Questions

Sunday 21 April 2024 4:27 AM

4.1 A supplier shipped a lot of six parts to a company. The lot contained three defective parts. Suppose the customer decided to randomly select two parts and test them for defects. How large a sample space is the customer potentially working with? List the sample space. Using the sample space list, determine the probability that the customer will select a sample with exactly one defect.

Sample space: (1,2), (1,3), (1,4), (1,5), (1,6)
(2,3), (2,4), (2,5), (2,6)
(3,4), (3,5), (3,6) $\quad {}^6C_2$
(4,5), (4,6)
(5,6)

Ways to choose 1 defective & 1 non-defective part:

$${}^3C_1 \times {}^3C_1 = 9$$

Probability that customer will choose a sample exactly one defect:

$$\frac{{}^3C_1 \times {}^3C_1}{{}^6C_2} = \frac{9}{15} = \frac{3}{5}$$

A bucket contains the following marbles: 4 red, 3 blue, 4 green, and 3 yellow making 14 total marbles. Each marble is labeled with a number so they can be distinguished.

1. How many sets / groups of 4 marbles are possible?
2. How many sets / groups of 4 are there such that each one is a different color?
3. How many sets of 4 are there in which at least 2 are red?
4. How many sets of 4 are there in which none are red, but at least one is green?

1. ${}^{14}C_4 = 1001$

2. ${}^4C_1 \times {}^3C_1 \times {}^4C_1 \times {}^3C_1 = 144$

3. ${}^4C_2 \times {}^{10}C_2 + {}^4C_3 \times {}^{10}C_1 + {}^4C_4 \times {}^{10}C_0 = 270 + 40 + 1$
$= 311$

4. We are dealing with only 10 marbles

$${}^4C_1 \times {}^6C_3 + {}^4C_2 \times {}^6C_2 + {}^4C_3 \times {}^6C_1 + {}^4C_4 \times {}^6C_0 = 195$$

OR

$${}^{10}C_4 - {}^6C_4 = 195$$

Symbol	Company	Price	Yield
T	AT&T	30.24	5.82%
VZ	Verizon	40.12	4.99%
MRK	Merck	37.70	4.46%
GE	General Electric	17.91	3.80%
PFE	Pfizer	21.64	3.70%
DD	DuPont	45.78	3.58%
JNJ	Johnson & Johnson	65.58	3.48%
INTC	Intel	24.25	3.46%
PG	Procter & Gamble	66.71	3.15%
KFT	Kraft	37.36	3.10%

1. How many different 5-stock portfolios are possible?
2. How many different stock portfolios contain GE and PG, but do not have INTC (Intel) nor KFT (Kraft)?
3. How many different portfolios contain at least four stocks with yields above 3.5%?

1. ${}^{10}C_5 = 252$

2. ${}^{6}C_3 \times 1 = 20$

3. ${}^{6}C_4 \times {}^{4}C_1 + {}^{6}C_5 \times {}^{4}C_0 = 66$

4.4 A company's customer service **800** telephone system is set up so that the caller has six options. Each of these six options leads to a menu with four options. For each of these four options, three more options are available. For each of these three options, another three options are presented. If a person calls the **800** number for assistance, how many total options are possible?

4.5 A bin contains six parts. Two of the parts are defective and four are acceptable. If three of the six parts are selected from the bin, how large is the sample space? Which counting rule did you use, and why? For this sample space, what is the probability that exactly one of the three sampled parts is defective?

4.6 A company places a seven-digit serial number on each part that is made. Each digit of the serial number can be any number from **0** through **9**. Digits can be repeated in the serial number. How many different serial numbers are possible?

4.7 A small company has **20** employees. Six of these employees will be selected randomly to be interviewed as part of an employee satisfaction program. How many different groups of six can be selected?

4.4) $6 \times 4 \times 3 \times 3 = 216$ options

4.5) D D A A A A

Sample Size $= {}^{6}C_3 = 20$

Ways to choose 1D & 2A: ${}^{2}C_1 \times {}^{4}C_2 = 12$

probability 1 of 3 samples is defective $= 12/20 = 3/5$

4.6) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^7$ serial numbers

4.7) ${}^{20}C_6 = 38,760$ groups of six

4.12 According to the U.S. Bureau of Labor Statistics, 75% of the women 25 through 49 years of age participate in the labor force. Suppose 78% of the women in that age group are married. Suppose also that 61% of women 25 through 49 years of age are married and are participating in the labor force.

a. What is the probability that a randomly selected woman in that age group is married or is participating in the labor force?

b. What is the probability that a randomly selected woman in that age group is married or is participating in the labor force but not both?

