Assignment #8

Due: Never

Question 1 (Properties of a sampling-based estimator, from Bishop p 556) Show that the finite sample estimator \hat{f} defined by

$$\hat{f} = \frac{1}{L} \sum_{l=1}^{L} f(\mathbf{z}^{(l)})$$

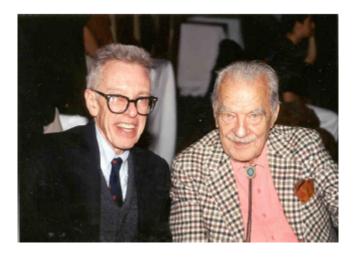
has mean equal to $\mathbb{E}[f]$ and variance given by

$$\operatorname{var}[\hat{f}] = \frac{1}{L} \mathbb{E}\left[(f - \mathbb{E}[f])^2 \right]$$

Question 2 (Markov-chain Monte Carlo)

Let p(**w**) correspond to a Gaussian mixture model with $\pi = \left\{\frac{3}{4}, \frac{1}{4}\right\}$, $\mu = \{0, 8\}$, and $\sigma = \{1, 1\}$. In this question, you will estimate E(**w**) using 3000 Metropolis-Hastings iterations, initialized at x = 0.

- (a) Estimate the expectation $E(\mathbf{w})$ using a proposal distribution $Q(x'|x) \sim \mathcal{N}(x,1)$. Plot your samples on the same graph as the true distribution $p(\mathbf{w})$, as a histogram. How many of the 3000 iterations resulted in successful samples? Hint: You may wish to run your code a few times but you need only report on one run.
- (b) Repeat using a *mixture proposal*: for each iteration, a proposal distribution $Q(x'|x) \sim \mathcal{N}(x, 10^2)$ is used with 50% probability, and with 50% probability the proposal in (a) is used instead.
- (c) Compare your two estimates for $E(\mathbf{w})$ to one another and to the true answer. State which of (a) or (b) is best for the current task, and justify your answer.



Wilfred Hastings (left) was a U of T student and prof until 1971. On the right is Nicholas Metropolis.