IIITS/M-2022/Mid Sem - 2

Date: Jan 2023

Indian Institute of Information Technology Sri City (IIITS)

Name of the Exam: DSMA

Duration: 1.5 hrs

Max. Marks: 25

Instructions:

1. **All questions are mandatory.**
2. Marks are indicated in [ ] after each question.
3. Rough Work should be done separately, not in the answer sheet.
4. **Answers should be reasoned and derived clearly, not a single word answer.**

##

1. Let n be a natural number such that

$$a_n = \begin{cases} 1 & \text{if } n = 1 \text{ and } 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$$

Prove by mathematical induction that,

$$a_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{\sqrt{5}\, 2^n}$$

[3M]

2. Two integers are called relatively prime if they have no common divisor greater than 1. Apply pigeonhole principle to prove that, if m and n are relatively prime positive integers, then there is a positive integer k such that $mk \equiv 1 \pmod{n}$. [3M]

3. Find the solution of the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 2^n$ with $a_0 = 1$ and $a_1 = 2$. [4M]

4. Let S be the subset of the set of ordered pairs of integers defined recursively by
Basis Step: $(0, 0) \in S$

Recursive Step: If $(a, b) \in S$, then $(a, b + 1) \in S$, $(a + 1, b + 1) \in S$, and $(a + 2, b + 1) \in S$

List the elements of S produced by the first two applications of the recursive definition. [2M]

5. Let $A = \{1, 2, 3, 4\}$, and $R = \{(1, 2), (2, 3), (3, 4), (2, 1)\}$ on A. Find the transitive closure of R by using relation matrix. [4M]

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6. Use the binomial theorem to show that

$$\sum_{r=0}^{k}\binom{n}{r}\binom{m}{k-r}=\binom{n+m}{k}.$$

[2M]

7. **Information:** A relation R on the set A is **irreflexive** if for every a ∈ A, (a, a) does not belong to R. That is, R is irreflexive if no element in A is related to itself.
   Consider the following relation R on the set {1, 2, 3, 4}:
   R= {(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)}
   R is neither reflexive nor irreflexive.

   Using the above **information**, solve the below problem.
   How many relations are there on a set with n elements that are

a) irreflexive
b) neither reflexive nor irreflexive? [2M+2M]

8. Consider the relation "~" on the set of integers where x~y if and only if x+y is divisible by 2.
   a) Prove that "~" is an equivalence relation on the set of integers.
   b) Clearly write all the different equivalence classes for this equivalence relation.

[1.5+1.5]