PCA

Principal Component Analysis (PCA) is one of the most widely used dimensionality reduction techniques in machine learning and data analysis. It's a linear transformation method that identifies the directions (principal components) along which the data varies the most.

Algorithm Steps

a. Standardization:

- Center the data by subtracting the mean of each feature
- (Optional) Scale the features to have unit variance

b. Compute the Covariance Matrix:

- Calculate the covariance between each pair of features

c. Compute Eigenvectors and Eigenvalues:

- Find the eigenvectors and eigenvalues of the covariance matrix

d. Sort Eigenvectors:

- Order eigenvectors by their corresponding eigenvalues (descending order)

e. Select Top k Eigenvectors:

- Choose the first k eigenvectors to form the new feature space

f. Project Data:

- Transform the original data onto the new k-dimensional space

Mathematical Formulation:

For a data matrix X with n samples and p features:

- Covariance matrix: $\Sigma = (1/n) X^T X$
- Eigenvalue equation: $\Sigma v = \lambda v$
- Where v are eigenvectors and λ are eigenvalues

Advantages:

- Reduces dimensionality while preserving maximum variance
- Removes correlations between features
- Can help in noise reduction
- Computationally efficient for many datasets

Limitations:

- Assumes linear relationships between features
- May not be suitable for non-linear data
- Can be sensitive to outliers
- Difficult to interpret transformed features

Applications:

- Image compression
- Feature extraction in machine learning
- Data visualization
- Noise reduction in signals
- Finance (e.g., portfolio optimization)

Choosing the Number of Components:

- Explained Variance Ratio: Select components that explain a certain percentage of variance
- Scree Plot: Plot eigenvalues and look for an "elbow"
- Cross-validation: Choose k that optimizes performance on a specific task