Design Theory for Relational Databases

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Introduction

- There are always many different schemas for a given set of data.
- E.g., you could combine or divide tables.
- How do you pick a schema?
  Which is better?
  What does "better" mean?
- Fortunately, there are some principles to guide us.

Database Design Theory

- It allows us to improve a schema systematically.
- General idea:
  - Express constraints on the relationships between attributes
  - Use these to decompose the relations
- Ultimately, get a schema that is in a "normal form" that guarantees good properties.
- "Normal" in the sense of conforming to a standard.
- The process of converting a schema to a normal form is called normalization.

Part I:
Functional Dependency Theory

A poorly designed table

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- In any domain, there are relationships between attribute values.
- Perhaps: functional dependencies
  - Every part has 1 manufacturer
  - Every manufacture has 1 address
  - Every seller has 1 address
- If so, this table will have redundant data.

Principle: Avoid redundancy

Redundant data can lead to anomalies.

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
~~8624~~	~~Lee Valley~~	~~102 Vaughn~~	~~ABC~~	~~1229 Bloor W~~	~~23.99~~
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- Update anomaly: if Hammers 'R Us moves and we update only one tuple, the data is inconsistent.
- Deletion anomaly: If ABC stops selling part 8624 and Lee Valley makes only that one part, we lose track of its address.

doing too much in one table

a type of integrity constraint

Definition of FD

- Suppose R is a relation, and X and Y are subsets of the attributes of R.
- $X \rightarrow Y$ asserts that:
  - If two tuples agree on all the attributes in set *X*, they must also agree on all the attributes in set *Y*.
- We say that "$X \rightarrow Y$ holds in R", or "*X* functionally determines *Y*."
- An FD constrains what can go in a relation.

More formally…

$A \rightarrow B$ means:

$\forall$ tuples t_1, t_2,

$(t_1[A] = t_2[A]) \Rightarrow (t_1[B] = t_2[B])$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$(t_1[A] = t_2[A]) \wedge (t_1[B] \neq t_2[B])$

Generalization to multiple attributes

$A_1A_2 \ldots A_m \rightarrow B_1B_2 \ldots B_n$ means:

$\forall$ tuples t_1, t_2,

$$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \Rightarrow$$
$$(t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \wedge$$
$$\neg (t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$$

Why "functional dependency"?

- "dependency" because the value of *Y* depends on the value of *X*.
- "functional" because there is a mathematical function that takes a value for X and gives a *unique* value for Y.
- (It's not a typical function; just a lookup.)

f: X -> Y

Equivalent sets of FDs

- When we write a set of FDs, we mean that all of them hold.
- We can very often rewrite sets of FDs in equivalent ways.
- When we say S_1 is equivalent to S_2 we mean that:
  - S_1 holds in a relation iff S_2 does.

Splitting rules for FDs

- Can we split the RHS of an FD and get multiple, equivalent FDs?

yes the splitting/combining rules

- Can we split the LHS of an FD and get multiple, equivalent FDs?

Coincidence or FD?

- An FD is an assertion about *every* instance of the relation.
- You can't know it holds just by looking at one instance.
- You must use knowledge of the domain to determine whether an FD holds.

FDs are closely related to keys

- Suppose K is a set of attributes for relation R.
- Our old definition of superkey:

  a set of attributes for which no two rows can have the same values.
- A claim about FDs:

  K is a *superkey* for *R* iff
  K functionally determines all of *R*.

FDs are a generalization of keys

- key:
  $X \rightarrow \boxed{R}$ Every attribute

- Functional dependency:
  $X \rightarrow \boxed{Y}$ Not necessarily every attribute

- An FD can be more subtle.

Inferring FDs

- Given a set of FDs, we can often infer further FDs.
- This will be handy when we apply FDs to the problem of database design.
- Big task: given a set of FDs,
  infer *every* other FD that must also hold.
- Simpler task: given a set of FDs,
  check whether *a given* FD must also hold.

