STA 247
Probability with Computer Applications

Professor K. H. Wong

Week 4 - Topic B

Negative Binomial Distribution

Consider the following example:

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

Negative Binomial Distribution

Consider the following example:

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Not interested in 5 defects in 100 parts

Negative Binomial Distribution

Consider the following example:

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Not interested in 5 defects in 100 parts
- Not interested in only 1 defect

Negative Binomial Distribution

Consider the following example:

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Not interested in 5 defects in 100 parts
- Not interested in only 1 defect
- In fact, it's the probability that the factory will produce *99 parts before the 5th defect occurs*

Negative Binomial Distribution

Consider the following example:

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Not interested in 5 defects in 100 parts
- Not interested in only 1 defect
- In fact, it's the probability that the factory will produce *99 parts before the 5th defect occurs*
- That is, the 100th trial results in a defect (fixed order)

Negative Binomial Distribution

Consider the following example:

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Not interested in 5 defects in 100 parts
- Not interested in only 1 defect
- In fact, it's the probability that the factory will produce *99 parts before the 5th defect occurs*
- That is, the 100th trial results in a defect (fixed order)
- And 4 defects occur in 99 trials (unfixed order)

so 99_C_4 * p^4 * (1-p)^95 which is P(X=4) when X ~ Bin(99, 0.02)

Negative Binomial Distribution

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

Negative Binomial Distribution

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Let $X =$ number of failures before the r^{th} success. In this case, there are $100 - 5 = 95$ "failures" i.e./ non-defective parts, and $r = 5$ as trials end once the 5th defect occurs on the 100th part.

Negative Binomial Distribution

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Let $X =$ number of failures before the r^{th} success. In this case, there are $100 - 5 = 95$ "failures" i.e./ non-defective parts, and $r = 5$ as trials end once the 5th defect occurs on the 100th part.
- The first 4 defective follow a binomial distribution $\sim Bin(99, 0.02)$

Negative Binomial Distribution

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Let X = number of failures before the r^{th} success. In this case, there are $100 - 5 = 95$ "failures" i.e./ non-defective parts, and $r = 5$ as trials end once the 5th defect occurs on the 100th part.
- The first 4 defective follow a binomial distribution $\sim Bin(99, 0.02)$
  $P(X = 95) = \binom{99}{4}(0.02)^4 \cdot (0.98)^{99-4} \cdot (0.02)$

note p represent probability of success or defective in this case

Negative Binomial Distribution

Example 5 A factory that produces car parts has a defect rate of 2%. What is the probability that the factory will produce 5 defects by the 100th part produced?

- Let X = number of failures before the r^{th} success. In this case, there are $100 - 5 = 95$ "failures" i.e./ non-defective parts, and $r = 5$ as trials end once the 5th defect occurs on the 100th part.
- The first 4 defective follow a binomial distribution $\sim Bin(99, 0.02)$
  $P(X = 95) = \binom{99}{4}(0.02)^4 \cdot (0.98)^{99-4} \cdot (0.02)$
  $P(X = 95) = \binom{99}{4}(0.02)^5 \cdot (0.98)^{95} = 0.00176 = 0.18\%$

Computing in R:

R code

```
dbinom(4, size = 99, p = 0.2)*0.2
[1] 0.00176734
```

Negative Binomial Distribution

Notice in the example that (x number of failures) + (r number of successes) = (total number of trials). Since the last success is guaranteed in the last trial, you need only find the probability of $(r-1)$ successes in the reamaining $(x+r-1)$ trials. This type of random variable X has a *negative binomial distribution*.

Examples

- The number of losing lotto numbers selected before you have 6 winning numbers
- The number of tails before getting 4 heads in a coin toss

Negative Binomial Distribution

Negative Binomial Distribution

Let $X =$ number of failures before r^{th} success and p the probability of success is fixed for each trial.

$$P(X = x) = \binom{x+r-1}{r-1} \cdot p^r \cdot (1-p)^x$$

A negative binomial distribution has expectation $E[X] = \frac{r(1-p)}{p}$ and variance $Var(X) = \frac{r(1-p)}{p^2}$

Note: There isn't a fixed number of trials, n as there is in a binomial distribution.

this is like finding combination of choose r from x+r:
which means finding unique combination of r-1
occurrences in x+r trials. then times probability of
success P^r then probability of failures (1-p)^x