Chain Rule Single Variable

The chain rule, even in its single-variable form, is a powerful tool in calculus for differentiating composite functions. It tells you how the derivative of a composite function (a function within another function) relates to the derivatives of the inner and outer functions.

Chain Rule Formula

The formula of chain rule for the function y = f(x), where f(x) is a composite function such that x = g(t), is given as:

$$dy/dx = dy/du * du/dx$$

Example 1:

Find the derivative of the function f(x) = sin(2x2 – 6x).

Solution:

The given can be expressed as a composite function as given below:

$$f(x) = sin(2x2 - 6x)$$

$$u(x) = 2x2 - 6x$$

$$v(t) = sint$$

$$Thus, t = u(x) = 2x2 - 6x$$

$$\Rightarrow f(x) = v(u(x))$$

According to the chain rule,

$$df(x)/dx = (dv/dt) \times (dt/dx)$$

Where,

$$dv/dt = d/dt(sint) = cost$$

$$dt/dx = d/dx[u(x)] = d/dx(2x2\ {-}6x) = 4x{-}6$$

$$Therefore, df/dx = cost \times (4x{-}6)$$

$$= cos(2x2\ {-}6x) \times (4x{-}6)$$

$$= (4x{-}6)cos(2x2\ {-}6x)$$