Indian Institute of Information Technology, Sri City, Chittoor

End-Term Exam - M2023

Name of the Subject: **ASDA** Duration: 90 mins Max Marks: 25

Roll Number: ___________________________ Section:_________

Read the Instructions before proceeding:

1. **Attach the question paper with the answer sheet.**
2. **Write all the answers in the answer sheet.**
3. This is a **closed book exam**. Electronics are not allowed during the examination.
4. **Please Write/Draw legibly**! If we can't understand what you have written, we can't grade it.
5. **Don't use Pencils** for answering/drawing. The final answer **must** be in blue or black ink.

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Part A: Objective [10 Marks]:

1. Which approach is commonly used to solve the Knapsack problem?
   - a) Divide and Conquer b) Greedy Algorithm
   - c) Bottom-Up (Tabulation) d) Backtracking

2. The Floyd-Warshall algorithm for all-pair shortest paths computation is based on:
   - a) Greedy paradigm
   - b) Divide-and-Conquer paradigm
   - c) c) Dynamic Programming paradigm
   - d) Neither Greedy nor Divide-and-Conquer nor Dynamic Programming paradigm.

3. Which of the problems written below cannot be solved by using dynamic programming?
   - a) Fractional knapsack problem b) Matrix chain multiplication problem
   - c) Edit distance problem d) 0/1 knapsack problem

4. You can use a greedy algorithm to solve all the dynamic programming problems.
   - a) True b) False

5. For any directed graph G with $n \geq 2$ vertices and any two vertices s,t in G, the number of distinct paths from s to t is $O(2^n)$
   - a) True b) False

6. Consider a complete undirected graph with a vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

   - a) 7
   - b) 8
   - c) 9
   - d) 10

7. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.

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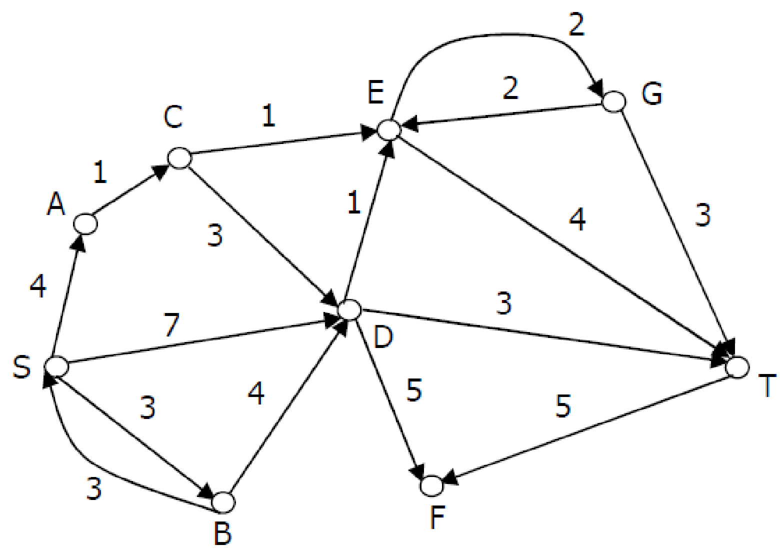

a) S D T
b) S B D T
c) S A C D T
d) S A C E T

8. How might a randomized algorithm be applied to the network flow problem?

a. By randomly selecting augmenting paths.

b. Randomizing the capacities of the edges.

c. Using a random starting point for the Ford-Fulkerson algorithm.

d. Randomly reordering the nodes in the flow network.

9. NP-hard problems are those that:

a. Cannot be solved in polynomial time.

b. Can be solved in exponential time.

c. Are at least as hard as the hardest problems in NP.

d. Are easy to verify.

