$$S_{xx}=\sum_{i=1}^{n}(x_i-\bar{x})^2,\qquad S_{xy}=\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}),\qquad \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x},\qquad \hat{\beta}_1=\frac{S_{xy}}{S_{xx}}=\frac{\sum_{i=1}^{n}x_iy_i-n\bar{x}\,\bar{y}}{\sum_{i=1}^{n}x_i^2-n\bar{x}^2}$$

$$\operatorname{var}(\hat{\beta}_0)=\sigma^2\left[\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right],\qquad \operatorname{var}(\hat{\beta}_1)=\frac{\sigma^2}{S_{xx}},\qquad \operatorname{cov}(\hat{\beta}_0,\,\hat{\beta}_1)=-\frac{\sigma^2\bar{x}}{S_{xx}}$$

$$\underbrace{\sum_{i=1}^{n}(y_i-\bar{y})^2}_{\text{SST}}=\underbrace{b_1^2\sum_{i=1}^{n}(x_i-\bar{x})^2}_{\text{SSReg}}+\underbrace{\sum_{i=1}^{n}\hat{e}_i^2}_{\text{RSS}},\qquad \text{SSReg}=\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2,\qquad F_{\text{obs}}=\frac{\text{MSReg}}{\text{MSE}}$$

$$\operatorname{var}(\hat{y}^*)=\sigma^2\left[\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right],\qquad \operatorname{var}(Y^*-\hat{y}^*)=\sigma^2\left[1+\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right]$$

$$r=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}},\qquad \hat{y}_i=\sum_{j=1}^{n}h_{ij}\,y_k\qquad h_{ij}=\frac{1}{n}+\frac{(x_i-\bar{x})(x_j-\bar{x})}{S_{xx}}$$

$$\text{DFBETA}_{ik}=\frac{\hat{\beta}_k-\hat{\beta}_{k(i)}}{\text{s.e. of }\hat{\beta}_{k(i)}},\qquad \text{DFFITS}_i=\frac{\hat{y}_i-\hat{y}_{i(i)}}{\text{s.e. of }\hat{y}_{i(i)}},\qquad D_i=\frac{\sum_{j=1}(\hat{y}_{j(i)}-\hat{y}_j)^2}{2S^2}=\frac{r_i^2h_{ii}}{2(1-h_{ii})}$$

$r_i=\frac{\hat{e}_i}{S\sqrt{1-h_{ii}}}$ where $S^2=\text{MSE}=\frac{\text{RSS}}{n-p-1}$ Criteria for ordinary data points on small datasets: $|r_i|<2$, $h_{ii}<2(p+1)/n$, DFBETA $<2/\sqrt{n}$, DFFITS $<2\sqrt{\frac{p+1}{n}}$, $D_i<4/(n-p-1)$

Transformations: $f'(\mu)\propto\dfrac{1}{\sqrt{V(\mu)}}$ Covariance matrix: $\operatorname{var}(\mathbf{X})=\operatorname{E}\left[(\mathbf{X}-\operatorname{E}(\mathbf{X})\,(\mathbf{X}-\operatorname{E}(\mathbf{X}))'\right]$

For $f(\boldsymbol{\theta})=\mathbf{c}'\boldsymbol{\theta}$, $\dfrac{\partial f(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}=\mathbf{c}$ For $f(\boldsymbol{\theta})=\boldsymbol{\theta}'\mathbf{A}\boldsymbol{\theta}$, $\dfrac{\partial f(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}=2\mathbf{A}\boldsymbol{\theta}$

$$\text{RSS}(\boldsymbol{\beta})=\sum_{i=1}^{n}(y_i-\beta_0-\beta_1x_i)^2=(\mathbf{Y}-\mathbf{X}\boldsymbol{\beta})'(\mathbf{Y}-\mathbf{X}\boldsymbol{\beta})\qquad \hat{\boldsymbol{\beta}}=(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}\quad\text{and}\quad\operatorname{var}\left(\hat{\boldsymbol{\beta}}\right)=\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$$

$$\mathbf{X}'\mathbf{X}=\begin{pmatrix}n & n\bar{x}\\ n\bar{x} & \sum x_i^2\end{pmatrix}\qquad\text{and}\qquad(\mathbf{X}'\mathbf{X})^{-1}=\frac{1}{S_{xx}}\begin{pmatrix}\frac{1}{n}\sum_{i=1}^{n}x_i^2 & -\bar{x}\\ -\bar{x} & 1\end{pmatrix}$$

$$\hat{\mathbf{e}}=(\mathbf{I}-\mathbf{H})\,\mathbf{Y}\quad\text{where}\quad\mathbf{H}=\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\qquad\operatorname{var}(\hat{\mathbf{e}})=(\mathbf{I}-\mathbf{H})\operatorname{E}(\mathbf{Y}\mathbf{Y}')(\mathbf{I}-\mathbf{H})$$

$$\underbrace{\mathbf{Y}'(\mathbf{I}-\frac{1}{n}\mathbf{J})\,\mathbf{Y}}_{\text{SST}}=\underbrace{\mathbf{Y}'\left(\mathbf{H}-\frac{1}{n}\mathbf{J}\right)\mathbf{Y}}_{\text{SSReg}}+\underbrace{\mathbf{Y}'(\mathbf{I}-\mathbf{H})\mathbf{Y}}_{\text{RSS}}$$

$$\text{Adj }R^2=1-\frac{n-1}{n-p-1}\,\frac{\text{RSS}}{\text{SST}}\qquad\text{Partial }F\text{-test}:F=\frac{(\text{RSS(reduced)}-\text{RSS(full)})\,/\,(\text{df}_{\text{reduced}}-\text{df}_{\text{full}})}{\text{RSS(full)}\,/\,\text{df}_{\text{full}}}$$

$$\text{VIF}_j=\frac{1}{1-R_j^2}\qquad r_{\text{partial}}(X_1,X_2)=r(\hat{e}_{X_1\text{ vs }Z},\,\hat{e}_{X_2\text{ vs }Z})\qquad\text{WRSS}=\sum_{i=1}^{n}w_i\,(y_i-\hat{y}_i)^2\ \text{ for }w_i\propto1/\sigma_i^2$$

$$\hat{\beta}_{1W}=\frac{\sum_{i=1}^{n}w_i\,(x_i-\bar{x}_W)\,(y_i-\bar{y}_W)}{\sum_{i=1}^{n}w_i\,(x_i-\bar{x}_W)^2}\qquad\hat{\beta}_{0W}=\bar{y}_W-\hat{\beta}_{1W}\,\bar{x}_W$$

$$\begin{pmatrix}\hat{\beta}_{0W}\\ \hat{\beta}_{1W}\end{pmatrix}=\hat{\boldsymbol{\beta}}_W=\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y}\quad\text{and var}\quad(\hat{\boldsymbol{\beta}}_W)=\sigma^2\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}$$