STA302/STA1001, Weeks 11-12

Mark Ebden, 28–30 November

With grateful acknowledgment to Alison Gibbs and Becky Lin

Overview

- Practical example: Analysing house-price data
- Chapter 7: Variable selection, and going with the flow
- Key MLR aspects

Exploring a house-price dataset

For 26 houses sold in Chicago a long time ago, we know the selling price Y as well as eight characteristics of each house: square footage, parking information, etc.

Reference: Ashish Sen and Muni Srivastava, *Regression Analysis: Theory, Methods and Applications*, 2013.

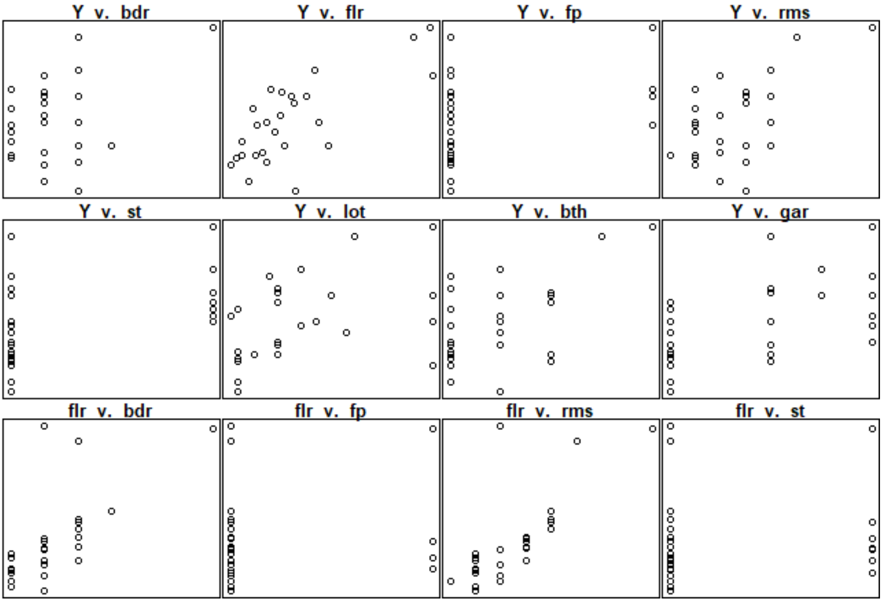
```
##     Y bdr  flr fp rms st lot bth gar
## 1  53   2  967  0   5  0  39 1.5 0.0
## 2  55   2  815  1   5  0  33 1.0 2.0
## 3  56   3  900  0   5  1  35 1.5 1.0
## 4  58   3 1007  0   6  1  24 1.5 2.0
## 5  64   3 1100  1   7  0  50 1.5 1.5
## 6  44   4  897  0   7  0  25 2.0 1.0
## 7  49   5 1400  0   8  0  30 1.0 1.0
## 8  70   3 2261  0   6  0  29 1.0 2.0
## 9  72   4 1290  0   8  1  33 1.5 1.5
## 10 82   4 2104  0   9  0  40 2.5 1.0
## 11 85   8 2240  1  12  1  50 3.0 2.0
## 12 45   2  641  0   5  0  25 1.0 0.0
## 13 47   3  862  0   6  0  25 1.0 0.0
## 14 49   4 1043  0   7  0  30 1.5 0.0
## 15 56   4 1325  0   8  0  50 1.5 0.0
## 16 60   2  782  0   5  1  25 1.0 0.0
## 17 62   3 1126  0   7  1  30 2.0 0.0
## 18 64   4 1226  0   8  0  37 2.0 2.0
## 19 66   2  929  1   5  0  30 1.0 1.0
## 20 35   4 1137  0   7  0  25 1.5 0.0
## 21 38   3  743  0   6  0  25 1.0 0.0
## 22 43   3  596  0   5  0  50 1.0 0.0
## 23 46   2  803  0   5  0  27 1.0 0.0
```

The response variable and eight predictors

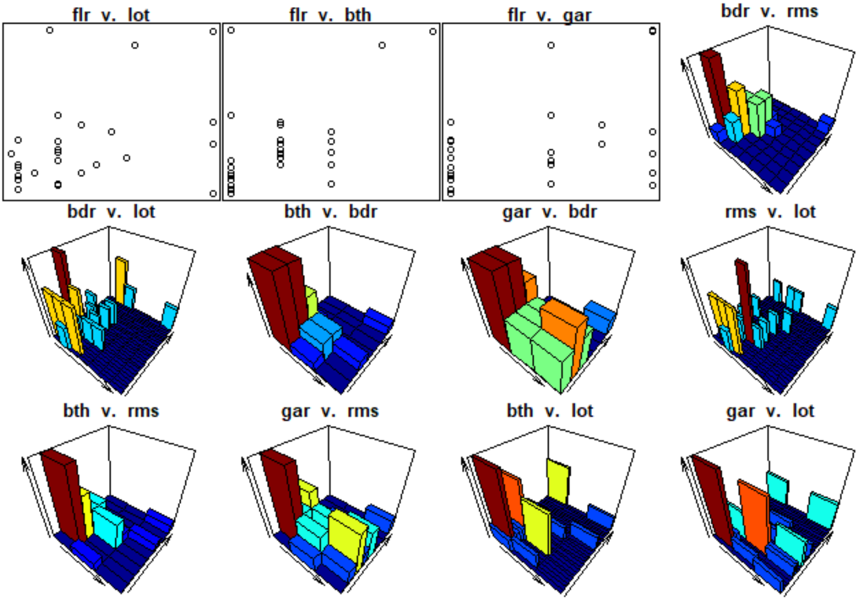
Variable	Meaning
`Y`	Selling price in thousands of dollars
`bdr`	Number of bedrooms
`flr`	Floor space in square feet
`fp`	Number of fireplaces
`rms`	Number of rooms
`st`	Storm windows present (indicator variable)
`lot`	Frontage in feet
`bth`	Number of bathrooms
`gar`	Number of garage parking spaces

We'll use the techniques on last week's slides to analyse the dataset. The data are available on Portal.

