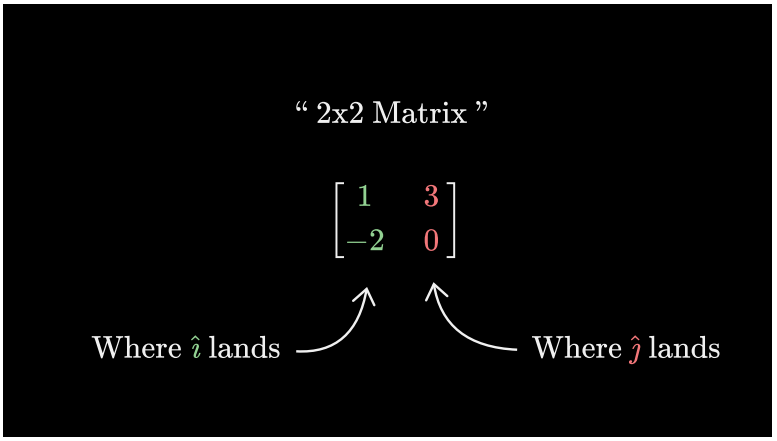
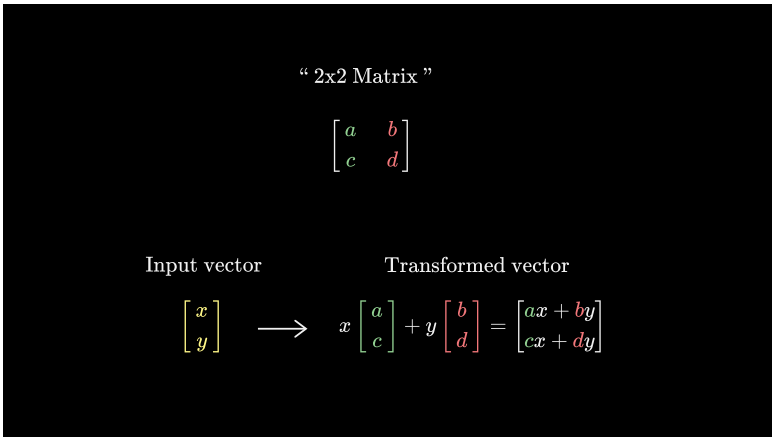
Linear transformations

The idea of transformation the basis is transformed then the vectors in this space is changed



If i have vector input and i want with any type of transformation get the out vector



Example

Given the input

[3 −4] to the linear transformation described by the matrix

[1 −6]

[4 2]

what vector will the transformation produce?

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 4\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

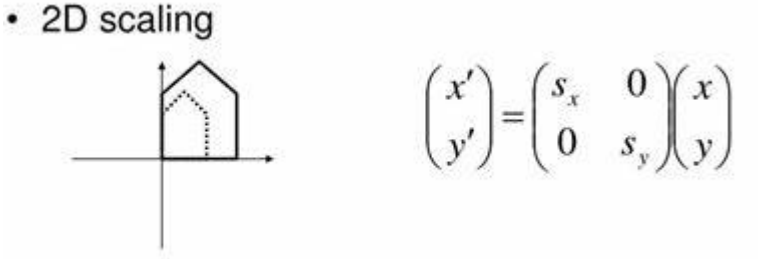
$$\begin{bmatrix} 1 & -6 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ -4 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 4 \end{bmatrix} - 4\begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1) - 4(-6) \\ 3(4) - 4(2) \end{bmatrix}$$

$$= \begin{bmatrix} 27 \\ 4 \end{bmatrix}$$

Type of transformations

1. Scaling

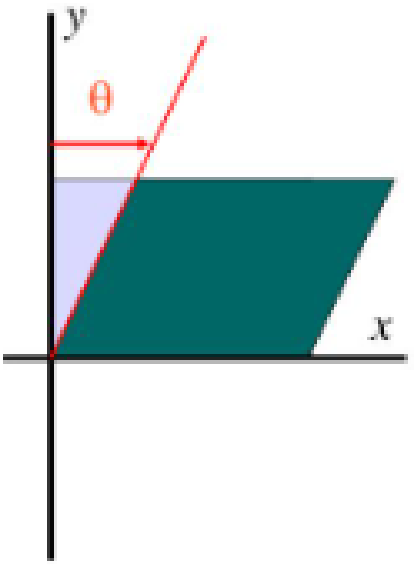


2. Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

3. Shear

The idea of the shear transformation is only one of the basis vector is rotate



Shear in x-axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear in y-axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

4. Reflection

The matrix to make reflection along to x-axis

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The matrix to make reflection along to y-axis

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$