How to solve linear equations

Problems

- Suppose we have a set of linear equations and we want to solve for X1, X2, . . . , XM

$$B_1 = A_{11}X_1 + A_{12}X_2 + \cdots + A_{1M}X_M$$

$$B_2 = A_{21}X_1 + A_{22}X_2 + \cdots + A_{2M}X_M$$

$$\vdots$$

$$B_N = A_{N1}X_1 + A_{N2}X_2 + \cdots + A_{NM}X_M.$$

Problems

- We can actually collapse this into matrix notation in the following way:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1M} \\ A_{21} & A_{22} & \dots & A_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NM} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix}$$

Problems

- We can simplify the notation equation, but still keep its semantic meaning

$$B = AX,$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix}, A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1M} \\ A_{21} & A_{22} & \dots & A_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NM} \end{bmatrix}, X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix}$$

Solution

- Matrix VS Real number

$$\text{If } B = AX, \text{ then } X = \frac{B}{A} = (\frac{1}{A})B.$$

Solution

- The idea here, is that even though we cannot invert A, we know that ATA is invertible.
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- Since we know that B = AX, we know that ATB = (ATA)X so we can isolate X by applying the inverse.

Solution

- We want to find some way of using a matrix inverse to isolate X

$$
\begin{aligned}
B &= AX, \\
A^T B &= A^T A X, \\
(A^T A)^{-1} A^T B &= (A^T A)^{-1} (A^T A) X, \\
X &= (A^T A)^{-1} A^T B.
\end{aligned}
$$