

```
> restart; read("newlib.m"); with(mylib): with(LinearAlgebra):
Поиск возможных замен из нормированных структурных форм во все им предшествующие:
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NSF 2-10

```
> M := zamproc(0,0,0,k,1,0,0,0, r1,s1,r2,s2):

$$\frac{kr^{23}s2 - rl^3s1}{rls2 - r2s1}, \frac{3kr^{22}s2 - 3rl^2s1}{rls2 - r2s1}, \frac{3kr2s23 - 3rls1^3}{rls2 - r2s1}, \frac{ks24 - s1^4}{rls2 - r2s1}$$


$$\frac{-kr^{24} + rl^4}{rls2 - r2s1}, \frac{-3kr^{23}s2 + 3rl^3s1}{rls2 - r2s1}, \frac{-3kr^{22}s2 + 3rl^2s1^2}{rls2 - r2s1}, \frac{-kr2s23 + rl^3s1^3}{rls2 - r2s1} \quad (1.1)$$

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {r1,s1,r2,s2});
```

NSF 3-1

```
> M := zamproc(u,1,0,0,0,0,0,1, r1,s1,r2,s2):

$$\frac{r12(r1u + r2)s2 - s1r23}{rls2 - r2s1}, \frac{3s2\left(\left(url2 + \frac{2}{3}rlr2 - r22\right)s1 + \frac{rl^2s2}{3}\right)}{rls2 - r2s1}, \frac{3s1s2\left(s2\left(\frac{2rl}{3} - r2\right) + \left(r1u + \frac{r2}{3}\right)s1\right)}{rls2 - r2s1}$$


$$\frac{s2s1(s1l2u + s1s2 - s22)}{rls2 - r2s1} \quad (2.1)$$


$$-\frac{r2rl\left(url2 + rl^2r2 - r22\right)}{rls2 - r2s1}, -\frac{3rl\left(r2\left(\frac{2s1}{3} - s2\right) + \left(s1u + \frac{s2}{3}\right)rl\right)r2}{rls2 - r2s1}, -\frac{3\left(\left(r1u + \frac{r2}{3}\right)s1^2 + \frac{2rls1s2}{3} - r1s2^2\right)r2}{rls2 - r2s1},$$


$$\frac{-r2s1^3u + r1s2^3 - s1^2r2s2}{rls2 - r2s1}$$

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

NSF 3-2

```
> M := zamproc(k,0,u,0,0,0,0,1, r1,s1,r2,s2):

$$\frac{s2(krl^3 + ur1r22) - s1r23}{rls2 - r2s1}, \frac{3\left(\frac{s1(u-3)r22}{3} + \frac{2rlr2s2u}{3} + krl^2s1\right)s2}{rls2 - r2s1}, \frac{3\left(\frac{rls22u}{3} + \frac{2r2s1\left(u - \frac{3}{2}\right)s2}{3} + krls1^2\right)s2}{rls2 - r2s1},$$


$$\frac{(u-1)s22 + ks1^2}{rls2 - r2s1} \quad (3.1)$$


$$-\frac{r2rl((u-1)r22 + krl^2)}{rls2 - r2s1}, -\frac{3r2\left(\frac{r22s1u}{3} + \frac{2s2rl\left(u - \frac{3}{2}\right)r2}{3} + krl^2s1\right)}{rls2 - r2s1}, -\frac{3r2\left(\frac{rl(u-3)s22}{3} + \frac{2r2s1s2u}{3} + krls1^2\right)}{rls2 - r2s1},$$


$$\frac{-kr2s1^3 - u s1 r2 s22 + r1s23}{rls2 - r2s1}$$

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

(3.2)

```
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

```
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}, {r1=r1,r2=0,s1=0,s2=s2,u=0}, {r1=r1,r2=0,s1=RootOf(2k_Z+1)s2,s2=s2,u=
-RootOf(2k_Z+1)^2k+1} \quad (3.3)
```

```
> u1 := 3/2;
s11 := s2/sqrt(2):
zamproc(-1,0,u1,0,0,0,0,1, r1,s11,0,s2):

$$-rl2, -\frac{3rls2\sqrt{2}}{2}, 0, 0$$


$$0, 0, 0, s22 \quad (3.4)$$

```

NSF 3-4

$$\begin{aligned} > M := \text{zamproc}(u, 0, 0, 1, 0, 0, 0, 1, r1, s1, r2, s2) : \\ & \frac{(-s1 + s2)r2^3 + r1s1s2u}{r1s2 - s1r2}, \frac{3((-s1 + s2)r2^2 + r1^2s1u)s2}{r1s2 - s1r2}, \frac{3s2(r1s1^2u - s1r2s2 + r2s2^2)}{r1s2 - s1r2}, \frac{s2(u s1^3 - s1s2^2 + s2^3)}{r1s2 - s1r2} \\ & - \frac{r2(u r1^3 - r1r2^2 + r2^3)}{r1s2 - s1r2}, - \frac{3r2(r1^2s1u - r1r2s2 + r2^2s2)}{r1s2 - s1r2}, - \frac{3((-r1 + r2)s2^2 + r1s1^2u)r2}{r1s2 - s1r2}, \frac{(r1 - r2)s2^3 - r2s1^3u}{r1s2 - s1r2} \end{aligned} \quad (4.1)$$

$$\begin{aligned} > 2-1: \\ & \text{solve}([M[1, 2], M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ & \{r1 = r1, r2 = 0, s1 = s2, s2 = s2, u = 0\}, \{r1 = r2, r2 = r2, s1 = s1, s2 = 0, u = 0\} \end{aligned} \quad (4.2)$$

$$> 2-10: \text{solve}([M[1, 1], M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{u, r1, s1, r2, s2\});$$

$$\begin{aligned} > 3-1: \\ & \text{solve}([M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ & \{r1 = r1, r2 = 0, s1 = s2, s2 = s2, u = 0\}, \{r1 = r2, r2 = r2, s1 = s1, s2 = 0, u = 0\} \end{aligned} \quad (4.3)$$

$$\begin{aligned} > 3-2: \\ & \text{solve}([M[1, 2], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ & \{r1 = r1, r2 = 0, s1 = s2, s2 = s2, u = 0\}, \{r1 = r2, r2 = r2, s1 = s1, s2 = 0, u = 0\} \end{aligned} \quad (4.4)$$

NSF 3-9

$$\begin{aligned} > M := \text{zamproc}(1, 0, 0, u, 0, 0, 1, 0, r1, s1, r2, s2) : \\ & \frac{s2(r2^3u + r1^3) - s1r1r2^2}{r1s2 - s1r2}, \frac{-s1^2r2^2 + 3r1\left(r1 - \frac{2r2}{3}\right)s2s1 + 3ur2^2s2^2}{r1s2 - s1r2}, \frac{3s2\left(\left(r1 - \frac{2r2}{3}\right)s1^2 - \frac{r1s1s2}{3} + r2s2^2u\right)}{r1s2 - s1r2}, \\ & - \frac{s2(u s2^3 + s1^3 - s1^2s2)}{r1s2 - s1r2}, - \frac{3\left(s1 - \frac{2s2}{3}\right)r1^2 - \frac{r1r2s1}{3} + r2^2s2u}{r1s2 - s1r2}, \frac{r1^2s2^2 - 3s1r2\left(s1 - \frac{2s2}{3}\right)r1 - 3ur2^2s2^2}{r1s2 - s1r2}, \\ & - \frac{-ur2s2^3 + r1s1s2^2 - r2s1^3}{r1s2 - s1r2} \end{aligned} \quad (5.1)$$

$$\begin{aligned} > 2-1: \\ & \text{solve}([M[1, 2], M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ > 2-10: \\ & \text{solve}([M[1, 1], M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{u, r1, s1, r2, s2\}); \\ > 3-1: \\ & \text{solve}([M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ & \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0\} \\ > 3-2: \\ & \text{solve}([M[1, 2], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ > 3-4: \\ & \text{solve}([M[1, 2], M[1, 3], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \end{aligned} \quad (5.2)$$

NSF 3-15

$$\begin{aligned} > M := \text{zamproc}(u, 0, 0, 1, 0, 1, 0, 0, r1, s1, r2, s2) : \\ & \frac{s2(ur1^3 + r2^3) - r2r1s1}{r1s2 - s1r2}, \frac{3s1s2\left(u - \frac{1}{3}\right)r1^2 - 2r2r1s1^2 + 3r2^2s2^2}{r1s2 - s1r2}, \frac{-r2s1^3 + 3r1s2\left(u - \frac{2}{3}\right)s1^2 + 3r2s2^3}{r1s2 - s1r2}, \\ & - \frac{s2((u - 1)s1^3 + s2^3)}{r1s2 - s1r2}, - \frac{r1^3s2 - 3s1r2\left(u - \frac{2}{3}\right)r1^2 - 3r2^3s2}{r1s2 - s1r2}, \frac{2s2r1^2s1 - 3s1^2r2\left(u - \frac{1}{3}\right)r1 - 3r2^2s2^2}{r1s2 - s1r2}, \\ & - \frac{-r2s1^3u + s2r1s1^2 - r2s2^3}{r1s2 - s1r2} \end{aligned} \quad (6.1)$$

$$\begin{aligned} > 2-1: \\ & \text{solve}([M[1, 2], M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, r1, s1, r2, s2\}); \\ > 2-10: \\ & \text{solve}([M[1, 1], M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{u, r1, s1, r2, s2\}); \\ > 3-1: \end{aligned}$$

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solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});

```

NSF 3-20

$$\begin{aligned}
&> M := \text{zamproc}(1,0,0,u,1,0,0,0, r1,s1,r2,s2) : \\
&\quad \frac{(-s1+s2)r1^3+ur2^3s2}{r1s2-s1r2}, \frac{-3s1(s1-s2)r1^2+3ur2^2s2^2}{r1s2-s1r2}, \frac{-3s1^2(s1-s2)r1+3ur2s2^3}{r1s2-s1r2}, \frac{s24u-s1^4+s2s1^3}{r1s2-s1r2} \\
&\quad \frac{-r24u+r1^4-r2r1^3}{r1s2-s1r2}, \frac{3r1^2(r1-r2)s1-3ur2^3s2}{r1s2-s1r2}, \frac{3r1(r1-r2)s1^2-3ur2^2s2^2}{r1s2-s1r2}, \frac{(r1-r2)s1^3-ur2s2^3}{r1s2-s1r2}
\end{aligned} \tag{7.1}$$

$$\begin{aligned}
&> 2-1:
&\quad \text{solve}([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=r2, r2=r2, s1=0, s2=s2, u=0\}, \{r1=0, r2=r2, s1=s2, s2=s2, u=0\}
\end{aligned} \tag{7.2}$$

$$\begin{aligned}
&> 2-10:
&\quad \text{solve}([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], \{u,r1,s1,r2,s2\}); \\
&> 3-1:
&\quad \text{solve}([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=r2, r2=r2, s1=0, s2=s2, u=0\}, \{r1=0, r2=r2, s1=s2, s2=s2, u=0\}
\end{aligned} \tag{7.3}$$

$$\begin{aligned}
&> 3-2:
&\quad \text{solve}([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=r2, r2=r2, s1=0, s2=s2, u=0\}, \{r1=0, r2=r2, s1=s2, s2=s2, u=0\}
\end{aligned} \tag{7.4}$$

$$\begin{aligned}
&> 3-4:
&\quad \text{solve}([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=0, r2=r2, s1=s1, s2=s2, u=0\}, \{r1=r2, r2=r2, s1=0, s2=s2, u=0\}
\end{aligned} \tag{7.5}$$

$$\begin{aligned}
&> 3-9:
&\quad \text{solve}([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=r2, r2=r2, s1=0, s2=s2, u=0\}
\end{aligned} \tag{7.6}$$

$$\begin{aligned}
&> 3-15:
&\quad \text{solve}([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=r2, r2=r2, s1=0, s2=s2, u=0\}
\end{aligned} \tag{7.7}$$

NSF 3-23

$$\begin{aligned}
&> M := \text{zamproc}(0,u,0,k,1,0,0,0, r1,s1,r2,s2) : \\
&\quad \frac{s2(kr2^3+r1^2r2u)-s1r1^3}{r1s2-r2s1}, \frac{(3kr2^2+r1^2u)s2^2+2r1r2s1s2u-3r1^2s1^2}{r1s2-r2s1}, \frac{3kr2s2^3+2r1s1s2^2u+r2s1^2s2u-3r1s1^3}{r1s2-r2s1}, \\
&\quad \frac{ks2^4+s1^2s2^2u-s1^4}{r1s2-r2s1} \\
&\quad \frac{-kr2^4-r1^2r2^2u+r1^4}{r1s2-r2s1}, \frac{-3kr2^3s2-r1^2r2s2u-2r1r2^2s1u+3s1r1^3}{r1s2-r2s1}, \frac{(-r2^2u+3r1^2)s1^2-2r1r2s1s2u-3kr2^2s2^2}{r1s2-r2s1}, \\
&\quad \frac{-kr2s2^3-r2s1^2s2u+r1s1^3}{r1s2-r2s1}
\end{aligned} \tag{8.1}$$

$$\begin{aligned}
&> 2-1:
&\quad \text{solve}([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&> 2-10:
&\quad \text{solve}([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], \{u,r1,s1,r2,s2\}); \\
&\quad \{r1=r1, r2=0, s1=0, s2=s2, u=0\}, \{r1=0, r2=r2, s1=s1, s2=0, u=0\}
\end{aligned} \tag{8.2}$$

$$\begin{aligned}
&> 3-1:
&\quad \text{solve}([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&> 3-2:
&\quad \text{solve}([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&> 3-4:
&\quad \text{solve}([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], \{u,r1,s1,r2,s2\}); \\
&> 3-9:
&\quad \text{solve}([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], \{u,r1,s1,r2,s2\}); \\
&> 3-15:
&\quad \text{solve}([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], \{u,r1,s1,r2,s2\});
\end{aligned}$$

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}, {r1=r1,r2=0,s1=0,s2=s2,u=0} (8.3)

```

NSF 3-24

```

> M := zamproc(0,0,1,u,1,0,0,0, r1,s1,r2,s2):
r22 (r2 u+r1) s2-s1 r1^3, (3 u r2^2+2 r1 r2) s22+s1 s2 r22-3 r1 s1 r2, (3 r2 u+r1) s23+2 s1 s22 r2-3 r1 s1^3, s24 u-s1^4+s1 s23
r1 s2-r2 s1, r1 s2-r2 s1, r1 s2-r2 s1, r1 s2-r2 s1
-r24 u+r1^4-r1 r23, (-3 s2 u-s1) r23-2 r1 r22 s2+3 s1 r1^3, (-3 u r22-r1 r2) s22-2 s1 s2 r22+3 r1 s1 r2, r1 s2-r2 s1
-u r2 s23+r1 s1^3-s1 s22 r2
r1 s2-r2 s1 (9.1)

```

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0} (9.2)

```

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

NSF 4-1

```

> M := zamproc(u,v,0,0,0,0,1,1, r1,s1,r2,s2):
r1^3 s2 u+r1^2 r2 s2 v-r1 r22 s1-r23 s1, -s1^2 r22+3 (u r1^2 + 2 r2 (v-1) r1
r1 s2-s1 r2 3 -r22) s2 s1+v r1^2 s22
r1 s2-s1 r2, 3 s1 (u r1 + r2 (v-2) 3 ) s1 + 2 ((v- 1 2 ) r1 - 3 r2 2 ) s2
r1 s2-s1 r2 s2, s1 s2 (s1^2 u+s2 (v-1) s1-s2^2)
r1 s2-s1 r2, -r1 r2 (u r1^2 + r2 (v-1) r1 - r22)
r1 s2-s1 r2, - 3 ((( 2 v 3 - 1 3 ) s1-s2) r2 + (s1 u + s2 (v-2) 3 ) r1) r1 r2
r1 s22-3 r2 (s1^2 u + 2 s2 (v-1) s1 3 - s2^2) r1 - r22 s1^2 v
r1 s2-s1 r2, -r2 s1^3 u - r2 s1^2 s2 v + r1 s1 s22 + r1 s23
r1 s2-s1 r2 (10.1)

```

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=-r1/3, s1=s1, s2=0, u=1/9, v=1}, {r1=0, r2=r2, s1=s1, s2=-2s1/3, u=-2/9, v=0}, {r1=0, r2=r2, s1=s1, s2=0, u=u, v=0}, {r1=r1, r2=-r1/3, s1=0, s2=s2, u=1/9, v=1} (10.2)
```

```

> r11 := -3*r2:
zamproc(1/9,1,0,0,0,1,1, r11,0,r2,s2):
-2 r22, -3 r2 s2, 0, 0, 0, s22 (10.3)
```

```
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=-s1/3,u=-2/9,v=0}, {r1=r1,r2=-r1/3,s1=s1,s2=s1/3,u=1/9,v=1} (10.4)
```

```
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=-s1/3,u=1/3,v=1}, {r1=r1,r2=0,s1=s1,s2=-s1,u=1/3,v=1} (10.5)
```

```
> s21 := -s1:
zamproc(1/3,1,0,0,0,0,1,1, r1,s1,0,s21):

$$\frac{rl^2}{3}, 0, 0, -\frac{2sI^3}{3rl}$$


$$0, 0, -sIrl, 0$$

```

(10.6)

```
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=3r2,r2=r2,s1=s1,s2=-s1/3,u=-1/9,v=-1} (10.7)
```

```
> r11 := 3*r2:
s11 := -3*s2:
zamproc(-1/9,-1,0,0,0,0,1,1, r11,s11,r2,s2):

$$0, 6r2s2, 0, -\frac{2s2^3}{r2}$$


$$\frac{4r2^3}{s2}, 0, 0, 0$$

(10.8)
```

```
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

NSF 4-2

```
> M := zamproc(u,1,v,0,0,0,0,1, r1,s1,r2,s2):

$$\frac{rl(url^2+vr22+r1r2)s2-s1r2^3}{s2rl-r2s1}, \frac{3s2\left(\left(u s1 + \frac{s2}{3}\right)rl^2 + \frac{2r2(s2v+s1)rl}{3} + \frac{r22s1(v-3)}{3}\right)}{s2rl-r2s1},$$


$$\frac{3\left(\left(r1u + \frac{r2}{3}\right)s1^2 + \frac{2\left(r1 + \left(v - \frac{3}{2}\right)r2\right)s2s1}{3} + \frac{vs22rl}{3}\right)s2}{s2rl-r2s1}, \frac{s1((v-1)s2^2+s1s2+us12)s2}{s2rl-r2s1}$$


$$-\frac{((v-1)r22+r1r2+ur1l^2)rlr2}{s2rl-r2s1}, -\frac{3r2\left(\left(u s1 + \frac{s2}{3}\right)rl^2 + \frac{2\left(s1 + \left(v - \frac{3}{2}\right)s2\right)r2rl}{3} + \frac{vr22s1}{3}\right)}{s2rl-r2s1},$$


$$-\frac{3r2\left(\left(r1u + \frac{r2}{3}\right)s1^2 + \frac{2s2(vr2+r1)s1}{3} + \frac{r1s22(v-3)}{3}\right)}{s2rl-r2s1}, \frac{(-u s1^3 - vs22s1 - s2s12)r2 + s2^3rl}{s2rl-r2s1} (11.1)$$

```

```
> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r1,r2=0,s1=s1,s2=-s1/3,u=1/9,v=3}, {r1=r1,r2=-r1/3,s1=s1,s2=0,u=1/9,v=3} (11.2)
```

```
> s11 := -3*s2:
zamproc(1/9,1,3,0,0,0,0,1, r1,s11,0,s2):

$$\frac{rl^2}{9}, 0, 0, 0$$


$$0, 0, 0, s2^2$$

(11.3)
```

```
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {v,r2,s2});
```

$$\left\{ r2=0, s2=\text{RootOf}(_Z + _Z + 2u) \right. \\ \left. s1, v = \frac{2 \text{RootOf}(_Z + _Z + 2u) + 3u}{\text{RootOf}(_Z + _Z + 2u) + 2u} \right\} \quad (11.4)$$

```
> solve(_Z^2+_Z+2*u, _Z);
z1 := -1/2+(1/2)*sqrt(1-8*u):
z2 := -1/2-(1/2)*sqrt(1-8*u):
- 1/2 + sqrt(1-8*u)/2, - 1/2 - sqrt(1-8*u)/2 \quad (11.5)
```

```
> s21 := z1*s1:
v1 := evala((2*z1+3*u)/(z1+2*u));
zamproc(u,1,v1,0,0,0,0,1, r1,s1,0,s21):
v1 := (sqrt(1-8*u)+12*u+1)/8*u
u r1^2, (6*u-1+sqrt(1-8*u))*r1*s1/2, 0, 0
0, 0, 0, (-1+sqrt(1-8*u))^2*s1^2/4 \quad (11.6)
```

```
> s22 := z2*s1:
v2 := evala((2*z2+3*u)/(z2+2*u));
zamproc(u,1,v2,0,0,0,0,1, r1,s1,0,s22):
v2 := (-sqrt(1-8*u)+12*u+1)/8*u
u r1^2, (-6*u+1+sqrt(1-8*u))*r1*s1/2, 0, 0
0, 0, 0, (1+sqrt(1-8*u))^2*s1^2/4 \quad (11.7)
```

```
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=-s2/3u, s2=s2, v=(9u+2)/9u} \quad (11.8)
```

```
> s21 := -3*u*s1:
v1 := (9*u+2)/(9*u):
zamproc(u,1,v1,0,0,0,0,1, r1,s1,0,s21):
u r1^2, 0, u s1^2 (9u-1), 0
0, 0, 0, 9 s1^2 u^2 \quad (11.9)
```

```
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {v,r1,r2,s2});
{r1=r1, r2=0, s2=-3 s1 u, v=1/(3u)} \quad (11.10)
```

```
> s21 := -3*s1*u:
v1 := 1/(3*u):
zamproc(u,1,v1,0,0,0,0,1, r1,s1,0,s21):
u r1^2, 0, 0, -s1^3 u (9u-1)/r1
0, 0, 0, 9 u^2 s1^2 \quad (11.11)
```

```
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}, {r1=-r2, r2=r2, s1=s1, s2=0, u=0, v=2} \quad (11.12)
```

```
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
{r1=RootOf(2u_Z+Z+1)r2, r2=r2, s1=s1, s2=0, v=-RootOf(2u_Z+Z+1)^2u-RootOf(2u_Z+Z+1)+1}, {r1=0, r2=r2, s1=s1, s2=0, v=0}, {r1=r1, r2=0, s1=0, s2=s2, v=0}, {r1=r1, r2=0, s1=RootOf(2u_Z+Z+1)s2, s2=s2, v=0} \quad (11.13)
```

$$\left. -\frac{\text{RootOf}(2 u _Z^2 + _Z + 1)}{2} + \frac{3}{2} \right\}$$

$$\begin{aligned} > \text{solve}(2 * _Z^2 * u + _Z + 1, _Z); \\ z1 := (-1 + \sqrt{1 - 8 * u}) / (4 * u); \\ z2 := -(1 + \sqrt{1 - 8 * u}) / (4 * u); \\ & \quad \frac{-1 + \sqrt{1 - 8 u}}{4 u}, -\frac{1 + \sqrt{1 - 8 u}}{4 u} \end{aligned} \tag{11.14}$$

$$\begin{aligned} > r11 := z1 * r2; \\ v1 := -z1^2 * u - z1 + 1; \\ \text{zamproc}(u, 1, v1, 0, 0, 0, 0, 1, r11, s1, r2, 0); \\ & \quad v1 := -\frac{(-1 + \sqrt{1 - 8 u})^2}{16 u} - \frac{-1 + \sqrt{1 - 8 u}}{4 u} + 1 \\ & \quad r22, 0, 0, 0 \\ & \quad 0, 0, \frac{r2 s1 (1 + 3 \sqrt{1 - 8 u})}{4}, u s1^2 \end{aligned} \tag{11.15}$$

$$\begin{aligned} > s11 := z1 * s2; \\ v1 := -(1/2) * z1 + 3/2; \\ \text{zamproc}(u, 1, v1, 0, 0, 0, 0, 1, r1, s11, 0, s2); \\ & \quad u r1^2, \frac{s2 r1 (1 + 3 \sqrt{1 - 8 u})}{4}, 0, 0 \\ & \quad 0, 0, 0, s22 \end{aligned} \tag{11.16}$$

NSF 4-3

$$\begin{aligned} > M := \text{zamproc}(u, 0, v, 0, 0, 0, 1, 1, r1, s1, r2, s2); \\ & \frac{u r1^3 s2 + r22 (s2 v - s1) r1 - s1 r23}{-s1 r2 + r1 s2}, \frac{s1 (-s1 + s2 (v - 3)) r22 + 2 r1 s2 (s2 v - s1) r2 + 3 s2 r1^2 s1 u}{-s1 r2 + r1 s2}, \\ & \frac{3 \left(\left(r1 u - \frac{2 r2}{3} \right) s1^2 + \frac{2 \left(-\frac{r1}{2} + \left(v - \frac{3}{2} \right) r2 \right) s2 s1}{3} + \frac{v r1 s22}{3} \right) s2}{-s1 r2 + r1 s2}, \frac{s1 ((v - 1) s22 - s2 s1 + u s1^2) s2}{-s1 r2 + r1 s2} \\ & \frac{r1 r2 ((v - 1) r22 - r2 r1 + u r1^2)}{-s1 r2 + r1 s2}, \frac{3 r2 \left(\left(s1 u - \frac{2 s2}{3} \right) r1^2 + \frac{2 \left(-\frac{s1}{2} + \left(v - \frac{3}{2} \right) s2 \right) r2 r1}{3} + \frac{v s1 r22}{3} \right)}{-s1 r2 + r1 s2}, \\ & \frac{s22 r1^2 - 3 r2 \left(\left(\frac{v}{3} - 1 \right) s22 - \frac{2 s2 s1}{3} + u s1^2 \right) r1 - 2 v s1 r22 s2}{-s1 r2 + r1 s2}, \frac{-u r2 s1^3 - s22 (r2 v - r1) s1 + s23 r1}{-s1 r2 + r1 s2} \end{aligned} \tag{12.1}$$

$$\begin{aligned} > 2-1: \\ & \text{solve}([M[1,2], M[1,3], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 2-10: \\ & \text{solve}([M[1,1], M[1,2], M[1,3], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-1: \\ & \text{solve}([M[1,3], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & \quad \left\{ r1 = 0, r2 = r2, s1 = -\frac{3 s2^2}{2}, s2 = s2, u = -\frac{2}{9}, v = 0 \right\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0 \} \end{aligned} \tag{12.2}$$

$$\begin{aligned} > 3-2: \\ & \text{solve}([M[1,2], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & \quad \left\{ r1 = 0, r2 = r2, s1 = -3 s2, s2 = s2, u = -\frac{2}{9}, v = 0 \right\} \end{aligned} \tag{12.3}$$

$$\begin{aligned} > 3-4: \\ & \text{solve}([M[1,2], M[1,3], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-9: \\ & \text{solve}([M[1,2], M[1,3], M[2,1], M[2,2], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \quad \{r1 = r1, r2 = 0, s1 = -s2, s2 = s2, u = 0, v = -1\} \end{aligned} \tag{12.4}$$

$$\begin{aligned} > 3-15: \\ & \text{solve}([M[1,2], M[1,3], M[2,1], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-20: \\ & \text{solve}([M[1,2], M[1,3], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-23: \\ & \text{solve}([M[1,1], M[1,3], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \quad \{r1 = -r2, r2 = r2, s1 = s1, s2 = 0, u = 0, v = -1\} \end{aligned} \tag{12.5}$$