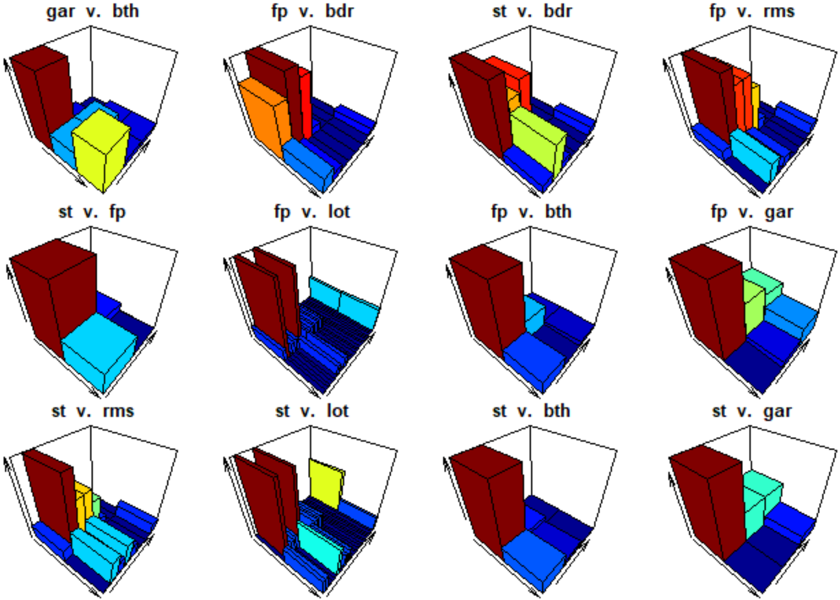
Pairwise scatter plots



Pairwise 3D histograms



Pairwise 3D histograms



Initial findings from the scatter plots

- All X's seem to have some relationship with Y
- No obvious nonlinear relationship between Y and any of the predictors; no obvious need for transformations
- For later when we do the 7 C's: perhaps there exists an influential point; namely, the very first plot shows a house with eight bedrooms
- There are also relationships among some of the X's: for example, between `bdr` and `rms` (middle slide of the three: top right)
- But first, let's continue to explore the dataset systematically

Examining the correlations r and their p-values

	Y	bdr	flr	fp	rms	st	lot	bth	gar
Y		0.41	0.74	0.39	0.58	0.46	0.42	0.55	0.54
bdr	1.41		0.68	0.17	0.92	0.23	0.40	0.63	0.24
flr	4.74	3.81		0.16	0.74	0.13	0.34	0.53	0.40
fp	1.32				0.19	-0.02	0.39	0.12	0.41
rms	2.73	10.37	4.81			0.23	0.43	0.69	0.30
st	1.72						-0.04	0.35	0.17
lot	1.50	1.39		1.34	1.55			0.37	0.14
bth	2.47	3.28	2.23		3.98				0.26
gar	2.39		1.40	1.44					

Upper right: Pearson's r. **Lower left:** $-\log_{10}(p\text{-value})$, e.g. 2 means $p = 0.01$

- Each correlation p-value is equal to an SLR β_1 t-test's
- We mainly consider the p-values and signs here, and needn't pay as much attention to the magnitude of each r
- All X's have a statistically significant r (or R^2) with Y
- Question: Have we run an appropriate test for `st`?

Performing MLR

The next slide describes the MLR. (What can we conclude from the F-test?)

The fitted equation is:

$$\begin{aligned}\hat{Y} =&18.6 - 7.70 \text{ bdr} + 0.0176 \text{ flr} + 6.91 \text{ fp} + 3.90 \text{ rms}+\\ &10.8187 \text{ st} + 0.2635 \text{ lot} + 2.37 \text{ bth} + 1.77 \text{ gar}\end{aligned}$$

Interpreting the coefficients:

- Recall that b_j represents the estimated change in mean selling price due to a unit change in X_j with all other variables held constant
- e.g. A 100-square-foot increase corresponds to a \$1760 increase in mean price, with everything else held constant
- A 1-bedroom increase corresponds to a \$7700 decrease in mean price (despite the fact that the correlation between price and `bdr` is positive)

Despite the significant correlation between `Y` and `bth`, the t-test on the next slide gives a p-value of $0.366 > 0.05$. Why? (Hint: slide 26 from last week)

```
## Call:
## lm(formula = Q)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.3058  -2.8417  -0.1511   3.2882   7.9518
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.637664   5.240957   3.556 0.002429 **
## bdr         -7.697444   1.829426  -4.208 0.000592 ***
## flr          0.017570   0.003235   5.431 4.49e-05 ***
## fp           6.909765   3.083583   2.241 0.038680 *
## rms          3.904374   1.615617   2.417 0.027194 *
## st          10.818663   2.300203   4.703 0.000205 ***
## lot          0.263522   0.135109   1.950 0.067808 .
## bth          2.374591   2.557865   0.928 0.366221
## gar          1.770861   1.404310   1.261 0.224334
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.717 on 17 degrees of freedom
## Multiple R-squared:  0.9044, Adjusted R-squared:  0.8595
## F-statistic: 20.11 on 8 and 17 DF,  p-value: 3.147e-07
```

