Problems to be marked: #1, 2, 4, 6, 7. Set all other problems to have a mark of $\emptyset$.

Solutions

Problem 1

a) $\Omega = \{2,3,4,5,6,7,8,9,10,11,12\}$ 1

↳ sample space should be sums, not outcomes of two dice tosses.

b) sum of 5 $= \{(1,4),(2,3),(3,2),(4,1)\}$ = event A (0.5)

Total possible outcomes $= 6 \times 6 = 36$ (0.5) 2

$P(A = \text{sum of } 5) = \frac{4}{36} = \frac{1}{9}$ (1)

c) $P(\text{sum of } 5 \mid \text{one die} = 4) = \frac{P(\text{sum}=5 \cap \text{one } 4)}{P(\text{one die} = 4)} = \frac{2/36}{11/36} = 2/11$ (0.5) (1) 2

$(n(\text{one die} = 4) = 5 + 6 = 11)$ (0.5)

d) $P(\text{one die} = 4 \mid \text{sum of } 5) = \frac{P(\text{sum}=5 \cap \text{one } 4)}{P(\text{sum}=5)} = \frac{2/36}{4/36} = 1/2$ 1

OR

They could use the restricted sample space as well:

$A = \text{sum of } 5 = \{(1,4), (2,3), (3,2), (4,1)\}$

e) $P(\text{sum} \geq 5) = 30/36 = 5/6$ 2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Total: $\frac{\quad}{9}$

Problem 2

$31{,}752 = 3^4 \cdot 2^3 \cdot 7^2$

Total: $\frac{}{3}$

a) # of divisors $\Rightarrow 5 \times 4 \times 3 = 60$ ✓ 1

b) # of divisors $\div$ by 7 $\Rightarrow 5 \times 4 \times 2 = 40$ ✓ ✓ 2

Problem 4

$\boxed{\begin{array}{l}\text{Given:} \\ P(A|B) = 0.25 \\ P(C|B) = 0.5 \\ P(A\cap C|B) = 0.10\end{array}}$

a) $P(C \cap A^c | B) = ?$

$P(C|B) = P(C\cap A|B) + P(C \cap A^c|B)$ ✓

$0.50 = 0.10 + P(C\cap A^c|B)$ 2

$P(C\cap A^c|B) = 0.40$ ✓

b) $P[(A \cup C^c) \cup (C\cap A^c)|B]$

$= P(A\cup C^c|B) + P(C\cap A^c|B) - P((A\cup C^c)\cap(C\cap A^c)|B)$ ✓ 0

$= P(A|B) - P(A\cap C|B) + 0.40 - 0$

$= 0.25 - 0.10 + 0.4$

$= 0.55$ ✓

$\boxed{\text{OR}}$ 2

$P(A\cup C|B) - P(A\cap C|B)$ ✓

$= P(A|B) + P(C|B) - 2P(A\cap C|B)$

$= 0.25 + 0.5 - 2(0.10)$

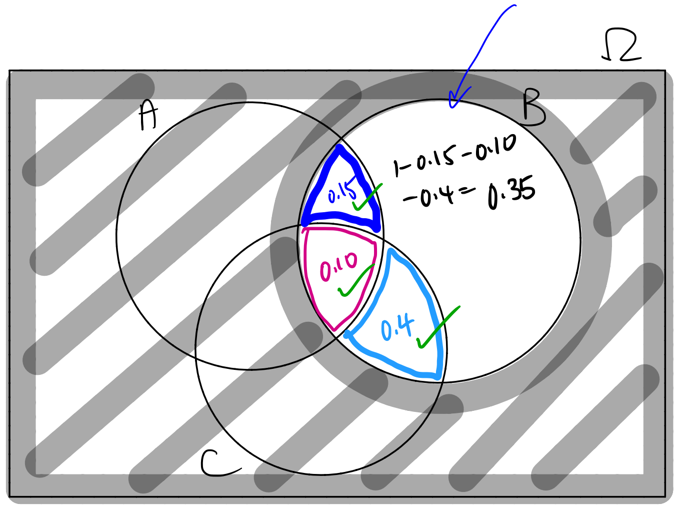
$= 0.55$ ✓

c) $P(A\cup C|B) = P(A|B) + P(C|B) - P(A\cap C|B)$ ✓ 2

$= 0.25 + 0.5 - 0.1$

$= 0.65$ ✓

Restricted sample space – only within B

d) 

