Physics in Julia: combining **Unitful.jl** and **DifferentialEquations.jl**

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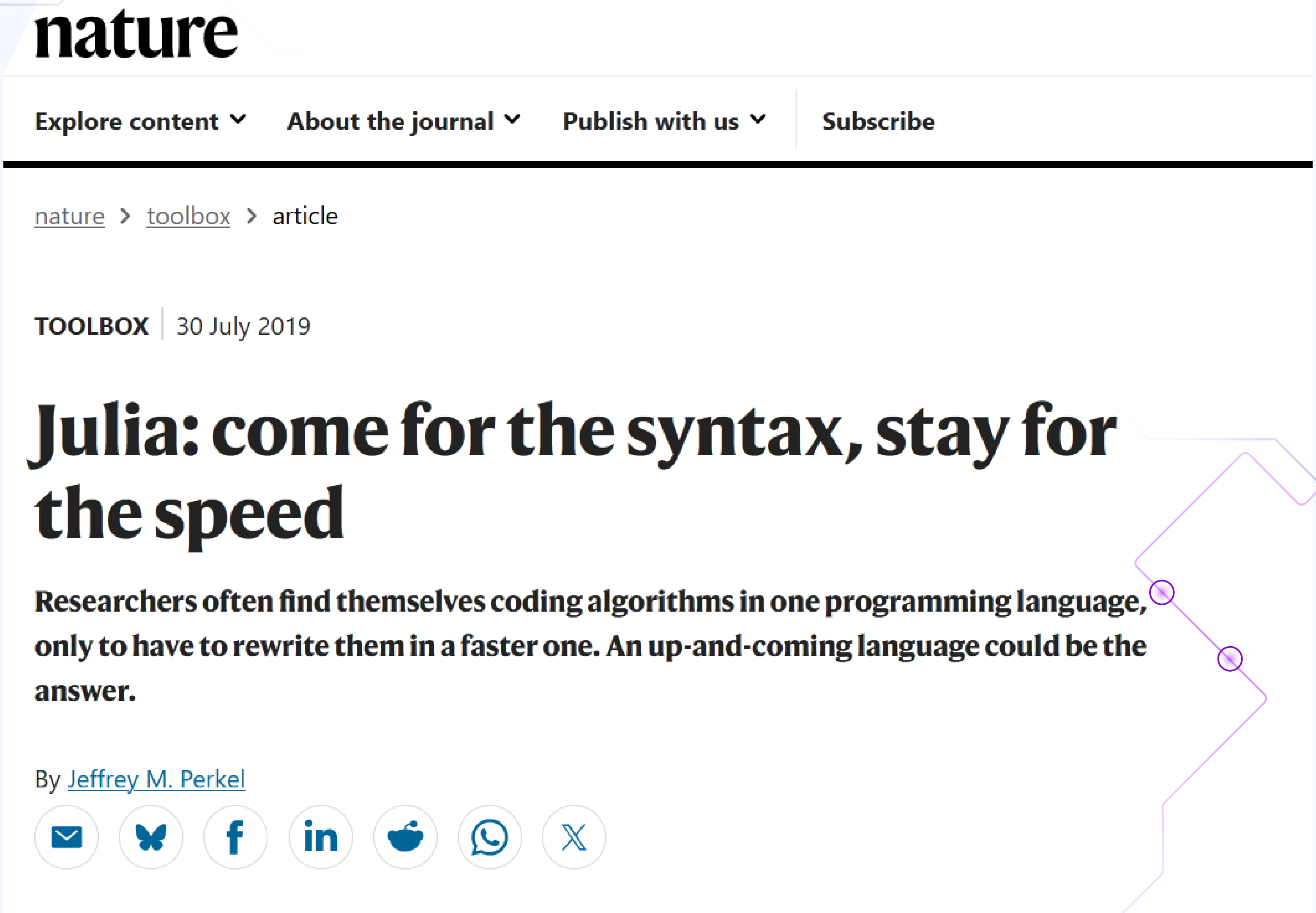
Departament of Physics and Applied Computer Science



TOOLBOX | 30 July 2019

Julia: come for the syntax, stay for the speed

Researchers often find themselves coding algorithms in one programming language, only to have to rewrite them in a faster one. An up-and-coming language could be the answer.

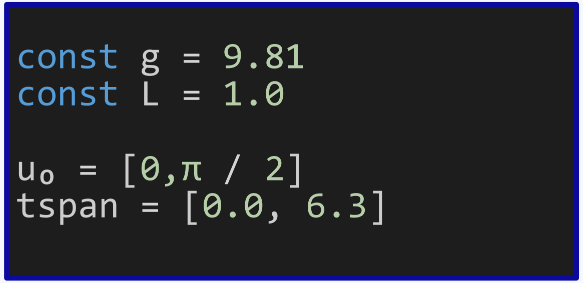
By Jeffrey M. Perkel

“For those who want some proof that physicists are human, the proof is in the idiocy of all the different units which they use for measuring energy”

— Richard P. Feynman (Nobel’65), The Character of Physical Law

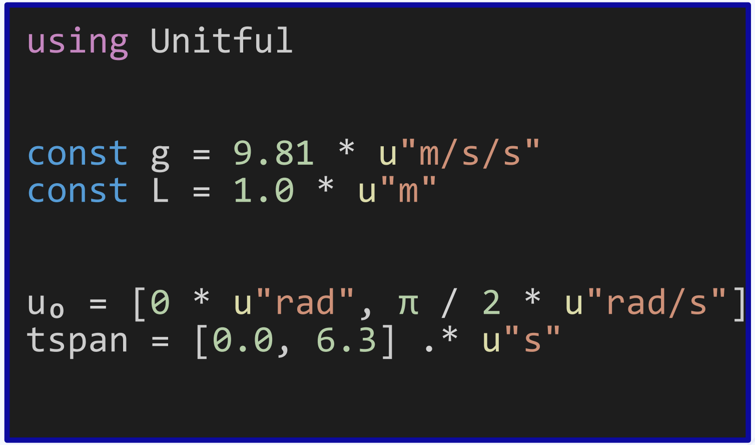
Unitful.jl

- **Code readability/maintainability**
- Conversions and prefixes
- Compile-time dimensionality check
- Tool for unit testing
- Automatic units on plots
- No runtime overhead



```julia
const g = 9.81
const L = 1.0

u₀ = [0,π / 2]
tspan = [0.0, 6.3]
```



```julia
using Unitful


const g = 9.81 * u"m/s/s"
const L = 1.0 * u"m"


u₀ = [0 * u"rad", π / 2 * u"rad/s"]
tspan = [0.0, 6.3] .* u"s"
```

