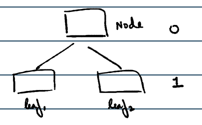
AdaBoost

Sunday 5 May 2024 2:40 PM

→ Stage wise Additive method

→ Weak learners (High Bias low variance)

→ Decision stumps



→ ~~1~~/0 $\overset{(+ve)}{1}$/$\overset{(-ve)}{-1}$

→ In Random Forest each tree has an equal vote on the final prediction but in a Forest of stumps made with AdaBoost, some stumps get more say in the final prediction than others.

→ In Random Forest each DT is made independently of each other. But, in a Forest of stumps made with AdaBoost, order is important. The errors that the first stump makes influence how the second stump is made and so on....

→ Trees are fit sequentially to improve error of previous trees

Algorithm 10.1 *AdaBoost.M1.*

1. Initialize the observation weights $w_i = 1/N,\ i = 1, 2, \ldots, N$.
2. For $m = 1$ to M:
   (a) Fit a classifier $G_m(x)$ to the training data using weights w_i.
   (b) Compute
   $$\text{err}_m = \frac{\sum_{i=1}^N w_i I(y_i \neq G_m(x_i))}{\sum_{i=1}^N w_i}.$$
   (c) Compute $\alpha_m = \log((1 - \text{err}_m)/\text{err}_m)$.
   (d) Set $w_i \leftarrow w_i \cdot \exp[\alpha_m \cdot I(y_i \neq G_m(x_i))],\ i = 1, 2, \ldots, N$.
3. Output $G(x) = \text{sign}\left[\sum_{m=1}^M \alpha_m G_m(x)\right]$.

Algorithm

Data

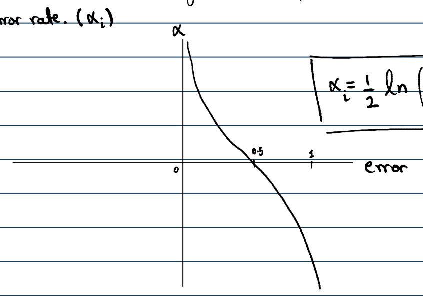
x_1	x_2	y
1	5	1
2	4	0
9	2	1
1	7	0
2	3	1

1) assign initial weight to each row.

weight = $\frac{1}{n}$ ← total # of rows

x_1	x_2	y	wt
1	5	1	1/5
2	4	0	1/5
9	2	1	1/5
1	7	0	1/5
2	3	1	1/5

2) train a Decision stump
- check performance on data [calc predictions]
- calculate model's weight, which depends on the error rate. (α_i)



$$\boxed{\alpha_i = \frac{1}{2} \ln\left(\frac{1-\text{error}}{\text{error}}\right)}$$

if error is 1 or 0 equation will panic add a very small number.

x_1	x_2	y	wt	$\hat{y}$
1	5	1	1/5	0
2	4	0	1/5	1
9	2	1	1/5	1
1	7	0	1/5	0
2	3	1	1/5	1

→ error = sum of weights of misclassified points

error = $\frac{2}{5}$ $\alpha_1 = \frac{1}{2} \ln\left(\frac{1 - 2/5}{2/5}\right) \approx 0.2$

3) Update weights [increasing the importance of misclassified points
decreasing the " " correctly classified "]

<u>For misclassified</u>

new_wt = curr_wt x e^{α_i}

<u>For correctly classified</u>

new_wt = curr_wt x $e^{-\alpha_i}$

x_1	x_2	y	wt	$\hat{y}$	new_wt	normalized
1	5	1	1/5	0	0.24	0.25
2	4	0	1/5	1	0.24	0.25
9	2	1	1/5	1	0.16	0.166
1	7	0	1/5	0	0.16	0.166
2	3	1	1/5	1	0.16	0.166
					0.96	

→ Normalize the weights

x_1	x_2	y	new_wt	$\hat{y}$	Range
1	5	1	0.25	0	0-0.25
2	4	0	0.25	1	0.25-0.50
9	2	1	0.166	1	0.50-0.666
1	7	0	0.166	0	0.666-0.832
2	3	1	0.166	1	0.832-1

