3)

Performing Lucas kanade

Given that equations for optical flow (u, v) are

$$u(x, y) = a_1 x + b_1 y + c_1$$
$$v(x, y) = a_2 x + b_2 y + c_2$$

for each pixel assume that optical flow (u, v) and field is constant within a small neighbourhood 'ω'.

For every point $(k, l) \in \omega$:

$$I_x(k, l)u + I_y(k, l)v + I_t(k, l) = 0$$

let the size of ω is $n \times n$

$$\Rightarrow \underbrace{\begin{pmatrix} I_x(1,1) & I_y(1,1) \\ I_x(k,l) & I_y(k,l) \\ \vdots & \vdots \\ I_x(n,n) & I_y(n,n) \end{pmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{u} = \underbrace{\begin{pmatrix} I_t(1,1) \\ I_t(k,l) \\ \vdots \\ I_t(n,n) \end{pmatrix}}_{B}$$

$$A^T \cdot A u = A^T \cdot B$$

Writing this in matrix form

$$\begin{bmatrix} \Sigma_\omega I_x I_y & \Sigma_\omega I_x I_y \\ \Sigma_\omega I_x I_y & \Sigma_\omega I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\Sigma_\omega I_x I_t \\ -\Sigma_\omega I_y I_t \end{bmatrix}$$

(known) (unknown) (known)

Suppose if pixel (x, y) displaced to $(x+v, y+$

$$E(u, v) = \Sigma \left[I(x+u, y+v) - T(x, y) \right]^2$$

$$\Rightarrow \Sigma \left[I(x, y) + u I_x(x, y) + v I_y(x, y) - T(x, y) \right]^2$$

$$= \Sigma \left[u I_x(x, y) + v I_y(x, y) + D(x, y) \right]^2$$

Taking partial derivation and equality to '0'

$$\frac{\delta E}{\delta u} = \Sigma \left[u I_x(x, y) + v I_y(x, y) + D(x, y) \right] I_x(x,$$

$$\frac{\delta f}{\delta v} = \Sigma \left[u I_x(x,y) + v I_y(x,y) + D(x,y) \right] I_y(x,y) = 0$$

writing in the Matrix form

$$\Sigma \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\Sigma \begin{bmatrix} I_x D \\ I_y D \end{bmatrix}$$

<u>Steps to perform</u>

→ Compute I_x, I_y, I_t for the images
→ for each pixel check whether the determinant of 'A' is zero (or) not

if det $|A| == 0$

calculate u and v for that pixels using least squares.