

# Normal Distribution

The normal distribution, also known as the Gaussian distribution, is a bell-shaped curve that describes the probability of finding a certain value in a continuous random variable. It's incredibly common across statistics and many other fields.

**There are two main things that define a normal distribution:**

- **Mean ( $\mu$ ):** This represents the center of the distribution, where the peak of the bell curve is. It indicates the average value you'd expect to find.
- **Standard deviation ( $\sigma$ ):** This controls how spread out the data is. A larger standard deviation means the data points are further from the mean and the curve is flatter, while a smaller standard deviation indicates the data points are clustered closer to the mean and the curve is steeper.

**The normal distribution is useful because it applies to many real-world phenomena where data tends to cluster around an average value with some variation. Examples include:**

- Heights of people
- IQ scores
- Test grades
- Errors in measurements

**Probability Density Function (PDF):**

The PDF describes the probability density of a specific value ( $x$ ) occurring in the normal distribution. The general form for a normal distribution with mean ( $\mu$ ) and standard deviation ( $\sigma$ ) is:

$$f(x) = (1/(\sigma * \sqrt{2 * \pi})) * \exp(-((x - \mu)^2)/(2 * \sigma^2))$$

Where:

- $f(x)$  is the probability density at value  $x$
- $\mu$  (mu) is the mean of the distribution
- $\sigma$  (sigma) is the standard deviation of the distribution
- $\pi$  (pi) is the mathematical constant pi

- $\exp()$  is the exponential function

### Cumulative Distribution Function (CDF):

For a normal distribution with any mean ( $\mu$ ) and standard deviation ( $\sigma$ ), the CDF can be calculated using the CDF of the standard normal distribution ( $\Phi$ ) and the following relationship:

$$\Phi\left(\frac{x - \mu}{\sigma}\right)$$

