Potential Review of Concepts. Ma 442 – Final Exam

1. Solve the Bessel equation of the first kind x^2 y'' + x y' + (x^2 - nu^2) y = 0 for nu = 1.

2. Find the general solution of the Bessel equation of the second kind.

3. Show that the Bessel functions J_nu(x) and Y_nu(x) are linearly independent solutions to the Bessel equation.

4. Find the asymptotic behavior of J_nu(x) for large x.

5. Solve the Bessel equation for nu = 3/2 and find the solution for x.

6. Derive the recurrence relation for the Bessel functions of the first kind J_nu(x).

7. Verify the orthogonality of Bessel functions J_nu(x) for different orders nu.

8. Solve the boundary value problem for J_nu(x) in the interval 0 < x < L with J_nu(0) = 0.

9. Show how the Bessel function J_nu(x) appears in problems of heat conduction in cylindrical coordinates.

10. Apply the solution of the Bessel equation to model vibration modes in a circular drumhead.

11. Compute the Fourier transform of f(x) = e^(-x^2).

12. Find the Fourier transform of a Dirac delta function.

13. Use the Fourier transform to solve the heat equation (partial u/partial t) = k (partial^2 u/partial x^2).

14. Derive the inverse Fourier transform of 1/(1 + xi^2).

15. Show the Fourier transform properties of the step function u(x).

16. Solve the diffusion equation (partial u/partial t) = D (nabla^2 u) in one dimension using Fourier transforms.

17. Compute the Fourier transform of sin(kx).

18. Find the Fourier transform of a rectangular pulse function.

19. Solve the wave equation (partial^2 u/partial x^2) = (1/v^2) (partial^2 u/partial t^2) using the Fourier transform.

20. Derive the convolution theorem for Fourier transforms and use it to solve a differential equation.

21. Solve the Schrödinger equation for a particle in a box using eigenfunctions.

22. Find the eigenfunctions and eigenvalues of a differential operator $A = d^2/dx^2$ with boundary conditions $u(0) = u(L) = 0$.

23. Use the method of separation of variables to solve Laplace's equation in a rectangular domain.

24. Determine the eigenvalues of the operator $A = d^2/dx^2 + x$ with appropriate boundary conditions.

25. Verify that the set of eigenfunctions of a Hermitian operator forms a complete orthonormal basis.

26. Find the eigenfunctions of the angular part of Laplace's equation in spherical coordinates.

27. Compute the eigenvalues of the Hamiltonian for a quantum harmonic oscillator.

28. Find the eigenfunctions and eigenvalues of the Laplace operator in polar coordinates.

29. Solve the time-independent Schrödinger equation in three dimensions for a free particle.

30. Prove that the eigenfunctions of a linear operator form a complete set if the operator is Hermitian.

31. Solve the second-order linear ODE $y'' + 4y = 0$.

32. Solve the first-order linear ODE $y' + y = e^{-x}$.

33. Solve the non-homogeneous ODE $y'' - 2y' + y = e^x$ using the method of undetermined coefficients.

34. Find the general solution to $y'' + y = 0$ with initial conditions $y(0) = 1$, $y'(0) = 0$.

35. Solve the ODE $y'' + 3y' + 2y = 0$ using the characteristic equation.

36. Solve the Cauchy problem for the wave equation $(\partial^2 u/\partial t^2) - c^2 (\partial^2 u/\partial x^2) = 0$ with given initial conditions.

37. Use Laplace transforms to solve the initial value problem $y'' + y = 0$ with $y(0) = 1$, $y'(0) = 0$.

38. Solve the Riccati equation $y' = y^2 + 1$ using separation of variables.

39. Solve the system of first-order linear differential equations $x' = 2x + y$, $y' = x - 3y$.

40. Use the method of variation of parameters to solve $y'' + 2y' + y = e^x$.

41. Normalize the function $f(x) = e^{-x^2}$ over $-\infty < x < \infty$.

42. Show that the Gaussian function is a normalized eigenfunction of the Fourier transform operator.

43. Find the orthonormal basis for the space of square-integrable functions on the interval [0, 1].

44. Show that the set of eigenfunctions of the Laplacian in a circle forms a complete basis.

45. Normalize the sine function $\sin(\pi x)$ on the interval [0, 1].

46. Find the inner product of the functions f(x) = x^2 and g(x) = e^(-x^2) on $[0, \infty)$.

47. Show that the Legendre polynomials form an orthogonal set on [-1, 1].

48. Use the orthonormal basis to expand the function f(x) = x^2 in terms of the Fourier sine series on $[0, \pi]$.

49. Prove that the functions e^(-x^2) and e^(-y^2) are orthogonal with respect to the inner product on $\mathbb{R}^2$.

50. Use the completeness relation to express a function as a Fourier series.

51. Solve the heat equation (partial u/partial t) = D (nabla^2 u) in a circular region.

52. Solve Laplace's equation in spherical coordinates for a point source.

53. Use separation of variables to solve the wave equation (u_tt) = c^2 u_xx in a finite domain with boundary conditions.

54. Solve the Laplace equation in a half-space using Fourier transforms.

55. Solve the 2D heat equation (partial u/partial t) = k (nabla^2 u) with an initial condition of a uniform temperature.

56. Solve the wave equation (u_tt) = c^2 u_xx with Dirichlet boundary conditions u(0, t) = u(L, t) = 0.

57. Solve the Helmholtz equation (nabla^2 u) + k^2 u = 0 in cylindrical coordinates.

58. Use the method of characteristics to solve the linear transport equation (u_t) + a u_x = 0.

59. Solve the wave equation on a finite interval using the Fourier transform.

60. Use the Green's function to solve the Poisson equation (nabla^2 u) = -f(x).

61. Apply the Fourier transform to solve the Poisson equation in a disk.

62. Use Bessel functions to solve the heat equation in cylindrical coordinates.

63. Solve the Maxwell equations for a monochromatic electromagnetic wave using Fourier transforms.

64. Solve a second-order non-homogeneous ODE using Green's functions.

65. Derive the Green's function for the Laplace equation in two dimensions.

66. Use the Fourier transform to solve the quantum mechanical problem for a free particle.

67. Compute the convolution of two functions using their Fourier transforms.

68. Derive the time evolution of a free particle using the Fourier transform.

69. Solve the Schrödinger equation in one dimension using separation of variables.

70. Use the method of separation of variables to solve Laplace's equation in spherical coordinates.

71. Prove that the Fourier transform of a Gaussian function is also Gaussian.

72. Prove that the Bessel functions form a complete set of solutions to the Bessel differential equation.

73. Prove that the eigenfunctions of a Hermitian operator are orthogonal.

74. Show that the Fourier transform of a derivative is F[d/dx f(x)] = i xi f(x).

75. Prove that the Fourier series of a periodic function converges to the function in the L2 norm.

76. Prove Parseval's theorem for Fourier transforms.

77. Prove the Plancherel theorem for Fourier transforms.