Design Theory for Relational Databases

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Introduction

- There are always many different schemas for a given set of data.
- E.g., you could combine or divide tables.
- How do you pick a schema?
  Which is better?
  What does "better" mean?
- Fortunately, there are some principles to guide us.

Database Design Theory

- It allows us to improve a schema systematically.
- General idea:
  - Express constraints on the relationships between attributes
  - Use these to decompose the relations
- Ultimately, get a schema that is in a "normal form" that guarantees good properties.
- "Normal" in the sense of conforming to a standard.
- The process of converting a schema to a normal form is called normalization.

Part I:
Functional Dependency Theory

A poorly designed table

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- In any domain, there are relationships between attribute values.
- Perhaps: functional dependencies
  - Every part has 1 manufacturer
  - Every manufacture has 1 address
  - Every seller has 1 address
- If so, this table will have redundant data.

Principle: Avoid redundancy

Redundant data can lead to anomalies.

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
~~8624~~	~~Lee Valley~~	~~102 Vaughn~~	~~ABC~~	~~1229 Bloor W~~	~~23.99~~
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- Update anomaly: if Hammers 'R Us moves and we update only one tuple, the data is inconsistent.
- Deletion anomaly: If ABC stops selling part 8624 and Lee Valley makes only that one part, we lose track of its address.

doing too much in one table

a type of integrity constraint

Definition of FD

- Suppose R is a relation, and X and Y are subsets of the attributes of R.
- $X \rightarrow Y$ asserts that:
  - If two tuples agree on all the attributes in set *X*, they must also agree on all the attributes in set *Y*.
- We say that "$X \rightarrow Y$ holds in R", or "*X* functionally determines *Y*."
- An FD constrains what can go in a relation.

More formally…

$A \rightarrow B$ means:

$\forall$ tuples t_1, t_2,

$(t_1[A] = t_2[A]) \Rightarrow (t_1[B] = t_2[B])$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$(t_1[A] = t_2[A]) \wedge (t_1[B] \neq t_2[B])$

Generalization to multiple attributes

$A_1A_2 \ldots A_m \rightarrow B_1B_2 \ldots B_n$ means:

$\forall$ tuples t_1, t_2,

$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \Rightarrow$

$(t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \wedge$

$\neg (t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$

Why "functional dependency"?

- "dependency" because the value of *Y* depends on the value of *X*.
- "functional" because there is a mathematical function that takes a value for X and gives a *unique* value for Y.
- (It's not a typical function; just a lookup.)

f: X -> Y

Equivalent sets of FDs

- When we write a set of FDs, we mean that all of them hold.
- We can very often rewrite sets of FDs in equivalent ways.
- When we say S_1 is equivalent to S_2 we mean that:
  - S_1 holds in a relation iff S_2 does.

Splitting rules for FDs

- Can we split the RHS of an FD and get multiple, equivalent FDs?

- Can we split the LHS of an FD and get multiple, equivalent FDs?

Coincidence or FD?

- An FD is an assertion about *every* instance of the relation.
- You can't know it holds just by looking at one instance.
- You must use knowledge of the domain to determine whether an FD holds.

FDs are closely related to keys

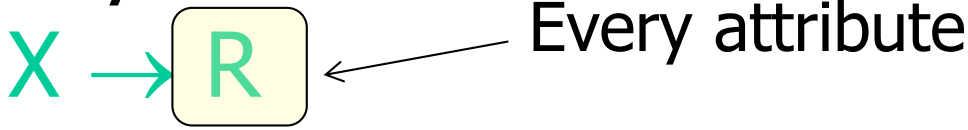
- Suppose K is a set of attributes for relation R.
- Our old definition of superkey:

  a set of attributes for which no two rows can have the same values.
- A claim about FDs:

  K is a *superkey* for *R* iff
  K functionally determines all of *R*.

FDs are a generalization of keys

- key:



- Functional dependency:

- An FD can be more subtle.

Inferring FDs

- Given a set of FDs, we can often infer further FDs.
- This will be handy when we apply FDs to the problem of database design.
- Big task: given a set of FDs,

  infer *every* other FD that must also hold.
- Simpler task: given a set of FDs,

  check whether *a given* FD must also hold.

