Complex HW3

Wednesday, 24 February 2021 12:06

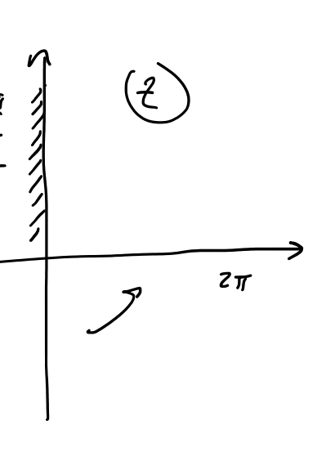
(1.) $\varphi(z)=\sqrt[3]{z}$

$z=|z|e^{i\varphi'}$

$z^{\frac{1}{3}}=\sqrt{|z|}\,e^{i\varphi'/3}$

$\varphi(-1)=e^{i\pi/3}$, =>

a)

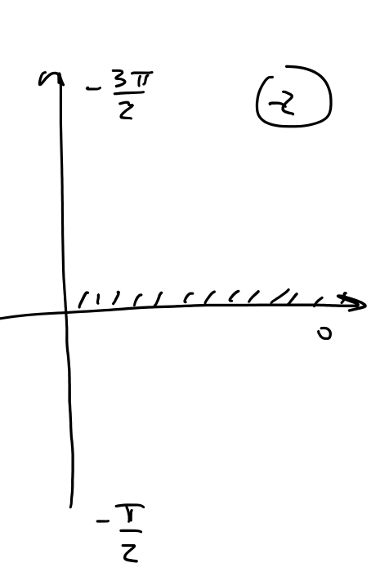


• $\varphi(1)=(*)$ $\Delta\arg z=\pi$, $\Delta\arg\varphi(z)=\frac{\pi}{3}$ $(*)=\left|\frac{\varphi(1)}{\varphi(-1)}\right|\cdot e^{i\pi/3}\cdot e^{i\pi/3}=e^{2i\pi/3}$

$\varphi(i+0)=(*)$ $\Delta\arg z=\frac{3\pi}{2}$, $\Delta\arg\varphi(z)=\frac{\pi}{2}$ $(*)=\left|\frac{\varphi(i+0)}{\varphi(-1)}\right|\cdot e^{i\pi/3}\cdot e^{i\pi/2}=e^{5i\pi/6}$

$\varphi(i-0)=(*)$ $\Delta\arg z=-\frac{\pi}{2}$, $\Delta\arg\varphi(z)=-\frac{\pi}{6}$ $(*)=\left|\frac{\varphi(i-0)}{\varphi(-1)}\right|\cdot e^{i\pi/3}\cdot e^{-i\pi/6}=e^{i\pi/6}$

б)



$\varphi(z)=\ln z$

$z=|z|e^{i\varphi'}$

$\varphi(1-i0)=0$

$\varphi(1+i0)=\varphi(1-i0)+\ln\left|\frac{1-i0}{1+i0}\right|+i(-2\pi)=-2\pi i$

$\varphi(i)=i\left(-\frac{3\pi}{2}\right)=-\frac{3\pi i}{2}$

$\varphi(-i)=i\cdot\left(-\frac{\pi}{2}\right)=-\frac{\pi i}{2}$

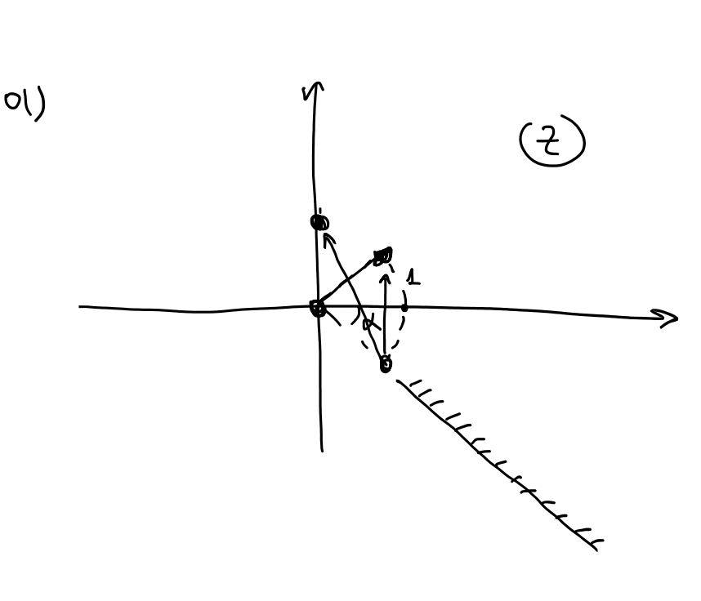
(2.) $\varphi_1(z)=\sqrt{z-e^{-i\alpha}}$

$\varphi_2(z)=\ln(z-e^{-i\alpha})$, $\alpha\in(0;\frac{\pi}{2})$

$\varphi_1(0)=ie^{-i\alpha/2}$

$\varphi_2(0)=-i\pi-i\alpha$

(a)