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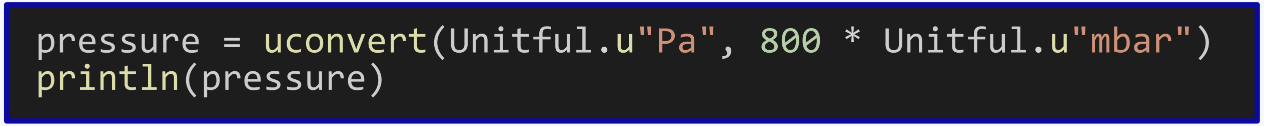
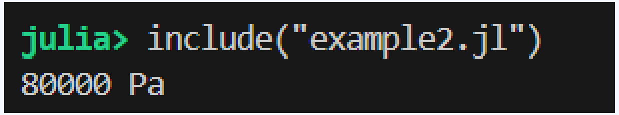


```julia
pressure = uconvert(Unitful.u"Pa", 800 * Unitful.u"mbar")
println(pressure)
```



```
julia> include("example2.jl")
80000 Pa
```

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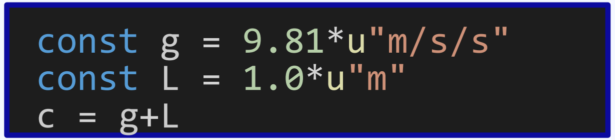
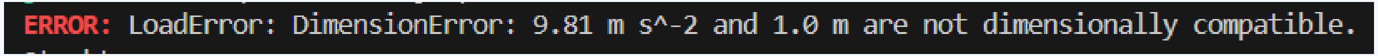


```julia
const g = 9.81*u"m/s/s"
const L = 1.0*u"m"
c = g+L
```



```
ERROR: LoadError: DimensionError: 9.81 m s^-2 and 1.0 m are not dimensionally compatible.
```

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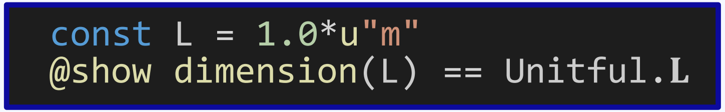
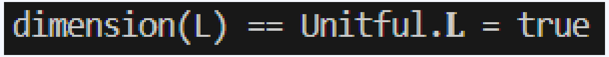


```julia
const L = 1.0*u"m"
@show dimension(L) == Unitful.L
```

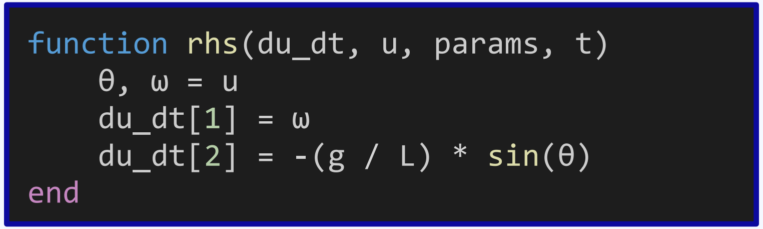


```
dimension(L) == Unitful.L = true
```

DifferentialEquations.jl

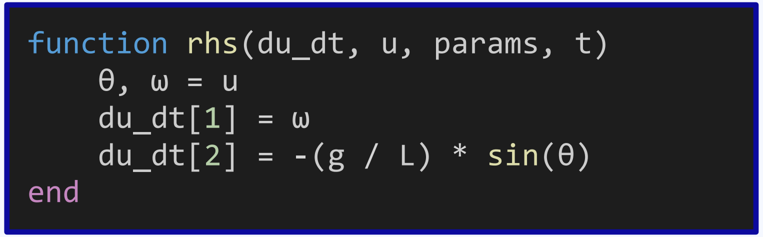
Image by Manfred Antranias Zimmer from Pixabay

$$\frac{d^2\theta}{dt^2} = \frac{g}{L} sin\theta$$



```julia
function rhs(du_dt, u, params, t)
    θ, ω = u
    du_dt[1] = ω
    du_dt[2] = -(g / L) * sin(θ)
end
```

DifferentialEquations.jl



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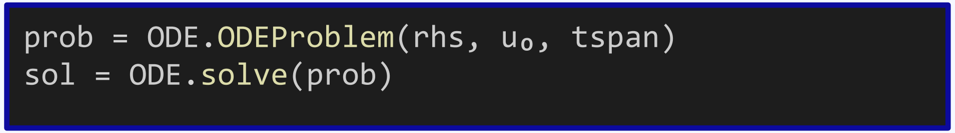
This package enables to solve the system of ordinary differential equations in just **two lines** of code:

