Chapter 4 Straight-Line Regression Based on Weighted Least Squares

1. **Strait linear regression model**

$$Y_i = \beta_0 + \beta_1 x_i + e_i$$

where $var\{e_i\} = \dfrac{\sigma^2}{w_i}$. When w_i is large then variance close to 0, the estimates of regression parameter should be such that the fitted line at x_i be very close to y_i. Conversely, if w_i is very small, then variance of e_i is large, in which case estimates of regression parameters should take little account of (x_i, y_i). We want to minimize weighted version of residual sum of squares

$$WRSS = \sum_i w_i(y_i - \hat{y}_{w_i})^2 = \sum_i w_i(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

the larger w_i for i-th sample (x_i, y_i), the more its taken into account. Minimizing WRSS yields **weighted least squares estimators**

$$\hat{\beta}_{1W} = \frac{\sum_i w_i(x_i - \overline{x}_W)(y_i - \overline{y}_W)}{\sum_i w_i(x_i - \overline{x}_W)^2} \qquad \hat{\beta}_{0W} = \overline{y}_W - \hat{\beta}_{1W}\overline{x}_W$$

$$\overline{x}_W = \frac{\sum_i w_i x_i}{\sum_i w_i} \quad \overline{y}_W = \frac{\sum_i w_i y_i}{\sum_i w_i}$$