UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2007 EXAMINATIONS

CSC320H1S : Introduction to Visual Computing

Duration: 3 hours

No aids allowed

There are 14 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

Question	Marks
1	___________/25
2	___________/15
3	___________/10
4	___________/20
5	___________/15
6	___________/15
7	___________/20
8	___________/35
9	___________/25
Total	__________/180

1 Masks and Template Matching (25 marks total)

(a) [7 marks] Give the definition of a *separable* $N \times N$ mask (or filter):

(b) [8 marks] Why is separability a useful property for a mask to have? Be as specific as possible.

(c) Suppose we want to perform template matching using the following image and template:

Image:

	6	6	9	3	3	4	20	10	2
pixel #	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*

Template:

6	6	8

Indicate the best-matching pixel when we use

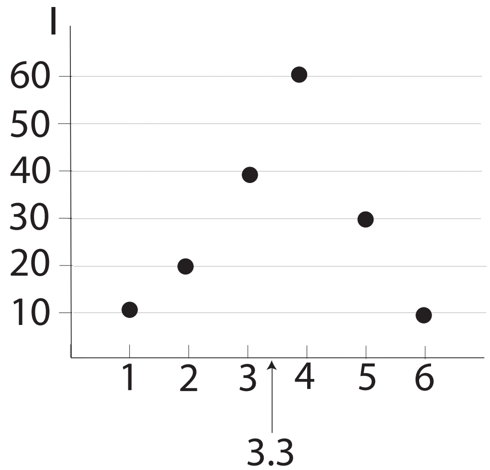
(c1) [5 marks] cross-correlation:

(c2) [5 marks] normalized cross-correlation:

In each case, either show your calculations or include a one-sentence explanation. No marks will be given without them.

2 Image Interpolation (15 marks total)

You are given a 6-pixel image I and are asked to interpolate the known pixels to compute an intensity for fractional pixel 3.3.



(a) [10 marks] Give the expression for $I(3.3)$ in terms of I and an interpolation kernel (or mask) M. Assume that $M(t) = 0$ for $t > 2$ and that $M(-t) = M(t)$.

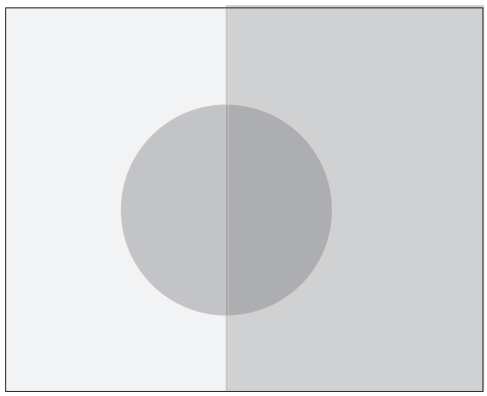
$I(3.3) =$

(b) [5 marks] Compute the interpolation result when M is the *linear interpolation kernel*.

$I(3.3) =$

3 Alpha Matting (10 marks total)

Suppose you are given the intensity of every pixel in the following grayscale photo and are asked to compute the alpha value of every pixel inside the circle:



Can you do this? If yes, explain how; if not, explain what else you have to assume to do it.

4 Least-Squares Estimation (20 marks total)

Let $I_1, I_2, \ldots, I_n$ be the intensities of an n-pixel image patch. Prove that the mean intensity of the patch is the patche's 0^{th}-order least-squares approximation, i.e., that it minimizes the least-squares error, $E(x) = \sum_{k=1}^{n}(I_k - x)^2$.

5 2D Curves (15 marks total)

Prove that

$$\frac{d\mathbf{T}}{ds}(s) \ \times \ \mathbf{N}(s) \ = \ 0$$

where $\times$ denotes the cross product of two vectors; $\mathbf{T}(s), \mathbf{N}(s)$ are the unit tangent and unit normal of a 2D curve, respectively; and s is the curve's arc-length parameter.

6 SIFT (15 marks total)

Give the main steps that the SIFT algorithm uses to *locate* keypoints in an image. Be as specific as possible, and focus only on keypoint detection and localization—do not discuss the definition and creation of keypoint descriptors, or how these descriptors are matched between images.

