

MID EXAMINATION – MONSOON 2025

## Discrete Structures and Matrix Algebra

CSE/ECE/AI&amp;DS:UG1 (IC)

Duration: 90 Mins (04:30-06:00 PM)

Date: 08-10-2025

Max. Marks: 25

Roll No: S20250010192Instructions:

1. All questions are compulsory.
2. Marks are indicated in [ ] after each question.
3. Return the question paper along with the answer sheet before leaving the exam hall.

1. Simplify the following compound proposition step by step to an equivalent expression containing no implications or negations of conjunctions/disjunctions:  
 $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$

Then, based on the derived equivalent expression, verify whether your final expression is a tautology, contradiction, or contingency. Please clearly state which law you are using at each step. [1M]

2. Let the universe of discourse be the set of all real numbers. Consider the statements:

$$P(x): x^2 > 4$$

$$Q(x): x > 2$$

$$R(x): x < -2$$

Express the following statements in English and simplify them. Determine whether these statements are true or false. Justify your answer with proper reasoning. [1M]

- I.  $\forall x[P(x) \rightarrow (Q(x) \vee R(x))]$
- II.  $\exists x[P(x) \wedge \neg(Q(x) \vee R(x))]$

3. Which of the following statements are logically equivalent? [1M]
  - (a) If it rains, then the ground gets wet.
  - (b) If the ground gets wet, then it rains.
  - (c) If the ground does not get wet, then it does not rain.
  - (d) It rains and the ground gets wet.
  - (e) Either it does not rain or the ground gets wet.

4. Express these statements using predicates, quantifiers, and logical connectives. [2M]
  - i. "No cats are dogs."
  - ii. "Every student in DSMA class has studied Math I." (Suppose domain of discourse is all the students of IIIT Sri City.)
  - iii. A negative real number does not have a square root that is a real number.
  - iv. Every insurance agent who sells a policy to at least one person who is not insured refuses to sell policies to anyone who is dishonest.

5. The  $n$ th statement in a list of 129 statements is “Exactly  $n$  of the statements in this list are true.” What conclusions can you draw from these statements? [1 M]
6. Either prove or disprove the following statements [2 M]
- Is  $\forall x \exists y \forall z P(x, y, z)$  equivalent or not equivalent to  $\exists y \forall x \forall z P(x, y, z)$ , where predicate  $P(x, y, z)$  is dependent on  $x, y$ , and  $z$ ?
  - Is  $\exists x \forall y \forall z Q(x, y, z)$  equivalent or not equivalent to  $\forall y \exists x \exists z Q(x, y, z)$ , where predicate  $Q(x, y, z)$  is dependent on  $x, y$ , and  $z$ ?
7. Determine whether this argument is valid. Prove your reasoning.

SRI is an advanced artificial intelligence created to protect humanity. If SRI were both rational and ethical, she would prevent disasters. If SRI were not rational, she would be malfunctioning; if she were not ethical, she would be dangerous. However, SRI does not prevent disasters. If SRI exists, she is neither malfunctioning nor dangerous. Therefore, SRI does not exist. [2M]

8. If  $3n + 2 \equiv 0 \pmod{4}$ , then  $n \equiv 2 \pmod{4}$ . Prove this statement using contradiction. Please clearly write your steps. [ $a \equiv b \pmod{n}$  implies  $n/(a - b)$ ] [1 M]
9. Let  $S$  be the set of all integers defined recursively as follows:  
*Basis step:*  $4 \in S$   
*Recursive step:* Given any integer  $n$  in  $S$ ,  $n + 3 \in S$  and  $n^2 \in S$   
 Use structural induction to prove that for every integer  $n$  in  $S$ ,  $n \equiv 1 \pmod{3}$ . [2 M]

10. Use Mathematical induction to show that

$$\frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n - 1)}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)} \leq \frac{1}{\sqrt{n+1}}$$

for  $n \geq 1$ .

[2M]

11. Let  $a_0, a_1, a_2, \dots$  be the sequence recursively defined by  $a_0 = 1$  and

$$a_n = a_{n-1} + 3n^2$$

for  $n \geq 1$ . Find a formula for  $a_n$  for all  $n \geq 0$  using iteration method and prove the obtained result using mathematical induction. [3M]

12. Which of the following sets are uncountable? Justify your answer? [3M]

- The set of all functions from  $\mathbb{R}$  to  $\{0,1\}$
- The set of all functions from  $\mathbb{N}$  to  $\{0,1\}$
- The set of all finite subset of  $\mathbb{N}$

13. Solve the recurrence relation

$$y_n = 9y_{n-1} - 26y_{n-2} + 24y_{n-3} + 4^n$$

with  $y_0 = 0$ ,  $y_1 = 1$ , and  $y_2 = 10$  to obtain an expression for  $y_n$ .

[3M]

14. Define a bijective function from  $(0, 1)$  to  $\mathbb{R}$  i.e. the set of real numbers. [1M]