Part 1

Thursday, 29 February 2024 2:39 pm

Terminoly

① Random Experiment — Has more than one possible outcome; the outcome is not possible to be predicted in advance

e.g. coin toss

② Trial - single execution of a random experiment. Each trial produces an outcome.

③ Outcome - single possible result of a trial

④ sample space - set of all possible outcomes that can occur.

⑤ Event - specific set of outcomes from a random experiment or process. subset of the sample space. An event can include a single outcome, or it can include multiple outcomes. One random experiment can have multiple events.

e.g. Tossing a coin twice

RE - Tossing a coin twice
Trial - Tossing a coin twice (once)
outcome - {H, T}
sample space - { (H,H), (H,T), (T,T), (T,H)}
Event - getting Two heads - {(H,H)}
getting atleast 1 tails - { (H,T), (T,T), (T,H)}

Types of events

① Simple event: is a event that consists of exactly one outcome

② Compound event: consists of two or more simple events.

③ Independent event: Two events are independent if the occurrence of one event doesn't affect the probability of the occurrence of the other event.

④ Dependent event: Two events are dependent if the occurrence of one event affects the probability of the occurrence of the other event.

⑤ Mutually Exclusive Events: Two events are mutually exclusive (or disjoint) if the cannot both occur at the same time.

⑥ Exhaustive Events : A set of events is exhaustive if at least one of the events must occur when the experiment is performed.

e.g. When rolling a die, the events "roll an even number" and "roll an odd number" are exhaustive b/c one or the other must occur on any roll.

⑦ Impossible event & Certain Event

→ Probability : measure of the likelihood that a particular event will occur
[0, 1]

Probability

- Empirical : is a probability measure that is based on observed data rather than theoretical assumptions. it is calculated as the ratio of the number of times a particular event occurs to the total number of trials.
- Theoretical : is used when each outcome in a sample space is equally likely to occur. let's take an event A,

Theoretical probability of event A = # of Favourable outcomes / Total # of outcomes in the sample space.

* # of trials → ∞ ; Emperical → Theoretical

→ Random ~~Variable~~ Function : A random variable is a function that maps the outcomes of a random process (sample space) to a set of real numbers.

input → $\boxed{\text{RV Logic}}$ → output

Function

output ↳ is a real number that we assign to each possible outcome

input ↳ is an outcome from the sample space of a random process.

→ Random Variable is denoted by a capital letter e.g. X Y Z

* The transformation from input to output in the function of a random variable is determined by how we choose to define the random variable.

And the choice of how to define a random variable often depends on the specific aspects of the random process (or event) that we are interested in studying.

e.g. RE → Rolling 2 dice together

Event → getting a sum of 7

Event → getting a sum of 7

Sample space → { (1,1), (1,2), (1,3) .....
.... (6,6) }

X = { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 }

↳ Logic: add outcome of each die.

→ Random variable — Discrete / Continuous

→ Probability distribution of a random variable: list of all possible outcomes of a random variable along with their corresponding probability values.

e.g. 1) Toss

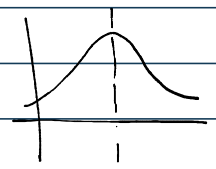
X	1	0
Pr(x)	1/2	1/2

↳ $Pr(X=1) = 1/2$

* Not always possible to draw up a table
we need to find a mathematical relationship b/w X and $Pr(X=x)$
X ↓ f(x), $Pr(X=x)$ ↓ Y, f(x) ← Y

$\boxed{y = f(x)}$ → graph [probability Distribution function]

PDF (continuous) / PMF (Discrete)

↳ 

→ Expected value of a Random Variable: is essentially the average outcome of a random process that is repeated many times. It's a weighted average of the possible outcomes of the random variables, where each outcome is weighted by it's probability of occurrence.

$$\boxed{E[X] = \sum_{i=1}^{n} Pr(X=x_i)\, x_i}$$

→ Variance of a Random Variable: The variance of a random variable is a statistical measurement that describes how much individual observations in a group differ from the expected value.

$$\boxed{Var(X) = E[(X - E[X])^2]}$$

$$\boxed{Var(X) = E[(X - E[X])^2]}$$

$$Var(X) = E[X^2 - 2XE[X] + (E[X])^2]$$

$* \ E[X+Y] = E[X] + E[Y]$

$$= E[X^2] - E[2XE[X]] + E[(E[X])^2]$$

$* \ E[XY] = E[X]E[Y]$ given X & Y independent

$* \ E[c] = c$

$$= E[X^2] - E[2]E[X]E[E[X]] + E[(E[X])^2]$$

$$= E[X^2] - 2(E[X])^2 + E[(E[X])^2]$$

$$= E[X^2] - 2(E[X])^2 + (E[X])^2$$

$$\boxed{Var(X) = E[X^2] - (E[X])^2}$$

→ Variance of a discrete random variable: $Var(X) = \sigma^2 = \sum (x_i - E[X])^2 P_r(X = x_i)$

$$= \sum [(x_i - \mu)^2 \cdot P(X = x_i)]$$

Part 2

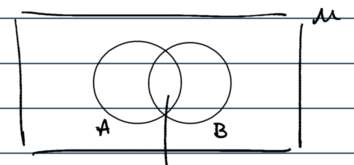
Thursday, 29 February 2024 5:28 pm

Venn diagrams

μ = sample set

$p(\mu) = 1$

$p(A)' = 1 - p(A)$



where A, B are events

$p(A \cap B)$

$\boxed{p(A \cup B) = p(A) + p(B) - p(A \cap B)}$ ← General law of addition

Joint probability

→ Let's say we have two random variables X and Y. The joint probability of X and Y, denoted as $p(X=x, Y=y)$ is the probability that X takes the value x and Y takes the value y at the same time.