$\varphi_1(e^{i\alpha})=(*)$ $\Delta\arg z=-\frac{\pi}{2}+\alpha$, $\Delta\arg\varphi=\frac{\alpha}{2}-\frac{\pi}{4}$ $(*)=\left|\frac{\sqrt{e^{i\alpha}-e^{-i\alpha}}}{ie^{-i\alpha/2}}\right|\cdot ie^{-i\alpha/2}\cdot e^{i(\frac{\alpha}{2}-\frac{\pi}{4})}=$

$=\sqrt{|2i\sin\alpha|}\cdot ie^{-i\frac{\pi}{4}}=e^{i\pi/2}e^{-i\frac{\pi}{4}}\sqrt{2\sin\alpha}=e^{i\pi/4}\sqrt{2\sin\alpha}$ //

$\varphi_1(i)=(*)$ $\Delta\arg z=-(\frac{\pi}{2}-\alpha)/2$, $\Delta\arg\varphi(z)=\frac{\pi}{8}-\frac{\alpha}{4}$ $(*)=|\sqrt{i-e^{-i\alpha}}|\cdot ie^{-i\alpha/2-i\frac{\pi}{8}+i\alpha/4}=$

$=(*)\ |i-e^{-i\alpha}|=|e^{i\pi/2}-e^{-i\alpha}|=\left|2ie^{i\pi/4-i\alpha/2}\left(\frac{e^{i\pi/4+i\alpha/2}-e^{-i\pi/4-i\alpha/2}}{2i}\right)\right|=2\sin(\frac{\pi}{4}+\frac{\alpha}{2})\ (*)=$

$=e^{i\pi/2}e^{i\alpha/4-i\frac{\pi}{8}}\sqrt{2\sin(\frac{\pi}{4}+\frac{\alpha}{2})}=e^{3i\pi/8-i\alpha/4}\sqrt{2\sin(\frac{\pi}{4}+\frac{\alpha}{2})}=(*)$

$\sin(\frac{\pi}{4}+\frac{\alpha}{2})=\sin\frac{\pi}{4}\cos\frac{\alpha}{2}+\cos\frac{\pi}{4}\sin\frac{\alpha}{2}=\cos\frac{\pi}{4}\cos\frac{\alpha}{2}+\sin\frac{\pi}{4}\sin\frac{\alpha}{2}=\cos(\frac{\pi}{4}-\frac{\alpha}{2})$

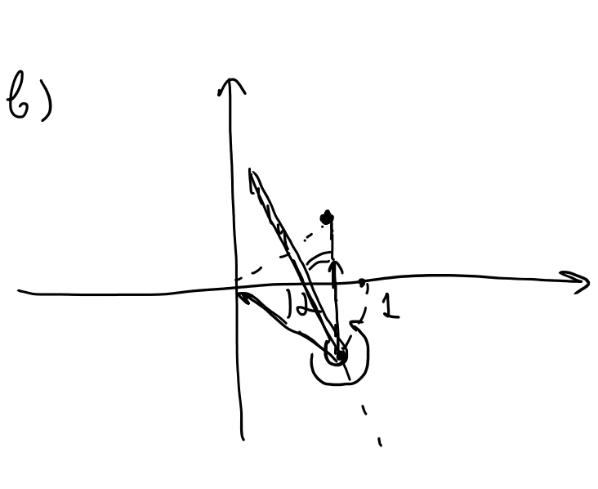
$(*)=e^{3i\pi/8-i\alpha/4}\sqrt{2\cos(\frac{\pi}{4}-\frac{\alpha}{2})}$ //

$\varphi_2(e^{i\alpha})=-i\pi-i\alpha+\ln\left|\frac{e^{i\alpha}-e^{-i\alpha}}{-1}\right|+i(-\frac{\pi}{2}+\alpha)=-\frac{3i\pi}{2}+\ln(2\sin\alpha)$ //

$\varphi_2(i)=-i\pi-i\alpha+\ln\left|\frac{i-e^{-i\alpha}}{-1}\right|-i(\frac{\pi}{2}-\alpha)/2=-\frac{5i\pi}{4}-\frac{i\alpha}{2}+\ln(2\sin(\frac{\pi}{4}+\frac{\alpha}{2}))=$

$=-\frac{5i\pi}{4}-\frac{i\alpha}{2}+\ln(2\cos(\frac{\pi}{4}-\frac{\alpha}{2}))$

(б)



$\varphi_1(e^{i\alpha})=(*)$ $\Delta\arg z=2\pi-\frac{\pi}{2}+\alpha$, $\Delta\arg\varphi=\frac{3\pi}{4}+\frac{\alpha}{2}$ $(*)=\left|\frac{\sqrt{e^{i\alpha}-e^{-i\alpha}}}{ie^{-i\alpha/2}}\right|ie^{-i\alpha/2}e^{3i\pi/4+\frac{i\alpha}{2}}=$

$=\sqrt{2\sin\alpha}\,e^{i\pi/2}e^{3i\pi/4}=e^{5i\pi/4}\sqrt{2\sin\alpha}$ //

$\varphi_2(e^{i\alpha})=-i\pi-i\alpha+\ln\left|\frac{e^{i\alpha}-e^{-i\alpha}}{-1}\right|+i(\frac{3\pi}{2}+\alpha)=\frac{i\pi}{2}+\ln(2\sin\alpha)$

