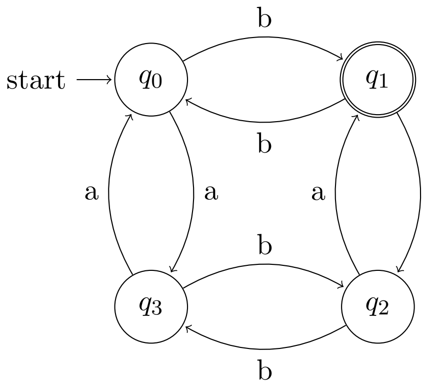
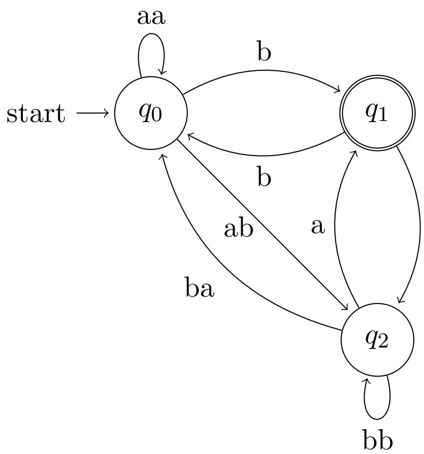
Consider language $L = \{s \in \{a,b\} : s$ has even a and odd $b\}$. Devise an FSA for L and then a RE for L. Provec that $L(R) = L$

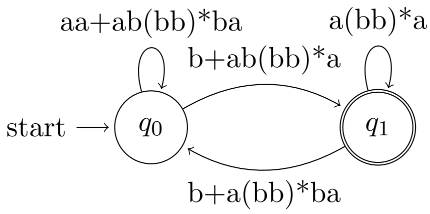
Solution.
We can construct DFSA L for by,



By state removal method we can construct a simpler FSA



Then,



Here we can derive the regular expression by going through the loop. A complete roundtrip from q_0 to q_0 via q_1 requires a regular expression of

$$E = (aa + ab(bb)^*ba) + ((b + ab(bb)^*a)(a(bb)^*a)^*(b + a(bb)^*ba))$$

This loop could be repeated many times before the string leads the automaton to its final q_1 accepting state via

$$F = (b + ab(bb)^*a)(a(bb)^*a)^*$$

So altogether the regular expression for L derived from FSA is

$$R = E^*F = ((aa+ab(bb)^*ba)+((b+ab(bb)^*a)(a(bb)^*a)^*(b+a(bb)^*ba)))^*(b+ab(bb)^*a)(a(bb)^*a)^*$$

Now we prove $L(R) = L$. Let $x \in$ Ł(R). If E is repeated zero times then $x \in L((b + ab(bb)^*a)(a(bb)^*a)^*) = (L(b) + L(ab(bb)^*a)) \circ L((a(bb)^*a)^*)$. So then either $x \in L(b) \circ L((a(bb)^*a)^*)$ or $x \in L(ab(bb)^*a) \circ L((a(bb)^*a)^*)$. Now consider $y \in L(a(bb)^*a)$, $y = a(bb)^k a$ for some $k \in \mathbb{N}$. We see that y has two , therefore even number, of a's; y has even number of b's by the exponentiation operation. ALso consider $z \in L(ab(bb)^k a)$, then z has two therefore even number of a and odd number of b. Since either $x = by^m$ or $x = zy^m$ for arbitrary $m \in \mathbb{N}$ We see that x has even number of a and odd number of b in both cases. The same rationale can be applied to the case where E is not an empty string, with the exact same result. Then $L(R) \subseteq L$

$\square$