Examples

- If $A \rightarrow B$ and $B \rightarrow C$ hold,
  must $A \rightarrow C$ hold?
- If $A \rightarrow H$, $C \rightarrow F$, and $F\,G \rightarrow A\,D$ hold,
  must $F\,A \rightarrow D$ hold?
  must $C\,G \rightarrow F\,H$ hold?
- If $H \rightarrow GD$, $HD \rightarrow CE$, and $BD \rightarrow A$ hold,
  must $EH \rightarrow C$ hold?
- Aside: we are not generating new FDs, but testing a specific possible one.

Method 1: Prove an FD follows using first principles

- You can prove it by referring back to
  - The FDs that you know hold, and
  - The definition of functional dependency.
- But the Closure Test is easier.

Method 2: Prove an FD follows using the Closure Test

- Assume you know the values of the LHS attributes, and figure out everything else that is determined.
- If it includes the RHS attributes, then you know that LHS $\rightarrow$ RHS
- This is called the closure test.

Y is a set of attributes, S is a set of FDs.

Return the closure of Y under S.

Attribute_closure(Y, S):

Initialize Y^+ to Y

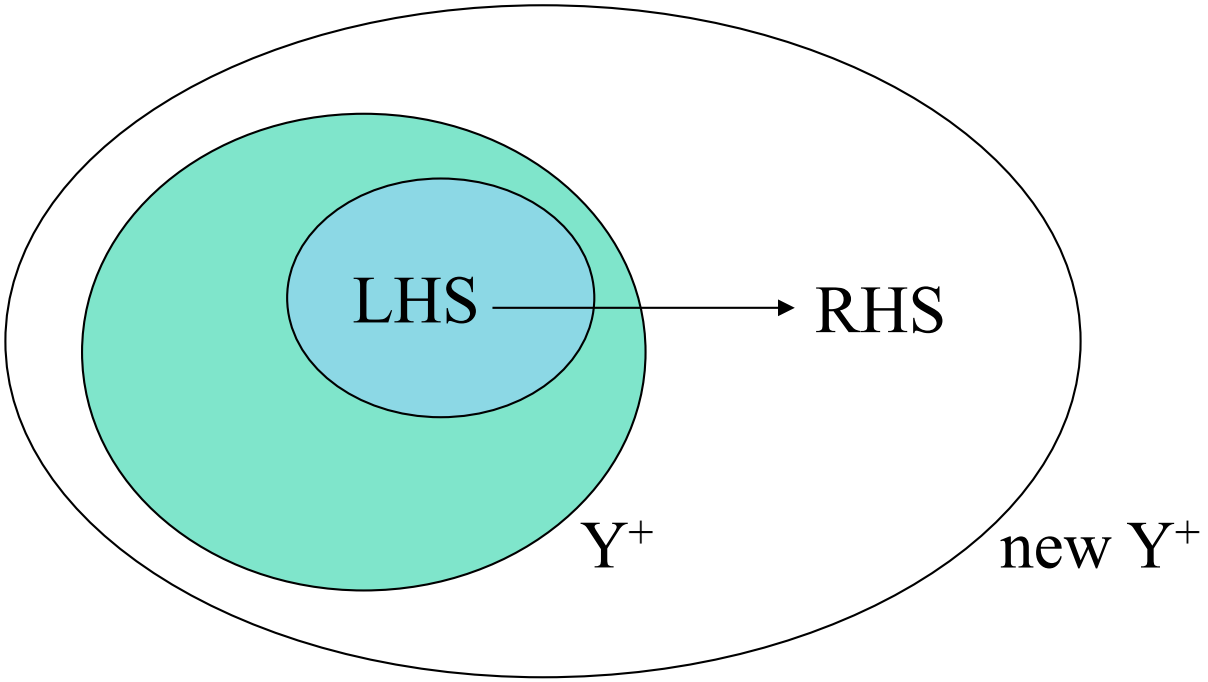
Repeat until no more changes occur:

If there is an FD LHS $\rightarrow$ RHS in S such that LHS is in Y^+:

Add RHS to Y^+

Return Y^+

Visualizing attribute closure



If LHS is in Y^+ and LHS $\rightarrow$ RHS holds, we can add RHS to Y^+

S is a set of FDs; LHS → RHS is a single FD.
Return true iff LHS → RHS follows from S.

