Notes

Tuesday, 5 March 2024 10:18 pm

→ Differentiation :- finding the derivative of a function

↳ instantaneous rate of change of the function w.r.t its variable

↳ $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \rightarrow \frac{f(x+dx) - f(x)}{dx} = \frac{df(x)}{dx} \rightarrow$ slope at that point

→ derivate of a constant : $\frac{d}{dx}(c) = 0$

→ Power Rule : $\frac{d}{dx}(x^n) = nx^{n-1}$

→ Sum Rule : $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

→ product Rule : $(f(x)\,g(x))' = f'(x)\,g(x) + f(x)\,g'(x)$

$(cf(x))' = c(f'(x))$

→ Quotient Rule : $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$

→ Chain Rule : $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)$

$\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$

→ Partial Differentiation :

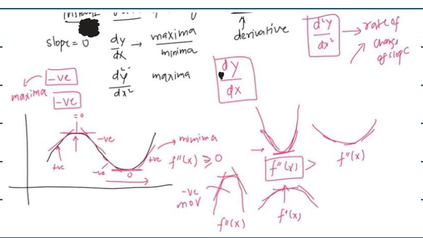
e.g. $z = x^2 + y^2$

$\frac{\partial z}{\partial x} = 2x + 0 = 2x$ $\quad \frac{\partial z}{\partial y} = 0 + 2y = 2y$

→ Higher Order Derivatives :

if slope = 0, $\frac{dy}{dx} \rightarrow$ maxima / minima

$\frac{d^2y}{dx^2} \rightarrow \uparrow$ or $\uparrow$



→ Matrix Differentiation :

$$\boxed{\frac{d}{dx} Ax}$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}}_{Ax} \begin{matrix} \rightarrow f_1(x_1, x_2) \\ \rightarrow f_2(x_1, x_2) \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{\frac{d}{dx}Ax} = A$$

A is a symmetric matrix

$y = x^T A x \quad , \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad , \quad A = \begin{bmatrix} a_{11} & a_x \\ a_x & a_{22} \end{bmatrix}$

$Y = x'Ax \quad , \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad , \quad A = \begin{bmatrix} a_{11} & a_x \\ a_x & a_{22} \end{bmatrix}$

$\boxed{\frac{dy}{dx}}$

$$x'Ax = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_x \\ a_x & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11}x_1 + a_x x_2 \\ a_x x_1 + a_{22} x_2 \end{bmatrix} = a_{11}x_1^2 + a_x x_1 x_2 + a_x x_1 x_2 + a_{22} x_2^2 \rightarrow \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2a_{11}x_1 + 2a_x x_2 \\ 2a_x x_1 + 2a_{22} x_2 \end{bmatrix}$$

$$a_{11}x_1^2 + 2a_x x_1 x_2 + a_{22}x_2^2 \rightarrow f(x_1, x_2)$$

$$= 2 \begin{bmatrix} a_{11}x_1 + a_x x_2 \\ a_x x_1 + a_{22} x_2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} a_{11} & a_x \\ a_x & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2Ax$$

$$\downarrow$$

$$(2Ax)^T = 2x^T A^T = 2x^T A$$