Chain Rule Two Variable

The chain rule for two variables extends the concept of the single-variable chain rule to functions of multiple variables. It allows you to differentiate composite functions where the inner function takes two arguments and the outer function takes one argument.

Formula:

$$\nabla z = (\nabla f(u)) * (J(u))$$

Explanation:

- ∇z is the gradient of the composite function z with respect to the original variables x and y.
- ∇f(u) is the gradient of the outer function f with respect to the intermediate variable u.
- J(u) is the Jacobian matrix of the inner function u with respect to the original variables x and y.

When you have a function (z) that is composed of other functions, like (z = f(u(x, y))), the rate at which (z) changes with respect to (x) and (y) can be found by multiplying two things:

1. **The Gradient of (f) with Respect to (u)**: This tells you how much (f) changes when (u) changes. It's like asking, "If I tweak (u) a little bit, how much does (f) move?"

2. **The Jacobian Matrix of (u) with Respect to (x) and (y)**: This tells you how much (u) changes when (x) and (y) change. It's like a summary of all the different ways (u) can wiggle when you jiggle (x) and (y).

So, the chain rule formula in simple terms is:

∇z =How much 'f' changes with 'u' multiplied by how much 'u' changes with 'x' and 'y'.

Here's the formula in matrix form:

$$\nabla z = \begin{bmatrix} \frac{\partial f}{\partial u_1} & \frac{\partial f}{\partial u_2} & \cdots & \frac{\partial f}{\partial u_n} \end{bmatrix} \begin{bmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial y} \\ \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial y} \\ \vdots & \vdots \\ \frac{\partial u_n}{\partial x} & \frac{\partial u_n}{\partial y} \end{bmatrix}$$