Jeopardy Review

- Form teams of 5
- If you prefer to solo it, sit on this side →

MISC \$500

x	0	1	2
	1/4	1/2	1/4

$E(X) = \frac{1}{2} + \frac{1}{2} = 1$

y	0	1	2
	1/3	1/3	1/3

$E(Y) = \frac{1}{3} + \frac{2}{3} = 1$

xy	0	1	2	4
	1/2	1/6	1/4	1/12

$E(XY) = \frac{1}{6} + \frac{1}{2} + \frac{1}{3}$

$= 1$

$\sigma_{XY} = 1 - 1 = 0$

MGF \$500

PDF

$Y \sim \Gamma(2,2), \quad y > 0$

$f_Y(y) = \frac{1}{(2-1)!\, 2^2} y^1 e^{-y/2} = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

$\Gamma(\alpha) = (\alpha-1)!$

$f_W(w) = \frac{1}{2\sqrt{w}} f_Y(\sqrt{w}) + \cancel{\frac{1}{2\sqrt{w}} f_Y(-\sqrt{w})} \to 0$

$= \frac{1}{2\sqrt{w}} \frac{1}{4} \sqrt{w}\, e^{-\sqrt{w}/2}$

$= \frac{1}{8\sqrt{w}} \sqrt{w}\, e^{-\sqrt{w}/2}$

in general

$Y = X^2$

PDF of Y in terms of PDF of X. (Week 11)

$\frac{1}{4} w^{1/2} e^{-\sqrt{w}/2} \cdot \frac{1}{2\sqrt{w}}$

Counting $300

Sticks & stones

— 11 stones

— 3 shelves "2 sticks"

$$\binom{11+3-1}{11} = 78$$

Arranging 11 identical stones
2 identical sticks.

$$_{13}C_2 = {}_{13}C_{11}$$

$$_nC_k = \frac{n!}{k!(n-k)!}$$

x x x x | x x x | x x x x

4 + 3 + 4 = 11

Counting $500

(1) $\dfrac{\binom{12}{2}}{(11)^3}$

↪ # of ways to select 3 #s w/ repetition.

(2) $_9C_2$ ✓

| |

x x | x x x x x x | x x   2+6+2 = 10

/ 0 _ 0 _ 0 _ 0 | 0 _ 0 _ 0 | 0 _ 0 _ 0 / → non zeros.

P ... $\dfrac{_9C_2}{(11)^3}$

Misc. for $400

$\pi_1 = \frac{15}{103}$ $\pi_2 = \frac{40}{103}$ $\pi_3 = \frac{48}{103}$

$\pi_1 = \frac{3}{19}$ $\pi_2 = \frac{8}{19}$ $\pi_3 = \frac{8}{19}$

$\pi_1 = \frac{9}{61}$ $\pi_2 = \frac{28}{61}$ $\pi_3 = \frac{24}{61}$ ✓

$$[\pi_1 \ \pi_2 \ \pi_3] \begin{bmatrix} 1/3 & 1/6 & 1/2 \\ 0 & 5/8 & 3/8 \\ 1/4 & 3/8 & 3/8 \end{bmatrix} = [\pi_1 \ \pi_2 \ \pi_3]$$

① $\frac{1}{3}\pi_1 + \frac{1}{4}\pi_3 = \pi_1 \rightarrow \frac{1}{4}\pi_3 = \frac{2}{3}\pi_1$

$$\boxed{\pi_1 = \frac{3}{8}\pi_3}$$

② $\frac{\pi_1}{2} + \frac{3\pi_2}{8} + \frac{3\pi_3}{8} = \pi_3$

$\frac{3}{16}\pi_3 + \frac{3\pi_2}{8} + \frac{3\pi_3}{8} = \pi_3$

$\frac{9}{16}\pi_3 + \frac{3\pi_2}{8} = \pi_3 \rightarrow \frac{3\pi_2}{8} = \frac{7}{16}\pi_3$