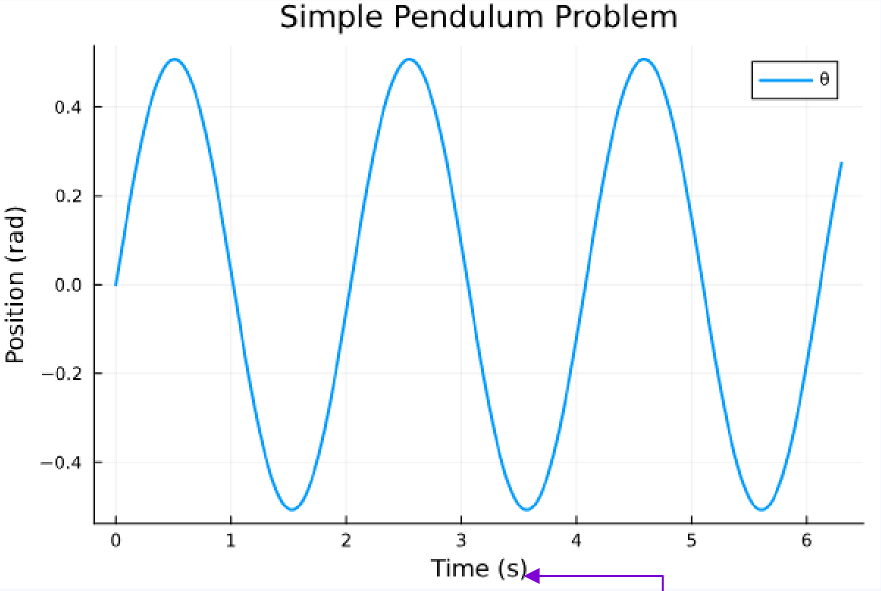


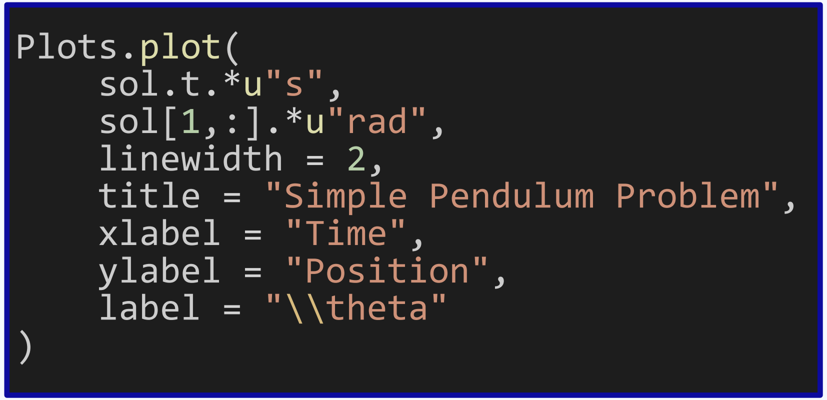
```julia
prob = ODE.ODEProblem(rhs, u₀, tspan)
sol = ODE.solve(prob)
```

← Create differentail equation problem

← Solve it!

DifferentialEquations.jl

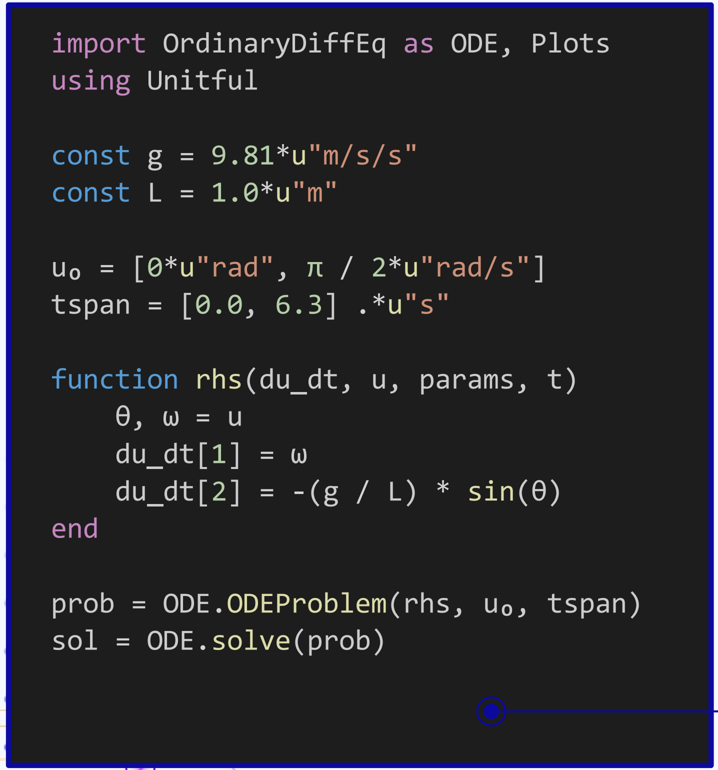




```
Plots.plot(
    sol.t.*u"s",
    sol[1,:].*u"rad",
    linewidth = 2,
    title = "Simple Pendulum Problem",
    xlabel = "Time",
    ylabel = "Position",
    label = "\\theta"
)
```

Automatic labeling!

Let’s try together: Unitful.jl + DiffEq.jl



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import OrdinaryDiffEq as ODE, Plots
using Unitful

const g = 9.81*u"m/s/s"
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u₀ = [0*u"rad", π / 2*u"rad/s"]
tspan = [0.0, 6.3] .*u"s"

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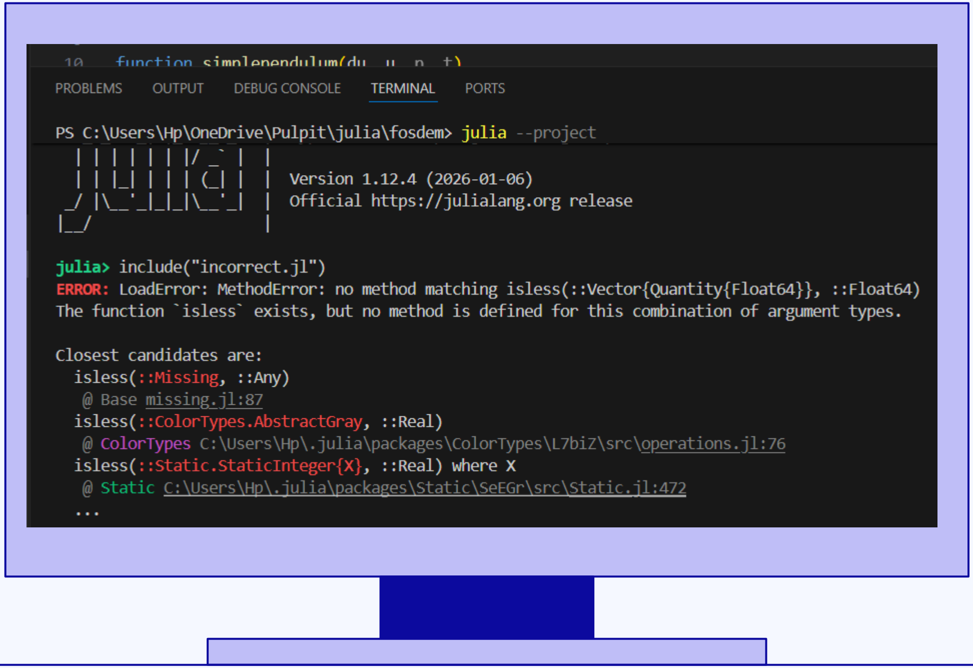


```
function simplependulum(du, u, p, t)
```

