1 Definitions (15 marks total)

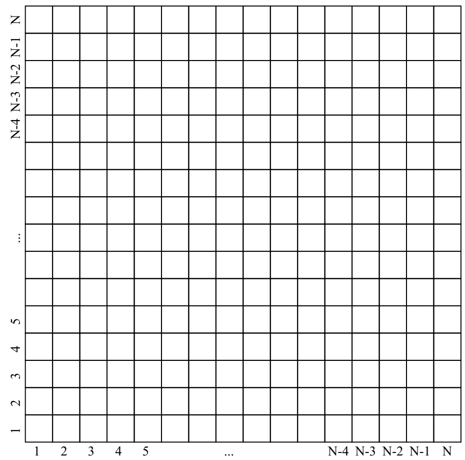
(a) **[5 Marks]** Give the second-order Taylor series expansion of a grayscale image I in the neighbourhood of pixel (x_0, y_0).

(b) **[5 Marks]** Give the definition of the Laplacian of image I at pixel (x, y) using standard calculus notation.

(c) **[5 Marks]** Define the principal curvatures at an image point $I(x, y)$. You may assume that (x, y) is an intensity extremum.

2 Gradient-based Image Reconstruction (15 marks total)

(a) **[2 Marks]** Suppose you are given an $N \times N$ grayscale image I. You are asked to compute ∇I using a first-order least-squares fit with a 1D, three-pixel sliding window. Mark on the grid below the pixels where it is possible to compute the gradient this way. Explain in a sentence.

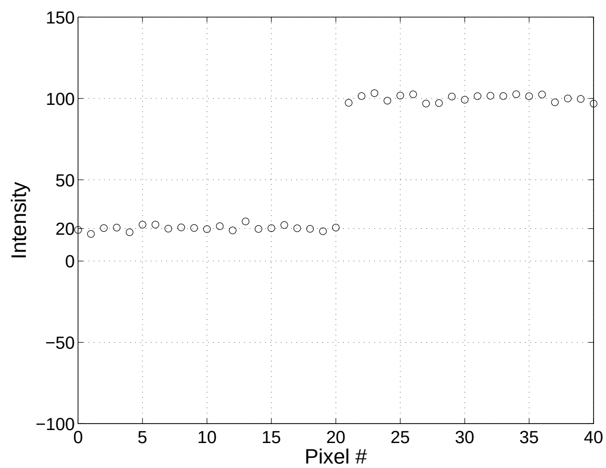


(b) **[10 Marks]** After computing ∇I, you somehow managed to erase the original image accidentally. Now you want to use ∇I to get the original image back. Formulate this computation as the solution of a linear system of equations of the form $\mathbf{Ax} = \mathbf{b}$. *Hint:* you should specify the dimensions of $\mathbf{A}, \mathbf{x}$ and $\mathbf{b}$ and the contents of at least one row of $\mathbf{A}$ and one element of $\mathbf{b}$.

(c) **[3 Marks]** Can I be computed from ∇I? Using your answer in (b), briefly explain why or why not. No marks will be awarded without an explanation.

3 Weighted Least Squares Estimation (20 marks total)

Consider the 1D image shown below, whose 41 pixels have intensities $I_0, \ldots, I_{40}$, respectively. We want to estimate the image intensity, $I(x)$, and its first derivative, $\frac{d}{dx}I(x)$, at pixel x using the sliding window algorithm with a first-order, weighted least squares fit. Assume the window has size 2*2+1 pixels and the weights are given by a function $\Omega(q)$, with $q \in [-2, 2]$.

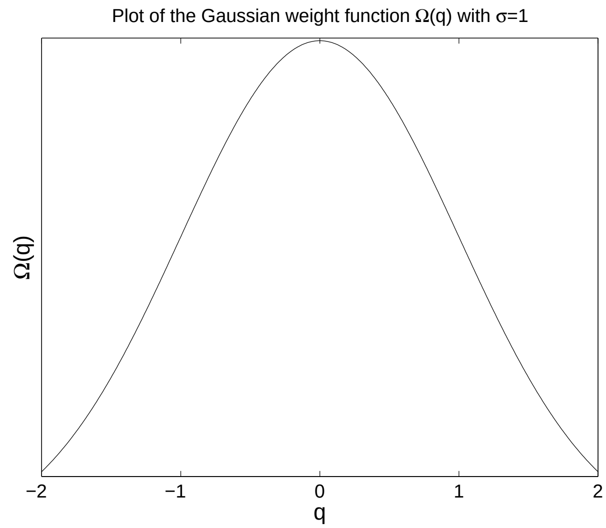


(a) **[5 Marks]** Using matrix notation, show the linear system that must be solved to compute the fit for pixel $x = 20$. Be sure to indicate the dimensions and contents of each matrix.

