ANOVA with multiple factors/variables

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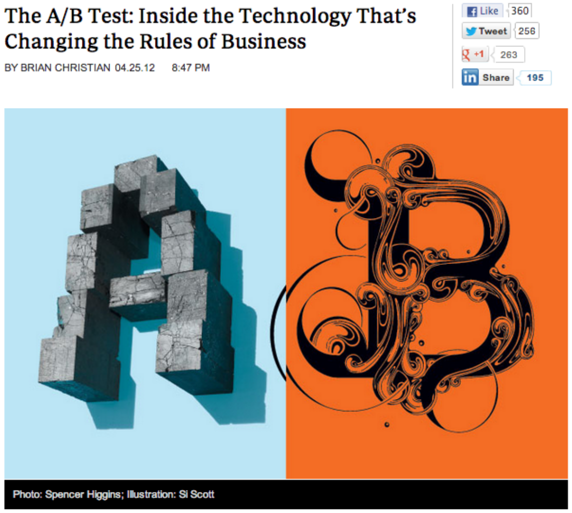
Key ideas

- Outcome is still quantitative
- You have multiple explanatory variables
- Goal is to identify contributions of different variables

A successful example



The A/B Test: Inside the Technology That's Changing the Rules of Business

BY BRIAN CHRISTIAN 04.25.12 8:47 PM

Photo: Spencer Higgins; Illustration: Si Scott

"For the button, an A/B test of three new word choices—"Learn More," "Join Us Now," and "Sign Up Now"—revealed that "Learn More" garnered 18.6 percent more signups per visitor than the default of "Sign Up." Similarly, a black-and-white photo of the Obama family outperformed the default turquoise image by 13.1 percent. Using both the family image and "Learn More," signups increased by a thundering 40 percent."

http://www.wired.com/business/2012/04/ff_abtesting/

Movie Data

```
download.file("http://www.rossmanchance.com/iscam2/data/movies03RT.txt",
              destfile="./data/movies.txt")
movies <- read.table("./data/movies.txt",sep="\t",header=T,quote="")
head(movies)
```

```
                 X score rating            genre box.office running.time
1 2 Fast 2 Furious  48.9  PG-13 action/adventure     127.15          107
2    28 Days Later  78.2      R           horror      45.06          113
3      A Guy Thing  39.5  PG-13       rom comedy      15.54          101
4      A Man Apart  42.9      R action/adventure      26.25          110
5    A Mighty Wind  79.9  PG-13           comedy      17.78           91
6 Agent Cody Banks  57.9     PG action/adventure      47.81          102
```

http://www.rossmanchance.com/

Relating score to rating

$$S_i = b_0 + b_1 \mathbb{1}(Ra_i = "\ PG\ ") + b_2 \mathbb{1}(Ra_i = "\ PG - 13\ ") + b_3 \mathbb{1}(Ra_i = "\ R\ ") + e_i$$

The notation $\mathbb{1}(Ra_i = "\ PG\ ")$ is a logical value that is one if the movie rating is "PG" and zero otherwise.

Average values

b_0 = average of the G movies

$b_0 + b_1$ = average of the PG movies

$b_0 + b_2$ = average of the PG-13 movies

$b_0 + b_3$ = average of the R movies

ANOVA in R

```
aovObject <- aov(movies$score ~ movies$rating)
aovObject
```

```
Call:
   aov(formula = movies$score ~ movies$rating)

Terms:
                movies$rating Residuals
Sum of Squares            570     28149
Deg. of Freedom             3       136

Residual standard error: 14.39
Estimated effects may be unbalanced
```

ANOVA in R

```
aovObject$coeff
```

```
        (Intercept)     movies$ratingPG movies$ratingPG-13      movies$ratingR
              67.65              -12.59              -11.81              -12.02
```

Adding a second factor

$$S_i = b_0 + b_1 \mathbb{1}(Ra_i = "\ PG\ ") + b_2 \mathbb{1}(Ra_i = "\ PG - 13\ ") + b_3 \mathbb{1}(Ra_i = "\ R\ ")$$

$$+\gamma_1 \mathbb{1}(G_i = "\ action\ ") + \gamma_2 \mathbb{1}(G_i = "\ animated\ ") + \ldots + e_i$$

The notation $\mathbb{1}(Ra_i = "\ PG\ ")$ is a logical value that is one if the movie rating is "PG" and zero otherwise.

Adding a second factor

$$S_i = b_0 + \underbrace{b_1 \mathbb{1}(Ra_i = "\ PG\ ") + b_2 \mathbb{1}(Ra_i = "\ PG - 13\ ") + b_3 \mathbb{1}(Ra_i = "\ R\ ")}_{rating}$$

$$+\gamma_1 \underbrace{\mathbb{1}(G_i = "\ action\ ") + \gamma_2 \mathbb{1}(G_i = "\ animated\ ") + \ldots}_{genre} + e_i$$

There are only 2 variables in this model. They have multiple levels.

Second variable

```
aovObject2 <- aov(movies$score ~ movies$rating + movies$genre)
aovObject2
```

```
Call:
   aov(formula = movies$score ~ movies$rating + movies$genre)

Terms:
                movies$rating movies$genre Residuals
Sum of Squares            570         3935     24214
Deg. of Freedom             3           12       124

Residual standard error: 13.97
Estimated effects may be unbalanced
```

ANOVA Summary

```
summary(aovObject2)
```

```
               Df Sum Sq Mean Sq F value Pr(>F)
movies$rating   3    570     190    0.97  0.408
movies$genre   12   3935     328    1.68  0.079 .
Residuals     124  24214     195
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Order matters

```
aovObject3 <- aov(movies$score ~ movies$genre + movies$rating)
summary(aovObject2)
```

```
               Df Sum Sq Mean Sq F value Pr(>F)
movies$rating   3    570     190    0.97  0.408
movies$genre   12   3935     328    1.68  0.079 .
Residuals     124  24214     195
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Order matters

```
aovObject3 <- aov(movies$score ~ movies$genre + movies$rating)
summary(aovObject3)
```

```
                Df Sum Sq Mean Sq F value Pr(>F)  
movies$genre    12   4222     352    1.80  0.055 .
movies$rating    3    284      95    0.48  0.694  
Residuals      124  24214     195                 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Adding a quantitative variable

$$S_i = b_0 + \underbrace{b_1 \mathbb{1}(Ra_i = "\ PG\ ") + b_2 \mathbb{1}(Ra_i = "\ PG - 13\ ") + b_3 \mathbb{1}(Ra_i = "\ R\ ")}_{rating}$$

$$+ \underbrace{\gamma_1 \mathbb{1}(G_i = "\ action\ ") + \gamma_2 \mathbb{1}(G_i = "\ animated\ ") + \dots}_{genre} + \eta_1 \underbrace{BO_i}_{box\ office} + e_i$$

There are three variables in this model - box office is quantitative so only has one term.

ANOVA with quantitative variable in R

```
aovObject4 <- aov(movies$score ~ movies$genre + movies$rating + movies$box.office)
summary(aovObject4)
```

```
                   Df Sum Sq Mean Sq F value  Pr(>F)    
movies$genre       12   4222     352    2.19   0.016 *  
movies$rating       3    284      95    0.59   0.624    
movies$box.office   1   4421    4421   27.47 6.7e-07 ***
Residuals         123  19793     161                    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Language and further resources

- Units - one observation
- Treatments - applied to units
- Factors - controlled by experimenters
- Replicates - multiple (independent) units with the same factors/treatments
- Wikipedia on Experimental Design
- Wikipedia on ANOVA
- Wikipedia on A/B Testing