The Seven C's (7 Checks) for STA302 MLR diagnostics

In their proper order this time:

1. Plot *standardized residuals* to help determine whether the proposed regression model is a valid model

2. Identify any *leverage points*

3. Identify any *outliers*

4. Assess the effect of each X on Y

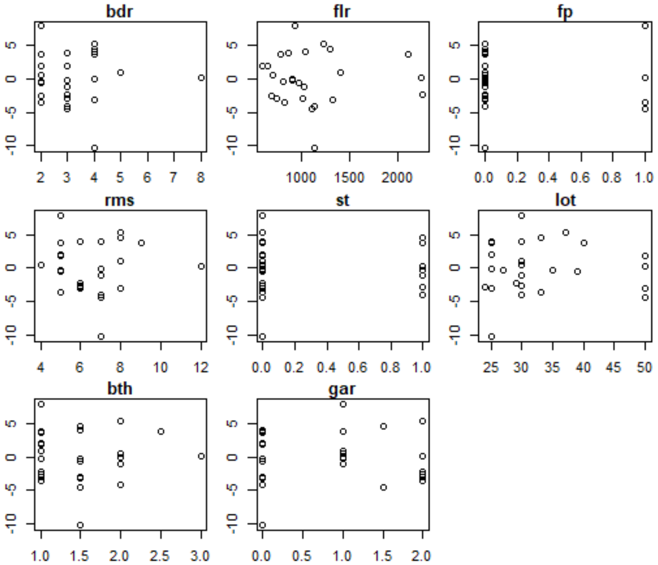
5. Assess *multicollinearity*

6. Assess the assumption of *error homoscedasticity*

7. For time series: examine whether the data are *correlated over time*

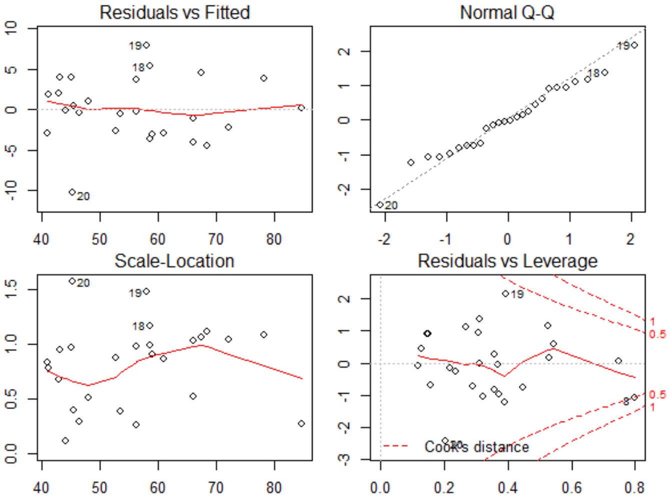
Residual plots in the house-price example

Recall that plotting residuals versus X_j for $j \in \{1, \dots p\}$ helps us to look for curvature, influential points, and outliers.

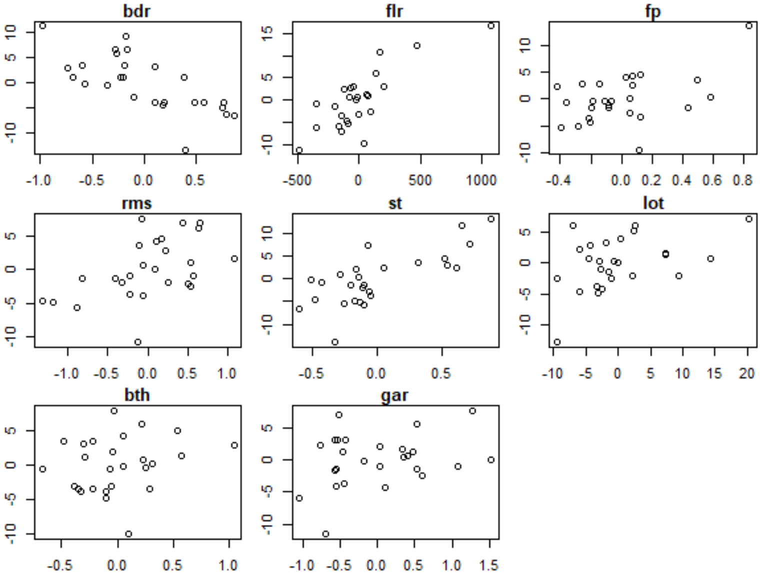


Output of `plot(lm(..))`

- One house with large negative residual (perhaps one large positive residual)
- At least one influential point according to the $4/(n-p-1)$ criterion: large Cook's distance in the bottom-right of the diagnostics panel



Check 4 in the house-price example: Added variable plots



Assessing assumptions in the House-price example

We view the added variable plots in context. Consider the plot of Y versus `bdr`:

- Not a strong linear relationship; otherwise, no concerns with non-linearity
- Potential influential point: one house has 8 bedrooms
- SLR may not be a good idea for the price - bedroom relationship but may be ok in multiple regression

Consider the added variable plot:

- The linear relationship is stronger (although it's now negative)
- We want to check if the influential is here. However, the large negative residual is not the house with 8 bedrooms

Summary of concerns:

- Large negative residual in the added variable plots
- Potential influential point in the added variable plot for `flr`

Summary of unusual observations

House 8:

- Influential
- Largest Cook's distance
- Very large floor space, everything else including price (Y) was of moderate space

House 19:

- Large positive residual
- Small house but middle-ish price; nice house?