Examples

- If $A \rightarrow B$ and $B \rightarrow C$ hold,
  must $A \rightarrow C$ hold?
- If $A \rightarrow H$, $C \rightarrow F$, and $F\,G \rightarrow A\,D$ hold,
  must $F\,A \rightarrow D$ hold?
  must $C\,G \rightarrow F\,H$ hold?
- If $\mathrm{H} \rightarrow \mathrm{GD}$, $\mathrm{HD} \rightarrow \mathrm{CE}$, and $\mathrm{BD} \rightarrow \mathrm{A}$ hold,
  must $\mathrm{EH} \rightarrow \mathrm{C}$ hold?
- Aside: we are not generating new FDs, but testing a specific possible one.

Method 1: Prove an FD follows using first principles

- You can prove it by referring back to
  - The FDs that you know hold, and
  - The definition of functional dependency.
- But the Closure Test is easier.

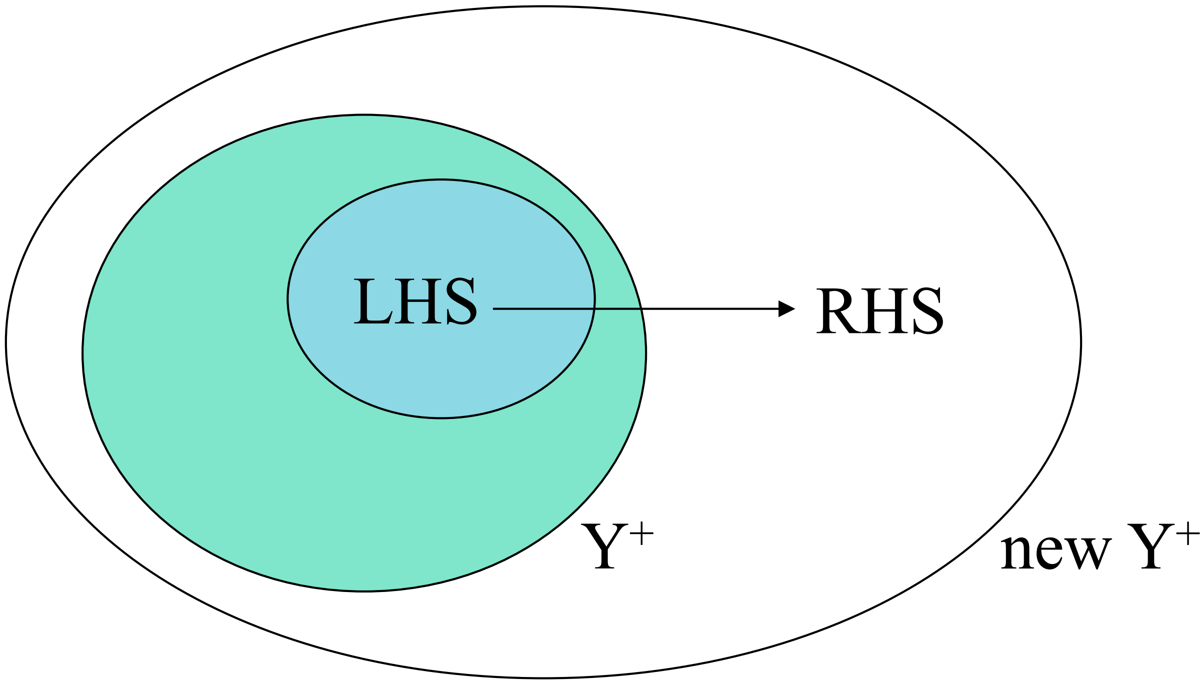
Method 2: Prove an FD follows using the Closure Test

- Assume you know the values of the LHS attributes, and figure out everything else that is determined. finding the closure
- If it includes the RHS attributes, then you know that LHS $\rightarrow$ RHS
- This is called the closure test.

Y is a set of attributes, S is a set of FDs.
Return the closure of Y under S.

