Determinant

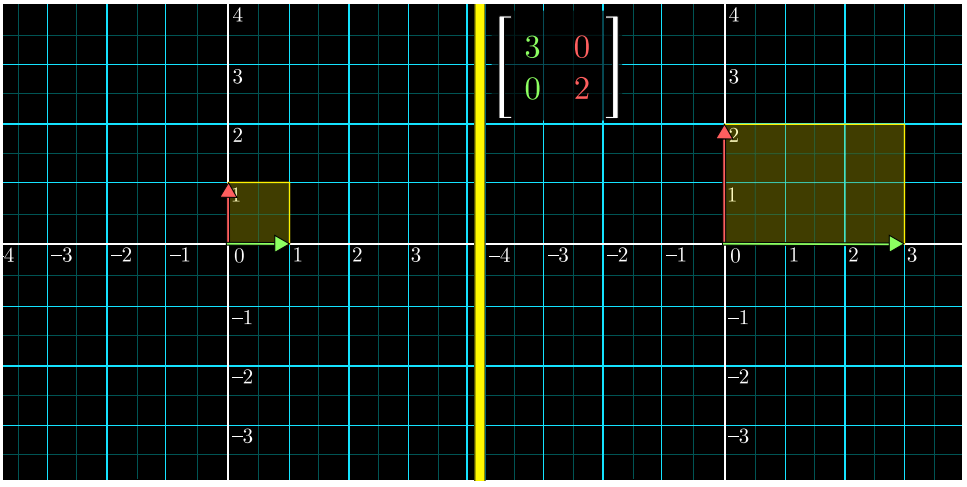
In Two Dimensions

Scaling Area

One thing that turns out to be surprisingly useful for understanding a given transformation is to measure exactly how much a given transformation stretches and squishes things. More specifically, to measure the factor by which the area of a given region increases or decreases.

Positive determinant

For Example



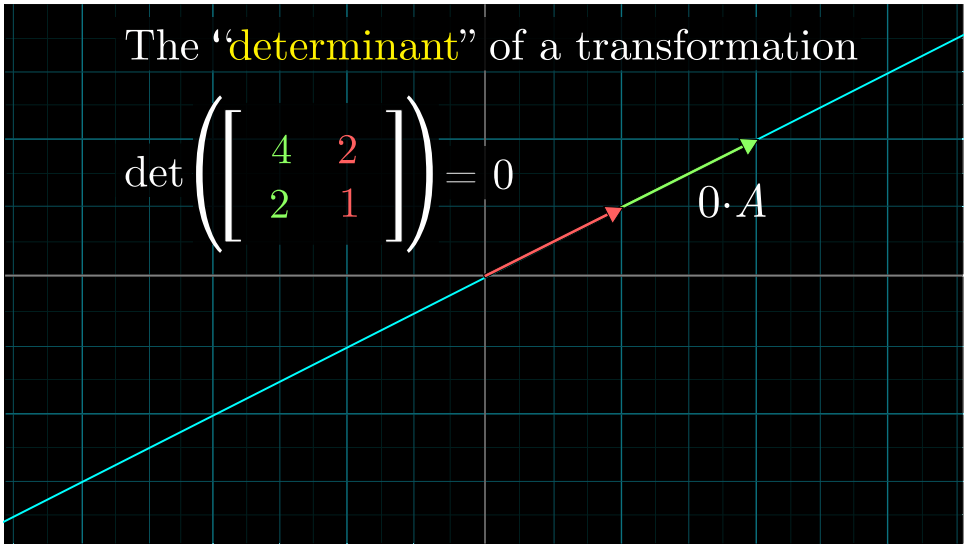
After the transformation, this turns into a 2×3 rectangle.

Since this region started with area 1 and ended up with area 2×3=6 we can say the linear transformation has scaled its area by a factor of 6.

Zero determinant

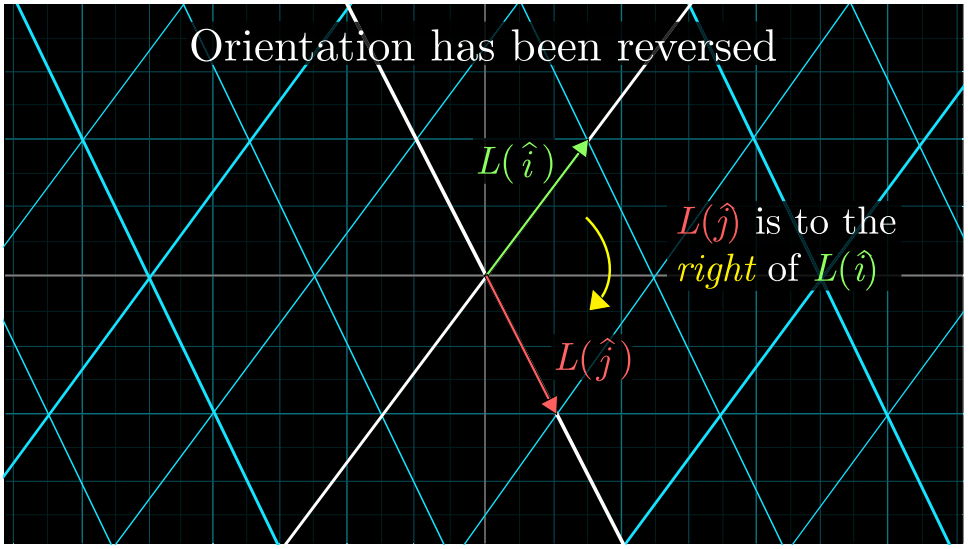
The determinant of a 2D transformation is 0 if it squishes all of space onto a line, or even onto a single point, since the area of every region would then become 0.

For Example



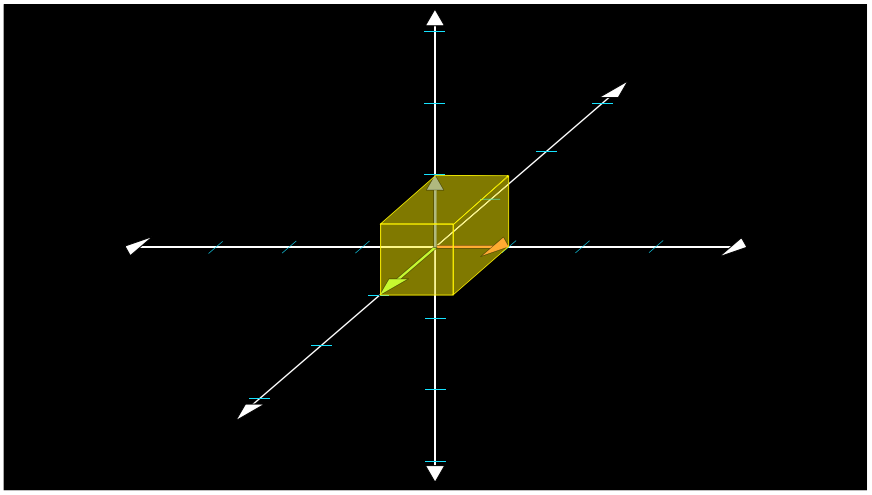
Negative determinant

If the basis is reversed



In Three Dimensions

It also tells you how much the transformation scales things, but this time, it tells you how much *volumes* get scaled.



A determinant of zero would mean that all of space is squished onto something with zero volume, meaning a flat plane, a line, or in the most extreme case into a single point at the origin.