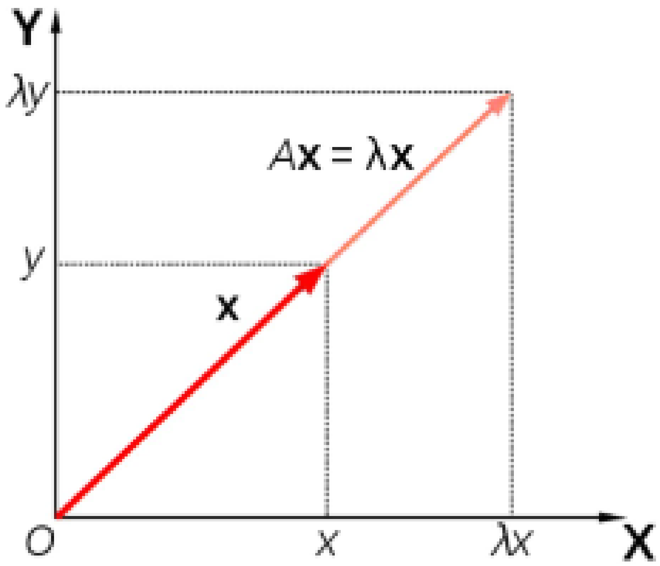
Eigenvalues and Eigenvectors

Eigenvalues and Eigenvectors are the scalar and vector quantities associated with Matrix used for linear transformation. The vector that does not change even after applying transformations is called the Eigenvector and the scalar value attached to Eigenvectors is called Eigenvalues



Eigenvector Equation

Av = λv

- ***A** is the given square matrix*
- ***v** is the eigenvector of matrix A*
- ***λ** is any scaler multiple*

Example:

Find the eigenvalues and the eigenvector for the matrix

A = [[1, 2], [5, 4]]

If eigenvalues are represented using λ and the eigenvector is represented as $v = \begin{bmatrix} a \\ b \end{bmatrix}$

Then the eigenvector is calculated by using the equation,

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 2 \\ 5 & 4-\lambda \end{bmatrix} = 0$$

$(1-\lambda)(4-\lambda) - 2.5 = 0$

$4 - \lambda - 4\lambda + \lambda^2 - 10 = 0$

$\lambda^2 - 5\lambda - 6 = 0$

$\lambda^2 - 6\lambda + \lambda - 6 = 0$

$\lambda(\lambda - 6) + 1(\lambda - 6) = 0$

$(\lambda - 6)(\lambda + 1) = 0$

$\lambda = 6$ *and* $\lambda = -1$

Thus, the eigenvalues are 6, and -1. Then the respective eigenvectors are,

For $\lambda = 6$

$(A - \lambda I)v = 0$

For λ = 6

(A-λI)v = 0

$\begin{bmatrix} 1-6 & 2 \\ 5 & 4-6 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}$ *= 0*

$\begin{bmatrix} -5 & 2 \\ 5 & -2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}$ *= 0*

-5a + 2b = 0

5a – 2b = 0

simplifying the above equation we get,

5a=2b

The required eigenvector is,

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

For λ = -1

(A-λI)v = 0

$\begin{bmatrix} 1-(-1) & 2 \\ 5 & 4-(-1) \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}$ *= 0*

$\begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}$ *= 0*

2a + 2b = 0

5a + 5b = 0

simplifying the above equation we get,

a = -b

The required eigenvector is,

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Then the eigenvectors of the given 2 × 2 matrix are $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$