

# CS 6150: HW0 – L<sup>A</sup>T<sub>E</sub>X Intro and Background

Submission date: Tuesday, Aug 30 2016

This assignment has 5 questions, for a total of 50 points. Unless otherwise specified, complete and reasoned arguments will be expected for all answers.

Question	Points	Score
Basic Background: Big Oh	10	
Removing Duplicates	10	
Proof Practice: Square vs. Multiply	10	
Background: Probability	10	
Array Sums	10	
Total:	50	

Question 1: Basic Background: Big Oh ..... [10]

Write each of the functions below ( $n$  is the positive integer variable) in the Big-Oh notation.

- (a) [2]  $f(n) = n^2 + 5n + 20$ .
- (b) [2]  $f(n) = 2 \log n + 4$ .
- (c) [2]  $f(n) = \frac{1}{n^2} + \frac{2}{n}$ .
- (d) [2]  $f(n) = \frac{1}{n^2} + 1$ .
- (e) [2]  $f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ .

Question 2: Removing Duplicates ..... [10]

Let  $A[1 \dots n]$  be an array of integers. Describe an algorithm that runs in time  $O(n \log n)$  and returns an array  $B$  whose entries are all the *distinct* elements of  $A$  (i.e., with no duplicates).

Question 3: Proof Practice: Square vs. Multiply ..... [10]

Suppose I tell you that there is an algorithm that can square any  $n$  digit number in time  $O(n \log n)$ , for all  $n \geq 1$ . Then, prove that there is an algorithm that can find the product of *any two*  $n$  digit numbers in time  $O(n \log n)$ . [*Hint*: think of using the squaring algorithm as a subroutine to find the product.]

Question 4: Background: Probability ..... [10]

Write the answers to the following as functions of  $k$ :

- (a) [4] Suppose we toss a fair coin  $k$  times. What is the probability that we see heads precisely once?
- (b) [6] Suppose we have  $k$  different boxes, and suppose that every box is colored uniformly at random with one of  $k$  colors (independently of the other boxes). What is the probability that all the boxes get distinct colors?

Question 5: Array Sums ..... [10]

Given an array  $A[1 \dots n]$  of integers (not necessarily distinct), find if there exist indices  $i, j, k$  such that  $A[i] = A[j] + A[k]$ . Can you find an algorithm with running time  $o(n^3)$ ? [NOTE: this is the little-oh notation, i.e., the algorithm should run in time  $< cn^3$ , for any constant  $c$ , as  $n \rightarrow \infty$ .] [*Hint*: aim for an algorithm with running time  $O(n^2 \log n)$ .]