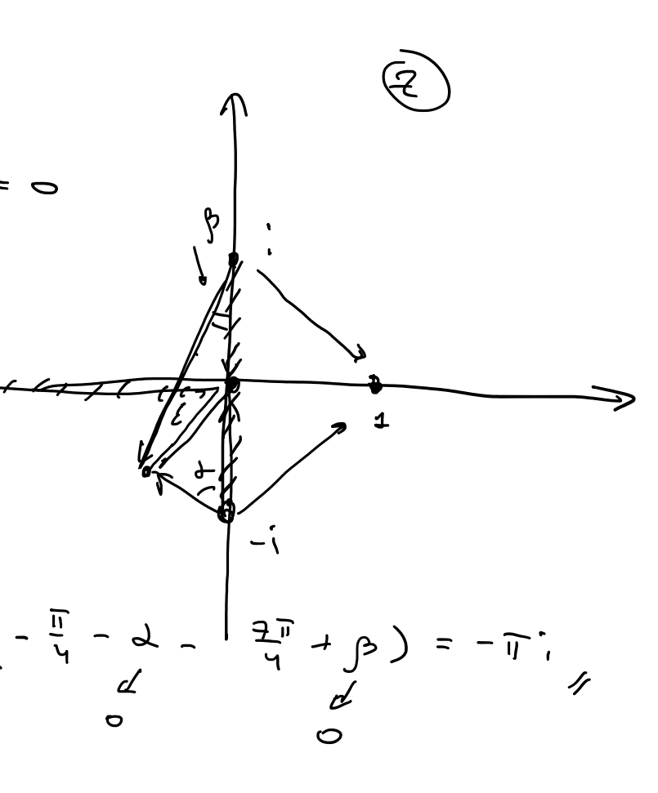
$\varphi_1(i)=(*)$ $\Delta\arg z=2\pi-\frac{\pi}{4}+\frac{\alpha}{2}$, $\Delta\arg\varphi=\frac{7\pi}{8}+\frac{\alpha}{4}$ $(*)=\left|\frac{\sqrt{e^{i\pi/2}-e^{-i\alpha}}}{ie^{-i\alpha/2}}\right|ie^{-i\alpha/2}e^{7i\pi/8+i\alpha/4}=$

$=\sqrt{2\cos(\frac{\pi}{4}-\frac{\alpha}{2})}\,e^{i\pi/2}e^{7i\pi/8-i\alpha/4}=e^{11i\pi/8-i\alpha/4}\sqrt{2\cos(\frac{\pi}{4}-\frac{\alpha}{2})}=e^{-5i\pi/8-i\alpha/4}\sqrt{2\cos(\frac{\pi}{4}-\frac{\alpha}{2})}$ //

$\varphi_2(i)=-i\pi-i\alpha+\ln\left|\frac{e^{i\pi/2}-e^{-i\alpha}}{-1}\right|+i(\frac{7\pi}{4}+\frac{\alpha}{2})=\frac{3i\pi}{4}-\frac{i\alpha}{2}+\ln(2\cos(\frac{\pi}{4}-\frac{\alpha}{2}))$ //

(7.) $f(x)=\ln[(1+x^2)^{1/2}]=\ln[(x+i)^{1/2}(x-i)^{1/2}]$

Пусть $f(1)=\frac{1}{2}\ln(2)$



1) $f(\varepsilon)=\ln\left|\frac{\sqrt{1+\varepsilon^2}}{\sqrt{2}}\right|+\frac{1}{2}\ln(2)+\frac{i}{2}(\frac{\pi}{4}-\overset{0}{\gamma}-\frac{\pi}{4}+\overset{0}{\gamma})=0$

2) $f(\varepsilon e^{3\pi i/4})=\ln\left|\frac{\sqrt{1+\varepsilon^2e^{3\pi i/2}}}{\sqrt{2}}\right|+\frac{1}{2}\ln(2)+\frac{i}{2}(\frac{\pi}{4}+\overset{0}{\beta}+\frac{7\pi}{4}-\overset{0}{\alpha})=\pi i$ //

3) $f(\varepsilon e^{-3i\pi/4})=\ln\left|\frac{\sqrt{1+\varepsilon^2e^{-3i\pi/2}}}{\sqrt{2}}\right|+\frac{1}{2}\ln 2+\frac{i}{2}(-\frac{\pi}{4}-\underset{0}{\alpha}-\frac{7\pi}{4}+\underset{0}{\beta})=-\pi i$ //

(12.) (i) $|z|\in(0;2\pi)$

$f(z)=\frac{e^{iz}}{\cos z-1}\simeq\frac{1+iz-\frac{z^2}{2}}{1-\frac{z^2}{2}+\frac{z^4}{4!}-1}=\frac{1+iz-\frac{z^2}{2}}{\frac{z^4}{4!}-\frac{z^2}{2!}}=\frac{12(1+iz-z^2)}{z^4-12z^2}=\frac{12}{z^2}\cdot\frac{1+iz-z^2}{z^2-12}=$