```

> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
{r1=RootOf(2_Z^2 u+1) r2, r2=r2, s1=s1, s2=0, v=-RootOf(2_Z^2 u+1)^2 u+RootOf(2_Z^2 u+1)+1}, {r1=r1, r2=0, s1=0, s2=s2, v=0}, {r1=r1, r2=0, s1=RootOf(2_Z^2 u+1) s2, s2=s2, v=RootOf(2_Z^2 u+1)+3/2}, {r1=0, r2=r2, s1=s1, s2=0, v=0}

```

(12.6)

```

> solve(2*_Z^2*u+1, _Z);
z1 := 1/sqrt(-2*u):
z2 := -1/sqrt(-2*u):

```

$$-\frac{1}{\sqrt{-2u}}, \frac{1}{\sqrt{-2u}}$$

```

> r11 := z1*r2:
v1 := -z1^2*u+z1+1;
zamproc(u,0,v1,0,0,0,1,1, r11,s1,r2,0):

```

$$\begin{aligned} v1 &:= \frac{3}{2} + \frac{1}{\sqrt{-2u}} \\ &\frac{r2^2 (\sqrt{2} \sqrt{-u} + 1) \sqrt{2}}{2 \sqrt{-u}}, s1 r2, 0, 0 \\ &0, 0, -\frac{3 \sqrt{2} \sqrt{-u} r2 s1}{2}, u s1^2 \end{aligned}$$

(12.8)

```

> r12 := z2*r2:
v2 := -z2^2*u+z2+1;
zamproc(u,0,v2,0,0,0,1,1, r12,s1,r2,0):

```

$$\begin{aligned} v2 &:= \frac{3}{2} - \frac{1}{\sqrt{-2u}} \\ &\frac{r2^2 (\sqrt{2} \sqrt{-u} - 1) \sqrt{2}}{2 \sqrt{-u}}, s1 r2, 0, 0 \\ &0, 0, \frac{3 \sqrt{2} \sqrt{-u} r2 s1}{2}, u s1^2 \end{aligned}$$

(12.9)

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0}, {r1=0, r2=r2, s1=s1, s2=s2, u=s2(s1+s2)/s1^2, v=0}

```

(12.10)

NSF 4-4 (не является CF)

```

> M := zamproc(u,v,0,1,0,0,0,1, r1,s1,r2,s2):

```

$$\begin{aligned} &\frac{(-s1+s2)r23+vr12r2s2+us2r13}{r1s2-s1r2}, \frac{3s2\left(s2\left(\frac{r12v}{3}+r22\right)+\left(-r22+\frac{2}{3}vr2r1+ur12\right)s1\right)}{r1s2-s1r2}, \\ &\frac{3\left(\left(r1u+\frac{vr2}{3}\right)s1^2+\frac{2\left(vr1-\frac{3r2}{2}\right)s2s1}{3}+r2s2^2\right)s2}{r1s2-s1r2}, \frac{s2(s1^2s2v+us1^3-s2^2s1+s2^3)}{r1s2-s1r2} \\ &-\frac{r2(vr12r2+r13u-r1r22+r23)}{r1s2-s1r2}, -\frac{3r2\left(\left(\frac{vs2}{3}+us1\right)r12+\frac{2r2\left(vs1-\frac{3s2}{2}\right)r1}{3}+s2r22\right)}{r1s2-s1r2}, \\ &-\frac{3r2\left((-r1+r2)s22+\frac{2vr1s1s2}{3}+\left(r1u+\frac{vr2}{3}\right)s1^2\right)}{r1s2-s1r2}, \frac{(-us1^3-s1^2s2v-s2^3)r2+s2^3r1}{r1s2-s1r2} \end{aligned}$$

(13.1)

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,r2,s2});
{r1=r2, r2=r2, s2=0, u=0, v=0}, {r1=r1, r2=0, s2=RootOf(_Z-_Z+v_Z+u)s1, u=u, v=v}

```

(13.2)

```

> s21 := theta*s1:
zamproc(u,v,0,1,0,0,0,1, r1,s1,0,s21):

```

$$u r1^2, r1 s1 (\theta v + 3u), s1^2 (2\theta v + 3u), \frac{s1^3 (\theta^3 - \theta^2 + \theta v + u)}{r1}$$

12.3

▼ NSF 4-6

$$\begin{aligned}
 > M := \text{zamproc}(u, 0, 0, v, 0, 0, 1, 1, r1, s1, r2, s2) : \\
 & \frac{(s2 v - s1) r2^3 - r1 s1 r2^2 + r1^3 s2 u}{r1 s2 - r2 s1}, \frac{(3 s2^2 v - s1^2 - 3 s1 s2) r2^2 - 2 r1 s1 r2 s2 + 3 u r1^2 s1 s2}{r1 s2 - r2 s1}, \\
 & \frac{3 s2 \left(\left(u r1 - \frac{2 r2}{3} \right) s1^2 - \frac{s2 (r1 + 3 r2) s1}{3} + r2 s2^2 v \right)}{r1 s2 - r2 s1}, \frac{s2 (s1^3 u + s2^3 v - s1 s2 - s1 s2^2)}{r1 s2 - r2 s1} \\
 & - \frac{r2 (r1^3 u + r2^3 v - r1^2 r2 - r1 r2^2)}{r1 s2 - r2 s1}, - \frac{3 r2 \left(\left(u s1 - \frac{2 s2}{3} \right) r1^2 - \frac{r2 (s1 + 3 s2) r1}{3} + r2^2 s2 v \right)}{r1 s2 - r2 s1}, \\
 & \frac{r1^2 s2^2 - 3 r2 \left(u s1^2 - \frac{2}{3} s1 s2 - s2^2 \right) r1 - 3 r2^2 s2^2 v}{r1 s2 - r2 s1}, \frac{(-r2 v + r1) s2^3 + r1 s1 s2^2 - r2 s1^3 u}{r1 s2 - r2 s1}
 \end{aligned} \tag{14.1}$$

$$\begin{aligned}
 > 2-1: \\
 & \text{solve}([M[1, 2], M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, v, r1, s1, r2, s2\}); \\
 > 2-10: \\
 & \text{solve}([M[1, 1], M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{u, v, r1, s1, r2, s2\}); \\
 > 3-1: \\
 & \text{solve}([M[1, 3], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, v, r1, s1, r2, s2\}); \\
 & \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0\}, \left\{ r1 = 0, r2 = r2, s1 = -\frac{3 s2}{2}, s2 = s2, u = -\frac{2}{9}, v = 0 \right\}
 \end{aligned} \tag{14.2}$$

$$\begin{aligned}
 > 3-2: \\
 & \text{solve}([M[1, 2], M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, v, r1, s1, r2, s2\}); \\
 & \left\{ r1 = 0, r2 = r2, s1 = -3 s2, s2 = s2, u = -\frac{2}{9}, v = 0 \right\}
 \end{aligned} \tag{14.3}$$

$$\begin{aligned}
 > 3-4: \\
 & \text{solve}([M[1, 2], M[1, 3], M[2, 1], M[2, 2], M[2, 3]], \{u, v, r1, s1, r2, s2\}); \\
 > 3-9: \\
 & \text{solve}([M[1, 2], M[1, 3], M[2, 1], M[2, 2], M[2, 4]], \{u, v, r1, s1, r2, s2\}); \\
 > 3-15: \\
 & \text{solve}([M[1, 2], M[1, 3], M[2, 1], M[2, 3], M[2, 4]], \{u, v, r1, s1, r2, s2\}); \\
 > 3-20: \\
 & \text{solve}([M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{u, v, r1, s1, r2, s2\}); \\
 > 3-23: \\
 & \text{solve}([M[1, 1], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{u, v, r1, s1, r2, s2\}); \\
 > 3-24: \\
 & \text{solve}([M[1, 1], M[1, 2], M[2, 2], M[2, 3], M[2, 4]], \{u, v, r1, s1, r2, s2\}); \\
 > 4-1: \\
 & \text{solve}([M[1, 3], M[1, 4], M[2, 1], M[2, 2]], \{v, r1, s1, r2, s2\}); \\
 & \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, v = 0\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, v = 0\}, \left\{ r1 = \frac{r2}{3 u}, r2 = r2, s1 = s1, s2 = 0, v = -\frac{\frac{2 r2^3}{27 u^2} - \frac{r2^3}{3 u}}{r2^3} \right\}, \left\{ r1 = \frac{r2^2}{3 u}, r2 = 0, s1 = \frac{s2}{3 u}, s2 = s2, v = \frac{9 u + 2}{27 u^2} \right\}
 \end{aligned} \tag{14.4}$$

$$\begin{aligned}
 > s11 := s2 / (3 * u); \\
 & v1 := (9 * u^2) / (27 * u^2); \\
 & \text{zamproc}(u, 0, 0, v1, 0, 0, 1, 1, r1, s11, 0, s2) : \\
 & \quad v1 := \frac{9 u + 2}{27 u^2} \\
 & \quad r1^2 u, r1 s2, 0, 0 \\
 & \quad 0, 0, r1 s2, \frac{s2^2 (3 u + 1)}{3 u}
 \end{aligned} \tag{14.5}$$

$$> v2 := \text{simplify}(-(-2 * r2^3 / (27 * u^2) - r2^3 / (3 * u)) / r2^3); \\
 & v2 := \frac{9 u + 2}{27 u^2} \tag{14.6}$$

$$> 4-2: \\
 & \text{solve}([M[1, 4], M[2, 1], M[2, 2], M[2, 3]], \{u, v, r1, s1, r2, s2\}); \\
 & \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0\}, \left\{ r1 = 0, r2 = r2, s1 = s1, s2 = s2, u = \frac{s2 (s1 + s2)}{s1^2}, v = 0 \right\}
 \tag{14.7}$$

```

> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1 = - $\frac{3 r2}{2}$ , r2 = r2, s1 = 0, s2 = s2, u = - $\frac{2}{9}$ , v = 0}, {r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0}, {r1 = r1, r2 = 0, s1 = s1, s2 = s2, u = 0, v = 0} (14.8)
=  $\frac{s1(s1+s2)}{s2^2}$ , {r1 = 0, r2 = r2, s1 = -3 s2, s2 = s2, u = - $\frac{2}{9}$ , v = 0}

```

NSF 4-8

```

> M := zamproc(u,0,v,0,0,1,0,k, r1,s1,r2,s2):
-k r2^3 s1 + r1^3 s2 u + r1 r2^2 s2 v - r1^2 r2 s1,  $\frac{-2 r1 s1 r2 r2 - 3 \left( \left( -u + \frac{1}{3} \right) r1^2 + r2^2 \left( k - \frac{v}{3} \right) \right) s2 s1 + 2 v r1 r2 s2^2}{r1 s2 - r2 s1},$ 
-s1^3 r2 + 3  $\left( u - \frac{2}{3} \right) s2 r1 s1^2 - 3 \left( k - \frac{2 v}{3} \right) r2 s2^2 s1 + v r1 s2^3$   $\frac{\left( \left( -u + 1 \right) s1^2 + s2^2 (k - v) \right) s2 s1}{r1 s2 - r2 s1}$ ,
((-u+1) r1^2 + r2^2 (k-v)) r2 r1,  $\frac{r1^3 s2 - 3 \left( u - \frac{2}{3} \right) r2 s1 r1^2 + 3 \left( k - \frac{2 v}{3} \right) r2^2 s2 r1 - r2^3 s1 v}{r1 s2 - r2 s1},$ 
2 r1^2 s1 s2 + 3 r2  $\left( \left( -u + \frac{1}{3} \right) s1^2 + s2^2 \left( k - \frac{v}{3} \right) \right) r1 - 2 r2^2 s1 s2 v,  $\frac{k r1 s2^3 - r2 s1^3 u - r2 s1 s2^2 v + r1 s1^2 s2}{r1 s2 - r2 s1}$$ 

```

(15.1)

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1 = - $\frac{3 r2 k}{RootOf(\mathcal{Z} - 3 k)}$ , r2 = r2, s1 = RootOf( $\mathcal{Z} - 3 k$ ) s2, s2 = s2, u =  $\frac{1}{3}$ , v = 3 k} (15.2)

```

```

> r11 := -3*r2/sqrt(3):
s11 := sqrt(3)*s2:
zamproc(1/3,0,3,0,0,1,0,1, r11,s11,r2,s2):
4 r2^2, 0, 0, 0
0, 0, 0, 4 s2^2

```

(15.3)

```

> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1 = - $\frac{3 r2 k}{RootOf(\mathcal{Z} - 3 k)}$ , r2 = r2, s1 = RootOf( $\mathcal{Z} - 3 k$ ) s2, s2 = s2, u = - $\frac{k}{RootOf(\mathcal{Z} - 3 k)^2}$ , v = - $\frac{9 k^2}{RootOf(\mathcal{Z} - 3 k)^2}$ } (15.4)

```

```

> r11 := -3*r2/sqrt(3):
s11 := sqrt(3)*s2:
zamproc(-1/3,0,-3,0,0,1,0,1, r11,s11,r2,s2):
0, 0, 0,  $\frac{4 s2^3}{r2}$ 
 $\frac{4 r2^3}{s2}$ , 0, 0, 0

```

(15.5)

```

> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1 = 0, r2 = r2, s1 = RootOf( $\mathcal{Z} + 3 k$ ) s2, s2 = s2, u =  $\frac{2}{3}$ , v = 0}, {r1 = - $\frac{3 r2 k}{RootOf(\mathcal{Z} + 3 k)}$ , r2 = r2, s1 = RootOf( $\mathcal{Z} + 3 k$ ) s2, s2 = s2, u =  $\frac{1}{3}$ , v = 3 k} (15.6)

```

```

> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1 = RootOf( $\mathcal{Z} - 3 k$ ) r2, r2 = r2, s1 = - $\frac{3 s2 k}{RootOf(\mathcal{Z} - 3 k)}$ , s2 = s2, u =  $\frac{1}{3}$ , v = 3 k}, {r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0} (15.7)

```

```

> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1 = RootOf( $\mathcal{Z} - 3 k$ ) r2, r2 = r2, s1 = - $\frac{3 s2 k}{RootOf(\mathcal{Z} - 3 k)}$ , s2 = s2, u =  $\frac{1}{3}$ , v = 3 k} (15.8)

```

```

> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

```

```

> 3-23:
  solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
  solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
  solve([M[1,3],M[1,4],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
   $\left\{ r1 = \text{RootOf}(\_Z^2(2u-1)+k) r2, r2 = r2, s1 = -s2 \text{RootOf}(\_Z^2(2u-1)+k), s2 = s2, v = \frac{k(3u-2)}{2u-1} \right\} \quad (15.9)$ 

```

```

> k1 := 1:
z1 := 1/sqrt(1-2*u);
r11 := z1*r2;
s11 := -z1*s2;
v1 := (3*u-2)/(2*u-1);
zamproc(u,0,v1,0,0,1,0,1, r11,s11,r2,s2):
  
$$z1 := \frac{1}{\sqrt{-2u+1}}$$

  
$$v1 := \frac{3u-2}{2u-1}$$

  
$$\frac{2r2^2(u-1)}{2u-1}, \frac{(6u-2)s2r2}{2u-1}, 0, 0$$

  
$$0, 0, \frac{(6u-2)s2r2}{2u-1}, \frac{2s2^2(u-1)}{2u-1} \quad (15.10)$$


```

```

> k2 := -1:
z2 := 1/sqrt(2*u-1);
r12 := z2*r2;
s12 := -z2*s2;
v2 := -(3*u-2)/(2*u-1);
zamproc(u,0,v2,0,0,1,0,-1, r12,s12,r2,s2):
  
$$z2 := \frac{1}{\sqrt{2u-1}}$$

  
$$v2 := -\frac{3u-2}{2u-1}$$

  
$$-\frac{2r2^2(u-1)}{2u-1}, \frac{(-6u+2)s2r2}{2u-1}, 0, 0$$

  
$$0, 0, \frac{(-6u+2)s2r2}{2u-1}, -\frac{2s2^2(u-1)}{2u-1} \quad (15.11)$$


```

```

> 4-2:
  solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0},  $\left\{ r1=0, r2=r2, s1=s1, s2=s2, u=\frac{s2^2k+s1^2}{s1^2}, v=0 \right\}, \left\{ r1=\text{RootOf}(\_Z-3k) r2, r2=r2, s1=0, s2=s2, u=\frac{1}{3}, v=3k \right\}, \left\{ r1=\text{RootOf}(\_Z-3k) r2, r2=r2, s1=-\text{RootOf}(\_Z-3k) s2, s2=s2, u=\frac{1}{3}, v=3k \right\} \quad (15.12)$ 
  = 0, s2=s2, u =  $\frac{1}{3}$ , v = 3k,  $\left\{ r1=\text{RootOf}(\_Z-3k) r2, r2=r2, s1=s1, s2=0, u=\frac{1}{3}, v=3k \right\}, \left\{ r1=\text{RootOf}(\_Z-3k) r2, r2=r2, s1=-\text{RootOf}(\_Z-3k) s2, s2=s2, u=\frac{1}{3}, v=3k \right\}$ 

```

```

> 4-3:
  solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0},  $\left\{ r1=\text{RootOf}(\_Z+3k) r2, r2=r2, s1=0, s2=s2, u=\frac{2}{3}, v=0 \right\}, \left\{ r1=\text{RootOf}(\_Z-3k) r2, r2=r2, s1=-\frac{3ks2}{\text{RootOf}(\_Z-3k)}, s2=s2, u=\frac{1}{3}, v=3k \right\} \quad (15.13)$ 
  r2=r2, s1 =  $-\frac{3ks2}{\text{RootOf}(\_Z-3k)}$ , s2=s2, u =  $\frac{1}{3}$ , v = 3k

```

```

> 4-6:
  solve([M[1,2],M[1,3],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
   $\left\{ r1=\text{RootOf}((2u-1)\_Z+k) r2, r2=r2, s1=-\frac{ks2(3u-2)}{(2u-1)\text{RootOf}((2u-1)\_Z+k)}, s2=s2, v=\frac{k(3u-2)}{2u-1} \right\} \quad (15.14)$ 

```

NSF 4-9 (не является CF)

```

> M := zamproc(u,0,v,1,0,0,0,1, r1,s1,r2,s2):
  
$$\frac{(-s1+s2)r2^3+vs2r1r22+us2r1^3}{r1s2-r2s1}, \frac{\frac{3s2}{3}\left(\left(\frac{v}{3}-1\right)s1+s2\right)r22+\frac{2vs2r1r2}{3}+us1r1^2}{r1s2-r2s1},$$


```

$$\begin{aligned}
& \frac{3 \left(\left(\frac{r1 v}{3} + r2 \right) s2^2 + \frac{2 r2 \left(v - \frac{3}{2} \right) s1 s2}{3} + u r1 s1^2 \right) s2}{r1 s2 - r2 s1}, \frac{s2 (s1^3 u + s2^3 + s1 (v-1) s2^2)}{r1 s2 - r2 s1} \\
& - \frac{(r2^3 + r1 (v-1) r2^2 + u r1^3) r2}{r1 s2 - r2 s1}, - \frac{3 r2 \left(\left(\frac{s1 v}{3} + s2 \right) r2^2 + \frac{2 s2 \left(v - \frac{3}{2} \right) r1 r2}{3} + u s1 r1^2 \right)}{r1 s2 - r2 s1}, \\
& - \frac{3 r2 \left(\left(\left(\frac{v}{3} - 1 \right) r1 + r2 \right) s2^2 + \frac{2 v r2 s1 s2}{3} + u r1 s1^2 \right)}{r1 s2 - r2 s1}, \frac{(-s1^3 u - v s1 s22 - s2^3) r2 + r1 s2^3}{r1 s2 - r2 s1}
\end{aligned} \tag{16.1}$$

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {r1,r2,s2});
{r1=r1, r2=0, s2=RootOf(_Z+u+(v-1)_Z)s1} \tag{16.2}

```

```

> s21 := theta*s1:
zamproc(u,0,v,1,0,0,0,1, r1,s1,0,s21):
ur1^2, 3 r1 s1 u, s1^2 (θ^2 v + 3 u), (θ^3 + (v-1) θ^2 + u) s1^3
r1 s2 - r2 s1
0, 0, 0, θ^2 s1^2 \tag{16.3}

```

NSF 4-10

$$\begin{aligned}
& M := \text{zamproc}(1, u, 0, v, 0, 0, 1, 0, r1, s1, r2, s2): \\
& \frac{s2 (u r1^2 r2 + r2^3 v + r1^3) - s1 r1 r2^2}{r1 s2 - r2 s1}, \frac{(r1^2 u + 3 r2^2 v) s2^2 + 2 r1 s1 \left(\frac{3 r1}{2} + r2 (u-1) \right) s2 - s1^2 r2^2}{r1 s2 - r2 s1}, \\
& \frac{2 \left(\left(\frac{3 r1}{2} + \left(\frac{u}{2} - 1 \right) r2 \right) s1^2 + r1 s2 \left(u - \frac{1}{2} \right) s1 + \frac{3 r2 s2^2 v}{2} \right) s2}{r1 s2 - r2 s1}, \frac{(v s2^3 + s1^2 (u-1) s2 + s1^3) s2}{r1 s2 - r2 s1} \\
& - \frac{(r2^3 v + r1^2 (u-1) r2 + r1^3) r2}{r1 s2 - r2 s1}, - \frac{r2 \left((3 s1 + (u-2) s2) r1^2 + 2 s1 r2 \left(u - \frac{1}{2} \right) r1 + 3 r2^2 s2 v \right)}{r1 s2 - r2 s1}, \\
& \frac{(-s1^2 u - 3 s2^2 v) r2^2 - 2 r1 s1 \left(\frac{3 s1}{2} + s2 (u-1) \right) r2 + r1^2 s2^2}{r1 s2 - r2 s1}, \frac{(-s1^2 s2 u - v s2^3 - s1^3) r2 + r1 s1 s2^2}{r1 s2 - r2 s1}
\end{aligned} \tag{17.1}$$

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}, {r1=-r2/3, r2=r2, s1=s1, s2=0, u=1, v=1/27} \tag{17.2}

```

```

> r21 := -3*r1:
zamproc(1,1,0,1/27,0,0,1,0, r1,s1,r21,0):
-3 r1^2, -3 r1 s1, 0, 0
0, 0, 0, s1^2 \tag{17.3}

```

```

> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r2/3, r2=r2, s1=-2 s2/3, s2=s2, u=0, v=2/27}, {r1=r1, r2=0, s1=0, s2=s2, u=0, v=v} \tag{17.4}

```

```

> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:

```

```

solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, v=0}, {r1=0, r2=r2, s1=s1, s2=0, v=0}, {r1=-\frac{(2 u-1) r2}{3}, r2=r2, s1=s1, s2=0, v=-\frac{4}{27} u3 + \frac{4}{9} u2 - \frac{1}{3} u + \frac{2}{27}}, {r1=r1, r2=0, s1=-\frac{2}{3} s2 u + \frac{1}{3} s2, s2=s2, v=-\frac{4}{27} u3 + \frac{4}{9} u2 - \frac{1}{3} u + \frac{2}{27}}
+ \frac{4}{9} u2 - \frac{1}{3} u + \frac{2}{27}}, {r1=r1, r2=0, s1=-\frac{2}{3} s2 u + \frac{1}{3} s2, s2=s2, v=-\frac{4}{27} u3 + \frac{4}{9} u2 - \frac{1}{3} u + \frac{2}{27}} \quad (17.5)

```

```

> r11 := -(1/3)*(2*u-1)*r2;
v1 := factor(-(4/27)*u^3+(4/9)*u^2-(1/3)*u+2/27);
zamproc(1,u,0,v1,0,0,1,0, r11,s1,r2,0):
v1 := -\frac{(u-2) (2 u-1)^2}{27}
-\frac{2}{3} r22 u + \frac{1}{3} r22, s1 r2, 0, 0
0, 0, -s1 r2 (u-1), s12 \quad (17.6)

