APM462 Syllabus Summer 2019

Instructor: Jonathan Korman

Lectures: Tuesday 6-8pm and Thursday 6-7pm in BA1240.

Office Hours: Tue 5-6pm in HU1024 (if building door closed, call

Jonathan's office 416-978-4174).

Office: HU1024

email: jkorman at math dot toronto dot edu

Teaching Assistant: Mykola Matviichuk

Office: BA 6135

email: mykola dot matviichuk at mail dot utoronto dot ca

Office Hours: Thr 11am-1pm in BA6135

Textbook:

The suggested textbook for this course is *Linear and Non-Linear Programming (4th Edition)* by David Luenberger & Yinyu Ye. Publisher: Springer. Should be availlable at the UT bookstore. The textbook is suggested and not required: I will try to make the lectures and HW self contained. We will cover some of the material from chapters 7-11 in the textbook. Lectures will also include supplementary material not in the textbook.

Supplementary texts:

- 1. "Calculus of Variations" by I.M. Gelfand and S.V. Fomin.
- 2. "Intro. To Calculus of Variations". Online lectures by Peter Olver. http://www-users.math.umn.edu/~olver/ln_/cv.pdf
- 3. "Mathematical Methods of Classical Mechanics" by V.I. Arnold.

Course grade:

HW - 20% Term Test - 30% Final Exam- 50%

Homework:

There will be 5 homework assignments. For each homework, 2 questions will be randomly selected for grading. Homework assignments are to be uploaded online on Quercus on the assigned dates. Late homework will not be accepted.

Concerns about HW grading should be directed to Mykola during office hours no later than 1 week after homework has been graded. Regrade means that you are requesting us to check your work again for a reason, after you have carefully articulated that reason. It does not mean "I don't like my grade and I hope you can give me a better grade." If you still have concerns regarding the grading after you discussed it with Mykola, please see Jonathan during office hours.

Term test:

There will be **no** make up term test! If you have a **valid** reason for missing the term test, the corresponding portion of the final exam will count as your term test grade.

Exam dates:

Term Test: Mon, June 17, 6-8pm.

Final Exam: TBA

How to Study:

Nonlinear Optimization is a 4th year math course and is a demanding course! You are expected to attend lectures, participate, and take detailed notes. After every lecture, you should read the lectures, the examples and proofs to make sure you have no gaps. You are encouraged to come to office hours to dicuss the material.

Attempting to solve problems is where much of your effort should be directed. It is normal to struggle with problem sets and it is normal not to be able to solve everything the first time you try. Take time to think things over, discuss problems with your classmates and come to office hours. If you are looking for an easy way out by copying others' solutions, you will learn little and almost certaily will fail the tests (apart from possibly comitting plagerism). Spending 10 hours working on a problem set is normal, even expected.

Preparing for tests. Start studying early (at least 10 days before the test). Start by making sure you understand all the definitions, concepts, theorems, examples and proofs. Next, do all the HW and practice problems yourself. Review and memorize all the concepts, theorems and formulas you need. In the days before the test, practice tests and exams from previous years.

Important information:

- Class behaviour: texting and surfing the internet disrupts the lecture, therefore it is not allowed. If you want to use your laptop to take notes, please sit at the back of the class so as not to distract other students. If you have special needs, you will be accommodated. Flat computers such as Tablets, iPads, etc. are fine as long as they are used with the screen sitting flat on the desktop.
- **Class participation:** you are expected to attend lectures. Participation is encouraged but not required. If you missed a class it is your responsibility to catch up.
- **Office hours** start on the week of May 15.
- **Copyrights:** course materials are provided for the use of enrolled students only. Students are not allowed to post, share, or sell course materials without instructor's permission.
- **Privacy:** students are not authorized to take photos during class or record lectures.
- **Email policy:** Please use the email addresses provided above only; make sure to include the word "APM462" in the subject of the email. Email to other accounts will not be answered. The

proper place to ask math questions is in class or during office hours, <u>not</u> on email.

Collaboration versus plagiarism: discussing problems sets
with each other is perfectly fine, but you must write your own
solutions independently. Copying solutions from the internet or
your friends is plangiarism. Please read the information in the
two links below about Academic misconduct and Pagiarism:
http://www.artsci.utoronto.ca/osai/The-rules/what-is-academic-misconduct

http://www.artsci.utoronto.ca/osai/students/avoidmisconduct/tips-for-avoiding-academic-misconduct

• In case you **missed the term test** for a medical reason you will need to provide an original copy of the following form within one week of the missed date:

http://www.illnessverification.utoronto.ca/

You must hand in the original form to Jonathan in person. Electronic forms will not be accepted.

• Accessibility Services: https://www.studentlife.utoronto.ca/as

Course Description:

We will discuss finite-dimensional optimization problems: unconstrained problems, equality constraints, inequality constraints. Next we will learn some algorithms for computing minimizers, including: steepest descent, conjugate gradients. We will end with an introduction to to the Calculus of Variations. Time permitting, we will discuss subgradients and convex optimization.

Course Calendar (tentative)

| # Week of Summer Semester: 1 May 7 Review. Finite dimentional optimization (unconstrained problems): 1st and 2nd order neccessary conditions for a minimum. [LY#7.1-7.2] Office hours begin. Finite dimentional optimization (unconstrained problems): 2nd order sufficient condition for a minimum. Convex functions: C ¹ and C ² characterizations. [LY#7.3-7.4] Convex functions: local minimum is a global minimum, maxumum is attained on boundary of compact convex domain. [LY#7.4-7.5] Introduction to Finite dimentional optimization (equality constraints): Lagrange multipliers. [LY#11.1-11.4] Finite dimentional optimization (equality constraints): 1st and 2nd order neccessary conditions for a local minimum. 2nd order sufficient condition for a local minimum. [LY#11.3-11.5] Finite dimentional optimization (inequality constraints): 1st and 2nd order neccessary conditions for a local minimum. 2nd order sufficient condition for a local minimum. [LY#11.8] June 17 No classes. Midterm: Mon, June 17. June 25 Summer break. July 2 Conjugate direction methods. [LY#9.1-9.3] 8 July 9 Conjugate direction methods. [LY#9.1-9.3] 2 Conjugate direction methods. [LY#9.1-9.3] 3 July 16 Calculus of Variations: introduction. [GF, chp.1], [O#1-3] July 23 Calculus of Variations: Ist order necc. conditions, Euler-Lagrange equation. [GF, chp.1], [O#1-3] July 30 Calculus of Variations: Examples, classical mechanics (least action principle). [GF#4.21-22] [A#13] 2 Aug 6 Calculus of Variations: equality constraints, sufficient conditions (convexity). [GF#2.12] Aug Final Exam. | Course Calendar (tentative) | | | |
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