Problem Set 10

You are strongly encouraged to solve the following exercise before the final exam:

To explore the potential effect of age on systolic blood pressure (SBP), data of 33 women, aged 22-81 was collected. It is presented here in Table 1.

(a) Figure 1 displays a scatter plot of the data in Table 1. What is your early assessment of the idea to fit a simple linear regression model to the data?

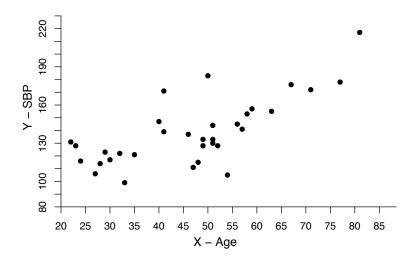


Figure 1: A scatter plot of the data displayed in Table 1.

a linear trend but maybe too much noise.

Use Least Square Estimators formula

- (b) Calculate the straight line equation for the simple linear regression model and interpret the estimated slope. What proportion of the variability in SBP can be explained by age?

 R^2 = r^2, so find sample correlation coefficient
- (c) Use Table 2 to estimate the random noise variance. Test the hypothesis on the existence of a linear trend (at the 5% level) and provide a 95% confidence interval for the slope.

 slope estimate follows t_{n-2} distribution
- (d) Provide a point estimate and a 95% confidence interval for the mean SBP of 37 year old women. (use $t_{31,0.975} = 2.04$)
- (e) Express your opinion on the validity of the model assumptions, based on the residual plots in Figure 2.

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2 23 128 3 24 116 4 27 106 5 28 114 6 29 123 7 30 117 8 32 122 9 33 99 10 35 121 11 40 147 12 41 139 13 41 171 14 46 137 15 47 111 16 48 115 17 49 133 18 49 128 19 50 183 20 51 130 21 51 133 22 51 144 23 52 128 24 54 105 25 56 145 26 57 141
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25 56 145 26 57 141
26 57 141
27 58 153
28 59 157
29 63 155
30 67 176
31 71 172
32 77 178
33 81 217
$\sum x_i = 1542$ $\sum x_i^2 = 79,716$
$\sum y_i = 4,575$ $\sum y_i^2 = 656,481$

