt$$

Apply $e^{\wedge}$ O.B.S

$$\frac{N}{C} = e^{-Kt}$$

$$\boxed{N = C \cdot e^{-Kt}} \quad \text{—(I)}$$

Case(i):- $t=0 \Rightarrow N=400$

(1) $\Rightarrow 400 = C \cdot e^0$

$$\boxed{C=400} \text{ — (9)}$$

Case(ii) at $t=10$ hours $\Rightarrow N=100$

$$100 = 400 \cdot e^{-10K}$$

$$e^{-10K} = \frac{100}{400}$$

$$(e^{-K})^{10} = \frac{1}{4}$$

$$\Rightarrow \boxed{e^{-K} = \left(\frac{1}{4}\right)^{1/10}}$$

put K & C in eqn (1)

(1) $\Rightarrow N = 400(e^{-K})^t$

$$\boxed{N = 400\left(\frac{1}{4}\right)^{t/10}}$$

$N=?$ at $t=3$ hours

$$N = 400\left(\frac{1}{4}\right)^{3/10}$$

$$N = 400 \times 0.659$$

$$N = 263.90$$

$\boxed{N = 264}$ at $t=3$ hours.

Chapter - III

*** **Equations of the first order does not first degree :-**

Mainly we have 4 methods are there those are given by

*** i) Method - I :- Clairaut's equation.

2) Method - II :- solvable for p

3) Method - III :- solvable for y

4) method - IV :- solvable for x.

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procedure:- (solving Clairaut's eq^n):-

→ write the Clairaut's eq^n which is of the form $\boxed{y = px + f(p)}$

→ Differentiating of Clairaut's eq^n w.r.to x we get reduced diff eq^n that is eq^n ②

put $\frac{dy}{dx} = p$

→ solving eq^n ② we get two sol^n's. Those are general sol^n and singular sol^n.

① solve $y = px + p^2$

sol:- The given eq^n is Clairaut's eq^n. that is of the form $y = px + f(p)$

$$y = px + p^2 \quad \text{—— ①}$$

Now diff eq^n ① w.r.to 'x'

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + 2p\frac{dp}{dx}$$

put $\boxed{\frac{dy}{dx} = p}$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}[x+2p]$$

$$\frac{dp}{dx}(x+2p) = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x+2p = 0$$

Apply S.O.B.S

$$\int \frac{dp}{dx} = 0 + c$$

$$\boxed{p = c}$$

Put $p=c$ in eqⁿ ①

① $\Rightarrow$ $\boxed{y = cx + c^2}$ is General solⁿ

Solve $x = -2p$

$$p = -\frac{x}{2}$$

Put 'p' in eqⁿ ①

① $\Rightarrow$ $y = px + p^2$

$$\boxed{y = \left(\frac{-x}{2}\right)x + \left(\frac{-x}{2}\right)^2}$$

is Singular solⁿ.

(2) Solve $y = px + \log p$

Sol: The given eq^n is clairaut's eq^n

that is of the form $y = p(x) + f(p)$

$$y = px + \log p \quad -(1)$$

Now diff w.r.to x to eq^n (1)

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + \frac{1}{p}\frac{dp}{dx}$$

put $\boxed{\frac{dy}{dx} = p}$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}\left(x + \frac{1}{p}\right)$$

$$\frac{dp}{dx}\left(x + \frac{1}{p}\right) = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x + \frac{1}{p} = 0$$

Apply $\int$ O.B.S

$$\int \frac{dp}{dx} = 0 + C$$

$$\boxed{p = C}$$

put p in eq^n (1)

$\boxed{y = Cx + \log C}$ is general sol^n

now $x+\frac{1}{p}=0$

$$x=\frac{-1}{p}$$

put $\boxed{p=\frac{-1}{x}}$ in eqⁿ (1)

$$(1) \Rightarrow y=px+\log p$$

$$y=\left(\frac{-1}{\cancel{x}}\right)\cancel{x}+\log\left(\frac{-1}{x}\right)$$

$\boxed{y=-1+\log\left(\frac{-1}{x}\right)}$ is singular solⁿ.

