Functions

Functions are fundamental concepts in calculus, representing a rule that assigns an output to an input. They are particularly useful for describing relationships between quantities that change, which is a core concept in calculus. There are two main types of functions we deal with in calculus: scalar functions and vector-valued functions.

Scalar Functions

A scalar function takes one or more numbers as inputs (which can be scalars or vectors themselves) and outputs a single number (a scalar).

- The input can be one number (like a simple function of x), or it can be multiple numbers grouped into a vector.

- The output is always a single number.

- Examples of scalar functions include:
  - f(x) = x^2 (univariable)
  - g(x, y) = x^2 + y^2 (multivariable)
  - h(t) = sin(t) (univariable)

Vector-Valued Functions

Vector-valued functions, on the other hand, can take one or more numbers as inputs (similar to scalar functions) but output a vector as the result.

- The input can be a scalar or a vector.

- The output is always a vector, with a dimension equal to the number of elements it contains.

- Here are a few examples of vector-valued functions:

1. Position vector of a moving object: Suppose a particle is moving in a 2D plane with its x-coordinate given by the function f(t) = 2t and its y-coordinate given by the function g(t) = 3t^2. The position vector of the particle at time t can be represented as a vector-valued function:

$$r(t) =< f(t), g(t) >=< 2t, 3t^2 >$$

2. Parametric equations of a curve: Consider the curve defined by the parametric equations:

$$x(t) = cos(t), y(t) = sin(t)$$

The vector-valued function representing this curve is:

$$r(t) =< x(t), y(t) >=< cos(t), sin(t) >$$

3. Space curve: In 3D space

$$r(t) =< f(t), g(t), h(t) >$$

4. Motion of a particle in 3D space :

$$x(t) = 2cos(t), y(t) = 2sin(t), z(t) = 3t$$

The vector-valued function representing the motion of the particle is:

$$r(t) =< x(t), y(t), z(t) >=< 2cos(t), 2sin(t), 3t >$$