Assignment #8

Due: Never

Question 1 (Properties of a sampling-based estimator, from Bishop p 556)

Show that the finite sample estimator $\hat{f}$ defined by

$$\hat{f} = \frac{1}{L}\sum_{l=1}^{L} f\left(\mathbf{z}^{(l)}\right)$$

has mean equal to $\mathbb{E}[f]$ and variance given by

$$\mathrm{var}[\hat{f}] = \frac{1}{L}\mathbb{E}\left[(f - \mathbb{E}[f])^2\right]$$

Question 2 (Markov-chain Monte Carlo)

Let $\mathrm{p}(\mathbf{w})$ correspond to a Gaussian mixture model with $\boldsymbol{\pi} = \left\{\frac{3}{4}, \frac{1}{4}\right\}$, $\boldsymbol{\mu} = \{0, 8\}$, and $\boldsymbol{\sigma} = \{1, 1\}$. In this question, you will estimate $\mathrm{E}(\mathbf{w})$ using 3000 Metropolis-Hastings iterations, initialized at $x = 0$.

(a) Estimate the expectation $\mathrm{E}(\mathbf{w})$ using a proposal distribution $Q(x'|x) \sim \mathcal{N}(x, 1)$. Plot your samples on the same graph as the true distribution $\mathrm{p}(\mathbf{w})$, as a histogram. How many of the 3000 iterations resulted in successful samples? Hint: You may wish to run your code a few times but you need only report on one run.

(b) Repeat using a *mixture proposal*: for each iteration, a proposal distribution $Q(x'|x) \sim \mathcal{N}(x, 10^2)$ is used with 50% probability, and with 50% probability the proposal in (a) is used instead.

(c) Compare your two estimates for $\mathrm{E}(\mathbf{w})$ to one another and to the true answer. State which of (a) or (b) is best for the current task, and justify your answer.

Wilfred Hastings (left) was a U of T student and prof until 1971. On the right is Nicholas Metropolis.