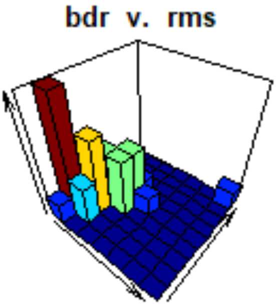
House 20:

- Large negative residual
- Smallest price, but all features middle-ish

House 11 *isn't* a concern:

- It has eight bedrooms but is neither an outlier nor influential
- It's expensive, but everything is big

Check 5 in the house-price example: Multicollinearity



- $\text{VIF}_{\text{rms}} = 8.5$, $\text{VIF}_{\text{bdr}} = 6.5$, $\text{VIF}_{\text{flr}} = 2.5$, $\text{VIF}_{\text{bth}} = 2.1$
- Indeed we saw what might have been the effects of multicollinearity: the coefficient for `bdr` wasn't of the expected sign
- Details of executing the two remedial measures introduced last week (ridge regression, principal component regression) are beyond STA302's scope
- For now, awareness of possible multicollinearity informs our choice of how to **simplify the model**

Model simplification

- If you remove variables during MLR, the β_j values of the remaining predictors become biased, and the p-values from t- and F- statistics generally shrink below their true values
- Therefore, when reducing the dimensionality of your MLR problem, take into account your reduction when reporting CIs and other results

Model simplification

Method 1: If there is multicollinearity in your data set, perform **variable selection**. e.g. choose the subset of predictors that maximizes R^2_{adj}.

Method 2: If there isn't multicollinearity, you can conduct a **partial *F*-test** on predictors you suspect of being unhelpful. Remove them if the result is non-significant.

Applying the two methods to the house-price data:

1. The subset turns out to be everything but `bth`. Therefore, `bth` would removed by Method 1.
   - Among the choices of removing `gar` and/or `bth`, the effect on R^2_{adj} is small in all cases (well under 1% change). Method 1 naively maximizes R^2_{adj} without regard to this; the method is known for its timidity in removing predictors.

2. Removing `bth` and `gar` leads to a non-significant partial F-test ($p > 0.05$). Therefore, `bth` and `gar` would be removed by Method 2. Removing any other predictor(s) instead of or in addition to these leads to a significant p-value for the partial F-test.

The MLR approach

1. Plot *standardized residuals* to help determine whether the proposed regression model is a valid model

2. Identify any *leverage points*

3. Identify any *outliers*

4. Assess the effect of each X on Y

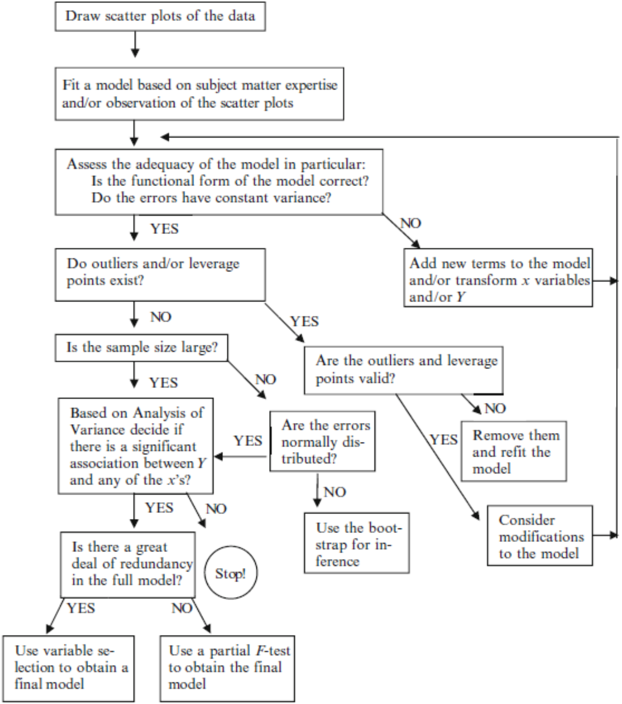
5. Assess *multicollinearity*

6. Assess the assumption of *error homoscedasticity*

7. For time series: examine whether the data are *correlated over time*

The Seven C's can also be viewed as fitting into a broader **flowchart approach to MLR** provided on the next slide.

- The flowchart mentions the "bootstrap", which you're not responsible for
- The bootstrap uses data resampling to numerically approximate the sampling distribution of the test statistic under H_0 (rather than using theoretical results, which we did assuming normally distributed errors)


Draw scatter plots of the data
Fit a model based on subject matter expertise and/or observation of the scatter plots
Assess the adequacy of the model in particular: Is the functional form of the model correct? Do the errors have constant variance?
YES
NO
Add new terms to the model and/or transform x variables and/or Y
Do outliers and/or leverage points exist?
NO
YES
Is the sample size large?
YES
NO
Are the outliers and leverage points valid?
NO
Remove them and refit the model
YES
Consider modifications to the model
Based on Analysis of Variance decide if there is a significant association between Y and any of the x's?
YES
Are the errors normally distributed?
NO
Use the bootstrap for inference
YES
NO
Is there a great deal of redundancy in the full model?
Stop!
YES
NO
Use variable selection to obtain a final model
Use a partial F-test to obtain the final model

Overview

- Practical example: Analysing house-price data
- Chapter 7: Variable selection, and going with the flow
- **Key MLR aspects**

