Regression with factor variables

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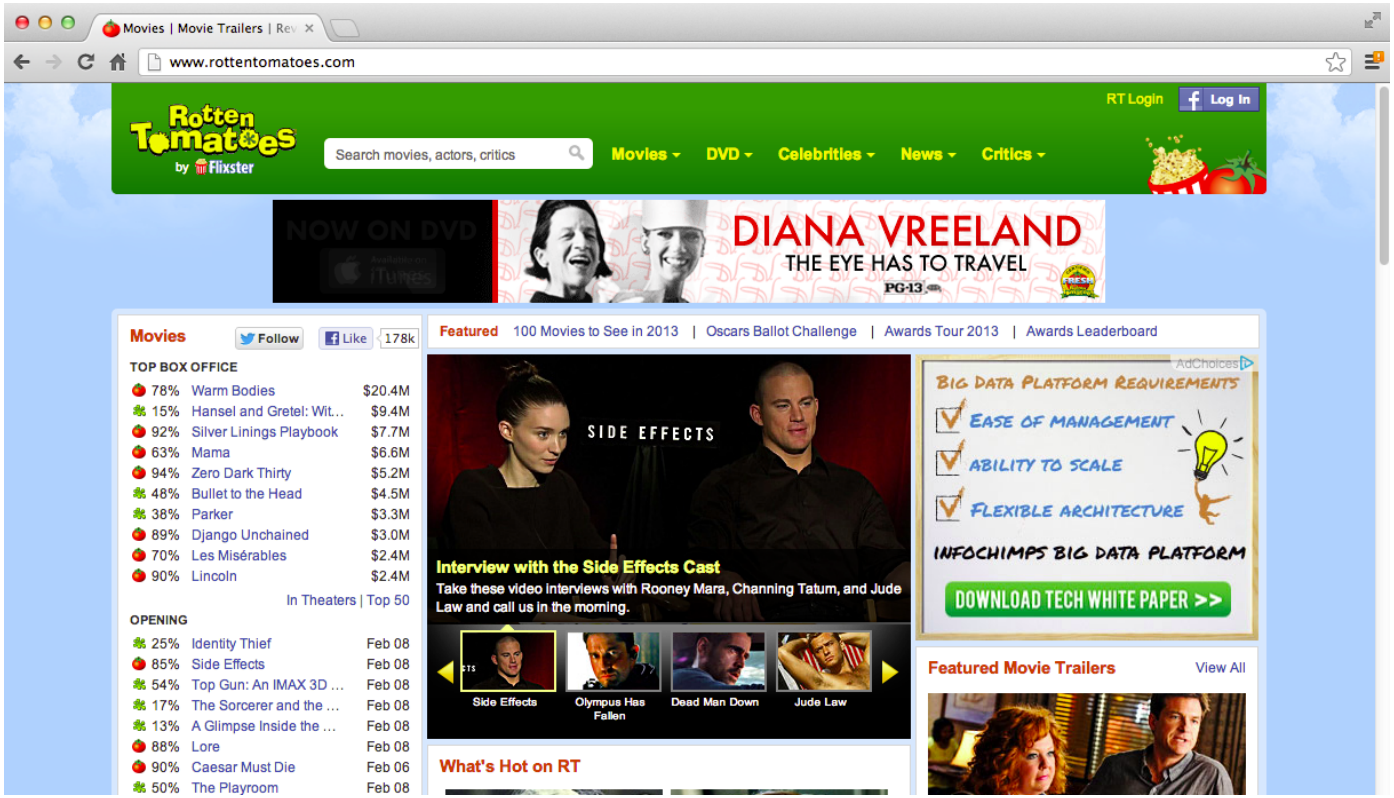
Key ideas

- Outcome is still quantitative
- Covariate(s) are factor variables
- Fitting lines = fitting means
- Want to evaluate contribution of all factor levels at once