$=-\frac{1}{z^2}(1+iz-z^2)\cdot\frac{1}{1-z^2/12}\simeq-\frac{1}{z^2}(1+iz-z^2)(1+\frac{z^2}{12}+\frac{z^4}{12^2})=$

$=-\frac{1}{z^2}-\frac{i}{z}-\frac{1}{12}+\dots$, => $C_{-3}\cdot\frac{1}{z^3}=0$

$C_{-3}=0$ //

(ii) $|z|\in(2\pi;4\pi)$

$C_{-3}=\frac{1}{2\pi i}\oint_\gamma\frac{e^{iz}}{\cos z-1}\cdot(z^{3-1})\,dz=\frac{1}{2\pi i}\oint_\gamma\frac{z^2e^{iz}}{\cos z-1}dz=\frac{1}{2\pi i}\cdot 2\pi i\sum\operatorname{Res}g(z)=$

$=\operatorname{Res}_{z=2\pi}g(z)+\operatorname{Res}_{z=-2\pi}g(z)+\operatorname{Res}_{z=0}g(z)=(*)\ \underset{z_0=2\pi}{f(z)}=\frac{z^2(1+(iz-2\pi i))}{1-\frac{(z-2\pi)^2}{2}+\frac{(z-2\pi)^4}{24}-1}\simeq$

$\simeq-2\frac{z^2(1-2\pi i+iz)}{(z-2\pi)^2}=-2\left[\frac{z^2-4\pi z+4\pi^2}{(z-2\pi)^2}+\frac{4\pi z-4\pi^2}{(z-2\pi)^2}-\frac{z^2i}{(z-2\pi)^2}(z-2\pi)\right]=$

$=-2\left[1+\frac{4\pi z-4\pi^2}{(z-2\pi)^2}-\frac{i(z^2-4\pi z+4\pi^2)}{z-2\pi}-i\frac{4\pi z-4\pi^2}{z-2\pi}\right]=-2\left[1+\frac{4\pi(z-2\pi)+4\pi^2}{(z-2\pi)^2}-\right.$

$\left.-i(z-2\pi)+i\frac{4\pi(z-2\pi)+4\pi^2}{z-2\pi}\right]=-2\left[1-i(z+2\pi)+\frac{1}{z-2\pi}(4\pi+4\pi^2i)+\frac{4\pi^2}{(z-2\pi)^2}\right]$, =>

$\operatorname{Res}_{z=2\pi}g(z)=-2(4\pi+4\pi^2i)=-8\pi(1+\pi i)$

$\operatorname{Res}_{z=-2\pi}g(z)=-2(4\pi-4\pi^2i)=-8\pi(\pi i-1)$

$\operatorname{Res}_{z=0}g(z)=0$

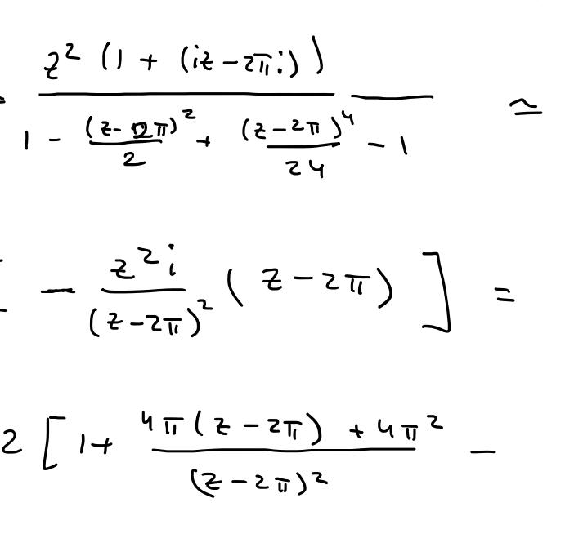
$(*)=-8\pi(1+\pi i+\pi i-1)=-16\pi^2i$ //

(3.) $f(z)=(1+z^2)^{1/2}=(z+i)^{1/2}(z-i)^{1/2}$

(i) $\Delta\arg(z+i)=2\pi$

$\Delta\arg(z-i)=2\pi$

$\Delta\arg f=\frac{1}{2}(2\pi+2\pi)=2\pi$, => ф-я опред. однозначно



$I_1=\int_{C_1}\sqrt{1+z^2}\,dz=\int_{C_{R\to\infty}}\sqrt{1+z^2}\,dz=2\pi i\operatorname{Res}_{z\to\infty}f(z)=(*)\ f(z)=\sqrt{1+z^2}=z\sqrt{1+\frac{1}{z^2}}\approx$

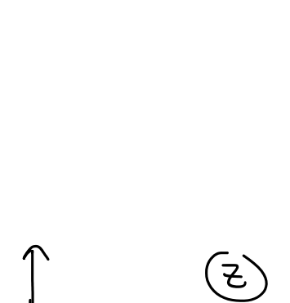
$\approx z(1+\frac{1}{2z^2})=z+\frac{1}{2z}$