Attribute_closure(Y, S):
  Initialize Y^+ to Y
  Repeat until no more changes occur:
    If there is an FD LHS $\rightarrow$ RHS in S
    such that LHS is in Y^+:
      Add RHS to Y^+
    Return Y^+

Visualizing attribute closure



If LHS is in Y^+ and LHS $\rightarrow$ RHS holds, we can add RHS to Y^+

S is a set of FDs; LHS $\rightarrow$ RHS is a single FD.
Return true iff LHS $\rightarrow$ RHS follows from S.

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FD_follows(S, LHS → RHS):
    Y+ = Attribute_closure(LHS, S)
    return (RHS is in Y+)
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Projecting FDs

- Later, we will learn how to normalize a schema by decomposing relations.
  - This is the whole point of this theory.
- We will need to know what FDs hold in the new, smaller, relations.
  - We must project our FDs onto the attributes of our new relations.

Example

R(A1, ..., An)   Set of attributes: A

Decompose into:

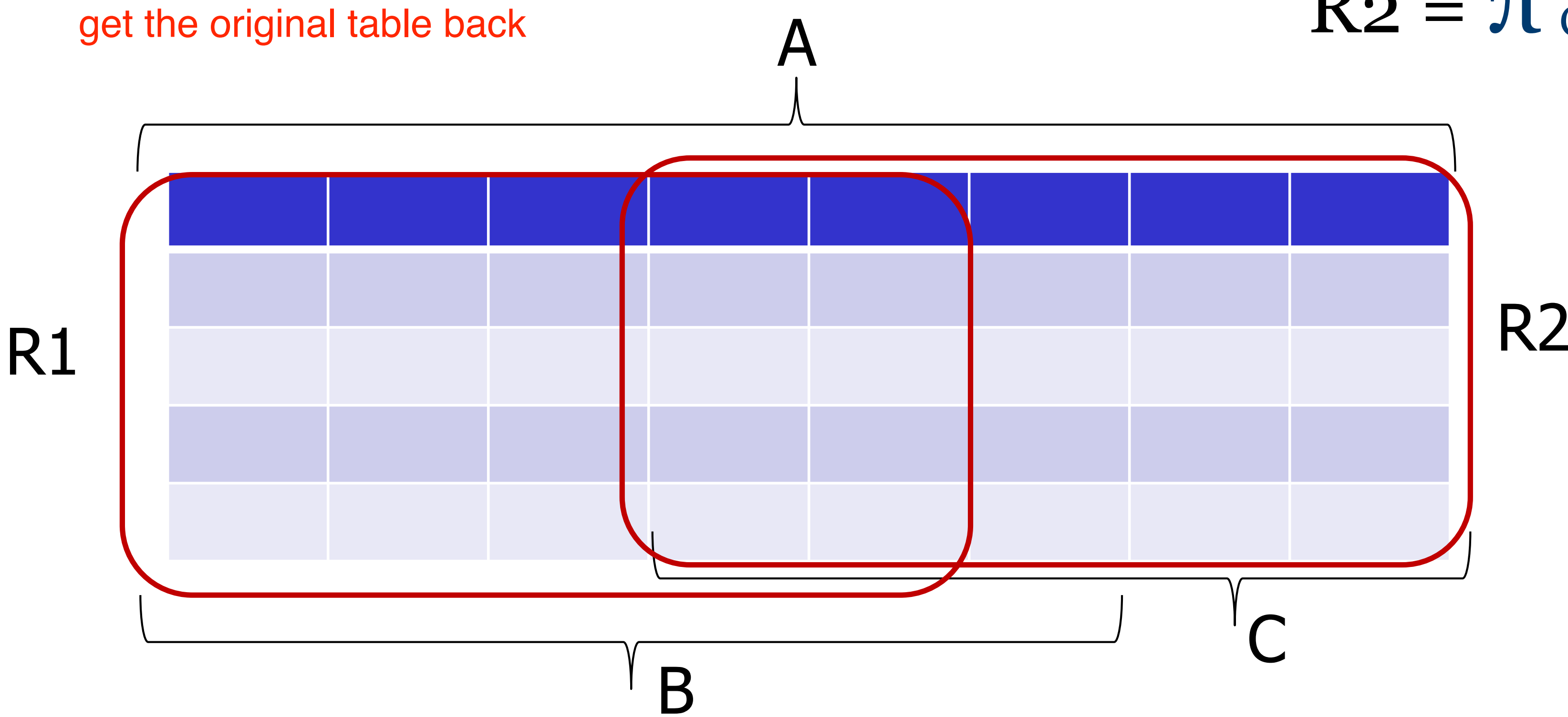
- R1(B1, ..., Bk)   Set of attributes: B, and
- R2(C1, ..., Cm)   Set of attributes: C

