Sheet 3 - Boolean Algebra & Logic Gates

ECE 222 - Digital Logic Design

120200033 CSE Section 01
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Questions 1:2

1 . Find the complement of the following expressions:

a . $xy' + x'y$

Solution:

$$\begin{aligned}\overline{(xy' + x'y)} &= \overline{(xy')} \cdot \overline{(x'y)} \\ &= (x' + y) \cdot (x + y')\end{aligned}$$

b . $(a + c)(a + b')(a' + b + c')$

Solution:

$$\begin{aligned}\overline{(a + c)(a + b')(a' + b + c')} &= \overline{(a + c)} + \overline{(a + b')} + \overline{(a' + b + c')} \\ &= (a' \cdot c') + (a' \cdot b) + (a \cdot b' \cdot c)\end{aligned}$$

c . $z + z'(v'w + xy)$

Solution:

$$\begin{aligned}\overline{z + z'(v'w + xy)} &= z' \cdot [z + \overline{(v'w + xy)}] \\ &= z' \cdot [z + (v + w') \cdot (x' + y')]\end{aligned}$$

2 . Simplify the following Boolean expressions to a minimum number of literals:

a . $ABC + A'B + ABC'$

Solution:

$$\begin{aligned}ABC + A'B + ABC' &= B(AC + A' + AC') \\ &= B(A(C + C') + A') \\ &= B(A + A') \\ &= B\end{aligned}$$

b . $x'yz + xz$

Solution:

$$\begin{aligned}x'yz + xz &= z(x'y + x) \\ &= z((x' + x) \cdot (y + x)) \\ &= z(y + x)\end{aligned}$$

c . $(x + y)' \cdot (x' + y')$

Solution:

$$\begin{aligned}(x + y)' \cdot (x' + y') &= (x' \cdot y') \cdot (x' + y') \\ &= x'y'\end{aligned}$$

d . $xy + x(wz + wz')$

Solution:

$$\begin{aligned} xy + x(wz + wz') &= xy + x(w(z + z')) \\ &= xy + xw \\ &= x(y + w) \end{aligned}$$

e . $(BC' + A'D)(AB' + CD')$

Solution:

$$\begin{aligned} (BC' + A'D)(AB' + CD') &= BC'AB' + CD'BC' + A'DAB' + A'DCD' \\ &= 0 + 0 + 0 + 0 \\ &= 0 \end{aligned}$$

f . $(a' + c')(a + b' + c')$

Solution:

$$\begin{aligned} (a' + c')(a + b' + c') &= (a'a + a'b' + a'c') + (c'a + c'b' + c'c') \\ &= a'b' + a'c' + c'a + c'b' + c' \\ &= a'b' + c'(a' + a + b' + 1) \\ &= a'b' + c' \end{aligned}$$

3 . Reduce the following Boolean expressions to the indicated number of literals:

a . $A'C' + ABC + AC'$ To three literals

Solution:

$$\begin{aligned} A'C' + ABC + AC' &= A'C' + A(BC + C') \\ &= A'C' + A(B + C') \cdot (C + C') \\ &= A'C' + AB + AC') \\ &= C'(A' + A) + AB \\ &= C' + AB \end{aligned}$$

b . $(x'y' + z)' + z + xy + wz$ To three literals

Solution:

$$\begin{aligned} (x'y' + z)' + z + xy + wz &= x + y \cdot z' + z + xy + wz \\ &= z'(x + y) + z(1 + w) + xy \\ &= z'(x + y) + z + xy \\ &= (z' + z)(x + y + z) + xy \\ &= x + y + z + xy \\ &= x + y(1 + x) + z \\ &= x + y + z \end{aligned}$$

c . $A'B(D'+C'D)+B(A+A'CD)$ To one literal

Solution:

$$\begin{aligned} A'B(D'+C'D)+B(A+A'CD) &= B(A'D'+A'C'D+A+A'CD) \\ &= B[A+A'(D'+C'D+CD)] \\ &= B[A+A'(D'+D(C'+C))] \\ &= B[A+A'(D'+D)] \\ &= B[A+A'] \\ &= B \end{aligned}$$

d . $(A'+C)(A'+C')(A+B+C'D)$ To four literals

Solution:

$$\begin{aligned} (A'+C)(A'+C')(A+B+C'D) &= (A'+CC')(A+B+C'D) \\ &= A'A+A'B+A'C'D \\ &= A'(B+C'D) \end{aligned}$$

e . $ABC'D+A'BD+ABCD$ To two literals

Solution:

$$\begin{aligned} ABC'D+A'BD+ABCD &= BD(AC'+AC+A') \\ &= BD[A(C'+C)+A'] \\ &= BD[A+A'] \\ &= BD \end{aligned}$$

