①

① After considering 10 images of chessboard. Let us consider one image and mark pixel and world coordinates of any '6' points on the chessboard

world coordinates (x_w, y_w, z_w)

(4, 2.3)

(5, 3.1)

(6, 3)

(4, 3.4)

(5, 4.1)

(6, 4.3)

pixel coordinate (x_i, y_i)

(256, 184)

(326, 131)

(422, 163)

(312, 116)

(289, 155)

(310, 202)

$$\begin{bmatrix} -x_c z_w & -x_c \\ -y_c z_w & -y_c \end{bmatrix} \begin{pmatrix} m_{11} & m_{31} \\ m_{12} & m_{32} \\ m_{13} & m_{33} \\ m_{14} & m_{34} \\ m_{21} & \\ m_{22} & \\ m_{23} & \\ m_{24} & \end{pmatrix}$$

We know that

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

After multiplying we get

$$x_i = \frac{m_{11}x_w + m_{12}y_w + m_{13}z_w + m_{14}}{m_{31}x_w + m_{32}y_w + m_{33}z_w + m_{34}} \quad -①$$

$$y_i = \frac{m_{21}x_w + m_{22}y_w + m_{23}z_w + m_{24}}{m_{31}x_w + m_{32}y_w + m_{33}z_w + m_{34}} \quad -②$$

$$① \Rightarrow x_w m_{11} + y_w m_{12} + z_w m_{13} + m_{14} - x_w x_i m_{31} - x_i y_w m_{32} - x_i z_w m_{33} - x_i m_{34} = 0$$

$$② \Rightarrow x_w m_{21} + y_w m_{22} + z_w m_{23} + m_{24} - x_w y_i m_{31} - y_w y_i m_{32} - z_w y_i m_{33} - y_i m_{34} = 0$$

Representing them in Matrix form

$$\begin{bmatrix} x_w & y_w & z_w & 1 & 0 & 0 & 0 & 0 & -x_c x_w \\ 0 & 0 & 0 & 0 & x_w & y_w & z_w & 1 & -y_c x_w \end{bmatrix}$$

Writing all the equations for the '6' points

$$
\begin{bmatrix}
4 & 2.3 & 0 & 1 & 0 & 0 & 0 & 0 & -1024 & -588.8 & 0 & -256 \\
0 & 0 & 0 & 0 & 4 & 2.3 & 0 & 1 & -736 & -423.2 & 0 & -184 \\
5 & 3.1 & 0 & 1 & 0 & 0 & 0 & 0 & -1630 & -1001.6 & 0 & -326 \\
0 & 0 & 0 & 0 & 5 & 3.1 & 0 & 1 & -655 & -406.1 & 0 & -131 \\
6 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & -2532 & -1266 & 0 & -422 \\
0 & 0 & 0 & 0 & 6 & 3 & 0 & 1 & -978 & -489 & 0 & -163 \\
4 & 3.4 & 0 & 1 & 0 & 0 & 0 & 0 & -1248 & -1060.8 & 0 & -312 \\
0 & 0 & 0 & 0 & 4 & 3.4 & 0 & 1 & -464 & -394.4 & 0 & -116 \\
5 & 4.1 & 0 & 1 & 0 & 0 & 0 & 0 & -1445 & -1184.9 & 0 & -289 \\
0 & 0 & 0 & 0 & 5 & 4.1 & 0 & 1 & -775 & -635.5 & 0 & -155 \\
6 & 4.3 & 0 & 1 & 0 & 0 & 0 & 0 & -1860 & -1333 & 0 & -310 \\
0 & 0 & 0 & 0 & 6 & 4.3 & 0 & 1 & -1212 & -868.3 & 0 & -202
\end{bmatrix}
$$

$A \rightarrow$ Matrix of all known values

Eigen vector corresponding to the smallest eigen value of $A \cdot A^T$ is Matrix 'M'

Smallest least eigen value of $A \cdot A^T \Rightarrow \lambda = \boxed{0.00}$

$$\begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{pmatrix} = 0$$

Its eigen vector is

$$M = \begin{bmatrix} -0.00 & -0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.0015 & 0.0245 & 3.111 \end{bmatrix}$$

QR decomposition of Matrix 'M' gives the intrinsic & Extrinsic Parameters

QR decomposition of 'M'

$$\text{Intrinsic Matrix} = \begin{bmatrix} 0.413 & -0.02 & 0.10 \\ 0 & 0.532 & -0.07 \\ 0 & 0 & 0.64 \end{bmatrix}$$

$$\text{Extrinsic Rotation Matrix} = \begin{bmatrix} -0.34 & -0.25 & 0.43 \\ 0.41 & -0.37 & 0.03 \\ 0.23 & 0.49 & 0.52 \end{bmatrix}$$