FD_follows(S, LHS → RHS):

Y+ = Attribute_closure(LHS, S)

return (RHS is in Y^+)

Projecting FDs

- Later, we will learn how to normalize a schema by decomposing relations.
  - This is the whole point of this theory.
- We will need to know what FDs hold in the new, smaller, relations.
  - We must project our FDs onto the attributes of our new relations.

Example

R(A1, ..., An) Set of attributes: A

Decompose into:

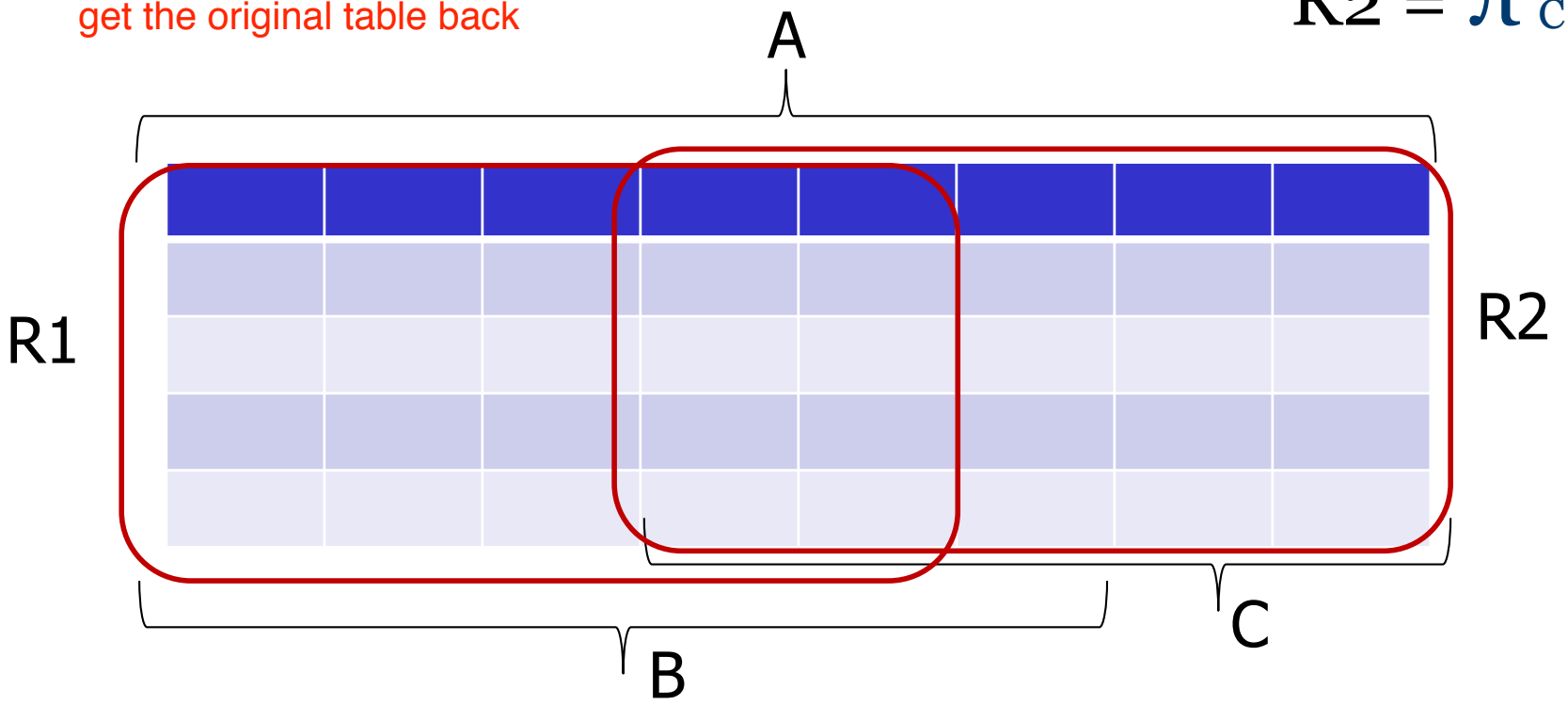
- R1(B1, ..., Bk) Set of attributes: B, and
- R2(C1, ..., Cm) Set of attributes: C