Table 1: Raw data for the SBP vs. Age example.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Age (X)	SBP(Y)	\hat{Y}	e
3 24 116 110.9 5.1 4 27 106 114.5 -8.5 5 28 114 115.7 -1.7 6 29 123 117.0 6.0 7 30 117 118.2 -1.2 8 32 122 120.6 1.4 9 33 99 121.9 -22.9 10 35 121 124.3 -3.3 11 40 147 130.4 16.6 12 41 139 131.6 7.4 13 41 171 131.6 39.4 14 46 137 137.7 -0.7 15 47 111 139.0 -28.0 16 48 115 140.2 -25.2 17 49 133 141.4 -8.4 18 49 128 141.4 -13.4 19 50	1	22	131	108.4	22.6
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5 28 114 115.7 -1.7 6 29 123 117.0 6.0 7 30 117 118.2 -1.2 8 32 122 120.6 1.4 9 33 99 121.9 -22.9 10 35 121 124.3 -3.3 11 40 147 130.4 16.6 12 41 139 131.6 7.4 13 41 171 131.6 39.4 14 46 137 137.7 -0.7 15 47 111 139.0 -28.0 16 48 115 140.2 -25.2 17 49 133 141.4 -8.4 18 49 128 141.4 -13.4 19 50 183 142.6 40.4 20 51 130 143.9 -13.9 21 51	3	24	116	110.9	5.1
6 29 123 117.0 6.0 7 30 117 118.2 -1.2 8 32 122 120.6 1.4 9 33 99 121.9 -22.9 10 35 121 124.3 -3.3 11 40 147 130.4 16.6 12 41 139 131.6 7.4 13 41 171 131.6 7.4 13 41 171 131.6 39.4 14 46 137 137.7 -0.7 15 47 111 139.0 -28.0 16 48 115 140.2 -25.2 17 49 133 141.4 -8.4 18 49 128 141.4 -13.4 19 50 183 142.6 40.4 20 51 130 143.9 -13.9 21 51	4	27	106	114.5	-8.5
7 30 117 118.2 -1.2 8 32 122 120.6 1.4 9 33 99 121.9 -22.9 10 35 121 124.3 -3.3 11 40 147 130.4 16.6 12 41 139 131.6 7.4 13 41 171 131.6 39.4 14 46 137 137.7 -0.7 15 47 111 139.0 -28.0 16 48 115 140.2 -25.2 17 49 133 141.4 -8.4 18 49 128 141.4 -13.4 19 50 183 142.6 40.4 20 51 130 143.9 -10.9 21 51 133 143.9 -10.9 22 51 144 143.9 0.1 23 52 </td <td>5</td> <td>28</td> <td>114</td> <td>115.7</td> <td>-1.7</td>	5	28	114	115.7	-1.7
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14 46 137 137.7 -0.7 15 47 111 139.0 -28.0 16 48 115 140.2 -25.2 17 49 133 141.4 -8.4 18 49 128 141.4 -13.4 19 50 183 142.6 40.4 20 51 130 143.9 -13.9 21 51 133 143.9 -10.9 22 51 144 143.9 0.1 23 52 128 145.1 -13.1 24 54 105 147.5 -42.5 25 56 145 150.0 -5.0 26 57 141 151.2 -10.2 27 58 153 152.4 0.6 28 59 157 153.6 3.4 29 63 155 158.5 -3.5 30 67 176 168.3 3.7 32 77 178 1	12	41	139	131.6	7.4
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32 77 178 175.6 2.4 33 81 217 180.5 36.5	30	67	176	163.4	12.6
33 81 217 180.5 36.5	31	71	172	168.3	3.7
	32	77	178	175.6	2.4
$\sum e_i^2 = 10769.7$	33	81	217	180.5	36.5
					$\sum e_i^2 = 10769.7$

Table 2: The original data table along with the fitted values and the model residuals.

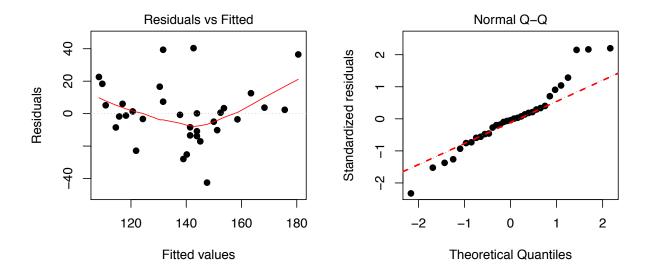


Figure 2: Residual plots for the linear fit.

Solution:

(a) There definitely appears to be an upward trend, but with the abundance of noise it is hard to tell whether or not it is linear. In any case, the ability of age alone to explain differences in SBP seems to be limited.

(b)
$$\overline{x} = \frac{1542}{33} = 46.73$$
, $S_x^2 = \frac{\sum x_i^2 - n\overline{x}^2}{n - 1} = \frac{79716 - 33 \times (46.73)^2}{32} = 239.45$, $\overline{y} = \frac{4575}{33} = 138.64$, $S_{xy} = \frac{\sum x_i y_i - n\overline{x}\overline{y}}{n - 1} = \frac{223144 - 33 \times 46.73 \times 138.64}{32} = 292.71$, $\widehat{\beta}_1 = \frac{S_{xy}}{S_x^2} = \frac{292.71}{239.45} = 1.222$ and $\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1 \overline{x} = 138.64 - 1.222 \times 46.73 = 81.52$,

and the linear fit equation is thus

$$\hat{y}(x) = 81.52 + 1.222x.$$

We estimate the increase in average systolic blood pressure at 1.222 per one year of aging. In addition,