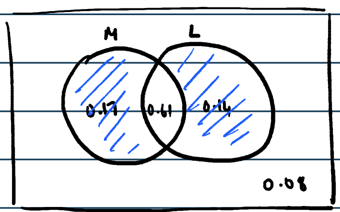
c. What is the probability that a randomly selected woman in that age group is neither married nor participating in the labor force?

	M	M'	
L	0.61	0.14	0.75
L'	0.17	0.08	0.25
	0.78	0.22	1

a) $P(M \cup L) = P(M) + P(L) - P(M \cap L)$

$= 0.78 + 0.75 - 0.61$

$= 92\%$

b) 

$0.17 + 0.14 = 31\%$

c) 8%

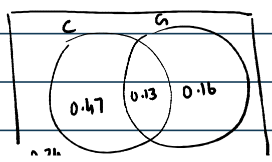
4.17 a. A batch of 50 parts contains six defects. If two parts are drawn randomly one at a time without replacement, what is the probability that both parts are defective?

b. If this experiment is repeated, with replacement, what is the probability that both parts are defective?

a) $\frac{6}{50} \times \frac{5}{49} = 1.2\%$ b) $\frac{6}{50} \times \frac{6}{50} = 1.4\%$

4.18 The U.S. Energy Department states that **60%** of all U.S. households have ceiling fans. In addition, **29%** of all U.S. households have an outdoor grill. Suppose 13% of all U.S. households have both a ceiling fan and an outdoor grill. A U.S. household is randomly selected.

a. What is the probability that the household has a ceiling fan or an outdoor grill?



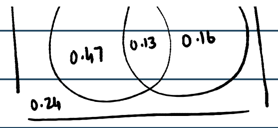
Suppose 13% of all U.S. households have both a ceiling fan and an outdoor grill. A U.S. household is randomly selected.

a. What is the probability that the household has a ceiling fan or an outdoor grill?

b. What is the probability that the household has neither a ceiling fan nor an outdoor grill?

c. What is the probability that the household does not have a ceiling fan and does have an outdoor grill?

d. What is the probability that the household does have a ceiling fan and does not have an outdoor grill?



a) 76% b) 24% c) 16% d) 47%

4.21 A study by Becker Associates, a San Diego travel consultant, found that 30% of the traveling public said that their flight selections are influenced by perceptions of airline safety. Thirty-nine percent of the traveling public wants to know the age of the aircraft. Suppose 87% of the traveling public who say that their flight selections are influenced by perceptions of airline safety wants to know the age of the aircraft.

a. What is the probability of randomly selecting a member of the traveling public and finding out that she says that flight selection is influenced by perceptions of airline safety and she does not want to know the age of the aircraft?

b. What is the probability of randomly selecting a member of the traveling public and finding out that she says that flight selection is neither influenced by perceptions of airline safety nor does she want to know the age of the aircraft?

c. What is the probability of randomly selecting a member of the traveling public and finding out that he says that flight selection is not influenced by perceptions of airline safety and he wants to know the age of the aircraft?

$P(S) = 30\%$

$P(A) = 39\%$

$P(A|S) = 87\%$

	S	S'	
A	0.261	0.129	0.39
A'	0.039	0.571	0.61
	0.3	0.7	1

$$\boxed{P(A|S) = \frac{P(A\cap S)}{P(S)}}$$

a) 3.9% b) 57.1% c) 12.9%

4.30 In a study undertaken by Catalyst, 43% of women senior executives agreed or strongly agreed that a lack of role models was a barrier to their career development. In addition, 46% agreed or strongly agreed that gender-based stereotypes were barriers to their career advancement. Suppose 77% of those who agreed or strongly agreed that gender-based stereotypes were barriers to their career advancement agreed or strongly agreed that the lack of role models was a barrier to their career development. If one of these female senior executives is randomly selected, determine the following probabilities:

a. What is the probability that the senior executive does not agree or strongly agree that a lack of role models was a barrier to her career development given that she does agree or strongly agree that gender-based stereotypes were barriers to her career development?

b. What is the probability that the senior executive does not agree or strongly agree that gender-based stereotypes were barriers to her career development given that she does agree or strongly agree that the lack of role models was a barrier to her career development?

c. If it is known that the senior executive does not agree or strongly agree that gender-based stereotypes were barriers to her career development, what is the probability that she does not agree or strongly agree that the lack of role models was a barrier to her career development?