$B \cup C = A$, $R1 \bowtie R2 = R$

get the original table back

$$R1 = \pi_B(R)$$

$$R2 = \pi_C(R)$$



S is a set of FDs; L is a set of attributes.

Return the projection of S onto L:

all FDs that follow from S and involve only attributes from L.

Project(S, L):

Initialize T to {}.

For each subset X of L:

Compute X^+ *Close X and see what we get.*

For every attribute A in X^+:

If A is in L: *$X \rightarrow A$ is only relevant if A is in L (we know X is).*

add $X \rightarrow A$ to T.

Return T.

A few speed-ups

- No need to add $X \rightarrow A$ if A is in X itself. It's a trivial FD.
- These subsets of X won't yield anything, so no need to compute their closures:
  - the empty set
  - the set of all attributes
- Neither are big savings, but …

A big speed-up

- If we find X^+ = all attributes, we can ignore any superset of X.
  - It can only give use "weaker" FDs (with more on the LHS).
- This is a big time saver!

Projection is expensive

- Even with these speed-ups, projection is still expensive.
- Suppose R_1 has n attributes. How many subsets of R_1 are there?

Minimal Basis

- We saw earlier that we can very often rewrite sets of FDs in equivalent ways.
- Example: $S_1 = \{A \rightarrow BC\}$ is equivalent to $S_2 = \{A \rightarrow B, A \rightarrow C\}$.
- Given a set of FDs S, we may want to find a minimal basis: A set of FDs that is equivalent, but has
  - no redundant FDs, and
  - no FDs with unnecessary attributes on the LHS.

S is a set of FDs. Return a minimal basis for S.

Minimal_basis(S):

1. Split the RHS of each FD
2. For each FD $X \rightarrow Y$ where $|X| \geq 2$:

   If you can remove an attribute from X and get an FD that follows from S:

   Do so! (It's a stronger FD.)
3. For each FD f:

   If S – {f} implies f:

   Remove f from S.

Some comments on minimal basis

- Often there are multiple possible results.
  - Depends on the order in which you consider the possible simplifications.
- After you identify a redundant FD, you must not use it when computing subsequent closures.

… and less intuitive

- When you are computing closures to decide whether the LHS of an FD

  $X \rightarrow Y$

  can be simplified, continue to use that FD.

- You must do (2) and (3) in that order.

  Otherwise, must repeat until no changes occur.

Part II:
Using FD Theory to do Database Design

Recall that poorly designed table?

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- We can now express the relationships as FDs:
  - part → manufacturer
  - manufacturer → address
  - seller → address
- The FDs tell us there can be redundancy, thus the design is bad.
- That's why we care about FDs.

Decomposition

- To improve a badly-designed schema $R(A_1, \ldots A_n)$, we will decompose it into smaller relations

  $R1(B_1, \ldots B_j)$ and $R2(C_1, \ldots C_k)$ such that:

  - $R1 = \pi_{B1, \ldots Bj}(R)$
  - $R2 = \pi_{C1, \ldots Ck}(R)$
  - $\{B_1, \ldots B_j\} \cup \{C_1, \ldots C_k\} = \{A_1, \ldots A_n\}$
  - $R1 \bowtie R2 = R$