$$\boxed{\pi_2 = \frac{7}{6}\pi_3}$$

③ $\pi_1 + \pi_2 + \pi_3 = 1$

$\frac{3}{8}\pi_3 + \frac{7}{6}\pi_3 + \pi_3 = 1$

$\frac{61}{24}\pi_3 = 1 \rightarrow \pi_3 = \frac{24}{61}$

Prob. & Discrete $500

X	-4	3
$P(X=x)$	0.7	0.3

$\mathbb{1}_F$	1	0
p	$1/2$	$1/2$

$X\mathbb{1}_F$	0	-4	3
	$\frac{1}{2}\cdot 0.7+\frac{1}{2}\cdot 0.3$ 0.5	0.35	0.15

$$\mathbb{1}_F = \begin{cases} 1 & \text{if } H \quad p=0.5 \\ 0 & \text{if } T \quad p=0.5 \end{cases}$$

$$E(X\mathbb{1}_F) = -0.95$$

$E(X)\,E(\mathbb{1}_F)$ → flipping coins are indep. of any other existing events.

$X\mathbb{1}_F$	0	-4	3
$P(X\mathbb{1}_F)$	0.5	0.35	0.15

$$E[X\mathbb{1}_F] = -0.95$$

$f(x) = \frac{1}{\theta} e^{-x/\theta}$

let $T \sim$ time to failure

$f(t) = \frac{1}{4} e^{-t/4}$

$\int_0^3 \frac{1}{4} e^{-t/4} dt = -e^{-t/4} \Big|_{t=0}^{3}$

$= 1 - e^{-3/4}$

$= 0.5276$

X = # of components, $X \sim NB(3, 0.5276)$

$P(X = 8) = ({}_7C_2)(0.5276)^3 (0.4724)^5 =$

T = time to failure, $T \sim exp(4)$

P(8 components b4 3 failures) = 7.26%

$P(\text{1 fail in 3}) = 1 - e^{-3/4} = 0.5276 = \int_0^3 f(t)\,dt$

$X \sim NB(3, 0.5276)$ $P(X=8) = ({}_7C_2)(0.5276)^3(1-0.5276)^5$

MGF : $400

$Y = X_1 + X_2 + 2X_3$

MGF of Y $(1-5t)^{-4}$

$E(e^{Yt}) = E(e^{X_1t + X_2t + 2X_3t})$

$= E(e^{X_1t}) E(e^{X_2t}) E(e^{2X_3t})$

$= M_{X_1}(t) \quad M_{X_2}(t) \quad M_{X_3}(2t)$

$= (1-5t)^{-1.2} (1-5t)^{-1.3} (1-2.5(2t))^{-1.5}$

$= (1-5t)^{-4}$

$Y \sim \Gamma(\alpha = 4, \beta = 5)$

Counting $400

1 # smaller {1, 2, 3}

4

3 #s larger {5, 6, 7, 8, 9}

$$\frac{{}_3C_1 \cdot {}_1C_1 \cdot {}_5C_3}{{}_9C_5}$$

P(4 is 2nd smallest) = $\frac{5}{21}$

The factory with the most days w/ defective > 20%.
↳ That has a higher daily prob. of a defective rate > 20%.

Factory X

$n_x = 300$

$p = 0.15$

$\hat{p}_x$ – RV

$\hat{p}_x \sim N\left(p, \frac{p(1-p)}{n}\right)$
↳ By CLT

$\sim N\left(0.15, \frac{0.15 \times 0.85}{300}\right)$

Factory Y

$n_y = 100$

$p = 0.15$

$\hat{p}_y$ = RV

$\hat{p}_y \sim N\left(0.15, \frac{0.15 \times 0.85}{100}\right)$

Factory Z

$n_z = 200$

$p = 0.15$

$\hat{p}_z \sim N\left(0.15, \frac{0.15 \times 0.85}{200}\right)$

$P(\hat{p}_x > 0.20)$ = STANDARDIZE

$= P\left(\frac{\hat{p} - p}{\sqrt{}} > \frac{0.15 - 0.20}{\sqrt{}}\right)$

$P(z > \_\_\_)$

$P\left(\frac{\hat{p}_y - p}{\sqrt{}} > \frac{0.15 - 0.2}{\sqrt{}}\right)$

$P(z > \_\_\_)$

The one with the largest prob. is the one w/ smallest z-score → largest SD. → factory w/ largest SD is the one that has smallest n.