1. The foundations logic & proofs (30)
2. Sets & Relations TF } FT } (30)
3. Algorithm Induction & Recursion
4. Discrete probability
5. Graphs & tree.

Unit 1

Logic & proofs

1. propositional logic T, F. (.)
2. propositional Equivalence.
3. predicates & Quantifiers.
4. Nested Quantifiers
5. Rules of Inference.
6. Introduction to proofs
7. proofs methods & strategy's

Compound proposition - when one or more propositions are connected through various connectives is called compound propositions

① Negation ⟶ T→F, ⟹T. (~p)
② Conjunction → T when both p & q r (T) ∧ (Both)
③ Disjunction - T when any p & q is (T) ∨
④ Conditional - p→Ⓠ→ q false unte motham false (in conclusion)
⑤. Biconditional - p⇄q → T T T, F F T (++|+ / --|+)
⑥ Tautology
⑦ Contradiction

Conjunction — Bride Bridegroom

Disjunction — left eye, right eye

Conditional — Q paper answersheet

Bicondition — ++, --

p	q	p∧q
T	T	T.
T	F	F
F	T	F.
F	F	F.

p	q	p∨q
T	T	T
T	F	T
F	T.	T.
F	F	F

p	q	p→q
T	T.	T
T	F	F
F	T	T
F	F	T

p	q	p⇄q
T	T	T
T	F	F
F	T	F
F	F	T

Find the truth table

$\boxed{\text{2 variable} = 2^2 = 4 \text{ rows}}$

① ~~p∧ p∨~~ $p \wedge (p \vee q)$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	F	F	F
F	T	T	F

$2^2 = 4$ variables.

② $p \wedge (\sim p \vee q)$

p	q	$\sim p \vee q$	$p \wedge (\sim p \vee q)$	$\sim p$
T	T	T	T	F
T	F	F	F	F
F	~~F~~ T	T	F	T
F	F	T	F	T

③ $\sim p \wedge (\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim p \wedge (\sim p \vee \sim q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	T

④ $p \land (p \to q)$.

p	q	$p \to q$	$p \land (p \to q)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

⑤ $(p \land q) \leftrightarrow (p \lor q)$.

t t = t
– – = t

p	q	$p \land q$	$p \lor q$	$(p \land q) \leftrightarrow (p \lor q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

Ⅱ Propositional Equivalence

① Tautology

② Contradiction

③ Contigency

* ④ logical equivalence

Tautology:- A compound position that is always True To.

$(p \vee q) \vee (p \leftrightarrow q)$

p	q	$p \vee q$	$p \leftrightarrow q$	$(p \vee q) \vee (p \leftrightarrow q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

✓

Contradiction:- A compound position that is always False.

$p \wedge (\sim p \wedge q)$

p	q	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

✓

Contigency:- A compound position that is neither a tautology nor a contradiction is called a contigency

$\boxed{(\sim p \wedge q) \vee (p \to \sim q)}$ mix

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \to \sim q$
T	T	F	F	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	F	F

$2^n = 2^3 = 8$ rows

$(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology

sol $p, q, r;\ p \to q,\ q \to r,\ p \to r,\ [(p \to q) \wedge (q \to r)],\ [(p \to q) \wedge (q \to r) \to (p \to r)]$

3T, 3F, 2T, ; 2T, 2F, 2T, 2F ; T, F, T, F, T, F

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)]$	$[(p \to q) \wedge (q \to r) \to (p \to r)]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	F	T	F	T
F	F	F	T	T	T	T	T
F	T	T	F	T	F	F	T
F	T	F	F	F	T	F	T
T	F	T	F	F	T	F	T
T	F	F	F	T	F	F	T

Logical Equivalence

* The compound propositions that have the same truth table values in all possible cases are called logical equivalence

* The compound positions p & q are called logical equivalence if $\boxed{p \Leftrightarrow q}$ is $\boxed{\text{Tautology}}$

* The logical equivalent of a compound propositions p & q is denoted by $p \equiv q$ or $p \Leftrightarrow q$

① logical equivalence of $\sim(p \vee q), (\sim p \wedge \sim q)$ $\quad 2^2 = 4$.

