Matrix

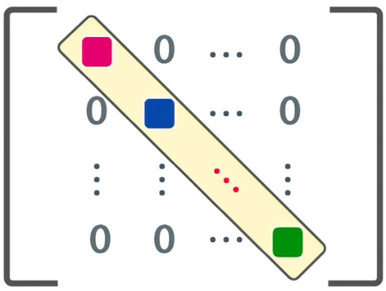
Popular type of matrix

Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2 x 2 3 x 3

Diagonal matrix



Scalar matrix

$$k \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

Triangular matrix

Upper Triangular Matrix

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}_{4\times 4}$$

Lower Triangular Matrix

$$L = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times 4}$$

Symmetric matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 8 \end{bmatrix}^T$$

Symmetric matrix

Matrix Operation

Addition and Subtraction

The two matrices must be the same size.

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Example

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

✗ We cannot add them

But we can add

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -9 & 2 \\ -1 & 3 \end{bmatrix}$$

✓ We can add them

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} -2 & 5 \\ -8 & 12 \\ 0 & -18 \end{bmatrix}$$

✓ We can add them

Multiplication

The first column of the matrix must be the same as the first row of the matrix.

If I have a matrix of size m x n and a second matrix of size n x c.

We discovered that the size of the first column of the matrix is identical to the size of the first row. Consequently, the output matrix has a size of m x c.

The operation of multiplication involves multiplying the first row by the first column to obtain the first element in the new matrix, and so on.

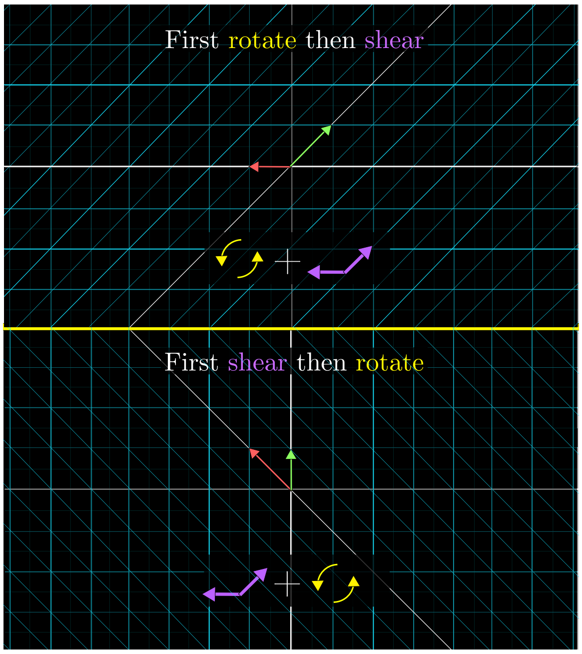
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 10 & 11 \\ 20 & 21 \\ 30 & 31 \end{bmatrix}$$

$$= \begin{bmatrix} 1\text{x}10 + 2\text{x}20 + 3\text{x}30 & 1\text{x}11 + 2\text{x}21 + 3\text{x}31 \\ 4\text{x}10 + 5\text{x}20 + 6\text{x}30 & 4\text{x}11 + 5\text{x}21 + 6\text{x}31 \end{bmatrix}$$

$$= \begin{bmatrix} 10+40+90 & 11+42+93 \\ 40+100+180 & 44+105+186 \end{bmatrix} = \begin{bmatrix} 140 & 146 \\ 320 & 335 \end{bmatrix}$$

The multiplication is noncommutativity

$M1\ M2 \neq M2\ M1$



We could show that matrix multiplication isn't communitive through numerical computation:

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{M_1} = \underbrace{\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}}_{M_2 M_1}$$

$$\underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{M_1} \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{M_2} = \underbrace{\begin{bmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{bmatrix}}_{M_1 M_2}$$

$$\underbrace{\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}}_{M_2 M_1} \neq \underbrace{\begin{bmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{bmatrix}}_{M_1 M_2}$$

I know we just said that matrices don't commute! That is true, *generally*. However there are some examples of matrices commuting, even if not every pair of matrices being multiplied is communitive like this

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

This example isn't intuitive in numeric form so let's think about it visually.

One matrix is rotating by $90^{\circ}90^{\circ}$ counterclockwise. The other matrix is scaling by -2 (reflecting through the $y=-x$ line and multiplying by 2). Does it matter if we first rotate and then scale, or can we also scale and then rotate?