Homework #7 - STA414

Winter 2018

Instructions: Do not submit your work. This assignment is for your edification only – not for credit.

Question 1.

yeah expandable …

Expanding the square in the squared exponential kernel, we have

write this as inner product of feature vectors

$$k(\mathbf{x}_m, \mathbf{x}_n) = \exp\left(-||\mathbf{x}_m - \mathbf{x}_n||^2/2\sigma^2\right) = \exp\left(-\mathbf{x}_m^T\mathbf{x}_m/2\sigma^2\right)\,\exp\left(-\mathbf{x}_m^T\mathbf{x}_n/\sigma^2\right)\,\exp\left(-\mathbf{x}_n^T\mathbf{x}_n/2\sigma^2\right)$$

By expanding one of the factors as a power series, show that the squared exponential kernel can be expressed as the inner product of an infinite-dimensional feature vector. (From Bishop p 321.)

expand the middle term as infinite sums, or polynomial with nonnegative coefficients

use property ii, and vi, and i to prove the constructed kernel is valid

Question 2.

We learned in class that kernels can be constructed from others. To be valid, the resulting kernel should be expressible as $\phi(\mathbf{x}_m)^T\phi(\mathbf{x}_n)$ or equivalently have a kernel matrix $\mathbf{K}$ that is *positive-definite*. If $k_1(\mathbf{x}_m, \mathbf{x}_n)$ and $k_2(\mathbf{x}_m, \mathbf{x}_n)$ are valid kernels, show that the following constructed kernels are valid. (From Bishop p 320.)

(i) $ck_1(\mathbf{x}_m, \mathbf{x}_n)$, for constant $c > 0$ just try to prove x^T K x >= 0

(ii) $f(\mathbf{x}_m)\,k_1(\mathbf{x}_m, \mathbf{x}_n)\,f(\mathbf{x}_n)$, for any function $f(\cdot)$

(iii) $k_1(\mathbf{x}_m, \mathbf{x}_n) + k_2(\mathbf{x}_m, \mathbf{x}_n)$ straight from adding two x^T K x

(iv) $k_1(\mathbf{x}_m, \mathbf{x}_n)\,k_2(\mathbf{x}_m, \mathbf{x}_n)$

(v) $q(k_1(\mathbf{x}_m, \mathbf{x}_n))$, for polynomial $q(\cdot)$ with nonnegative coefficients

(vi) $\exp\left(k_1(\mathbf{x}_m, \mathbf{x}_n)\right)$

Question 3.

Using your results in Question 2, or otherwise, show that the kernel in Question 1 is a valid kernel.