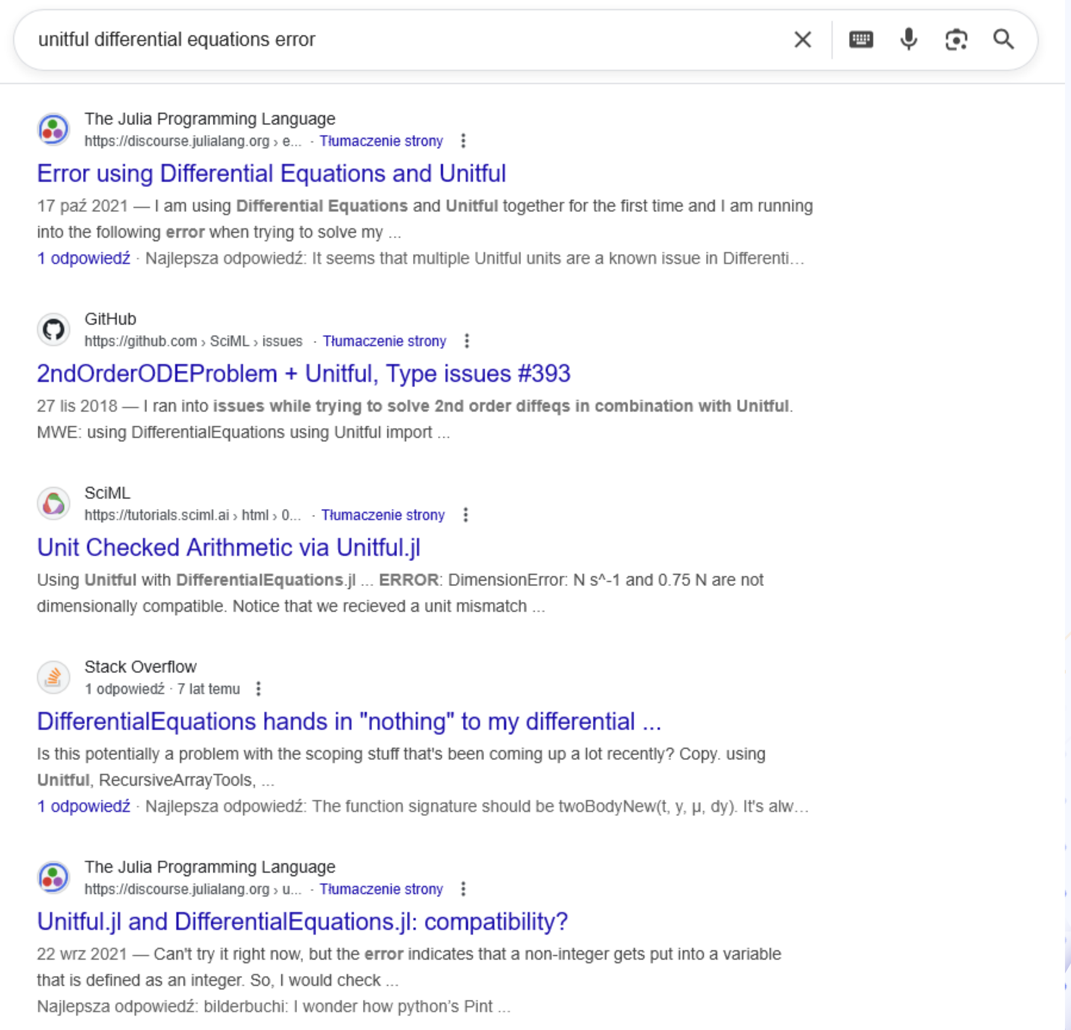
```
PS C:\Users\Hp\OneDrive\Pulpit\julia\fosdem> julia --project
  | | | | | | |/ _` |  |
  | | |_| | | | (_| |  |  Version 1.12.4 (2026-01-06)
 _/ |\__'_|_|_|\__'_|  |  Official https://julialang.org release
|__/                   |

julia> include("incorrect.jl")
ERROR: LoadError: MethodError: no method matching isless(::Vector{Quantity{Float64}}, ::Float64)
The function `isless` exists, but no method is defined for this combination of argument types.

Closest candidates are:
  isless(::Missing, ::Any)
   @ Base missing.jl:87
  isless(::ColorTypes.AbstractGray, ::Real)
   @ ColorTypes C:\Users\Hp\.julia\packages\ColorTypes\L7biZ\src\operations.jl:76
  isless(::Static.StaticInteger{X}, ::Real) where X
   @ Static C:\Users\Hp\.julia\packages\Static\SeEGr\src\Static.jl:472
  ...
```

What does the community say?

- errors inevitable
- „solution”: drop units
- only basic solvers compatible



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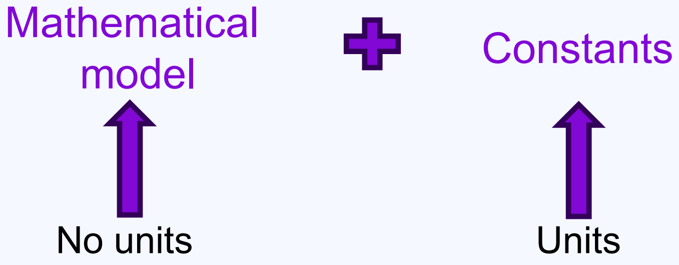
Najlepsza odpowiedź: bilderbuchi: I wonder how python's Pint ...

"… the astute observer (Fourier was the first astute observer) notices that the equations … **remain true when** the size of the fundamental **units is changed**"

— Percy W. Bridgman (Nobel'46), Dimensional Analysis

Separation of Concerns

Physics:



Mathematical model + Constants

No units → Mathematical model

Units → Constants

Software engineering:

- Make **injectible dependency** holding all constants
- Use **dependency with units** for dimensionality calculations
- Inject **dependecy stripped** of units for numerical calculations

What to change so it does work?

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import OrdinaryDiffEq as ODE, Plots
using Unitful

const g = 9.81*u"m/s/s"
const L = 1.0*u"m"

u₀ = [0*u"rad", π / 2*u"rad/s"]
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```julia
import OrdinaryDiffEq as ODE, Plots
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const_u = (
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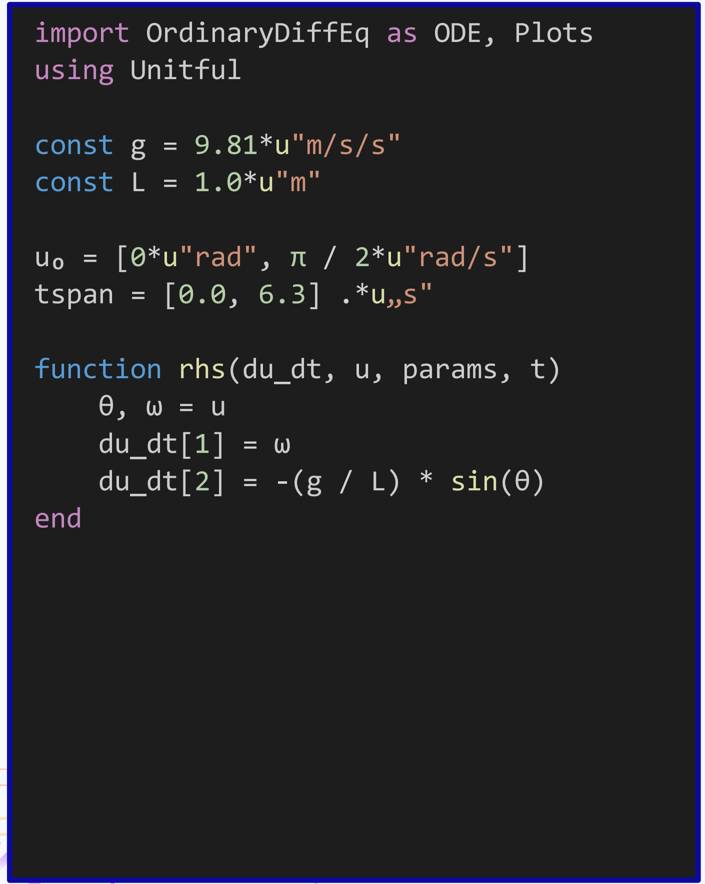