$\boxed{\therefore \sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q}$

p	q	$\sim p$	$\sim q$	(or) $(p \vee q)$	$\sim(p \vee q)$	and $(\sim p \wedge \sim q)$	$\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$
T	T	F	F	T	F	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	T	T

$\therefore \sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$ is a tautology Same

$\therefore \sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent.

Show that $p \wedge (q \vee r)$, $(p \wedge q) \vee (p \wedge r)$ are logically equivalent.

$2^3 = 8$ $\left[\underline{p \wedge (q \vee r)} \Leftrightarrow \underline{(p \wedge q) \vee (p \wedge r)} \right]$

p	q	r	$p \wedge q$	$p \wedge r$	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	T	T	T
F	F	F	F	F	F	F	F	T
F	T	T	F	F	T	F	F	T
F	T	F	F	F	T	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	F	F	F	T

$\Leftrightarrow$

logical equivalence

Tautology

The law of logic

① Laws of Double negation

For any proposition P, $(\sim\sim p) \Leftrightarrow p$

② Idempotent laws

For any proposition P. ⓐ $(p \vee p) \Leftrightarrow p$ ⓑ $(p \wedge p) \Leftrightarrow p$

③ Inverse laws (AA^{-1})

ⓐ $(p \vee \sim p) \Leftrightarrow T_0$ ⓑ $(p \wedge \sim p) \Leftrightarrow F_0$

④ Domination Laws

ⓐ $(p \vee T_0) \Leftrightarrow T_0$ ⓑ $(p \wedge F_0) \Leftrightarrow F_0$

⑤ Commutative laws

ⓐ $(p \vee q) \Leftrightarrow (q \vee p)$ ⓑ $(p \wedge q) \Leftrightarrow (q \wedge p)$

⑥ Absorption laws

ⓐ $[p \vee (p \wedge q)] \Leftrightarrow p$ ⓑ $[p \wedge (p \vee q)] \Leftrightarrow p$

⑦ Demorgan law

ⓐ $\sim(p \vee r) \Leftrightarrow \sim p \wedge \sim q$ ⓑ $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

⑧ Associative laws

ⓐ $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ ⓑ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

⑨ Distributive laws

ⓐ $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ ⓑ $p \vee (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Simplify Compound Propositions by law of Logic.

① $(p \vee q) \wedge [\sim(\sim p) \wedge q]$.

$(p \vee q) \wedge [\sim(\underline{\sim p}) \wedge q]$ [∵ from demorgans Law]

$(p \vee q) \wedge [\underline{\sim(\sim p)} \vee \sim q)]$.

$(p \vee q) \wedge [p \vee \underline{\sim q}]$ [∵ from Double negation law]

a b c

$p \vee (\underline{q} \wedge \sim q)$ [from distributive law].

$p \vee F_0$ [∴ Inverse law].

p. (

$\sim\sim p \to p$.

② $\sim \{[\sim(p \vee q) \wedge r] \vee \sim q\}$

$\sim \{[\sim(p \vee q) \wedge r] \wedge q\}$. Demorgan.

$[(p \vee q) \wedge r] \wedge q$ Double negation

$(p \vee q) \wedge (q \wedge r)$. Associative & Commutative

$\{(p \vee q) \wedge q\} \wedge r$. Associative

$\equiv q \wedge r$ [Absorption law

(2)

$p \vee [p \wedge (p \vee q)] \Leftrightarrow p$. Absorption law.

p or Absolute

$p \vee p \Rightarrow p$. Idempotent.

p

$p \vee [p \wedge (p \vee q)] \Leftrightarrow p$.

Predicates

$x \boxed{>3}$ → predicate

x is greater than '3'.

x
↓
subject / variable.

Ex]:- Let $P(x)$ denotes the statement $\boxed{x > 3}$ what are the truth values of $P(4)$ & $P(2)$

∴ $P(x)$ — variable.
↓
predicate

The predicate is $x > 3$. → $P(x)$.

If $P(4)$, then $x = 4$

$4 > 3$

∴ $P(4)$ is true.

The predicate $x > 3$ → $P(x)$.

If $P(2)$, then $2 = x$.

