Problem 3

a) $({}_7C_3)({}_8C_2)({}_5C_2)({}_{10}C_4) = 2,058,000$ 0.5 each

(1)

$\therefore$ 2,058,000 ways to select the furniture

Adjusting choices

b) $\dfrac{({}_6C_2)({}_8C_2)({}_5C_2)({}_9C_3)}{2,058,000} = \dfrac{352,800}{2,058,000} = 0.1714$ 0.5 each

(2)

$\therefore$ 17.14% chance you'll randomly select what your boss really likes.

Total: $\overline{3}$

Problem 5

16 shoes to choose from

Guarantee mismatch

$\dfrac{16 \times 1 \times 14 \times 12}{4!} \times 3! = 672$ ✓ $\Rightarrow$ all ways to match 1 pair

↑ Remove order

0.5 each

$P(1 \text{ matching pair}) = \dfrac{672}{{}_{16}C_4}$ ✓ $= \dfrac{672}{1820} = 0.3692$

$\therefore$ 36.92% chance to have exactly one match.

Total: $\overline{3}$

Problem 8

binomial expansion (in slides)

• Show $\sum_{x=0}^{n} f(x) = 1$ ✓

$\sum_{x=0}^{n} f(x) = \sum_{x=0}^{n} \frac{n!}{x!(n-x)!} p^x (1-p)^x = (p+(1-p))^n = 1^n = 1$ ✓

Must provide some justification to why $f(x) \geq 0$ and $f(x) \leq 1$ to earn all 3 marks

• $0 \leq P(X=x) = f(x) \leq 1$

since $0 \leq p \leq 1$, then ✓ $0 \leq (1-p) \leq 1$, ${}_nC_x > 0$ always. Then $f(x) \geq 0$

since $\sum_{x=0}^{n} f(x) = 1$ ✓, each $0 \leq f(x) \leq 1$ ✓

Problem 9

a) $P(A|B) = P(A|B^c) \Rightarrow A \perp B$.

using Law of Total Probability:

$$P(A) = P(A\cap B) + P(A\cap B^c)$$
$$= P(A|B)\cdot P(B) + P(A|B^c)\cdot P(B^c)$$
$$= P(A|B)\cdot P(B) + P(A|B)\cdot P(B^c)$$
$$= P(A|B)\,[P(B) + P(B^c)]$$
$$= P(A|B)$$

$\therefore$ $P(A) = P(A|B) \Rightarrow A \perp B$

'$P(B) \neq 0$ can be assumed.

3

b) $P(A|C) > P(B|C)$ and $P(A|C^c) > P(B|C^c) \Rightarrow P(A) > P(B)$

$$P(A|C) > P(B|C) \Rightarrow \frac{P(A\cap C)}{P(C)} > \frac{P(B\cap C)}{P(C)}$$

$$\Rightarrow P(A\cap C) > P(B\cap C)$$

Need to show this connection.

$$P(A|C^c) > P(B|C^c) \Rightarrow \frac{P(A\cap C^c)}{P(C^c)} > \frac{P(B\cap C^c)}{P(C^c)}$$

$$\Rightarrow P(A\cap C^c) > P(B\cap C^c)$$

3

Then $P(A\cap C) + P(A\cap C^c) > P(B\cap C) + P(B\cap C^c)$

disjoint events, combine as union

$$P((A\cap C)\cup(A\cap C^c)) > P((B\cap C)\cup(B\cap C^c))$$

$$P(A) > P(B)$$

Also immediate due to Law of Total Probability

Total: $\frac{}{6}$