

Definition. The goal of **regression** is to summarize observed data as simply, usefully, and elegantly as possible. **Simple Linear Regression** has a summarizing model

$$\mathbb{E}(Y|X = x) = \beta_0 + \beta_1 x_i$$

$$V(Y|X = x) = \sigma^2$$

while making some error assumptions.

Definition. **parameter** a population quantity

statistic a quantity based on a sample drawn from the population

Definition. **Central limit theorem** if X_1, X_2, \dots is an independent sequence of identically distributed random variables with mean $\mu = \mathbb{E}(X_i)$ and variance $\sigma^2 = V(X_i)$ then

$$\lim_{n \rightarrow \infty} \mathbb{P}\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq x\right) = \phi(x)$$

where $\bar{x} = \sum_i X_i/n$ and $\phi(x)$ is standard normal CDF

Definition. **Relationship between normal and χ^2 distribution** Let $X_1, \dots \sim \mathcal{N}(\mu, \sigma^2)$ be independent, then distribution of sample variance $S^2 = \sum_{i=1}^n (X_i - \bar{X})^2 / (n-1)$

$$S^2 \sim \frac{\sigma^2}{(n-1)} \chi_{n-1}^2$$

Definition. **t distribution**

Definition. **F distribution**

Definition. A linear regression model of mortality versus temperature is by estimating intercept β_0 and slope β_1

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

for $i \in \{1, \dots, n\}$ and $\epsilon \sim \mathcal{N}(0, \sigma^2)$. Try to find least-square estimators β_0 and β_1 that minimize the sum of squares

$$\sum_{i=1}^n (y_i - (\beta_0 + \beta_1 x_i))^2$$

The solution is given by

$$\hat{\beta}_1 = \frac{S_{XY}}{S_{XX}} = r \frac{S_Y}{S_X}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$