INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, SRI CITY

END EXAMINATION -M 2025

Discrete Structures and Matrix Algebra

Date: 28-11-2025

Duration: 180 Mins (9.30AM-12:30 PM) **Max. Marks: 45**

Instructions: **Roll No:______________**

1. All questions are compulsory. Write the answers legibly.
2. Rough work should be done separately on the last page, not with the answer.
3. Answers should be reasoned and derived clearly, not a single word answer.
4. Return the question paper along with your answer sheet before leaving the exam hall.

1	(a) Let the propositional formula be: $$[(p \vee q) \rightarrow (r \wedge \neg q)] \vee \neg(r \rightarrow p)$$ Determine whether it is a tautology, contingency, or a contradiction. **[1M]**
	(b) Consider the following argument involving predicates and quantifiers over domain $\mathbb{R}$. **P1:** $\forall x \in \mathbb{R},\ P(x) \rightarrow (Q(x) \vee R(x))$ **P2:** $\exists x \in \mathbb{R}$ such that $\neg R(x)$ **P3:** $\forall x \in \mathbb{R},\ Q(x) \rightarrow S(x)$ **Conclusion:** $\exists x \in \mathbb{R}$ such that $(P(x) \rightarrow S(x))$ Determine whether the above argument is valid or invalid. If the argument is valid, provide a formal proof using rules of inference and quantifier logic (truth tables are not allowed). **[2M]**
	(c) State the following sentences into logical form. **[2M]** i. "The sum of two positive integers is always positive." ii. "If a person is male and is a parent, then this person is someone's father."
2	(a) Let $S = \left\{(x,y)/x^2 + y^2 = \frac{1}{n^2}\right\}$ where $n \epsilon N$ and either $x \epsilon Q$ or $y \epsilon Q$. Here Q is the set of rational numbers and N is the set of positive integers. Check S is countable or not? Why? **[2M]**
	(b) Prove by the Principle of Mathematical Induction that for every non-negative integer n, the integer $n^3 + 3n^2 + 2n$ is divisible by 6. **[3M]**
3	(a) Solve the recurrence relation: $$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + 2^n$$ with initial conditions $a_0 = 5$, $a_1 = 7$, and $a_2 = 2$. **[3M]**
	(b) Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive 0s or two consecutive 1s. What are the initial conditions? (**Hint:** A ternary string is a sequence (string) made up of three possible symbols, typically: {0,1,2}). **[2M]**
	(c) In a railway network, 60 cities are connected to 15 major junction stations through direct railway lines. What is the minimum number of railway lines required to guarantee that every group of 15 cities has direct rail access to 15 different junction stations (i.e., no two cities in the group depend on the same junction)? **[2M]**

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(a) Answer the following questions for the set $(\{1,2,3,5,6,10,15,30\}, |)$. **[5M]**

i. Is it a partially ordered set (POSET)?
ii. Is it a totally ordered set (TOSET)?
iii. Draw Hasse diagram.
iv. What is the greatest element and least element?
v. Find the least upper bound of $\{2, 5\}$ and the greatest lower bound of $\{6, 15\}$, if it exists.
vi. Is it a lattice or not?

(b) If an equivalence relation $\Re$ *on* $A = \{1,2,3,4,5,6,7\}$ induces a partition

$$A = \{1,2\} \cup \{3\} \cup \{4,5,7\} \cup \{6\}.$$

What is $\Re$ and $|\Re|$? **[1M]**

(c) For $A = \{1,2,3,4\}$, let R and S be the relations on A defined by

$$R = \{(1,2)\ (1,3)\ (2,4)\ (4,4)\} \text{ and } S = \{(1,1)\ (1,2)\ (1,3)\ (2,3)\ (2,4)\}$$

Find RoS, SoR, R^2, and S^2. **[1M]**

(d) Consider the relation R = {(a, a), (a, b), (b, c), (c, c)} on the set A = {a, b, c}. Find the reflexive, symmetric, and transitive Closure of R. **[1M]**

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(a) Let $A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 2 & 1 & 2 & 1 \\ 6 & 5 & 4 & 2 \end{bmatrix}$.

i. Use elementary row operations to decompose A into the form $A = LU$, where L is a lower triangular matrix and U is an upper triangular matrix.
ii. Write the reduced row echelon form of the matrix. **[2M]**

(b) For what values of k, the following matrix has rank (i) 1, (ii) 2, and (iii) 3 **[2M]**

$$\begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & k \end{bmatrix}$$

(c) Is the plane $x - 6y + 2z = 0$ a subspace of $\mathbb{R}^3$. If yes, what is the basis and dimension of the subspace? **[2M]**

(d) Consider the following linear system $Ax = b$ **[4M]**

$$x_1 - x_2 + 2x_3 + 4x_4 = b_1$$
$$2x_1 - 2x_2 + 5x_3 + 9x_4 = b_2$$
$$3x_1 - 3x_2 + 5x_3 + 11x_4 = b_3$$

i. Determine the condition on b_1, b_2, and b_3 to have a solution.
ii. Describe the column space of A. Which geometric object in $\mathbb{R}^3$ does it represent?
iii. Describe the null space of A. List the special solutions in $\mathbb{R}^4$.
iv. Let $b = (1,3,2)$. Find one particular solution x_p and then write the complete solution $x_p + x_n$.

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a. Do the vectors $u_1 = (1,1,0)$, $u_2 = (0,1,1)$, $u_1 = (1,0,1)$ form a basis for $\mathbb{R}^3$? If yes, apply Gram- Schmidt orthogonalization to obtain on orthogonal basis of $\mathbb{R}^3$. **[2M]**

b. Consider the matrix $\begin{bmatrix} 2 & 2 \\ -4 & 8 \end{bmatrix}$ and the vectors $v = [1 \quad 1]$, $w = [2 \quad 1]$. Which of these are eigenvectors, and what are their eigenvalues? **[2M]**

c. Compute the SVD of $\begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & -2 \end{bmatrix}$ **[4M]**

d. Let $v = (a,b,c) \in \mathbb{R}^3$ be a non-zero vector that lies in the orthogonal complement of the row space of the matrix $\begin{bmatrix} 2 & 2 & 7 \\ 3 & 1 & 4 \end{bmatrix}$. If a, b, c are all integers, then what is the smallest possible values of $|a + b + c|$ =? **[2M]**