# CSC321 Lecture 21: Policy Gradient

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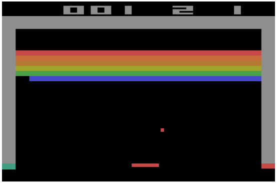
# Overview

- Most of this course was about supervised learning, plus a little unsupervised learning.
- Final 3 lectures: reinforcement learning
  - Middle ground between supervised and unsupervised learning
  - An agent acts in an environment and receives a reward signal.
- Today: policy gradient (directly do SGD over a stochastic policy using trial-and-error)
- Next lecture: Q-learning (learn a value function predicting returns from a state)
- Final lecture: policies and value functions are way more powerful in combination

# Reinforcement learning



- An **agent** interacts with an **environment** (e.g. game of Breakout)
- In each time step $t$,
  - the agent receives **observations** (e.g. pixels) which give it information about the **state** $\mathbf{s}_t$ (e.g. positions of the ball and paddle)
  - the agent picks an **action** $\mathbf{a}_t$ (e.g. keystrokes) which affects the state
- The agent periodically receives a **reward** $r(\mathbf{s}_t, \mathbf{a}_t)$, which depends on the state and action (e.g. points)
- The agent wants to learn a **policy** $\pi_{\boldsymbol{\theta}}(\mathbf{a}_t \mid \mathbf{s}_t)$
  - Distribution over actions depending on the current state and parameters $\boldsymbol{\theta}$

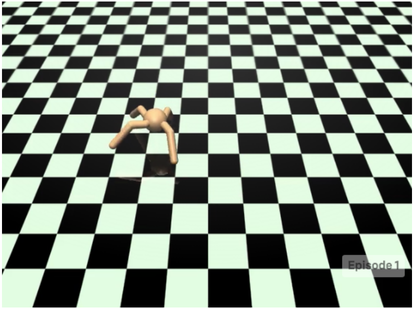
# Markov Decision Processes

- The environment is represented as a Markov decision process $\mathcal{M}$.
- Markov assumption: all relevant information is encapsulated in the current state; i.e. the policy, reward, and transitions are all independent of past states given the current state
- Components of an MDP:
  - initial state distribution $p(\mathbf{s}_0)$
  - policy $\pi_{\boldsymbol{\theta}}(\mathbf{a}_t \mid \mathbf{s}_t)$
  - transition distribution $p(\mathbf{s}_{t+1} \mid \mathbf{s}_t, \mathbf{a}_t)$
  - reward function $r(\mathbf{s}_t, \mathbf{a}_t)$
- Assume a fully observable environment, i.e. $\mathbf{s}_t$ can be observed directly
- Rollout, or trajectory $\tau = (\mathbf{s}_0, \mathbf{a}_0, \mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)$
- Probability of a rollout

$$p(\tau) = p(\mathbf{s}_0)\, \pi_{\boldsymbol{\theta}}(\mathbf{a}_0 \mid \mathbf{s}_0)\, p(\mathbf{s}_1 \mid \mathbf{s}_0, \mathbf{a}_0) \cdots p(\mathbf{s}_T \mid \mathbf{s}_{T-1}, \mathbf{a}_{T-1})\, \pi_{\boldsymbol{\theta}}(\mathbf{a}_T \mid \mathbf{s}_T)$$

# Markov Decision Processes

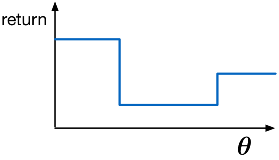
Continuous control in simulation, e.g. teaching an ant to walk



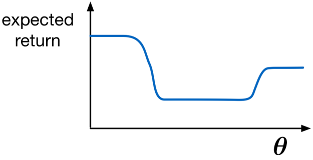
- State: positions, angles, and velocities of the joints
- Actions: apply forces to the joints
- Reward: distance from starting point (policy network) state -> action
- Policy: output of an ordinary MLP, using the state as input
- More environments: `https://gym.openai.com/envs/#mujoco`

# Markov Decision Processes

- Return for a rollout: $r(\tau) = \sum_{t=0}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$
  - Note: we're considering a finite horizon $T$, or number of time steps; we'll consider the infinite horizon case later.
- Goal: maximize the expected return, $R = \mathbb{E}_{p(\tau)}[r(\tau)]$
- The expectation is over both the environment's dynamics and the policy, but we only have control over the policy.
- The stochastic policy is important, since it makes $R$ a continuous function of the policy parameters.
  - Reward functions are often discontinuous, as are the dynamics (e.g. collisions)



deterministic policies



stochastic policies

# REINFORCE

- REINFORCE is an elegant algorithm for maximizing the expected return $R = \mathbb{E}_{p(\tau)}[r(\tau)]$.
- Intuition: trial and error
  - Sample a rollout $\tau$. If you get a high reward, try to make it more likely. If you get a low reward, try to make it less likely.
- Interestingly, this can be seen as stochastic gradient ascent on $R$.