$R1 = \pi_B(R)$

$R2 = \pi_C(R)$

$B \cup C = A$, $R1 \bowtie R2 = R$

get the original table back



S is a set of FDs; L is a set of attributes.

Return the projection of S onto L:

all FDs that follow from S and involve only attributes from L.

Project(S, L):

Initialize T to {}.

For each subset X of L: any combination…

Compute X^+ *Close X and see what we get.*

For every attribute A in X^+:

If A is in L: *$X \rightarrow A$ is only relevant if A is in L (we know X is).*

add $X \rightarrow A$ to T.

Return T.

A few speed-ups

- No need to add $X \rightarrow A$ if A is in X itself. It's a trivial FD.
- These subsets of X won't yield anything, so no need to compute their closures:
  - the empty set
  - the set of all attributes is just everything
- Neither are big savings, but …

A big speed-up

a key already, so superset of X are superkeys

- If we find X^+ = all attributes, we can ignore any superset of X.
  - It can only give use "weaker" FDs (with more on the LHS).
- This is a big time saver!

Projection is expensive

- Even with these speed-ups, projection is still expensive.
- Suppose R_1 has n attributes.
  How many subsets of R_1 are there?

Minimal Basis

- We saw earlier that we can very often rewrite sets of FDs in equivalent ways.
- Example: $S_1 = \{A \rightarrow BC\}$ is equivalent to $S_2 = \{A \rightarrow B, A \rightarrow C\}$.
- Given a set of FDs S, we may want to find a minimal basis: A set of FDs that is equivalent, but has
  - no redundant FDs, and
  - no FDs with unnecessary attributes on the LHS.

S is a set of FDs. Return a minimal basis for S.

Minimal_basis(S):

1. Split the RHS of each FD
2. For each FD $X \rightarrow Y$ where $|X| \geq 2$:

   If you can remove an attribute from X and get an FD that follows from S:

   Do so! (It's a stronger FD.)

3. For each FD f:

   If S – {f} implies f:

   Remove f from S.

compute closure of LHS of f, see if RHS of f implied by S

repeat 2,3 until cant update

Some comments on minimal basis

- Often there are multiple possible results.

  Depends on the order in which you consider the possible simplifications.

- After you identify a redundant FD, you must not use it when computing subsequent closures.

step 3

… and less intuitive

- When you are computing closures to decide whether the LHS of an FD

  $X \rightarrow Y$

  can be simplified, continue to use that FD.

- You must do (2) and (3) in that order.

  Otherwise, must repeat until no changes occur.

Part II:
Using FD Theory to do Database Design

Recall that poorly designed table?

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers ‘R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers ‘R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers ‘R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- We can now express the relationships as FDs:
  - part → manufacturer
  - manufacturer → address
  - seller → address
- The FDs tell us there can be redundancy, thus the design is bad.
- That’s why we care about FDs.

Decomposition

- To improve a badly-designed schema $R(A_1, \ldots A_n)$, we will decompose it into smaller relations

  $R1(B_1, \ldots B_j)$ and $R2(C_1, \ldots C_k)$ such that:

  - $R1 = \pi_{B1, \ldots Bj}(R)$
  - $R2 = \pi_{C1, \ldots Ck}(R)$
  - $\{B_1, \ldots B_j\} \cup \{C_1, \ldots C_k\} = \{A_1, \ldots A_n\}$
  - $R1 \bowtie R2 = R$

