UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2010 EXAMINATIONS

CSC320H1S : Introduction to Visual Computing

Duration: 2 hours

No aids allowed

There are 10 pages total (including this page)

Given name(s): _____

Family name: _____

Student number: _____

Question	Marks
1	_____/25
2	_____/30
3	_____/30
4	_____/15
5	_____/10
6	_____/10
Total	_____/120

1 Estimation by 2D Polynomial Fitting (25 marks total)

(a) **[5 Marks]** Give the 2nd-order Taylor series expansion of the image $I(x, y)$ in the neighborhood of pixel $(0, 0)$:

$I(x, y) \;=$

(b) **[10 Marks]** Consider the image below. We want to use polynomial fitting to estimate the intensity, I, and its first and second partial derivatives at the center of the highlighted 21×21-pixel patch. To do this, we will use least squares to fit a *single* second-degree polynomial in x *and* y to the patch intensities.

Using matrix notation, show the linear system that must be solved to compute the fit for the patch center. Be sure to indicate the dimensions and contents of each matrix.

(c) **[5 Marks]** Now suppose that we slide the 21×21-pixel window over the entire image, solving the system in (b) for every possible window center (x, y) in order to estimate $I(x, y)$. Ignoring the pixels near the image border, what differences do you expect to see between the estimated image I and the original photo? Explain briefly.

(d) **[5 Marks]** Finally, suppose that we use RANSAC fitting, rather than standard least squares, to estimate $I(x, y)$. Ignoring the pixels near the image border, what differences do you expect to see between the estimated image I and the original photo in this case? Explain briefly.

2 Hessians, Principal Curvatures, and Corner Detection (30 marks total)

(a) **[5 Marks]** Define the Hessian of an image I using standard calculus notation.

(b) **[15 Marks]** Suppose that the intensities in the neighborhood of the central pixel, $(0, 0)$, of a 2D patch I are well-approximated by the polynomial

$$I(x, y) = 2\,x^2 - 2\,y^2 + 4\,xy.$$

Compute the principal curvatures of I at pixel $(0, 0)$.

(c) **[10 Marks]** Suppose that pixel (x_0, y_0) corresponds to a local extremum of the intensity surface, $I(x, y)$. Which of the conditions below are strong evidence that pixel (x_0, y_0) is a poor candidate for being a corner feature? Explain briefly.

(a) (x_0, y_0) corresponds to a parabolic point of the intensity surface.

(b) (x_0, y_0) corresponds to a hyperbolic point of the intensity surface.

(c) (x_0, y_0) corresponds to an elliptical point of the intensity surface.

(d) $\frac{\min(|\kappa_1|,|\kappa_2|)}{\max(|\kappa_1|,|\kappa_2|)}$ is close to 1, where κ_1, κ_2 are the principal curvatures of I at (x_0, y_0).

(e) $\frac{\min(|\kappa_1|,|\kappa_2|)}{\max(|\kappa_1|,|\kappa_2|)}$ is close to 0.

3 Masks and Template Matching (30 marks total)

(a) **[7 Marks]** Give the definition of a *separable* $N \times N$ mask (or filter):

(b) **[8 Marks]** Why is separability a useful property for a mask to have? Be as specific as possible.

(c) Suppose we want to perform template matching using the following image and template:

Image:

row/col #	1	2	3	4	5	6	7	8	9
1	1	6	1	20	13	4	20	20	20
2	6	7	6	21	21	1	21	20	21
3	0	6	1	22	3	4	20	22	20
4	1	6	1	1	3	1	0	20	0

Template:

18	21	18
1	20	2
2	21	4

Indicate the best-matching pixel when we use

(1) **[5 Marks]** cross-correlation:

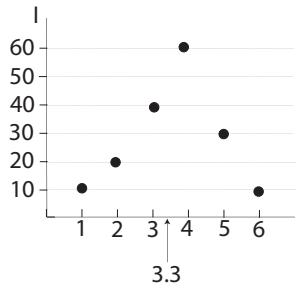
(2) **[5 Marks]** normalized cross-correlation:

In each case, include an explanation. No marks will be given without it.

(d) **[5 Marks]** What is the geometrical interpretation of the normalized cross-correlation of two $N \times N$ patches P and Q?

4 Image Interpolation (15 marks total)

You are given a 6-pixel image I and are asked to interpolate the known pixels to compute an intensity for fractional pixel 3.3.



(a) **[10 Marks]** Give the expression for $I(3.3)$ in terms of I and an interpolation kernel (or mask) M. Assume that $M(t) = 0$ for $t > 2$ and that $M(-t) = M(t)$.

$I(3.3) =$

(b) **[5 Marks]** Compute the interpolation result when M is the *linear interpolation kernel*.

$I(3.3) =$

5 Wavelets (10 marks total)

Pictured below is an image and its partially-completed 2D wavelet transform. Complete the transform by filling in the entries left blank.

Image

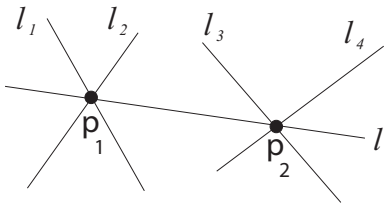
16	12	4	8
8	12	4	16
4	16	8	12
12	8	16	4

Haar Transform

	1	0	-4
0	1	-2	2
2	-2	2	
0	0	-4	-4

6 Homogeneous Coordinates (10 marks total)

Give a single formula that expresses the *homogeneous coordinates* of line l in terms of the homogeneous coordinates of lines $l_1, \ldots, l_4$.



$l \cong$

END OF EXAM