$2 > 3$.
false

Ex ② Let $Q(x,y)$ denotes the statement $\boxed{x=y+3}$, what are the truth tables of proposition $\boxed{Q(1,2)}$ & $\boxed{Q(3,0)}$

$\downarrow$	$\downarrow$
$x=1,\ y=2$.	$x=3,\ y=0$.
$1=2+3$.	$3=0+3$
$1=5$	$3=3$
$Q(1,2)$ is false	$Q(3,0)$ is true.

predicates :- Compound statements in predicate logic. (&, or, then) [$\vee \wedge$, $\Rightarrow$ $\Leftrightarrow$]

Quantifiers :- All, some, ~~oh~~ no one, for every, there exists. "Quantity"

① Universal Quantifier $\forall$.

② Existential Quantifiers $\exists x Q(x)$

Rules of inferences

a Inconsistent & consistent

eg p & $(\sim p \wedge q)$.

p	q	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

The conjuction of all premisis p & $(\sim p \wedge q)$ is false in all cases.

$\therefore$ p and $(\sim p \wedge q)$ are inconsistant.

② $p \vee q$ and $\sim p$.

$(p \vee q) \wedge \sim p$.

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	~~F~~ T

$p \vee q$ and $\sim p$ all premisis result is false but in 1 case it is true.

$\therefore$ It is consistant

Rules of Inferences

Rule	Tautology	Name
① p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Rule of detachment (modus ponens)
② $p \to q$ $\sim q$ $\therefore \sim p$	$[p \to q \wedge (\sim q)] \to \sim p$	Modus Tollen
③ $p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to p \to r$	Syllogism
④ $p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge (\sim p)] \to q$	Disjunction Syllogism
⑤ p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
⑥ $p \wedge q$ $\therefore p$	$p \wedge q \to p$	Simplification
⑦ p q $- p \wedge q$	(	Conjuction
⑧ $\sim p$		Resoluti

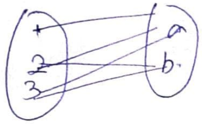
Set is a collection of well defined objects & it is represented by "{ }".

Roaster $\{2, 4, 6 - - -\}$

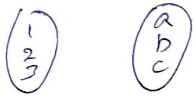
set buil $\{x / x$ is a even no $x < 20)$

$A \times B$.

Cartesian



Equality



$=$.

Subset

(A B) A lo unami matili B to man

Powerset. $A = \{1, 2\}$. $(2^n) = 2^2 = 4$

$P(A) = \{\phi, \{1\}, \{2\} \{1, 2\}\}$

① If $A = \{1, 2, 3\}$ $B = \{a, b, c\}$

Find $A \times B$ & $B \times A$.

$A \times B = \{(1,a) (1,b), (1,c) (2,a) (2,b), (2,c) (3,a) (3,b) (3,c)\}$

$B \times A = \{(a,1), (a,2) (a,3) (b,1) (b,2) (b,3) (c,1) (c,2) (c,3)\}$

$A \times B \neq B \times A$.

$P = \{\phi\}$.

Cartesian product of '3' sets A, B, & C.

Let A, B, C are 3 non-empty sets. Then the cartesian product of A, B, C is defined as

$A \times B \times C = \{(a,b,c) / a \in A, b \in B, c \in C\}$

Eg $A = \{0,1\}$. $B = \{1,2\}$ $C = \{0,1,2\}$.

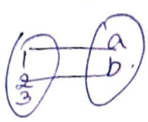
find $A \times B \times C$

$A \times B = \{(0,1)(0,2)(1,1)(1,2)\}$.

$A \times B \times C = \{(0,1,0)(0,2,0)(1,1,0)(1,2,0)$
$(0,1,1)(0,2,1)(1,1,1)(1,2,1)$
$(0,1,2)(0,2,2)(1,1,2)(1,2,2)\}$

<u>Relations</u>: If A & B are

2^{mn}.



$\therefore f\{(1,a)(2,b)\}$.

composition of functions:- consider 3 non empty sets A, B, C are the functions $f: A \to B$ and $g: B \to C$, The composition of these 2 functions are defined as the function $g \circ f: A \to C$ with $g \circ f(a) = g[f(a)]$ $\forall a \in A$.

Let $A = \{1, 2, 3, 4\}$.
$B = \{a, b, c\}$
$C = \{w, x, y, z\}$.
with $f: A \to B$ & $g: B \to C$ given by

$f = \{(1, a), (2, a), (3, b), (4, c)\}$
$g = \{(a, x), (b, y), (c, z)\}$ find $g \circ f$.

