Linear Equations

Solution of a system of linear equations:

Linear equations can have three kind of possible solutions:

1. Unique Solution

Form 1: Unique Solution
(Consistent and Independent)

$$\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right]$$

2. Infinite Solution

Form 2: Infinitely Many Solutions
(Consistent and Dependent)

$$\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & 0 \end{array}\right]$$

3. No Solution

Form 3: No Solution (Inconsistent)

$$\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & p \end{array}\right]$$

GAUSSIAN ELIMINATION (REF)

The Gaussian elimination method refers to a strategy used to obtain the row-echelon form of a matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{After\ Gaussian\ elimination} A = \begin{bmatrix} 1 & b_{12} & b_{13} \\ 0 & 1 & b_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Example

Use **Gaussian elimination** to solve the given 2×2 **system of equations.**

$$\begin{aligned} 2x + y &= 1 \\ 4x + 2y &= 6 \end{aligned}$$

Solution

Write the system as an **augmented matrix.**

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 4 & 2 & 6 \end{array}\right]$$

Obtain a 1 in row 1, column 1. This can be accomplished by multiplying the first row by $\frac{1}{2}$.

$$\frac{1}{2}R_1 = R_1 \rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 4 & 2 & 6 \end{array}\right]$$

Next, we want a 0 in row 2, column 1. Multiply row 1 by -4 and add row 1 to row 2.

$$-4R_1 + R_2 = R_2 \rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 4 \end{array}\right]$$

The second row represents the equation $0 = 4$. Therefore, the system is inconsistent and has no solution.

GAUSS-JORDAN ELIMINATION (RREF)

The Gauss-Jordan elimination method refers to a strategy used to obtain the reduced row-echelon form of a matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{After\ Gauss-Jordan\ elimination} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example

Solve the system of equations.

$$\begin{aligned} 6x + 4y + 3z &= -6 \\ x + 2y + z &= \frac{1}{3} \\ -12x - 10y - 7z &= 11 \end{aligned}$$

Solution

Write the augmented matrix for the system of equations.

$$\left[\begin{array}{ccc|c} 6 & 4 & 3 & -6 \\ 1 & 2 & 1 & \frac{1}{3} \\ -12 & -10 & -7 & 11 \end{array}\right]$$

On the matrix page of the calculator, enter the augmented matrix above as the matrix variable $[A]$.

$$[A] = \left[\begin{array}{ccc|c} 6 & 4 & 3 & -6 \\ 1 & 2 & 1 & \frac{1}{3} \\ -12 & -10 & -7 & 11 \end{array}\right]$$

Use the **rref(** function in the calculator, calling up the matrix variable $[A]$.

rref([A])

Use the MATH --> FRAC option in the calculator to express the matrix elements as fractions.

Evaluate

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{2}{3} \\ 0 & 1 & 0 & \frac{5}{2} \\ 0 & 0 & 1 & -4 \end{array}\right] \rightarrow \begin{aligned} x + 0y + 0z &= -\frac{2}{3} \\ y + 0z &= \frac{5}{2} \\ z &= -4 \end{aligned}$$

Thus the solution, which can easily be read from the right column of the reduced row-echelon form of the matrix, is $\left(-\frac{2}{3}, \frac{5}{2}, -4\right)$.