10. If a problem is NP-hard and in P, what is the implication?

a. P = NP

b. P is a subset of NP

c. NP is a subset of P

d. NP and P are unrelated classes

Part B : Descriptive [15 marks]

Question 1: [5 marks]

A. Determine the longest common subsequence between the given strings "stone" and "Longest" using the

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Tabulation method. Please note that providing only the answer may not result in receiving full marks; make sure to demonstrate the Tabulation method in your solution. [2-marks]

str1 = stone

str2 : longest

		l	o	n	g	e	s	t
	0	1	2	3	4	5	6	7
φ	0	0	0	0	0	0	0	0
s 1	0	0	0	0	0	0	1	1
t 2	0	0	0	0	0	0	1	2
o 3	0	0	1	1	1	1	1	2
n 4	0	0	1	2	2	2	2	2
e 5	0	0	1	2	2	3	3	3

if (A[i] = B[j])

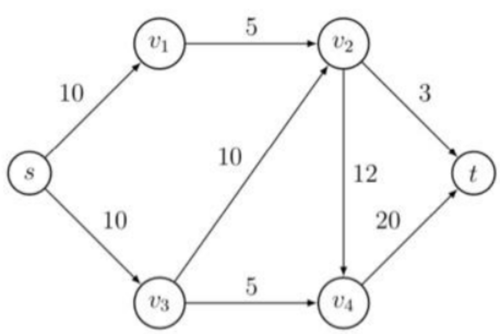
LCS[i, j] = 1 + LCS[i-1, j-1]

else

LCS[i, j] = max(LCS[i-1, j], LCS[i, j-1])

$\boxed{\text{One}}$

B. Determine the shortest path between all pairs of vertices in the given graph using the Floyd-Warshall algorithm. Ensure you present the step-by-step application of the algorithm in your solution, as merely stating the final result may not warrant full credit. [3-marks]



$$A^0 = \begin{array}{c} \\ s \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} \begin{array}{c} \begin{array}{cccccc} s & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{bmatrix} 0 & 10 & \infty & 10 & \infty & \infty \\ \infty & 0 & 5 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c} \\ s \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} \begin{array}{c} \begin{array}{cccccc} s & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{bmatrix} 0 & 10 & 15 & 10 & \infty & \infty \\ \infty & 0 & 5 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ s \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} \begin{array}{c} \begin{array}{cccccc} s & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{bmatrix} 0 & 10 & 15 & 10 & 27 & \infty \\ \infty & 0 & 5 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^k[i,j] = \min\left[A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}[k,j]\right]$$

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$$A^{3} = \begin{array}{c c} & \begin{array}{cccccc} s & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{array}{c} s \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} & \left[\begin{array}{cccccc} 0 & 10 & 15 & 40 & 27 & 18 \\ \infty & 0 & 5 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{array}\right] \end{array}$$

$$A^{4} = \begin{array}{c c} & \begin{array}{cccccc} s & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{array}{c} s \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} & \left[\begin{array}{cccccc} 0 & 10 & 15 & 10 & 27 & 18 \\ \infty & 0 & 5 & \infty & 17 & \infty \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{array}\right] \end{array}$$

$$A^{5} = \begin{array}{c c} & \begin{array}{cccccc} s & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{array}{c} s \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} & \left[\begin{array}{cccccc} 0 & 10 & 15 & 10 & 27 & 18 \\ \infty & 0 & 5 & \infty & 17 & 8 \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{array}\right] \end{array}$$

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$$A^6 = \begin{array}{c} \\ S \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ t \end{array} \begin{array}{c} \begin{array}{cccccc} S & v_1 & v_2 & v_3 & v_4 & t \end{array} \\ \begin{bmatrix} 0 & 10 & 15 & 10 & 15 & 18 \\ \infty & 0 & 5 & \infty & 17 & 8 \\ \infty & \infty & 0 & \infty & 12 & 3 \\ \infty & \infty & 10 & 0 & 5 & 13 \\ \infty & \infty & \infty & \infty & 0 & 20 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \end{array}$$