4) Upsampling
- choose n random # b/w 0-1

e.g. 0.54, 0.2, 0.99, 0.06, 0.31
(3) (1) (5) (1) (2)

- check which range they fall in & add that row to new dataset

x_1	x_2	y	wt
9	2	1	1/5
1	5	1	1/5
2	3	1	1/5
1	5	1	1/5
2	4	0	1/5

→ reset weights to 1/n

sklearn.ensemble.AdaBoostClassifier

class sklearn.ensemble.**AdaBoostClassifier**(*estimator=None, *, n_estimators=50, learning_rate=1.0, algorithm='SAMME.R', random_state=None*) [source]

lr x α_i

↑ n_estimators ↓ learning_rate

5) Redo steps 2-4 for as many decision stumps in model.

6) $\hat{y} = \text{sign}(\alpha_1 h_1(x) + \alpha_2 h_2(x) + \alpha_3 h_3(x) + \ldots + \alpha_n h_n(x))$

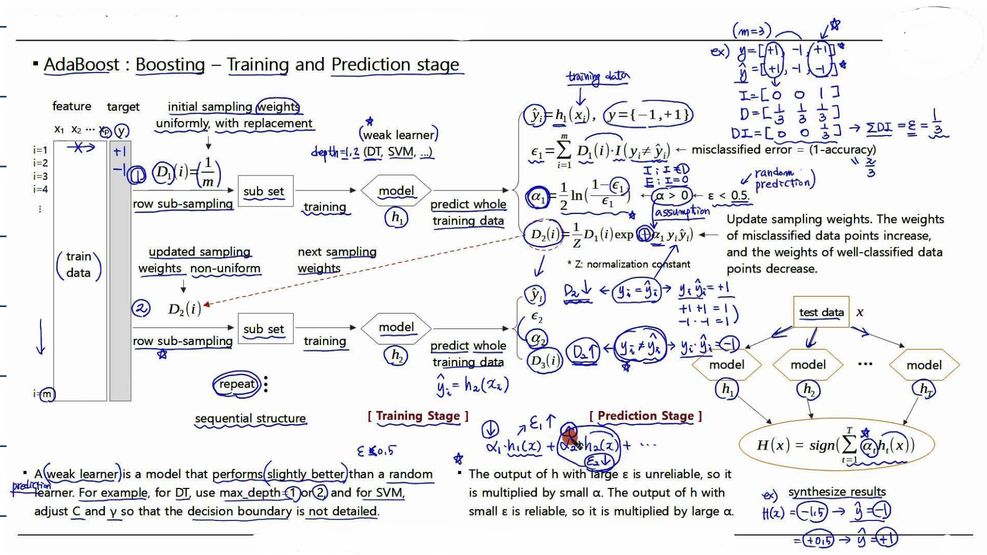
AdaBoost Revisited

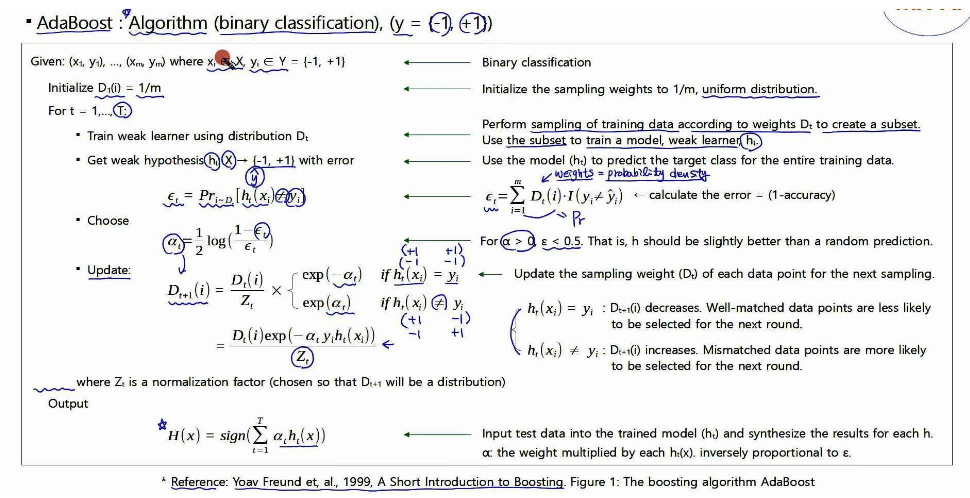
Saturday 25 May 2024 1:01 AM

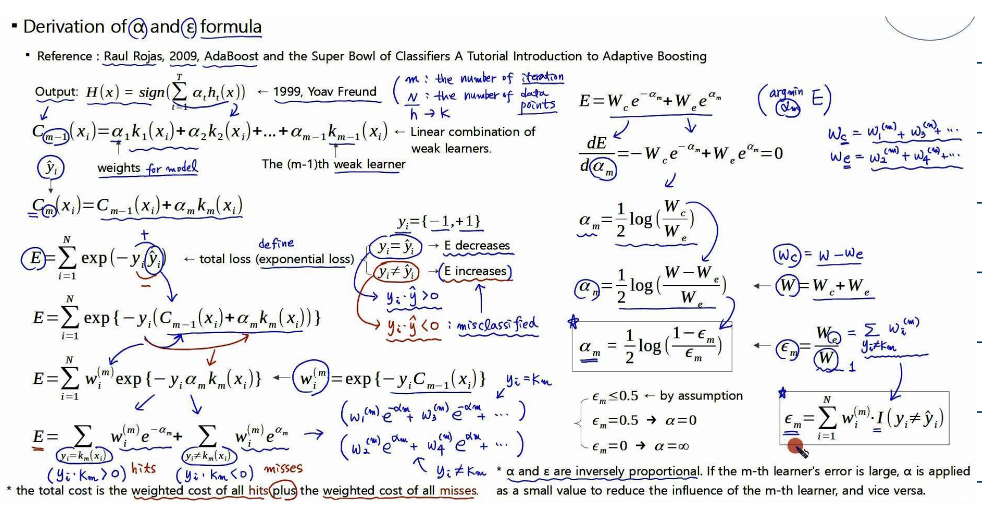
→ Boosting is a method of gradually strengthening a weak learner model.

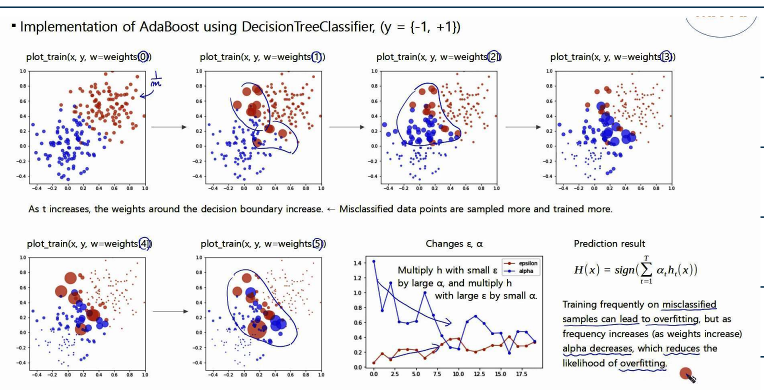
→ In estimation stage, the results of multiple models are combined.

→ AdaBoost is an algorithm that increases accuracy by focusing more on misclassified data points during training stage.









▪ AdaBoost : Algorithm (binary classification), (y = {0, 1})

- Use y = {0, 1} instead of y = {-1, +1}. It can be easily extended to multiclass classification.