$R(A_1, \ldots A_n)$ Set of attributes: A

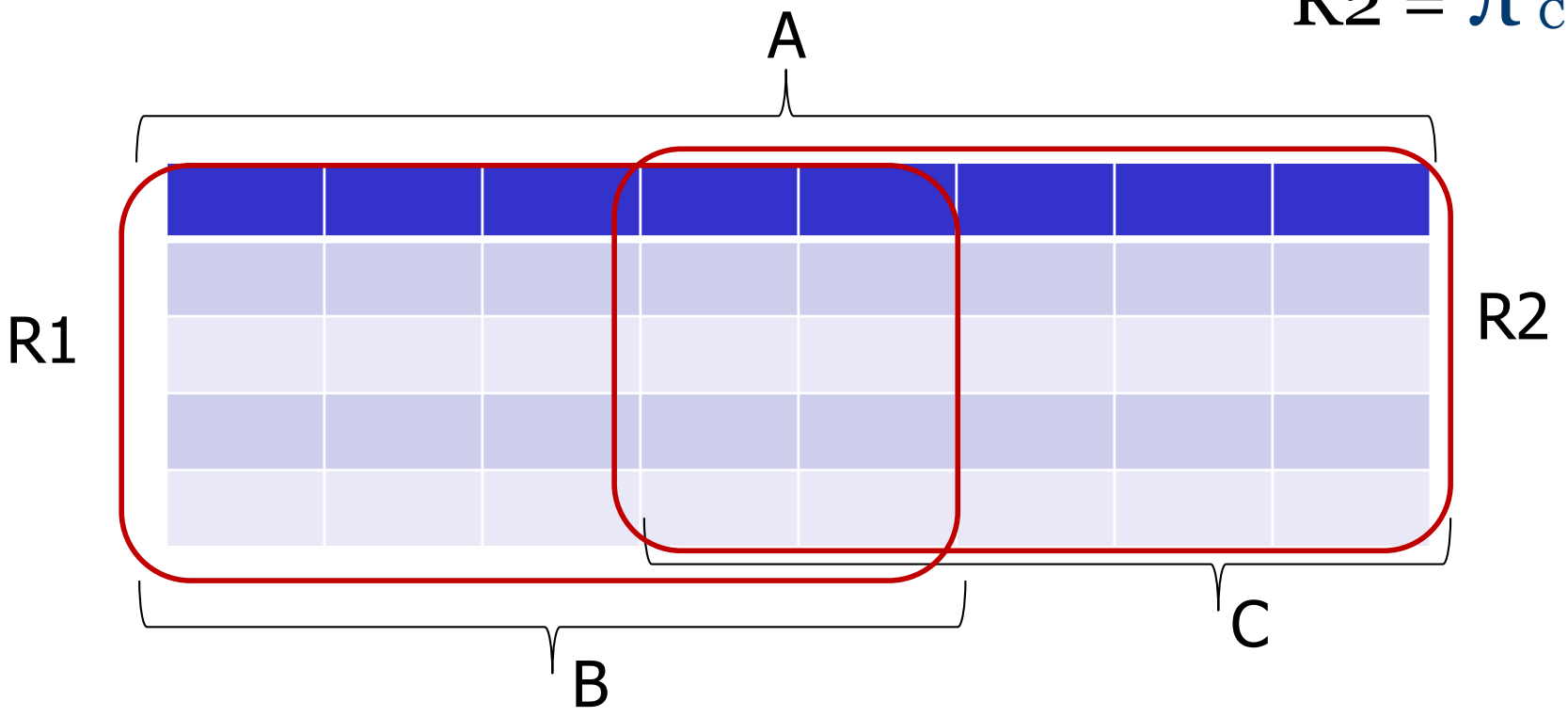
Decompose into:

- $R1(B_1, \ldots B_j)$ Set of attributes: B, and
- $R2(C_1, \ldots C_k)$ Set of attributes: C

$B \cup C = A$, $R1 \bowtie R2 = R$

$$R1 = \pi_B(R)$$

$$R2 = \pi_C(R)$$



But *which* decomposition?

- Decomposition can definitely improve a schema.
- But which decomposition?
  There are many possibilities.
- And how can we be sure a new schema doesn't exhibit other anomalies?
- Boyce-Codd Normal Form *guarantees* it.

Boyce-Codd Normal Form

- We say a relation *R* is in *BCNF* if for every nontrivial FD $X \rightarrow Y$ that holds in *R*, *X* is a superkey.
  - Remember: *nontrivial* means *Y* is not contained in *X*.
  - Remember: a *superkey* doesn't have to be minimal.
- [Exercise]

Intuition

In other words, BCNF requires that:

Only things that functionally determine ***everything*** can functionally determine ***anything***.

