Inner product : $x^T y$

Outer product : $x y^T$

$Cov(A,B) = \mathbb{E}(AB) - \mathbb{E}(A)\,\mathbb{E}(B)$

Eigen decomposition

$$A = \begin{bmatrix} 1 & 4 \\ 9 & 1 \end{bmatrix} \qquad \lambda_1 = 7,\ V_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \lambda_2 = -5,\ V_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{bmatrix} \qquad u_2 = \begin{bmatrix} 2/\sqrt{13} \\ -3/\sqrt{13} \end{bmatrix}$$

$$\begin{matrix} Au_1 = \lambda_1 u_1 \\ Au_2 = \lambda_2 u_2 \end{matrix} \longrightarrow A \begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix} = \begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\downarrow$$

$$AU = U\Lambda$$

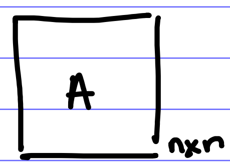
$$\downarrow$$

$$\boxed{A = U\Lambda U^{-1}}$$

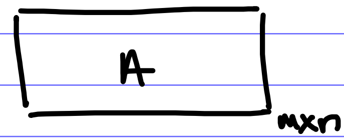
given A is square & diagonalizable

$$A^3 = U\Lambda^3 U^{-1}$$

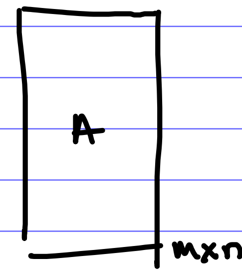
2 multiplications



Inverse can exist
&
unique



Inverse can't exist
$n > m$



Inverse can exist
BUT
not unique
$m > n$

→ referring to left inverse

Rank of a Matrix:

$$\begin{bmatrix} | & | & | & | & | \\ a_1 & a_2 & a_3 & a_4 & a_5 \\ | & | & | & | & | \end{bmatrix} = \begin{bmatrix} | & | \\ a_1 & a_2 \\ | & | \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 & 0 \end{bmatrix}$$

A is $N \times P$
A' is $N \times \underline{\underline{k}}$ → Rank
R is $K \times P$

$\left.\begin{matrix} \\ \\ \end{matrix}\right] k(N+P)$

→ # of linearly independent columns

Reduction: $\dfrac{k(N+P)}{NP}$

Note: in real world datasets we won't find exact linear combination but in many applications we can afford a close approx while requiring less storage.