Why do regression? Two main reasons

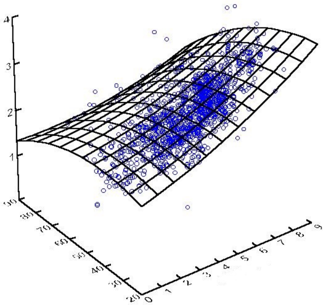
1. Predict the value of a variable given the value(s) of 1+ others
   - For interpolation or extrapolation, for example in house prices
   - It doesn't matter which variables are in the model or how they were chosen (model building/variable selection OK)
   - However, we must consider the level of smoothing that's appropriate. If we overfit, predictions may not work well on new data

2. Understanding an underlying mechanism
   - Describe the relationships among variables
   - What are the effects of predictor(s) on the response?
   - Describe differences in relationships between different sets of data
   - For example, data sets for the Montreal Protocol (CFC data), Meadowfoam, etc
   - This is *inferential modelling*. The form of the model is assumed to be known but there is uncertainty about the value(s) of the coefficients

It can be risky doing inferential modelling on the *same* data that you used to *build* a model (to describe how Y might originate in general).

Why do *multiple* regression?

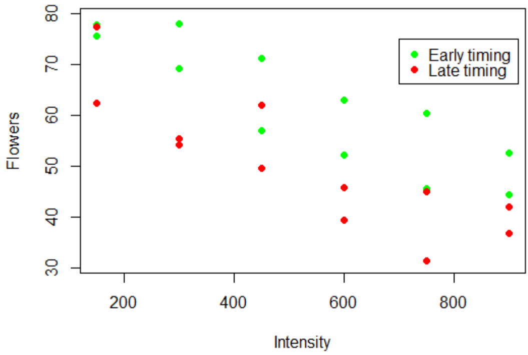
- Multiple X's give a better prediction of Y, e.g. house prices
- MLR allows polynomial fits
- MLR allows comparison of regression line for two groups, e.g. meadowfoam data
- You can control for some X's, e.g. a baseline value in a study, such as mammals and brain size



Experiments vs Observational Studies

Experiment (e.g. for the meadowfoam dataset):
A treatment is imposed on experimental units. Advantages:

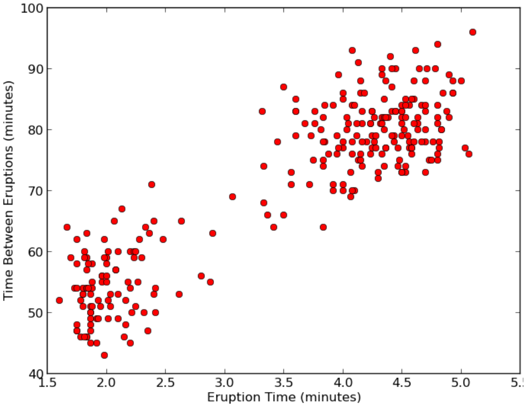
- Strength of conclusion
- If properly randomized, you can make cause-and-effect conclusions
- e.g. you can say that increasing light intensity causes a decrease in the number of flowers per plant on average



Experiments vs Observational Studies

Observational study (e.g. for the CFC data or house-price data):
Data are measured without an intervention.

- You can say that there's an association, but you can't say that X causes Y
- Perhaps they are both related to a third variable
- e.g. Old Faithful data: you can't say that a longer duration causes a large interval: perhaps both are related to some other geological phenomenon



Multiple Regression

Comparison of:

- t-tests ($H_0 : \beta_j = 0$, all other X's in the model)
- The ANOVA F-test ($H_0 : \beta_1 = \cdots = \beta_p = 0$, a test for the overall model)

It's possible to have:

F-test significant ($p < 0.05$), and all or some t-tests significant:

- *The model has useful variables for explaining Y*

F-test non-significant, and all t-tests non-significant:

- *There are no explanatory variables useful for explaining Y*

F-test significant, and all t-tests non-significant:

- *Multicollinearity; individual X's don't predict Y over and above the others*

F-test non-significant, and some t-tests significant:

- *There are two possible ways this can happen (next slide)*

Significant t-test results accompanying a non-significant F-test result

1. The model has no predictive value, but in your t-test analysis there were some Type I errors (falsely rejecting H_0). This is always a risk with multiple tests

2. The predictors were chosen poorly. e.g. if there was one useful predictor among many not useful predictors, the contribution from the useful one may not be enough for the F-test to be significant

Problems with Regression Models and their Consequences

You might have the wrong model for several reasons:

1. **The true relationship is nonlinear:**

- Biased $\hat{\beta}$'s
- Meaningless t- and F-tests / CIs

2. **You omitted an important predictor variable:**

- Biased $\hat{\beta}$'s
- Meaningless t- and F-tests / CIs

$$\text{Actual model: } Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$$

$$\text{Fitted model: } \hat{Y} = b_0 + b_1 x_1 \quad \text{and thus } \mathrm{E}(b_1) \neq \beta_1$$

3. **You included an unimportant predictor variable:**

- $\hat{\beta}$'s aren't biased
- The variance is unnecessarily large (inefficient)

Problems with Regression Models and their Consequences

The Gauss-Markov conditions might not be met:

1. **Non-constant variance:**

- $\hat{\beta}$'s aren't biased
- S^2 is a biased estimate
- Invalid t- and F-tests / CIs

2. **Correlated errors:**

- $\hat{\beta}$'s aren't biased
- S^2 isn't estimating the right thing
- Invalid t- and F-tests / CIs