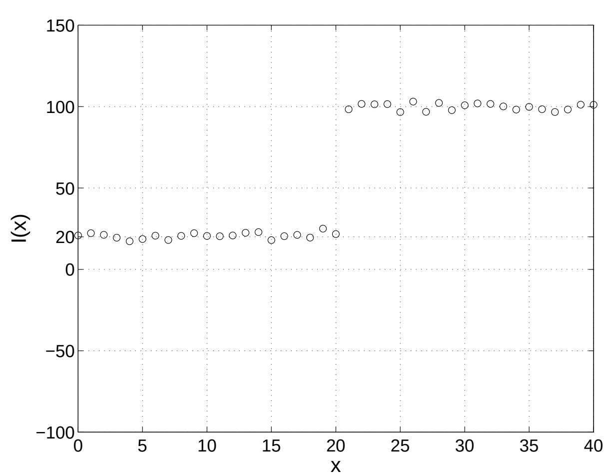
(b) Now suppose that the weight function is a Gaussian

$$\Omega(q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{q^2}{2\sigma^2}} \ ,$$

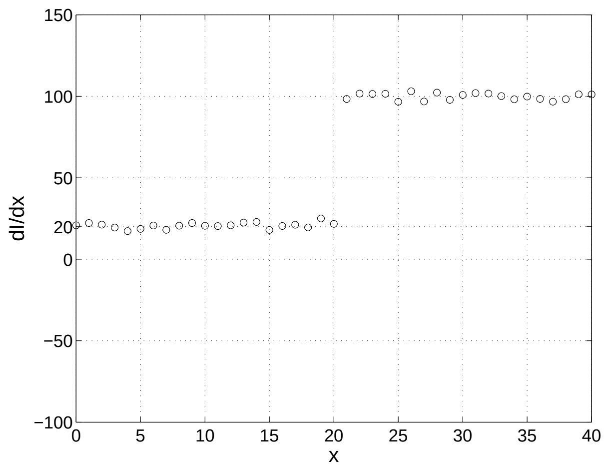
with $\sigma = 1$ (plotted below). Observe that the function is maximized at $q = 0$ and is almost zero when q is outside the range $[-2\sigma, 2\sigma]$.



(b1) **[5 Marks]** Plot the estimated intensity $I(x)$ on the graph below for $x \in [5, 35]$ and indicate the x values where important transitions in the shape of $I(x)$ will occur. For reference, the original pixel intensities are shown as well.



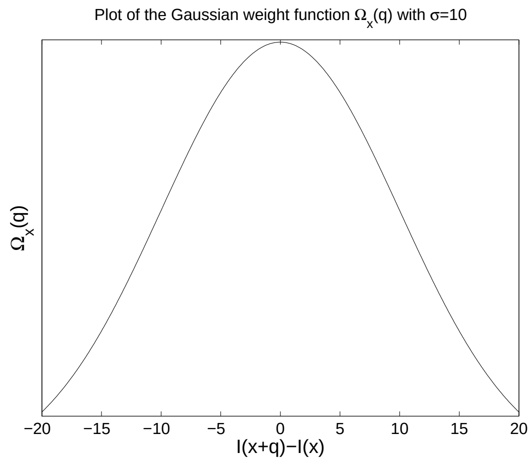
(b2) **[5 Marks]** Plot the estimated intensity derivative $\frac{d}{dx}I(x)$ for $x \in [5, 35]$. Indicate the x values where important transitions in the shape of $\frac{d}{dx}I(x)$ will occur and indicate the (approximate) value of $\frac{d}{dx}I(x)$ at those locations. For reference, the original pixel intensities are shown as well.



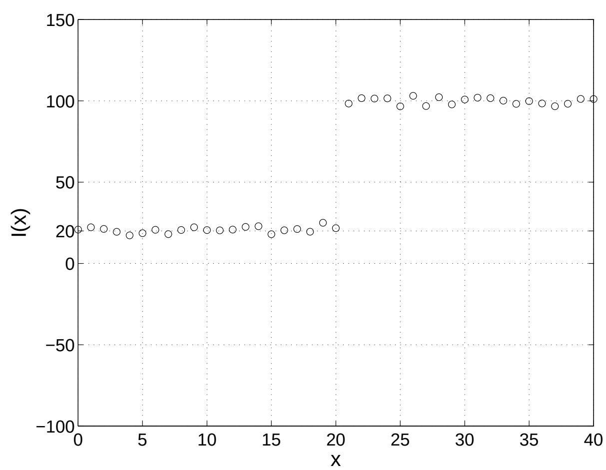
(c) **[5 Marks]** Finally, suppose that we do our estimation with a Gaussian weight function that *changes* from window to window and depends on pixel *intensities* within the window. Specifically, for the window centered at pixel x, we use the weight function

$$\Omega_x(q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(I(x+q)-I(x))^2}{2\sigma^2}} ,$$

with $\sigma = 10$ (plotted below).



Plot the estimated intensity $I(x)$ on the graph below for $x \in [5, 35]$ and indicate the x values where important transitions in the shape of $I(x)$ will occur. For reference, the original pixel intensities are shown as well.



END OF EXAM

Midterm Test

February 24th, 2010

CSC320H1S : Introduction to Visual Computing

Duration: 50 minutes

No aids allowed

There are 8 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

Question	Marks
1	__________/15
2	__________/15
3	__________/20
Total	__________/50