$R(A_1, \ldots A_n)$ Set of attributes: A

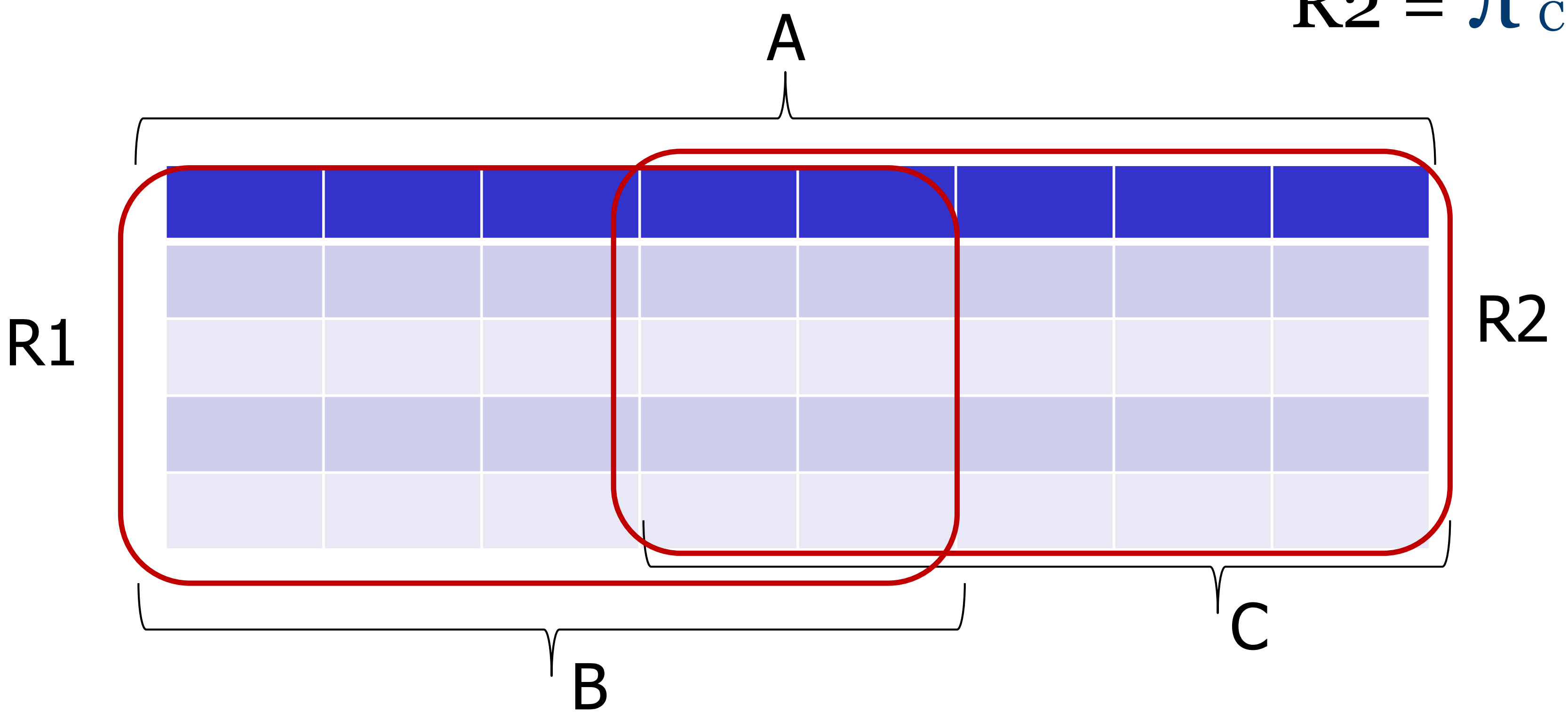
Decompose into:

- $R1(B_1, \ldots B_j)$ Set of attributes: B, and
- $R2(C_1, \ldots C_k)$ Set of attributes: C

$$B \cup C = A, \quad R1 \bowtie R2 = R$$

$$R1 = \pi_B(R)$$

$$R2 = \pi_C(R)$$



But *which* decomposition?

- Decomposition can definitely improve a schema.
- But which decomposition?
  There are many possibilities.
- And how can we be sure a new schema doesn't exhibit other anomalies?
- Boyce-Codd Normal Form *guarantees* it.