```

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=-\frac{r2}{3}, r2=r2, s1=s1, s2=0, u=1, v=\frac{1}{27}}, {r1=0, r2=r2, s1=s2, s2=s2, u=0, v=0}, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}, {r1=-\frac{r2}{3}, r2=r2, s1=\frac{\text{RootOf}(\text{Z}^2-\text{Z}+1) s2}{3}, s2=s2, u=1, v=\frac{1}{27}} \quad (17.7)

```

```

> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
{r1=-\frac{r2}{9}, r2=r2, s1=\frac{2 s2}{9}, s2=s2, u=\frac{2}{3}, v=\frac{4}{729}}, {r1=r1, r2=0, s1=s1, s2=s2, u=-\frac{3 s1}{s2}, v=\frac{s1^2 (2 s1+s2)}{s2^3}}
{r1=r1, r2=0, s1=-\frac{s2 u}{3}, s2=s2, v=-\frac{2}{27} u3 + \frac{1}{9} u2} \quad (17.8)

```

```

> r21 := -9*r1;
s11 := 2*s2/9;
zamproc(1,2/3,0,4/729,0,0,1,0, r1,s11,r21,s2):
-9 r12, 0, \frac{2 s22}{9}, 0
0, 0, -r1 s2, \frac{2 s22}{9} \quad (17.9)

```

```

> s11 := -(1/3)*s2*u;
v1 := -(2/27)*u^3+(1/9)*u^2;
zamproc(1,u,0,v1,0,0,1,0, r1,s11,0,s2):
r12, 0, -\frac{s22 u (u-1)}{3}, 0
0, 0, r1 s2, -\frac{s22 u}{3} \quad (17.10)

```

```

> subs(u=2/3, -(2/27)*u^3+(1/9)*u^2);
\frac{20}{729} \quad (17.11)

```

```

> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=\frac{r2}{3}, r2=r2, s1=-\frac{2 s2}{3}, s2=s2, u=0, v=\frac{2}{27}}, {r1=r1, r2=0, s1=0, s2=s2, u=0, v=v}, {r1=r1, r2=0, s1=-\frac{s2}{3}, s2=s2, u=1, v=v} \quad (17.12)

```

```

> s21 := -3*s1;
zamproc(1,1,0,v,0,0,1,0, r1,s1,0,s21):
r12, 0, 0, \frac{(-27 v+1) s1^3}{r1}
0, 0, -3 r1 s1, -3 s12 \quad (17.13)

```

```

> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=-\frac{r2 (5 \text{RootOf}(\text{Z}^2-2 \text{Z}-2)+4)}{3 (7 \text{RootOf}(\text{Z}^2-2 \text{Z}-2)+5)}, r2=r2, s1=\frac{\text{RootOf}(\text{Z}^2-2 \text{Z}-2) s2}{3}, s2=s2, u=0, v=\frac{(5 \text{RootOf}(\text{Z}^2-2 \text{Z}-2)+4)^2 (26 \text{RootOf}(\text{Z}^2-2 \text{Z}-2)+19)}{27 (7 \text{RootOf}(\text{Z}^2-2 \text{Z}-2)+5)^3}} \quad (17.14)

```

NSF 4-13

$$\begin{aligned}
 > M := \text{zamproc}(u, 0, 0, 1, 0, 1, 0, v, r1, s1, r2, s2) : \\
 & \frac{(-s1 v + s2) r23 - r12 s1 r2 + r13 s2 u}{r1 s2 - s1 r2}, \frac{-2 r1 s12 r2 + 3 \left(\left(u - \frac{1}{3} \right) r12 - r22 v \right) s2 s1 + 3 s22 r22}{r1 s2 - s1 r2}, \\
 & \frac{-s13 r2 + 3 \left(u - \frac{2}{3} \right) s2 r1 s12 - 3 s1 s22 r2 v + 3 r2 s23}{r1 s2 - s1 r2}, \frac{s2 ((u - 1) s13 - s1 s22 v + s23)}{r1 s2 - s1 r2} \\
 & - \frac{((u - 1) r13 - r1 r22 v + r23) r2}{r1 s2 - s1 r2}, \frac{r13 s2 - 3 \left(u - \frac{2}{3} \right) r2 s1 r12 + 3 r1 s2 r22 v - 3 s2 r23}{r1 s2 - s1 r2}, \\
 & \frac{2 r12 s1 s2 - 3 \left(\left(u - \frac{1}{3} \right) s12 - s22 v \right) r2 r1 - 3 s22 r22}{r1 s2 - s1 r2}, \frac{(v r1 - r2) s23 + r1 s12 s2 - r2 s13 u}{r1 s2 - s1 r2}
 \end{aligned} \tag{18.1}$$

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=u, v=0}, \left\{ r1 = -\frac{r2}{2 \text{RootOf}(6 \underline{Z} + 1) s1}, u = -\frac{1}{3}, v = 0 \right\} \tag{18.2}

```

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0} \tag{18.3}

```

```

> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
\left\{ r1 = \text{RootOf}(\underline{Z} + \text{RootOf}(\underline{Z} + 3 v \underline{Z} - 3)^2 + 3 v + \text{RootOf}(\underline{Z} + 3 v \underline{Z} - 3) \underline{Z}) r2, r2 = r2, s1 = \text{RootOf}(\underline{Z} + 3 v \underline{Z} - 3) s2, s2 = s2, u = \frac{2}{3} \right\} \tag{18.4}

```

```

> solve(_Z^3+3*_Z*v-3, _Z);
z1 := (1/2) * (12+4*sqrt(4*v^3+9))^(1/3)-2*v/(12+4*sqrt(4*v^3+9))^(1/3);
z1 := theta/2 - 2*v/theta;
\frac{(12+4\sqrt{4v^3+9})^{1/3}}{2} - \frac{2v}{(12+4\sqrt{4v^3+9})^{1/3}}, -\frac{(12+4\sqrt{4v^3+9})^{1/3}}{4} + \frac{v}{(12+4\sqrt{4v^3+9})^{1/3}}
+ \frac{I\sqrt{3}\left(\frac{(12+4\sqrt{4v^3+9})^{1/3}}{2} + \frac{2v}{(12+4\sqrt{4v^3+9})^{1/3}}\right)}{2}, -\frac{(12+4\sqrt{4v^3+9})^{1/3}}{4} + \frac{v}{(12+4\sqrt{4v^3+9})^{1/3}}
- \frac{I\sqrt{3}\left(\frac{(12+4\sqrt{4v^3+9})^{1/3}}{2} + \frac{2v}{(12+4\sqrt{4v^3+9})^{1/3}}\right)}{2}
z1 := \frac{(12+4\sqrt{4v^3+9})^{1/3}}{2} - \frac{2v}{(12+4\sqrt{4v^3+9})^{1/3}}
z1 := \frac{\theta}{2} - \frac{2v}{\theta} \tag{18.5}

```

```
> solve(_Z^2+z^2+3*v+z*_Z, _Z);
```

```

z2 := ((12+4*sqrt(4*v^3+9))^(1/3)*v+sqrt(-(12+4*sqrt(4*v^3+9))^(4/3)*v-3*(12+4*sqrt(4*v^3+9))^(2/3)*v^2-2*(12+4*sqrt(4*v^3+9))^(1/3)*sqrt(4*v^3+9)*v-12*v^3-6*(12+4*sqrt(4*v^3+9))^(1/3)*v-54-18*sqrt(4*v^3+9))-sqrt(4*v^3+9)-3)/(12+4*sqrt(4*v^3+9))^(2/3):
subs(z=z1,-3*z^2-12*v);

```

$$-\frac{z}{2} + \frac{\sqrt{-3z^2 - 12v}}{2}, \quad -\frac{z}{2} - \frac{\sqrt{-3z^2 - 12v}}{2}$$

$$-3 \left(\frac{\theta}{2} - \frac{2v}{\theta} \right)^2 - 12v \quad (18.6)$$

```
> r11 := z2*r2;
  s11 := z1*s2;
  zamproc(2/3,0,0,1,0,1,0,v, r11,s11,r2,s2);
```

$$\begin{aligned}
& - \left(4 \left(\left(\left(-\frac{\nu^3}{12} - \frac{\sqrt{4\nu^3+9}}{8} - \frac{3}{8} \right) (12 + 4\sqrt{4\nu^3+9})^{2/3} \right. \right. \right. \\
& \left. \left. \left. + \frac{4 \left(-\frac{(12 + 4\sqrt{4\nu^3+9})^{1/3}(3 + \sqrt{4\nu^3+9})\nu}{16} + \nu^3 + \frac{3\sqrt{4\nu^3+9}}{2} + \frac{9}{2} \right) \nu}{3} \right) \right. \\
& \left. \sqrt{-3(12 + 4\sqrt{4\nu^3+9})^{2/3}\nu^2 - 6(12 + 4\sqrt{4\nu^3+9})^{1/3}(3 + \sqrt{4\nu^3+9})\nu - 12\nu^3 - 18\sqrt{4\nu^3+9} - 54} + \left(-\frac{3\nu^3}{2} \right. \right. \\
& \left. \left. - \frac{9\sqrt{4\nu^3+9}}{4} - \frac{27}{4} \right) (12 + 4\sqrt{4\nu^3+9})^{2/3} + \left(\nu \left(\nu^3 + \frac{3\sqrt{4\nu^3+9}}{2} + \frac{9}{2} \right) (12 + 4\sqrt{4\nu^3+9})^{1/3} - \sqrt{4\nu^3+9}\nu^3 - 9\nu^3 \right. \right. \\
& \left. \left. - 9\sqrt{4\nu^3+9} - 27 \right) \nu \right) r_{22} \right) \Bigg/ \left(\left((12 + 4\sqrt{4\nu^3+9})^{1/3}\nu - \sqrt{4\nu^3+9} \right. \right. \\
& \left. \left. + \frac{\sqrt{-3(12 + 4\sqrt{4\nu^3+9})^{2/3}\nu^2 - 6(12 + 4\sqrt{4\nu^3+9})^{1/3}(3 + \sqrt{4\nu^3+9})\nu - 12\nu^3 - 18\sqrt{4\nu^3+9} - 54}}{3} - 3 \right) (3 \right. \\
& \left. + \sqrt{4\nu^3+9})^2 \right), \left(240 s_2 r_2 \left(\left(\frac{\nu(3 + \sqrt{4\nu^3+9})(12 + 4\sqrt{4\nu^3+9})^{2/3}}{40} + \left(-\frac{\nu^3}{20} - \frac{3\sqrt{4\nu^3+9}}{40} - \frac{9}{40} \right) (12 + 4\sqrt{4\nu^3+9})^{1/3} \right. \right. \right. \\
& \left. \left. \left. - \frac{\nu^2(3 + \sqrt{4\nu^3+9})}{20} \right) \right) \right. \\
& \left. \sqrt{-3(12 + 4\sqrt{4\nu^3+9})^{2/3}\nu^2 - 6(12 + 4\sqrt{4\nu^3+9})^{1/3}(3 + \sqrt{4\nu^3+9})\nu - 12\nu^3 - 18\sqrt{4\nu^3+9} - 54} + \left(\nu \right. \right. \\
& \left. \left. - \frac{\nu(12 + 4\sqrt{4\nu^3+9})^{2/3}}{4} + \frac{3(12 + 4\sqrt{4\nu^3+9})^{1/3}}{10} \right) \left(\nu^3 + \frac{3\sqrt{4\nu^3+9}}{2} + \frac{9}{2} \right) \right) \Bigg) \Bigg/ \left((12 + 4\sqrt{4\nu^3+9})^{2/3}(3 \right. \right. \\
& \left. \left. + \sqrt{4\nu^3+9}) \left(3(12 + 4\sqrt{4\nu^3+9})^{1/3}\nu + \sqrt{-3(12 + 4\sqrt{4\nu^3+9})^{2/3}\nu^2 - 6(12 + 4\sqrt{4\nu^3+9})^{1/3}(3 + \sqrt{4\nu^3+9})\nu - 12\nu^3 - 18\sqrt{4\nu^3+9} - 54} - 3\sqrt{4\nu^3+9} - 9 \right) \right. \right. \\
& \left. \left. \right), 0, 0 \right)
\end{aligned}$$

$$0, 0, - \left(16 s2 r2 \left(\left(\left(\frac{\sqrt{4 v^3 + 9}}{64} + \frac{3}{64} \right) (12 + 4 \sqrt{4 v^3 + 9})^2 \right)^{1/3} + \frac{v ((12 + 4 \sqrt{4 v^3 + 9})^{1/3} v - 3 \sqrt{4 v^3 + 9} - 9)}{16} \right) \right. \\ \left. \sqrt{-3 (12 + 4 \sqrt{4 v^3 + 9})^2 v^2 - 6 (12 + 4 \sqrt{4 v^3 + 9})^{1/3} (3 + \sqrt{4 v^3 + 9}) v - 12 v^3 - 18 \sqrt{4 v^3 + 9} - 54} + \left(\frac{3 \sqrt{4 v^3 + 9}}{32} \right. \right. \\ \left. \left. + \frac{9}{32} \right) (12 + 4 \sqrt{4 v^3 + 9})^2 v^3 + \left(-\frac{(12 + 4 \sqrt{4 v^3 + 9})^{1/3} (3 + \sqrt{4 v^3 + 9}) v}{4} + v^3 + \frac{3 \sqrt{4 v^3 + 9}}{2} + \frac{9}{2} \right) v \right) \right) \Bigg/ \left(\left((12 \right. \right. \\ \left. \left. + 4 \sqrt{4 v^3 + 9})^{1/3} v - \sqrt{4 v^3 + 9} \right. \right. \\ \left. \left. + \frac{\sqrt{-3 (12 + 4 \sqrt{4 v^3 + 9})^2 v^2 - 6 (12 + 4 \sqrt{4 v^3 + 9})^{1/3} (3 + \sqrt{4 v^3 + 9}) v - 12 v^3 - 18 \sqrt{4 v^3 + 9} - 54}}{3} - 3 \right) \right. \\ \left. \left. + \sqrt{4 v^3 + 9} \right) \right), -24 \left(\left(\frac{v (12 + 4 \sqrt{4 v^3 + 9})^{2/3}}{24} + \left(-\frac{\sqrt{4 v^3 + 9}}{24} - \frac{1}{8} \right) (12 + 4 \sqrt{4 v^3 + 9})^{1/3} \right. \right.$$

```


$$\left( \frac{\nu^2}{6} \right) \sqrt{-3 (12 + 4\sqrt{4\nu^3 + 9})^2 \nu^2 - 6 (12 + 4\sqrt{4\nu^3 + 9})^{1/3} (3 + \sqrt{4\nu^3 + 9}) \nu - 12\nu^3 - 18\sqrt{4\nu^3 + 9} - 54} + \left( \nu^2 \right.$$


$$- \frac{\nu (12 + 4\sqrt{4\nu^3 + 9})^{2/3}}{4} + \frac{3 (12 + 4\sqrt{4\nu^3 + 9})^{1/3}}{4} \left. (3 + \sqrt{4\nu^3 + 9}) \right) s2^2 \Bigg) \Big/ \left( (12 + 4\sqrt{4\nu^3 + 9})^{2/3} (3 (12 + 4\sqrt{4\nu^3 + 9})^{1/3} \nu \right.$$


$$+ \sqrt{-3 (12 + 4\sqrt{4\nu^3 + 9})^2 \nu^2 - 6 (12 + 4\sqrt{4\nu^3 + 9})^{1/3} (3 + \sqrt{4\nu^3 + 9}) \nu - 12\nu^3 - 18\sqrt{4\nu^3 + 9} - 54} - 3\sqrt{4\nu^3 + 9} - 9 \Big) \right)$$


```

> 4-2:
 solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-3:
 solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-6:
 solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-8:
 solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-10:
 solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});

NSF 4-16

$$\begin{aligned} > M := \text{zamproc}(u, 0, 1, v, 0, 0, 1, 0, r1, s1, r2, s2): \\ & \frac{r1^3 s2 u - r2^2 (s1 - s2) r1 + v r2^3 s2}{r1 s2 - r2 s1}, \frac{(3 s2^2 v - s1^2 + s1 s2) r2^2 - 2 r1 s2 (s1 - s2) r2 + 3 u r1^2 s1 s2}{r1 s2 - r2 s1}, \\ & \frac{s2 (3 r1 s1^2 u + 3 r2 s2^2 v - r1 s1 s2 + r1 s2^2 - 2 r2 s1^2 + 2 r2 s1 s2)}{r1 s2 - r2 s1}, \frac{s2 (s1^3 u + s2^3 v - s1^2 s2 + s1 s2^2)}{r1 s2 - r2 s1} \\ & - \frac{r2 (r1^3 u + r2^3 v - r1^2 r2 + r1 r2^2)}{r1 s2 - r2 s1}, - \frac{3 r2 \left(\left(v s2 + \frac{s1}{3} \right) r2^2 - \frac{r1 (s1 - 2 s2) r2}{3} + r1^2 \left(u s1 - \frac{2 s2}{3} \right) \right)}{r1 s2 - r2 s1}, \end{aligned} \quad (19.1)$$

$$\begin{aligned} > 2-1: \\ & \text{solve}([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,v,r1,s1,r2,s2\}); \\ > 2-10: \\ & \text{solve}([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], \{u,v,r1,s1,r2,s2\}); \\ > 3-1: \\ & \text{solve}([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,v,r1,s1,r2,s2\}); \\ & \{r1=r2, r2=r2, s1=s1, s2=0, u=0, v=0\} \end{aligned} \quad (19.2)$$

$$\begin{aligned} > 3-2: \\ & \text{solve}([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], \{u,v,r1,s1,r2,s2\}); \\ > 3-4: \\ & \text{solve}([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], \{u,v,r1,s1,r2,s2\}); \\ > 3-9: \\ & \text{solve}([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], \{u,v,r1,s1,r2,s2\}); \\ > 3-15: \\ & \text{solve}([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], \{u,v,r1,s1,r2,s2\}); \\ & \left\{ r1 = 2 r2, r2 = r2, s1 = -s2, s2 = s2, u = \frac{1}{3}, v = -\frac{2}{3} \right\} \end{aligned} \quad (19.3)$$

$$\begin{aligned} > r11 := 2 * r2: \\ & s11 := -s2: \\ & \text{zamproc}(1/3, 0, 1, -2/3, 0, 0, 1, 0, r11, s11, r2, s2): \\ & \quad 2 r2^2, 0, 0, -\frac{s2^3}{r2} \\ & \quad 0, 3 r2^2, 0, 0 \end{aligned} \quad (19.4)$$

> 3-20:
 solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
 solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:

NSE 4-17

```
> M := zamproc(u,v,0,1,0,1,0,0,r1,s1,r2,s2);
```

$$\begin{aligned}
& \frac{r1^3 s2 u + r2 (s2 v - s1) r1^2 + r2^3 s2}{r1 s2 - r2 s1}, \frac{(v s22 + s1 (3 u - 1) s2) r1^2 + 2 r2 s1 (s2 v - s1) r1 + 3 r2^2 s22}{r1 s2 - r2 s1}, \\
& \frac{-r2 s1^3 + 3 \left(\left(u - \frac{2}{3} \right) r1 + \frac{v r2}{3} \right) s2 s1^2 + 2 r1 s1 s22 v + 3 r2 s2^3}{r1 s2 - r2 s1}, \frac{((u - 1) s1^3 + s1^2 s2 v + s2^3) s2}{r1 s2 - r2 s1}, \\
& \frac{-r2 ((u - 1) r1^3 + r1^2 r2 v + r2^3)}{r1 s2 - r2 s1}, \frac{r1^3 s2 - 3 r2 \left(\left(u - \frac{2}{3} \right) s1 + \frac{s2 v}{3} \right) r1^2 - 2 r1 r2^2 s1 v - 3 r2^3 s2}{r1 s2 - r2 s1}, \\
& \frac{(-s1^2 v - 3 s22) r22 - 3 s1 r1 \left(s1 \left(u - \frac{1}{3} \right) + \frac{2 s2 v}{3} \right) r2 + 2 r1^2 s1 s2}{r1 s2 - r2 s1}, \frac{(-s1^3 u - s1^2 s2 v - s2^3) r2 + r1 s1^2 s2}{r1 s2 - r2 s1}
\end{aligned} \tag{20.1}$$

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1 = -\frac{r2}{2 RootOf(6 Z^3 + 1)}, r2 = r2, s1 = s1, s2 = RootOf(6 Z^3 + 1) s1, u = -\frac{1}{3}, v = 0}, {r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0} \tag{20.2}

```

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0} \tag{20.3}

```

```

> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1 = RootOf(_Z + RootOf(_Z - 3)^2 + RootOf(_Z - 3) _Z), r2 = r2, s1 = RootOf(_Z - 3) s2, s2 = s2, u = \frac{2}{3}, v = 0} \tag{20.4}

```

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1 = RootOf(2 Z^6 + 30 Z^3 - 9) r2, r2 = r2, s1 = -\frac{RootOf(2 Z^6 + 30 Z^3 - 9) s2 (14 RootOf(2 Z^6 + 30 Z^3 - 9)^3 - 3)}{52 RootOf(2 Z^6 + 30 Z^3 - 9)^3 - 15}, s2 = s2, u = \frac{24 RootOf(2 Z^6 + 30 Z^3 - 9)^6 + 95 RootOf(2 Z^6 + 30 Z^3 - 9)^3 - 30}{RootOf(2 Z^6 + 30 Z^3 - 9)^3 (52 RootOf(2 Z^6 + 30 Z^3 - 9)^3 - 15)}, v = \frac{28 RootOf(2 Z^6 + 30 Z^3 - 9)^6 - 162 RootOf(2 Z^6 + 30 Z^3 - 9)^3 + 45}{(52 RootOf(2 Z^6 + 30 Z^3 - 9)^3 - 15) RootOf(2 Z^6 + 30 Z^3 - 9)^2}} \tag{20.5}

```

```

> solve(2 * Z^6 + 30 * Z^3 - 9, Z);
z1 := -(1/2) * (36 * sqrt(3) + 60)^(1/3);
z2 := (1/2) * (-60 + 36 * sqrt(3))^(1/3);
-\frac{(36 \sqrt{3} + 60)^{1/3}}{2}, \frac{(36 \sqrt{3} + 60)^{1/3}}{4} - \frac{i \sqrt{3} (36 \sqrt{3} + 60)^{1/3}}{4}, \frac{(36 \sqrt{3} + 60)^{1/3}}{4} + \frac{i \sqrt{3} (36 \sqrt{3} + 60)^{1/3}}{4},
\frac{(-60 + 36 \sqrt{3})^{1/3}}{2}, -\frac{(-60 + 36 \sqrt{3})^{1/3}}{4} + \frac{i \sqrt{3} (-60 + 36 \sqrt{3})^{1/3}}{4}, -\frac{(-60 + 36 \sqrt{3})^{1/3}}{4} - \frac{i \sqrt{3} (-60 + 36 \sqrt{3})^{1/3}}{4} \tag{20.6}

```

```

> r11 := z1 * r2;
s11 := -z1 * s2 * (14 * z1^3 - 3) / (52 * z1^3 - 15);
u1 := evala((24 * z1^6 + 95 * z1^3 - 30) / (z1^3 * (52 * z1^3 - 15)));

```

```

v1 := evala(((28*z1^6-162*z1^3+45)/((52*z1^3-15)*z1^2)));
zamproc(u1,v1,0,1,0,1,0,0, r11,s11,r2,s2):

$$sII := \frac{(36\sqrt{3}+60)^{1/3}s2(-108-63\sqrt{3})}{2(-405-234\sqrt{3})}$$


$$uI := \frac{1}{3}$$


$$vI := -\frac{\sqrt{3}(36\sqrt{3}+60)^{1/3}}{2} + \frac{(36\sqrt{3}+60)^{1/3}}{2}$$


$$\frac{(36\sqrt{3}+60)^{2/3}r22}{4}, 0, -\frac{(36\sqrt{3}+60)^{2/3}(-3+2\sqrt{3})s22}{4}, 0$$


$$0, \frac{(36\sqrt{3}+60)^{2/3}(-3+2\sqrt{3})r22}{4}, 0, -\frac{(36\sqrt{3}+60)^{2/3}(-7+4\sqrt{3})s22}{4}$$

(20.7)

```

```

> s1 = evala(-(z1*(14*z1^3-3)/(52*z1^3-15))^3)^(1/3)*s2;
v = evala(((28*z1^6-162*z1^3+45)/((52*z1^3-15)*z1^2))^3)^(1/3);

$$sI = \left( -\frac{15}{2} + \frac{9\sqrt{3}}{2} \right)^{1/3}s2$$


$$v = (-6)^{1/3}$$

(20.8)

```

```

> r12 := z2*r2;
s12 := -z2*s2*(14*z2^3-3)/(52*z2^3-15);
u2 := evala((24*z2^6+95*z2^3-30)/(z2^3*(52*z2^3-15)));
v2 := evala((28*z2^6-162*z2^3+45)/((52*z2^3-15)*z2^2));
zamproc(u2,v2,0,1,0,1,0,0, r12,s12,r2,s2):

$$u2 := \frac{1}{3}$$


$$v2 := -\frac{(-60+36\sqrt{3})^{1/3}}{2} - \frac{\sqrt{3}(-60+36\sqrt{3})^{1/3}}{2}$$


$$\frac{(-60+36\sqrt{3})^{2/3}r22}{4}, 0, \frac{(-60+36\sqrt{3})^{2/3}(3+2\sqrt{3})s22}{4}, 0$$


$$0, -\frac{(-60+36\sqrt{3})^{2/3}(3+2\sqrt{3})r22}{4}, 0, \frac{(-60+36\sqrt{3})^{2/3}(7+4\sqrt{3})s22}{4}$$

(20.9)