$$S_Y^2 = \frac{\sum y_i^2 - n\overline{y}^2}{n - 1} = \frac{656481 - 33 \times (138.64)^2}{32} = 694.36$$

$$S_Y^2 = \frac{\sum y_i^2 - n\overline{y}^2}{n - 1} = \frac{656481 - 33 \times (138.64)^2}{32} = 694.36$$

$$\Longrightarrow R^2 = r_{XY}^2 = \frac{\overline{S_{XY}^2}}{S_X^2 S_Y^2} = \frac{(292.71)^2}{239.45 \times 694.36} = 0.5153,$$

hence only 51.53% of the variation in the SBP values can be explained by age, as expected perhaps.

(c) As argued in class, the noise variance can be estimated by

$$S^{2} = \frac{1}{n-2} \sum_{i=1}^{n} e_{i}^{2} = \frac{10769.71}{31} = 347.41.$$

Testing for a linear trend is based on evaluating $\mathcal{T} = \frac{\widehat{\beta}_1}{\frac{S}{\sqrt{\sum_j (x_j - \overline{x})^2}}}$ with respect to the t_{n-2} distribution. Here

$$\mathcal{T} = \frac{1.222}{\frac{\sqrt{347.41}}{\sqrt{(33-1)\cdot 239.45}}} = 5.73 > 2.04 = t_{31,0.975}$$

(no row for 31 degrees of freedom in the table, but clearly $t_{31,0.975} < t_{30,0.975} = 2.042$), and it can be verified that the p-value is 2.57×10^{-6} , so if we trust the model, the existence of a linear trend is undeniable. Similarly,

$$\widehat{\beta}_1 \pm \frac{S}{\sqrt{\sum_j (x_j - \overline{x})^2}} t_{n-2,1-\alpha/2} = 1.222 \pm \frac{\sqrt{347.41}}{\sqrt{(33-1) \cdot 239.45}} \underbrace{t_{31,0.975}}_{2.04}$$

$$= 1.222 \pm 0.434 = [0.788, 1.656]$$

is a 95% confidence interval for β_1 .

(d) A point estimate for the mean SBP of 37 year old women would be

$$\hat{y}(37) = 81.52 + 1.222 \times 37 = 126.73,$$

whereas a 95% confidence interval for the mean is given by

$$\widehat{y}(x_0) \pm t_{n-2,1-\alpha/2} S \sqrt{\frac{1}{n} + \frac{(x_0 - \overline{x})^2}{\sum_j (x_j - \overline{x})^2}}$$

$$= 126.73 \pm \underbrace{t_{31,0.975}}_{2.04} \sqrt{\frac{1}{347.41}} \sqrt{\frac{1}{33} + \frac{(37 - 46.73)^2}{(33 - 1) \cdot 239.45}} = 126.73 \pm 7.85$$

$$= [118.88, 134.58].$$

The fitted line and the 95% confidence bands are displayed in Figure 3.

(e) The "belly" in the Residuals vs. Fitted plot implies model misspecification. This is perhaps the reason why the standardized residuals in the Q-Q plot appear to have heavier tails than expected. It is possible that a quadratic model of the form

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i$$

could prove a better fit for this particular dataset.

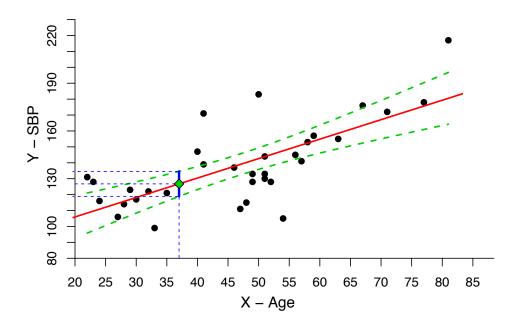


Figure 3: Point estimate and 95% confidence bands for the mean SBP of women, based on the linear regression model.