Boyce-Codd Normal Form

- We say a relation *R* is in *BCNF* if for every nontrivial FD $X \rightarrow Y$ that holds in *R*, *X* is a superkey.
  - Remember: *nontrivial* means *Y* is not contained in *X*.
  - Remember: a *superkey* doesn't have to be minimal.
- [Exercise]

the only FD allowed are superkeys, but since relation guarantee non-duplicate rows, there is no duplicate data

Intuition

In other words, BCNF requires that:

Only things that functionally determine ***everything***
can functionally determine ***anything***.

Why is the BCNF property valuable?

Note:

- FDs are not the problem. They are facts!
- The schema (in the context of the FDs) is the problem.

R is a relation; F is a set of FDs.
Return the BCNF decomposition of R, given these FDs.

BCNF_decomp(R, F):

If an FD $X \rightarrow Y$ in F violates BCNF

Compute X^+.

Replace R by two relations with schemas:

$$R_1 = X^+$$

$$R_2 = R - (X^+ - X)$$

Project the FD's F onto R_1 and R_2.

Recursively decompose R_1 and R_2 into BCNF.

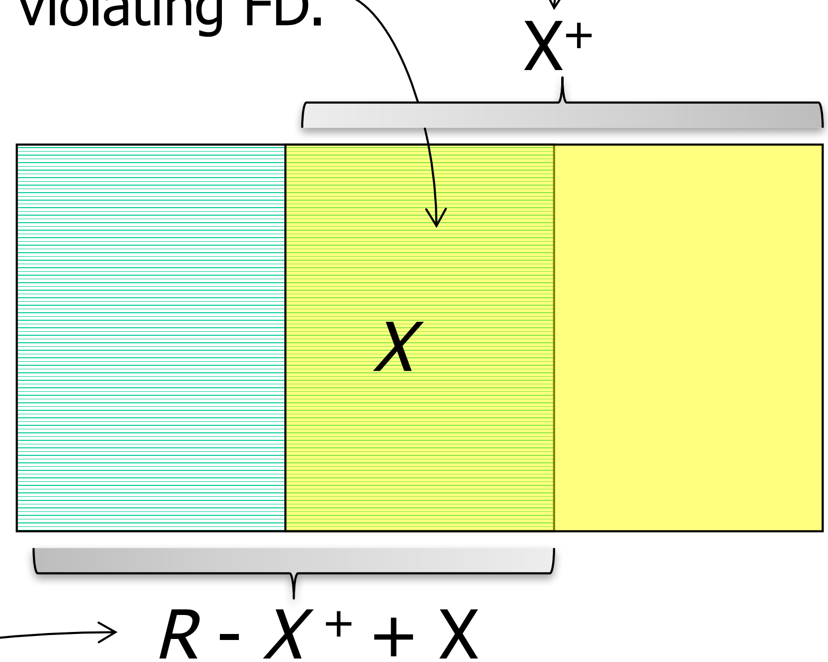
relation with <= attr is in BCNF

[Example]

Decomposition Picture

1) Start with the LHS
of the violating FD.

2) Close the LHS
to get one new relation



3) Everything except the new stuff is the other new relation.
X is in both new relations to make a connection between them.

Some comments on BCNF decomp

- If more than one FD violates BCNF, you may decompose based on any one of them.
  - So there may be multiple results possible.
- The new relations we create may not be in BCNF. We must recurse.
  - We only keep the relations at the "leaves".
- How does the decomposition step help? [Exercise]

Speed-ups for BCNF decomposition

- Don't need to know any keys.
  - Only superkeys matter.
- And don't need to know *all* superkeys.
  - Only need to check whether the LHS of each FD is a superkey.
  - Use the closure test (simple and fast!).

BCNF

- Every attribute depends on:
  - The key
  - The whole key
  - And nothing but the key…

so help me Codd….

More speed-ups

- When projecting FDs onto a new relation, check each new FD:
  - Does the new relation violate BCNF because of this FD?
- If so, abort the projection.
  - You are about to discard this relation anyway (and decompose further).

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