Pixel density of camera $(m_x) = 0.175$

Comparing intrinsic Matrix with

$$\begin{bmatrix} mf & 0 & 0.23 \\ 0 & mf & -0.18 \\ 0 & 0 & 1 \end{bmatrix}$$

$$mf = 0.52$$

$$f = \frac{0.52}{0.175} \text{ cm}$$

focal length (f) = 29.7 cm

Principle point $P(P_x, P_y) = (0.23, -0.18)$

(2) Let us consider pair of images as source and destination.

Consider four points from the source image and corresponding destination image.

Source	destination
P (2, 2)	(1, 2)
Q (7, 6)	(3, 6)
R (9, 3)	(8, 4)
S (10, 5)	(5, 3)

As we know that

$$
\underbrace{\begin{bmatrix}
x_s & y_s & 1 & 0 & 0 & 0 & -x_d x_s & -x_d y_s & -x_d \\
0 & 0 & 0 & x_s & y_s & 1 & -y_d x_s & -y_d y_s & -y_d \\
 & & \vdots & & & & & & \\
x_s & y_s & 1 & 0 & 0 & 0 & -x_d y_s & -x_d y_s & -x_d \\
0 & 0 & 0 & x_s & y_s & 1 & -y_d x_s & -y_d y_s & -y_d
\end{bmatrix}}_{A}
\underbrace{\begin{bmatrix}
h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33}
\end{bmatrix}}_{H} =
$$

$AH = 0$

where 'H' is homography matrix

Now we simplify the above 'A' with the help of x, y coordinates

Eigen vector of smallest eigen value of $A^T \cdot A$ gives homography matrix 'H'.

$$A = \begin{bmatrix} 4 & 2 & 1 & 0 & 0 & 0 & -4 & -2 & -1 \\ 0 & 0 & 0 & 4 & 2 & 1 & -8 & -4 & -2 \\ 7 & 6 & 1 & 0 & 0 & 0 & -21 & -18 & -3 \\ 0 & 0 & 0 & 7 & 6 & 1 & -42 & -36 & -6 \\ 9 & 3 & 1 & 0 & 0 & 0 & -72 & -24 & -8 \\ 0 & 0 & 0 & 9 & 3 & 1 & -24 & -12 & -4 \\ 10 & 5 & 1 & 0 & 0 & 0 & -50 & -25 & -5 \\ 0 & 0 & 0 & 10 & 5 & 1 & -15 & -15 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$A \cdot A^T$ is calulated.

0

$$A^T = \begin{bmatrix} 4 & 0 & 7 & 0 & 9 & 0 & 10 & 0 \\ 2 & 0 & 8 & 0 & 3 & 0 & 5 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 4 & 0 & 7 & 0 & 9 & 0 & 10 \\ 0 & 2 & 0 & 6 & 0 & 3 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ -4 & -8 & -21 & -42 & -72 & -24 & -50 & -15 \\ -2 & -4 & -18 & -36 & -24 & -12 & -25 & -15 \\ -1 & -2 & -3 & -6 & -8 & -4 & -5 & -3 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 42 & 42 & 164 & 246 & 387 & 124 & 306 & 93 \\ 42 & 105 & 246 & 533 & 688 & 291 & 520 & 237 \\ 164 & 246 & 860 & 1548 & 2050 & 732 & 1616 & 591 \\ 246 & 533 & 1548 & 3182 & 3936 & 1546 & 3030 & 1289 \\ 387 & 688 & 2050 & 3936 & 5915 & 2048 & 4346 & 1464 \\ 124 & 291 & 732 & 1546 & 2048 & 827 & 1520 & 658 \\ 306 & 520 & 1616 & 3030 & 4346 & 1520 & 3276 & 1140 \\ 93 & 237 & 594 & 1289 & 1464 & 658 & 1140 & 555 \end{bmatrix}$$

Least eigen value of $A \cdot A^T$ is

$$\lambda = 0.002$$

Eigen vector corresponding to $\lambda = 0.002$

is $[0.431, 0.427, -0.691, -0.973, -0.710, -0.57, 0.97, 1.0]$

Homography Matrix (H) $= \begin{bmatrix} 0.431 & 0.427 & -0.641 \\ -0.473 & -0.710 & -0.57 \\ 0.99 & 1 & 0 \end{bmatrix}$