3

Total: $\frac{}{9}$

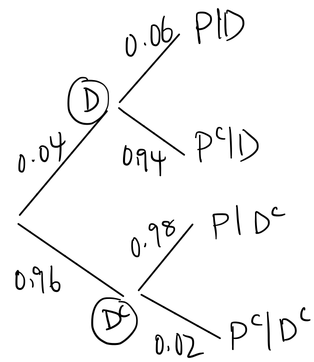
Problem 6

(1) Let D = event board is defective ✓
P = event passes test

students MUST define their random variables / events, you shouldn't have to "guess" at what they mean!

(2) a) $P(P \mid D^c) = 49/50 = 0.98$ ✓ (0.5)

$P(P^c \mid D) = 47/50 = 0.94$ ✓ (0.5)



(3) b) $P(P) = P(P \cap D) + P(P \cap D^c)$ ✓

$= P(P \mid D) \cdot P(D) + P(P \mid D^c) \cdot P(D^c)$

↳ Apply Law of Total Probability

$= 0.06 \times 0.04 + 0.98 \times 0.96$

$= 0.9432$ ✓

∴ 94.32% chance the test will pass a board. ✓

Tree diagram is not required, however, if they did use a tree diagram, verify/correct any mistakes they may have made and make a brief comment.

(4) c) $P(D \mid P) = \dfrac{P(D \cap P)}{P(P)} = \dfrac{P(P \mid D) \cdot P(D)}{P(P)}$ ✓

$= \dfrac{0.06 \times 0.04}{0.9432}$

$= 0.002545$ ✓

Total: $\frac{}{9}$

$n(D \mid P) = P(D \mid P) \times n(P)$

$= 0.002545 \times 875 = 2.23$ ✓

∴ Anticipate there are still 2-3 defective circuit boards in the lot that passed. ✓

↳ They need to make sense / interpret their results in context problems.

Problem 7 - order matters

a) $\Omega = \{ (i,j,k),\ 1 \le i,j,k \le 15,\ i,j,k \in \mathbb{Z} \}$ (0.5) (0.5) 1

↳ If they use another notation that is clear and represents the same sample space is fine. Mark leniently. However, the conditions must be present in all cases.

b) $P(i \ne j \ne k) = \dfrac{15 \times 14 \times 13}{15^3} = \dfrac{182}{225} = 0.8089 \approx 80.89\%$ 2

c) $P(i = j = k) = \dfrac{15 \times 1 \times 1}{15^3} = \dfrac{1}{15^2} = 0.00444 \approx 0.44\%$ 2

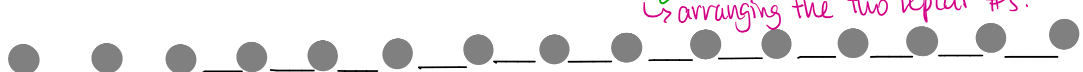
d) $P(2 \text{ are the same}) = 1 - P(\text{all diff}) - P(\text{all same})$

$$= 1 - \frac{182}{225} - \frac{1}{225} = \frac{42}{225} = 0.1867 \approx 18.67\%$$ 2

OR

Direct counting: $P(2 \text{ are the same}) = \dfrac{15 \times 1 \times 14 \times 3!}{15^3 \times 2!} = 0.1867 \approx 18.67\%$ (0.5) (0.5)

↳ arranging the two repeat #s.

e) 

They need to justify their method.

$\#$ of sums of 15 $= {}_{14}C_2 = 91$ 3

$P(i + j + k = 15) = \dfrac{{}_{14}C_2}{15^3} = \dfrac{91}{3375} = 0.02696 \approx 2.70\%$

Total: 10