Diagonalization

Concept of Diagonalization:

1. **Visualizing Transformations:** Diagonal entries represent the scaling factors applied along each basis vector direction. A diagonal matrix with positive entries stretches vectors along their corresponding axes, while negative entries indicate reflections across those axes. Entries of zero signify no change in that direction.
2. **Simplifying Calculations:** Diagonalization allows you to analyze the behavior of a transformation by examining only the diagonal entries. This can be much easier than working with the original, potentially complex matrix.
3. **Eigenvalues and Eigenvectors:** Diagonalization is closely linked with eigenvalues and eigenvectors. Eigenvalues are special scalar values associated with a matrix, and eigenvectors are non-zero vectors that, when multiplied by the matrix, are scaled by their corresponding eigenvalue. Diagonalization essentially reveals the eigenvalues and eigenvectors of a matrix.

Process of Diagonalization:

1. **Finding Eigenvalues:** The first step involves finding the eigenvalues of the matrix. These are the solutions to the characteristic equation `det(A - λI) = 0`, where A is the matrix, λ is the eigenvalue, I is the identity matrix, and det represents the determinant.
2. **Finding Eigenvectors:** For each eigenvalue, you need to find the corresponding eigenvector by solving the equation `(A - λI)v = 0`, where v is the eigenvector. If a matrix has distinct eigenvalues, there will be a unique eigenvector for each eigenvalue.
3. **Diagonalization Matrix (Optional):** If the matrix has a complete set of linearly independent eigenvectors (one for each eigenvalue), you can construct a diagonalization matrix (D) containing the eigenvalues on the diagonal and a matrix (P) whose columns are the eigenvectors. The original

matrix (A) can then be expressed as the product of P, D, and the inverse of P ($A = P * D * P^{-1}$).

Example

Diagonalize the following matrix, if possible.

$A = [[1 , -3 , 3] , [3 , -5 , 3] , [3 , -3 , 1]]$

Step 1: Find the eigenvalues of A

$$0=\det(A-\lambda I)$$

$$0=-\lambda^3-3\lambda^2+4$$

$$0=-(\lambda-1)(\lambda+2)^2$$

So the eigenvalues are $\lambda=1$ and $\lambda=-2$ (with multiplicity two)

Step 2: Find three linearly independent eigenvectors of A

$$v_1 = (1, -1, 1) , v_2 = (-1, 1 ,0) , v_3 = (-1, 0, 1)$$

Step 3: Construct P from the vectors in Step 2.

$P = [v_1\ v_2\ v_3]$

Step 4: Construct D from the corresponding eigenvalues

$D = [[1, 0 ,0] , [0 , -2 , 0] , [0 , 0 , -2]]$

$A = P * D * P^{-1}$