UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2011 EXAMINATIONS

CSC320H1S : Introduction to Visual Computing

Duration: 2 hours

No aids allowed

There are 9 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

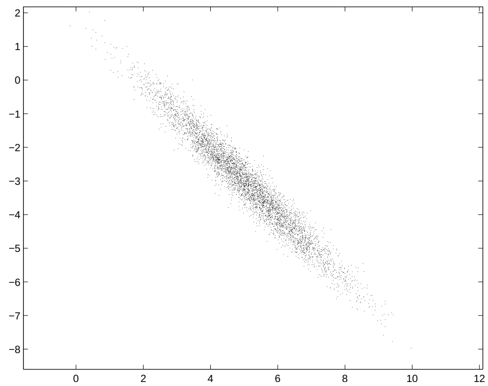
Question	Marks
1	__________/32
2	__________/15
3	__________/20
4	__________/18
5	__________/20
6	__________/15
Total	_________/120

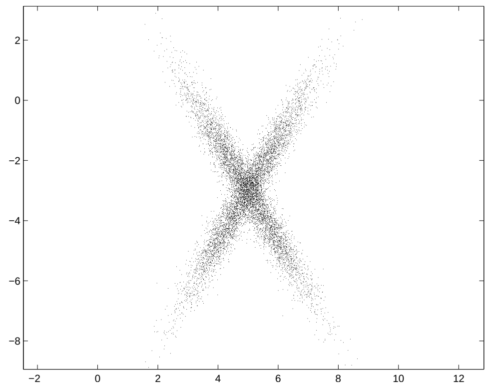
1 PCA & Eigenfaces (32 marks total)

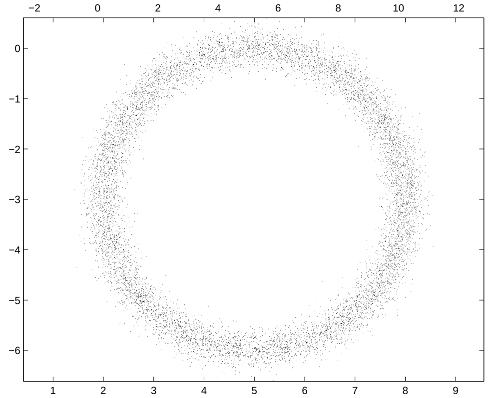
Let $I_1, \ldots, I_n$ be a set of face images, represented as $M \times 1$ column vectors.

(a) **[15 Marks]** Give the main steps of the algorithm for computing the eigenfaces of $I_1, \ldots, I_n$. Be as specific as possible, showing the relevant equations.

(b) **[12 Marks]** Suppose that $M = 2$. In this case, we can represent the n 2-pixel images with a scatter plot, where each point in the scatter plot corresponds to one of the images. For each of the three scatter plots below, indicate the location of the mean image and draw the corresponding principal components.







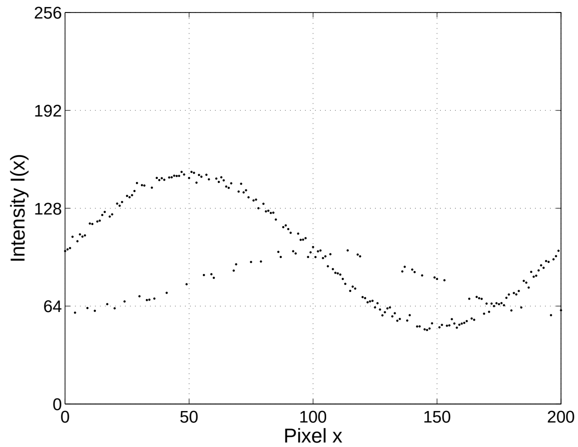
(c) **[5 Marks]** Are the principal components of a set of images always unique? Explain briefly.

2 Robust Estimation (15 marks total)

Suppose you are given a 1D image I containing n pixels that is generated by randomly "interleaving" two other n-pixel images, I_1 and I_2:

$$I(x) = \begin{cases} I_1(x), & \text{or} \\ I_2(x) \,. \end{cases}$$

An example is shown below (in this example, I_1 is shaped like a sinusoid):



The images I_1 and I_2 are *not* known and neither is the way by which they were interleaved. Moreover, since the interleaving was random, there is no fixed interleaving pattern. You are told, however, that in any window of size $w > 10$, approximately 70% of the pixels come from image I_1 and the rest come from I_2.