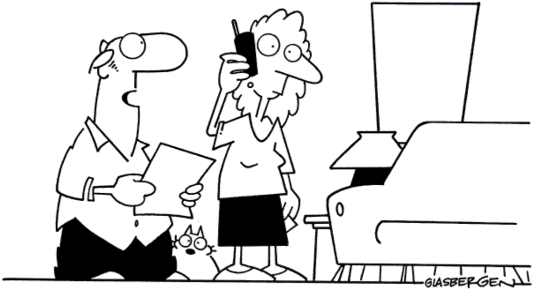
3. **Non-normal errors:**

- $\hat{\beta}$'s aren't biased
- S^2 is an unbiased estimate for σ^2
- Invalid t- and F-tests / CIs, except for large sample sizes
- Invalid PIs

Other problems with Regression Models and their Consequences

1. **Multicollinearity:**
   - $\text{var}(\hat{\beta})$ will be high — inefficient

2. **The predictor variables were measured with error:**
   - Biased $\hat{\beta}$ and S^2
   - Invalid t- and F-tests / CIs
   - Generally ok provided the variance in X's is small
   - To fix: take multiple measurements, or use Structural Equation Modelling



"Ask the realtor if we can list the litter box as a third bathroom."

Exam results by Section

After the exams are marked, I'll check for any significant differences between the grades of Sections 1 and 2. If one exists then the marks may be tweaked to eliminate this.

If you're in Section 1 but have been attending Section 2's lectures and wish to be viewed within Section 2 for the purposes of exam marks adjustment (or vice versa), please let me know directly by Friday 8 December and give your reasons.

The expectation is that Sections 1 and 2 will perform similarly on the shared exam, having been taught the same. Therefore, hopefully this slide is pointless!

Course Evaluations

UNIVERSITY OF TORONTO

FAS Fall 2017 Undergrad for Meth Data Analysis STA302H1-F-LEC0101

Medium	Online
Timing	Scheduled

- Start Date 2017-11-23 00:00
- End Date 2017-12-08 23:59

Response Rate

	Responded	Invited	% Rate
Students	39	341	11.44%

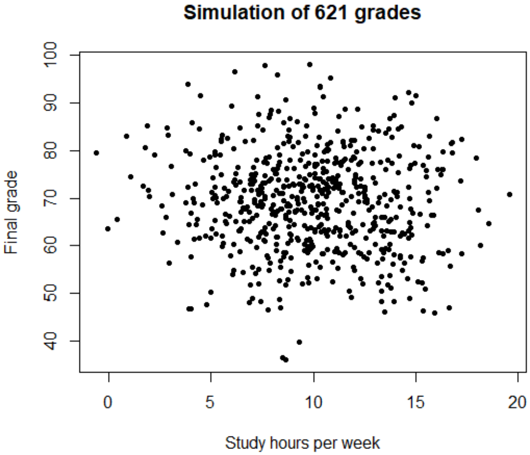
Please visit https://courseevaluations.utoronto.ca

Next steps

- Solutions to Chapter 6 have been posted on Portal under 'Homework'
- Chapter 7: You may wish to attempt the R^2_{adj} portion of 1(a), 2(a), or 3(a), but not all three
- ~~No more homework!~~

Appendix

Detailed example of partial correlation



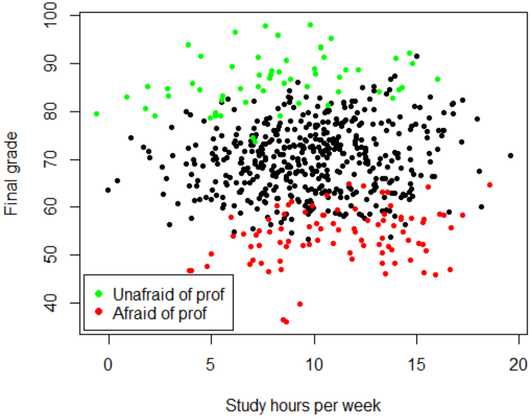
It may appear that studying more doesn't improve your final grade:

- No pattern is distinguishable
- $r \approx -0.06$, with a p-value of ~ 0.14

Detailed example of partial correlation

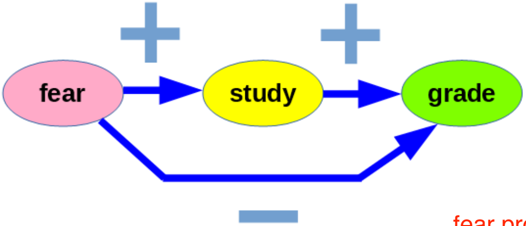
Students answer a question from 1 to 10: “Were you afraid of the prof?”

- Green points: Score of 2.5 or lower
- Red points: Score of 6 or higher



What happened?

As a graphical model (which you aren't responsible for in this course):



fear prof…
more motivated to study
but might panic on tests

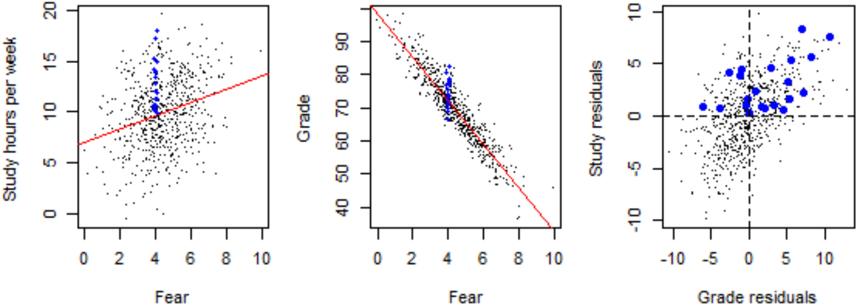
Partial correlations:

