UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2006 EXAMINATIONS

CSC320H1S : Introduction to Visual Computing

Duration: 2 hours

No aids allowed

There are 10 pages total (including this page)

Given name(s): ______________________________

Family name: ______________________________

Student number: ______________________________

Question	Marks
1	__________/15
2	__________/20
3	__________/30
4	__________/10
5	__________/30
6	__________/15
Total	_________/120

1 Normalized Cross-Correlation and Convolution (15 marks total)

(a) [7 marks] Give the formula for the *normalized cross-correlation* of two column vectors, $\mathbf{v_1}$ and $\mathbf{v_2}$, of equal length.

$NCC(\mathbf{v_1}, \mathbf{v_2}) =$

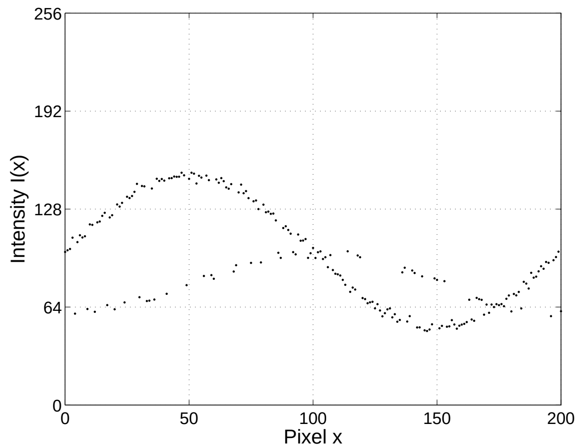
(b) [8 marks] Give an expression for the *convolution* of a filter $h = [\ h(0) \ \ \ldots \ \ h(2k)\ ]$ with an image $I = [\ I(0) \ \ \ldots \ \ I(n)\ ]$. Assume $2k + 1 < n + 1$.

2 Robust Estimation (20 marks total)

Suppose you are given a 1D image I containing n pixels that is generated by randomly "interleaving" two other n-pixel images, I_1 and I_2:

$$I(x) = \begin{cases} I_1(x), & \text{or} \\ I_2(x) . \end{cases}$$

An example is shown below (in this example, I_1 is shaped like a sinusoid):



The images I_1 and I_2 are *not* known and neither is the way by which they were interleaved. Moreover, since the interleaving was random, there is no fixed interleaving pattern. You are told, however, that in any window of size $w > 10$, approximately 70% of the pixels come from image I_1 and the rest come from I_2.

(a) [10 marks] Show how to estimate the derivative of I in a way that is completely unaffected by the pixels coming from image I_2. That is, your derivative estimate should be (almost) identical to the estimate that you would have computed if $I(x)$ were equal to $I_1(x)$ for *every* pixel x. If your method requires any additional assumptions, be sure to state them.

(b) [10 marks] Alternatively, suppose you are asked to compute, for every pixel x, the derivative of the image it came from. That is, without knowing *a priori* which pixel came from which image, you must estimate the function

$$\frac{d}{dx}I(x) = \begin{cases} \frac{d}{dx}I_1(x) & \text{if } I(x) = I_1(x) \\ \\ \frac{d}{dx}I_2(x) & \text{if } I(x) = I_2(x) \ . \end{cases}$$

If your method requires any additional assumptions, be sure to state them. (Note: you do not need to know the answer to (a) in order to answer this question).

3 Hessians, Principal Curvatures, and Corner Detection (30 marks total)

(a) [5 marks] Define the Hessian of an image I using standard calculus notation.

(b) [15 marks] Suppose that the intensities in the neighborhood of the central pixel, $(0, 0)$, of a 2D patch I are well-approximated by the polynomial

$$I(x, y) = 100\, x^2 - 20\, y^2 + 80\, x^3 y^3.$$

Compute the principal curvatures of I at pixel $(0, 0)$.

(c) [10 marks] Suppose that pixel (x_0, y_0) corresponds to a local extremum of the intensity surface, $I(x, y)$. Which of the conditions below are strong evidence that pixel (x_0, y_0) is a poor candidate for being a corner feature?

1. (x_0, y_0) corresponds to a parabolic point of the intensity surface.
2. (x_0, y_0) corresponds to a hyperbolic point of the intensity surface.
3. (x_0, y_0) corresponds to an elliptical point of the intensity surface.
4. $\frac{\min(|\kappa_1|,|\kappa_2|)}{\max(|\kappa_1|,|\kappa_2|)}$ is close to 1, where κ_1, κ_2 are the principal curvatures of I at (x_0, y_0).
5. $\frac{\min(|\kappa_1|,|\kappa_2|)}{\max(|\kappa_1|,|\kappa_2|)}$ is close to 0.