Example: Movie ratings



http://www.rottentomatoes.com/

Movie Data

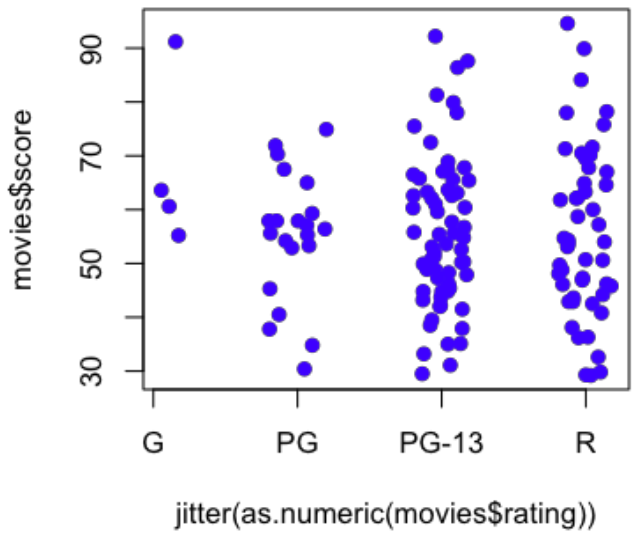
```
download.file("http://www.rossmanchance.com/iscam2/data/movies03RT.txt",destfile="./data/movies.txt")
movies <- read.table("./data/movies.txt",sep="\t",header=T,quote="")
head(movies)
```

```
                X score rating            genre box.office running.time
1 2 Fast 2 Furious  48.9  PG-13 action/adventure     127.15          107
2    28 Days Later  78.2      R           horror      45.06          113
3      A Guy Thing  39.5  PG-13       rom comedy      15.54          101
4      A Man Apart  42.9      R action/adventure      26.25          110
5    A Mighty Wind  79.9  PG-13           comedy      17.78           91
6 Agent Cody Banks  57.9     PG action/adventure      47.81          102
```

http://www.rossmanchance.com/

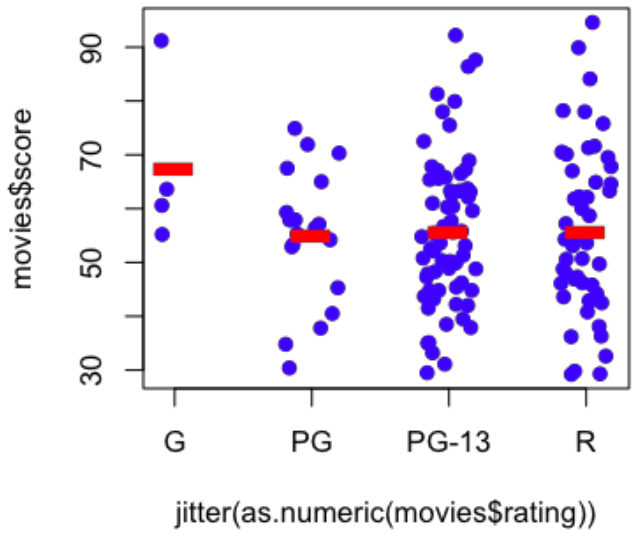
Rotton tomatoes score vs. rating

```
plot(movies$score ~ jitter(as.numeric(movies$rating)),col="blue",xaxt="n",pch=19)
axis(side=1,at=unique(as.numeric(movies$rating)),labels=unique(movies$rating))
```



Average score by rating

```
plot(movies$score ~ jitter(as.numeric(movies$rating)),col="blue",xaxt="n",pch=19)
axis(side=1,at=unique(as.numeric(movies$rating)),labels=unique(movies$rating))
meanRatings <- tapply(movies$score,movies$rating,mean)
points(1:4,meanRatings,col="red",pch="-",cex=5)
```



Another way to write it down

$$S_i = b_0 + b_1 \mathbb{1}(Ra_i = \text{" } PG \text{ "}) + b_2 \mathbb{1}(Ra_i = \text{" } PG-13 \text{ "}) + b_3 \mathbb{1}(Ra_i = \text{" } R \text{ "}) + e_i$$

The notation $\mathbb{1}(Ra_i = \text{" } PG \text{ "})$ is a logical value that is one if the movie rating is "PG" and zero otherwise.

