STA 247 - Discrete Activity

Names:______________________________ Topic letter: ____________

Complete by: October 7, 2016

Instructions:

1. Return to your original group of three.
2. You will have 10 minutes to go over your answers and compare, discuss.
3. We will then take up the problems via Kahoots!

Problem 1. Determine whether the following problems are best modeled with a(n)... **(1) indicator variable, (2) binomial distribution, (3) geometric distribution, (4) negative binomial distribution, or (5) hypergeometric distribution**.

Winning 3 out of 5 games in a series. B

Number of hits a baseball player will have until the first miss. G

Number of female members in a committee of 5 when selecting from a pool of 30 candidates: 17 males and 13 females. H

The outcome of answering a multiple choice problem randomly. I

The number of swings a baseball player will make until he strikes out. NB

Testing 18 patients before finding one with type B- blood. G

Randomly grabbing two batteries from a box of 20 batteries of which 8 are dead. H

Problem 2. Suppose the Toronto Blue Jays have a 54.9% probability of winning against the Boston Red Sox and their performance per game is independent of others. Given these statistics (found on baseball-reference.com a few months ago),

_ _ _ W

a. What is the probability that they will beat the Red Sox by the 4th game in the American League Division Series (which has a total of 5 games)? $\rightarrow r=3$ = # of successes

let X = # of losses before 3rd win : $P(X=1) = \binom{3}{2}(0.549)^3(0.451)$

b. What is the expected number of losses before the third win?

$E(X) = \frac{r(1-p)}{p} = 3(0.451)/0.549$

c. What is the variance for the number of losses before the third win?

$V(X) = \frac{r(1-p)}{p^2} = \frac{3(0.451)}{(0.549)^2}$

"success"

p

Problem 3. A factory that produces car parts has a defect rate of 2%.

a. What is the probability the factory will produce 100 parts before a defective product is produced? Let $X = \#$ of failures before first success

$P(X=100) = (1-p)^{100}\, p = (0.98)^{100}(0.02)$

b. What is the probability that the factory will produce 50 parts before a defective product is produced, given that the first 40 parts were not defective?

$P(X=50 \mid X\geq 40) = \frac{P(X=50 \cap X\geq 40)}{P(X\geq 40)} = \frac{P(X=50)}{P(X\geq 40)} = \frac{(0.98)^{50}(0.02)}{(0.98)^{40}} = (0.98)^{10}(0.02)$

Problem 4. In an assembly-line production of industrial robots, gearbox assemblies can be installed in 1 minute each, if the holes have been properly drilled in the boxes, and in 10 minutes each if the holes must be redrilled. There are 20 $\to N$ gearboxes in stock and 2 $\to k$ of these have improperly drilled holes. From the 20 gearboxes available, 5 $= n$ are selected at random for installation in the next 5 robots in line.

18 - Functional
2 - Defective

Let $D = \#$ of defectives in a random draw
$D = \{0, 1, 2\}$

a. Find the probability that all 5 gearboxes will fit properly.

$P(D=0) = ({}_{18}C_5)({}_2C_0)/({}_{20}C_5)$

b. Find the expected value, the variance, and the standard deviation of the time it will take to install these 5 gearboxes.

$E(D) = n\cdot\frac{k}{N} = 5\times\frac{2}{20} = \frac{1}{2}$ | $V(D) = n\frac{k}{N}\cdot\left(1-\frac{k}{N}\right)\left(\frac{N-n}{N-1}\right) = \left(\frac{1}{2}\right)\left(\frac{18}{20}\right)\left(\frac{15}{19}\right)$

Let $T =$ time to install $= 10(D) + 1(5-D)$
$= 9D+5$

$E(T) = E(9D+5) = 9E(D)+5 = 9.5$

$V(T) = V(9D+5) = 9^2 V(D) = 81\, V(D)$

Problem 5. An applicant for a driver's license has a 75% probability of passing the road test.

a. What is the probability that an applicant passes the test on their fifth try?

b. What is the average and variance for the number of trials until the applicant passes?

c. What is the probability that it will take the applicant less than 3 tries **before** passing the road test?

1 pass, 2 or fewer fails

x x x x ✓

Let $F = \#$ of fails before first pass
$F \sim \text{Geometric}(0.75)$

a) $P(F=4) = (0.25)^4(0.75)$

b) $E(F) = \frac{1-p}{p} = \frac{0.25}{0.75} = \frac{1}{3}$ (Between 0 & 1 tries before first pass)

c) $P(F\leq 2) = P(F=0) + P(F=1) + P(F=2)$

$= 0.75 + (0.25)(0.75) + (0.25)^2(0.75)$