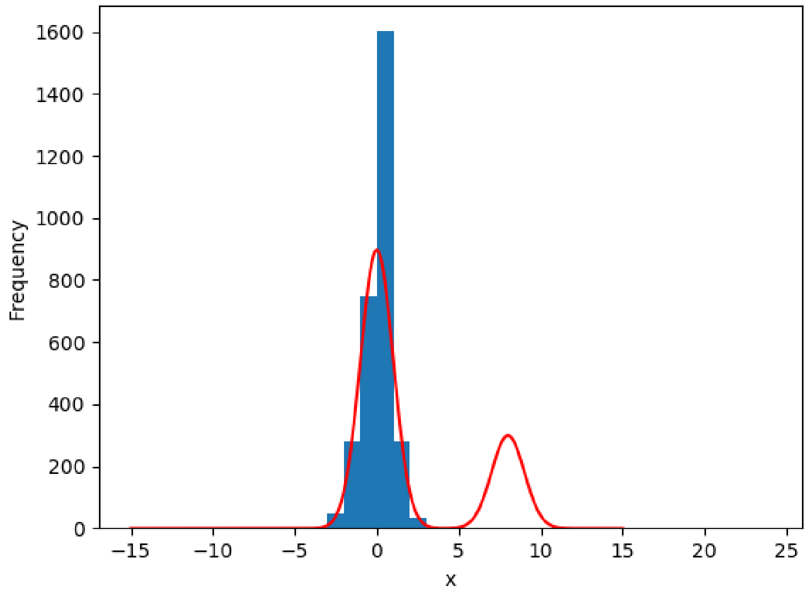
Assignment #8: Example solution

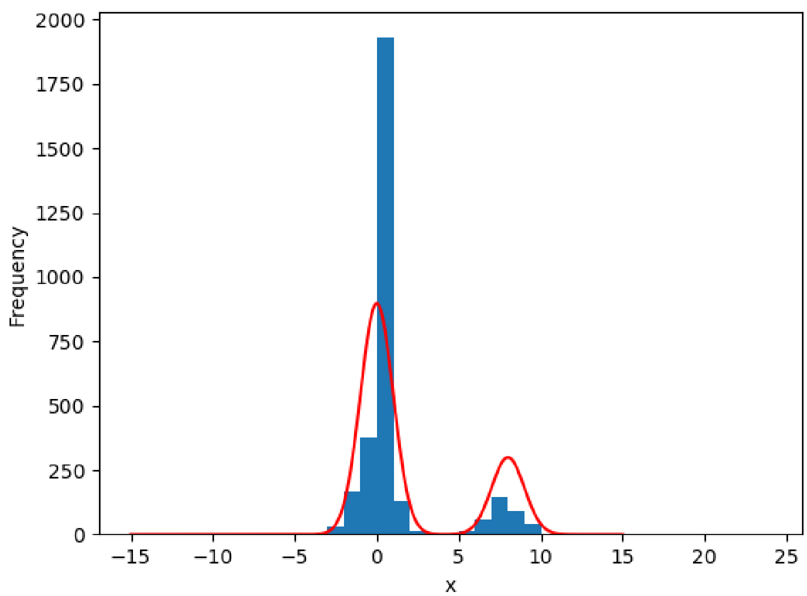
Due: Never

Question 2

(a) The average of 2093 samples was approx. -0.08.



(b) The average of 1371 samples was approx. 1.92.



(c) The true answer is $\mathrm{E}(\mathbf{w}) = 2$, noting

$$p(\mathbf{w}) = \frac{0.75}{\sqrt{2\pi}} \exp\left(-\frac{w_1^2}{2}\right) + \frac{0.25}{\sqrt{2\pi}} \exp\left(-\frac{(w_2-8)^2}{2}\right).$$

Part (b) tends to come closest to 2. Part (a) accepts a greater number of samples, but it sometimes gets stuck in one of the two modes, as shown in the example figure. Part (b) is best because it comes closer to the answer.

```
import numpy as np
import random
from scipy.stats import norm
import matplotlib.pyplot as plt

pi = np.array([0.75,0.25])
mu = np.array([0,8])
sigma = [1,1]
S = 3000 # number of iterations
wVec = np.arange(301)/10-15
realPDF = pi[0]*norm.pdf(wVec,mu[0],sigma[0]) + pi[1]*norm.pdf(wVec,mu[1],sigma[1])

for qpart in range(2): # Loop over parts a and b
    r = 0 # old guess for x
    s = 0 # sample index
    Ss = np.zeros(S) # vector of S samples indexed by s
    rejects = 0 # number of rejections
    while s+rejects<S:
        if qpart==0:
            q = np.random.normal(r,1) # q is the proposal
        else:
            if np.random.random() < 0.5:
                q = np.random.normal(r,1)
            else:
                q = np.random.normal(r,10)
        accpt = sum(pi * norm.pdf([q,q],mu,sigma))
        accpt = accpt / sum(pi * norm.pdf([r,r],mu,sigma))
        if random.random() < accpt:
            Ss[s]=q
            s+=1
            r=q
        else:
            rejects += 1

    print(qpart, s, np.mean(Ss[:s]),sum(pi*mu),rejects)
    plt.hist(Ss,bins=range(-15,25))
    plt.plot (wVec,S*realPDF,'r')
    plt.xlabel('x')
    plt.ylabel('Frequency')
    plt.show()
```