```

```

> s1 = evala(-(z2*(14*z2^3-3)/(52*z2^3-15))^3)^(1/3)*s2;
v = evala(((28*z2^6-162*z2^3+45)/((52*z2^3-15)*z2^2))^3)^(1/3);

$$sI = \left( -\frac{15}{2} - \frac{9\sqrt{3}}{2} \right)^{1/3}s2$$


$$v = (-6)^{1/3}$$

(20.10)

```

```

> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-13:
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=rI, r2=0, s1=0, s2=s2, u=u, v=0},  $\left\{ rI = \text{RootOf}(4\text{ }_Z^3-3) \text{ } r2, r2=r2, sI=sI, s2=-\frac{2\text{ }sI \text{ RootOf}(4\text{ }_Z^3-3)^2}{3}, u=-\frac{1}{3}, v=0 \right\}$ 
(20.11)

```

```

> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {v,s1,r2,s2});

$$\left\{ r2 = \text{RootOf}(2\text{ }_Z^3-u) \text{ } rI, sI=sI, s2=-2 \text{ RootOf}(2\text{ }_Z^3-u) \text{ } sI, v=-\frac{3\text{ }u-2}{2 \text{ RootOf}(2\text{ }_Z^3-u)} \right\}$$

(20.12)

```

```

> z1 := (u/2)^(1/3):
r21 := z1*r1;
s21 := -2*s1*z1;
v1 := -(3*u-2)/(2*z1);
zamproc(u,v1,0,1,0,1,0,0, r1,s1,r21,s21):

$$vI := -\frac{(3\text{ }u-2)\text{ }2^{1/3}}{2\text{ }u^{1/3}}$$


$$rI^2, 0, (9\text{ }u-3)\text{ }sI^2, -\frac{2\text{ }sI^3}{rI}$$


$$0, 0, \frac{9\text{ }sI\text{ }rI\text{ }u}{2}, 0$$

(20.13)

```

NSF 4-21

```
> M := zamproc(u,0,0,v,1,0,0,1, r1,s1,r2,s2):
```

$$\begin{aligned}
& \frac{(s2 u - s1) r1^3 + r2^3 (s2 v - s1)}{r1 s2 - r2 s1}, \frac{-3 r1^2 s1^2 + 3 s2 (r1^2 u - r2^2) s1 + 3 v r2^2 s2^2}{r1 s2 - r2 s1}, \frac{3 r1 s1^2 s2 u + 3 r2 s2^3 v - 3 r1 s1^3 - 3 r2 s1 s2^2}{r1 s2 - r2 s1}, \\
& \frac{s1^3 s2 u + s2^4 v - s1^4 - s1 s2^3}{r1 s2 - r2 s1} \\
& \frac{-r1^3 r2 u - r2^4 v + r1^4 + r1 r2^3}{r1 s2 - r2 s1}, \frac{-3 r1^2 r2 s1 u - 3 r2^3 s2 v + 3 r1^3 s1 + 3 r1 r2^2 s2}{r1 s2 - r2 s1}, \frac{3 r1^2 s1^2 - 3 r2 (s1^2 u - s2^2) r1 - 3 v r2^2 s2^2}{r1 s2 - r2 s1}, \\
& \frac{-r2 (s1^3 u + s2^3 v) + r1 (s1^3 + s2^3)}{r1 s2 - r2 s1}
\end{aligned} \tag{21.1}$$

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
(r1=r2,r2=r2,s1=s1,s2=-s1,u=1,v=1)

```

```

> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=u,v=0)

```

```

> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=0,v=0)

```

```

> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=0,v=0)

```

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=0,v=v)

```

```

> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=s2,s2=s2,u=-1,v=-1}, {r1 =  $\frac{r2}{RootOf(\mathcal{Z}^2 + Z + 1) + 1}$ , r2=r2,s1=RootOf(\mathcal{Z}^2 + Z + 1)s2,s2=s2,u =  $\frac{1}{RootOf(\mathcal{Z}^2 + Z + 1) + 1}$ , v =  $\frac{1}{RootOf(\mathcal{Z}^2 + Z + 1) + 1}$ }

```

```

> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r1,r2=r2,s1=s1,s2=RootOf(\mathcal{Z}^2 - Z + 1)s1,u =  $\frac{r1}{r2}$ ,v =  $\frac{r1}{r2}$ }, {r1=0,r2=r2,s1=s1,s2=s2,u =  $\frac{s1^3 + s2^3}{s1^2 s2}$ ,v=0}, {r1=r1,r2 =  $\frac{r1}{r2}$ ,s1=s1,s2=-s1,u =  $\frac{r1}{r2}$ ,v =  $\frac{r1}{r2}$ }

```

```

> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
(r1=r2,r2=r2,s1=s1,s2=-s1,u=1,v=1)

```

```

> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=u,v=0)

```

```

> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {v,r1,s1,r2,s2});
{r1 = - $\frac{r2}{RootOf(\mathcal{Z}^2 - u)}$ , r2=r2,s1=s1,s2=RootOf(\mathcal{Z}^2 - u)s1,v = - $\frac{RootOf(\mathcal{Z}^2 - u)^3 - RootOf(\mathcal{Z}^2 - u)u - 1}{RootOf(\mathcal{Z}^2 - u)^4}$ }

```

```

> z1 := sqrt(u):
z2 := -sqrt(u):
> r21 := -z1*r1:
s21 := z1*s1:
v1 := evala(-(z1^3-z1*u-1)/z1^4);
zamproc(u,0,0,v1,1,0,0,1, r1,s1,r21,s21):

```

$$\begin{aligned}
v1 &:= \frac{1}{u^2} \\
\frac{rI^2 (\sqrt{u} - 1) (u + \sqrt{u} + 1)}{\sqrt{u}}, 0, \frac{3 sI^2 (\sqrt{u} - 1) (u + \sqrt{u} + 1)}{\sqrt{u}}, 0 \\
0, -\frac{3 rI^2 (\sqrt{u} + 1) (-u + \sqrt{u} - 1)}{\sqrt{u}}, 0, \frac{sI^2 (\sqrt{u} + 1) (u - \sqrt{u} + 1)}{\sqrt{u}}
\end{aligned} \tag{21.12}$$

> v2 := evala(-(z2^3-z2*u-1)/z2^4);

$$v2 := \frac{1}{u^2} \tag{21.13}$$

> 4-10:

$$\begin{aligned}
&\text{solve}([\mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\
&\left\{ r1 = r1, r2 = r2, s1 = \text{RootOf}(\mathcal{Z} r1 + r2) s2, s2 = s2, u = \frac{r1}{r2}, v = \frac{r1}{r2} \right\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0\}
\end{aligned} \tag{21.14}$$

> 4-13:

$$\begin{aligned}
&\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\
&\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0\}, \{r1 = -r2, r2 = r2, s1 = s2, s2 = s2, u = 1, v = 1\}, \{r1 = \text{RootOf}(\mathcal{Z} - \mathcal{Z} + 1) r2, r2 = r2, s1 = -s2, s2 = s2, u = -\text{RootOf}(\mathcal{Z} - \mathcal{Z} + 1), v = -\text{RootOf}(\mathcal{Z} - \mathcal{Z} + 1)\}
\end{aligned} \tag{21.15}$$

> 4-16:

$$\begin{aligned}
&\text{solve}([\mathbf{M}[1,2], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\
&\left\{ r1 = r1, r2 = r2, s1 = s1, s2 = -\frac{s1 r1^2}{r2^2}, u = \frac{r1}{r2}, v = \frac{r1}{r2} \right\}
\end{aligned} \tag{21.16}$$

> 4-17:

$$\begin{aligned}
&\text{solve}([\mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\
&\left\{ r1 = r1, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0 \right\}, \left\{ r1 = r1, r2 = r2, s1 = \text{RootOf}(\mathcal{Z} r1 + r2) s2, s2 = s2, u = \frac{r1}{r2}, v = \frac{r1}{r2} \right\}
\end{aligned} \tag{21.17}$$

NSF 4-22

$$\begin{aligned}
&\text{M} := \text{zamproc}(u, 0, 0, v, 0, 1, 1, 0, r1, s1, r2, s2): \\
&\frac{rI^3 s2 u + vr23 s2 - s1 rI2 r2 - r1 s1 r22}{-s1 r2 + r1 s2}, \frac{(-2 rI r2 - r22) sI2 + 3 rI \left(\left(u - \frac{1}{3}\right) rI - \frac{2 r2}{3} \right) s2 s1 + 3 vr22 s22}{-s1 r2 + r1 s2}, \\
&\frac{-r2 sI3 + 3 \left(\left(u - \frac{2}{3}\right) rI - \frac{2 r2}{3} \right) s2 sI2 - rI s1 s22 + 3 vr2 s23}{-s1 r2 + r1 s2}, \frac{((u - 1) sI3 - s2 s1 s2 + vs23) s2}{-s1 r2 + r1 s2} \\
&\frac{((u - 1) rI3 - rI2 r2 + vr23) r2}{-s1 r2 + r1 s2}, \frac{rI^3 s2 - 3 r2 \left(\left(u - \frac{2}{3}\right) sI - \frac{2 s2}{3} \right) rI2 + rI s1 r22 - 3 vr23 s2}{-s1 r2 + r1 s2}, \\
&\frac{(2 s1 s2 + s22) rI2 - 3 r2 s1 \left(sI \left(u - \frac{1}{3}\right) - \frac{2 s2}{3} \right) rI - 3 vr22 s22}{-s1 r2 + r1 s2}, \frac{-r2 sI3 u - vr2 s23 + rI sI2 s2 + rI s1 s22}{-s1 r2 + r1 s2}
\end{aligned} \tag{22.1}$$

> 2-1:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

> 2-10:

$$\text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

> 3-1:

$$\begin{aligned}
&\text{solve}([\mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\
&\left\{ r1 = 0, r2 = r2, s1 = -2 s2, s2 = s2, u = \frac{1}{2}, v = 0 \right\}
\end{aligned} \tag{22.2}$$

> 3-2:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

> 3-4:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

> 3-9:

$$\begin{aligned}
&\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\
&\left\{ r1 = -2 r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{1}{2}, v = 0 \right\}
\end{aligned} \tag{22.3}$$

> 3-15:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

> 3-20:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

```

> 3-23:
  solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = 0, r2 = r2, s1 = s1, s2 = -\frac{s1}{2}, u = 0, v = 0 \right\}$  (22.4)

> 3-24:
  solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
  solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = 0, r2 = r2, s1 = -2s2, s2 = s2, u = \frac{1}{2}, v = 0 \right\}, \left\{ r1 = -2r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{1}{2}, v = 0 \right\}$  (22.5)

> 4-2:
  solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = 0, r2 = r2, s1 = s1, s2 = s2, u = \frac{s1 + s2}{s1}, v = 0 \right\}, \left\{ r1 = -r2, r2 = r2, s1 = \frac{s2}{2}, s2 = s2, u = \frac{1}{3}, v = \frac{1}{3} \right\}, \left\{ r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0 \right\}, \left\{ r1 = -r2, r2 = r2, s1 = s1, s2 = 0, u = \frac{1}{3}, v = \frac{1}{3} \right\}$  (22.6)

> 4-3:
  solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = r2, r2 = r2, s1 = -s2, s2 = s2, u = 1, v = 1 \right\}, \left\{ r1 = -2r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{1}{2}, v = 0 \right\}, \left\{ r1 = -\frac{r2}{2}, r2 = r2, s1 = s1, s2 = 0, u = 0, v = \frac{1}{8} \right\}$  (22.7)

> 4-6:
  solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = -2r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{1}{2}, v = 0 \right\}$  (22.8)

> 4-8:
  solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = -\frac{r2}{2}, r2 = r2, s1 = s1, s2 = 0, u = \frac{1}{3}, v = \frac{1}{6} \right\}, \left\{ r1 = r1, r2 = 0, s1 = -\frac{s2}{2}, s2 = s2, u = \frac{1}{3}, v = \frac{1}{6} \right\}$  (22.9)

> s21 := -2*s1:
  zamproc(1/3,0,0,1/6,0,1,1,0, r1,s1,0,s21):
   $\frac{rl^2}{3}, 0, s12, 0$ 
   $0, rl2, 0, -s12$  (22.10)

> 4-10:
  solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = -2r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{1}{2}, v = 0 \right\}, \left\{ r1 = 0, r2 = r2, s1 = s1, s2 = -\frac{s1}{2}, u = 0, v = 0 \right\}$  (22.11)

> 4-13:
  solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
   $\left\{ r1 = -r2, r2 = r2, s1 = s1, s2 = s1, u = -\frac{1}{3}, v = -\frac{1}{3} \right\}$  (22.12)

> 4-16:
  evala([solve([M[1,2],M[2,1],M[2,2],M[2,4]], {v,r1,r2,s2})]);
   $\left[ \left\{ r1 = \frac{r2}{3u-2}, r2 = r2, s2 = -\frac{s1u(3u-2)}{2u-1}, v = \frac{2u-1}{(3u-2)^3} \right\} \right]$  (22.13)

> r21 := (3*u-2)*r1:
  s21 := -s1*u*(3*u-2)/(2*u-1):
  v1 := (2*u-1)/(3*u-2)^3:
  zamproc(u,0,0,v1,0,1,1,0, r1,s1,r21,s21):
   $r1/2(3u-1), 0, \frac{s1^2(3u-1)}{2u-1}, \frac{us1^3(3u-1)(u-1)}{r1(2u-1)^2}$ 
   $0, 0, \frac{s1(3u-1)^2r1}{2u-1}, 0$  (22.14)

> 4-17:
  factor(evala([solve([M[1,3],M[2,1],M[2,3],M[2,4]], {v,r2,s2})]));
   $\left[ \left\{ r2 = \frac{r1(3u+1)(u-1)}{4u}, s2 = -\frac{(3u+1)s1}{2}, v = -\frac{16u^2}{(u-1)(3u+1)^3} \right\} \right]$  (22.15)

> r21 := r1*(3*u+1)*(u-1)/(4*u):
  s21 := -(1/2)*s1*(3*u+1):
  v1 := -16*u^2/((u-1)*(3*u+1)^3):

```

$$\begin{aligned} \text{zamproc}(u, 0, 0, v1, 0, 1, 1, 0, r1, s1, r21, s21) : \\ \frac{r1^2(u+1)(3u-1)}{4u}, \frac{s1r1(9u^2-1)}{4u}, 0, \frac{s1^3(3u-1)u}{r1(u-1)} \\ 0, \frac{r1^2(3u-1)^2}{4u}, 0, 0 \end{aligned} \quad (22.16)$$

$$\begin{aligned} > 4-21: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ \left\{ r1 = r1, r2 = \text{RootOf}(\mathcal{Z} + \mathcal{Z} + 1), r1, s1 = s1, s2 = -\frac{s1(\text{RootOf}(\mathcal{Z} + \mathcal{Z} + 1) - 1)}{2 \text{RootOf}(\mathcal{Z} + \mathcal{Z} + 1) + 1}, u = 0, v = 0 \right\} \end{aligned} \quad (22.17)$$

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$$\begin{aligned} > M := \text{zamproc}(u, 0, v, 1, 0, 1, 0, 0, r1, s1, r2, s2); \\ \frac{s2(r1^3u + vr1r22 + r23) - s1r2r1l2}{s2rl - r2s1}, \frac{(2vr1r2 + 3r22)s22 + 3\left(u - \frac{1}{3}\right)r1l2 + \frac{vr22}{3}s1s2 - 2r2rls1l2}{s2rl - r2s1}, \\ \frac{(vr1 + 3r2)s23 + 2vs1r2s22 + 3s1l2r1\left(u - \frac{2}{3}\right)s2 - r2s1l3}{s2rl - r2s1}, \frac{((u-1)s1l3 + vs1s22 + s23)s2}{s2rl - r2s1} \\ \frac{r2((u-1)r1l3 + vr1r22 + r23)}{s2rl - r2s1}, \frac{(-vs1 - 3s2)r23 - 2vr1r22s2 - 3s1rl2\left(u - \frac{2}{3}\right)r2 + rl^3s2}{s2rl - r2s1}, \\ \frac{(-2s1s2v - 3s22)r22 - 3\left(u - \frac{1}{3}\right)s1l2 + \frac{vs22}{3})r1r2 + 2s2rl2s1}{s2rl - r2s1}, \frac{(-us1l3 - vs1s22 - s23)r2 + s2rls1l2}{s2rl - r2s1} \end{aligned} \quad (23.1)$$

$$\begin{aligned} > 2-1: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 2-10: \\ \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-1: \\ \text{solve}([\mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-2: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-4: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-9: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-15: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0\}, \left\{ r1 = -\frac{\text{RootOf}(\mathcal{Z} + 6)r2}{2}, r2 = r2, s1 = \text{RootOf}(\mathcal{Z} + 6)s2, s2 = s2, u = \frac{\text{RootOf}(\mathcal{Z} + 6)^3}{12} + \frac{1}{6}, v = 0 \right\} \quad (23.2) \end{aligned}$$

$$\begin{aligned} > 3-20: \\ \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-23: \\ \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-24: \\ \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0\} \quad (23.3) \end{aligned}$$

$$\begin{aligned} > 4-1: \\ \text{solve}([\mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2]], \{u, v, r1, s1, r2, s2\}); \\ \left\{ r1 = \text{RootOf}(\mathcal{Z} + \text{RootOf}(\mathcal{Z} - 3)^2 + \text{RootOf}(\mathcal{Z} - 3)\mathcal{Z})r2, r2 = r2, s1 = \text{RootOf}(\mathcal{Z} - 3)s2, s2 = s2, u = \frac{2}{3}, v = 0 \right\} \quad (23.4) \end{aligned}$$

$$\begin{aligned} > 4-2: \\ \text{solve}([\mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ \left\{ r1 = \text{RootOf}(\mathcal{Z} + 3)r2, r2 = r2, s1 = s1, s2 = 0, u = \frac{1}{3}, v = -\frac{3}{\text{RootOf}(\mathcal{Z} + 3)} \right\} \quad (23.5) \end{aligned}$$

$$\begin{aligned} > r11 := -3^{(1/3)} * r2; \\ v1 := 3^{(2/3)}; \\ \text{zamproc}(1/3, 0, v1, 1, 0, 1, 0, 0, r11, s1, r2, 0); \\ v1 := 32 \sqrt[3]{3} \\ 32 \sqrt[3]{3} r22, -2 \sqrt[3]{3} r2 s1, s1 l2, 0 \end{aligned}$$

$$0, 0, 0, \frac{sI^2}{3} \quad (23.6)$$

```
> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1});
{r1=RootOf(_Z^3-9) r2, s1=-RootOf(_Z^3-9) s2, u=5/9, v=3/RootOf(_Z^3-9)} (23.7)
```

```
> z1 := 3^(2/3):
r11 := z1*r2:
s11 := -z1*s2/2:
zamproc(5/9,0,3^(1/3),1,0,1,0,0, r11,s11,r2,s2):
3 r22 31^(1/3), 0, 3 s22 31^(1/3)
3 r22 31^(1/3), 0, 3 s22 31^(1/3) (23.8)
```

```
> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,s2});
{r1=RootOf(_Z^6-_Z^6+15 _Z^3+9) r2, s1=s1, s2=-s1 (RootOf(_Z^6-_Z^6+15 _Z^3+9)^3+1)/2 RootOf(_Z^6-_Z^6+15 _Z^3+9), u = RootOf(_Z^6-_Z^6+15 _Z^3+1)/2 RootOf(_Z^6-_Z^6+15 _Z^3+9)^3, v = RootOf(_Z^6-_Z^6+15 _Z^3-3)/2 RootOf(_Z^6-_Z^6+15 _Z^3+9)} (23.9)
```

```
=> r11 := theta*r2:
s21 := -s1*(theta^3+1)/(2*theta):
u1 := (theta^3+1)/(2*theta^3):
v1 := (theta^3-3)/(2*theta):
zamproc(u1,0,v1,1,0,1,0,0, r11,s1,r2,s21):
θ² r2², -(θ³-3) θ r2 s1 , (θ⁹-θ⁶+15 θ³+9) sI² , -(θ+1) (θ²-θ+1) sI³
θ² r2², -(θ³-3) θ r2 s1 , (θ⁹-θ⁶+15 θ³+9) sI² , -(θ+1) (θ²-θ+1) sI³
0, 0, (θ³+3)² sI r2 , 0 (23.10)
```

```
> 4-13:
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=u, v=0}, {r1=RootOf(4 _Z^3-3) r2, r2=r2, s1=-2 s2 RootOf(4 _Z^3-3), s2=s2, u=-1/3, v=0} (23.11)
```

```
> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=RootOf(_Z^3-3) r2, r2=r2, s1=s1, s2=-2 s1 RootOf(_Z^3-3)^2/3, u=2/3, v=0} (23.12)
```

```
> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=-r2/2 RootOf(6 _Z^3+1), r2=r2, s1=s1, s2=RootOf(6 _Z^3+1) s1, u=-1/3, v=0}, {r1=r1, r2=0, s1=0, s2=s2, u=u, v=0} (23.13)
```

```
> 4-21:
solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=u, v=0}, {r1=-RootOf(_Z^3+6) r2/2, r2=r2, s1=RootOf(_Z^3+6) s2, s2=s2, u=RootOf(_Z^3+6)^3/12 + 1/6, v=0} (23.14)
```

NSF 4-26

```
> M := zamproc(1,u,0,v,1,0,0,0, r1,s1,r2,s2):
(-s1+s2) r1^3+u r1^2 r2 s2+v r2^3 s2 , (s2^2 u-3 s1^2+3 s1 s2) r1^2+2 s2 r2 r1 s1 u+3 v s2^2 r2^2 ,
s2 r1-r2 s1 , s2 r1-r2 s1
-3 r1 s1^3+s2 (u r2+3 r1) s1^2+2 r1 u s1 s2^2+3 v s2^3 r2 , s1^2 s2^2 u+s2^4 v-s1^4+s1^3 s2
s2 r1-r2 s1 , s2 r1-r2 s1 (24.1)
```

$$\frac{-r1^2 r22 u - r24 v + r14 - r2 r13}{s2 r1 - r2 s1}, \frac{3 s1 r13 - r2 (s2 u + 3 s1) r12 - 2 r1 u s1 r22 - 3 v r23 s2}{s2 r1 - r2 s1}, \quad (24.1)$$

$$\frac{(-s12 u - 3 v s22) r22 - 2 \left(s2 u + \frac{3 s1}{2} \right) s1 r1 r2 + 3 r12 s12}{s2 r1 - r2 s1}, \frac{(-s12 u s2 - v s23 - s13) r2 + r1 s13}{s2 r1 - r2 s1}$$

> 2-1:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}, \{r1=0, r2=r2, s1=s2, s2=s2, u=0, v=0\}$

> 2-10:
 $\text{solve}([\text{M}[1, 1], \text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 2], \text{M}[2, 3], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$

> 3-1:
 $\text{solve}([\text{M}[1, 3], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=0, r2=r2, s1=s1, s2=s1, u=0, v=0\}, \{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}$

> 3-2:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}, \{r1=0, r2=r2, s1=s1, s2=s1, u=0, v=0\}$

> 3-4:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=0, r2=r2, s1=s1, s2=s2, u=0, v=0\}, \{r1=r1, r2=r1, s1=0, s2=s2, u=0, v=0\}$

> 3-9:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}$

> 3-15:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 3], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}$

> 3-20:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 2], \text{M}[2, 3], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r1, r2=r2, s1=0, s2=s2, u=0, v=0\}, \{r1=r1, r2=0, s1=0, s2=s2, u=0, v=v\}$

> 3-23:
 $\text{solve}([\text{M}[1, 1], \text{M}[1, 3], \text{M}[2, 2], \text{M}[2, 3], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$

> 3-24:
 $\text{solve}([\text{M}[1, 1], \text{M}[1, 2], \text{M}[2, 2], \text{M}[2, 3], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$

> 4-1:
 $\text{solve}([\text{M}[1, 3], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}, \{r1=0, r2=r2, s1=s1, s2=s1, u=0, v=0\}$

> 4-2:
 $\text{solve}([\text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=0, r2=r2, s1=s1, s2=s1, u=0, v=0\}, \{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}$

> 4-3:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2]], \{u, v, r1, s1, r2, s2\});$
 $\left\{ r1=0, r2=r2, s1=s1, s2=s2, u=\frac{s1(s1-s2)}{s2^2}, v=0 \right\}, \{r1=r1, r2=r1, s1=0, s2=s2, u=0, v=0\}$

> 4-6:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 2]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0\}, \{r1=0, r2=r2, s1=s1, s2=s2, u=0, v=0\}, \{r1=r1, r2=r1, s1=0, s2=s2, u=0, v=0\}$

> 4-8:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=r1, r2=r1, s1=0, s2=s2, u=0, v=0\}, \{r1=0, r2=r2, s1=s1, s2=s1, u=0, v=0\}$

> 4-10:
 $\text{solve}([\text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 4]], \{u, v, r1, s1, r2, s2\});$
 $\left\{ r1=\frac{3 r2}{2}, r2=r2, s1=-\frac{3 s2}{4}, s2=s2, u=\frac{9}{8}, v=-\frac{27}{32} \right\}, \{r1=r2, r2=r2, s1=0, s2=s2, u=0, v=0\}$

> r11 := 3*r2*(1/2):
 $s11 := -3*s2*(1/4):$
 $\text{zamproc}(1, 9/8, 0, -27/32, 1, 0, 0, 0, r11, s11, r2, s2):$
 $\frac{27 r22}{8}, -\frac{81 s2 r2}{16}, 0, -\frac{27 s23}{64 r2}$
 $0, 0, \frac{81 s2 r2}{32}, 0$