$g \circ f(1) = g[f(1)] = g[a] = x$.
$g \circ f(2) = g[f(2)] = g[a] = x$.
$g \circ f(3) = g[f(3)] = g[b] = y$.
$g \circ f(4) = g[f(4)] = g[c] = z$.

$\therefore g \circ f = \{(1, x)(2, x)(3, y)(4, z)\}$

Consider a functions f & g defined by.
$f(x) = x^3$ & $g(x) = x^2+1$, $\forall x \in R$.

Find gof, fog, f^2 & g^2.

$gof = gof(x)$.

$\Rightarrow$ ~~gof~~ $g[f(x)] = g[x^3]$

but $g(x) = x^2+1$ — sub x^3 in x^2.

$= [x^3]^2+1$.

x^6+1.

$fog = f[g(x)]$

$= f[x^2+1]$

but $f(x) = x^3$. — sub x^3 in x^2.

$f[(x^3)^2+1]$.

$f[x^6+1]$

x^6+1.

$g^2 = gog = g[g(x)]$

$= g[(x^2+1)]$.

$(x^2+1)^2+1$

x^4+2x^2+1+1

x^4+2x^2+2.

$f^2 = fof = f[f(x)]$ — $f(x) = x^3$.

$f[x^3]$.

$= f[(x)^3]^3$

x^9.

$f^2 =$

Let f & g be 2 functions from R to R defined by $f(x) = ax+b$

$g(x) = 1-x+x^2$

If $fog(x) = 9x^2 - 9x + 3$.

determine a & b

$fog(x) = f[g(x)]$

$= f(1-x+x^2)$.

w.k.t $f(x) = ax+b$.

$a(1-x+x^2)+b$.

$a-ax+ax^2+b$.

$9x^2-9x+3 = a-ax+ax^2+b$.

Comparing x^2 co-effs on Both sides

$\boxed{9 = a}$

x: $-a = -9$.

const.

$a+b = 3$.

$9+b = 3$

$\boxed{b = -6}$

Q). Let f, g, h be functions from ...
defined by $f(x) = x-1$, $g(x) = 3x$
Determine $f\circ(g\circ h)(x)$ & $(f\circ g)\circ h(x)$ & verify.
$f\circ(g\circ h) = (f\circ g)\circ h$.

$h(x) = \begin{cases} 0, \text{ if its even} \\ 1, \text{ if its odd} \end{cases}$

Sol LHS

$f\circ(g\circ h)(x) = f[g(h(x))] = f[3h(x)] = 3h(x-1)$

$3h(x)-1$.

$\begin{cases} -1 \text{ if } x \text{ is even} \\ 2 \text{ if } x \text{ is odd} \end{cases}$

RHS

$(f\circ g)\circ h(x) = f[g[h(x)]]$

$= f[3h(x)]$

$= 3h(x-1)$

$\{-1$

//.

8). Let f, g, h be functions from $\mathbb{Z}$ to $\mathbb{Z}$
defined by $f(x) = x-1$, $g(x) = 3x$
Determine $f\circ(g\circ h)(x)$ & $(f\circ g)\circ h(x)$ & verify.
$f\circ(g\circ h) = (f\circ g)\circ h$.

$h(x) = \begin{cases} 0, \text{ if its even} \\ 1, \text{ if its odd} \end{cases}$

Sol LHS

$f\circ(g\circ h)(x) = f[g(h(x))] = f[3h(x)] = 3h(x-1)$

$3h(x)-1$

$\begin{cases} -1 \text{ if } x \text{ is even} \\ 2 \text{ if } x \text{ is odd} \end{cases}$

RHS

$(f\circ g)\circ h(x) = f[g[h(x)]]$

$= f[3h(x)]$

$= 3h(x-1)$

$\{-1$

//.

Inverse functions

① Let $A = \{1, 2, 3, 4\}$ and f and g the functions from A to A given by

$f = \{ (1,4) (2,1) (3,2) (4,3) \}$

$g = \{ (1,2) (2,3) (3,4) (4,1) \}$

prove that they are inverse of each other

$gof(1) = g[f(1)] = g[4] = 1 = I_A(1)$

$gof(2) = g[f(2)] = g[1] = 2 = I_A(2)$.

