Mini-Problems 9

1. Calculate the limit

$$\lim_{x \to 0} \frac{x^3 - \sin(x^3)}{x^5(1 - \cos(x^2/3))}.$$

2. Find a polynomial which approximates the function $\ln(1 + x)$ to within 10^{-2} on the interval $[0, 1]$.

3. Consider the following lemma: If $P(x)$ is a polynomial of degree $\leq k$ such that $P(x)/||x||^k \to 0$ as $x \to 0$, then P is identically zero. (You may also prove this lemma as an exercise).

Using this, explain how to show that the Taylor polynomial of a multivariable function centred at a point $x = a$ is unique. That is, if we have a polynomial $Q(x)$ of degree k such that $f(x) = Q(x) + E(x)$ where $E(x)/||x - a||^k \to 0$ as $x \to a$ then $Q(x)$ is necessarily the kth Taylor polynomial of f at $x = a$.

4. Calculate the specified Taylor polynomials of the following 2-variable functions: (i) $T_1(e^{x+y} \cos(x + y))$ (ii) $T_3(\ln(1 + x^2y)y)$ (iii) $T_2(x^4 + 3x^2y^2 + 4y^3 + 5y^5)$ (iv) $T_3(\sin(e^{x+2y}x))$ (v) $T_{72}(x^4 + 3x^2y^2 + 4y^3 + 5y^5)$.

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