> 4-13:
 $\text{solve}([\text{M}[1, 2], \text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 3]], \{u, v, r1, s1, r2, s2\});$
 $\{r1=0, r2=r2, s1=s1, s2=s2, u=0, v=0\}, \{r1=r1, r2=r1, s1=0, s2=s2, u=0, v=0\}$

```

> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r2,r2=r2,s1=0,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=s1,s2=s2,u=-s1/s2,v=0} (24.17)

> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1,r2=r2,s1=0,s2=s2,u=-r1^2+r1 r2/r2^2,v=0} (24.18)

> 4-21:
solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r1,r2=r2,s1=0,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=s1,s2=0,u=0,v=v}, {r1=r1,r2=0,s1=0,s2=s2,u=0,v=v} (24.19)

> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r2,r2=r2,s1=0,s2=s2,u=0,v=0} (24.20)

> 4-25:
solve([M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r2,r2=r2,s1=0,s2=s2,u=0,v=0}, {r1=3 r2,r2=r2,s1=s1,s2=-s1/6,u=-18,v=216} (24.21)

> r11 := 3*r2;
s11 := -6*s2;
zamproc(1,-18,0,216,1,0,0,0, r11,s11,r2,s2):
27 r2^2, 0, 324 s2^2, -216 s2^3
0, -162 r2^2, 0, 0 (24.22)

```

NSF 4-28

```

> M := zamproc(0,u,0,v,1,0,0,1, r1,s1,r2,s2):
(s2 v - s1) r2^3 + u r1^2 r2 s2 - s1 r1^3, (u r1^2 + 3 v r2^2) s2^2 + 2 (r1 u - 3 r2^2/2) r2 s1 s2 - 3 r1^2 s1^2
s2 r1 - r2 s1, s2 r1 - r2 s1,
-3 r1 s1^3 + s1^2 u r2 s2 + 2 (r1 u - 3 r2^2/2) s2^2 s1 + 3 v s2^3 r2
s2 r1 - r2 s1, s2 r1 - r2 s1,
-r1^2 r2^2 u - r2^4 v + r1^4 + r1 r2^3, 3 s1 r1^3 - u r1^2 r2 s2 - 2 (s1 u - 3 s2^2/2) r2^2 r1 - 3 v r2^3 s2
s2 r1 - r2 s1, s2 r1 - r2 s1,
(-s1^2 u - 3 v s2^2) r2^2 - 2 (s1 u - 3 s2^2/2) r1 s2 r2 + 3 r1^2 s1^2
s2 r1 - r2 s1, s2 r1 - r2 s1 (25.1)

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0,v=0} (25.2)

> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (25.3)

> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0,v=0} (25.4)

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} (25.5)

> 3-23:

```

```

solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=RootOf(_Z-2)r2,r2=r2,s1=0,s2=s2,u=3 RootOf(_Z-2)^2/2,v=0}, {r1=0,r2=r2,s1=RootOf(_Z-2)s2,s2=s2,u=0,v=0}
= 3/RootOf(_Z-2), v=0} (25.6)

```

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s2 RootOf(_Z-_Z+1),s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=-s2,s2=s2,u=0,v=0} (25.7)

```

```

> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,s2});
{r1=RootOf(_Z^6-4_Z^3+2)r2,s1=-s2,u=3 RootOf(_Z^6-4_Z^3+2)^2/2,v=
- RootOf(_Z^6-4_Z^3+2)(RootOf(_Z^6-4_Z^3+2)^3-2)/2} (25.8)

```

```

> solve(_Z^6-4*_Z^3+2, _Z);
z1 := (2+sqrt(2))^(1/3):
z2 := (2-sqrt(2))^(1/3):
(2-sqrt(2))^(1/3), -(2-sqrt(2))^(1/3) + sqrt(3)(2-sqrt(2))^(1/3), -(2-sqrt(2))^(1/3) - sqrt(3)(2-sqrt(2))^(1/3), (2+sqrt(2))^(1/3), -(2+sqrt(2))^(1/3)
+ sqrt(3)(2+sqrt(2))^(1/3), -(2+sqrt(2))^(1/3) - sqrt(3)(2+sqrt(2))^(1/3) (25.9)

```

```

> r11 := z1*r2:
s21 := -z1^2*s1:
u1 := 3*z1^2*(1/2):
v1 := -(1/2)*z1*(z1^3-2):
zamproc(0,u1,0,v1,1,0,0,1, r11,s1,r2,s21):
u1 := 3(2+sqrt(2))^2^(1/3)/2
v1 := -(2+sqrt(2))^(1/3)sqrt(2)
(3+sqrt(2))r2, 0, -3(2+sqrt(2))^(1/3)(3sqrt(2)+2)s12, 0
0, 0, -3(3+sqrt(2))s1r2(2+sqrt(2))^(1/3)/2, (2+sqrt(2))^(1/3)(3sqrt(2)+2)s12
(25.10)

```

```

> r12 := z2*r2:
s22 := -z2^2*s1:
u2 := 3*z2^2*(1/2):
v2 := -(1/2)*z2*(z2^3-2):
zamproc(0,u2,0,v2,1,0,0,1, r12,s1,r2,s22):
u2 := 3(2-sqrt(2))^2^(1/3)/2
v2 := (2-sqrt(2))^(1/3)sqrt(2)
(-3+sqrt(2))r2, 0, 3(2-sqrt(2))^(1/3)(3sqrt(2)-2)s12, 0
0, 0, 3(-3+sqrt(2))s1r2(2-sqrt(2))^(1/3)/2, -(2-sqrt(2))^(1/3)(3sqrt(2)-2)s12
(25.11)

```

```

> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (25.12)

```

```

> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=RootOf(2_Z^6+10_Z^3-1)r2,r2=r2,s1=-RootOf(2_Z^6+10_Z^3-1)s2(RootOf(2_Z^6+10_Z^3-2)/5 RootOf(2_Z^6+10_Z^3-1)),s2=s2,u=3(4 RootOf(2_Z^6+10_Z^3-1)^6+6 RootOf(2_Z^6+10_Z^3-1)^3-1)/2(5 RootOf(2_Z^6+10_Z^3-1)^3-1),v=0} (25.13)

```

```
> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (25.14)
```

```
> 4-13:
evala([solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1})]);
[[{r1=RootOf(_Z^6+7_Z^3+1)r2,s1=-((RootOf(_Z^6+7_Z^3+1)^4+8RootOf(_Z^6+7_Z^3+1))s2)/3,u=-2RootOf(_Z^6+7_Z^3+1)^5 (25.15)
-13RootOf(_Z^6+7_Z^3+1)^2,v=-RootOf(_Z^6+7_Z^3+1)}]]
```

```
> solve(_Z^6+7*_Z^3+1, _Z);
z1 := -(1/2)*(12*sqrt(5)+28)^(1/3):
z2 := -(1/2)*(-12*sqrt(5)+28)^(1/3):

$$\frac{(12\sqrt{5}+28)^{1/3}}{2}, \frac{(12\sqrt{5}+28)^{1/3}}{4} - \frac{i\sqrt{3}(12\sqrt{5}+28)^{1/3}}{4}, \frac{(12\sqrt{5}+28)^{1/3}}{4} + \frac{i\sqrt{3}(12\sqrt{5}+28)^{1/3}}{4},$$


$$-\frac{(-12\sqrt{5}+28)^{1/3}}{2}, \frac{(-12\sqrt{5}+28)^{1/3}}{4} - \frac{i\sqrt{3}(-12\sqrt{5}+28)^{1/3}}{4}, \frac{(-12\sqrt{5}+28)^{1/3}}{4} + \frac{i\sqrt{3}(-12\sqrt{5}+28)^{1/3}}{4} (25.16)$$

```

```
> r11 := z1*r2:
s11 := -(1/3)*(z1^4+8*z1)*s2:
u1 := root(evala((-2*z1^5-13*z1^2)^3), 3);
v1 := -z1;
zamproc(0,u1,0,v1,1,0,0,1, r11,s11,r2,s2):
s11 := -
$$\frac{\left(\frac{(12\sqrt{5}+28)^4}{16}-4(12\sqrt{5}+28)^{1/3}\right)s2}{3}$$

u1 := 
$$\frac{3(4\sqrt{5}-4)^{1/3}}{2}$$

v1 := 
$$\frac{(12\sqrt{5}+28)^{1/3}}{2}$$


$$-\frac{3r22\sqrt{5}}{2} - \frac{5r22}{2}, 0, 0, -\frac{(3\sqrt{5}-5)s2^3}{2r2}$$


$$0, \frac{3(5+\sqrt{5})r22}{2}, 0, \sqrt{5}s2^2 (25.17)$$

```

```
> s1 = evala((-1/3)*(z1^4+8*z1))^3^(1/3)*s2;
s1 = 
$$\left(-\frac{\sqrt{5}}{2} + \frac{3}{2}\right)^{1/3}s2 (25.18)$$

```

```
> r12 := z2*r2:
s12 := -(1/3)*(z2^4+8*z2)*s2:
u2 := -2*z2^5-13*z2^2;
v2 := -z2;
zamproc(0,u2,0,v2,1,0,0,1, r12,s12,r2,s2):
u2 := 
$$\frac{(-12\sqrt{5}+28)^5}{16} - \frac{13(-12\sqrt{5}+28)^2}{4}$$

v2 := 
$$\frac{(-12\sqrt{5}+28)^{1/3}}{2}$$


$$\frac{(3\sqrt{5}-5)r22}{2}, 0, 0, \frac{(3\sqrt{5}+5)s2^3}{2r2}$$


$$0, -\frac{3(\sqrt{5}-5)r22}{2}, 0, -\sqrt{5}s2^2 (25.19)$$

```

```
> u = evala((-2*z2^5-13*z2^2)^3)^(1/3);
s1 = evala((-1/3)*(z2^4+8*z2))^3^(1/3)*s2;
u = 
$$\left(-\frac{27\sqrt{5}}{2} - \frac{27}{2}\right)^{1/3}$$

s1 = 
$$\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^{1/3}s2 (25.20)$$

```

```
> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (25.21)
```

```
> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0,v=0} (25.22)
```

> 4-21:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

$$\{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = 0, v = v\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = v\}$$
 (25.23)

```
> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0 r2=r2 s1=s1 s2=0 u=u v=0} (25.24)
```

```
> 4-25:
solve([M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$$\{r1 = -r2, r2 = r2, s1 = 2s2, s2 = s2, u = -3, v = 3\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0\}, \left\{ r1 = RootOf(_Z^3 + 3) r2, r2 = r2, s1 = 0, \right. \\ \left. s2 = s2, u = -\frac{3}{RootOf(_Z^3 + 3)}, v = RootOf(_Z^3 + 3) \right\} \quad (23.23)$$

```

> r11 := -3^(1/3)*r2:
u1 := 3^(2/3):
v1 := -3^(1/3):
zamproc(0,u1,0,v1,1,0,0,1, r11,0,r2,s2):
                                         -2 r22, 0, 3 s22,  $\frac{s23}{r2}$ 
                                         0, 3 r22, 0, 0

```

(25.26)

> 4-26:

$$\text{solve}([\text{M}[1,3], \text{M}[2,2], \text{M}[2,3], \text{M}[2,4]], \{\text{u}, \text{v}, \text{r1}, \text{s1}, \text{r2}, \text{s2}\});$$

$$\{r1=0, r2=r2, s1=s1, s2=0, u=0, v=v\}$$
 (25.27)

NSF 4-31

$$\begin{aligned}
& M := \text{zamproc}(u, 0, 1, v, 1, 0, 0, 0, r1, s1, r2, s2) : \\
& \frac{(s2 u - s1) r1^3 + r1 r22 s2 + v r2^3 s2}{r1 s2 - s1 r2}, \frac{3 u r1^2 s1 s2 + 3 v r22 s22 - 3 r1^2 s1^2 + 2 r1 r2 s22 + s1 r22 s2}{r1 s2 - s1 r2}, \\
& \frac{(3 v r2 + r1) s2^3 + 2 s1 r2 s22 + 3 u r1 s1^2 s2 - 3 r1 s1^3}{r1 s2 - s1 r2}, \frac{s1^3 s2 u + s2^4 v - s1^4 + s1 s2^3}{r1 s2 - s1 r2} \\
& \frac{-r1^3 r2 u - r24 v + r1^4 - r1 r23}{r1 s2 - s1 r2}, \frac{(-3 s2 v - s1) r23 - 2 r1 r22 s2 - 3 u r1^2 s1 r2 + 3 s1 r1^3}{r1 s2 - s1 r2}, \\
& \frac{(-3 s22 v - 2 s1 s2) r22 + (-3 s1^2 u - s22) r1 r2 + 3 r1^2 s1^2}{r1 s2 - s1 r2}, \frac{(-u s1^3 - v s2^3 - s1 s22) r2 + r1 s1^3}{r1 s2 - s1 r2}
\end{aligned} \tag{26.1}$$

```
> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

```
> 2-10: solve([M[1,1], M[1,2], M[1,3], M[2,1], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

SOLVE([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,t1});

```
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]],{u,v,r1,s1,r2,s2});
```

$\geq 3-2:$

```
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

> 3-4:

```
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

> 3-9:

```
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
```

> 3-15:

```
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}
```

$$\{r_1=0, r_2=r_2, s_1=s_1, s_2=0, u=0, v=0\}$$

```
> 3-20:
solve([M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], {u, u, x1, x1, x2, x2});
```

```
SOLVE([M[1,Z],M[1,S],M[Z,Z],M[Z,S],M[Z,4]], {u,v,II,S1,I2,S2}),  
S3, S2;
```

```
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

> 4-1:

```
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
```

> 4-2:

```
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

> 4-3:

```

> 4-6:
  solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-8:
  solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-10:
  evala([solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,s1,r2})]);
  
$$\left\{ r2 = \text{RootOf}(\mathcal{Z}-4) r1, s1 = -\frac{s2 \text{RootOf}(\mathcal{Z}-4)^2}{2}, u = -\frac{\text{RootOf}(\mathcal{Z}-4)^2}{12}, v = -\frac{\text{RootOf}(\mathcal{Z}-4)^2}{6} \right\}$$
 (26.4)

> z1 := 4^(1/3):
r21 := z1*r1:
s11 := -2^(1/3)*s2:
u1 := -2^(1/3)/6;
v1 := -2^(1/3)/3;
zamproc(u1,0,1,v1,1,0,0,0, r1,s11,r21,s2):
  
$$uI := -\frac{21^{1/3}}{6}$$

  
$$vI := -\frac{21^{1/3}}{3}$$

  
$$\frac{21^{1/3} r1^2}{2}, -\frac{3 22^{1/3} r1 s2}{2}, 0, -\frac{s2^3 21^{1/3}}{r1}$$

  
$$0, 0, 3 22^{1/3} r1 s2, 0$$
 (26.5)

> 4-13:
  solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
  
$$\{r1=0, r2=r2, s1=s1, s2=0, u=u, v=v=0\}$$
 (26.6)

> 4-16:
  solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-17:
  solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
  
$$\{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0\}$$
 (26.7)

> 4-21:
  solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-22:
  solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});
  
$$\{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0\}$$
 (26.8)

> 4-25:
  solve([M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,s2});
  
$$\{r1=0, s1=s1, s2=0, u=0, v=0\}, \left\{ r1 = \text{RootOf}(\mathcal{Z}-2) r2, s1 = -\frac{s2 \text{RootOf}(\mathcal{Z}-2)}{2}, s2 = s2, u = \frac{\text{RootOf}(\mathcal{Z}-2)}{3}, v = \frac{\text{RootOf}(\mathcal{Z}-2)}{3} \right\}$$
 (26.9)

> r11 := 2^(1/3)*r2:
s21 := -2^(2/3)*s1:
u1 := 2^(1/3)/3;
v1 := 2^(1/3)/3;
zamproc(u1,0,1,v1,1,0,0,0, r11,s1,r2,s21):
  
$$uI := \frac{21^{1/3}}{3}$$

  
$$vI := \frac{21^{1/3}}{3}$$

  
$$2 r2^2, 0, 3 21^{1/3} s1^2, \frac{s1^3}{r2}$$

  
$$0, -3 r2^2, 0, 0$$
 (26.10)

> 4-26:
  solve([M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-28:
  solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

NSF 4-32

```

> M := zamproc(0,0,u,v,1,0,0,1, r1,s1,r2,s2):

$$\frac{(s2 v - s1) r2^3 + u r1 r2^2 s2 - s1 r1^3}{r1 s2 - s1 r2}, \frac{s2 (3 s2 v + s1 (u - 3)) r2^2 + 2 u r1 r2 s2^2 - 3 r1^2 s1^2}{r1 s2 - s1 r2},$$

```

$$\begin{aligned}
& \frac{(u r l + 3 v r 2) s 23 + 2 r 2 \left(u - \frac{3}{2}\right) s 1 s 22 - 3 r l s 13}{r l s 2 - s l r 2}, \frac{-s l 4 + s 23 (u - 1) s l + s 24 v}{r l s 2 - s l r 2} \\
& \frac{r l 4 - r 23 (u - 1) r l - r 24 v}{r l s 2 - s l r 2}, \frac{(-u s l - 3 s 2 v) r 23 - 2 s 2 r l \left(u - \frac{3}{2}\right) r 22 + 3 s l r l 3}{r l s 2 - s l r 2}, \\
& \frac{(-2 s l s 2 u - 3 s 22 v) r 22 - r l s 22 (u - 3) r 2 + 3 r l 2 s l 2}{r l s 2 - s l r 2}, \frac{(-v r 2 + r l) s 23 - u s l r 2 s 22 + r l s l 3}{r l s 2 - s l r 2}
\end{aligned} \tag{27.1}$$

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=0,v=0} \tag{27.2}

```

```

> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=0,v=0} \tag{27.3}

```

```

> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} \tag{27.4}

```

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} \tag{27.5}

```

```

> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
    {r1=RootOf(2_Z - 1) r2, r2=r2, s1=RootOf(_Z + RootOf(2_Z - 1)^2 + RootOf(2_Z - 1)_Z) s2, s2=s2, u = \frac{3}{2}, v=0} \tag{27.6}

```

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=-s2,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=RootOf(_Z - _Z + 1) s2, s2=s2,u=0,v=0} \tag{27.7}

```

```

> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
    {r1=r2, r2=r2, s1=-s2, s2=s2, u=3, v=-1}, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=0} \tag{27.8}

```

```

> r11 := r2;
s11 := -s2;
zamproc(0,0,3,-1,1,0,0,1, r11,s11,r2,s2):
    2 r22, 0, 0, -\frac{2 s 23}{r 2}
    0, 0, 6 s 2 r 2, 2 s 22 \tag{27.9}

```

```

> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=0,v=0}, {r1=-r2,r2=r2,s1=2 s2,s2=s2,u=-3,v=-3} \tag{27.10}

```

```

> 4-13:
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} \tag{27.11}

```

```

> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
    {r1=0,r2=r2,s1=s1,s2=0,u=0,v=0}, {r1=RootOf(2_Z - 1) r2, r2=r2, s1=-2 s 2 RootOf(2_Z - 1), s2=s2, u = \frac{3}{2}, v=0} \tag{27.12}

```

```

> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (27.13)

> 4-21:
solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
(r1=r1,r2=0,s1=0,s2=s2,u=0,v=v}, {r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} (27.14)

> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (27.15)

> 4-25:
solve([M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (27.16)

> 4-26:
solve([M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} (27.17)

> 4-28:
solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
(r1=r1,r2=0,s1=0,s2=s2,u=0,v=v} (27.18)

> 4-31:
solve([M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} (27.19)

```

NSF 4-34

```

> M := zamproc(0,u,0,v,1,0,k,0, r1,s1,r2,s2):

$$\frac{-kr1r22s1+r12r2s2u+r23s2v-r13s1}{r1s2-r2s1}, \frac{(s22u-3s12)r12-2r2s1s2(k-u)r1-r22(ks12-3s22)v}{r1s2-r2s1},$$


$$\frac{-3r1s13-2\left(k-\frac{u}{2}\right)r2s2s12-r1s22(k-2u)s1+3r2s23v}{r1s2-r2s1}, \frac{-s14-s22(k-u)s12+s24v}{r1s2-r2s1}$$


$$\frac{r14+r22(k-u)r12-r24v}{r1s2-r2s1}, \frac{3r13s1+2\left(k-\frac{u}{2}\right)r2s2r12+r22s1(k-2u)r1-3r23s2v}{r1s2-r2s1},$$


$$\frac{(ks22+3s12)r12+2r2s1s2(k-u)r1-r22(s12u+3s22)v}{r1s2-r2s1}, \frac{kr1s1s22-r2s12s2u-r2s23v+r1s13}{r1s2-r2s1} (28.1)$$


```

```

> 2-1:
solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=RootOf(3_Z^2-k)r2, r2=r2, s1=-RootOf(3_Z^2-k)s2, s2=s2, u=k, v=  $\frac{k^2}{9}$ } (28.2)

```

```

> r11 := r2/sqrt(3):
s11 := -s2/sqrt(3):
zamproc(0,1,0,1/9,1,0,1,0, r11,s11,r2,s2):

$$\frac{4r22\sqrt{3}}{9}, 0, 0, 0$$


$$0, 0, 0, -\frac{4s22\sqrt{3}}{9} (28.3)$$


```

```

> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=RootOf(3_Z^2-k)r2, r2=r2, s1=-RootOf(3_Z^2-k)s2, s2=s2, u=-k, v=-RootOf(3_Z^2-k)^4} (28.4)

```

```

> r11 := r2/sqrt(3):
s11 := -s2/sqrt(3):
zamproc(0,-1,0,-1/9,1,0,1,0, r11,s11,r2,s2):

$$0, 0, 0, -\frac{4s23\sqrt{3}}{9r2}$$


$$\frac{4r23\sqrt{3}}{9s2}, 0, 0, 0 (28.5)$$


```

```

> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=RootOf(3_Z^2-k)r2, r2=r2, s1=-RootOf(3_Z^2-k)s2, s2=s2, u=k, v=  $\frac{k^2}{9}$ } (28.6)

```

```
> 3-2:
```

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

$$\left\{ r1 = \text{RootOf}(3 _Z^2 - k) r2, r2 = r2, s1 = -\text{RootOf}(3 _Z^2 - k) s2, s2 = s2, u = k, v = \frac{k^2}{9} \right\} \quad (28.7)$$

$$> 3-4:$$

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

$$\left\{ r1 = \text{RootOf}(3 _Z^2 - k) r2, r2 = r2, s1 = -\text{RootOf}(3 _Z^2 - k) s2, s2 = s2, u = k, v = \frac{k^2}{9} \right\} \quad (28.8)$$

$$> 3-9:$$

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

$$> 3-15:$$

$$\text{evalu}([\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, s2\})]);$$

$$\left[\left\{ r1 = \text{RootOf}(3 _Z^4 + 6 _Z^2 k - k^2) r2, s1 = -\frac{\text{RootOf}(3 _Z^4 + 6 _Z^2 k - k^2) (3 \text{RootOf}(3 _Z^4 + 6 _Z^2 k - k^2)^2 + 7 k) s2}{2 k}, s2 = s2, u = -k, v = \frac{k^2}{3} \right\} \right] \quad (28.9)$$

$$> \text{solve}(3 _Z^4 + 6 _Z^2 k - k^2, _Z);$$

$$z1 := (1/3) * \text{sqrt}(6 * \text{sqrt}(3) - 9 * k);$$

$$z2 := -(1/3) * \text{sqrt}(6 * \text{sqrt}(3) - 9 * k);$$

$$\frac{\sqrt{6 k \sqrt{3} - 9 k}}{3}, -\frac{\sqrt{6 k \sqrt{3} - 9 k}}{3}, \frac{\sqrt{-6 k \sqrt{3} - 9 k}}{3}, -\frac{\sqrt{-6 k \sqrt{3} - 9 k}}{3}$$

$$z1 := \frac{\sqrt{6 \sqrt{3} - 9 k}}{3}$$

$$z2 := -\frac{\sqrt{6 \sqrt{3} - 9 k}}{3} \quad (28.10)$$

$$> k1 := 1;$$

$$z11 := (1/3) * \text{sqrt}(6 * \text{sqrt}(3) - 9 * k1);$$

$$r11 := z11 * r2;$$

$$s11 := -z11 * (3 * z11^2 + 7 * k1) * s2 / (2 * k1);$$

$$\text{zamproc}(0, -k1, 0, 1/3, 1, 0, k1, 0, r11, s11, r2, s2);$$

$$k1 := 1$$

$$z11 := \frac{\sqrt{6 \sqrt{3} - 9}}{3}$$

$$s11 := -\frac{\sqrt{6 \sqrt{3} - 9} (4 + 2 \sqrt{3}) s2}{6}$$

$$\frac{2 \sqrt{6 \sqrt{3} - 9} \sqrt{3} r22}{9}, 0, 0, -\frac{2 \sqrt{6 \sqrt{3} - 9} (9 + 5 \sqrt{3}) s23}{9 r2}$$

$$0, -\frac{2 \sqrt{6 \sqrt{3} - 9} \sqrt{3} r22}{3}, 0, 0 \quad (28.11)$$

$$> k2 := -1;$$

$$z12 := (1/3) * \text{sqrt}(6 * \text{sqrt}(3) - 9 * k2);$$

$$r11 := z12 * r2;$$

$$s11 := -z12 * (3 * z12^2 + 7 * k2) * s2 / (2 * k2);$$

$$\text{zamproc}(0, -k2, 0, 1/3, 1, 0, k2, 0, r11, s11, r2, s2);$$

$$k2 := -1$$

$$z12 := \frac{\sqrt{6 \sqrt{3} + 9}}{3}$$

$$s11 := \frac{\sqrt{6 \sqrt{3} + 9} (2 \sqrt{3} - 4) s2}{6}$$

$$\frac{2 \sqrt{6 \sqrt{3} + 9} \sqrt{3} r22}{9}, 0, 0, -\frac{2 \sqrt{6 \sqrt{3} + 9} (-9 + 5 \sqrt{3}) s23}{9 r2}$$

$$0, -\frac{2 \sqrt{6 \sqrt{3} + 9} \sqrt{3} r22}{3}, 0, 0 \quad (28.12)$$

$$> k1 := 1;$$

$$z21 := -(1/3) * \text{sqrt}(6 * \text{sqrt}(3) - 9 * k1);$$

$$r12 := z21 * r2;$$

$$s12 := -z21 * (3 * z21^2 + 7 * k1) * s2 / (2 * k1);$$

```

zamproc(0,-k1,0,1/3,1,0,k1,0, r12,s12,r2,s2):
  k1 := 1
  
$$z21 := -\frac{\sqrt{6\sqrt{3}-9}}{3}$$

  
$$s12 := \frac{\sqrt{6\sqrt{3}-9}(4+2\sqrt{3})s2}{6}$$

  
$$-\frac{2\sqrt{6\sqrt{3}-9}\sqrt{3}r22}{9}, 0, 0, \frac{2\sqrt{6\sqrt{3}-9}(9+5\sqrt{3})s23}{9r2}$$

  
$$0, \frac{2\sqrt{6\sqrt{3}-9}\sqrt{3}r22}{3}, 0, 0$$