# REINFORCE

- Recall the derivative formula for log:

$$\frac{\partial}{\partial \boldsymbol{\theta}} \log p(\tau) = \frac{\frac{\partial}{\partial \boldsymbol{\theta}} p(\tau)}{p(\tau)} \qquad \Longrightarrow \qquad \frac{\partial}{\partial \boldsymbol{\theta}} p(\tau) = p(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log p(\tau)$$

- Gradient of the expected return:

$$\begin{aligned}
\frac{\partial}{\partial \boldsymbol{\theta}} \mathbb{E}_{p(\tau)}\left[r(\tau)\right] &= \frac{\partial}{\partial \boldsymbol{\theta}} \sum_{\tau} r(\tau) p(\tau) \\
&= \sum_{\tau} r(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} p(\tau) \\
&= \sum_{\tau} r(\tau) p(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log p(\tau) \\
&= \mathbb{E}_{p(\tau)}\left[ r(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log p(\tau) \right]
\end{aligned}$$

- Compute stochastic estimates of this expectation by sampling rollouts.

# REINFORCE

- For reference:

$$\frac{\partial}{\partial \boldsymbol{\theta}} \mathbb{E}_{p(\tau)}\left[r(\tau)\right] = \mathbb{E}_{p(\tau)}\left[r(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log p(\tau)\right]$$

- If you get a large reward, make the rollout more likely. If you get a small reward, make it less likely.
- Unpacking the REINFORCE gradient:

only pi matters here

$$\begin{aligned}
\frac{\partial}{\partial \boldsymbol{\theta}} \log p(\tau) &= \frac{\partial}{\partial \boldsymbol{\theta}} \log \left[p(\mathbf{s}_0) \prod_{t=0}^{T} \pi_{\boldsymbol{\theta}}(\mathbf{a}_t \,|\, \mathbf{s}_t) \prod_{t=1}^{T} p(\mathbf{s}_t \,|\, \mathbf{s}_{t-1}, \mathbf{a}_{t-1})\right] \\
&= \frac{\partial}{\partial \boldsymbol{\theta}} \log \prod_{t=0}^{T} \pi_{\boldsymbol{\theta}}(\mathbf{a}_t \,|\, \mathbf{s}_t) \\
&= \sum_{t=0}^{T} \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t \,|\, \mathbf{s}_t)
\end{aligned}$$

- Hence, it tries to make *all* the actions more likely or less likely, depending on the reward. I.e., it doesn't do credit assignment.
  - This is a topic for next lecture.

# REINFORCE

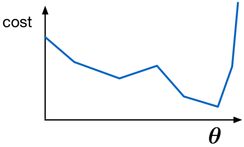
```
Repeat forever:
    Sample a rollout $\tau = (\mathbf{s}_0, \mathbf{a}_0, \mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)$ from the distribution
    $r(\tau) \leftarrow \sum_{k=0}^{T} r(\mathbf{s}_k, \mathbf{a}_k)$
    For $t = 0, \ldots, T$:
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha r(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_k \mid \mathbf{s}_k)$
```

- Observation: actions should only be reinforced based on future rewards, since they can't possibly influence past rewards.
- You can show that this still gives unbiased gradient estimates.

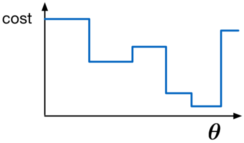
```
Repeat forever:
    Sample a rollout $\tau = (\mathbf{s}_0, \mathbf{a}_0, \mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T)$
    For $t = 0, \ldots, T$:
        $r_t(\tau) \leftarrow \sum_{k=t}^{T} r(\mathbf{s}_k, \mathbf{a}_k)$
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha r_t(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_k \mid \mathbf{s}_k)$
```

# Optimizing Discontinuous Objectives

- Edge case of RL: handwritten digit classification, but maximizing accuracy (or minimizing 0–1 loss)
- Gradient descent completely fails if the cost function is discontinuous:



Non-differentiable: OK



Discontinuous: not OK

- Original solution: use a surrogate loss function, e.g. logistic-cross-entropy
- RL formulation: in each episode, the agent is shown an image, guesses a digit class, and receives a reward of 1 if it's right or 0 if it's wrong
- We'd never actually do it this way, but it will give us an interesting comparison with backprop

# Optimizing Discontinuous Objectives

- RL formulation
  - one time step
  - state $\mathbf{x}$: an image
  - action $\mathbf{a}$: a digit class
  - reward $r(\mathbf{x}, \mathbf{a})$: 1 if correct, 0 if wrong
  - policy $\pi(\mathbf{a} \,|\, \mathbf{x})$: a distribution over categories
    - Compute using an MLP with softmax outputs – this is a policy network

# Optimizing Discontinuous Objectives

- Let $z_k$ denote the logits, $y_k$ denote the softmax output, $t$ the integer target, and $t_k$ the target one-hot representation.
- To apply REINFORCE, we sample $\mathbf{a} \sim \pi_{\boldsymbol{\theta}}(\cdot \mid \mathbf{x})$ and apply:

$$\begin{aligned} \boldsymbol{\theta} &\leftarrow \boldsymbol{\theta} + \alpha r(\mathbf{a}, \mathbf{t}) \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a} \mid \mathbf{x}) \\ &= \boldsymbol{\theta} + \alpha r(\mathbf{a}, \mathbf{t}) \frac{\partial}{\partial \boldsymbol{\theta}} \log y_a \\ &= \boldsymbol{\theta} + \alpha r(\mathbf{a}, \mathbf{t}) \sum_k (a_k - y_k) \frac{\partial}{\partial \boldsymbol{\theta}} z_k \end{aligned}$$

- Compare with the logistic regression SGD update:

$$\begin{aligned} \boldsymbol{\theta} &\leftarrow \boldsymbol{\theta} + \alpha \frac{\partial}{\partial \boldsymbol{\theta}} \log y_t \\ &\leftarrow \boldsymbol{\theta} + \alpha \sum_k (t_k - y_k) \frac{\partial}{\partial \boldsymbol{\theta}} z_k \end{aligned}$$

# Reward Baselines

- For reference:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha r(\mathbf{a}, \mathbf{t}) \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a} \,|\, \mathbf{x})$$

- Clearly, we can add a constant offset to the reward, and we get an equivalent optimization problem.
- Behavior if $r = 0$ for wrong answers and $r = 1$ for correct answers
  - wrong: do nothing
  - correct: make the action more likely
- If $r = 10$ for wrong answers and $r = 11$ for correct answers
  - wrong: make the action more likely
  - correct: make the action more likely (slightly stronger)
- If $r = -10$ for wrong answers and $r = -9$ for correct answers
  - wrong: make the action less likely
  - correct: make the action less likely (slightly weaker)

# Reward Baselines

- Problem: the REINFORCE update depends on arbitrary constant factors added to the reward.
- Observation: we can subtract a baseline $b$ from the reward without biasing the gradient.

$$\begin{aligned}
\mathbb{E}_{p(\tau)}\left[(r(\tau)-b)\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau)\right] &= \mathbb{E}_{p(\tau)}\left[r(\tau)\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau)\right] - b\mathbb{E}_{p(\tau)}\left[\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau)\right] \\
&= \mathbb{E}_{p(\tau)}\left[r(\tau)\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau)\right] - b\sum_{\tau} p(\tau)\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau) \\
&= \mathbb{E}_{p(\tau)}\left[r(\tau)\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau)\right] - b\sum_{\tau}\frac{\partial}{\partial\boldsymbol{\theta}}p(\tau) \\
&= \mathbb{E}_{p(\tau)}\left[r(\tau)\frac{\partial}{\partial\boldsymbol{\theta}}\log p(\tau)\right] - 0
\end{aligned}$$

still unbiased gradient estimator

- We'd like to pick a baseline such that good rewards are positive and bad ones are negative.
- $\mathbb{E}[r(\tau)]$ is a good choice of baseline, but we can't always compute it easily. There's lots of research on trying to approximate it.

# More Tricks

- We left out some more tricks that can make policy gradients work a lot better.
  - Evaluate each action using only future rewards, since it has no influence on past rewards. It can be shown this gives unbiased gradients.
  - Natural policy gradient corrects for the geometry of the space of policies, preventing the policy from changing too quickly.
  - Rather than use the actual return, evaluate actions based on estimates of future returns. This is a class of methods known as actor-critic, which we'll touch upon next lecture.
- Trust region policy optimization (TRPO) and proximal policy optimization (PPO) are modern policy gradient algorithms which are very effective for continuous control problems.

