1. **Binary Addition with RNN**
   Idea is to have h_1, h_2, h_3 activate when the sum of inputs to the hidden layer is at least 1, 2, or 3, respectively. h_2^{t-1} represents the carry over to the sume at step t

$$\mathbf{U} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \mathbf{W} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \mathbf{b}_h = \begin{pmatrix} -0.5 \\ -1.5 \\ -2.5 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \quad h_y = -0.5$$

2. **LSTM Gradient**

   (a)

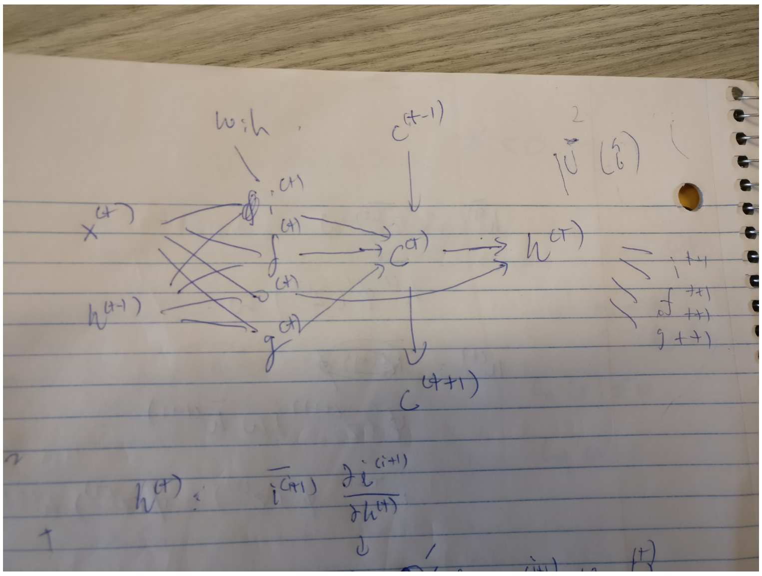
$$\begin{aligned}
\overline{h^{(t)}} &= \overline{i^{(t+1)}} w_{ih} \sigma(w_{ix} x^{(t+1)} + w_{ih} h^{(t)})(1 - \sigma(w_{ix} x^{(t+1)} + w_{ih} h^{(t)})) \\
&\quad + \overline{f^{(t+1)}} w_{fh} \sigma(w_{fx} x^{(t+1)} + w_{fh} h^{(t)})(1 - \sigma(w_{fx} x^{(t+1)} + w_{fh} h^{(t)})) \\
&\quad + \overline{o^{(t+1)}} w_{oh} \sigma(w_{ox} x^{(t+1)} + w_{oh} h^{(t)})(1 - \sigma(w_{ox} x^{(t+1)} + w_{oh} h^{(t)})) \\
&\quad + \overline{g^{(t+1)}} w_{gh} (1 - \tanh^2(w_{ox} x^{(t+1)} + w_{oh} h^{(t)})) \\
\overline{c^{(t)}} &= \overline{c^{(t+1)}} f^{(t+1)} + \overline{h^{(t)}} o^{(t)} (1 - \tanh^2(c^{(t)})) \\
\overline{g^{(t)}} &= \overline{c^{(t)}} i^{(t)} \\
\overline{o^{(t)}} &= \overline{h^{(t)}} \tanh(c^{(t)}) \\
\overline{f^{(t)}} &= \overline{c^{(t)}} c^{(t-1)} \\
\overline{i^{(t)}} &= \overline{c^{(t)}} g^{(t)}
\end{aligned}$$

   (b)

$$\overline{w_{ix}} = \sum_t \overline{i^{(t)}} x^{(t)} \sigma(w_{ix} x^{(t)} + w_{ih} h^{(t-1)})(1 - \sigma(w_{ix} x^{(t)} + w_{ih} h^{(t-1)}))$$

   (c) In case where forget gate has value very close to 1 and input/output gate close to 0, we have a LSTM block that acts as an identity function

$$c^{(t)} \approx c^{(t-1)} \qquad \overline{c^{(t)}} \approx 1$$


w:h
c(t-1)
x(t)
h(t-1)
i(t)
f(t)
o(t)
g(t)
c(t)
h(t)
c(t+1)
h(t):
i(t+1)
∂i(t+1)/∂h(t)