STA 247 - Assignment #1

Name:________________________ Student #:____________________

Due: October 3, 2016 @ 3:15PM - Submit through Crowdmark on Blackboard

Remember to **show all your work** in the assignment. Solutions without justifications will not earn any marks.

The goal of the assignments is to assess whether you can **apply the concepts discussed in lecture**. While there may be different paths to the solution, it is your responsibility to demonstrate in your solutions that you have learned the course material.

Problem 1 [9 marks]. A game involves tossing two fair six-sided dice and observing the sum.

a) Using set notation, write out the sample space of this experiment

b) What is the probability of rolling a sum of 5?

c) What is the probability of rolling a sum of 5 if one of the dice is a 4?

d) What is the probability one of the dice is a 4 if you roll a sum of 5?

e) what is the probability of rolling a sum of 5 or greater?

Problem 2 [3 marks]. How many divisors does $31,752$ have? How many divisors are multiples of 7?

Problem 3 [3 marks]. A local furniture store has a stock consisting of 7 models of desks, 8 models of chairs, 5 models of filing cabinets, and 10 models of bookcases. Your boss has asked you to go and purchase 3 desks, 2 chairs, 2 filing cabinets, and 4 bookcases.

a) In how many ways can you make this purchase if your boss does not want to select more than one of any model?

b) There is a particular bookcase and desk that your boss really likes. What is the probability that those two pieces were selected during your random selection?

Problem 4 [9 marks]. Suppose events A, B, C have the following probabilities: $P(A|B) = 0.25, P(C|B) = 0.5, P(A \cap C|B) = 0.10$. Given that B has occurred,

a) Find the probability that only C has occurred.

b) Find the probability that only A or only C has occurred, but not both.

c) Find the probability that A or C has occurred.

d) Represent all the probabilities in an appropriately labeled Venn diagram

Problem 5 [3 marks]. A closet has 8 pairs of shoes. If you randomly select 4 shoes, what is the probability that you will have exactly 1 complete pair?

Problem 6 [9 marks]. A manufacturer of circuit boards will test all of their products via a verification test. Unfortunately, the test is not 100% accurate. For every 50 defective circuit boards, the test will pass 3 of them. For every 50 functional boards, the test will pass 49 of them. The manufacturer also knows that they tend to have a 4% defective rate. In one day, the manufacturer produced 890 circuit boards of which 875 passed. A properly labeled tree diagram might be helpful here.

a) Using probability notation (and appropriate definitions of variables), write down the effectiveness of the verification test. i.e./ How likely is the test to identify a defective circuit board when it's defective, and vice versa if the circuit board is functional.

b) Calculate the probability that the verification test will pass a circuit board.

c) Of the 875 circuit boards that passed, how many would you expect to be defective? (Hint: If it passes, what are the chances it's defective?)

Problem 7 [10 marks]. A TV show has a weekly game where 1 audience member is selected to participate. The participant must choose 3 numbers from $\{1, 2, 3, ..., 15\}$, with replacement. You can assume order matters here.

a) What is the sample space? *Hint: You don't need to list out every single element, but you should be able to notate it in a compact form.*

b) What is the probability that all three numbers are different?

c) What is the probability that all three numbers are the same?

d) What is the probability that only two of the numbers are the same?

e) What is the probability that the three numbers sum to 15? *Hint: Find a way to apply the sticks and stones method here.*

Problem 8 [6 marks]. Verify that the probability mass function of a binomial random variable $X \sim Bin(n, p)$ where $n > 1$ is a valid PMF.

Problem 9 [6 marks]. Use the law of total probability to prove the following:

a) If $P(A|B) = P(A|B^c)$, then A and B are independent.

b) If $P(A|C) > P(B|C) and P(A|C^c) > P(B|C^c)$, then $P(A) > P(B)$.