Algorithm 1: AdaBoost (Freund & Schapire 1997)

1. Initialize the observation weights $w_i = 1/n, \ i=1,2,\dots,n$
2. **For** m = 1 **to** M:
   a) Fit a classifier $T^{(m)}(x)$ to the training data using weights w_i
   b) Compute

   actual class (y) · training data

   $$err^{(m)} = \sum_{i=1}^{n} w_i \cdot I(c_i \neq T^{(m)}(x_i)) \ / \ \sum_{i=1}^{n} w_i$$

   ε · predicted class ($\hat{y}$)

   c) Compute

   $$\alpha^{(m)} = \log \frac{1-err^{(m)}}{err^{(m)}}$$

   d) Set

   $$w_i \leftarrow w_i \cdot \exp(\alpha^{(m)} \cdot I(c_i \neq T^{(m)}(x_i))), \quad i=1,2,\dots,n$$

   e) Re-normalize w_i

   $y \neq \hat{y} \rightarrow I=1 \rightarrow w \uparrow$
   $y = \hat{y} \rightarrow I=0 \rightarrow w \rightarrow$ ↓

3. Output

$$C(x) = \underset{k}{argmax} \sum_{m=1}^{M} \alpha^{(m)} \cdot I(T^{(m)}(x)=k)$$

* Reference: Ji Zhu, et, al., 2006, Multi-class AdaBoost, Algorithm 1

In this form, multiple class classification is also possible, but α <0 problem occurs if $err^{(m)} > 0.5$. In binary classification, α <0.5 is possible even in weak learners, but in three classes, α <0 occurs easily because the accuracy of random prediction is 1/3 and $err^{(m)} = 0.67$. → Need to be improved → SAMME algorithm in the next video.

It is always 1 because it is normalized.

Not multiplied by 1/2. The reason was not explained.

Use argmax instead of the sign function.

$\hat{y}$ · $\varepsilon\uparrow$ · α
m=1 : 0 · [1 0] × 0.1 = [0.1 0]
2 : 0 · [1 0] × 0.1 = [0.1 0]
3 : 1 · $\varepsilon\downarrow$ · [0 1] × 0.9 = [0 0.9]
4 : 0 · [1 0] × 0.1 = [0.1 0]
majority = 0 · Σ = [0.3 0.9]
($\hat{y}$ = 1)

(example)

test data · $\hat{y}$

$T^{(m)}(x)$ · $I(T^{(m)}(x)=k)$

	$\hat{y}$		k=0 1	$\alpha^{(m)}$		
m=1:	0	→	[1 0]	* 0.5	=	[0.5 0]
m=2:	0	→	[1 0]	* 0.7	=	[0.7 0]
m=3:	1	→	[0 1]	* 0.2	=	[0 0.2]
m=4:	0	→	[1 0]	* 0.6	=	[0.6 0]
					sum =	[1.8 0.2]

$\hat{y}$ → OHE · argmax = 0 ← C(x)

▪ Multiclass Classification – SAMME Algorithm (Stagewise Additive Modeling using a Multi-class Exponential loss function)

- In 2006, Ji Zhu et al. proposed the following algorithm to solve the multiclass problem of Freund's (1995) AdaBoost algorithm.
- The err(m) < 0.5 condition in Freund's model was modified to err(m) < 1 – 1/K (K: the number of classes). If K=3, then err(m) < 0.67.

ε

Multi-class AdaBoost

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Abstract

Boosting has been a very successful technique for solving the two-class classification problem. In going from two-class to multi-class classification, most algorithms have been restricted to reducing the multi-class classification problem to multiple two-class problems. In this paper, we propose a new algorithm that naturally extends the original AdaBoost algorithm to the multiclass case without reducing it to multiple two-class problems. Similar to AdaBoost in the two-class case, this new algorithm combines weak classifiers and only requires the performance of each weak classifier be better than random guessing (rather than 1/2). We further provide a statistical justification for the new algorithm using a novel multi-class exponential loss function and forward stage-wise additive modeling. As shown in the paper, the new algorithm is extremely easy to implement and is highly competitive with the best currently available multi-class classification methods.