$gof(3) = g[f(3)] = g[2] = 3 = I_A(3)$

$gof(4) = g[f(4)] \quad g[3] = 4 - I_A(4)$

$fog(1) = f[g(1)] = f[2] = 1 = I_A(1)$

$fog(2) = f[g(2)] = f[3] = 2 = I_A(2)$

$fog(3) = f[g(3)] = f\{4\} = 3 = I_A(3)$.

$fog(4) = f[g(4)] = f[1] = 4 = I_A(2)$

$gof(x) = I_A(x)$.

$fog(x) = I_A(x)$.

both are equal!

② Consider the function $f: R \to R$ defined by $f(x) = 2x+5$, let the function $g: R \to R$ be defined by $g(x) = \frac{1}{2}(x-5)$. Prove that g is an inverse of f.

Sol

$f(x) = 2x+5$

$g(x) = \frac{1}{2}(x-5)$.

$\boxed{fog(x)} = f[g(x)] = f[\frac{1}{2}(x-5)]$

$= \cancel{\frac{1}{2}(2x+5)} = \cancel{\frac{2x+5}{2}}$

$= \frac{1}{2}($

w.k.t. $f(x) = 2x+5$.

$2x+5$

$= \cancel{2}[\frac{1}{\cancel{2}}(x-5)] + 5$.

$= x$.

$fog(x) = x = I_R(x)$

$gof(x) = g[f(x)] = g[2x+5]$.

w.k.t $g(x) = \frac{1}{2}(x-5)$.

$\frac{1}{2}(x-5)$.

$\frac{1}{\cancel{2}}(\cancel{2}x+\cancel{5}-\cancel{5})$.

$= x$.

$\boxed{gof(x)} = x = I_R(x)$.

$\therefore$ Both f&g are inv

Properties of Relations

① Relative property : A relation R on a set A is said to be reflexive if $(a,a) \in R, \forall a \in A$.

Ex:- Let us consider that set $A = \{1, 2, 3, 4\}$ A relation R is defined on a set A as. $R = \{(1,1),(2,2)(3,3)(4,4),(1,3)(2,4)\}$ is relation / R is reflex=?

yes is $1 = (1,1)$ ..... etc. (a,a).

② Symmetric Relation :- A relation R is said to be symmetric if whenever $(a,b) \in$ then $(b,a) \in R, \forall a,b \in A$

Ex:- Let a consider a set $A = \{1,2,3,4\}$. A relation R is defined on set A. as $R = \{(1,2)(2,1)(3,3)(4,2)(2,4)\}$ is symmetric or not.

Transitive property - A relation R on a set A is called transitive if whenever $(a,b) \in R$ & $(b,c) \in R$

Then $(a,c) \in R$ $\forall\ a,b,c \in A$

Ex: Let us consider the set $A = \{1,2,3,4\}$

A relation R is defined on set A as

$R = \{(1,2)(2,1)(3,3)(4,2)(2,4)(1,4)\}$ is

=

Representation of relations

A relation can be represented in terms of matrix

Let $R: A \to B$ be a relation where A & B are finite sets containing elements

$A = \{a_1, a_2, a_3 \ldots a_n\}$

$B = \{b_1, b_2, b_3 \ldots b_m\}$

Then 'R' can be represented as $(m \times n)$

$M_R = [m_{ij}]$ where $M_{ij} \begin{cases} 1 \text{ if } (a_i, b_j) \in R \\ 0 \text{ if } (a_i, b_j) \in R \end{cases}$

Find the matrix of relation R on set $A = \{1, 3, 4\}$ where $R = \{(1,1)(1,3)(3,3)($

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

* Q) Find the relation R from set $A = \{a_1, a_2, a_3\}$ to set $\{b_1, b_2, b_3, b_4\}$ with the matrix relation given by

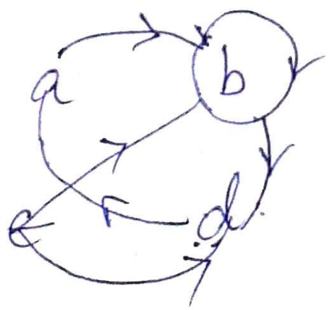
$$M_R = \begin{array}{c|c} & \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$(a_1, b_2)\ (a_2, b_1)\ (a_2, b_3)\ (a_3, b_1)$.

