Homework #6 - STA414

Winter 2018

Instructions: Do not submit your work. This assignment is for your edification only – not for credit.

Question 1.

a	b	c	p(a, b, c)
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

Consider three binary variables $a, b, c \in \{0, 1\}$ having the joint distribution given in the table above. Show by direct evaluation that this distribution has the property that a and b are marginally dependent, so that $p(a, b) \neq p(a)p(b)$, but that they become independent when conditioned on c, so that $p(a, b|c) = p(a|c)p(b|c)$ for both $c = 0$ and $c = 1$. (From Bishop, p 419.)

just assume some c=0 and check, try to write in terms of joint distributions …

Question 2.

Evaluate the distributions $p(a)$, $p(b|c)$, and $p(c|a)$ corresponding to the joint distribution given in the table in Question 1. Hence show by direct evaluation that $p(a, b, c) = p(a)p(c|a)p(b|c)$. Draw the corresponding directed graph. (From Bishop, p 419.)

Question 3.

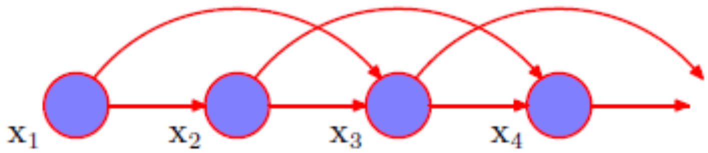
Sketch a Bayesian network representing a HMM over five observed variables in a sequence $\{x_1, x_2, x_3, x_4, x_5\}$. Label the latent state variables $\{z_1, z_2, z_3, z_4, z_5\}$.

Question 4.

Use the Bayes-ball algorithm (a.k.a. d-separation) on the model described by the graph in the figure below, in which there are N nodes in total, to show that the model satisfies the conditional independence properties

$$p(x_n|x_1,\ldots,x_{n-1}) = p(x_n|x_{n-1},x_{n-2})$$

for $n = 3,\ldots,N$. (From Bishop p 648.)



Question 5.

Verify the M-step equations:

$$\pi_k = \frac{\gamma(z_{1k})}{\sum_{j=1}^{K}\gamma(z_{1j})}$$

$$A_{jk} = \frac{\sum_{n=2}^{N}\xi(z_{n-1,j},z_{nk})}{\sum_{l=1}^{K}\sum_{n=2}^{N}\xi(z_{n-1,j},z_{nl})}$$

for the initial state probabilities and transition probability parameters of the hidden Markov model by maximization of the expected complete-data log likelihood function:

$$Q(\boldsymbol{\theta},\boldsymbol{\theta}^{\text{old}}) = \sum_{k=1}^{K}\gamma(z_{1k})\ln\pi_k + \sum_{n=2}^{N}\sum_{j=1}^{K}\sum_{k=1}^{K}\xi(z_{n-1,j},z_{nk})\ln A_{jk} + \sum_{n=1}^{N}\sum_{k=1}^{K}\gamma(z_{nk})\ln p(\mathbf{x}_n|\phi_k).$$

using appropriate Lagrange multipliers to enforce the summation constraints on the components of $\boldsymbol{\pi}$ and $\mathbf{A}$. (From Bishop p 648.)

Question 6.

Show that if any elements of the parameters $\boldsymbol{\pi}$ or A for a hidden Markov model are initially set to zero, then those elements will remain zero in all subsequent updates of the EM algorithm. (From Bishop p 648.)