4 PCA and Eigenfaces (10 marks total)

Let $I_1, \ldots, I_n$ be a set of face images, represented as column vectors. Give the main steps of the algorithm for computing the eigenfaces of $I_1, \ldots, I_n$. Be as specific as possible.

5 Multi-Scale Representations (30 marks total)

(a) [10 marks] Prove that

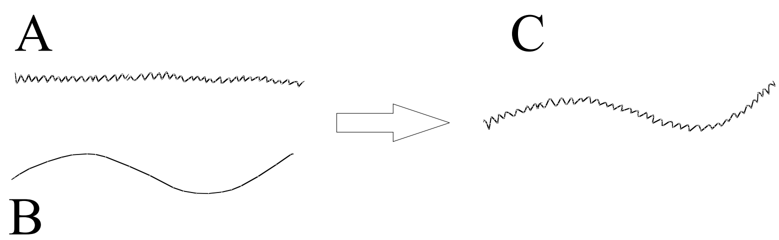
$$\frac{d}{dx}\left[I * G_\sigma\right](x) = \left[I * \frac{d}{dx} G_\sigma\right](x)$$

where $I = [\ I(1) \ \ \ldots \ \ I(n)\ ]$ is a 1D image containing n pixels and $G_\sigma(x)$ is the (continuous) 1D Gaussian function of standard deviation σ.

(b) [10 marks] Compute the Haar wavelet transform of the following image:

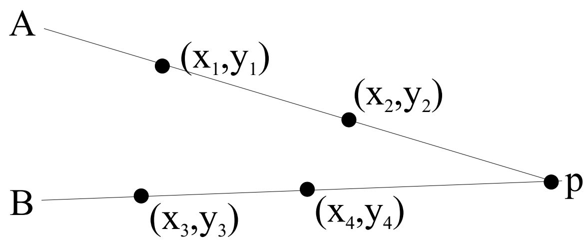
6	8	5	1	1	1	4	6

(c) [10 marks] Give an algorithm that performs the following *curve merging* operation: you are given two curves, A and B, each of which is 256 pixels long, and your goal is to create a new curve C, also 256 pixels long, that preserves the fine details of curve A but has the overall shape of curve B. Be as specific as possible.



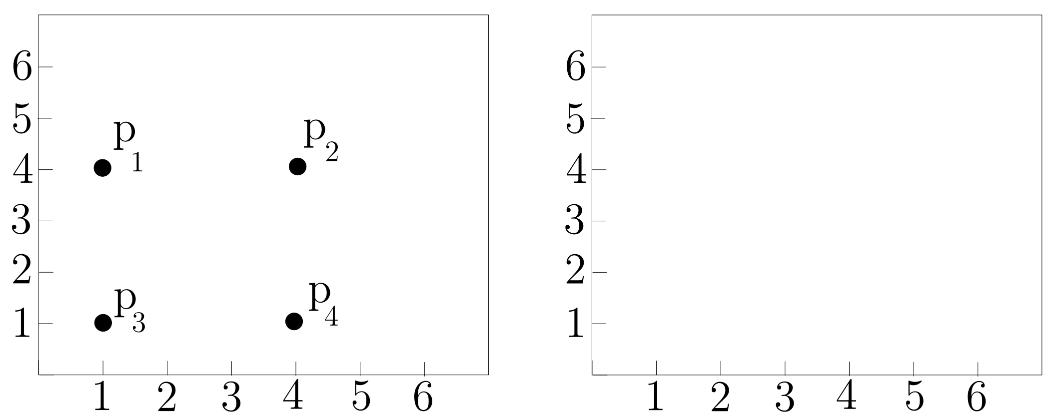
6 Homogeneous Coordinates (15 marks total)

(a) [7 marks] Give a single formula that expresses the *homogeneous coordinates* of the intersection of lines A and B in terms of the 2D coordinates of points $p_1, \ldots, p_4$.



$\mathbf{p} \cong$

(b) [8 marks] Indicate on the right image the location of points $p_1, \ldots, p_4$ after transformation with the homography $H = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 5 \\ 1 & 0 & 2 \end{bmatrix}$.



END OF EXAM