

Date: November 2023

IIITS/RANAC SQ II_2

Indian Institute of Information Technology, Sri City, Chittoor

Duration: 25 Minutes

Name of the Exam: RANAC SQ II

Max. Marks: 15 Marks

Roll No. :

Name:

Instructions:

1. Calculators are allowed.
2. All questions are mandatory and carry equal mark.
3. Tick the right option.
4. Rough work should not be done in the question paper.
5. Return the question paper along with the rough sheet before leaving the exam hall.

1. Choose the correct option containing all the horizontal and vertical asymptotes for the function

$$y=f(x), \text{ where } f(x) \text{ is given by } \frac{x^3 + 5x + 3}{x^3 - x}$$

- a) $x=0, x=1, y=0$
c) $x=-1, x=0, x=1, y=1$

- b) $x=-1, x=1, y=1, y=0$
d) $x=-1, x=0, x=1, y=0$

2. The value of $\int_0^1 (\log x)^5 dx = \dots$

- a) 5 b) 24 c) -120

- d) None of the above

3. By using the transformation $x+y=2u$ and $y=uv$. Then $dx dy$ is replaced by $\dots du dv$.

- a) $2u$ b) u c) v d) $2v$

4. The function $f(u, v) = v^2 + 4uv + 3u^2 + u^3$ has

- a) has saddle point at $(2/3, -4/3)$
c) has no saddle point
- b) has maximum at $(2/3, -4/3)$
d) has saddle point at $(0,0)$

5. The value of the integral $\int_0^{\frac{\pi}{2}} \sin^2 t \cos^4 t dt = \dots$

- a) $\pi/24$ b) $\pi/32$ c) π d) None of the above

6. Find the area enclosed between $y=\sin x$ and x-axis in $[0, \pi]$

- a) 0 b) 4 c) 1 d) 2

7. Let $f(x, y, z) = xe^{yz}$. Then f_{zxy} is

- a) $(1 + yz)e^{yz}$ b) $(1 + z)e^{yz}$ c) $(1 + z)e^y$ d) $(1 + yz)e^y$

8. The value of $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}} dx = \dots$

- a) 2 b) 5 c) 0 d) 1

9. Which of the following improper integral is convergent?

a) $\int_1^{\infty} \frac{dx}{x^{\frac{3}{2}}}$ b) $\int_2^{\infty} \frac{dx}{x}$ c) $\int_1^{\infty} \frac{dx}{\sqrt{x}}$ d) None of these

10. What does the region of integration $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx dy$ represent?

- a) Semicircle with radius 1 in Q I and II
c) Quarter circle with radius 1 in Q I

- b) Semicircle with radius 1 in Q I and IV
d) Quarter circle with radius 1 in Q IV

11. Which of the following is incorrect?

a) $\beta(m, n) = \beta(n, m)$

b) $\beta(m, n) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(m+n)}$

c) $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$

d) $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$

12. Evaluate $\int_{-10}^{10} \int_{-1}^1 \int_1^3 xyz dy dz dx$

a) -1

b) 10

c) 0

d) 3

13. Evaluate the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$

a) 10

b) Limit does not exist

c) 1/2

d) 0

14. The iterated integral for $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} f(x, y) dy dx$ after changing the order of integration is

a) $\int_0^2 \int_0^{\sqrt{9-y^2}} f(x, y) dx dy$

b) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx dy$

c) $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y) dx dy$

d) $\int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y) dx dy$

15. Suppose that $f(x)$ is increasing and concave up everywhere. Further, $f(2) = 2$ and the tangent line at the point $(2, 2)$ is the line $y = x$. Which of the following can you conclude?

a) $f(5) = 5$

b) $f(5) < 5$

c) $f(5) > 5$

d) none of the above.