UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2017 EXAMINATIONS

CSC320H1S : Introduction to Visual Computing

Duration: 2 hours

No aids allowed

There are 10 pages total (including this page)

Given name(s): _____

Family name: _____

Student number: _____

Question	Marks
1	_____/30
2	_____/25
3	_____/30
4	_____/35
Total	_____/120

1 Circle Fitting (30 marks total)

As you know, points $[x, y]$ on a circle satisfy the following general equation:

$$(x - a)^2 + (y - b)^2 \;=\; r^2$$

where $[a, b]$ is the circle's center and r is its radius.

(a) **[15 Marks]** Suppose you are given the coordinates of 100 distinct image points, $[x_1, y_1], \ldots, [x_{100}, y_{100}]$, that lie on a circle or very close to it. Provide full details of an algorithm that fits a circle to these points using least squares. In particular, specify the linear system of equations, its unknowns and coefficients, the dimensions of the associated matrices, and the formula that expresses the system's solution in terms of these matrices.

Note: If you don't know how to answer this question, then specify the linear system of equations, unknowns, coefficients and solution formula for least-squares fitting of a second-degree polynomial to $x_1, \ldots, x_{100}$. Although this is *not* the expected answer for (a), doing this correctly will give you 8 marks.

(b) **[5 Marks]** How can one compute the *curvature* of the best-fit circle computed in (a)?

(c) **[10 Marks]** Finally, suppose that the 100 points actually come from *two* distinct circles in the image, with approximately half the points coming from each circle. You do not know, however, which point comes from which circle. Briefly describe an algorithm that should be able to handle this case, *i.e.,* it can compute a good least-squares-fit for each of the two circles and correctly identify which point belongs to which circle. Be sure to explain why you think your algorithm will be successful.

Four or five sentences will suffice for your answer. You do not need to provide any explicit equations.

2 Image Tools (25 marks total)

(a) **[7 Marks]** Recall that in the *intelligent scissors* algorithm, one of the terms that control the weight of the link between two adjacent pixels p and q is the following:

$$f_D(p,q) \;=\; \frac{\text{angle}(q-p, \nabla p^{\perp}) \;+\; \text{angle}(q-p, \nabla q^{\perp})}{\pi}$$

where angle() is the angle between two vectors in radians and $a^{\perp}$ denotes the vector perpendicular to a. Explain the purpose of this particular numerator and denominator in the definition of $f_D(p,q)$.

(b) **[10 Marks]** Describe the algorithm for the *random search* step of PatchMatch.

(c) **[8 Marks]** Give the pseudocode of the *backward mapping algorithm* employed by image morphing.

3 SIFT (30 marks total)

(a) **[15 Marks]** Describe how the *scale* of a SIFT keypoint is determined, prior to refinement. Be as specific as possible: describe the steps of the computation, give relevant formulas, and draw diagrams/plots if needed.

(b) **[15 Marks]** Describe the algorithm used to assign a *2D orientation* to a SIFT keypoint. Again, be as specific as possible: describe each step of the computation, give relevant formulas, and draw diagrams/plots as needed. Be sure to focus only on keypoint orientation assignment, *i.e.,* assume that the keypoint's 2D position and scale are known and do not discuss the definition and creation of keypoint descriptors.

4 Interpolation, Wavelets & Curve Editing (35 marks total)

(a) **[8 Marks]** Give the expression for the *convolution* of a discrete 1D image $I = [I_1, \ldots, I_n]$ and a continuous interpolation kernel $K(x)$:

$[I * K](x) \; =$

(b) **[7 Marks]** Prove that

$$\frac{d^2}{dx^2}[I * K](x) = \left[I * \frac{d^2}{dx^2}K\right](x) \; .$$

(c) **[10 Marks]** Consider the following two vectors A and B:

$A =$

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}

$B =$

a_0	a_1	a_2	a_3	a_4	a_5	a_6	$2a_7$	a_8	$3a_9$	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}

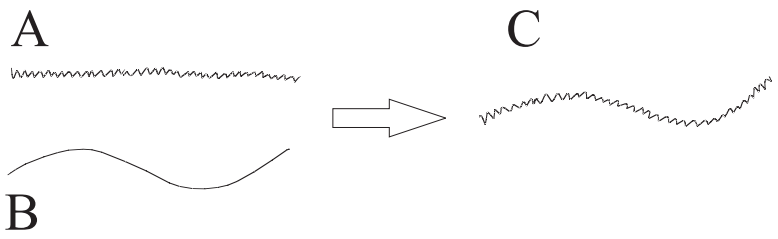
where a_7 and a_9 are not zero.

Now suppose A and B are the (unnormalized) 1D Haar wavelet transforms of two images I_A and I_B, respectively. Since A and B are not identical, I_A and I_B are not identical either. On the image below, mark with an X all the pixels where images I_A and I_B will have different intensities:

You need to justify your answer and/or show your calculations. No points will be awarded without this.

(d) **[10 Marks]** Suggest an algorithm that performs the following *curve merging* operation: you are given two image curves A and B, and your goal is to create a new curve C that preserves the fine details of curve A but has the overall shape and image location of curve B.

Both A and B contain 256 pixels whose x- and y-coordinates are stored in vectors X_A, Y_A and X_B, Y_B, respectively. Your algorithm should output the 256-dimensional vectors X_C, Y_C that store the coordinates of the pixels in curve C. Be as specific as possible.



END OF EXAM