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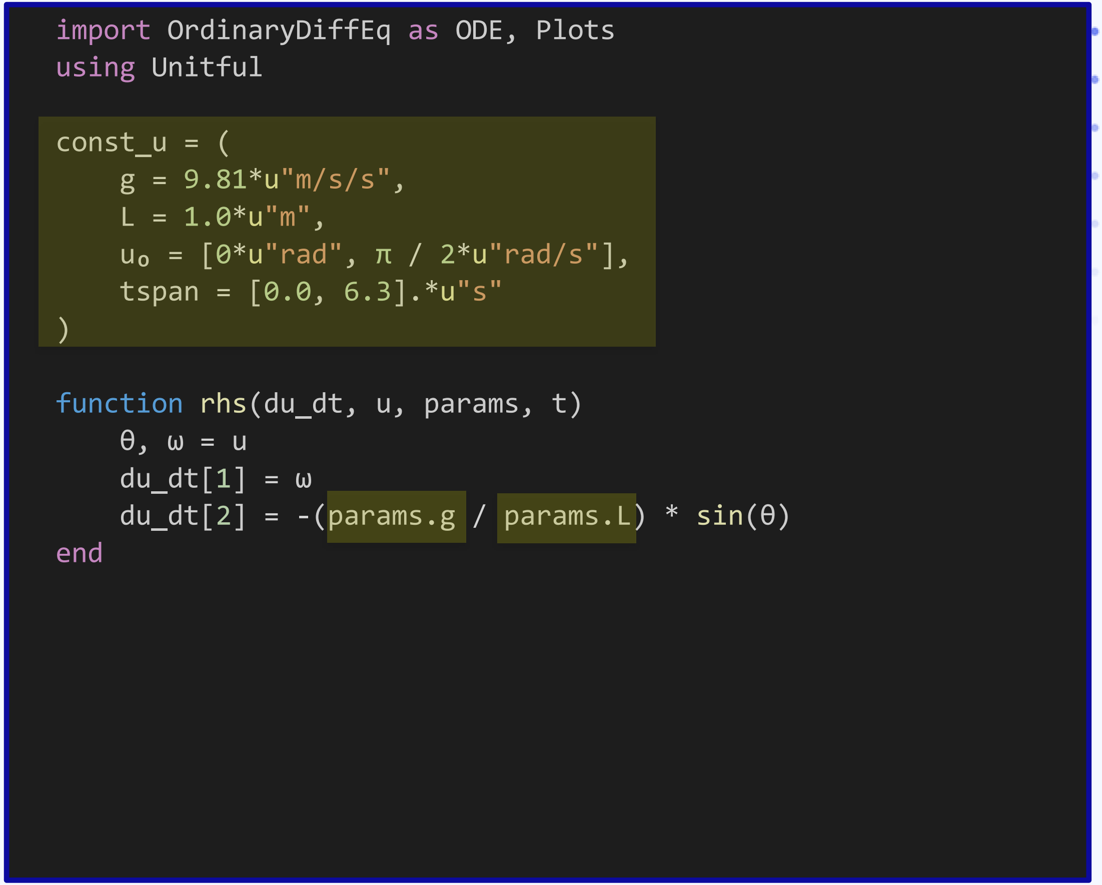

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)

function rhs(du_dt, u, params, t)
    θ, ω = u
    du_dt[1] = ω
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    du_dt[2] = -(params.g / params.L) * sin(θ)
end

strip_units(x) = ustrip(x)
strip_units(x::AbstractArray) = ustrip.(x)

