



Sheet 3 - Bisection & Fixed Point

CSE 213- Numerical Analysis

120200033

CSE Section 01

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Question 2: For root finding – Textbook (9th edition) – Problem Set 2.2 (Page 64). Problems 12 b) and d) are to do as homework.

12. For each of the following equations, use the given interval or determine an interval $[a, b]$ on which fixed-point iteration will converge. Estimate the number of iterations necessary to obtain approximations accurate to within 10^{-5} , and perform the calculations.

b. $x^3 - 2x - 5 = 0$ use $[2, 3]$

Solution:

$$g_1(x) = \frac{x^3 - 5}{2} \quad \text{or} \quad g_2(x) = \sqrt[3]{2x + 5}$$

$g(x)$	$g_1(x) = \frac{x^3 - 5}{2}$	$g_2(x) = \sqrt[3]{2x + 5}$
$g'(x)$	$g'_1(x) = \frac{3x^2}{2}$	$g'_2(x) = \frac{2}{3(2x + 5)^{\frac{2}{3}}}$
$\max(k)$	13.5 (not acceptable)	0.15408028319

$$n > \frac{\ln(10^{-5}) - \ln(\max(p_0 - a, b - p_0))}{\ln(k)}$$

$$n > \frac{\ln(10^{-5}) - \ln(1)}{\ln(0.15408028319)}$$

$$n > 6.1557$$

\therefore we expect $n > 6$, however; the desired tolerance is met with exactly 6 iterations.

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+---+-----+-----+-----+
| n |      xn_1      |      xn = g(xn_1)      |      abs(xn - xn_1)      |
+---+-----+-----+-----+
| 1 |          2      | 2.080083823051904      | 0.08008382305190409      |
| 2 | 2.080083823051904 | 2.092350677797578      | 0.012266854745674127      |
| 3 | 2.092350677797578 | 2.0942169960125234      | 0.0018663182149452062      |
| 4 | 2.0942169960125234 | 2.0945006521946543      | 0.00028365618213088695      |
| 5 | 2.0945006521946543 | 2.0945437575328114      | 4.310533815710116e-05      |
| 6 | 2.0945437575328114 | 2.0945503078082703      | 6.550275458927501e-06      |
+---+-----+-----+-----+
2.0945503078082703

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d. $x - \cos(x) = 0$ using $[0,1]$

Solution:

$$g_1(x) = \cos(x) \quad \text{or} \quad g_2(x) = \cos^{-1}(x)$$

$g(x)$	$g_1(x) = \cos(x)$	$g_2(x) = \cos^{-1}(x)$
$g'(x)$	$g'_1(x) = -\sin(x)$	$g'_2(x) = \frac{1}{\sqrt{1-x^2}}$
$\max(k)$	0.84147098481	∞ (not acceptable)

$$n > \frac{\ln(10^{-5}) - \ln(\max(p_0 - a, b - p_0))}{\ln(k)}$$

$$n > \frac{\ln(10^{-5}) - \ln(1)}{\ln(0.84147098481)}$$

$$n > 66.70148078$$

\therefore we expect $n > 66$, however; the tolerance is met with exactly 30 iterations.

n	xn_1	xn = g(xn_1)	abs(xn - xn_1)
1	0	1.0	1.0
2	1.0	0.5403023058681398	0.45969769413186023
3	0.5403023058681398	0.8575532158463934	0.31725090997825367
4	0.8575532158463934	0.6542897904977791	0.2032634253486143
5	0.6542897904977791	0.7934803587425656	0.13919056824478648
6	0.7934803587425656	0.7013687736227565	0.0921115851198091
7	0.7013687736227565	0.7639596829006542	0.06259090927789768
8	0.7639596829006542	0.7221024250267077	0.04185725787394645
9	0.7221024250267077	0.7504177617637605	0.028315336737052776
10	0.7504177617637605	0.7314040424225098	0.019013719341250734
11	0.7314040424225098	0.7442373549005569	0.012833312478047088
12	0.7442373549005569	0.7356047404363474	0.008632614464209487
13	0.7356047404363474	0.7414250866101092	0.0058203461737618145
14	0.7414250866101092	0.7375068905132428	0.003918196096866389
15	0.7375068905132428	0.7401473355678757	0.0026404450546329006
16	0.7401473355678757	0.7383692041223232	0.0017781314455525
17	0.7383692041223232	0.7395672022122561	0.0011979980899329279
18	0.7395672022122561	0.7387603198742113	0.0008068823380448231
19	0.7387603198742113	0.7393038923969059	0.0005435725226945465
20	0.7393038923969059	0.7389377567153445	0.0003661356815614081
21	0.7389377567153445	0.7391843997714936	0.00024664305614918725
22	0.7391843997714936	0.7390182624274122	0.00016613734408144065
23	0.7390182624274122	0.7391301765296711	0.00011191410225885878
24	0.7391301765296711	0.7390547907469174	7.538578275367858e-05
25	0.7390547907469174	0.7391055719265363	5.078117961887507e-05
26	0.7391055719265363	0.7390713652989449	3.420662759134885e-05
27	0.7390713652989449	0.7390944073790913	2.3042080146362665e-05
28	0.7390944073790913	0.739078885994992	1.552138409921522e-05
29	0.739078885994992	0.7390893414033927	1.0455408400611432e-05
30	0.7390893414033927	0.7390822985224024	7.0428809902933764e-06
31	0.7390822985224024		

Tools used in creating this document:

- Texmaker 5.0.4
- Google Colab with python 3.7.13 [GCC 7.5.0]

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