```

(28.13)

```

> k2 := -1;
z22 := -(1/3)*sqrt(6*sqrt(3)-9*k2);
r12 := z22*r2;
s12 := -z22*(3*z22^2+7*k2)*s2/(2*k2);
zamproc(0,-k2,0,1/3,1,0,k2,0, r12,s12,r2,s2):
  k2 := -1
  
$$z22 := -\frac{\sqrt{6\sqrt{3}+9}}{3}$$

  
$$s12 := -\frac{\sqrt{6\sqrt{3}+9}(2\sqrt{3}-4)s2}{6}$$

  
$$-\frac{2\sqrt{6\sqrt{3}+9}\sqrt{3}r22}{9}, 0, 0, \frac{2\sqrt{6\sqrt{3}+9}(-9+5\sqrt{3})s23}{9r2}$$

  
$$0, \frac{2\sqrt{6\sqrt{3}+9}\sqrt{3}r22}{3}, 0, 0$$


```

(28.14)

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,r2,s2});
  
$$\left\{ r1 = -\frac{r2 \operatorname{RootOf}(\mathcal{Z}^2 k - 3) k}{3}, r2 = r2, s2 = \operatorname{RootOf}(\mathcal{Z}^2 k - 3) s1, u = -k, v = -\frac{k^2}{9} \right\}$$


```

(28.15)

```

> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0,v=v}, {r1=RootOf(3_Z^2-k)r2,r2=r2,s1=-RootOf(3_Z^2-k)s2,s2=s2,u=-k,v=-RootOf(3_Z^2-k)^4}

```

(28.16)

```

> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,s1,r2,s2});
  
$$\left\{ r2 = \operatorname{RootOf}(\mathcal{Z}^2 k - 3) r1, s1 = -\frac{s2 \operatorname{RootOf}(\mathcal{Z}^2 k - 3) k}{3}, s2 = s2, u = -k, v = -\frac{k^2}{9} \right\}$$


```

(28.17)

```

> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
  
$$\left\{ r1 = -r2 \operatorname{RootOf}(3_Z^2 + k - 2u), r2 = r2, s1 = \operatorname{RootOf}(3_Z^2 + k - 2u) s2, s2 = s2, v = -\frac{2}{9} k^2 + \frac{5}{9} u k - \frac{2}{9} u^2 \right\}$$


```

(28.18)

```

> solve(3*_Z^2+k-2*u, _Z);
z1 := (1/3)*sqrt(6*u-3*k):
z2 := -(1/3)*sqrt(6*u-3*k):

$$\frac{\sqrt{-3k+6u}}{3}, -\frac{\sqrt{-3k+6u}}{3}$$


```

(28.19)

```

> r11 := -z1*r2:
s11 := z1*s2:
v1 := factor(-(2/9)*k^2+(5/9)*u*k-(2/9)*u^2);
zamproc(0,u,0,v1,1,0,k,0, r11,s11,r2,s2):
  v1 := -
$$\frac{(2k-u)(k-2u)}{9}$$

  
$$\frac{2r22(k+u)(k-2u)}{3\sqrt{-3k+6u}}, \frac{2r2s2(k-u)(k-2u)}{\sqrt{-3k+6u}}, 0, 0$$

  
$$0, 0, -\frac{2r2s2(k-u)(k-2u)}{\sqrt{-3k+6u}}, -\frac{2s22(k+u)(k-2u)}{3\sqrt{-3k+6u}}$$


```

(28.20)

```
> r12 := -z2*r2:
```

$$\begin{aligned}
s12 &:= z2*s2; \\
v1 &:= \text{factor}(-(2/9)*k^2+(5/9)*u*k-(2/9)*u^2); \\
\text{zamproc}(0,u,0,v1,1,0,k,0, r12,s12,r2,s2): \\
&\quad vI := -\frac{(2k-u)(k-2u)}{9} \\
&\quad -\frac{2r22(k+u)(k-2u)}{3\sqrt{-3k+6u}}, -\frac{2r2s2(k-u)(k-2u)}{\sqrt{-3k+6u}}, 0, 0 \\
&\quad 0, 0, \frac{2r2s2(k-u)(k-2u)}{\sqrt{-3k+6u}}, \frac{2s22(k+u)(k-2u)}{3\sqrt{-3k+6u}}
\end{aligned} \tag{28.21}$$

$$\begin{aligned}
> 4-2: \\
&\text{solve}([\text{M}[1,4],\text{M}[2,1],\text{M}[2,2],\text{M}[2,3]], \{u,v,r1,s1,r2,s2\}); \\
\left\{ r1 = -\text{RootOf}(3\text{Z}^2-k)r2, r2 = r2, s1 = \text{RootOf}(3\text{Z}^2-k)s2, s2 = s2, u = k, v = \frac{k^2}{9} \right\}, \{r1 = 0, r2 = r2, s1 = \text{RootOf}(\text{Z}^2+k)s2, s2 = s2, u = 0, v = 0 \} \tag{28.22}
\end{aligned}$$

$$\begin{aligned}
> 4-3: \\
&\text{evala}([\text{solve}([\text{M}[1,2],\text{M}[1,4],\text{M}[2,1],\text{M}[2,2]], \{u,v,r1,s1,r2\})]); \\
\left[\left\{ r1 = -r2 \text{RootOf}(3\text{Z}^2-k), r2 = r2, s1 = \text{RootOf}(3\text{Z}^2-k)s2, u = k, v = \frac{k^2}{9} \right\}, \left\{ r1 = \frac{\text{RootOf}(9\text{Z}^4+3k\text{Z}^2-k^2)(3\text{RootOf}(9\text{Z}^4+3k\text{Z}^2-k^2)^2+2k)r2}{k}, r2 = r2, s1 = \text{RootOf}(9\text{Z}^4+3k\text{Z}^2-k^2)s2, u = 3\text{RootOf}(9\text{Z}^4+3k\text{Z}^2-k^2)^2+2k, v = -\frac{k(3\text{RootOf}(9\text{Z}^4+3k\text{Z}^2-k^2)^2+2k)}{9} \right\} \right] \tag{28.23}
\end{aligned}$$

$$\begin{aligned}
> \text{solve}(9*\text{Z}^4+3*\text{Z}^2*k-k^2, \text{Z}); \\
z1 &:= (1/6)*\text{sqrt}(6*\text{sqrt}(5)-6*k); \\
z2 &:= -(1/6)*\text{sqrt}(6*\text{sqrt}(5)-6*k); \\
&\quad \frac{\sqrt{6k\sqrt{5}-6k}}{6}, -\frac{\sqrt{6k\sqrt{5}-6k}}{6}, \frac{\sqrt{-6k\sqrt{5}-6k}}{6}, -\frac{\sqrt{-6k\sqrt{5}-6k}}{6} \\
&\quad z1 := \frac{\sqrt{6\sqrt{5}-6k}}{6} \\
&\quad z2 := -\frac{\sqrt{6\sqrt{5}-6k}}{6}
\end{aligned} \tag{28.24}$$

$$\begin{aligned}
> k1 := 1; \\
z11 &:= (1/6)*\text{sqrt}(6*\text{sqrt}(5)-6*k1); \\
r11 &:= -z11*(3*z11^2+2*k1)*r2/k1; \\
s11 &:= z11*s2; \\
u1 &:= 3*z11^2+2*k1; \\
v1 &:= -(1/9)*k1*(3*z11^2+2*k1); \\
\text{zamproc}(0,u1,0,v1,1,0,k1,0, r11,s11,r2,s2): \\
&\quad k1 := 1 \\
&\quad zII := \frac{\sqrt{6\sqrt{5}-6}}{6} \\
&\quad rII := -\frac{\sqrt{6\sqrt{5}-6}\left(\frac{\sqrt{5}}{2}+\frac{3}{2}\right)r2}{6} \\
&\quad uI := \frac{\sqrt{5}}{2}+\frac{3}{2} \\
&\quad vI := -\frac{\sqrt{5}}{18}-\frac{1}{6} \\
&\quad -\frac{\sqrt{6\sqrt{5}-6}(5+2\sqrt{5})r22}{9}, 0, \frac{\sqrt{6\sqrt{5}-6}(5+\sqrt{5})s22}{12}, 0 \\
&\quad 0, 0, -\frac{\sqrt{6\sqrt{5}-6}(5+3\sqrt{5})r2s2}{12}, \frac{\sqrt{6\sqrt{5}-6}(5+\sqrt{5})s22}{36}
\end{aligned} \tag{28.25}$$

```

> k2 := -1;
z12 := (1/6)*\text{sqrt}(6*\text{sqrt}(5)-6*k2);
r11 := -z12*(3*z12^2+2*k2)*r2/k2;
s11 := z12*s2;
u1 := 3*z12^2+2*k2;
v1 := -(1/9)*k2*(3*z12^2+2*k2);

```

```

zamproc(0,u1,0,v1,1,0,k2,0, r11,s11,r2,s2):
  k2 := -1
  
$$z12 := \frac{\sqrt{6\sqrt{5}+6}}{6}$$

  
$$r11 := \frac{\sqrt{6\sqrt{5}+6}}{6} \left( \frac{\sqrt{5}}{2} - \frac{3}{2} \right) r2$$

  
$$u1 := \frac{\sqrt{5}}{2} - \frac{3}{2}$$

  
$$v1 := \frac{\sqrt{5}}{18} - \frac{1}{6}$$

  
$$-\frac{\sqrt{6\sqrt{5}+6}}{9} (-5 + 2\sqrt{5}) r2^2, 0, \frac{\sqrt{6\sqrt{5}+6}}{12} (\sqrt{5} - 5) s2^2, 0$$

  
$$0, 0, -\frac{\sqrt{6\sqrt{5}+6}}{12} (3\sqrt{5} - 5) r2 s2, \frac{\sqrt{6\sqrt{5}+6}}{36} (\sqrt{5} - 5) s2^2$$

(28.26)

```

```

> k1 := 1;
z21 := -(1/6)*sqrt(6*sqrt(5)-6*k1);
r12 := -z21*(3*z21^2+2*k1)*r2/k1;
s12 := z21*s2;
u2 := 3*z21^2+2*k1;
v2 := -(1/9)*k1*(3*z21^2+2*k1);
zamproc(0,u2,0,v2,1,0,k1,0, r12,s12,r2,s2):
  k1 := 1
  
$$z21 := -\frac{\sqrt{6\sqrt{5}-6}}{6}$$

  
$$r12 := \frac{\sqrt{6\sqrt{5}-6}}{6} \left( \frac{\sqrt{5}}{2} + \frac{3}{2} \right) r2$$

  
$$u2 := \frac{\sqrt{5}}{2} + \frac{3}{2}$$

  
$$v2 := -\frac{\sqrt{5}}{18} - \frac{1}{6}$$

  
$$\frac{\sqrt{6\sqrt{5}-6}}{9} (5 + 2\sqrt{5}) r2^2, 0, -\frac{\sqrt{6\sqrt{5}-6}}{12} (5 + \sqrt{5}) s2^2, 0$$

  
$$0, 0, \frac{\sqrt{6\sqrt{5}-6}}{12} (5 + 3\sqrt{5}) r2 s2, -\frac{\sqrt{6\sqrt{5}-6}}{36} (5 + \sqrt{5}) s2^2$$

(28.27)

```

```

> k2 := -1;
z22 := -(1/6)*sqrt(6*sqrt(5)-6*k2);
r12 := -z22*(3*z22^2+2*k2)*r2/k2;
s12 := z22*s2;
u2 := 3*z22^2+2*k2;
v2 := -(1/9)*k2*(3*z22^2+2*k2);
zamproc(0,u2,0,v2,1,0,k2,0, r12,s12,r2,s2):
  k2 := -1
  
$$z22 := -\frac{\sqrt{6\sqrt{5}+6}}{6}$$

  
$$r12 := -\frac{\sqrt{6\sqrt{5}+6}}{6} \left( \frac{\sqrt{5}}{2} - \frac{3}{2} \right) r2$$

  
$$u2 := \frac{\sqrt{5}}{2} - \frac{3}{2}$$

  
$$v2 := \frac{\sqrt{5}}{18} - \frac{1}{6}$$

  
$$\frac{\sqrt{6\sqrt{5}+6}}{9} (-5 + 2\sqrt{5}) r2^2, 0, -\frac{\sqrt{6\sqrt{5}+6}}{12} (\sqrt{5} - 5) s2^2, 0$$


```

$$0, 0, \frac{\sqrt{6\sqrt{5}+6}}{12} (3\sqrt{5}-5) r2 s2, -\frac{\sqrt{6\sqrt{5}+6}}{36} (\sqrt{5}-5) s2^2 \quad (28.28)$$

```
> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=RootOf(3_Z^2-k)r2,r2=r2,s1=s1,s2=-s1/RootOf(3_Z^2-k),u=k,v=k^2/9} \quad (28.29)
```

```
> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
evala([solve([M[1,2],M[1,4],M[2,1],M[2,3]], {u,v,s1,r2})]);
{r1=-r2 RootOf(_Z^4-v),r2=r2,s1=RootOf(_Z^4-v)s2,s2=s2,u=k}
{r2=-r1 s2/s1,s1=s1,u=k,v=s1^4/s2^4}, {r2=3 RootOf(9_Z^4+18 k_Z^2+k^2)r1/k,s1=RootOf(9_Z^4+18 k_Z^2+k^2)s2,u=-k,v=-k^2/9} \quad (28.30)
```

```
> r11 := -v^(1/4)*r2;
s11 := v^(1/4)*s2;
zamproc(0,k,0,v,1,0,k,0, r11,s11,r2,s2):
- r22 (sqrt(v)k+v)/sqrt(v)^4, 0, s22 (sqrt(v)k-3v)/sqrt(v)^4, 0
0, - r22 (sqrt(v)k-3v)/sqrt(v)^4, 0, s22 (sqrt(v)k+v)/sqrt(v)^4 \quad (28.31)
```

```
> r12 := v^(1/4)*r2;
s12 := -v^(1/4)*s2;
zamproc(0,k,0,v,1,0,k,0, r12,s12,r2,s2):
r22 (sqrt(v)k+v)/sqrt(v)^4, 0, -s22 (sqrt(v)k-3v)/sqrt(v)^4, 0
0, r22 (sqrt(v)k-3v)/sqrt(v)^4, 0, -s22 (sqrt(v)k+v)/sqrt(v)^4 \quad (28.32)
```

```
> solve(9*_Z^4+18*_Z^2*k+k^2, _Z);
z1 := (1/3)*sqrt(6*k*sqrt(2)+9);
z2 := -(1/3)*sqrt(6*k*sqrt(2)+9);
sqrt(6k*sqrt(2)-9k)/3, -sqrt(6k*sqrt(2)-9k)/3, sqrt(-6k*sqrt(2)-9k)/3, -sqrt(-6k*sqrt(2)-9k)/3
z1 := sqrt(6k*sqrt(2)+9)/3
z2 := -sqrt(6k*sqrt(2)+9)/3 \quad (28.33)
```

```
> k1 := 1;
z11 := (1/3)*sqrt(6*k1*sqrt(2)+9);
r21 := -3*z11*r1;
s11 := z11*s2;
u1 := 1;
zamproc(0,u1,0,-1/9,1,0,-1,0, r1,s11,r21,s2):
z11 := sqrt(6)/3 + sqrt(3)/3
2*sqrt(3)*(2+sqrt(2))/3*r21, 0, -2*sqrt(3)*(2+sqrt(2))/3*s22, 0
0, -2*sqrt(3)*(2+sqrt(2))/3*r21, 0, 2*sqrt(3)*(2+sqrt(2))/9*s22 \quad (28.34)
```

```
> k2 := -1;
z12 := (1/3)*sqrt(6*k2*sqrt(2)+9);
r21 := -3*z12*r1;
s11 := z12*s2;
u1 := 1;
zamproc(0,u1,0,-1/9,1,0,-1,0, r1,s11,r21,s2):
z12 := sqrt(6)/3 - sqrt(3)/3
```

$$\begin{aligned} & \frac{2(-2+\sqrt{2})\sqrt{3}rl2}{3}, 0, -\frac{2(-2+\sqrt{2})s22\sqrt{3}}{3}, 0 \\ & 0, -2(-2+\sqrt{2})\sqrt{3}rl2, 0, \frac{2(-2+\sqrt{2})s22\sqrt{3}}{9} \end{aligned} \quad (28.35)$$

```
> k1 := 1:
z21 := -(1/3)*sqrt(6*k1*sqrt(2)+9);
r21 := -3*z21*r1;
s11 := z21*s2;
u1 := 1:
zamproc(0,u1,0,-1/9,1,0,-1,0, r1,s11,r21,s2):
z21:=-\frac{\sqrt{6}}{3}-\frac{\sqrt{3}}{3}
-\frac{2\sqrt{3}(2+\sqrt{2})rl2}{3}, 0, \frac{2\sqrt{3}(2+\sqrt{2})s22}{3}, 0
0, 2\sqrt{3}(2+\sqrt{2})rl2, 0, -\frac{2\sqrt{3}(2+\sqrt{2})s22}{9}
```

$$(28.36)$$

```
> k2 := -1:
z22 := -(1/3)*sqrt(6*k2*sqrt(2)+9);
r21 := -3*z22*r1;
s11 := z22*s2;
u1 := 1:
zamproc(0,u1,0,-1/9,1,0,-1,0, r1,s11,r21,s2):
z22:=-\frac{\sqrt{6}}{3}+\frac{\sqrt{3}}{3}
-\frac{2(-2+\sqrt{2})\sqrt{3}rl2}{3}, 0, \frac{2(-2+\sqrt{2})s22\sqrt{3}}{3}, 0
0, 2(-2+\sqrt{2})\sqrt{3}rl2, 0, -\frac{2(-2+\sqrt{2})s22\sqrt{3}}{9}

$$(28.37)$$


```

```
> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=RootOf(_Z-k)r2, r2=r2, s1=0, s2=s2, u=2k, v=0}, {r1=RootOf(_Z+k)r2, r2=r2, s1=\frac{ks2}{RootOf(_Z+k)}, s2=s2, u=-k, v=-k2}, {r1=0, r2=r2, s1=s1, s2=0, u=u, v=0}
```

$$(28.38)$$

```
> 4-13:
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r2 RootOf(3_Z^2-k), r2=r2, s1=-RootOf(3_Z^2-k)s2, s2=s2, u=k, v=\frac{k^2}{9}}, {r1=RootOf(3_Z^4+6_Z^2k-k^2)r2, r2=r2, s1=\frac{RootOf(3_Z^4+6_Z^2k-k^2)s2}{3 RootOf(3_Z^4+6_Z^2k-k^2)^2+k}, s2=s2, u=-k, v=\frac{k^2}{3}}

$$(28.39)$$


```

```
> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=RootOf(3_Z^4-6_Z^2k-k^2)r2, r2=r2, s1=-\frac{s2(RootOf(3_Z^4-6_Z^2k-k^2)^2-k)}{2 RootOf(3_Z^4-6_Z^2k-k^2)}, s2=s2, u=\frac{3 RootOf(3_Z^4-6_Z^2k-k^2)}{2}+\frac{k}{2}, v=-\frac{k(3 RootOf(3_Z^4-6_Z^2k-k^2)^2+k)}{6}}

$$(28.40)$$


```

```
> solve(3*_Z^4-6*_Z^2*k-k^2, _Z);
\frac{\sqrt{6k\sqrt{3}+9k}}{3}, -\frac{\sqrt{6k\sqrt{3}+9k}}{3}, \frac{\sqrt{-6k\sqrt{3}+9k}}{3}, -\frac{\sqrt{-6k\sqrt{3}+9k}}{3}

$$(28.41)$$


```

```
> z1 := (1/3)*sqrt(6*sqrt(3)+9*k);
z2 := -(1/3)*sqrt(6*sqrt(3)+9*k);
z1 := \frac{\sqrt{6\sqrt{3}+9k}}{3}
z2 := -\frac{\sqrt{6\sqrt{3}+9k}}{3}

$$(28.42)$$


```

```
> k1 := 1:
z11 := (1/3)*sqrt(6*sqrt(3)+9*k1);
r11 := z11*r2;
```

```

s11 := -s2*(z11^2-k1)/(2*z11);
u1 := 3*z11^2*(1/2)+(1/2)*k1;
v1 := -(1/6)*k1*(3*z11^2+k1);
zamproc(0,u1,0,v1,1,0,k1,0, r11,s11,r2,s2):

```

$$z11 := \frac{\sqrt{6\sqrt{3}+9}}{3}$$

$$s11 := -\frac{s2\sqrt{3}}{\sqrt{6\sqrt{3}+9}}$$

$$u1 := 2+\sqrt{3}$$

$$v1 := -\frac{2}{3} - \frac{\sqrt{3}}{3}$$

$$\frac{2\sqrt{6\sqrt{3}+9}(\sqrt{3}+3)r22}{9}, 0, \frac{2\sqrt{6\sqrt{3}+9}(-3+\sqrt{3})s22}{3}, \frac{2\sqrt{2\sqrt{3}+3}(\sqrt{3}-2)s23}{3r2}$$

$$0, 0, \frac{2\sqrt{6\sqrt{3}+9}\sqrt{3}r2s2}{3}, 0 \quad (28.43)$$

```

> k2 := -1:
z12 := (1/3)*sqrt(6*sqrt(3)+9*k2);
r11 := z12*r2;
s11 := -s2*(z12^2-k2)/(2*z12);
u2 := 3*z12^2*(1/2)+(1/2)*k2;
v2 := -(1/6)*k2*(3*z12^2+k2);
zamproc(0,u2,0,v2,1,0,k2,0, r11,s11,r2,s2):

```

$$z12 := \frac{\sqrt{6\sqrt{3}-9}}{3}$$

$$s11 := -\frac{s2\sqrt{3}}{\sqrt{6\sqrt{3}-9}}$$

$$u2 := \sqrt{3}-2$$

$$v2 := \frac{\sqrt{3}}{3} - \frac{2}{3}$$

$$\frac{2\sqrt{6\sqrt{3}-9}(-3+\sqrt{3})r22}{9}, 0, \frac{2\sqrt{6\sqrt{3}-9}(\sqrt{3}+3)s22}{3}, -\frac{2\sqrt{2\sqrt{3}-3}(2+\sqrt{3})s23}{3r2}$$

$$0, 0, \frac{2\sqrt{6\sqrt{3}-9}\sqrt{3}r2s2}{3}, 0 \quad (28.44)$$

```

> k1 := 1:
z21 := -(1/3)*sqrt(6*sqrt(3)+9*k1);
r11 := z21*r2;
s11 := -s2*(z21^2-k1)/(2*z21);
u1 := 3*z21^2*(1/2)+(1/2)*k1;
v1 := -(1/6)*k1*(3*z21^2+k1);
zamproc(0,u1,0,v1,1,0,k1,0, r11,s11,r2,s2):

```

$$z21 := -\frac{\sqrt{6\sqrt{3}+9}}{3}$$

$$s11 := \frac{s2\sqrt{3}}{\sqrt{6\sqrt{3}+9}}$$

$$u1 := 2+\sqrt{3}$$

$$v1 := -\frac{2}{3} - \frac{\sqrt{3}}{3}$$

$$-\frac{2\sqrt{6\sqrt{3}+9}(\sqrt{3}+3)r22}{9}, 0, -\frac{2\sqrt{6\sqrt{3}+9}(-3+\sqrt{3})s22}{3}, -\frac{2\sqrt{2\sqrt{3}+3}(\sqrt{3}-2)s23}{3r2}$$

$$0, 0, -\frac{2\sqrt{6\sqrt{3}+9}\sqrt{3}r2s2}{3}, 0 \quad (28.45)$$