Average values

b_0 = average of the G movies

$b_0 + b_1$ = average of the PG movies

$b_0 + b_2$ = average of the PG-13 movies

$b_0 + b_3$ = average of the R movies

Here is how you do it in R

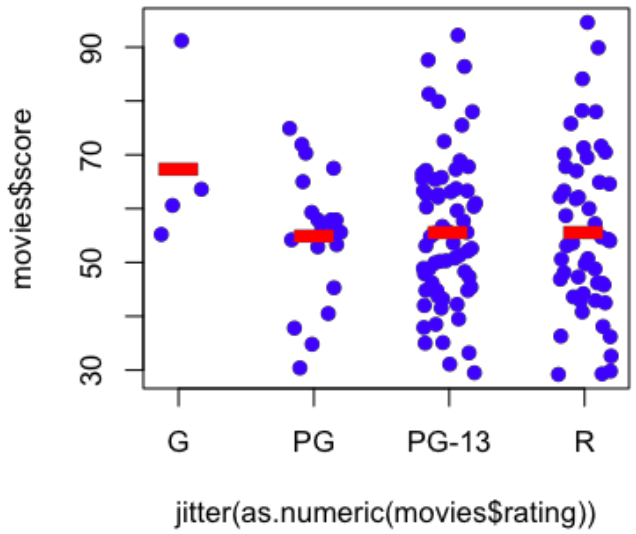
```
lm1 <- lm(movies$score ~ as.factor(movies$rating))
summary(lm1)
```

```

Call:
lm(formula = movies$score ~ as.factor(movies$rating))

Residuals:
   Min     1Q Median     3Q    Max
-26.43  -9.98  -0.98   9.34  38.97

Coefficients:
                              Estimate Std. Error t value Pr(>|t|)
(Intercept)                      67.65       7.19    9.40   <2e-16 ***
as.factor(movies$rating)PG      -12.59       7.85   -1.60     0.11
as.factor(movies$rating)PG-13   -11.81       7.41   -1.59     0.11
as.factor(movies$rating)R       -12.02       7.48   -1.61     0.11
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.4 on 136 degrees of freedom
Multiple R-squared:  0.0199,    Adjusted R-squared:  -0.00177
F-statistic: 0.918 on 3 and 136 DF,  p-value: 0.434
```

8/20

Plot fitted values

```
plot(movies$score ~ jitter(as.numeric(movies$rating)),col="blue",xaxt="n",pch=19)
axis(side=1,at=unique(as.numeric(movies$rating)),labels=unique(movies$rating))
points(1:4,lm1$coeff[1] + c(0,lm1$coeff[2:4]),col="red",pch="-",cex=5)
```



Question 1

Average values

b_0 = average of the G movies

$b_0 + b_1$ = average of the PG movies

$b_0 + b_2$ = average of the PG-13 movies

$b_0 + b_3$ = average of the R movies

What is the average difference in rating between G and R movies?

$b_0 + b_3 - b_0 = b_3$

Question 1 in R

```
lm1 <- lm(movies$score ~ as.factor(movies$rating))
summary(lm1)
```

```

Call:
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as.factor(movies$rating)R       -12.02       7.48   -1.61     0.11
---
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Multiple R-squared:  0.0199,    Adjusted R-squared:  -0.00177
F-statistic: 0.918 on 3 and 136 DF,  p-value: 0.434
```

Question 1 in R

```
lm1 <- lm(movies$score ~ as.factor(movies$rating))
confint(lm1)
```

```
                              2.5 % 97.5 %
(Intercept)                   53.42 81.875
as.factor(movies$rating)PG    -28.11  2.928
as.factor(movies$rating)PG-13 -26.47  2.842
as.factor(movies$rating)R     -26.80  2.763
```

Question 2

Average values

b_0 = average of the G movies

$b_0 + b_1$ = average of the PG movies

$b_0 + b_2$ = average of the PG-13 movies

$b_0 + b_3$ = average of the R movies

What is the average difference in rating between $PG-13$ and R movies?

$b_0 + b_2 - (b_0 + b_3) = b_2 - b_3$

We could rewrite our model

$$S_i = b_0 + b_1 \mathbb{1}(Ra_i = \text{" } G \text{ "}) + b_2 \mathbb{1}(Ra_i = \text{" } PG \text{ "}) + b_3 \mathbb{1}(Ra_i = \text{" } PG-13 \text{ "}) + e_i$$

Average values

b_0 = average of the R movies

$b_0 + b_1$ = average of the G movies

$b_0 + b_2$ = average of the PG movies

$b_0 + b_3$ = average of the PG-13 movies

What is the average difference in rating between $PG-13$ and R movies?