Why is the BCNF property valuable?

Note:

- FDs are not the problem. They are facts!
- The schema (in the context of the FDs) is the problem.

R is a relation; F is a set of FDs.
Return the BCNF decomposition of R, given these FDs.

BCNF_decomp(*R, F*):

If an FD $X \rightarrow Y$ in F violates BCNF

Compute X^+.

Replace R by two relations with schemas:

$$R_1 = X^+$$

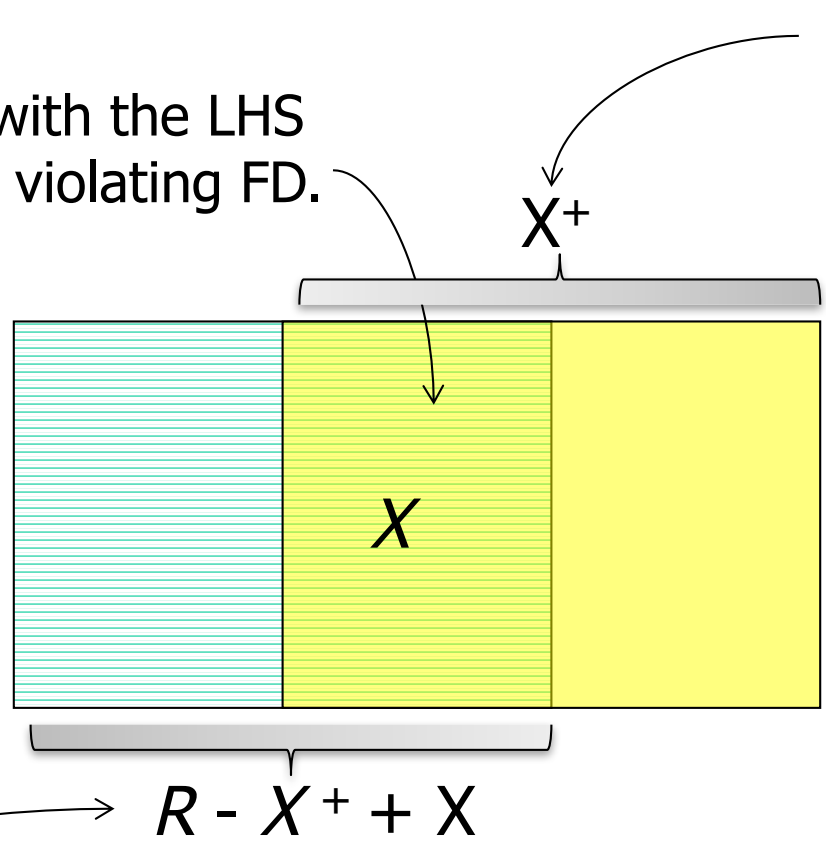
$$R_2 = R - (X^+ - X)$$

Project the FD's F onto R_1 and R_2.

Recursively decompose R_1 and R_2 into BCNF.

[Example]

Decomposition Picture



3) Everything except the new stuff is the other new relation.
X is in both new relations to make a connection between them.

Some comments on BCNF decomp

- If more than one FD violates BCNF, you may decompose based on any one of them.
  - So there may be multiple results possible.
- The new relations we create may not be in BCNF. We must recurse.
  - We only keep the relations at the "leaves".
- How does the decomposition step help? [Exercise]

Speed-ups for BCNF decomposition

- Don't need to know any keys.
  - Only superkeys matter.
- And don't need to know *all* superkeys.
  - Only need to check whether the LHS of each FD is a superkey.
  - Use the closure test (simple and fast!).

BCNF

- Every attribute depends on:
  - The key
  - The whole key
  - And nothing but the key…

so help me Codd….

More speed-ups

- When projecting FDs onto a new relation, check each new FD:
  - Does the new relation violate BCNF because of this FD?
- If so, abort the projection.
  - You are about to discard this relation anyway (and decompose further).