Problem Set 1

You are strongly encouraged to try to solve the following exercises before next week's tutorial:

- Exercises 4, 5, 6 and 9 on page 198 (end of chapter 6).
- Exercises 2, 4 ((a) and (c) only) and 16 ((a) and (b) only) on Section 8.10, page 312 onwards (end of chapter 8).

Additional exercise:

Suppose that $X_1, \ldots, X_n$ is a random sample from some distribution (not necessarily normal) with mean μ and variance σ^2, and recall the definition of the sample variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2.$$

(a) Show that

$$(n-1)\mathbb{E}[S^2] = \mathbb{E}\left[\sum_{i=1}^{n} X_i^2\right] - n\mathbb{E}\left[\overline{X}^2\right].$$

(b) Show that $\mathbb{E}\left[X_i^2\right] = \mu + \sigma^2$, $i = 1, \ldots, n$, and that $\mathbb{E}\left[\overline{X}^2\right] = \mu + \sigma^2/n$.

(c) Conclude that $\mathbb{E}\left[S^2\right] = \sigma^2$. This means that the sample variance is an *unbiased estimator* of the population variance, regardless of the distribution. We will talk about this property in greater detail in class.