1. Suppose we have a relation on attributes A, B, C, D, E, and F , and these functional dependencies hold: $S = \{B \to DE, BF \to C, CF \to B, DF \to AE\}$.

   Solution. □

   (a)
   $$B^+ = BDE$$

   (b)
   $$CF^+ = ABCDEF$$

   (c)
   $$DF^+ = ADEF$$

   (d)
   $$BC^+ = BCDE$$

   (e)
   $$ABC^+ = ABCDE$$

2. Again,suppose we have a relation on attributes A, B, C, D, E, and F,and these functional dependencies hold : $S = \{B \to DE, BF \to C, CF \to B, DF \to AE\}$.

   Solution. □

   (a) Does it follow from S that $B \to A$?
   No, because A not in B^+ from Q1

   (b) Does it follow from S that $CF \to E$?
   Yes, because E in CF^+ from Q1

   (c) Does it follow from S that $DF \to B$?
   No, because B not in DF^+ from Q1

   (d) Does it follow from S that $BD \to C$?

   $$BD^+ = BDE$$

   So No, C not in BD^+

   (e) Does it follow from S that $BFC \to A$?

   $$BFC^+ = ABCDEF$$

   Yes, since A in BFC^+

3. Suppose we have a relation with attributes ABCDE and these functional dependencies $S = \{A \to D, B \to A, C \to A, D \to CE\}$ Project functional dependencies onto ABD

Solution. □

$$A^+ = ACDE \text{ so } A \to D$$
$$B^+ = ABCDE \text{ so } B \to AD$$
$$D^+ = ACDE \text{ so } D \to A$$

Dont have to consider superset of B, since it functionally determines all of ABD.

$$AD^+ = ACDE$$

which does not yield new FDs so the projection of S onto ABD is

$$\{A \to D, B \to AD, D \to A\}$$

4. Consider relation $R(A, B, C, D, E, F)$ with functional dependencies

$$S = \{CD \to A, B \to EF, A \to BC, F \to D\}$$

Create an instance of R that satisfies its FDs and has redundant data. Identify redundancy by circling a single value in the table that could be erased and yet we would know what its value must be. (what does it have to do with FDs)?

Solution. □

A	B	C	D	E	F
3	1	4	1	5	6
3	1	4	1	5	6

Note the cell is colored instead of circled due to technical difficulty with latex. We can deduce the minimal basis for the set of FDs.