STA 247
Probability with Computer Applications

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Week 4 - Topic A

Geometric Distribution

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- $P(X = 100) = 0.98^{100} \cdot (0.2) = 0.00265 = 0.27\%$

Geometric Distribution

The random variable described in **example 1** follows a geometric distribution.

Geometric Distribution

Let X be the random variable representing the *number of failures before the* ***first*** *success*. The probability, p, is fixed for each trial.

p is probability of success

$$P(X = x) = (1 - p)^x \cdot p$$

A geometric distribution has expectation $E[X] = \frac{1-p}{p}$ and variance $Var(X) = \frac{1-p}{p^2}$

E and V does not depend on n....

Note: There isn't a fixed number of trials, n as there is in a binomial distribution. **Be attentive of the wording in problems!!**

Note: $P(X \geq k) = q^k$ - If the number of trials is more than k, then the first k trials must have all been failures.

this is really important

Geometric Distribution

Memoryless Property

Suppose you have observed j consecutive failures. The probability that we will observe at least another k failures given that the first j must be failures is the probability of observing k failures. In other words:

$$P(X \geq j + k | X \geq j) = P(X \geq k)$$

Textbook Example 4.15 & 4.17

A recruiting firm finds that 20% of the applicants for a particular sales position are fluent in both English and Spanish. Applicants are selected at random from a pool and interviewed sequentially. Suppose that 10 applicants have been interviewed and no person fluent in both English and Spanish has been identified. What is the probability that 15 unqualified applicants will be interviewed before finding the first applicant who is fluent in English and Spanish?

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- Let X be the number of people interviewed before the first applicant is fluent in both English and Spanish.
- Then we are interested in the probability:

$$P(X = 15 | X \geq 10) = \frac{P(X = 15 \cap X \geq 10)}{P(X \geq 10)} = \frac{P(X = 15)}{P(X \geq 10)}$$

- Computing these probabilities, we have: $\dfrac{pq^{15}}{q^{10}} = pq^5 = P(X = 5)$

here, P(X>=10) = q^10 guarantees 10 failures