Algorithm 2: **SAMME** · y = {0, 1} → y = {0, 1, 2 ⋯ K-1}

1. Initialize the observation weights $w_i = 1/n, \ i=1,2,\dots,n$
2. **For** m = 1 **to** M:
   a) Fit a classifier $T^{(m)}(x)$ to the training data using weights w_i · if K=2, err < 1 − 1/2 = 0.5
   b) Compute · if K=3, err < 1 − 1/3 = 0.67

   $$err^{(m)} = \sum_{i=1}^{n} w_i \cdot I(c_i \neq T^{(m)}(x_i)) \ / \ \sum_{i=1}^{n} w_i$$

   c) Compute

   $$\alpha^{(m)} = \log \frac{1-err^{(m)}}{err^{(m)}} + \log(K-1)$$

   This part was added to the Freund (1997) model.

   For $\alpha^{(m)} > 0, \quad err^{(m)} < 1 - \frac{1}{K}$

   d) Set

   $$w_i \leftarrow w_i \cdot \exp(\alpha^{(m)} \cdot I(c_i \neq T^{(m)}(x_i))), \ i=1,2,\dots,n$$

   e) Re-normalize w_i
3. Output

$$C(x) = \underset{k}{argmax} \sum_{m=1}^{M} \alpha^{(m)} \cdot I(T^{(m)}(x)=k)$$

proof

$$\log \frac{1-err^{(m)}}{err^{(m)}} + \log(K-1) > 0$$

$$\frac{1-err^{(m)}}{err^{(m)}} > \frac{1}{K-1}$$

$$\frac{1}{err^{(m)}} > \frac{K}{K-1}$$

▪ AdaBoost Regression - Algorithm

3. BOOSTING: In bagging, each training example is equally likely to be picked. In boosting, the probability of a particular example being in the training set of a particular machine depends on the performance of the prior machines on that example. The following is a modification of Adaboost.R [Freund and Schapire (1996a)].

1. Initialize, to each training pattern we assign a weight $w_i=1/N_1$, i =1, …,N_1 (uniform)
   Pick N_1 samples (with replacement) to form the training set (= subset)

Repeat the following while the average loss L defined below is less than 0.5 .

2. Construct a regression machine t from that training set. Each machine makes a hypothesis: h_t: x→y
3. Pass every member of the (entire) training set through this machine to obtain a prediction $y_i^{(p)}(x_i)$, i =1, …N_1.
4. Calculate a loss for each training sample $L_i = L[|y_i^{(p)} - y_i|]$ (predicted, actual value). The loss L may be of any functional form as long as L ∈ [0, 1]. If we let

$$D = \sup |y_i^{(p)}(x_i) - y_i| \quad i=1,\dots,N_1$$

then we have three candidate loss functions:

$$L_i = \frac{|y_i^{(p)} - y_i|}{D} \quad \text{(linear)}$$

$$L_i = \frac{|y_i^{(p)} - y_i|^2}{D^2} \quad \text{(square law)}$$

$$L_i = 1 - \exp\left(\frac{-|y_i^{(p)} - y_i|}{D}\right) \quad \text{(exponential)}$$

* classification

$$err^{(m)} = \sum_{i=1}^{n} w_i \cdot I(c_i \neq T^{(m)}(x_i)) \ / \ \sum_{i=1}^{n} w_i$$

$$\alpha^{(m)} = \log \frac{1-err^{(m)}}{err^{(m)}}$$

$$w_i \leftarrow w_i \cdot \exp(\alpha^{(m)} \cdot I(c_i \neq T^{(m)}(x_i)))$$

5. Calculate an average loss: $\bar{L} = \sum_{i=1}^{N_1} L_i w_i$
6. Form $\beta = \frac{\bar{L}}{1-\bar{L}}$ ← Proportional to average loss

   β ($\bar{L}$) · β^{1-L_i} (L_i)

   β is a measure of confidence in the predictor. Low β means high confidence in the prediction.

