Change of basis

Taking a vector represented in one basis (with its specific coordinates) and expressing it in terms of another basis. It's like translating the coordinates from one language to another.

To achieve this, we can:

1. **Express the new basis vectors (u and w) in terms of the old basis (i and j).** This gives us a relationship between the two bases.
2. **Solve for the scalars (a' and b')** that represent v in the new basis using the equation v = a'u + b'w.

Why Change Basis?

Changing basis can be useful for:

- **Simplifying calculations:** Sometimes, a specific linear transformation might be easier to represent in a different basis.
- **Solving problems with different viewpoints:** In physics, you might describe motion using a basis of forces or a basis of momentum depending on the problem.

Example:

Changing Basis with Coordinates

- **Basis 1:** Standard basis with vectors i = (1, 0) and j = (0, 1).
- **Vector in Basis 1:** v = (3, 4). This means the vector v starts at the origin and goes 3 units to the right and 4 units up.
- **Basis 2:** New basis with vectors u = (2, 1) and w = (1, -1). These vectors are tilted compared to the standard basis.

We want to find the coordinates of v in this new basis, say v' = a'u + b'w (where a' and b' are scalars).

Steps:

1. **Express new basis vectors in terms of Basis 1:**
    - u = 2i + 1j
    - w = 1i - 1j
2. **Solve for a' and b':**
   We can rewrite the equation for v as: (3, 4) = a'(2i + j) + b'(i - j)

   This translates to a system of equations:

    - 3 = 2a' + b'
    - 4 = a' - b'

   Solving this system, we get a' = 7/3 and b' = -5/3.

   In the new basis, vector v can be represented as v' = 7/3 u - 5/3 w. This means v can be reached by going 7/3 units in the direction of vector u and 5/3 unit in the direction of vector w.

   v' = (7/3, 5/3) (respect to u , w)