$(*)=2\pi i\cdot\frac{1}{2}=\pi i$ //

$\Delta\arg(z+i)=0$

$\Delta\arg(z-i)=0$

$\Delta\arg f=\frac{1}{2}\cdot 0=0$, => ф-я опред. однозначно

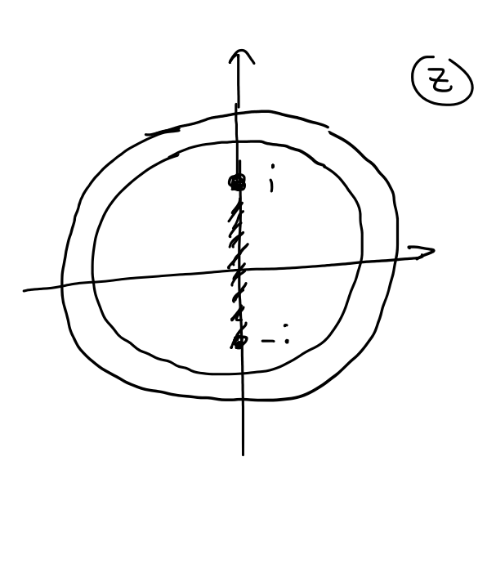


$I_2=\int_{C_2}\sqrt{1+z^2}\,dz=\int_{C_{2\to0}}\sqrt{1+z^2}\,dz=0$

$\Delta\arg(z+i)=2\pi$

$\Delta\arg(z-i)=0$

$\Delta\arg f=\frac{1}{2}\cdot2\pi=\pi$, => ф-я не определена однозначно



(ii) Пусть $f(1)=\sqrt{2}$, =>

a) $f(-1)=(*)$ $\Delta\arg(z+i)=\frac{3\pi}{2}$, $\Delta\arg(z-i)=\frac{\pi}{2}$, $\Delta\arg f=\pi$ $(*)=\left|\frac{\sqrt{2}}{\sqrt{2}}\right|\cdot\sqrt{2}\cdot e^{i\pi}=-\sqrt{2}$ //

б) $f(-1)=(*)$ $\Delta\arg(z-i)=\frac{\pi}{2}$, $\Delta\arg(z+i)=-\frac{\pi}{2}$, $\Delta\arg f=0$ $(*)=\left|\frac{\sqrt{2}}{\sqrt{2}}\right|\cdot\sqrt{2}\cdot e^{i\cdot0}=\sqrt{2}$

(4.) $\varphi(z)=z^{\mu}(1-z)^{1-\mu}$, $\varphi(\frac{1}{2}+i0)=\frac{1}{2}$

1) $\varphi(2)=\left|\frac{\varphi(2)}{\varphi(1/2)}\right|\cdot\varphi(\frac{1}{2})\cdot e^{i\cdot0\cdot\mu}\cdot e^{i\pi(1-\mu)}=\left|\frac{2^{\mu}\cdot(-1)^{1-\mu}}{1/2}\right|\cdot\frac{1}{2}\cdot e^{i\pi}\cdot e^{i\pi\mu}=$

$=-2^{\mu}\cdot e^{i\pi\mu}$ //

2) $\varphi(-1)=|(-1)^{\mu}\cdot2^{1-\mu}|\cdot e^{i\pi\mu}\cdot e^{i\cdot0\cdot(1-\mu)}=2^{1-\mu}e^{i\pi\mu}$ //

3) $f(z)=\frac{\varphi(z)}{z}=z^{\mu-1}(1-z)^{1-\mu}$

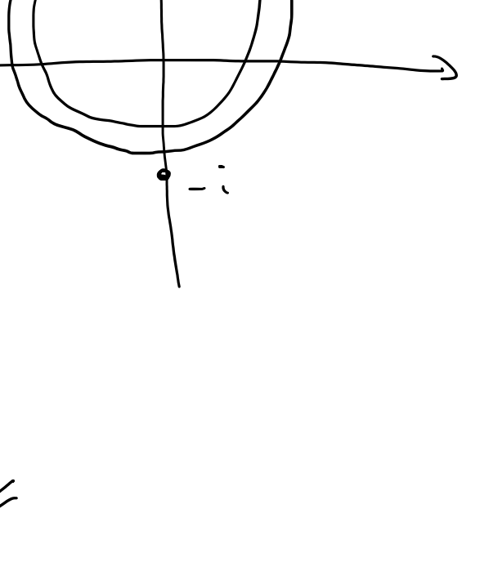
$\varphi(z_0)=\frac{\varphi(z_0)}{z_0}=1$, =>

$\lim_{z\to\infty}f(z)=\lim_{z\to\infty}\left|\frac{z^{\mu-1}(1-z)^{1-\mu}}{1}\right|\cdot1\cdot e^{i\cdot0\cdot\mu}\cdot e^{-i\pi(1-\mu)}=\lim_{z\to\infty}|z^{\mu-1}\cdot z^{1-\mu}|\cdot e^{i\pi(\mu-1)}=$