II) Digraph of a Relation

eg: $A = \{a, b, c, d\}$

$R = \{(a,b), (b,b)\ (b,d)\ (c,b)\ (c,d)\ (d,a)$



(29m)

Equivalence Relation :- A relation R on a set A is said to be an equivalence relation on A if

① R is Reflexive. $\{(a,a) \in R, a \in A$

② R is Symmetric $\{(a,b) \in R, (b,a) \in R, a \in R$

③ R is Transitive on A. $\{(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$.

①. Let $A = \{1,2,3,4\}$ & $R = \{(1,1), (1,2) (2,1) (2,2) (3,4) (4,3) (3,3) (4,4)\}$ be a relation on A. Verify that 'R' is an equivalence relation.

Sol. i) R is reflexive $(a,a) \in R, \forall a \in A$.

$(1,1) \in R$
$(2,2) \in R$.
$(3,3) \in R$
$(4,4) \in R$.

$\therefore$ R is reflexive

2) R is symmetric $(a,b) \in R$ & $(b,a) \in R$.

$(1,2) \in R$.	$(3,4) \in R$.
$(2,1) \in R$.	$(4,3) \in R$.

$\therefore$ R is symmetric.

(iii) R: Transitive.

$(a,b) \in R$, $(b,c) \in R$

$(a,c) \in R$.

$(1,2) \in R$

$(2,1) \in R$

$\therefore (1,1) \in R$

$\therefore$ R is transitive

② Let $A = \{1,2,3,4\}$

$R = \{(1,1)(1,2)(2,1)(2,2)(3,1)(3,3)(1,3)$
$(4,1)(4,4)\}$.

Relation is equivalent or not

i) Reflexive $(a,a) \in R$.

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$(4,4) \in R$

$\therefore$ It is reflexive

ii) Symmetric $(a,b) \in R$, $(b,a) \in R$

$(1,2) \in R$	$(4,1)$
$(2,1) \in R$	$(1,4) \times \in R$
$(3,1) \in R$	
$(1,3) \in R$	

$\therefore$ It's symmetric

iii) ~~Tran~~ $(a,b) \in R$, $(b,c) \in R$, then $(a,c) \in R$.

$(4,1) \in R$

$\therefore$ not ~~eq~~ $(4,4) \in R$ It is transitive

③ Let $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. on this set define the relation R by $(x,y) \in R$. If and only if $x-y$ is a multiple of 5. Verify that R is an equivalence relation

Sol Given that

$A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ on the set define the relation R is

$(x,y) \in R \Leftrightarrow x-y$ is a multiple of 6.

i). for every $\boxed{x \in A}$

$x - x = 0$

is a multiple of 5. $0 = 5 \cdot 0$

$(x,x) \in R$

for every $(x,x) \in R$.

$\therefore$ $\boxed{\text{R is reflexive}}$

ii) for every $\boxed{(x,y) \in R}$

$(y,x) \in R$.

$x-y$ is a multiple of 5.

$y-x$ is a muliple of 5.

$-(y-x)$ is a multiple of 5.

$\therefore (y,x) \in R$.

$\therefore$ for every $(x,y) \in R$.

$(y,x) \in R$.

$\therefore$ $\boxed{\text{R is symmetric}}$

iii). for every $\boxed{x,y,z \in A}$

If $(x,y) \in R$, $(y,z) \in R$.

$(x-y) \in R \Rightarrow x-y$ is a multiple of 5

$\boxed{x-y = 5k_1}$

$(y,-z) \in R$

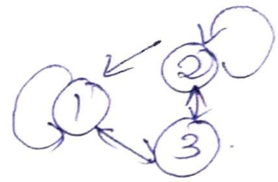
$\boxed{y-z = 5k_2}$

$x-y = (x-y)+(y-z)$

$5k_1 + 5k_2 = 5(k_1+k_2)$

$(x,z) \in R$

4. The diagraph of the relation 'R' on set $A = \{1,2,3\}$ is given as below, Determine whether 'R' is equivalence relation.



$R = \{(1,1)(1,3)(2,1)(2,3)(2,2)(3,1)(3,2)\}$

i) <u>Reflexive</u>

$(a,a) \in R$.
$(1,1) \in R$.
$(2,2) \in R$.
$(3,3) \notin R$.