c = NamedTuple{keys(const_u)}(strip_units.(values(const_u)))
```
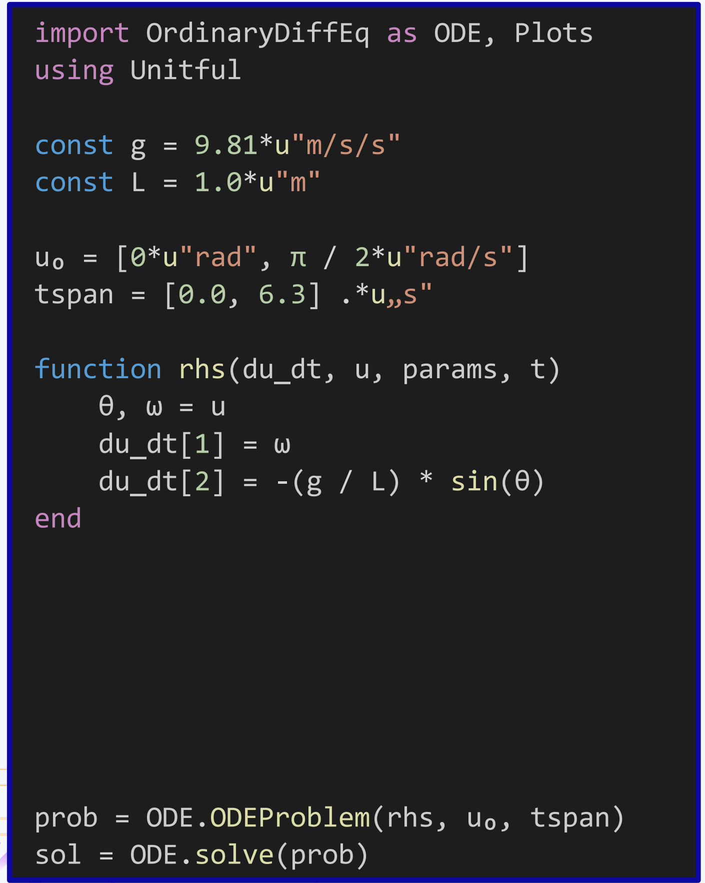

What to change so it does work?



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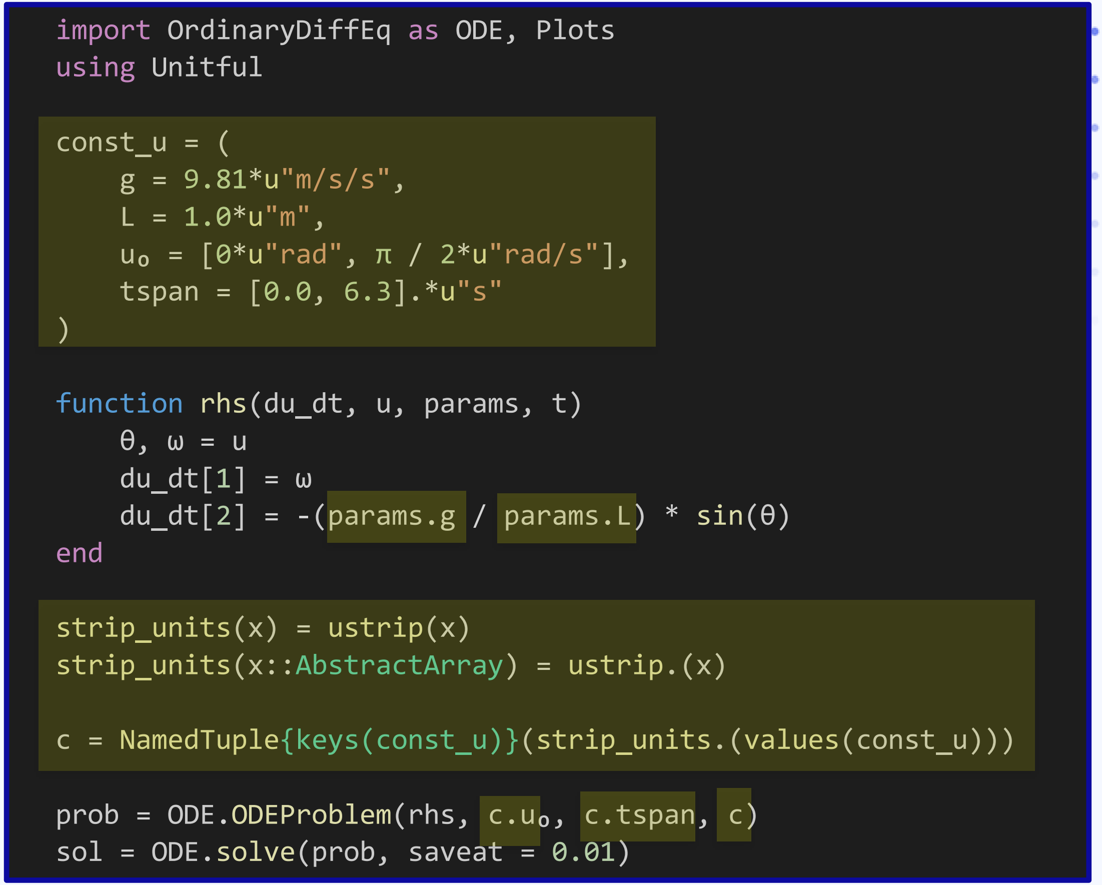
sol = ODE.solve(prob)
```



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function rhs(du_dt, u, params, t)
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end

strip_units(x) = ustrip(x)
strip_units(x::AbstractArray) = ustrip.(x)

c = NamedTuple{keys(const_u)}(strip_units.(values(const_u)))

prob = ODE.ODEProblem(rhs, c.u₀, c.tspan, c)
sol = ODE.solve(prob, saveat = 0.01)
```
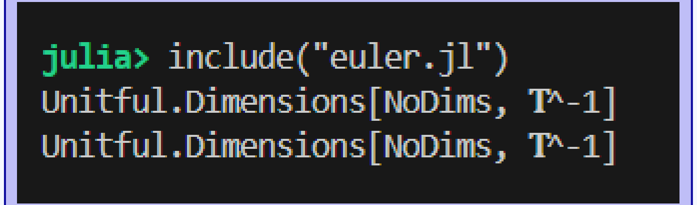
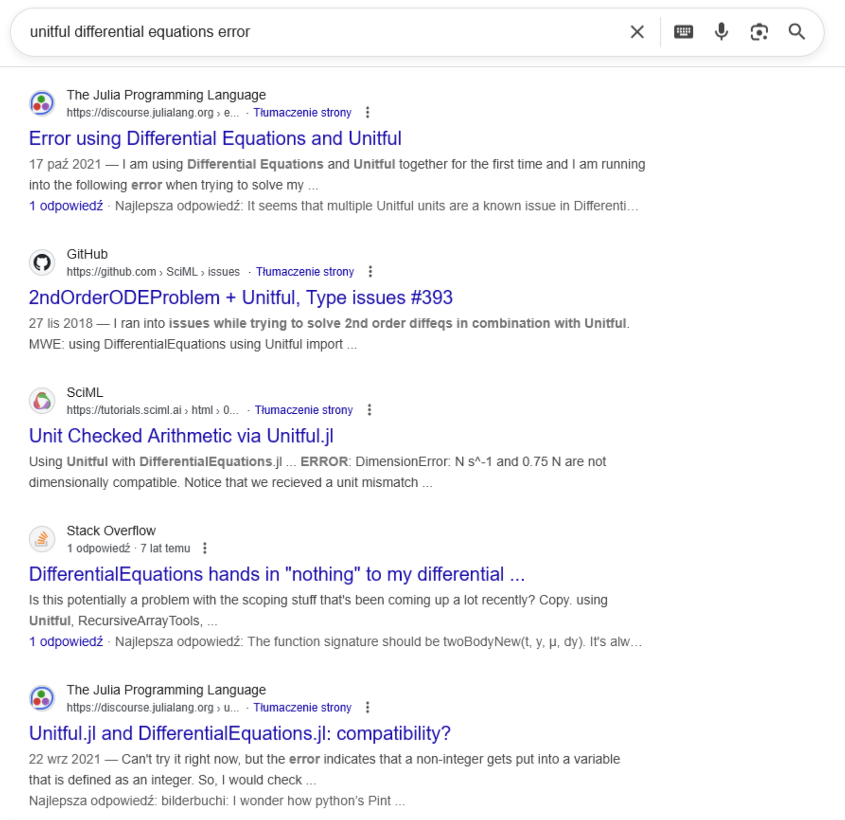

Note: the function works with units too

```julia
du_dt = similar(const_u.u₀)

rhs(du_dt, const_u.u₀, const_u, 0.1u"s")

u = const_u.u₀ + du_dt * 0.1u"s"