- Between Fear and Study: Significantly positive
- Between Fear and Grade: Significantly negative
- Between Study and Grade: Significantly positive

Regressing to produce the two sets of residuals

The partial correlation between `Study` and `Grade` given `Fear` is the r between:

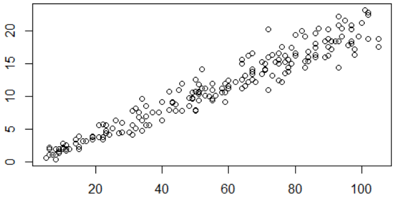
- The residuals $\hat{e}_{\text{Study}}$ resulting from the regression of `Study` versus `Fear`
- The residuals $\hat{e}_{\text{Grade}}$ resulting from the regression of `Grade` versus `Fear`



by comparing residuals, we see quite many spots in 1st quadrant

Why do WLS? Heteroscedasticity, i.e. nonconstant variance

When to use it? Example:



- A linear relationship is apparent
- The distribution of residuals isn't skewed
- There is nonconstant variance

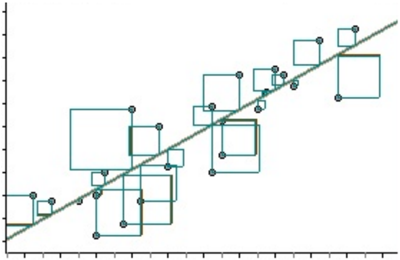
The idea: Downweight points which have more variability

Weighted Least Squares

We find the least squares estimates by minimizing the Weighted Residual Sum of Squares, WRSS:

$$\text{WRSS} = \sum_{i=1}^{n} w_i \, (y_i - \hat{y}_i)^2$$

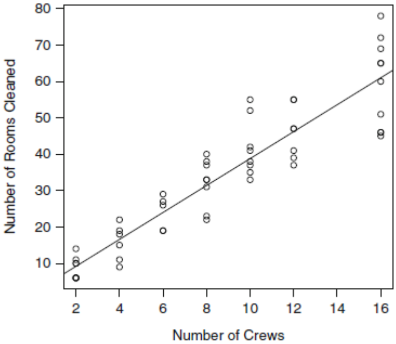
where $w_i \propto 1/\sigma_i^2$ are weights, in which σ_i^2 is the error variance for the ith observation, i.e. $\text{var}(e_i) = \sigma_i^2$.



Allows nonconstant variance, but effect of each point scaled by a weight

To find the weights:

1. If you have multiple observations for each value of x, you can estimate σ_i^2
2. Example choices in SLR might be $w_i = 1/x_i$ or $w_i = 1/x_i^2$. That is, the variance increases in proportion to the mean or mean^2 special use case
3. Often, the weights are chosen in an iterative fitting process with $\hat{y}_i$. Iteratively reweighted least squares (IRLS) is popular in logistic regression — STA303
4. If you know that y_i is the average (or median) of n_i observations, then $\text{var}(y_i) \propto 1/n_i$. This is a frequent source of unequal variance in datasets, and naturally $w_i = n_i$ is a good weight

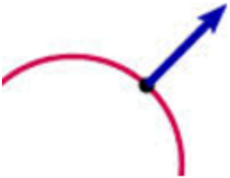


Minimizing WRSS

Solving the normal equations that result from the equation on the previous slide, we obtain the *weighted* (W) least squares estimates:

$$\hat{\beta}_{1W} = \frac{\sum_{i=1}^{n} w_i \,(x_i - \bar{x}_W)\,(y_i - \bar{y}_W)}{\sum_{i=1}^{n} w_i \,(x_i - \bar{x}_W)^2} \qquad \hat{\beta}_{0W} = \bar{y}_W - \hat{\beta}_{1W}\,\bar{x}_W$$

where $\bar{y}_W = \sum w_i \, y_i \,/ \sum w_i$ and $\bar{x}_W = \sum w_i \, x_i \,/ \sum x_i$ are weighted averages.



In matrix terms

Suppose $\text{var}(\mathbf{e}) = \sigma^2\mathbf{V}$ where $\mathbf{V}$ is a diagonal matrix in which the ith entry is $1/w_i$. Then,

$$\begin{pmatrix} \hat{\beta}_{0W} \\ \hat{\beta}_{1W} \end{pmatrix} = \hat{\boldsymbol{\beta}}_W = \left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y}$$

inverse is a diagonal matrix of w_i

and $\text{var}(\hat{\boldsymbol{\beta}}_W) = \sigma^2\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}$.

Compare these to what we derived earlier:

- On Week 8, slide 9: $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$
- In Weeks 8–9, docucam/board work: $\text{var}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$

Example: Question 1 from page 122

"A full professor of statistics at a major US university is interested in estimating the third quartile of salaries for full professors with 6 years of experience in that rank. [Data provided below or on Portal.] Using weighted least squares, estimate the 2005-2006 third quartile for salary of full professors with 6 years of experience."