$\therefore$ R is not reflexive.

$\therefore$ It is not an equivalence relation.

100

5. For a fixed Integer $n > 1$, prove that the relation "Congruent modules 'n'" is an equivalence relation on the set of all integers $\mathbb{Z}$.

i). R is reflexive.

For $a, b \in \mathbb{Z}$

"a is congruent modulo "n" means.

i.e $a \equiv b \pmod n$.

i.e $a-b$ is multiple of n

i.e $a-b = kn$, for some $k \in \mathbb{Z}$.

for any $a \in \mathbb{Z}$.

$a-a=0$.

$a-a$ is a multiple of n.

$\boxed{a \equiv a \pmod n}$.

$\therefore$ R is reflexive.

ii) R is symmetric.

for all $a, b \in \mathbb{Z}$

aRb.

$\Rightarrow a \equiv b \pmod n$.

$\Rightarrow a-b$ is a multiple of x.

$\Rightarrow b-a$ is a multiple of x.

$\Rightarrow -(b-a)$ is a multiple of x

$\Rightarrow b \equiv a \pmod n$.

$\Rightarrow bRa$.

for any $a, b \in \mathbb{Z}$.

$aRb = bRa$

$\therefore$ R is Symmetric

i). R is reflexive.

For $a, b \in \mathbb{Z}$.

"a is congruent modulo "n" means.

i.e $a \equiv b \pmod{n}$.

i.e $a-b$ is multiple of n.

i.e $a-b = kn$, for some $k \in \mathbb{Z}$.

for any $a \in \mathbb{Z}$.

$a-a=0$.

$a-a$ is a multiple of n.

$\boxed{a \equiv a \pmod{n}}$

$\therefore$ R is reflexive.

ii) R is symmetric.

for all $a, b \in \mathbb{Z}$

aRb.

$\Rightarrow a \equiv b \pmod{n}$.

$\Rightarrow a-b$ is a multiple of x.

$\Rightarrow b-a$ is a multiple of x.

$\Rightarrow -(b-a)$ is a multiple of x

$\Rightarrow b \equiv a \pmod{n}$.

$\Rightarrow bRa$.

for any $a, b \in \mathbb{Z}$.

$aRb = bRa$

$\therefore$ R is symmetric

iii) Transitive

for all $a, b, c \in \mathbb{Z}$

aRb, bRc

$\Rightarrow a \equiv b \pmod{n}$ & $b \equiv a \pmod{n}$

$\Rightarrow a-b, b-c$ are multiple of n

$\Rightarrow (a-b)+(b-c)$ is multiple of n.

$\Rightarrow a-c$ is multiple of n

$\Rightarrow a \equiv c \pmod{n}$.

$\Rightarrow aRc$

$\therefore aRb, bRc \Rightarrow aRc$.

R is transitive

$\therefore$ R is reflexive, Symmetric & Trans

$\therefore$ R is equivalence Relation.

Partially Ordered relation POSET.

* A relation R on a set A is said to be partially ordered relation. If a only if

i). R is reflexive.

ii) R is anti symmetric.

iii) R is transitive on A.

A set A with partial order R is defined on it is called partially ordered set.

* Symbol $\leq$, $\subseteq$ / $\geq$, $\supseteq$.

2). Partially Ordered set or (POSET): If $\leq$ is a partially ordering on a set A, then the ordered pair $(P, \leq)$ is called partially ordered (or) POSET.

Totally Ordered Relation

Let $(A, \leq)$ is a partially ordered set. If every 2 elements $a, b \in A$, we have either $a \leq b$ (or) $b \leq a$. Then it is called ordering or linear ordering on A i.e $(A, \leq)$ is called totally ordered set

Note :- $(P, \leq)$ is a dual of $(P, \geq)$
$(P, \geq)$ is a dual of $(P, \leq)$

* A partially ordering $(\leq)$ on a set A can be represented by the means of Hasse Diagram

(1) Show that the relation "greater than or equal" is a partially ordering relation on a set of integers

Sol Given that $Z = \{ ---- -3, -2, -1, 0, 1, 2, 3, 4 \}$

A relation $\geq$ is a poset.

