6 Heap Sort

Definition. *A **sorting problem** is one where*

- ***Input**: A sequence of n numbers $< a_1, a_2, \cdots, a_n >$*
- ***Output**: A permutation (reordering) $< a'_1, a'_2, \cdots, a'_n >$ of the input sequence such that $a'_1 \leq \cdots \leq a'_n$*

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Definition. *A **free tree** is a connected, acyclic, undirected graph. A **forest** is an undirected, acyclic but possibly disconnected graph.*

Definition. *A **rooted tree** is a free tree in which one of the vertices is distinguished from the others. This distinguished verex is the **root** of the tree. Vertex of a rooted tree is a **node** of the tree. Assume a node x in a rooted tree T with root r. Any node y on the unique simple path from r to x an **ancestor** of x. If y is an ancestor of x then x is a **descendent** of y. The **subtree rooted at** x is the tree induced by descendents of x, rooted at x. The root is the only node in T with no parent. If two nodes have the same parent they are **siblings**, A node with no children is a **leaf**. A nonleaf node is an **internal node**. The number of children of a node x in a rooted tree T equals the **degree** of x. The length of the simple path from the root r to a node x is the **depth** of x in T. A **level** of a tree consists of all nodes at the same depth. The **height** of a node in a tree is the number of edges on the longest simple downward path from the node to a leaf. The height of a tree is the height of its root. An **ordered tree** is a rooted tree in which the children of each node are ordered. That is if a node has k children, then there is a kth child.*

Definition. *A **binary tree** T is a structure defined on finite set of nodes that either*

- *contains no nodes, or*
- *is composed of three disjoint sets of nodes;*
  1. *a root node*
  2. *a binary tree called its **left subtree***
  3. *a binary tree called its **right subtree***

*The binary tree that contains no nodes is called the **empty / null tree**, denoted with* Nil. *A **full binary tree** is a binary tree that each node is either*

- *a leaf or*
- *has degree exactly 2*