CSC320: Visual Computing
Term Test 1 March 9th, 2007 9:10-10:00

Student Number:______________________

Last Name:______________________ First Name:______________________

This exam consists of 3 questions on 6 single-sided pages (including cover page).
Aids allowed: None.

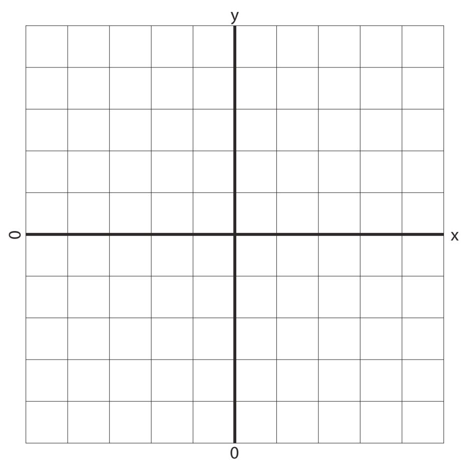
Total Marks: 50
Minutes: 50

Question	Marks
1a	______________/5
1b	______________/10
2a	______________/5
2b	______________/5
2c	______________/5
2d	______________/5
3a	______________/5
3b	______________/5
3c	______________/5
Total	______________/50

1. **2D Curves [15 Marks]**

Consider the 2D curve $\gamma(t) = (\alpha e^{\beta t} \cos t, \alpha e^{\beta t} \sin t)$ with $\alpha > 0$, $\beta < 0$, and $t \in [0, \infty)$.

(a) [5 Marks] Draw the curve in the grid provided below. Be as precise as possible.

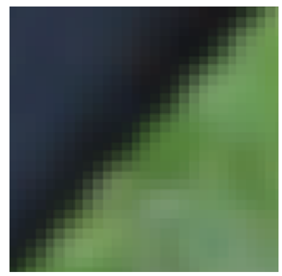


(b) [10 Marks] Prove that if t is the arc-length parameter of a curve $\gamma(t)$, then

$$\left\| \frac{d\gamma}{dt}(t) \right\| = 1 \quad \text{for all } t.$$

2. **Laplacian extrema & zero crossings [20 Marks]**

Consider the image I shown on the left, along with the zoomed-in portion shown on the right.



(a) [5 Marks] Draw on the zoomed-in image the location(s) of the Laplacian zero crossings, if any.

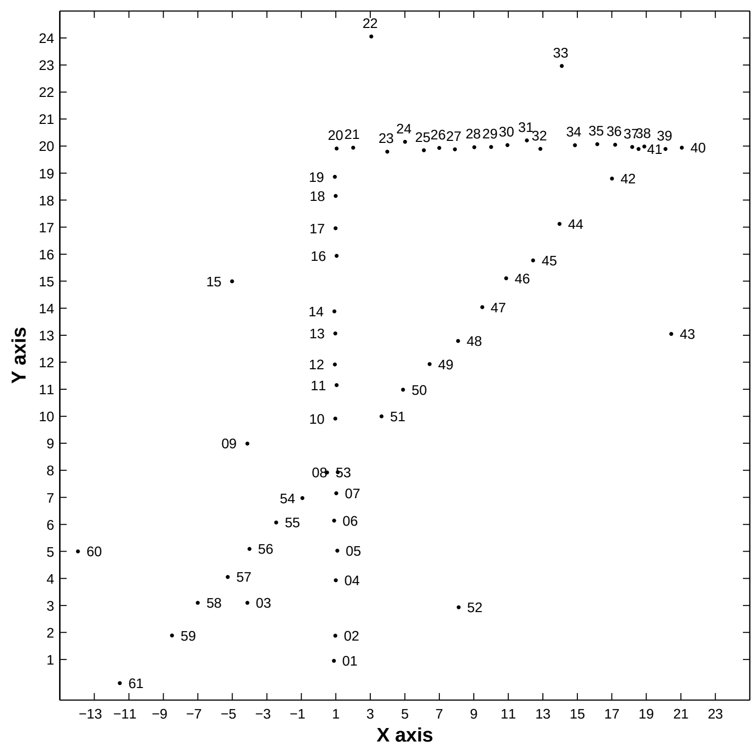
(b) [5 Marks] Draw on the zoomed-in image the location(s) of the local extrema of the image Laplacian.

(c) [5 Marks] Will the extrema in (b) be elliptical, hyperbolic, or cylindrical? Explain your choice.

(d) [5 Marks] Give the definition of the Laplacian of an image using standard calculus notation.

3. **RANSAC-based 2D Curve Estimation [15 Marks]**

Suppose we are given 61 points, $\gamma_1, \gamma_2, \ldots, \gamma_{61}$ along a 2D curve, shown below as dots with their index next to them. We want to estimate the 2D position, $\gamma(t)$, and unit tangent vector, $\mathbf{T}(t)$, using the sliding window algorithm with RANSAC fitting. Assume that t is in the range $[1, 61]$, corresponding to the points' index, that the window has size 2*5+1, the fit threshold is 1, and the number of RANSAC iterations is large enough to ensure success with a very high probability.

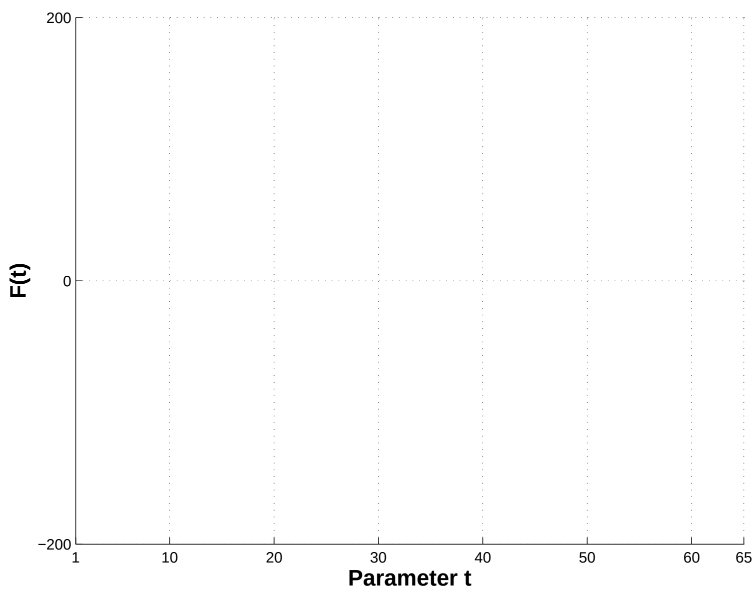


(a) [5 Marks] Draw the estimated position of $\gamma(t)$ for $t \in \{6, 7, \ldots, 55\}$ obtained by first-order RANSAC fitting. You can draw invididual dots or, for convenience, a continuous curve that contains them. Be as precise as possible.

(b) [5 Marks] Now plot the function

$$F(t) = \frac{180}{\pi} \arccos\left(\mathbf{v} \cdot \mathbf{T}(t)\right)$$

where $t \in \{6, 7, \ldots, 55\}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\mathbf{T}(t)$ is estimated by the RANSAC fitting in (a).



(c) [5 Marks] For comparison, draw the estimated position of $\gamma(t)$ for $t \in \{6, 7, \ldots, 55\}$ obtained by first-order weighted least-squares fitting, rather than RANSAC, with the sliding window algorithm. Assume a Gaussian weight function

$$\Omega(q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{q^2}{2\sigma^2}}$$

with $\sigma = 2$, and be as precise as possible.