Mini Problems 18

1.a. Is $F(x, y, z) = (-2x, 3y, -3)$ a gradient vector field? (That is, of the form ∇f for some scalar function $f : \mathbb{R}^3 \to \mathbb{R}$). **b.** Show that the vector field $F(x, y, z) = (e^x \cos(y) + e^{-x} \sin(z), -e^x \sin(y), e^{-x} \cos(z))$ is not the gradient of any vector field.

2. How much work is done by the force field $F(x, y, z) = (xy^2, z, 3x - z)$ on a particle moving along the straight line from $(1, 0, 1)$ to $(2, 3, -1)$?

3. Prove the following identity: $\nabla \cdot (f \nabla f) = |\nabla f|^2 + f \nabla^2 f$. Here f is a scalar function and ∇^2 denotes the Laplacian operator.

4.a. Show that for a vector field F, and a parametrized path $\bar{x}$, we have

$$\int_{\bar{x}} F \cdot d\bar{x} = \int_{\bar{x}} F \cdot T \, ds$$

where T is the unit tangent vector to $\bar{x}$. **b.** Use this to prove the following result: if C is a level curve of the function $f(x, y)$, then the line integral of ∇f along C vanishes.