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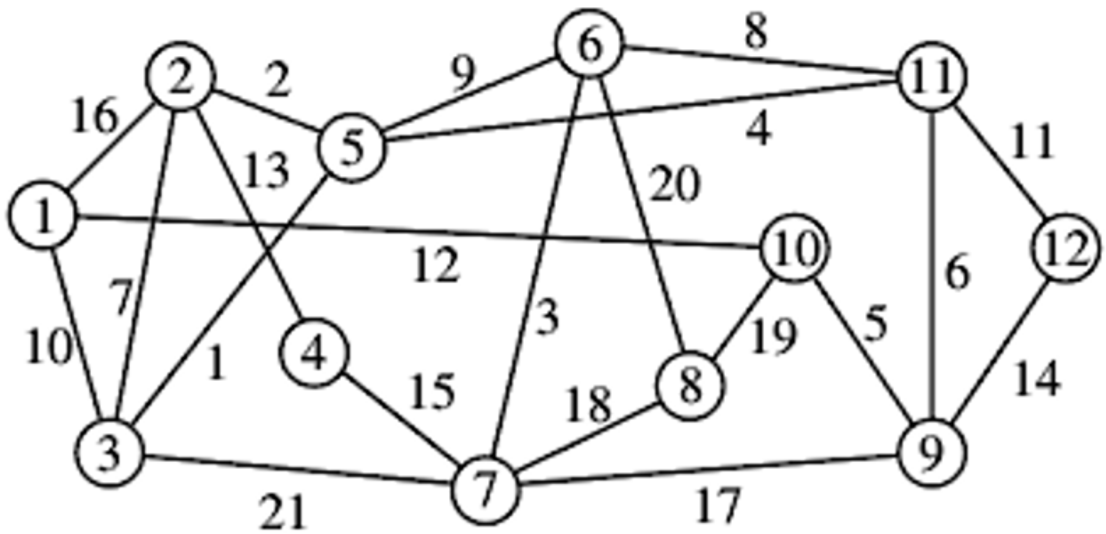
Question 2: [5 marks]

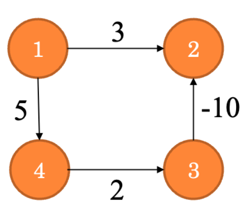
A. Consider the following graph. If we start with node 10 as the starting node and use Prim's algorithm to construct the minimum spanning tree, give the order in which nodes are visited. Also, give the minimum total weight. [3 marks]



Answer: The order is 10, 9, 11, 5, 3, 2, 6, 7, 1, 12, 4, 8. Total weight is 81.

B. Suppose, if you use Dijkstra's algorithm to find the single source shortest path on an undirected graph, what constraint you must use for the algorithm to work and why? Explain with an example. [2 marks]

Answer: Dijkstra's greedy algorithm will not work with negative weight edges. Being a greedy algorithm once a node is added to a path we cannot undo it. If we have negative weight edges then the greedy algorithm cannot work. For example,



When we try to relax the nodes after v time, the other way ends up with better results.

Since it follows a greedy approach, it will not check for all possible solutions to choose the best one.

Question 3: [5 marks]

A. Consider a decision problem Y that is NP-complete. Is it possible for Y to be polynomial-time reducible to another NP-complete problem? Justify your answer [2M]

Yes, it is possible for a decision problem Y that is NP-complete to be polynomial-time reducible to another NP-complete problem. This is allowed and consistent with the definition of NP-completeness.

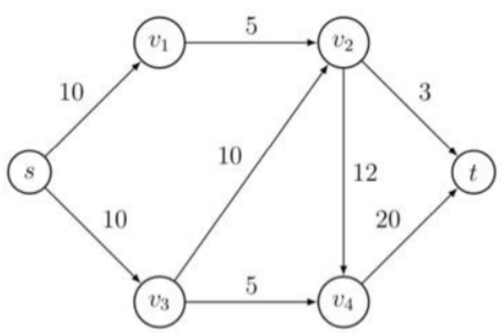
A problem Y is NP-complete if it is in the set NP (nondeterministic polynomial time) and every problem in NP is polynomial-time reducible to Y. If Y is NP-complete and Z is another NP-complete problem, it implies that Y and Z are polynomial-time reducible to each other.

B. List the limitations of the basic Ford-Fulkerson algorithm, especially in the context of finding an optimal solution.[1M]

The Ford-Fulkerson algorithm relies on augmenting paths to increase the flow in the network.

- the algorithm may not always find the optimal solution. It might get stuck in a local optimum and fail to reach the global optimum.
- The initial flow assignment can affect the performance of the algorithm. Depending on how the initial flow is set, the algorithm may converge to different solutions, and it may not always find the optimal solution.
- Non-Deterministic Running Time: The running time of the Ford-Fulkerson algorithm can vary based on the order in which augmenting paths are selected. I

C. Determine the Minimum Cut and Maximum flow capacity for a given graph [1.5]



MinCut ={(V1,V2),(S,V3)}

mAX FLOW= 10+5=15