# Discussion

- What's so great about backprop and gradient descent?
    - Backprop does credit assignment – it tells you exactly which activations and parameters should be adjusted upwards or downwards to decrease the loss on some training example.
    - REINFORCE doesn't do credit assignment. If a rollout happens to be good, all the actions get reinforced, even if some of them were bad.
    - Reinforcing all the actions as a group leads to random walk behavior.

# Discussion

- Why policy gradient?
  - Can handle discontinuous cost functions
  - Don't need an explicit model of the environment, i.e. rewards and dynamics are treated as black boxes
    - Policy gradient is an example of model-free reinforcement learning, since the agent doesn't try to fit a model of the environment
    - Almost everyone thinks model-based approaches are needed for AI, but nobody has a clue how to get it to work

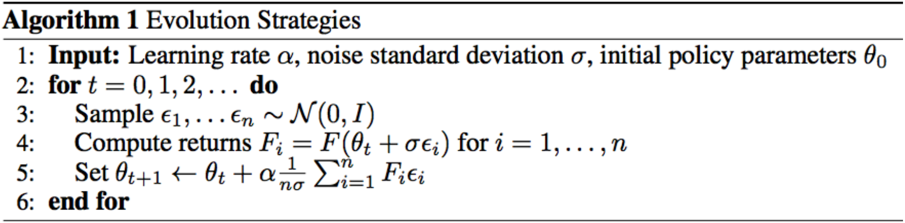
# Evolution Strategies (optional)

- REINFORCE can handle discontinuous dynamics and reward functions, but it requires a differentiable network since it computes $\frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t \mid \mathbf{s}_t)$
- Evolution strategies (ES) take the policy gradient idea a step further, and avoid backprop entirely.
- ES can use deterministic policies. It randomizes over the choice of policy rather than over the choice of actions.
    - I.e., sample a random policy from a distribution $p_{\boldsymbol{\eta}}(\boldsymbol{\theta})$ parameterized by $\boldsymbol{\eta}$ and apply the policy gradient trick

$$\frac{\partial}{\partial \boldsymbol{\eta}} \mathbb{E}_{\boldsymbol{\theta} \sim p_{\boldsymbol{\eta}}} \left[ r(\tau(\boldsymbol{\theta})) \right] = \mathbb{E}_{\boldsymbol{\theta} \sim p_{\boldsymbol{\eta}}} \left[ r(\tau(\boldsymbol{\theta})) \frac{\partial}{\partial \boldsymbol{\eta}} \log p_{\boldsymbol{\eta}}(\boldsymbol{\theta}) \right]$$

- The neural net architecture itself can be discontinuous.

# Evolution Strategies (optional)

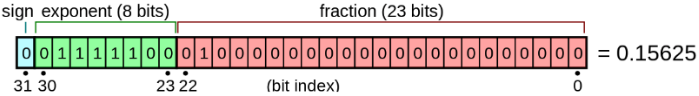


```
Algorithm 1 Evolution Strategies
Input: Learning rate $\alpha$, noise standard deviation $\sigma$, initial policy parameters $\theta_0$
for $t = 0, 1, 2, \dots$ do
    Sample $\epsilon_1, \dots \epsilon_n \sim \mathcal{N}(0, I)$
    Compute returns $F_i = F(\theta_t + \sigma\epsilon_i)$ for $i = 1, \dots, n$
    Set $\theta_{t+1} \leftarrow \theta_t + \alpha \frac{1}{n\sigma} \sum_{i=1}^{n} F_i \epsilon_i$
end for
```

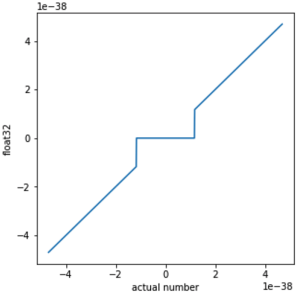
https://arxiv.org/pdf/1703.03864.pdf

# Evolution Strategies (optional)

- The IEEE floating point standard is nonlinear, since small enough numbers get truncated to zero.



- This acts as a discontinuous activation function, which ES is able to handle.
- ES was able to train a good MNIST classifier using a “linear” activation function.
- `https://blog.openai.com/nonlinear-computation-in-linear-`


1e−38
float32
actual number
1e−38