

Algebra II

Midterm Exam Information

The Midterm Exam will be held on **Thursday, February 15th**, 11:10–1 in our regular Thursday classroom, SF 1105.

The exam will cover the material from the following section, as covered in class:
2.2, 2.4, 2.5, 5.1, 5.2, (excluding direct sums), 5.4, (excluding direct sums), 6.1, 6.2.

You should know how to do problems similar to those assigned for homework. You should know precise definitions of important things. In the following list, you should be able to state and prove the Theorems marked with \star ,

Important topics are the following:

- columns of matrix repr as $T(v_j)$ where v_j is in β
- The matrix $[T]_\beta$ for a linear transformation with respect to a basis β
- the change of basis formula relating $[T]_\beta$ and $[T]_{\beta'}$.
- the characteristic polynomials $P_T(t)$ and $P_A(t)$ for a linear transformation and a matrix. det($T - \lambda I$) eigenvalue: root of $P_T(t)$
- how to find the eigenvalues and eigenvectors of a linear transformation. eigenvector: basis of eigenspaces
- diagonalizable transformations $D = Q^{-1}AQ$
- solution of linear systems of differential equations,
- invariant subspaces and T -cyclic subspaces, Theorem 5.22, p.315.
- the Cayley-Hamilton Theorem \star
- inner product spaces, Cauchy-Schwarz \star and triangle inequalities \star
- orthogonal and orthonormal bases and the Gram-Schmidt process
- orthogonal complements, the decomposition theorem \star (Theorem 6.6, p.350)
 $V = W \oplus W^\perp$, projections

diagonalizable iff

1. exists a basis such that matrix representation of transformation is diagonalizable
2. exists a basis of eigenvectors for V
3. satisfies 2 conditions

1. characteristic polynomial splits

2. multiplicity of each eigenvalues equals to dimension of corresponding eigenspace, i.e. $\dim(V) - \text{rank}(T - \lambda I)$

if diagonalizable, exists Q whose columns are eigenvectors $D = Q^{-1}AQ$ where j th diagonal entry of D is eigenvalue of j th column of Q