Contingency table / Cross table → normalize = 'all' → Joint probability

→ Joint probability distribution

Marginal / Simple / Unconditional probability

→ Refers to the probability of an event occuring irrespective of the outcome of some other event. When dealing with random variables, the marginal probability of a random variable is simply the probability of that variable taking a certain value, regardless of the values of other variables.

→ Contingency / Crosstab → Margins = True, Normalize = 'all'

Conditional probability

→ is a measure of the probability of an event occurring, given that another event has already occurred. $p(A|B)$ ← probability A given B.

$$p(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B)$ → joint probability

$P(B)$ ← marginal probability of B

→ reduces sample space based on B

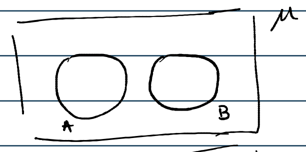
→ Contingency table / Crosstab → normalize = 'columns' / 'index'

'columns' → pr(index | col)

'index' → pr(col | index)

Independent vs Mutually Exclusive Events

	If statistically independent	If mutually exclusive
$P(A \mid B) =$	$P(A)$	0
$P(B \mid A) =$	$P(B)$	0
$P(A \cap B) =$	$P(A)P(B)$	0



$\boxed{Pr(A \cap B) = 0}$

Bayes Theorem

→ Let A & B be two events

$$Pr(B) \times Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} \times Pr(B)$$

$$Pr(B) \times Pr(A \mid B) = Pr(A \cap B) \quad — (1)$$

$$Pr(A) \times Pr(B \mid A) = \frac{Pr(B \cap A)}{P(A)} \times P(A')$$

$$Pr(A) \times Pr(B \mid A) = Pr(B \cap A) \quad — (2)$$

Note: $Pr(B \cap A) = Pr(A \cap B)$

$$Pr(B) \times Pr(A \mid B) = Pr(A) \times Pr(B \mid A)$$

(4) $$\boxed{Pr(B \mid A) = \frac{Pr(A \mid B) \times Pr(B)}{Pr(A)}}$$

(3) $$\boxed{Pr(A \mid B) = \frac{Pr(B \mid A) \times Pr(A)}{Pr(B)}}$$

Bayes Theorem

Likelyhood

Prior

Marginal / evidence

Posterior

Bayes Theorem Extended: Revision of original probabilities in the light of new information.

$$P(X_i \mid Y) = \frac{P(X_i)\,P(Y \mid X_i)}{\sum_{i=1}^{n} P(X_i)\,P(Y \mid X_i)}$$

* Laws & Rules of probability are for the 'LONG RUN'

Classical probabilities could be used unethically to lure a company or client into a potential short-run investement with the expectation of getting at least something in return, when in actuality the investor will either win or lose.

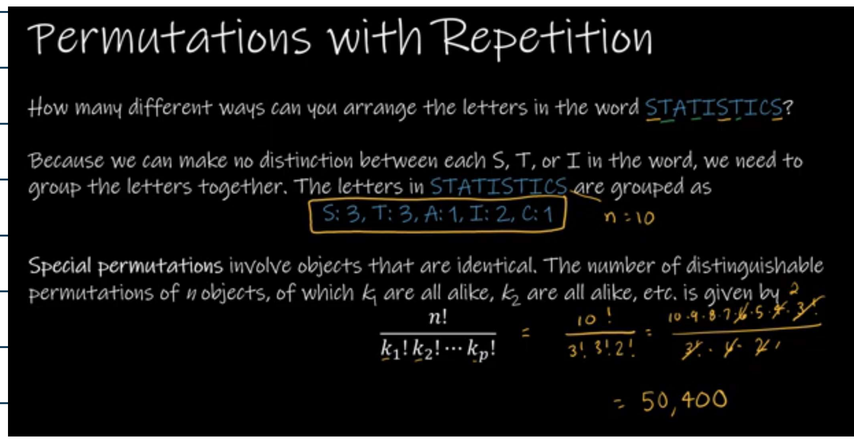
Counting

Friday 19 April 2024 2:39 AM

→ Counting made complex

→ Permutations - ORDER matters $P(n,r)$

Combinations - ORDER DOESNT MATTER $C(n,r)$



$$\boxed{C(n,r) = \frac{n!}{r!(n-r)!}} \qquad \boxed{P(n,r) = \frac{n!}{(n-r)!}}$$

Experiment:

→ Normal curve

→ Area under the curve

n	r	expr	comb	prop.
2	0	$C(2,0)$	1	$1/4$
2	1	$C(2,1)$	2	$1/2$
2	2	$C(2,2)$	1	$1/4$
Total			4	1