$=e^{i\pi(\mu-1)}$ //

(5.) $\varphi(z)=\ln(1-z^2)$

$\varphi(0)=-2\pi i$



1) $\varphi(-2)=-2\pi i+\ln\left|\frac{-4}{1}\right|-\pi i=-3\pi i+\ln4$ //

2) $\varphi(-i)=-2\pi i+\ln\left|\frac{2}{1}\right|-\frac{\pi i}{4}+\frac{\pi i}{4}=-2\pi i+\ln2$ //

3) $\varphi(\frac{-1+i\sqrt{3}}{2})=\varphi(-\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})=\varphi(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})=\varphi(e^{2\pi i/3})=$

$=-2\pi i+\ln\left|\frac{1-e^{4\pi i/3}}{1}\right|-\frac{\pi i}{6}+\frac{\pi i}{3}=-\frac{11\pi i}{6}+\ln|1-\overset{-\frac{1}{2}}{\cos\frac{4\pi}{3}}-i\overset{-\frac{\sqrt{3}}{2}}{\sin\frac{4\pi}{3}}|=-\frac{11\pi i}{3}+\ln\sqrt{\frac{9}{4}+\frac{3}{4}}=$

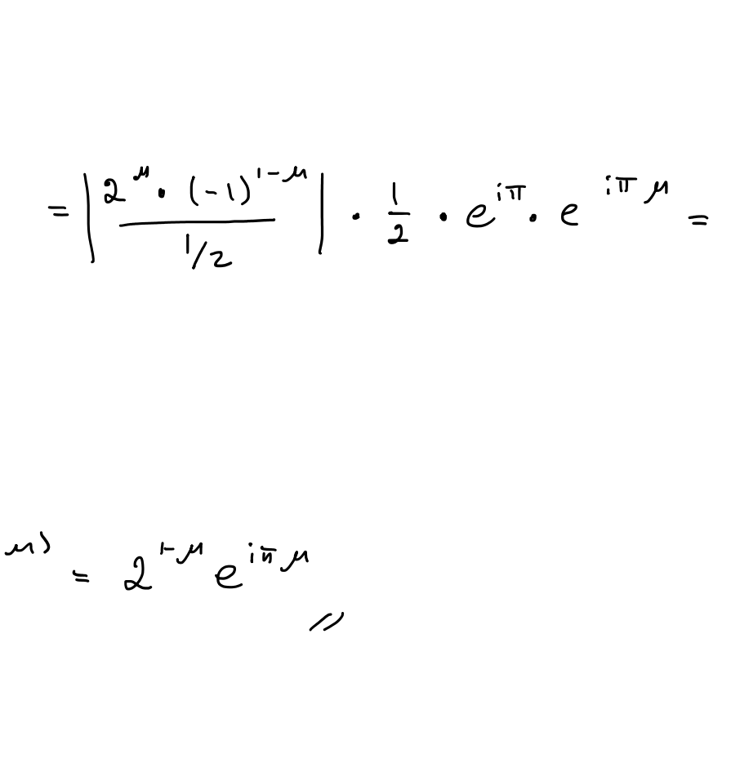
$=-\frac{11\pi i}{3}+\frac{1}{2}\ln3$ //

(6.) $\varphi(z)=\sqrt[3]{1+z^2}$

$\varphi(0)=1$

1) $\varphi(3i)=\left|\frac{\sqrt[3]{1-9}}{1}\right|e^{i\cdot0/3}\cdot e^{-\pi i/3}=2e^{-i\pi/3}$ //

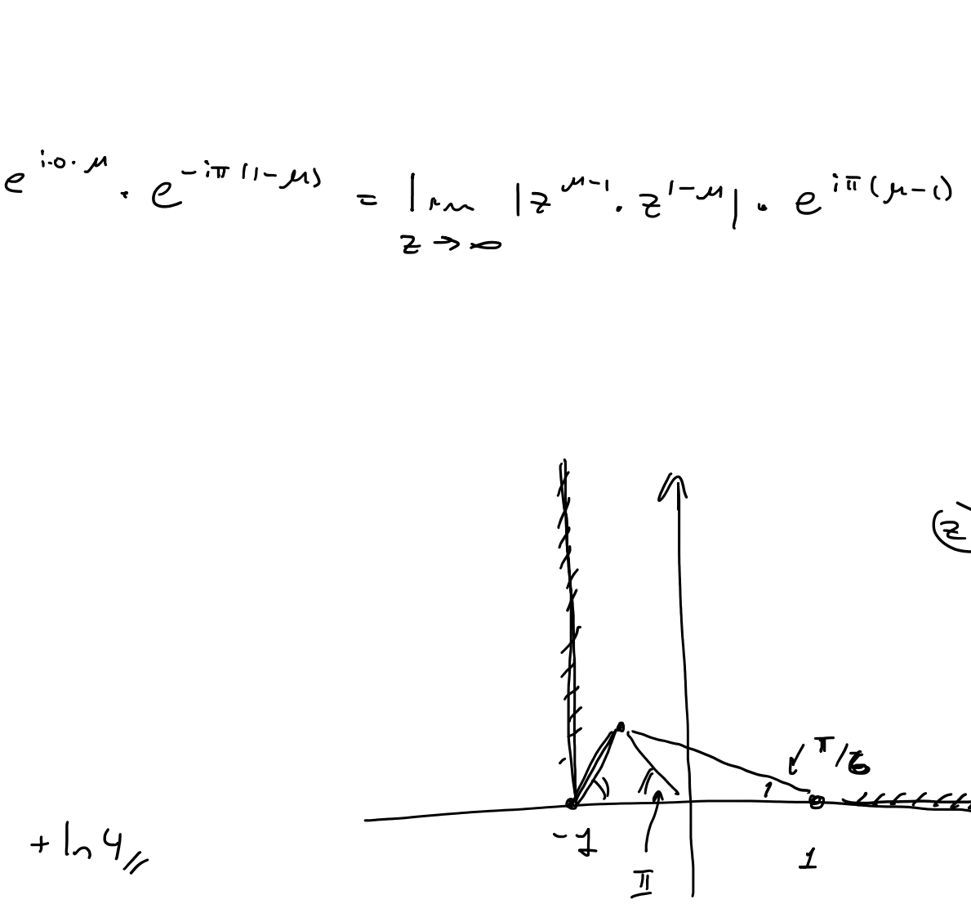
1)