** (3) solve $p=\log(px-y)$

Solⁿ:- Let $p=\log(px-y)$

Apply eⁿ O.B.S

$$e^{p}=e^{\log(px-y)}$$

$$e^{p}=px-y$$

$$\boxed{y=px-e^{p}} \quad -(1)$$

It is the clairaut's eqⁿ, it is in the form $\boxed{y=px+f(p)}$

diff w.r.to 'x' eqⁿ (1)

$$\frac{dy}{dx}=p+x\frac{dp}{dx}-e^{p}\frac{dp}{dx}$$

put $\boxed{\frac{dy}{dp} = p}$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}(x - e^p)$$

$$\frac{dp}{dx}(x - e^p) = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x - e^p = 0$$

Apply $\int$ o.B.S

$$\int \frac{dp}{dx} = 0 + c$$

$$\boxed{c = p}$$

put 'p' in eqn ①

$\boxed{y = cx - e^c}$ is the General soln.

$$x - e^p = 0$$

$$x = e^p$$

Apply log o.B.S

$$\log x = \log e^p$$

$$\boxed{\log x = p}$$

put 'p' in eqn ① then

$\boxed{y = (\log x)x - e^{\log x}}$ is singular soln.

H.W

(1) $y = px + p^3$

(2) $y = px - p^2$

(3) $xp^2 - yp + 2 = 0.$

H.W

(1) Let $y = px + p^3$.

Sol:- The given eqn is clauriat's eqn that is of the form $y = px + f(p)$

$$y = px + p^3 \quad \text{--- (1)}$$

diff w.r.to x to eqn (1)

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + 3p^2\frac{dp}{dx}$$

put $\boxed{\frac{dy}{dx} = p}$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}(x + 3p^2)$$

$$\frac{dp}{dx}(x + 3p^2) = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x + 3p^2 = 0$$

I.O.B.S

$\boxed{p = c}$

put 'p' in eqn ①

$\boxed{y = pc + c^3}$ is General soln.

$x = -3p^2$

$\boxed{p = \sqrt{\frac{-x}{3}}}$

sub 'p' in eqn ①

$$y = \left(\sqrt{\frac{-x}{3}}\right)x + \left(\sqrt{\left(\frac{-x}{3}\right)}\right)^3$$

② Let $y = px - p^2$ —①

Sol: It is clairaut's eqn, it is of the form $y = px + f(p)$

diff w.r.to 'x' to eqn ①

$$\frac{dy}{dx} = p(1) + x\frac{dp}{dx} - 2p\frac{dp}{dx}$$

put $\boxed{\frac{dy}{dx} = p}$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}(x - 2p)$$

$$\frac{dp}{dx}(x - 2p) = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x - 2p = 0$$

Apply S.O.B.S

$$\int \frac{dp}{dx} = 0$$

$$\boxed{p = c}$$

Sub 'p' in eqⁿ ①

$$① \Rightarrow \boxed{y = pc - c^2}$$

is General solⁿ

$$x = 2p$$

$$\boxed{p = \frac{x}{2}}$$

put 'p' in eqⁿ ①

$$① \Rightarrow \boxed{y = \left(\frac{x}{2}\right)x - \left(\frac{x}{2}\right)^2}$$

is singular solⁿ.

③

Soln: Let $xp^2 - yp + 2 = 0$

$$yp = xp^2 + 2$$

$$y = \frac{xp^2 + 2}{p} \Rightarrow y = \frac{2}{p} + xp \quad — ①$$

~~It~~ is Clairaut's eqⁿ of the form

$$y = px + f(p)$$

diff w.r.to 'x'

$$\frac{dy}{dx} = \cancel{2\log p}\; 2\left(\frac{-1}{p^2}\right)\frac{dp}{dx} + x\frac{dp}{dx} + p$$

put $\boxed{\frac{dy}{dx} = p}$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}\left(x - \frac{2}{p^2}\right).$$

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$$\frac{dp}{dx}\left(x - \frac{2}{p^2}\right) = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x - \frac{2}{p^2} = 0.$$

$\Rightarrow \int \frac{dp}{dx} = 0 + C$

$\boxed{P = C}$ in eqn ①

$$\boxed{y = xC + \frac{2}{C}}$$

is General soln.

