UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2009 EXAMINATIONS

CSC320H1S : Introduction to Visual Computing

Duration: 2 hours

No aids allowed

There are 12 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

Question	Marks
1	___________/15
2	___________/8
3	___________/15
4	___________/22
5	___________/20
6	___________/25
7	___________/15
Total	__________/120

1 Matting (15 marks total)

(a) **[5 Marks]** State the matting equation and define all necessary terms.

(b) **[5 Marks]** Suppose you are given a grayscale composite image C that satisfies the matting equation. Express the gradient of C in terms of the foreground, the background, the alpha matte, and their gradients:

$\nabla C \ =$

(c) **[5 Marks]** Now you are told that the pixel intensities of the foreground and background in (b) are nearly constant but that their actual values are unknown. What information about the gradient of the alpha matte can we obtain from the composite C in this case? Be as specific as possible.

2 Camera Response Function (8 marks total)

Define the camera's response function.

3 Principal Directions & Principal Curvatures (15 marks total)

Suppose that the intensities in the neighborhood of the central pixel, $(0, 0)$, of a 2D patch I are well-approximated by the polynomial

$$I(x, y) = x^2 + y^2 + 5xy^3 + 2xy \ .$$

Compute the principal directions and principal curvatures of I at pixel $(0, 0)$.

4 Image Reconstruction (22 marks total)

(a) **[10 Marks]** Let $I = [I_1, \ldots, I_n]$ be a 1D grayscale image and let $D = [D_1, \ldots, D_n]$ be the second derivative of I. Suppose you are given the four pixel intensities I_1, I_2, I_{n-1}, I_n and the second derivative values $D_3, \ldots, D_{n-2}$. Show how to compute the rest of I's intensities from this information.

(b) **[12 Marks]** Now suppose you are given a 2D filter mask M of size $m \times m$ and the result, $I * M$, of convolving M with an unknown $n \times n$ image I. You are now asked to "invert" the convolution process, i.e., to compute the unknown image I from the mask M and the convolution result, $I * M$. This procedure is called *deconvolution*.

How can we determine whether or not deconvolution is possible for a given mask M and image size $n \times n$? Be as concise as possible. You should assume that I and $I * M$ have the same size and that I is zero "outside" the image boundaries, i.e., $I(r, c) = 0$ if $r <= 0, c <= 0, r > n$ or $c > n$.

5 PCA (20 marks total)

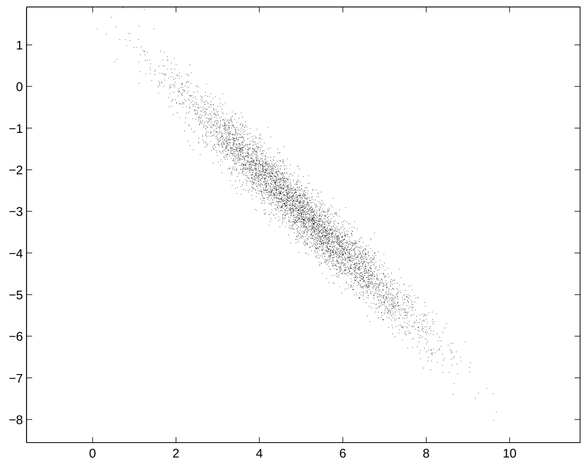
Let $\mathbf{X} = [\mathbf{X}_1 \;\; \mathbf{X}_2 \;\; \ldots \;\; \mathbf{X}_N]$ be a matrix of M-dimensional column vectors. Moreover, let $\mathbf{X}^j$ be the row vector corresponding to the j^{th} row of matrix $\mathbf{X}$.

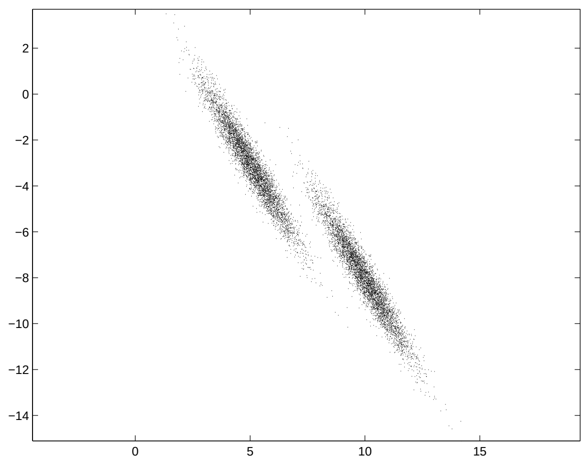
(a) **[5 Marks]** Define the *covariance* of vectors $\mathbf{X}^j, \mathbf{X}^k$ using vector notation:

$$\mathrm{cov}(\mathbf{X}^j, \mathbf{X}^k) \;=$$

(b) **[10 Marks]** Describe how to compute the principal components of $\mathbf{X}_1, \ldots, \mathbf{X}_N$ from matrix $\mathbf{X}$.

(c) **[5 Marks]** Suppose that $N = 2$. In this case, we can represent matrix $\mathbf{X}$ with a scatter plot, where each point in the scatter plot corresponds to a specific column of $\mathbf{X}$. For each of the two plots below, draw the corresponding principal components.





6 Multiscale Image Representations (25 marks total)

(a) **[5 Marks]** Prove that

$$\frac{d^2}{dx^2}\left[I * G_\sigma\right](x) = \left[I * \frac{d^2}{dx^2} G_\sigma\right](x)$$

where $I = [I_1, \ldots, I_n]$ is a 1D image containing n pixels and $G_\sigma(x)$ is the (continuous) 1D Gaussian function of standard deviation σ.

(b) **[10 Marks]** Compute the Haar wavelet transform of the following 1D image:

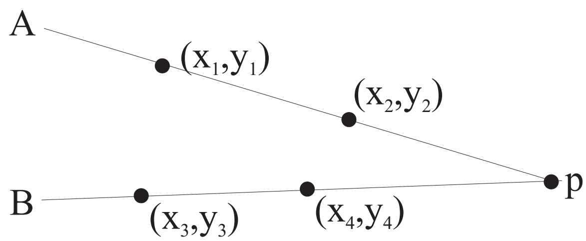
16	12	11	9	4	4	17	7

(c) **[10 Marks]** Compute the image whose Haar wavelet transform is given by the following vector:

13	7	6	5	4	3	2	1

7 Homogeneous Coordinates (15 marks total)

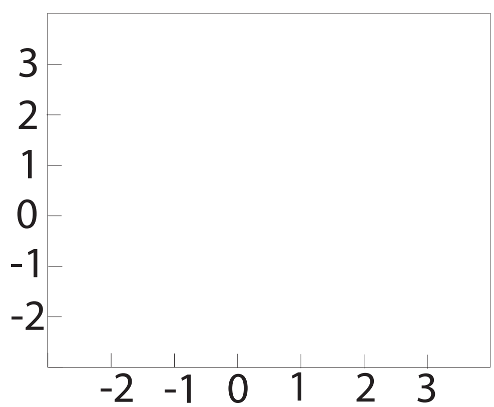
(a) **[7 Marks]** Give a single formula that expresses the *homogeneous coordinates* of the intersection of lines A and B in terms of the 2D coordinates of points $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$.



$p =$

(b) **[8 Marks]** Indicate on the plot below the 2D location of points $p_1, \ldots, p_4$:

$$p_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \quad p_2 = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}, \quad p_3 = \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}, \quad p_4 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$



END OF EXAM