2) $\varphi(3i)=\left|\frac{\sqrt[3]{1-9}}{1}\right|\cdot e^{i\pi/3}\cdot e^{i\cdot0/3}=$

$=2e^{i\pi/3}$ //

2)



(8.) 1) $f(z)=\ln z$, $\xi=\frac{1}{z}$, =>

$f(\xi)=\ln\frac{1}{\xi}=-\ln\xi$

Пусть $f(\xi_0)=-\ln\xi_0$, тогда

$f(\xi)=-\ln\left|\frac{\xi}{\xi_0}\right|-\ln\xi_0+\underbrace{\Delta\arg\xi}$

ф-я не возвращается к исходному значению, =>

$z=\infty$ – точка ветв.

2) $f(z)=\ln\frac{z-1}{z+1}$, $\xi=\frac{1}{z}$, =>

$f(\xi)=\ln\frac{1/\xi-1}{\frac{1}{\xi}+1}=\ln\frac{1-\xi}{1+\xi}$

$\lim_{\xi\to0}\ln\frac{1-\xi}{1+\xi}=\ln1=0$, => ф-я опред. однозначно при $\xi\to0$, $z\to\infty$

3) $f(z)=\ln(z^2-1)$, $\xi=\frac{1}{z}$, =>

$f(\xi)=\ln(\frac{1}{\xi^2}-1)=2\ln\xi+\ln(1-\xi^2)$

опред. однозн. при $\xi\to0$

опред. неодн. при $\xi\to0$, => $f(z)$ не опред. однозначно при $z\to\infty$, =>

$z=\infty$ – точка ветвления

4) $f(z)=\sqrt{z^2-1}$, $\xi=\frac{1}{z}$, =>

$f(\xi)=\sqrt{\frac{1}{\xi^2}-1}=\xi\sqrt{1-\xi^2}$

Пусть $f(\xi_0)=\xi_0\sqrt{1-\xi_0^2}$, тогда

$f(\xi)=\left|\frac{\xi\sqrt{1-\xi^2}}{\xi_0\sqrt{1-\xi_0}}\right|\xi_0\sqrt{1-\xi_0}\cdot\underbrace{e^{i\Delta\arg\xi}}$ => ф-я опред. однозначно при $\xi\to0$, $z\to\infty$

периодич. с периодом 2π

(9.) $f(z)=z^a(z-1)^b$

1) $a=1$, $b=1$, => $f(z)=z^2-z$

$f(z\to\infty)=z^2$ – точек ветв. нет, => $N(1,1)=0$ //

2) $a=1$, $b=1/2$, => $f(z)=z\sqrt{z-1}$ одна точка

$f(z\to\infty)=f(\xi\to0)=\left(\frac{1}{\xi}\right)^{3/2}$ – точка ветвления, => $N(1,\frac{1}{2})=2$ //

3) $a=1/2$, $b=1/3$, => $f(z)=\sqrt{z}(z-1)^{1/3}$ 2 точки ветв.

$f(z\to\infty)=f(\xi\to0)=\left(\frac{1}{\xi}\right)^{1/2}\cdot\left(\frac{1}{\xi}\right)^{1/3}=\left(\frac{1}{\xi}\right)^{5/6}$ – точка ветв., =>

$N(\frac{1}{2};\frac{1}{3})=3$ //

4) $a=2/3$, $b=1/3$, => $f(z)=\sqrt[3]{z^2}\cdot\sqrt[3]{z-1}$ 2 точки ветв.

$f(z\to\infty)=f(\xi\to0)=\left(\frac{1}{\xi^2}\right)^{1/3}\cdot\left(\frac{1}{\xi}\right)^{1/3}=\frac{1}{\xi}$, => $N(\frac{2}{3};\frac{1}{3})=2$ //

не точка ветв.