$x = \frac{2}{p^2}$

$P = \sqrt{\frac{2}{x}}$

sub p in eqn ①

$$\boxed{y = \sqrt{\frac{2}{x}}\, x + \frac{2}{\sqrt{\frac{2}{x}}}}$$

is singular soln.

2) method - II :- Solvable for P

→ write the given eqn for P

→ put $\boxed{P = \frac{dy}{dx}}$

→ use variable separable method, separating the variables as 'x' variables and 'y' variables

→ Apply $\int$ O.B.S and solving we get reduced soln.

① solve $p^2 = ax^3$

Sol:- Let $\boxed{p^2 = ax^3}$ — ①

O.B.S we apply power ½

$(p^2)^{1/2} = (ax^3)^{1/2}$

$\boxed{p = a^{1/2} x^{3/2}}$ — ②

put $\boxed{p = \frac{dy}{dx}}$

$\frac{dy}{dx} = a^{1/2} x^{3/2}$

use variable separable method

$dy = a^{1/2} x^{3/2} dx$

Apply $\int$ O.B.S

$\int dy = a^{1/2} \int x^{3/2} dx + C$

$y = a^{1/2} \left[\frac{x^{3/2+1}}{\frac{3}{2}+1} \right] + C$

$= a^{1/2} \frac{x^{5/2}}{5/2} + C$

$\boxed{y = a^{1/2} \frac{2}{5} x^{5/2} + C}$

required solⁿ

(2) solve $p^2-5p+6=0$

Sol:- Let $p^2-5p+6=0$.

$p^2-3p-2p+6=0$.

$p(p-3)-2(p-3)=0$

$(p-2)(p-3)=0$

$p-2=0$ & $p-3=0$

$p=2$

put $\boxed{p=\frac{dy}{dx}}$

$\Rightarrow \frac{dy}{dx}=2$

$dy=2dx$.

Apply S.O.B.S.

$\int dy=2\int dx+c$

$y=2x+c$.

$\boxed{y-2x-c=0}$

$p-3=0$

put $p=\frac{dy}{dx}$

$\frac{dy}{dx}=3$

$dy=3dx$.

$\int dy=3\int dx+c$

$y=3x+c$

$\boxed{y-3x-c=0}$

Here the required solⁿ is

$$\boxed{(y-2x-c)(y-3x-c)=0}$$

③ Solve $p^2 - 2p\cosh x + 1 = 0$

Sol:- Let $p^2 - 2p\cosh x + 1 = 0$

$$p^2 - 2p\left[\frac{e^x + e^{-x}}{2}\right] + 1 = 0 \quad \because \left[\cosh x = \frac{e^x + e^{-x}}{2}\right]$$

$$p^2 - pe^x - pe^{-x} + 1 = 0 \quad \because \left[e^x \cdot e^{-x} = \frac{e^x}{e^x}\right]$$