$P(LR) = 43\%$

$P(GS) = 46\%$

$P(LR|GS) = 77\%$

$$\boxed{P(LR|GS) = \frac{P(LR\cap GS)}{P(GS)} = \frac{P(LR)\,P(GS|LR)}{P(GS)}}$$

a) $P(\neg LR \mid GS) = \dfrac{P(\neg LR \cap GS)}{P(GS)}$

$= 23\%$

b) $P(\neg GS \mid LR) = \dfrac{P(\neg GS \cap LR)}{P(LR)}$

$= 17.6\%$

	LR	LR'	
GS	0.3542	0.1058	0.46
GS'	0.0758	0.4442	0.54
	0.43	0.57	1

c) $P(\neg LR \mid \neg GS) = \dfrac{P(\neg LR \cap \neg GS)}{P(\neg GS)} = 85.9\%$

4.31 In a manufacturing plant, machine A produces 10% of a certain product, machine B produces 40% of this product, and machine C produces 50% of this product. Five percent of machine A products are defective, 12% of machine B products are defective, and 8% of machine C products are defective. The company inspector has just sampled a product from this plant and has found it to be defective. Determine the revised probabilities that the sampled product was produced by machine A, machine B, or machine C.

Prior	Conditional	Joint probability	Posterior
$P(A) = 0.1$	$P(\text{defective}\mid A) = 0.05$	$P(A\cap \text{defective}) = P(A)\,P(\text{defective}\mid A) = 0.005$	$P(A\mid \text{defective}) = P(A)\,P(\text{defective}\mid A)/P(\text{defective}) = 0.0537$
$P(B) = 0.4$	$P(\text{defective}\mid B) = 0.12$	$P(B\cap \text{defective}) = P(B)\,P(\text{defective}\mid B) = 0.048$	$P(B\mid \text{defective}) = P(B)\,P(\text{defective}\mid B)/P(\text{defective}) = 0.5161$
$P(C) = 0.5$	$P(\text{defective}\mid C) = 0.08$	$P(C\cap \text{defective}) = P(C)\,P(\text{defective}\mid C) = 0.04$	$P(C\mid \text{defective}) = P(C)\,P(\text{defective}\mid C)/P(\text{defective}) = 0.4301$

$P(\text{defective}) = 0.093$

	Prior	Posterior
A	0.1	$\downarrow$ 0.0537
B	0.4	$\uparrow$ 0.5161
C	0.5	$\downarrow$ 0.4301

4.34 Suppose 70% of all companies are classified as small companies and the rest as large companies. Suppose further, 82% of large companies provide training to employees, but only 18% of small companies provide training. A company is randomly selected without knowing if it is a large or small company; however, it is determined that the company provides training to employees. What are the prior probabilities that the company is a large company or a small company? What are the revised probabilities that the company is large or small? Based on your analysis, what is the overall percentage of companies that offer training?

Prior	conditional	Joint probabilities	Posterior
$P(S) = 0.7$	$P(\text{Training}\mid S) = 0.18$	$P(S \cap \text{Training}) = P(S)P(\text{Training}\mid S) = 0.126$	$P(S\mid\text{Training}) = P(S \cap \text{Training})/P(\text{Training}) = 0.33$ $\downarrow$
$P(L) = 0.3$	$P(\text{Training}\mid L) = 0.82$	$P(L \cap \text{Training}) = P(L)P(\text{Training}\mid L) = 0.246$	$P(L\mid\text{Training}) = P(L \cap \text{Training})/P(\text{Training}) = 0.66$ $\uparrow$
		$P(\text{Training}) = 0.372$	