7 PCA (20 marks total)

Let $\mathbf{Z} = [\mathbf{Z}_1 \quad \mathbf{Z}_2 \quad \ldots \quad \mathbf{Z}_N]$ be a matrix of M-dimensional column vectors whose mean is zero (i.e., $\frac{1}{N}\sum_{i=1}^{N}\mathbf{Z}_i = 0$). Moreover, let $\mathbf{Z}^j$ be the row vector corresponding to the j^{th} row of matrix $\mathbf{Z}$.

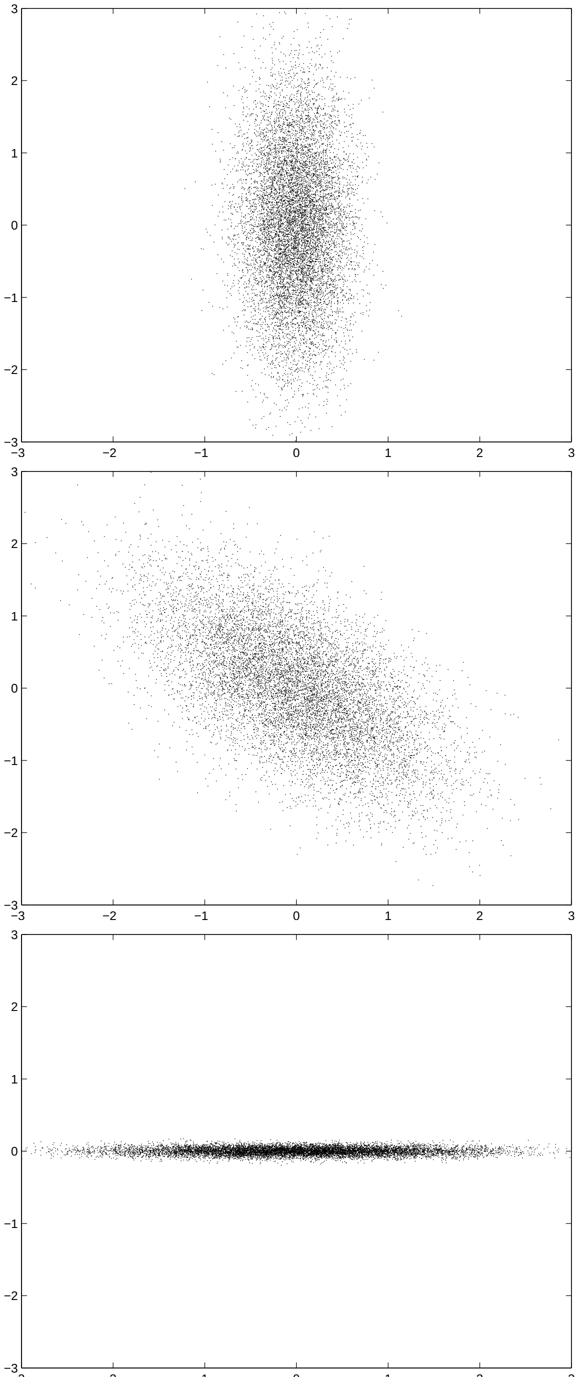
(a) [4 marks] Define the *variance* of vector $\mathbf{Z}^j$ using vector notation:

$$\text{var}(\mathbf{Z}^j) \;=$$

(b) [4 marks] Define the *covariance* of vectors $\mathbf{Z}^j, \mathbf{Z}^k$ using vector notation:

$$\text{cov}(\mathbf{Z}^j, \mathbf{Z}^k) \;=$$

(c) [12 marks] Suppose that $M = 2$. In this case, we can represent matrix $\mathbf{Z}$ with a scatter plot, where each point in the scatter plot corresponds to a specific column of $\mathbf{Z}$. For each of the three plots below, draw the vectors $(\lambda_1 e_1)$ and $(\lambda_2 e_2)$, where e_1, e_2 are the eigenvectors of matrix $\mathbf{Z}\mathbf{Z}^{\mathbf{T}}$ and λ_1, λ_2 are the corresponding eigenvalues.