$$p^2 - pe^x - pe^{-x} + e^x \cdot e^{-x} = 0$$

$$p(p - e^x) - e^{-x}(p - e^x) = 0$$

$$\boxed{(p - e^{-x})(p - e^x) = 0} \quad -②$$

$p - e^{-x} = 0$ & $p - e^x = 0$

$p = e^{-x}$

put $p = \frac{dy}{dx}$

$\frac{dy}{dx} = e^{-x}$

$dy = e^{-x}\,dx$

$\int dy = \int e^{-x}\,dx$

$y = -e^{-x} + c$

$\boxed{y + e^{-x} - c = 0}$

$p = e^x$

put $p = \frac{dy}{dx}$

$\frac{dy}{dx} = e^x$

$dy = e^x\,dx$

$\int dy = \int e^x\,dx + c$

$y = e^x + c$

$\boxed{y - e^x - c = 0}$

The req^d sol^n is $\boxed{(y + e^{-x} - c)(y - e^x - c) = 0}$

H.W

1. $p^2 - 7p + 12 = 0$
2. $p^2 = x^5$
3. $p^2 + p - 6 = 0$

Sol1: Let $p^2 - 7p + 12 = 0$

$$p^2 - 4p - 3p + 12 = 0$$

$$p(p-4) - 3(p-4) = 0$$

$$(p-3)(p-4) = 0$$

$p = 3$

Put $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = 3$$

$\Rightarrow dy = 3\,dx$

Apply $\int$ O.B.S

$$\int dy = 3\int dx + C$$

$$y = 3x + C$$

$$\boxed{y - 3x - C = 0}$$

$p = 4$

Put $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = 4$$

$$dy = 4\,dx$$

Apply $\int$ O.B.S

$$\int dy = 4\int dx + C$$

$$y = 4x + C$$

$$\boxed{y - 4x - C = 0}$$

Here the required solⁿ is

$$\boxed{(y - 3x - C)(y - 4x - C) = 0}$$

② Sol^n

Let $p^2 = x^5$ — ①

∴ O.B.S we apply power $1/2$

$$(p^2)^{1/2} = (x^5)^{1/2}$$

$$\boxed{p = x^{5/2}} \quad — ②$$

put $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = x^{5/2}$$

$$dy = x^{5/2}dx$$

Apply $\int$ O.B.S

$$\int dy = \int x^{5/2}dx + C$$

$$y = \frac{x^{5/2+1}}{5/2+1} + C$$

$$y = \frac{x^{7/2}}{7/2} + C$$

$$\boxed{y = \frac{2}{7}x^{7/2} + C}$$

It is the required sol^n.

③

Sol. Let $p^2 + p - 6 = 0$

$$p^2 + 3p - 2p - 6 = 0$$

$$p(p+3) - 2(p+3) = 0$$

$$(p-2)(p+3) = 0$$

H.W

① $p^2 - 7p + 12 = 0$

② $p^2 = x^5$

③ $p^2 + p - 6 = 0$

①Sol: let $p^2 - 7p + 12 = 0$

$$p^2 - 4p - 3p + 12 = 0$$

$$p(p-4) - 3(p-4) = 0$$

$$(p-3)(p-4) = 0$$

$p = 3$

put $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = 3$$

$\Rightarrow dy = 3\,dx$

Apply $\int$ O.B.S

$$\int dy = 3\int dx + C$$

$$y = 3x + C$$

$$\boxed{y - 3x - C = 0}$$

$p = 4$

put $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = 4$$

$$dy = 4\,dx$$

Apply $\int$ O.B.S

$$\int dy = 4\int dx + C$$

$$y = 4x + C$$

$$\boxed{y - 4x - C = 0}$$

Here the required solⁿ is

$$\boxed{(y - 3x - C)(y - 4x - C) = 0}$$

(2) Sol:- Let $p^2 = x^5$ — (1)

O.B.S we apply power $1/2$

$$(p^2)^{1/2} = (x^5)^{1/2}$$

$$\boxed{p = x^{5/2}} \quad \text{— (2)}$$

put $p = \frac{dy}{dx}$

$$\frac{dy}{dx} = x^{5/2}$$

$$dy = x^{5/2}\, dx$$

Apply $\int$ O.B.S

$$\int dy = \int x^{5/2}\, dx + C$$

$$y = \frac{x^{5/2+1}}{5/2+1} + C$$

$$y = \frac{x^{7/2}}{7/2} + C$$

$$\boxed{y = \frac{2}{7} x^{7/2} + C}$$

It is the required solⁿ.