i) $\geq$ is a reflexive

$a \in Z$

$aRa \Rightarrow (a, a) \in R$

$a \geq a \Rightarrow (a, a) \in R$

$\therefore$ given relation is reflexive

ii) $(a,b) \in R, (b,a) \in R \Rightarrow a = b$

(OR)

$(a,b) \in R, (b,a) \notin R \Rightarrow a \neq b$

$aRb, bRa \Rightarrow a = b$

$a \geq b$ & $b \geq a \Rightarrow a = b$

$\therefore$ given relation satisfies anti symmetric

(iii) $(a,b) \in R, (b,c) \in R, (a,c) \in R$

aRb, bRc, aRc

$a \geq b$ & $b \geq c$ & $a \geq c$

$\therefore$ given relation satisfies transitive

$\therefore$ It is a POSET

... Hasse Diagram for
$[\{1,2,3,4,6,9\}, |]$.

sol

$A = \{1,2,3,4,6,9\}$.

R, is divisible relation.

$[A, R]$ — Division relation.

↓

partially ordered set

$1 \to 1,2,3,4,6,9. = (1,1)(1,2)(1,3)(1,4)(1,6)(1,9).$

$2 \to 4,6,2. \quad = (2,4)(2,6):(2,2).$

$3 \to 3,6,9 \quad = (3,3)(3,6)(3,9).$

$4 \to 4 \quad = (4,4).$

$6 \to 6. \quad = (6,6).$

$9 \to 9. \quad = (9,9).$

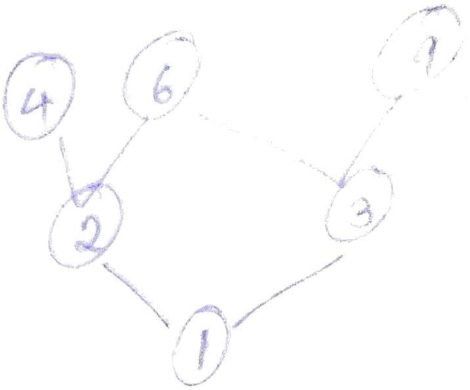
Combinations of all relation

$R = \{(1,1)(1,2)(1,3)(1,4)(1,6)(1,9)(2,4)(2,6).(2,2)$
$(3,3)(3,6)(3,9)(4,4)(6,6)(9,9)\}$

i) (a,a), $a/a \in R$.
∴ It satisfies Reflexive

ii) $aRb, bRa \Rightarrow a=b$. $\quad (1,2) \notin 2R1 \Rightarrow 1=2$

iii) $(1,2) \in R, \quad (1,4) \in R$



① Let $X = \{2, 3, 6, 12, 24, 36\}$ & relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse Diagram.

$2 \rightarrow \{(\cancel{2,2})(2,6)(2,12)(2,24)(2,36)\}$

$3 \rightarrow \{(\cancel{3,3})(3,6)(3,12)(3,24)(3,36)\}$

$6 \rightarrow \{(\cancel{6,6})(6,12)(6,24)(6,36)\}$

$12 \rightarrow \{(\cancel{12,12})\cancel{(12,24)}(12,36)\}$

$24 \rightarrow \{(24,24)\}$

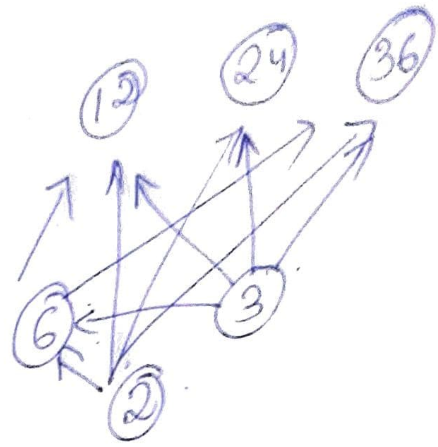
$36 \rightarrow \{(36,36)\}$

In hasse diagram we $\boxed{\text{don't have}}$ $\boxed{\text{self loop}}$

$R = \{(2,6)(2,12)(2,24)(2,36)(3,6)(3,12)(3,24)$
$(3,36)(6,12)(6,24)(6,36)(12,36)\}$.

$\leq$ relation satisfies reflexive, Anti symmetric & transitive

$\therefore$ given set is POSET



12
24
36
6
3
2