

# Linear transformations

The idea of transformation the basis is transformed then the vectors in this space is changed

“ 2x2 Matrix ”

$$\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$$

Where  $\hat{i}$  lands      Where  $\hat{j}$  lands

If i have vector input and i want with any type of transformation get the out vector

“ 2x2 Matrix ”

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Input vector      Transformed vector

$$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

## Example

Given the input

$\begin{bmatrix} 3 & -4 \end{bmatrix}$  to the linear transformation described by the matrix

[1 -6]

[4 2]

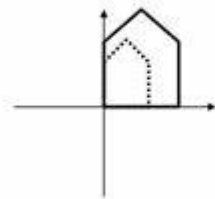
what vector will the transformation produce?

$$\begin{aligned}\begin{bmatrix} 3 \\ -4 \end{bmatrix} &= 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 1 & -6 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix} &= 3 \begin{bmatrix} 1 \\ 4 \end{bmatrix} - 4 \begin{bmatrix} -6 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 3(1) - 4(-6) \\ 3(4) - 4(2) \end{bmatrix} \\ &= \begin{bmatrix} 27 \\ 4 \end{bmatrix}\end{aligned}$$

## Type of transformations

### 1. Scaling

- 2D scaling



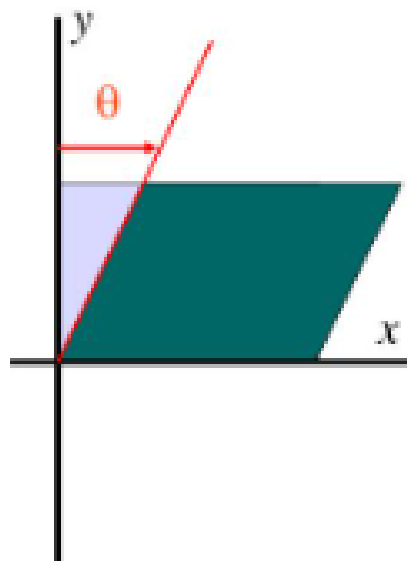
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

### 2. Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

### 3. Shear

The idea of the shear transformation is only one of the basis vector is rotate



Shear in x-axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear in y-axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

## 4. Reflection

The matrix to make reflection along to x-axis

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The matrix to make reflection along to y-axis

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$