(3)

Sol:- Let $p^2 + p - 6 = 0$

$$p^2 + 3p - 2p - 6 = 0$$

$$p(p+3) - 2(p+3) = 0$$

$$(p-2)(p+3) = 0$$

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$p-2=0$

$p=2$

put $p=\frac{dy}{dx}$

$\frac{dy}{dx}=2$

$dy=2dx$

Apply $\int$ O.B.S.

$\int dy=2\int dx+c$

$y=2x+c$

$\boxed{y-2x-c=0}$

$p+3=0.$

$p=-3$

put $p=\frac{dy}{dx}$

$\frac{dy}{dx}=-3$

$dy=-3dx$

Apply $\int$ O.B.S

$\int dy=-3\int dx+c$

$y=-3x+c$

$\boxed{y+3x-c=0}$

The required soln is $\boxed{(y-2x-c)(y+3x-c)=0}$

Method - III

Equations solvable for y

→ write the given eqn for y

→ Diff above eqn w.r.to 'x' and put $\frac{dy}{dx}=p$

→ Eliminating 'p' from the above two eqns we get reqd soln.

Method - (ii) :-

solvable for x

procesess :-

→ write the given eq^n for 'x'

→ Diff above eq^n w.r.to 'y' and put $\frac{dx}{dy} = \frac{1}{p}$

→ Eliminating 'p' from above two eq^n's we get the req^d. sol^n.

method - (iii) :-

① $y = (x-a)p - p^2$

Sol :- Let $y = (x-a)p - p^2$ — ①

which is of the form

$y = f(x, p)$

diff eq^n ① w.r.to 'x' O.B.S

$$\frac{dy}{dx} = (x-a)\frac{dp}{dx} + p(1-0) - 2p\frac{dp}{dx}$$

put $\frac{dy}{dx} = p$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}(x-a-2p)$$

$$\frac{dp}{dx}(x-a-2p) = 0$$

$\frac{dp}{dx} = 0.$

I O.B.S

$\int \frac{dp}{dx} = 0 + C.$

$\boxed{p = C}$

put $p = C$ in eq^n ①

① ⇒ $\boxed{y = (x-a)C - C^2}$

② Solve $y = xp^2 + p$

Sol:- The given $y = xp^2 + p$ — ①

is of the form $y = f(x, p)$

diff. Eq^n ① w.r.to x O.B.S

$$\frac{dy}{dx} = x \cdot 2p \frac{dp}{dx} + p^2(1) + \frac{dp}{dx}.$$

put $\boxed{\frac{dy}{dx} = p}$

$$p = p^2 + \frac{dp}{dx}(2px + 1)$$

$$\boxed{p - p^2 = \frac{dp}{dx}(1 + 2px)} \quad \text{— ②}$$

Solving ① & ② we get req^d sol^n.

(3) Solve $y = 3x + \log p$

Sol: Let $y = 3x + \log p$ —(1)

$y = f(x, p)$

diff eqⁿ (1) w.r.to x O.B.S

$$\frac{dy}{dx} = 3 + \frac{1}{p}\frac{dp}{dx}$$

put $\frac{dy}{dx} = p$

$$\boxed{p = 3 + \frac{1}{p}\frac{dp}{dx}} \quad \text{—(2)}$$

Solving (1) & (2) we get reqd solⁿ

m-4

(1) Solve $x - 3y - \log p$

Sol: Let $x = 3y - \log p$ —(1)

which is of the form $x = f(y, p)$

diff eqⁿ (1) w.r.to y O.B.S

$$\frac{dx}{dy} = 3 - \frac{1}{p}\frac{dp}{dy}.$$

put $\left(\frac{dx}{dy} = \frac{1}{p}\right)$

$$\boxed{\frac{1}{p} = \left(3 - \frac{1}{p}\right)\frac{dy}{dx}} \quad \text{—(2)}$$

Solving (1) & (2) we get required solution.

Good