(10.) $f(z)=\sum_{n=0}^{\infty}\frac{1}{(n+1)(n+2)}z^n=(*)$

$\frac{A}{n+1}+\frac{B}{n+2}=\frac{1}{(n+1)(n+2)}$

$A+B=0$

$2A+B=1$

$A=1$; $B=-1$

$(*)=\sum_{n=0}^{\infty}\frac{z^n}{n+1}-\sum_{n=0}^{\infty}\frac{z^n}{n+2}=$

$=\frac{1}{z}\sum_{n=0}^{\infty}\frac{z^{n+1}}{n+1}-\frac{1}{z^2}\sum_{n=0}^{\infty}\frac{z^{n+2}}{n+2}=\frac{1}{z}\sum_{n=1}^{\infty}\frac{z^n}{n}-\frac{1}{z^2}\sum_{n=2}^{\infty}\frac{z^n}{n}-\frac{1}{z}+\frac{1}{z}=$

$=-\frac{1}{z}\ln(1-z)+\frac{1}{z^2}\ln(1-z)+\frac{1}{z}$

Пусть $f(0)=0$

$f_{\gamma_1}(2)=\frac{1}{2}-\frac{1}{2}\left[\ln\left|\frac{1-2}{1}\right|+i(-\pi)\right]+\frac{1}{4}\left[\ln\left|\frac{1}{1}\right|+i(-\pi)\right]=\frac{1}{2}+\frac{\pi i}{4}$ //

$f_{\gamma_2}(2)=\frac{1}{2}-\frac{1}{2}[\ln1+i\pi]+\frac{1}{4}[\ln1+i\pi]=\frac{1}{2}-\frac{i\pi}{4}$ //

(13.) $I=\mathrm{PV}\int_0^{\infty}\frac{x\,dx}{(x^2+a^2)\sin bx}$

$x\to z$:

$I=\frac{1}{2}\int_{-\infty}^{+\infty}\frac{z\,dz}{(z+ia)(z-ia)\sin bz}=\frac{2\pi i}{2}\sum\operatorname{Res}f(z)=$

$=\pi i\left[\operatorname{Res}_{z=ia}f(z)+\sum\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)\right]=$

$=\pi i\left[\lim_{z\to ia}\frac{z}{\sin bz\cdot(z+ia)}+\operatorname{Res}_{z=\frac{\pi}{b}}f(z)+\operatorname{Res}_{z=-\frac{\pi}{b}}f(z)+\operatorname{Res}_{z=0}f(z)+\right.$

$\left.+\sum_{n=2}^{\infty}\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)+\sum_{n=-\infty}^{2}\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)\right]=$

$=\pi i\left[\frac{1}{2\sin iab}+\lim_{|z|\to0}\int_0^{\pi}\frac{\frac{\pi}{b}+|z|e^{i\varphi}}{a^2+(\frac{\pi}{b}+|z|e^{i\varphi})^2}\cdot\frac{b|z|e^{i\varphi}\,d\varphi}{b\underset{0}{\sin\pi}\cos|z|e^{i\varphi}b+\cos\pi\sin|z|e^{i\varphi}b}+\right.$

$+\lim_{|z|\to0}\int_0^{\pi}\frac{-\pi/b+|z|e^{i\varphi}}{a^2+(-\frac{\pi}{b}+|z|e^{i\varphi})^2}\cdot\frac{b|z|e^{i\varphi}\,d\varphi}{-b\sin\pi\cos|z|e^{i\varphi}b+\cos\pi\sin|z|e^{i\varphi}b}+$

$\left.+\lim_{|z|\to0}\int_0^{\pi}\frac{|z|e^{i\varphi}}{a^2+|z|^2e^{2i\varphi}}\cdot\frac{i|z|e^{i\varphi}}{\sin|z|e^{i\varphi}}d\varphi+\sum_{n\neq\pm1,0}\operatorname{Res}_{z=\frac{\pi n}{b}}f\right]=$

$=\pi i\left[-\frac{i}{2\operatorname{sh}ab}+\int_0^{\pi}\frac{\pi/b}{a^2+\pi^2/b^2}\cdot\frac{1}{b}d\varphi+\int_0^{\pi}\frac{-\pi/b}{a^2+\pi^2/b^2}\cdot\frac{1}{b}d\varphi+\right.$

$\left.+0+\sum_{n\neq\pm1,0}\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)\right]=(*)$ Заметим, что $\lim_{|z|\to0}\int_0^{\pi}f(\frac{\pi n}{b}+z)\,dz=$

$=\int_0^{\pi}\frac{\pi n/b}{a^2+\frac{\pi^2n^2}{b^2}}\cdot\frac{1}{b}d\varphi$, => $\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)=-\operatorname{Res}_{z=-\frac{\pi n}{b}}f(z)$, =>

$\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)+\operatorname{Res}_{z=-\frac{\pi n}{b}}f(z)=0$, => $\sum_n\operatorname{Res}_{z=\frac{\pi n}{b}}f(z)=0$

$(*)=\pi i\cdot\frac{-i}{2\operatorname{sh}ab}=\frac{\pi}{2\operatorname{sh}ab}$ //