2 Random Variable

Poisson Distribution

Definition. *The **Poisson frequency function** with parameter $\lambda > 0$ is*

$$P(X = k) = \frac{\lambda^k}{k!}e^{-\lambda},\ k = 0, 1, 2. \cdots$$

*where λ is the average number of events per interval. Since $e^\lambda = \sum_{k=0}^{n} \frac{\lambda^k}{k!}$ it follows that the frequency sum up to 1. The **distribution function** is derived as limit of a binomial distribution as number of trials n approaches infinity and the probability of success on each trial, p, approaches zero such that $np = \lambda$. Poisson distribution is a good model if*

1. *K is the number of times an event occurs in an interval and K can take values $0, 1, 2 \cdots$*
2. *Events occur independently.*
3. *The rate at which events occur is constant.*
4. *Two events cannot occur at exactly the same instant.*

Remark. In the partical emission case, λ is total number of emission divided by total time

Gamma Density

Definition. *$\sim Gamma(\alpha, \lambda)$ The **gamma density function** is defined to be*

$$g(t) = \frac{\lambda^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{\lambda t}\ t \geq 0$$

where

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du$$

and

$$E[X] = \frac{\alpha}{\lambda} = \alpha\theta\ Var(X) = \frac{\alpha}{\lambda^2} = \alpha\theta^2$$

Multinominal Distribution

Definition. *For n independent trials each of which leads to a success for exactly one of k categories, with each category having a given fixed success probability, the multinomial distribution gives the probability of any particular combination of numbers of successes for*

the various categories.
The probability mass function is defined by

$$f(x_1, \cdots, x_k; p_1, \cdots, p_k) = \begin{cases} \dfrac{n!}{\prod_{i=1}^k x_i!} \displaystyle\prod_{i=1}^k p_i^{x_i} & \displaystyle\sum_{i=1}^n x_i = n \\ 0 & otherwise \end{cases}$$

Remark. If $n = 1$ $k = 2$, the multinomial distribution is Bernoulli distribution. When k is 2 and number of trials are more than 1 it is the binomial distribution.

Cauchy distribution

Definition.

$$f(x|\theta) = \frac{1}{\pi[1 + (x - \theta)^2]}$$