IIITS/PS-2023/MID-II

Exam Date: 10-05-2023

Indian Institute of Information Technology Sri City (IIITS)

Name of the Exam: PS_MID - II | Duration: 1.5 hrs | Max. Marks: 25

Instructions:

1. **All questions are mandatory.**
2. **Marks are indicated in [ ] after each question.**
3. **Rough Work should be done separately, not in the answer sheet.**
4. **Answers should be reasoned and derived clearly, not a single word answer.**
5. **Preferably use a ballpoint pen.**
6. **You can use a calculator.**

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1. Suppose each random variable X and Y follow the exponential distribution with parameter 1, and are independent. Find the PDF of $U = X - Y$. [4 M]

2. Let the probability density function of the joint random variables X_1 and X_2 be given by

$$f_{X_1 X_2}(x_1, x_2) = \begin{cases} e^{-(x_1 + x_2)} & x_1 \geq 0,\ x_2 \geq 0 \\ 0 & elsewhere \end{cases}$$

If $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1 + X_2}$. Find the joint distribution of Y_1 and Y_2. [3 M]

3. Let Y be a random variable, and the PDF given is by $f_Y(y) = 2(1-y)$ for $0 \leq y \leq 1$. Find the density function of $U = 2Y - 1$. [2 M]

4. Show by Chebyshev's inequality that in 2000 throws of a coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$. $P(|X-\mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$ [2 M]

5. Examine if the condition in the Law of Large Numbers is satisfied by the sequence of independent random variables $\{X_N\}$, where $X_N = \pm\sqrt{2n-1}$ with probability $\frac{1}{2}$. [2 M]

6. The joint probability density function is given by

$$f_{XYZ}(x, y, z) = (x+y)\, e^{-z} \text{ for } 0 < x < 1,\ 0 < y < 1,\ z > 0$$

$$= 0 \quad \text{elsewhere.}$$

Show that the random variables X, Y and Z are not independent but the random variables X and Z are pairwise independent. [4 M]

7. The joint probability mass function of X and Y is given below. [3 M]

X \ Y	1	2	3
1	0.2	0.3	0.1
2	0.3	0.1	0

Find the Cov(X,Y) and Correlation (X,Y). Comment upon the nature of the correlation between X and Y.

8. Let X be distributed as $N_3(\mu, \Sigma)$ where $\mu^T = (2, 1, 1)$ and $\Sigma = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & -1 \end{pmatrix}$

Verify whether the random variables (X_1, X_3) and X_2 are Statistically Independent or not.

[2 M]

9. A bank employee serves customers standing in the queue one by one. Suppose that the service time X_i for customer i has mean $E(X_i) = 2$ (minutes) and $Var(X_i) = 1$. We assume that service times for different bank customers are independent. Let Y be the total time the bank employee spends serving 50 customers. Find $P(90 < Y < 110)$. [3 M]

(Here $\phi(\sqrt{2}) \approx 0.9207302$)