8 Multi-Scale Representations (35 marks total)

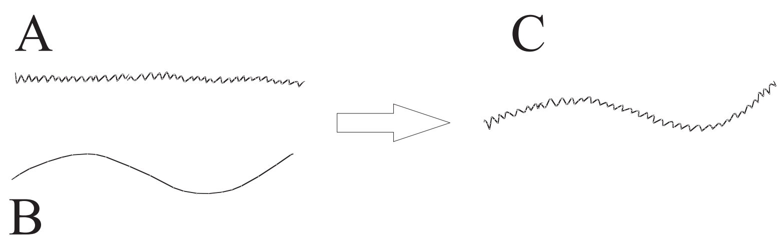
(a) [10 marks] Compute the Haar wavelet transform of the following image:

18	2	0	12	9	3	17	19

(b) [15 marks] Compute the image whose Haar wavelet transform is given by the following vector:

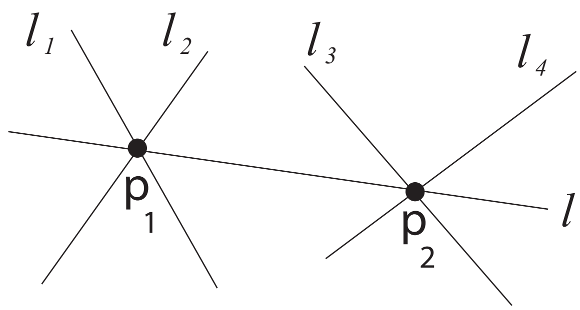
4	1	-2	2	-1	2	0	1

(c) [10 marks] Give an algorithm that performs the following *curve merging* operation: you are given two curves, A and B, each of which is 256 pixels long, and your goal is to create a new curve C, also 256 pixels long, that preserves the fine details of curve A but has the overall shape of curve B. Be as specific as possible.



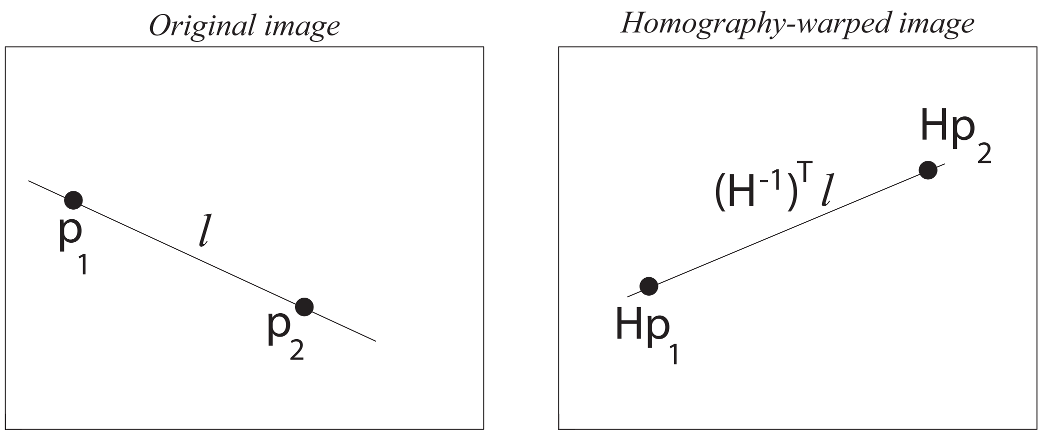
9 Homogeneous Coordinates (25 marks total)

(a) [10 marks] Give a single formula that expresses the *homogeneous coordinates* of line l in terms of the homogeneous coordinates of lines $l_1, \ldots, l_4$.



$l \cong$

(b) [15 marks] Let p_1, p_2 be the homogeneous coordinates of two points and let l be the homogeneous coordinates of the line connecting them. Now suppose that we warp the original image using a homography H. Prove that the line connecting the homography-warped points, Hp_1 and Hp_2, has homogeneous coordinates $(H^{-1})^T l$. Assume that p_1, p_2 and l are column vectors.



END OF EXAM