Table 4.3 Data on salaries

x Years of experience as a full professor	Sample size, n_i	Third quartile ($) y
0	17	101,300
2	33	111,303
4	19	98,000
6	25	124,000
8	18	128,475
12	60	117,410
17	58	115,825
22	31	134,300
28	34	128,066
34	19	164,700

sample size uneven, probably have unequal variance

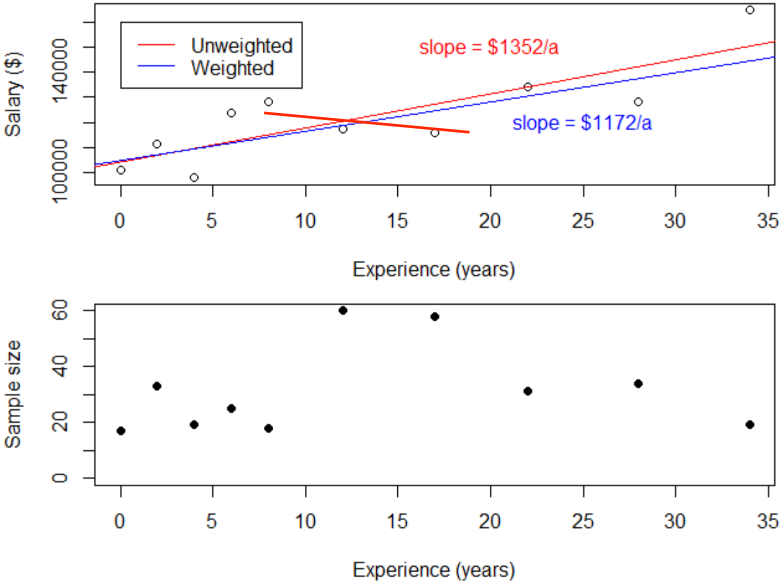
R code for Question 1

```r
Q <- read.csv("ch4q1.txt",sep="")
x <- Q[,1] # years of experience
y <- Q[,3] # salary
w <- Q[,2] # weights
m <- lm(y~x); mW <- lm(y~x,weights=w)
print(mW$coefficients[1] + 6*mW$coefficients[2])   beta0, beta1
x1 = c(-2,40)
y1 = m$coefficients[1] + x1*m$coefficients[2]
y1w = mW$coefficients[1] + x1*mW$coefficients[2]
plot(x,y,xlab="Experience (years)",ylab="Salary ($)")
lines(x1,y1,type="l",col="red")
lines(x1,y1w,type="l",col="blue")
```

```
## (Intercept)
##    111793.8
```

R output for Question 1

there is a trend of going down, hence the effect



Tests and confidence intervals

If our usual assumptions of normality and independence hold, the same general procedures for tests and CIs apply as in unweighted least squares. For example, recall from Week 3 slides 17–19 that our test statistic was

$$\frac{\hat{\beta}_1}{\mathrm{se}\left(\hat{\beta}_1\right)} = \frac{\hat{\beta}_1}{S/\sqrt{Sxx}}$$

Now in weighted least squares:

$$t_{\mathrm{obs}} = \frac{\hat{\beta}_{1W}}{S_W/\sqrt{\mathrm{WS}xx}}$$

where $\mathrm{WS}xx = \sum_{i=1}^{n} w_i \left(x_i - \bar{x}\right)^2$ w is the "weighted Sxx", and the weighted residual standard error S_W is such that

$$S_W^2 = \frac{\sum_{i=1}^{n} w_i \left(y_i - \hat{\beta}_{0W} - \hat{\beta}_{1W}\, x_i\right)^2}{n-2}$$

Under $H_0 : \beta_1 = 0$, the test statistic t_{obs} is an observation from the t_{n-2} distribution.

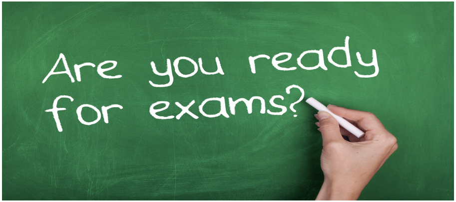
Chapter 4: What do you need to know?

STA302 gives an introduction to this method, but as per the syllabus, we skip §4.1.1 (PIs for weighted least squares), §4.1.2 (leverage for WLS), and §4.1.3 (using LS to calculate WLS).

Try homework questions 2 and 3 — they are quick, and solutions are already online.

Reminder about Datacamp: until 22 March

Name	Enrolled	Completed
Introduction to R	109	49
Intermediate R	26	11
Intro to Python for Data Science	27	9
Correlation and Regression	17	2
Intro to SQL for Data Science	10	5
Intermediate Python for Data Science	11	3
Intro to Statistics with R: Correlation and Linear Regression	8	0



Some suggestions:

- Practise all proofs in slides and extra assigned questions
- Know how to read and interpret R output
- Summary, ANOVA output
- Diagnostics plots
- Other output that you have seen from slides.
- Review the non-credit homework assignments and the midterm papers (both sections)
- Doing some old exams might help you to see which topics are important

Exam format

- A cover page and formula page will appear on Portal
- Question types: multiple choice, short answer, proofs, data analysis
- The coverage is on the entire course with emphasis (75%) on material taught from §3.3 (mid-October+) onwards

- This Friday, Monday, Tuesday and Wednesday there will be TA office hours and instructor office hours as per normal
- On Friday 8 December, there's the Exam Jam
- Limited TA office hours will occur after Wednesday 6 December, at days and times to be announced over Portal. The usual times will no longer apply by default
- On Wednesday 13 December there will be a bonus instructor office hours at the usual Wednesday time (bookable online as usual)
- TAs and the instructor won't generally be available to meet at other times but we'll continue to check Piazza

Final Steps

On Tuesday **5 December** in OI G162:

- At 10 am we may start with a lecture for Section 1 as normal (details to come on Friday 1 December), but it would be brief
- For most of the 10 am to 12 pm slot, there will be an optional **drop-in help session**, open to all
- You may like to meet among your Recognized Study Groups, work individually, etc

Wishing you all the best...

- On the exam
- For the Christmas holidays
- In 2018+