7. Update the weights: $w_i \rightarrow w_i \beta$ ** $(1-L_i)$, where ** indicates exponentiation. The smaller the loss the more the weight is reduced making the probability smaller that this pattern will be picked as a member of the training set for the next machine in the ensemble.

training stage

prediction stage

8. For a particular input x_i, each of the T machines makes a prediction h_t, t=1, …,T. Obtain the cumulative prediction h_f using the T predictors:

$$h_f = \inf\left\{ y \in Y : \sum_{t:h_t \leq y} \log\left(\frac{1}{\beta_t}\right) \geq \frac{1}{2}\sum_t \log\left(\frac{1}{\beta_t}\right)\right\}$$

This is the weighted median.

Weight of each machine (α role in classification). It is inversely proportional to the average loss. The weight of machines with large losses is reduced.

▪ AdaBoost Regression - Algorithm

- In algorithm step 8, the weighted median is used for prediction.
- Here we will try both weighted average and weighted median.

model weights

$$w_t = \underbrace{\log\left(\frac{1}{\beta_t}\right)}_{\alpha} \Big/ \sum_t \log\left(\frac{1}{\beta_t}\right) \quad \leftarrow \text{Weight for the value estimated by each model}$$

(normalize)

1. Using weighted average

$$h_f = \sum_t \hat{y}_t w_t \quad \leftarrow \text{weighted average}$$

test data → h_0, h_1 ⋯ h_{T-1} models → ($\hat{y}_0$ $\hat{y}_1$ $\hat{y}_{T-1}$) × $\log\frac{1}{\beta}$ → (w_0 w_1 w_{T-1})

```
# Method-1: Using weighted average
wavg_pred = np.sum(y_pred * w, axis=1)
```

y_pred → 2D: ($\hat{y}_0^{(1)}$ $\hat{y}_1^{(1)}$ ⋯ $\hat{y}_{T-1}^{(1)}$ / $\hat{y}_0^{(2)}$ $\hat{y}_1^{(2)}$ ⋯ $\hat{y}_{T-1}^{(2)}$) × [w_0 w_1 ⋯ w_{T-1}]

2. Using weighted median as in the paper

$$h_f = \inf\left\{ y \in Y : \sum_{t:h_t \leq y} \log\left(\frac{1}{\beta_t}\right) \geq \frac{1}{2}\sum_t \log\left(\frac{1}{\beta_t}\right)\right\}$$

$$h_f = \inf\left\{ y \in Y : \sum_{t:h_t \leq y} w_t \geq \frac{1}{2}\sum_t w_t\right\}$$

If y is sorted in ascending order, the y at the position where the sum of the lower w is half of the total sum of w becomes the final output.

The y value estimated by the first model.

the number of models (ex: T=50)

ex)

	0 1 2 3 4 5 6 7 8 9 10 11 … 49
y_pred	$\hat{y}_0$ $\hat{y}_1$ $\hat{y}_2$ …
	$\hat{y}_3$ $\hat{y}_0$ argsort, ascending · 23 $\hat{y}_{34}$
i_pred	3 0 9 11 5 4 8 6 1 10 2 7 … 34 …
w	w_3 w_0 w_9 w_{11} w_5 …
accumulated w (w-acc)	a_0 a_1 a_2 a_3 a_4 … … a_{49}
	is $(a_i \geq 0.5 \times a_{49})$? 23
False or True	F F F … … F T T T …

$\sum_{i=1}^{T} w_i$

the position of weighted median (ex: 23)

weighted median of y_pred = $\hat{y}_{34}$ ← final estimate

```
# weighted median: (sum of the lower w ≥ half of the total sum of w)
i_pred = np.argsort(y_pred, axis=1)
w_acc = np.cumsum(w[i_pred], axis=1)        # accumulated w
is_med = w_acc >= 0.5 * w_acc[:, -1][:, np.newaxis]
i_med = is_med.argmax(axis=1)               # 23
y_med = i_pred[np.arange(n_test), i_med]   # 34
wmed_pred = np.array(y_pred[np.arange(n_test), y_med]) # final estimate
```