```
> k2 := -1:
```

```

z22 := -(1/3)*sqrt(6*sqrt(3)+9*k2);
r11 := z22*r2;
s11 := -s2*(z22^2-k2)/(2*z22);
u1 := 3*z22^2*(1/2)+(1/2)*k2;
v1 := -(1/6)*k2*(3*z22^2+k2);
zamproc(0,u1,0,v1,1,0,k2,0, r11,s11,r2,s2):

```

$$z22 := -\frac{\sqrt{6\sqrt{3}-9}}{3}$$

$$s11 := \frac{s2\sqrt{3}}{\sqrt{6\sqrt{3}-9}}$$

$$u1 := \sqrt{3}-2$$

$$v1 := \frac{\sqrt{3}}{3} - \frac{2}{3}$$

$$-\frac{2\sqrt{6\sqrt{3}-9}(-3+\sqrt{3})r22}{9}, 0, -\frac{2\sqrt{6\sqrt{3}-9}(\sqrt{3}+3)s22}{3}, \frac{2\sqrt{2\sqrt{3}-3}(2+\sqrt{3})s23}{3r2}$$

$$0, 0, -\frac{2\sqrt{6\sqrt{3}-9}\sqrt{3}r2s2}{3}, 0 \quad (28.46)$$

```

> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

```

$$\left\{ r1 = -\frac{r2(\text{RootOf}(3_Z^4-6_Z^2k-k^2)^2-k)}{2\text{RootOf}(3_Z^4-6_Z^2k-k^2)}, r2=r2, s1=s2 \text{RootOf}(3_Z^4-6_Z^2k-k^2), s2=s2, u=-k, v=\frac{k^2}{3} \right\}, \{r1=0, r2=r2, s1=s2, s2=0, u=0, v=0\} \quad (28.47)$$

```

> 4-21:
solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});

```

$$\left\{ r1=r2 \text{RootOf}(3_Z^2-k), r2=r2, s1=-\text{RootOf}(3_Z^2-k) s2, s2=s2, u=u, v=\frac{ku}{9} \right\}, \left\{ r1=r1, r2=-\frac{r1}{\text{RootOf}(3_Z^2-k)}, s1=\text{RootOf}(3_Z^2-k) s2, s2=s2, u=-k, v=-\frac{k^2}{9} \right\} \quad (28.48)$$

```

> r11 := r2/sqrt(3):
s11 := -s2/sqrt(3):
v1 := (1/9)*u;
zamproc(0,u,0,v1,1,0,1,0, r11,s11,r2,s2):

```

$$v1 := \frac{u}{9}$$

$$\frac{2r22(1+u)\sqrt{3}}{9}, 0, 0, \frac{2s23(u-1)\sqrt{3}}{9r2}$$

$$-\frac{2r23(u-1)\sqrt{3}}{9s2}, 0, 0, -\frac{2s22(1+u)\sqrt{3}}{9} \quad (28.49)$$

```

> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});

```

$$\left\{ r1=\text{RootOf}(3_Z^4+6_Z^2k-k^2)r2, r2=r2, s1=-\frac{\text{RootOf}(3_Z^4+6_Z^2k-k^2)s2(3\text{RootOf}(3_Z^4+6_Z^2k-k^2)^2+k)}{9\text{RootOf}(3_Z^4+6_Z^2k-k^2)^2-k}, s2=s2, u=-k, v=\frac{k^2}{3} \right\}, \{r1=\text{RootOf}(_Z^2+k)r2, r2=r2, s1=0, s2=s2, u=0, v=0\} \quad (28.50)$$

```

> 4-25:
solve([M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

```

$$\left\{ r1=\text{RootOf}(_Z^2+k)r2, r2=r2, s1=-\text{RootOf}(_Z^2+k)s2, s2=s2, u=k, v=k \right\}, \left\{ r1=\text{RootOf}(3_Z^4+6_Z^2k-k^2)r2, r2=r2, s1=-\frac{\text{RootOf}(3_Z^4+6_Z^2k-k^2)s2(3\text{RootOf}(3_Z^4+6_Z^2k-k^2)^2+5k)}{3\text{RootOf}(3_Z^4+6_Z^2k-k^2)^2+k}, s2=s2, u=-k, v=\frac{k^2}{3} \right\} \quad (28.51)$$

```

> 4-26:
solve([M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

```

$$\left\{ r1=r1, r2=-\frac{r1}{\text{RootOf}(3_Z^2-k)}, s1=\text{RootOf}(3_Z^2-k)s2, s2=s2, u=-k, v=-\frac{k^2}{9} \right\}, \{r1=0, r2=r2, s1=s1, s2=0, u=0, v=v\} \quad (28.52)$$

```

> 4-28:
solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});

```

$$\left\{ r1=r2 \text{RootOf}(3_Z^2-k), r2=r2, s1=s1, s2=-\frac{s1}{\text{RootOf}(3_Z^2-k)}, u=-k, v=-\frac{k^2}{9} \right\}, \{r1=0, r2=r2, s1=s1, s2=0, u=0, v=v\} \quad (28.53)$$

$$> 4-31: \text{solve}([\mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

$$\left\{ r1 = -r2 \text{RootOf}(3 _Z^2 - k), r2 = r2, s1 = \text{RootOf}(3 _Z^2 - k) s2, s2 = s2, u = -k, v = -\frac{k}{9} \right\} \quad (28.54)$$

$$> 4-32: \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, r1, s1, r2, s2\});$$

$$\left\{ r1 = \text{RootOf}(k _Z^4 - v k + (-k^2 - 3 v) _Z^2) r2, r2 = r2, s1 = s1, s2 = \right.$$

$$\left. -\frac{s1 \text{RootOf}(k _Z^4 - v k + (-k^2 - 3 v) _Z^2) (\text{RootOf}(k _Z^4 - v k + (-k^2 - 3 v) _Z^2)^2 - k)}{2 v}, u = -k \right\} \quad (28.55)$$

$$> \text{solve}(k^* _Z^4 - k^* v + (-k^2 - 3 v) * _Z^2, _Z);$$

$$\frac{\sqrt{2} \sqrt{k (k^2 + 3 v + \sqrt{k^4 + 10 k^2 v + 9 v^2})}}{2 k}, \frac{-\sqrt{2} \sqrt{k (k^2 + 3 v + \sqrt{k^4 + 10 k^2 v + 9 v^2})}}{2 k}, \frac{\sqrt{-2 k (-k^2 + \sqrt{k^4 + 10 k^2 v + 9 v^2} - 3 v)}}{2 k},$$

$$-\frac{\sqrt{-2 k (-k^2 + \sqrt{k^4 + 10 k^2 v + 9 v^2} - 3 v)}}{2 k} \quad (28.56)$$

$$> z1 := \text{sqrt}(2) * \text{sqrt}(k * (1 + 3 * v + \text{sqrt}(1 + 10 * v + 9 * v^2))) / (2 * k);$$

$$z2 := -\text{sqrt}(2) * \text{sqrt}(k * (1 + 3 * v + \text{sqrt}(1 + 10 * v + 9 * v^2))) / (2 * k);$$

$$z3 := \text{sqrt}(-2 * k * (-1 + \text{sqrt}(1 + 10 * v + 9 * v^2) - 3 * v)) / (2 * k);$$

$$z4 := -\text{sqrt}(-2 * k * (-1 + \text{sqrt}(1 + 10 * v + 9 * v^2) - 3 * v)) / (2 * k);$$

$$z1 := \frac{\sqrt{2} \sqrt{k (1 + 3 v + \sqrt{9 v^2 + 10 v + 1})}}{2 k}$$

$$z2 := -\frac{\sqrt{2} \sqrt{k (1 + 3 v + \sqrt{9 v^2 + 10 v + 1})}}{2 k}$$

$$z3 := \frac{\sqrt{-2 k (-1 + \sqrt{9 v^2 + 10 v + 1} - 3 v)}}{2 k}$$

$$z4 := -\frac{\sqrt{-2 k (-1 + \sqrt{9 v^2 + 10 v + 1} - 3 v)}}{2 k} \quad (28.57)$$

$$> \text{solve}(1 + 3 * v - \text{sqrt}(9 * v^2 + 10 * v + 1) < 0, v); \quad [-\infty, -1], (0, \infty] \quad (28.58)$$

$$> k1 := 1;$$

$$z11 := \text{sqrt}(2) * \text{sqrt}(k1 * (1 + 3 * v + \text{sqrt}(1 + 10 * v + 9 * v^2))) / (2 * k1);$$

$$r11 := z11 * r2;$$

$$s21 := -s1 * z11 * (z11^2 - k1) / (2 * v);$$

$$u1 := -k1;$$

$$\text{zamproc}(0, u1, 0, v, 1, 0, k1, 0, r11, s1, r2, s21); \quad k1 := 1$$

$$z11 := \frac{\sqrt{2} \sqrt{1 + 3 v + \sqrt{9 v^2 + 10 v + 1}}}{2}$$

$$s21 := -\frac{s1 \sqrt{2} \sqrt{1 + 3 v + \sqrt{9 v^2 + 10 v + 1}} \left(-\frac{1}{2} + \frac{3 v}{2} + \frac{\sqrt{9 v^2 + 10 v + 1}}{2} \right)}{4 v}$$

$$0, 0, \frac{3 s1^2 \sqrt{1 + 3 v + \sqrt{9 v^2 + 10 v + 1}} \sqrt{2} (27 v^2 + 9 v \sqrt{9 v^2 + 10 v + 1} + 12 v - \sqrt{9 v^2 + 10 v + 1} + 1) (v + 1)}{8 v (\sqrt{9 v^2 + 10 v + 1} + 3 v + 3)},$$

$$-\frac{3 (3 v - 1) s1^3 (9 v \sqrt{9 v^2 + 10 v + 1} + 27 v^2 + \sqrt{9 v^2 + 10 v + 1} + 18 v - 1) (v + 1)}{8 v r2 (\sqrt{9 v^2 + 10 v + 1} + 3 v + 3)}$$

$$-\frac{2 r2^3 (1 + 3 v + \sqrt{9 v^2 + 10 v + 1}) (v + 1)}{s1 (\sqrt{9 v^2 + 10 v + 1} + 3 v + 3)}, 0, 0, \quad (28.59)$$

$$-\frac{s1^2 \sqrt{1 + 3 v + \sqrt{9 v^2 + 10 v + 1}} \sqrt{2} (27 v^2 + 9 v \sqrt{9 v^2 + 10 v + 1} + 12 v - \sqrt{9 v^2 + 10 v + 1} + 1) (v + 1)}{8 v (\sqrt{9 v^2 + 10 v + 1} + 3 v + 3)}$$

$$> k2 := -1;$$

$$z12 := \text{sqrt}(2) * \text{sqrt}(k2 * (1 + 3 * v + \text{sqrt}(1 + 10 * v + 9 * v^2))) / (2 * k2);$$

$$r12 := z12 * r2;$$

$$s22 := -s1 * z12 * (z12^2 - k2) / (2 * v);$$

$$u2 := -k2;$$

$$\text{zamproc}(0, u2, 0, v, 1, 0, k2, 0, r12, s1, r2, s22);$$

$$\begin{aligned}
k2 &:= -1 \\
z12 &:= -\frac{\sqrt{2} \sqrt{-1-3 v-\sqrt{9 v^2+10 v+1}}}{2} \\
s22 &:= \frac{s1 \sqrt{2} \sqrt{-1-3 v-\sqrt{9 v^2+10 v+1}}}{4 v} \left(\frac{1}{2} - \frac{3 v}{2} - \frac{\sqrt{9 v^2+10 v+1}}{2} \right) \\
0, 0, -\frac{3 s12 \sqrt{-1-3 v-\sqrt{9 v^2+10 v+1}} \sqrt{2} (27 v^2+9 v \sqrt{9 v^2+10 v+1}+12 v-\sqrt{9 v^2+10 v+1}+1) (v+1)}{8 v (\sqrt{9 v^2+10 v+1}+3 v+3)}, \\
-\frac{3 (3 v-1) s13 (9 v \sqrt{9 v^2+10 v+1}+27 v^2+\sqrt{9 v^2+10 v+1}+18 v-1) (v+1)}{8 v r2 (\sqrt{9 v^2+10 v+1}+3 v+3)} \\
-\frac{2 r23 (1+3 v+\sqrt{9 v^2+10 v+1}) (v+1)}{s1 (\sqrt{9 v^2+10 v+1}+3 v+3)}, 0, 0,
\end{aligned} \tag{28.60}$$

$$\begin{aligned}
> k1 := 1; \\
z41 &:= -\text{sqrt}(-2 * (-1 + \text{sqrt}(9 * v^2 + 10 * v + 1) - 3 * v)) / 2; \\
r11 &:= z41 * r2; \\
s21 &:= -s1 * z41 * (z41^2 - k1) / (2 * v); \\
u1 &:= -k1; \\
\text{zamproc}(0, u1, 0, v, 1, 0, k1, 0, r11, s1, r2, s21): & \quad k1 := 1 \\
z41 &:= -\frac{\sqrt{2-2 \sqrt{9 v^2+10 v+1}+6 v}}{2} \\
s21 &:= \frac{s1 \sqrt{2-2 \sqrt{9 v^2+10 v+1}+6 v} \left(-\frac{1}{2}-\frac{\sqrt{9 v^2+10 v+1}}{2}+\frac{3 v}{2} \right)}{4 v} \\
0, 0, -\frac{3 (v+1) s12 (9 \sqrt{9 v^2+10 v+1} v-27 v^2-\sqrt{9 v^2+10 v+1}-12 v-1) \sqrt{2-2 \sqrt{9 v^2+10 v+1}+6 v}}{8 v (\sqrt{9 v^2+10 v+1}-3 v-3)}, \\
-\frac{3 s13 (v+1) (9 \sqrt{9 v^2+10 v+1} v-27 v^2+\sqrt{9 v^2+10 v+1}-18 v+1) (3 v-1)}{8 v r2 (\sqrt{9 v^2+10 v+1}-3 v-3)} \\
-\frac{2 (-1-3 v+\sqrt{9 v^2+10 v+1}) (v+1) r23}{s1 (\sqrt{9 v^2+10 v+1}-3 v-3)}, 0, 0,
\end{aligned} \tag{28.61}$$

$$\begin{aligned}
> k2 := -1; \\
z42 &:= \text{sqrt}(2 * (-1 + \text{sqrt}(9 * v^2 + 10 * v + 1) - 3 * v)) / 2; \\
r11 &:= z42 * r2; \\
s21 &:= -s1 * z42 * (z42^2 - k2) / (2 * v); \\
u1 &:= -k2; \\
\text{zamproc}(0, u1, 0, v, 1, 0, k2, 0, r11, s1, r2, s21): & \quad k2 := -1 \\
z42 &:= \frac{\sqrt{-2+2 \sqrt{9 v^2+10 v+1}-6 v}}{2} \\
s21 &:= -\frac{s1 \sqrt{-2+2 \sqrt{9 v^2+10 v+1}-6 v} \left(\frac{1}{2}+\frac{\sqrt{9 v^2+10 v+1}}{2}-\frac{3 v}{2} \right)}{4 v} \\
0, 0, \frac{3 (v+1) s12 (9 \sqrt{9 v^2+10 v+1} v-27 v^2-\sqrt{9 v^2+10 v+1}-12 v-1) \sqrt{-2+2 \sqrt{9 v^2+10 v+1}-6 v}}{8 (\sqrt{9 v^2+10 v+1}-3 v-3) v}, \\
-\frac{3 s13 (v+1) (9 \sqrt{9 v^2+10 v+1} v-27 v^2+\sqrt{9 v^2+10 v+1}-18 v+1) (3 v-1)}{8 v r2 (\sqrt{9 v^2+10 v+1}-3 v-3)}
\end{aligned} \tag{28.62}$$

$$\begin{aligned} & \frac{2(-1-3v+\sqrt{9v^2+10v+1})(v+1)r^{23}}{s1(\sqrt{9v^2+10v+1}-3v-3)}, 0, 0, \\ & -\frac{(v+1)s12(9\sqrt{9v^2+10v+1}v-27v^2-\sqrt{9v^2+10v+1}-12v-1)\sqrt{-2+2\sqrt{9v^2+10v+1}-6v}}{8(\sqrt{9v^2+10v+1}-3v-3)v} \end{aligned} \quad (28.62)$$

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$$\begin{aligned} > M := \text{zamproc}(0, u, 1, v, 1, 0, 0, 0, r1, s1, r2, s2): \\ & \frac{r2(u r l^2 + v r 22 + r l r 2) s2 - r l^3 s1}{r l s2 - s1 r2}, \frac{(u r l^2 + 3 v r 22 + 2 r l r 2) s22 + 2 r2 s1 \left(u r l + \frac{r2}{2} \right) s2 - 3 r l^2 s12}{r l s2 - s1 r2}, \\ & \frac{(3 v r 2 + r l) s23 + 2 s1 (u r l + r 2) s22 + r2 s12 s2 u - 3 r l s13}{r l s2 - s1 r2}, \frac{s12 s22 u + s24 v - s l^4 + s1 s23}{r l s2 - s1 r2} \\ & - r l^2 r 22 u - r 24 v + r l^4 - r l r 23, \frac{(-3 s2 v - s1) r 23 - 2 r l (s1 u + s2) r 22 - r l^2 r 2 s2 u + 3 r l^3 s1}{r l s2 - s1 r2}, \\ & \frac{(-u s12 - 3 s22 v - 2 s1 s2) r 22 - 2 s2 r l \left(s1 u + \frac{s2}{2} \right) r 2 + 3 r l^2 s12}{r l s2 - s1 r2}, \frac{(-u s12 s2 - v s23 - s1 s22) r 2 + r l s13}{r l s2 - s1 r2} \end{aligned} \quad (29.1)$$

$$\begin{aligned} > 2-1: & \text{solve}([M[1,2], M[1,3], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 2-10: & \text{solve}([M[1,1], M[1,2], M[1,3], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-1: & \text{solve}([M[1,3], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-2: & \text{solve}([M[1,2], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-4: & \text{solve}([M[1,2], M[1,3], M[2,1], M[2,2], M[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = \text{RootOf}(3 _Z^3 + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = -\frac{1}{\text{RootOf}(3 _Z^3 + 1)}, v = -\frac{\text{RootOf}(3 _Z^3 + 1)}{3} \right\} \end{aligned} \quad (29.2)$$

$$\begin{aligned} > z1 := -1/(3^(1/3)); & z1 := -1/(3^(1/3)); \\ & r11 := z1 * r2; \\ & u1 := -1/z1; \\ & v1 := -z1/3; \\ & \text{zamproc}(0, u1, 1, v1, 1, 0, 0, 0, r11, 0, r2, s2): \\ & z1 := -\frac{32 \sqrt[3]{3}}{3} \\ & u1 := 31 \sqrt[3]{3} \\ & v1 := \frac{32 \sqrt[3]{3}}{9} \\ & -\frac{r22}{3}, 0, 0, -\frac{s23}{3 r2} \\ & 0, 0, 0, \frac{s22}{3} \end{aligned} \quad (29.3)$$

$$\begin{aligned} > 3-9: & \text{solve}([M[1,2], M[1,3], M[2,1], M[2,2], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-15: & \text{solve}([M[1,2], M[1,3], M[2,1], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0\} \end{aligned} \quad (29.4)$$

$$\begin{aligned} > 3-20: & \text{solve}([M[1,2], M[1,3], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-23: & \text{solve}([M[1,1], M[1,3], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-24: & \text{solve}([M[1,1], M[1,2], M[2,2], M[2,3], M[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1=r1, r2=0, s1=0, s2=s2, u=0, v=v\} \end{aligned} \quad (29.5)$$

$$\begin{aligned} > 4-1: & \text{solve}([M[1,3], M[1,4], M[2,1], M[2,2]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = \text{RootOf}(45 _Z^6 + 30 _Z^3 + 1) r2, r2 = r2, s1 = \frac{2 \text{RootOf}(45 _Z^6 + 30 _Z^3 + 1) s2 (15 \text{RootOf}(45 _Z^6 + 30 _Z^3 + 1)^3 + 1)}{3 (25 \text{RootOf}(45 _Z^6 + 30 _Z^3 + 1)^3 + 1)}, s2 = s2, u = u \right\} \end{aligned} \quad (29.6)$$

$$= \frac{3 \operatorname{RootOf}(45 _Z^6 + 30 _Z^3 + 1)^3 - 1}{2 \operatorname{RootOf}(45 _Z^6 + 30 _Z^3 + 1)}, v = -\frac{\operatorname{RootOf}(45 _Z^6 + 30 _Z^3 + 1) (\operatorname{RootOf}(45 _Z^6 + 30 _Z^3 + 1)^3 + 1)}{2} \} \quad (29.7)$$

$$\begin{aligned} > \text{solve}(45 _Z^6 + 30 _Z^3 + 1, _Z); \\ & \frac{(450 \sqrt{5} + 1125)^{1/3}}{15}, \frac{(450 \sqrt{5} + 1125)^{1/3}}{30} - \frac{1\sqrt{3}}{30} \frac{(450 \sqrt{5} + 1125)^{1/3}}{30}, \frac{(450 \sqrt{5} + 1125)^{1/3}}{30} \\ & + \frac{1\sqrt{3}}{30} \frac{(450 \sqrt{5} + 1125)^{1/3}}{30}, -\frac{(-450 \sqrt{5} + 1125)^{1/3}}{15}, \frac{(-450 \sqrt{5} + 1125)^{1/3}}{30} - \frac{1\sqrt{3}}{30} \frac{(-450 \sqrt{5} + 1125)^{1/3}}{30}, \\ & \frac{(-450 \sqrt{5} + 1125)^{1/3}}{30} + \frac{1\sqrt{3}}{30} \frac{(-450 \sqrt{5} + 1125)^{1/3}}{30} \end{aligned} \quad (29.7)$$

$$\begin{aligned} > z1 := -(1/15) * (450 * \operatorname{sqrt}(5) + 1125)^{(1/3)}; \\ & z2 := -(1/15) * (-450 * \operatorname{sqrt}(5) + 1125)^{(1/3)}; \\ & z1 := -\frac{(450 \sqrt{5} + 1125)^{1/3}}{15} \\ & z2 := -\frac{(-450 \sqrt{5} + 1125)^{1/3}}{15} \end{aligned} \quad (29.8)$$

$$\begin{aligned} > r11 := z1 * r2; \\ & s11 := \operatorname{evala}(2 * z1 * s2 * (15 * z1^3 + 1) / (3 * (25 * z1^3 + 1))); \\ & u1 := \operatorname{simplify}(((3 * z1^3 - 1) / (2 * z1))^3)^{(1/3)}; \\ & v1 := \operatorname{simplify}((-1/2) * z1 * (z1^3 + 1))^3)^{(1/3)}; \\ & \operatorname{zamproc}(0, u1, 1, v1, 1, 0, 0, 0, r11, s11, r2, s2); \\ & s11 := \frac{((450 \sqrt{5} + 1125)^{1/3} \sqrt{5} - 3 (450 \sqrt{5} + 1125)^{1/3}) s2}{30} \\ & u1 := \frac{241^{1/3} 52^{1/3}}{5} \\ & v1 := \frac{81^{1/3} 20252^{1/3}}{2025} \\ & -\frac{2 r22 \sqrt{5}}{15} - \frac{r22}{3}, -\frac{(5 + \sqrt{5}) s2 r2}{10}, 0, 0 \\ & 0, 0, \frac{(-5 + \sqrt{5}) s2 r2}{10}, \frac{(2 \sqrt{5} - 5) s22}{15} \end{aligned} \quad (29.9)$$

$$\begin{aligned} > s1 = \operatorname{evala}(((1/30) * ((450 * \operatorname{sqrt}(5) + 1125)^{(1/3)} * \operatorname{sqrt}(5) - 3 * (450 * \operatorname{sqrt}(5) + 1125)^{(1/3)})^3)^{(1/3)} * s2; \\ & s1 = \left(-\frac{1}{3} + \frac{2 \sqrt{5}}{15} \right)^{1/3} s2 \end{aligned} \quad (29.10)$$

$$\begin{aligned} > 4-2: \\ & \text{solve}([\mathbf{M}[1, 4], \mathbf{M}[2, 1], \mathbf{M}[2, 2], \mathbf{M}[2, 3]], \{u, v, r1, s1, s2\}); \\ & \left\{ r1 = \operatorname{RootOf}(3 _Z^3 + 1) r2, s1 = \operatorname{RootOf}(3 _Z^3 - 1 - 6 \operatorname{RootOf}(3 _Z^3 + 1)^2 _Z + 3 \operatorname{RootOf}(3 _Z^3 + 1) _Z) s2, s2 = s2, u = 3 \operatorname{RootOf}(3 _Z^3 + 1)^2, v = -\frac{\operatorname{RootOf}(3 _Z^3 + 1)}{3} \right\} \end{aligned} \quad (29.11)$$

$$\begin{aligned} > 4-3: \\ & \text{solve}([\mathbf{M}[1, 2], \mathbf{M}[1, 4], \mathbf{M}[2, 1], \mathbf{M}[2, 2]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = -r2, r2 = r2, s1 = 0, s2 = s2, u = 2, v = 0 \right\}, \left\{ r1 = \operatorname{RootOf}(_Z^3 - _Z + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = -\frac{2}{\operatorname{RootOf}(_Z^3 - _Z + 1)}, v = 0 \right\} \end{aligned} \quad (29.12)$$

$$\begin{aligned} > 4-6: \\ & \text{solve}([\mathbf{M}[1, 2], \mathbf{M}[1, 3], \mathbf{M}[2, 1], \mathbf{M}[2, 2]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = \operatorname{RootOf}(3 _Z^3 + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = -\frac{1}{\operatorname{RootOf}(3 _Z^3 + 1)}, v = -\frac{\operatorname{RootOf}(3 _Z^3 + 1)}{3} \right\} \end{aligned} \quad (29.13)$$

$$\begin{aligned} > 4-8: \\ & \text{solve}([\mathbf{M}[1, 2], \mathbf{M}[1, 4], \mathbf{M}[2, 1], \mathbf{M}[2, 3]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = \operatorname{RootOf}(18 _Z^6 - 6 _Z^3 + 1) r2, r2 = r2, s1 = \frac{s2 (3 \operatorname{RootOf}(18 _Z^6 - 6 _Z^3 + 1)^3 - 1)}{3 \operatorname{RootOf}(18 _Z^6 - 6 _Z^3 + 1)^2}, s2 = s2, u = \frac{3 \operatorname{RootOf}(18 _Z^6 - 6 _Z^3 + 1)^3 - 1}{\operatorname{RootOf}(18 _Z^6 - 6 _Z^3 + 1)}, v = -2 \operatorname{RootOf}(18 _Z^6 - 6 _Z^3 + 1)^4 \right\} \end{aligned} \quad (29.14)$$

$$\begin{aligned} > \text{solve}(18 _Z^6 - 6 _Z^3 + 1, _Z); \\ & \left(\frac{1}{6} - \frac{I}{6} \right)^{1/3}, -\frac{\left(\frac{1}{6} - \frac{I}{6} \right)^{1/3}}{2} + \frac{I\sqrt{3}}{2} \left(\frac{1}{6} - \frac{I}{6} \right)^{1/3}, -\frac{\left(\frac{1}{6} - \frac{I}{6} \right)^{1/3}}{2} - \frac{I\sqrt{3}}{2} \left(\frac{1}{6} - \frac{I}{6} \right)^{1/3}, \left(\frac{1}{6} + \frac{I}{6} \right)^{1/3}, \\ & -\frac{\left(\frac{1}{6} + \frac{I}{6} \right)^{1/3}}{2} + \frac{I\sqrt{3}}{2} \left(\frac{1}{6} + \frac{I}{6} \right)^{1/3}, -\frac{\left(\frac{1}{6} + \frac{I}{6} \right)^{1/3}}{2} - \frac{I\sqrt{3}}{2} \left(\frac{1}{6} + \frac{I}{6} \right)^{1/3} \end{aligned} \quad (29.15)$$

> 4-10:

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, s2\}); \\ & \left\{ r1 = \text{RootOf}(3 \cdot Z^6 - 1), r2, s1 = -\frac{s2(\text{RootOf}(3 \cdot Z^6 - 1)^3 + 1)}{2 \cdot \text{RootOf}(3 \cdot Z^6 - 1)^2}, s2 = s2, u = \frac{3 \cdot \text{RootOf}(3 \cdot Z^6 - 1)^3 - 1}{2 \cdot \text{RootOf}(3 \cdot Z^6 - 1)}, v = \right. \\ & \quad \left. -\frac{\text{RootOf}(3 \cdot Z^6 - 1) (\text{RootOf}(3 \cdot Z^6 - 1)^3 + 1)}{2} \right\} \end{aligned} \quad (29.16)$$

