CI : y +- s t_{1-alpha/2, n-2}

$$S_{xx} = \sum_{i=1}^{n}(x_i-\bar{x})^2, \qquad S_{xy} = \sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}), \qquad \hat{\beta}_0 = \bar{y}-\hat{\beta}_1\bar{x}, \qquad \hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\,\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$

$$\mathrm{var}(\hat{\beta}_0) = \sigma^2\left[\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right], \qquad \mathrm{var}(\hat{\beta}_1) = \frac{\sigma^2}{S_{xx}}, \qquad \mathrm{cov}(\hat{\beta}_0,\,\hat{\beta}_1) = -\frac{\sigma^2\bar{x}}{S_{xx}}$$

$$\underbrace{\sum_{i=1}^{n}(y_i-\bar{y})^2}_{\mathrm{SST}} = \underbrace{b_1^2\sum_{i=1}^{n}(x_i-\bar{x})^2}_{\mathrm{SSReg}} + \underbrace{\sum_{i=1}^{n}\hat{e}_i^2}_{\mathrm{RSS}}, \qquad \mathrm{SSReg} = \sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2, \qquad F_{\mathrm{obs}} = \frac{\mathrm{MSReg}}{\mathrm{MSE}}$$

$$\mathrm{var}(\hat{y}^*) = \sigma^2\left[\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right], \qquad \mathrm{var}(Y^*-\hat{y}^*) = \sigma^2\left[1+\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right]$$

Sxy estimates Cov(x,y) Sxx estimates Var(x)

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}, \qquad \hat{y}_i = \sum_{j=1}^{n} h_{ij}\,y_k \qquad h_{ij} = \frac{1}{n}+\frac{(x_i-\bar{x})(x_j-\bar{x})}{S_{xx}}$$

$$\mathrm{DFBETA}_{ik} = \frac{\hat{\beta}_k-\hat{\beta}_{k(i)}}{\text{s.e. of }\hat{\beta}_{k(i)}}, \qquad \mathrm{DFFITS}_i = \frac{\hat{y}_i-\hat{y}_{i(i)}}{\text{s.e. of }\hat{y}_{i(i)}}, \qquad D_i = \frac{\sum_{j=1}(\hat{y}_{j(i)}-\hat{y}_j)^2}{2S^2} = \frac{r_i^2 h_{ii}}{2(1-h_{ii})}$$

residual standard error = s

$r_i = \frac{\hat{e}_i}{S\sqrt{1-h_{ii}}}$ where $S^2 = \mathrm{MSE} = \frac{\mathrm{RSS}}{n-p-1}$ Criteria for ordinary data points on small datasets: $|r_i| < 2$, not outlier

estimated var(e_hat)

$h_{ii} < 2(p+1)/n$, DFBETA $< 2/\sqrt{n}$, DFFITS $< 2\sqrt{\frac{p+1}{n}}$, $D_i < 4/(n-p-1)$

not leverage

Transformations: $f'(\mu) \propto \dfrac{1}{\sqrt{V(\mu)}}$ Covariance matrix: $\mathrm{var}(\mathbf{X}) = \mathrm{E}\left[(\mathbf{X}-\mathrm{E}(\mathbf{X})\,(\mathbf{X}-\mathrm{E}(\mathbf{X}))'\right]$

used to solve for least squared estimator in matrix forms

For $f(\boldsymbol{\theta}) = \mathbf{c}'\boldsymbol{\theta}$, $\dfrac{\partial f(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} = \mathbf{c}$ For $f(\boldsymbol{\theta}) = \boldsymbol{\theta}'\mathbf{A}\boldsymbol{\theta}$, $\dfrac{\partial f(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} = 2\mathbf{A}\boldsymbol{\theta}$

A symmetric

$$\mathrm{RSS}(\boldsymbol{\beta}) = \sum_{i=1}^{n}(y_i-\beta_0-\beta_1 x_i)^2 = (\mathbf{Y}-\mathbf{X}\boldsymbol{\beta})'(\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}) \qquad \hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \quad \text{and} \quad \mathrm{var}\left(\hat{\boldsymbol{\beta}}\right) = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$$

has to be invertible

$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} n & n\bar{x} \\ n\bar{x} & \sum x_i^2 \end{pmatrix} \quad \text{and} \quad (\mathbf{X}'\mathbf{X})^{-1} = \frac{1}{S_{xx}}\begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n} x_i^2 & -\bar{x} \\ -\bar{x} & 1 \end{pmatrix}$$

$$\hat{\mathbf{e}} = (\mathbf{I}-\mathbf{H})\,\mathbf{Y} \quad \text{where} \quad \mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \qquad \mathrm{var}(\hat{\mathbf{e}}) = (\mathbf{I}-\mathbf{H})\,\mathrm{E}(\mathbf{Y}\mathbf{Y}')\,(\mathbf{I}-\mathbf{H})$$

= Var(Y) = \sigma^2 (I - H)

indemp -> trace = rank
indemp <=> rank(A) + rank(I-A) = n
indemp all A=B+C-> rank(A) = rank(B) + rank(C)

$$\underbrace{\mathbf{Y}'\,(\mathbf{I}-\frac{1}{n}\mathbf{J})\,\mathbf{Y}}_{\substack{\mathrm{SST}\\ \text{df = n-1}}} = \underbrace{\mathbf{Y}'\left(\mathbf{H}-\frac{1}{n}\mathbf{J}\right)\mathbf{Y}}_{\substack{\mathrm{SSReg}\\ \text{df = (p+1) - 1 = p}}} + \underbrace{\mathbf{Y}'\,(\mathbf{I}-\mathbf{H})\mathbf{Y}}_{\substack{\mathrm{RSS}\\ \text{n - p - 1}}}$$

$$\text{Adj } R^2 = 1-\frac{n-1}{n-p-1}\frac{\mathrm{RSS}}{\mathrm{SST}} \qquad \text{Partial } F\text{-test}: F = \frac{(\mathrm{RSS(reduced)}-\mathrm{RSS(full)})\,/\,\overbrace{(\mathrm{df}_{\mathrm{reduced}}-\mathrm{df}_{\mathrm{full}})}^{k}}{\mathrm{RSS(full)}\,/\,\underbrace{\mathrm{df}_{\mathrm{full}}}_{n-p-1}}$$

$$\mathrm{VIF}_j = \frac{1}{1-R_j^2} \qquad r_{\mathrm{partial}}(X_1,X_2) = r(\hat{e}_{X_1 \text{ vs } Z},\,\hat{e}_{X_2 \text{ vs } Z}) \qquad \mathrm{WRSS} = \sum_{i=1}^{n} w_i\,(y_i-\hat{y}_i)^2 \text{ for } w_i \propto 1/\sigma_i^2$$

how much variance of beta estimate increased because of collinearity, R^2_j is regressing x_j on all other x'sf

$$\hat{\beta}_{1W} = \frac{\sum_{i=1}^{n} w_i\,(x_i-\bar{x}_W)\,(y_i-\bar{y}_W)}{\sum_{i=1}^{n} w_i\,(x_i-\bar{x}_W)^2} \qquad \hat{\beta}_{0W} = \bar{y}_W-\hat{\beta}_{1W}\,\bar{x}_W$$

$$\begin{pmatrix}\hat{\beta}_{0W}\\ \hat{\beta}_{1W}\end{pmatrix} = \hat{\boldsymbol{\beta}}_W = \left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y} \quad \text{and var} \quad (\hat{\boldsymbol{\beta}}_W) = \sigma^2\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}$$