Show how to estimate the derivative of I in a way that is completely unaffected by the pixels coming from image I_2. That is, your derivative estimate should be (almost) identical to the estimate that you would have computed if $I(x)$ were equal to $I_1(x)$ for *every* pixel x. If your method requires any additional assumptions, be sure to state them.

3 Image Reconstruction (20 marks total)

(a) **[10 Marks]** Let $I = [I_1, \ldots, I_n]$ be a 1D grayscale image and let $D = [D_1, \ldots, D_n]$ be the second derivative of I. Suppose you are given the four pixel intensities I_1, I_2, I_{n-1}, I_n and the second derivative values $D_3, \ldots, D_{n-2}$. Show how to compute the rest of I's intensities from this information.

(b) **[10 Marks]** Now suppose you are given a 2D filter mask M of size $m \times m$ and the result, $J * M$, of convolving M with an unknown $n \times n$ image J. You are now asked to "invert" the convolution process, i.e., to compute the unknown image J from the mask M and the convolution result, $J * M$. This procedure is called *deconvolution*.

How can we determine whether or not deconvolution is possible for a given mask M and image size $n \times n$? Be as concise as possible. You should assume that J and $J * M$ have the same size and that J is zero "outside" the image boundaries, i.e., $J(r, c) = 0$ if $r <= 0, c <= 0, r > n$ or $c > n$. ***Hint:*** *It might be useful to treat image J as an n^2-dimensional vector.*

4 SIFT (18 marks total)

Give the main steps that the SIFT algorithm uses to *locate* keypoints in an image. Be as specific as possible, and focus only on keypoint detection and localization (*i.e.*, do not discuss the definition and creation of keypoint descriptors, or how these descriptors are matched between images).

5 Multi-Scale Representations (20 marks total)

(a) **[10 Marks]** Prove that

$$\frac{d}{dx}\left[I * G_\sigma\right](x) = \left[I * \frac{d}{dx}G_\sigma\right](x)$$

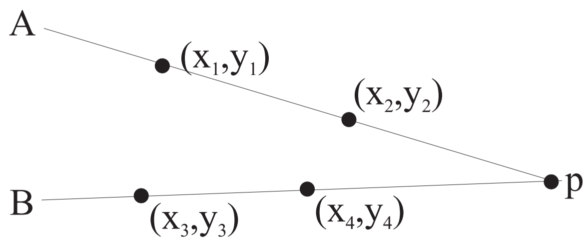
where $I = [\ I(1) \ \ \ldots \ \ I(n)\ ]$ is a 1D image containing n pixels and $G_\sigma(x)$ is the (continuous) 1D Gaussian function of standard deviation σ.

(b) **[10 Marks]** Compute the wavelet transform of the following 1D image:

6	8	5	1	1	1	4	6

6 Homogeneous Coordinates (15 marks total)

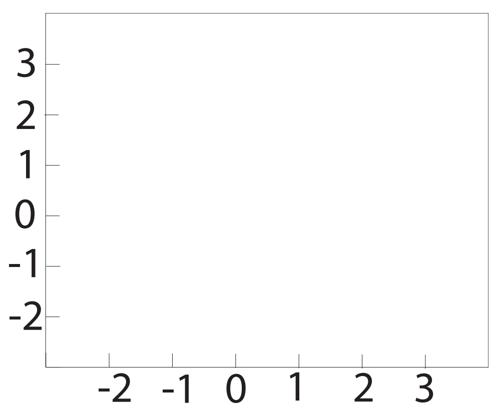
(a) **[7 Marks]** Give a single formula that expresses the *homogeneous coordinates* of the intersection of lines A and B in terms of the 2D coordinates of points $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$.



$p\ =$

(b) **[8 Marks]** Indicate on the plot below the 2D location of points $p_1, \ldots, p_4$:

$$p_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix},\ p_2 = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix},\ p_3 = \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix},\ p_4 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$



END OF EXAM