4 . List the truth table of the function:

a . $F=xy+xy'+y'z \xrightarrow{=x+y'z}$

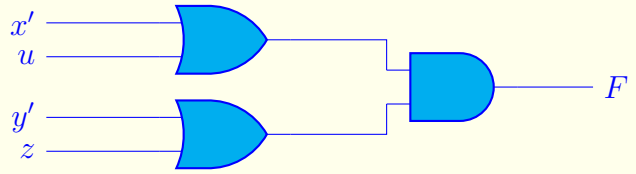
No.	x y z	F
1	0 0 0	0
2	0 0 1	1
3	0 1 0	0
4	0 1 1	0
5	1 0 0	1
6	1 0 1	1
7	1 1 0	1
8	1 1 1	1

b . $F=bc+a'c' \longrightarrow$

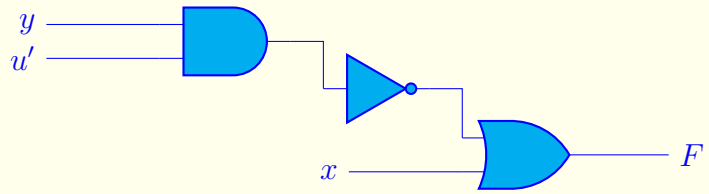
No.	a b c	F
1	0 0 0	1
2	0 0 1	0
3	0 1 0	1
4	0 1 1	1
5	1 0 0	0
6	1 0 1	0
7	1 1 0	0
8	1 1 1	1

5 . Draw logic diagrams to implement the following Boolean expressions:

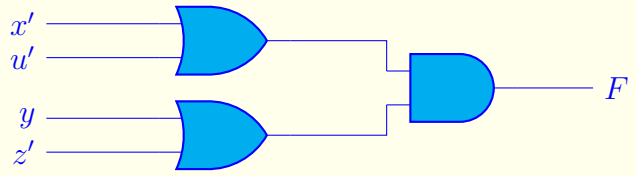
a . $F = (u + x')(y' + z)$



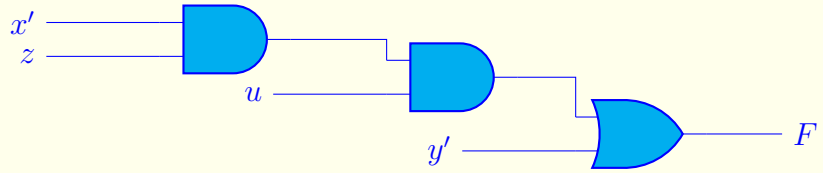
b . $F = (u'y)' + x$



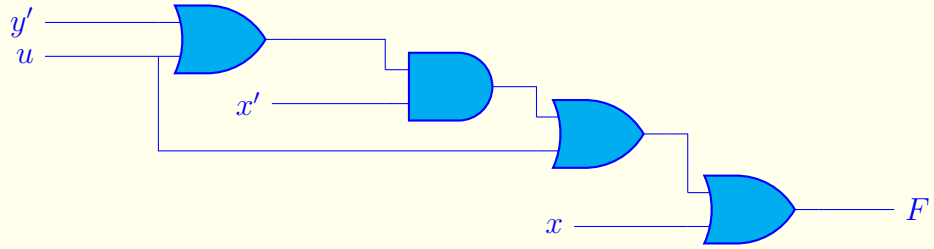
c . $F = (u' + x')(y + z')$



d . $F = u(x'z) + y'$

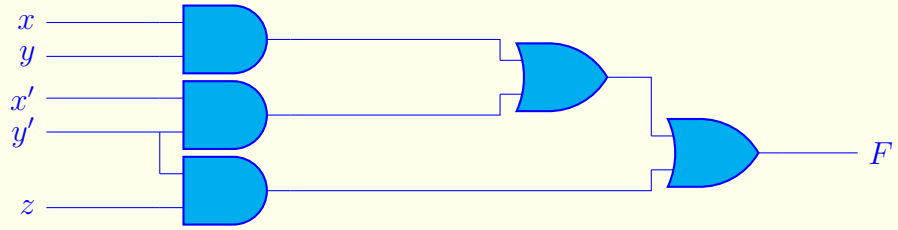


e . $F = u + x + x'(u + y')$

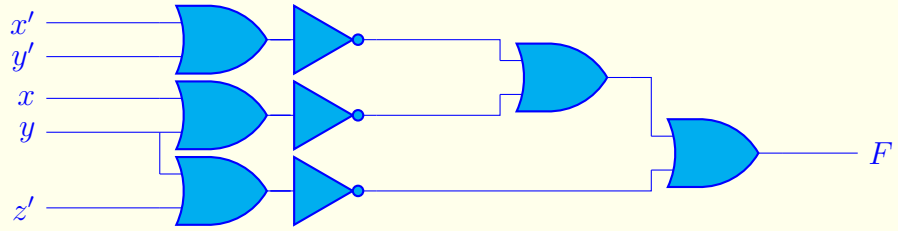


6 . Implement the Boolean function $F = xy + x'y' + y'z$ with:

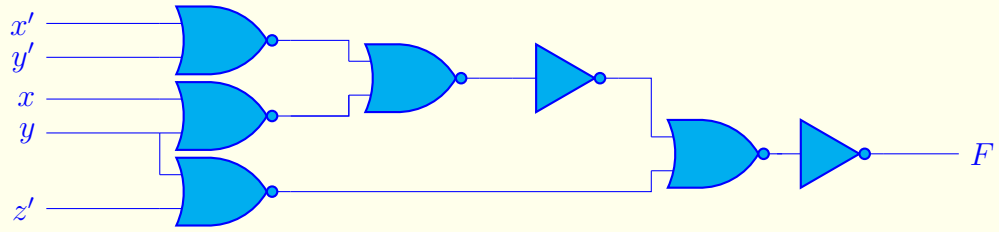
a . AND, OR, and inverter gates.



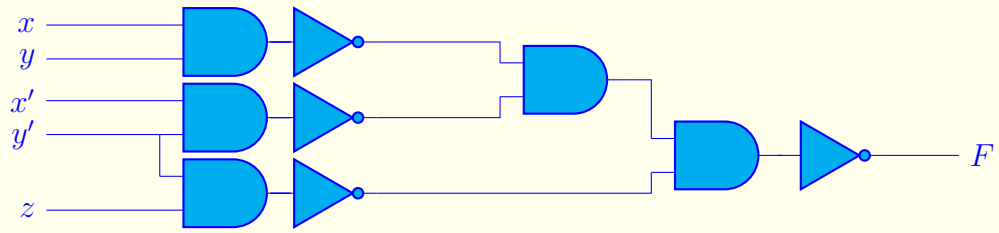
b . OR and inverter gates → $(x' + y')' + (x + y)' + (y + z')'$



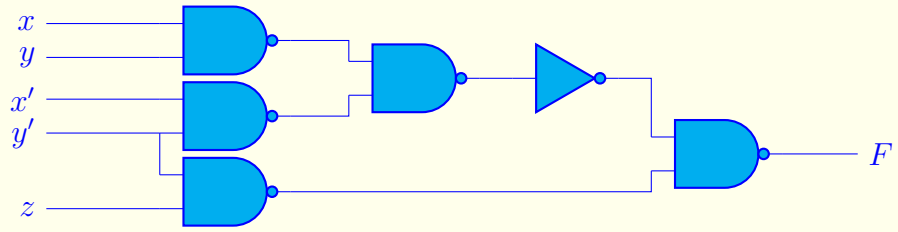
c . NOR and inverter gates.



d . AND and inverter gates.



e . NAND and inverter gates.



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- Texmaker 5.0.4

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