@test dimension.(u) == dimension.(const_u.u₀)

println(dimension.(u))

println(dimension.(const_u.u₀))
```



```
julia> include("euler.jl")
Unitful.Dimensions[NoDims, 𝐓^-1]
Unitful.Dimensions[NoDims, 𝐓^-1]
```

Take-home message:
there is no Unitful.jl + DiffEq.jl problem!



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- ODE definition should be **invariant to units**

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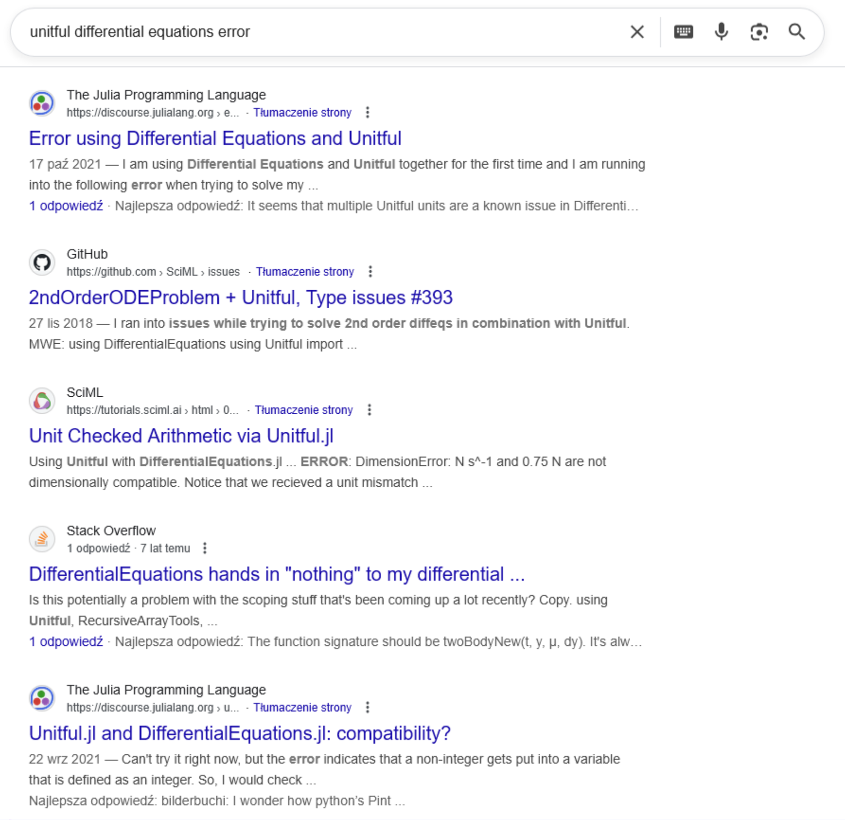

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- ODE definition should be **invariant to units**
- **units** injected through constants only

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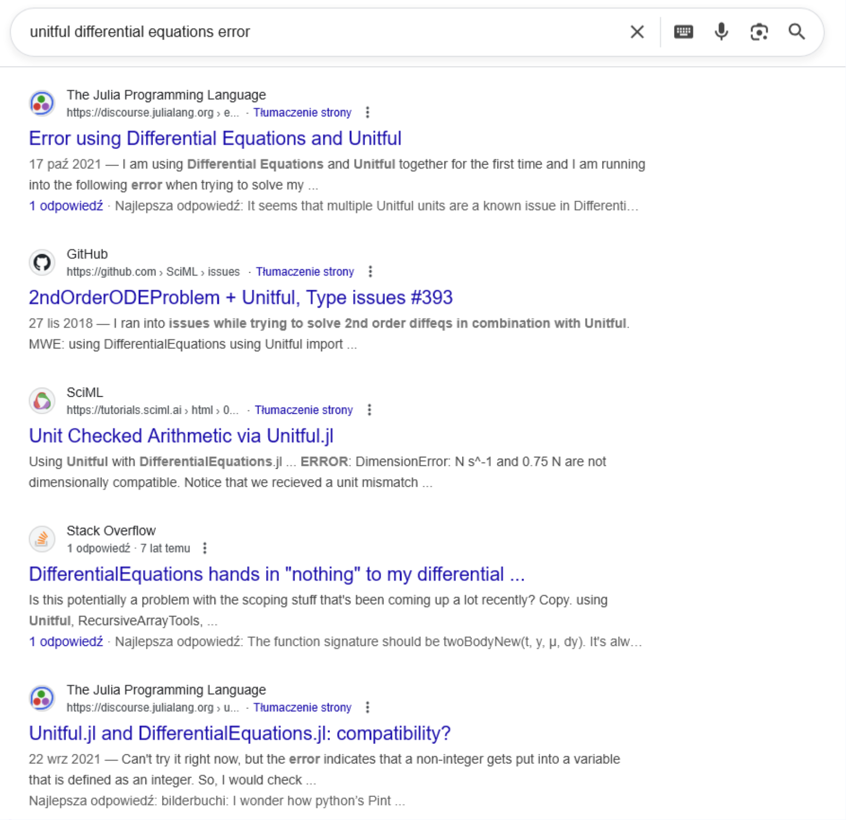

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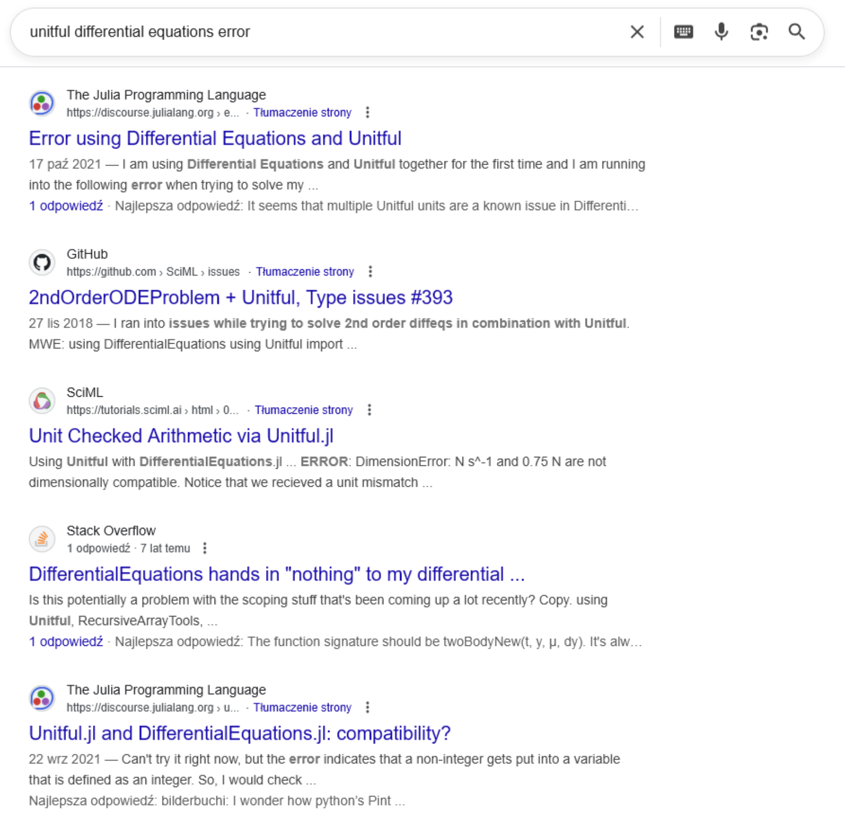
Take-home message: there is no Unitful.jl + DiffEq.jl problem!



unitful differential equations error

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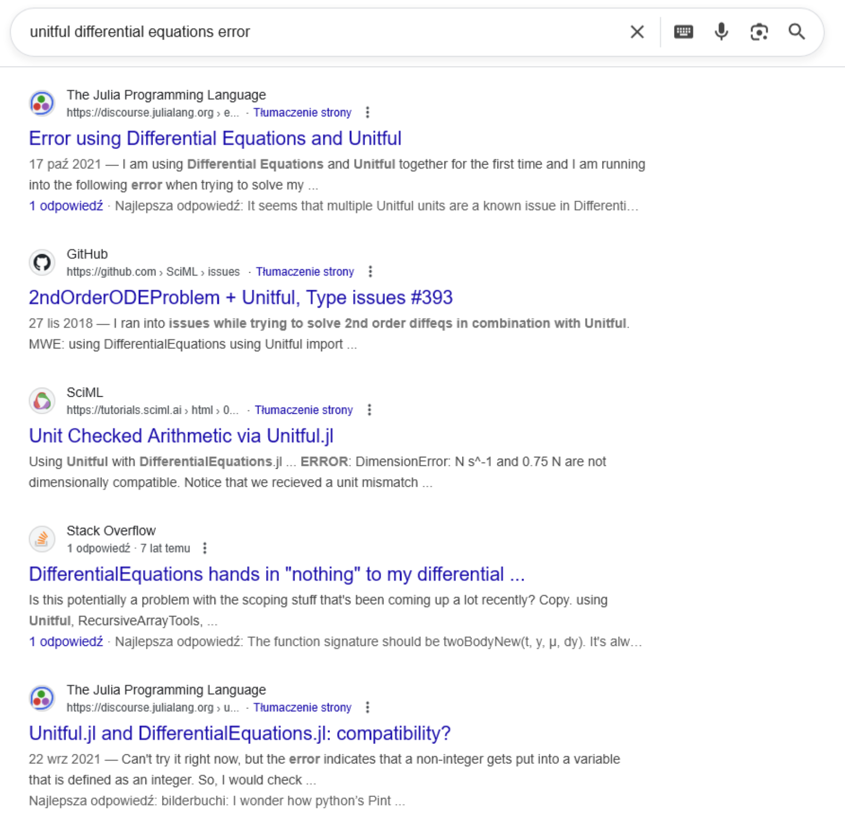

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- **It's not a workaround, it's coding physics thinking like physicist!**

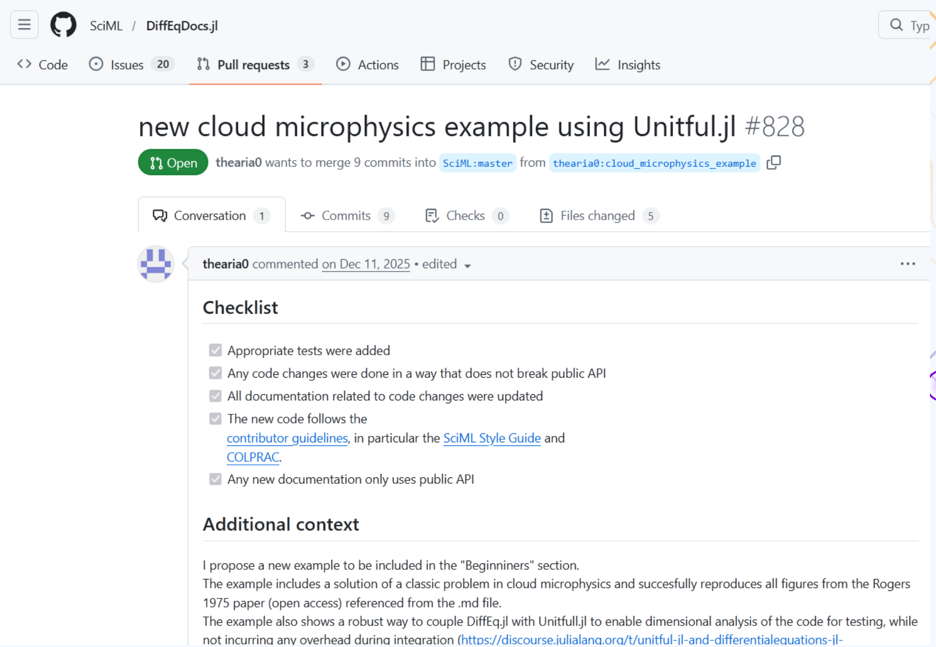
Thanks!

klimaszewska@student.agh.edu.pl

…/daria-klimaszewska

@thearia0

DiffEqDocs.jl PR



new cloud microphysics example using Unitful.jl #828

thearia0 wants to merge 9 commits into SciML:master from thearia0:cloud_microphysics_example

thearia0 commented on Dec 11, 2025 • edited

Checklist

- ☑ Appropriate tests were added
- ☑ Any code changes were done in a way that does not break public API
- ☑ All documentation related to code changes were updated
- ☑ The new code follows the contributor guidelines, in particular the SciML Style Guide and COLPRAC.
- ☑ Any new documentation only uses public API

Additional context

I propose a new example to be included in the "Beginniners" section.
The example includes a solution of a classic problem in cloud microphysics and succesfully reproduces all figures from the Rogers 1975 paper (open access) referenced from the .md file.
The example also shows a robust way to couple DiffEq.jl with Unitfull.jl to enable dimensional analysis of the code for testing, while not incurring any overhead during integration (https://discourse.julialang.org/t/unitful-jl-and-differentialequations-jl-

/ SciML / DiffEqDocs.jl / pull / 828

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