$b_0 + b_3 - b_0 = b_3$

Question 2 in R

```
lm2 <- lm(movies$score ~ relevel(movies$rating,ref="R"))
summary(lm2)
```

```

Call:
lm(formula = movies$score ~ relevel(movies$rating, ref = "R"))

Residuals:
   Min     1Q Median     3Q    Max 
-26.43  -9.98  -0.98   9.34  38.97 

Coefficients:
                                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)                           55.630      2.035   27.34   <2e-16 ***
relevel(movies$rating, ref = "R")G    12.020      7.476    1.61     0.11    
relevel(movies$rating, ref = "R")PG   -0.573      3.741   -0.15     0.88    
relevel(movies$rating, ref = "R")PG-13  0.205      2.706    0.08     0.94    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.4 on 136 degrees of freedom
Multiple R-squared:  0.0199,    Adjusted R-squared:  -0.00177 
F-statistic: 0.918 on 3 and 136 DF,  p-value: 0.434
```

Question 2 in R

```
lm2 <- lm(movies$score ~ relevel(movies$rating,ref="R"))
confint(lm2)
```

```
                                    2.5 % 97.5 %
(Intercept)                        51.606 59.654
relevel(movies$rating, ref = "R")G -2.763 26.803
relevel(movies$rating, ref = "R")PG -7.971  6.825
relevel(movies$rating, ref = "R")PG-13 -5.146  5.557
```

Question 3

$$S_i = b_0 + b_1 \mathbb{1}(Ra_i = \text{" } PG \text{ "}) + b_2 \mathbb{1}(Ra_i = \text{" } PG-13 \text{ "}) + b_3 \mathbb{1}(Ra_i = \text{" } R \text{ "}) + e_i$$

Average values

b_0 = average of the G movies

$b_0 + b_1$ = average of the PG movies

$b_0 + b_2$ = average of the PG-13 movies

$b_0 + b_3$ = average of the R movies

Is there any difference in score between any of the movie ratings?

Question 3 in R

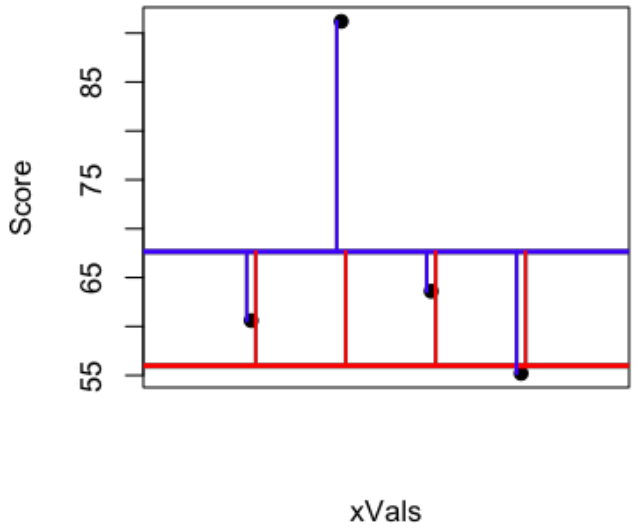
```
lm1 <- lm(movies$score ~ as.factor(movies$rating))
anova(lm1)
```

```
Analysis of Variance Table

Response: movies$score
                          Df Sum Sq Mean Sq F value Pr(>F)
as.factor(movies$rating)   3    570     190    0.92   0.43
Residuals                136  28149     207
```

Sum of squares (G movies)

```
gMovies <- movies[movies$rating=="G",]; xVals <- seq(0.2,0.8,length=4)
plot(xVals,gMovies$score,ylab="Score",xaxt="n",xlim=c(0,1),pch=19)
abline(h=mean(gMovies$score),col="blue",lwd=3); abline(h=mean(movies$score),col="red",lwd=3)
segments(xVals+0.01,rep(mean(gMovies$score),length(xVals)),xVals+0.01,
        rep(mean(movies$score),length(xVals)),col="red",lwd=2)
segments(xVals-0.01,gMovies$score,xVals-0.01,rep(mean(gMovies$score),length(xVals)),col="blue",lwd=2)
```



Tukey's (honestly significant difference test)

```
lm1 <- aov(movies$score ~ as.factor(movies$rating))
TukeyHSD(lm1)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = movies$score ~ as.factor(movies$rating))

$`as.factor(movies$rating)`
             diff     lwr    upr  p adj
PG-G     -12.5929 -33.008  7.822 0.3795
PG-13-G  -11.8146 -31.092  7.463 0.3854
R-G      -12.0200 -31.464  7.424 0.3776
PG-13-PG   0.7782  -8.615 10.171 0.9964
R-PG       0.5729  -9.158 10.304 0.9987
R-PG-13   -0.2054  -7.245  6.834 0.9998
```

http://en.wikipedia.org/wiki/Tukey's_range_test