```

> z1 := 1/3^(1/6);
r11 := z1*r2;
s11 := -s2*(z1^3+1)/(2*z1^2);
u1 := (3*z1^3-1)/(2*z1);
v1 := -(1/2)*z1*(z1^3+1);
zamproc(0,u1,1,v1,1,0,0,0, r11,s11,r2,s2):

$$z1 := \frac{35\sqrt[6]{6}}{3}$$


$$s11 := -\frac{s2 \left(1 + \frac{\sqrt{3}}{3}\right) 31\sqrt[3]{3}}{2}$$


$$u1 := \frac{(\sqrt{3}-1) 31\sqrt[6]{6}}{2}$$


$$v1 := -\frac{35\sqrt[6]{6} \left(1 + \frac{\sqrt{3}}{3}\right)}{6}$$


$$\frac{\sqrt{3} r2^2}{3}, -\frac{(3+\sqrt{3}) s2 r2}{2}, 0, -\frac{(9+5\sqrt{3}) s2^3}{12 r2}$$


$$0, 0, \frac{(3+2\sqrt{3}) s2 r2}{2}, 0$$


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(29.17)

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> z2 := -1/3^(1/6);
r12 := z2*r2;
s12 := -s2*(z2^3+1)/(2*z2^2);
u2 := (3*z2^3-1)/(2*z2);
v2 := -(1/2)*z2*(z2^3+1);
zamproc(0,u2,1,v2,1,0,0,0, r12,s12,r2,s2):

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$$z2 := -\frac{35^{1/6}}{3}$$

$$s12 := -\frac{s2 \left(-\frac{\sqrt{3}}{3} + 1\right) 31^{1/3}}{2}$$

$$u2 := -\frac{(-\sqrt{3} - 1) 31^{1/6}}{2}$$

$$v2 := \frac{35^{1/6} \left(-\frac{\sqrt{3}}{3} + 1\right)}{6}$$

$$-\frac{\sqrt{3} r22}{3}, \frac{(-3 + \sqrt{3}) r2 s2}{2}, 0, \frac{(-9 + 5\sqrt{3}) s23}{12 r2}$$

$$0, 0, -\frac{(-3 + 2\sqrt{3}) r2 s2}{2}, 0$$

(29.18)

> 4-13:

$$\text{solve}([\text{M}[1,2], \text{M}[1,3], \text{M}[2,1], \text{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

$$\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0\}, \left\{ r1 = \text{RootOf}(3 _Z^3 + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = -\frac{1}{\text{RootOf}(3 _Z^3 + 1)}, v = -\frac{\text{RootOf}(3 _Z^3 + 1)}{3} \right\} \quad (29.19)$$

> 4-16:

$$\text{solve}([\text{M}[1,2], \text{M}[2,1], \text{M}[2,2], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

$$\{r1 = -r2, r2 = r2, s1 = 0, s2 = s2, u = 2, v = 0\}, \left\{ r1 = \text{RootOf}(\underline{Z} - \underline{Z} + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = -\frac{2}{\text{RootOf}(\underline{Z} - \underline{Z} + 1)}, v = 0 \right\} \quad (29.20)$$

```
> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

$$\begin{cases} r1 = 0, & v^2 - u^2, & s1 = s2, & s2 = 0, & u = 0, & v = 0 \end{cases}$$


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$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{v, r1, s1, r2, s2\});$$

$$\left\{ r1=0, r2=r2, s1=-\frac{s2^2}{u}, s2=s2, v=\frac{1}{3 u} \right\}, \left\{ r1=-\frac{r2^2}{u}, r2=r2, s1=0, s2=s2, v=\frac{1}{3 u} \right\} \quad (29.22)$$

$$> r21 := -u*r1;$$

$$v1 := 1/(3*u);$$

$$\text{zamproc}(0, u, 1, v1, 1, 0, 0, 0, r1, 0, r21, s2):$$

$$-\frac{u^2 r1^2}{3}, 0, 0, \frac{s2^3}{3 u r1}$$

$$-\frac{r1^3 (u^3 - 3)}{3 s2}, 0, 0, \frac{s2^2}{3} \quad (29.23)$$

$$> 4-22:$$

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

$$\{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0\} \quad (29.24)$$

$$> 4-25:$$

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, s1, r2, s2\});$$

$$\left\{ r2 = \text{RootOf}(\mathbf{Z}^6 - 3 \mathbf{Z}^3 + 3)^5 r1, s1 = -\frac{s2 \text{RootOf}(\mathbf{Z}^6 - 3 \mathbf{Z}^3 + 3)^5}{3}, s2 = s2, u = -\frac{2 \text{RootOf}(\mathbf{Z}^6 - 3 \mathbf{Z}^3 + 3)^3 - 3}{\text{RootOf}(\mathbf{Z}^6 - 3 \mathbf{Z}^3 + 3)^2}, v = \frac{\text{RootOf}(\mathbf{Z}^6 - 3 \mathbf{Z}^3 + 3)^3 - 2}{\text{RootOf}(\mathbf{Z}^6 - 3 \mathbf{Z}^3 + 3)^4} \right\} \quad (29.25)$$

$$> \text{solve}(\mathbf{Z}^{6-3} \mathbf{Z}^{3+3}, \mathbf{Z});$$

$$\frac{(12 - 4 \sqrt[3]{-3})^{1/3}}{2}, -\frac{(12 - 4 \sqrt[3]{-3})^{1/3}}{4} - \frac{I \sqrt{3} (12 - 4 \sqrt[3]{-3})^{1/3}}{4}, -\frac{(12 - 4 \sqrt[3]{-3})^{1/3}}{4} + \frac{I \sqrt{3} (12 - 4 \sqrt[3]{-3})^{1/3}}{4},$$

$$\frac{(12 + 4 \sqrt[3]{-3})^{1/3}}{2}, -\frac{(12 + 4 \sqrt[3]{-3})^{1/3}}{4} + \frac{I \sqrt{3} (12 + 4 \sqrt[3]{-3})^{1/3}}{4}, -\frac{(12 + 4 \sqrt[3]{-3})^{1/3}}{4} - \frac{I \sqrt{3} (12 + 4 \sqrt[3]{-3})^{1/3}}{4} \quad (29.26)$$

$$> 4-26:$$

$$\text{solve}([\mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

$$> 4-28:$$

$$\text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

$$> 4-31:$$

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

$$\{r1=r1, r2=0, s1=0, s2=s2, u=0, v=v\} \quad (29.27)$$

$$> 4-32:$$

$$\text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\});$$

$$\{r1=r1, r2=0, s1=0, s2=s2, u=0, v=v\} \quad (29.28)$$

$$> 4-34:$$

$$\text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\});$$

NSF 4-36

$$> M := \text{zamproc}(0, 0, 1, u, 1, 0, v, 0, r1, s1, r2, s2):$$

$$\frac{-r1^3 s1 - r2^2 (v s1 - s2) r1 + u r2^3 s2}{-s1 r2 + r1 s2}, \frac{(-s1^2 v + 3 s2 u + s1 s2) r2^2 - 2 s2 r1 (v s1 - s2) r2 - 3 s1^2 r1^2}{-s1 r2 + r1 s2},$$

$$\frac{(3 u r2 + r1) s2^3 - s1 (v r1 - 2 r2) s2^2 - 2 v s1^2 r2 s2 - 3 r1 s1^3}{-s1 r2 + r1 s2}, \frac{s2^4 u - v s1^2 s2^2 - s1^4 + s1 s2^3}{-s1 r2 + r1 s2}$$

$$\frac{-u r2^4 + v r1^2 r2^2 + r1^4 - r1 r2^3}{-s1 r2 + r1 s2}, \frac{(-3 s2 u - s1) r2^3 + r1 (v s1 - 2 s2) r2^2 + 2 v r1^2 r2 s2 + 3 r1^3 s1}{-s1 r2 + r1 s2},$$

$$\frac{(v r1^2 - 3 u r2^2 - r1 r2) s2^2 + 2 r2 s1 (v r1 - r2) s2 + 3 s1^2 r1^2}{-s1 r2 + r1 s2}, \frac{r1 s1^3 + s2^2 (v r1 - r2) s1 - r2 u s2^3}{-s1 r2 + r1 s2} \quad (30.1)$$

$$> 2-1:$$

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s2\});$$

$$\left\{ r1 = \text{RootOf}(2 \mathbf{Z} + \text{RootOf}(\mathbf{Z} - 2) - 2 \text{RootOf}(\mathbf{Z} - 2)^2 \mathbf{Z}), r2, s2 = \text{RootOf}(\mathbf{Z} - 2) s1, u = \frac{3 \text{RootOf}(\mathbf{Z} - 2)^2}{2} - 4 \text{RootOf}(2 \mathbf{Z} + \text{RootOf}(\mathbf{Z} - 2) - 2 \text{RootOf}(\mathbf{Z} - 2)^2 \mathbf{Z}), v = \frac{-\text{RootOf}(\mathbf{Z} - 2) (8 \text{RootOf}(\mathbf{Z} - 2) \text{RootOf}(2 \mathbf{Z} + \text{RootOf}(\mathbf{Z} - 2) - 2 \text{RootOf}(\mathbf{Z} - 2)^2 \mathbf{Z}) - 7)}{2} \right\} \quad (30.2)$$

$$> z1 := 2^{(1/3)};$$

$$\text{solve}(2^2 \mathbf{Z}^2 + z1 - 2 * z1^2 * \mathbf{Z}, \mathbf{Z});$$

$$z2 := (1/2) * 2^{(2/3)};$$

$$z1 := 2^{1/3}$$

$$\frac{22^{1/3}}{2}, \frac{22^{1/3}}{2}$$

$$z2 := \frac{22\sqrt[3]{r2}}{2} \quad (30.3)$$

```
> r11 := z2*r2;
s21 := z1*s1;
r11 :=  $\frac{22\sqrt[3]{r2}}{2}$ 
s21 :=  $21\sqrt[3]{s1}$  (30.4)
```

```
> 2-10:
solve([M[1,1],M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-2:
solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0} (30.5)
```

```
> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=u, v=0} (30.6)
```

```
> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=RootOf(18_Z^6-6_Z^3+1)r2, r2=r2, s1=- $\frac{\text{RootOf}(18_Z^6-6_Z^3+1)s2(12\text{RootOf}(18_Z^6-6_Z^3+1)^3-1)}{3(2\text{RootOf}(18_Z^6-6_Z^3+1)^3-1)}$ , s2=s2, u=-2RootOf(18_Z^6-6_Z^3+1)^4, v=- $\frac{3\text{RootOf}(18_Z^6-6_Z^3+1)^3-1}{\text{RootOf}(18_Z^6-6_Z^3+1)}\} (30.7)$ 
```

```
> solve(18*_Z^6-6*_Z^3+1, -_Z);

$$\left(\frac{1}{6}-\frac{I}{6}\right)^{1/3}, -\frac{\left(\frac{1}{6}-\frac{I}{6}\right)^{1/3}}{2}+\frac{I\sqrt{3}\left(\frac{1}{6}-\frac{I}{6}\right)^{1/3}}{2}, -\frac{\left(\frac{1}{6}-\frac{I}{6}\right)^{1/3}}{2}-\frac{I\sqrt{3}\left(\frac{1}{6}-\frac{I}{6}\right)^{1/3}}{2}, \left(\frac{1}{6}+\frac{I}{6}\right)^{1/3},$$


$$-\frac{\left(\frac{1}{6}+\frac{I}{6}\right)^{1/3}}{2}+\frac{I\sqrt{3}\left(\frac{1}{6}+\frac{I}{6}\right)^{1/3}}{2}, -\frac{\left(\frac{1}{6}+\frac{I}{6}\right)^{1/3}}{2}-\frac{I\sqrt{3}\left(\frac{1}{6}+\frac{I}{6}\right)^{1/3}}{2} (30.8)$$

```

```
> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,s1,r1,s2});
{r1=RootOf(54_Z^9-18_Z^6+1)r2, s1=2RootOf(54_Z^9-18_Z^6+1)s2(3RootOf(54_Z^9-18_Z^6+1)^3-1), s2=s2, u=-2RootOf(54_Z^9-18_Z^6+1)^4, v=- $\frac{3\text{RootOf}(54_Z^9-18_Z^6+1)^3-1}{\text{RootOf}(54_Z^9-18_Z^6+1)}\} (30.9)$ 
```

```
> solve(54*_Z^9-18*_Z^6+1, -_Z):
> r11 := theta*r2;
s11 := 2*theta*s2*(3*theta^3-1);
u1 := -2*theta^4;
v1 := -(3*theta^3-1)/theta;
zamproc(0,0,1,u1,1,0,v1,0, r11,s11,r2,s2):
s11 :=  $2\theta s2(3\theta^3-1)$ 
u1 :=  $-2\theta^4$ 
v1 :=  $-\frac{3\theta^3-1}{\theta}$ 

$$-2r2\theta^3+r22, 0, 3(2\theta^3-1)(18\theta^6-6\theta^3+1)s22, \frac{2(2\theta^3-1)(54\theta^9-18\theta^6+1)s23}{r2}$$

0, 0, -9(2\theta^3-1)\theta^3r2s2, -36s22\theta^9+18s22\theta^6 (30.10)
```

> 4-6:

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,2]], \{u, v, s1, r2, s2\}); \\ r2 = \text{RootOf}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18) r1, s1 = -\frac{2s2(\text{RootOf}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18)^3 - 3)}{\text{RootOf}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18)^4}, s2 = s2, u = -\frac{2}{\text{RootOf}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18)^4}, v = \frac{\text{RootOf}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18)^3 - 3}{\text{RootOf}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18)^2} \quad (30.11)$$

$$> \text{solve}(\mathbf{Z}^6 - 6\mathbf{Z}^3 + 18, \mathbf{Z}); \\ (3 - 3\mathbf{I})^{1/3}, -\frac{(3 - 3\mathbf{I})^{1/3}}{2} + \frac{1\sqrt{3}(3 - 3\mathbf{I})^{1/3}}{2}, -\frac{(3 - 3\mathbf{I})^{1/3}}{2} - \frac{1\sqrt{3}(3 - 3\mathbf{I})^{1/3}}{2}, (3 + 3\mathbf{I})^{1/3}, -\frac{(3 + 3\mathbf{I})^{1/3}}{2} \\ + \frac{1\sqrt{3}(3 + 3\mathbf{I})^{1/3}}{2}, -\frac{(3 + 3\mathbf{I})^{1/3}}{2} - \frac{1\sqrt{3}(3 + 3\mathbf{I})^{1/3}}{2} \quad (30.12)$$

$$> 4-8: \\ \text{evala}([\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\})]); \\ \left[\begin{array}{l} \left\{ r1 = \text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4) r2, r2 = r2, s1 = \frac{(3\text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4)^4 - 10\text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4))s2}{4}, s2 = s2, u \right. \\ \left. = \frac{\text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4)^4}{4} - \frac{\text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4)}{2}, v = \frac{9\text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4)^5}{8} \right. \\ \left. - \frac{15\text{RootOf}(9\mathbf{Z}^6 - 24\mathbf{Z}^3 - 4)^2}{4} \right] \quad (30.13) \end{array} \right]$$

$$> \text{solve}(9*\mathbf{Z}^6 - 24*\mathbf{Z}^3 - 4, \mathbf{Z}); \\ \frac{(18\sqrt{5} - 36)^{1/3}}{3}, \frac{(18\sqrt{5} - 36)^{1/3}}{6} - \frac{1\sqrt{3}(18\sqrt{5} - 36)^{1/3}}{6}, \frac{(18\sqrt{5} - 36)^{1/3}}{6} + \frac{1\sqrt{3}(18\sqrt{5} - 36)^{1/3}}{6}, \\ \frac{(36 + 18\sqrt{5})^{1/3}}{3}, -\frac{(36 + 18\sqrt{5})^{1/3}}{6} + \frac{1\sqrt{3}(36 + 18\sqrt{5})^{1/3}}{6}, -\frac{(36 + 18\sqrt{5})^{1/3}}{6} - \frac{1\sqrt{3}(36 + 18\sqrt{5})^{1/3}}{6} \quad (30.14)$$

$$> z1 := (1/3)*(36+18*sqrt(5))^(1/3); \\ z2 := -(1/3)*(18*sqrt(5)-36)^(1/3); \\ z1 := \frac{(36+18\sqrt{5})^{1/3}}{3} \\ z2 := -\frac{(18\sqrt{5}-36)^{1/3}}{3} \quad (30.15)$$

$$> r11 := z1*r2; \\ s11 := (1/4)*(3*z1^4 - 10*z1)*s2; \\ u1 := \text{evala}((1/4)*z1^4 - (1/2)*z1)^3)^(1/3); \\ v1 := 9*z1^5*(1/8) - 15*z1^2*(1/4); \\ \text{zamproc}(0, 0, 1, u1, 1, 0, v1, 0, r11, s11, r2, s2, \text{full}): \\ s11 := \left(\frac{(36+18\sqrt{5})^4}{27} - \frac{10(36+18\sqrt{5})^{1/3}}{3} \right) s2 \\ u1 := \frac{21\sqrt{3}812\sqrt{3}}{81} \\ v1 := \frac{(36+18\sqrt{5})^5}{216} - \frac{5(36+18\sqrt{5})^2}{12} \\ \text{Initial system:} \\ 0, 0, 1, \frac{21\sqrt{3}32\sqrt{3}}{9} \\ 1, 0, \frac{(\sqrt{5}-3)(36+18\sqrt{5})^{2/3}}{12}, 0 \\ \text{substitution: } \frac{(36+18\sqrt{5})^{1/3}r2}{3}, \frac{(36+18\sqrt{5})^{1/3}(\sqrt{5}-3)s2}{6}; r2, s2 \\ \text{det: } -\frac{(36+18\sqrt{5})^{1/3}r2s2(-5+\sqrt{5})}{6} \\ \text{system after substitution:} \\ \frac{(5+\sqrt{5})r22}{6}, 0, \frac{(3\sqrt{5}-5)s22}{2}, 0 \\ 0, -\frac{3r22\sqrt{5}}{2} - \frac{5r22}{2}, 0, -\frac{(-5+\sqrt{5})s22}{6} \quad (30.16)$$

$$> s1 = \text{evala}((1/4)*(3*z1^4 - 10*z1))^3)^(1/3)*s2; \\ v = \text{evala}((9*z1^5*(1/8) - 15*z1^2*(1/4))^3)^(1/3); \\ s1 = \left(\frac{4}{3} - \frac{2\sqrt{5}}{3} \right)^{1/3} s2$$

$$v = \frac{(-3)^{1/3} z^{22/3}}{2} \quad (30.17)$$

```

> r12 := z2*r2;
s12 := (1/4)*(3*z2^4-10*z2)*s2;
u2 := evala(((1/4)*z2^4-(1/2)*z2)^3)^(1/3);
v2 := 9*z2^5*(1/8)-15*z2^2*(1/4);
zamproc(0,0,1,u2,1,0,v2,0, r12,s12,r2,s2,full):
s12 :=  $\left( \frac{(18\sqrt{5}-36)^{4/3}}{27} + \frac{10(18\sqrt{5}-36)^{1/3}}{3} \right) s2$ 
u2 :=  $\frac{21^{1/3} 81^{1/3}}{81}$ 
v2 :=  $-\frac{(18\sqrt{5}-36)^{5/3}}{216} - \frac{5(18\sqrt{5}-36)^{2/3}}{12}$ 
Initial system:
0, 0, 1,  $\frac{21^{1/3} 32^{1/3}}{9}$ 
1, 0,  $-\frac{(18\sqrt{5}-36)^{2/3}(3+\sqrt{5})}{12}, 0$ 
substitution:  $-\frac{(18\sqrt{5}-36)^{1/3} r2}{3}, \frac{(18\sqrt{5}-36)^{1/3}(3+\sqrt{5}) s2}{6}; r2, s2$ 
det:  $-\frac{(18\sqrt{5}-36)^{1/3} r2 s2 (5+\sqrt{5})}{6}$ 
system after substitution:
 $-\frac{(-5+\sqrt{5}) r22}{6}, 0, -\frac{3 s22 \sqrt{5}}{2} - \frac{5 s22}{2}, 0$ 
0,  $\frac{(3\sqrt{5}-5) r22}{2}, 0, \frac{(5+\sqrt{5}) s22}{6}$ 

```

(30.18)

```

> s1 = evala(((1/4)*(3*z2^4-10*z2))^3)^(1/3)*s2;
v = evala((9*z2^5*(1/8)-15*z2^2*(1/4))^3)^(1/3);
s1 =  $\left( \frac{4}{3} + \frac{2\sqrt{5}}{3} \right)^{1/3} s2$ 
v =  $\frac{(-3)^{1/3} z^{22/3}}{2}$ 

```

(30.19)

```

> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,s1,r2});
{r2=RootOf(_Z^3-4) r1, s1=- $\frac{2 s2}{RootOf(_Z^3-4)}$ , u=- $\frac{1}{2 RootOf(_Z^3-4)}$ , v= $\frac{RootOf(_Z^3-4)}{4}$ } \quad (30.20)

```

```

> r21 := 4^(1/3)*r1;
s11 := -2*s2/4^(1/3);
u1 := -1/(2*4^(1/3));
v1 := (1/4)*4^(1/3);
zamproc(0,0,1,u1,1,0,v1,0, r1,s11,r21,s2):
s11 := - $\frac{s2 42^{1/3}}{2}$ 
u1 := - $\frac{42^{1/3}}{8}$ 
v1 :=  $\frac{41^{1/3}}{4}$ 
21^{1/3} r12, - $\frac{3 r1 22^{1/3} s2}{2}, 0, -\frac{5 s23 21^{1/3}}{4 r1}$ 
0, 0,  $\frac{9 r1 22^{1/3} s2}{4}, 0$ 

```

(30.21)

```

> 4-13:
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1});
{r1=RootOf(9 _Z^6-24 _Z^3+8) r2, s1=- $\frac{RootOf(9 _Z^6-24 _Z^3+8) s2 (3 RootOf(9 _Z^6-24 _Z^3+8)^3-2)}{9 RootOf(9 _Z^6-24 _Z^3+8)^3-4}$ , u=
 $-\frac{RootOf(9 _Z^6-24 _Z^3+8)^4}{9 RootOf(9 _Z^6-24 _Z^3+8)^3-4}, v=-\frac{9 RootOf(9 _Z^6-24 _Z^3+8)^6-12 RootOf(9 _Z^6-24 _Z^3+8)^3+4}{(9 RootOf(9 _Z^6-24 _Z^3+8)^3-4) RootOf(9 _Z^6-24 _Z^3+8)}$ } \quad (30.22)
> solve(9*_Z^6-24*_Z^3+8, _Z);

```

$$\begin{aligned} & \frac{(36-18\sqrt{2})^{1/3}}{3}, -\frac{(36-18\sqrt{2})^{1/3}}{6} + \frac{1\sqrt{3}(36-18\sqrt{2})^{1/3}}{6}, -\frac{(36-18\sqrt{2})^{1/3}}{6} - \frac{1\sqrt{3}(36-18\sqrt{2})^{1/3}}{6}, \\ & \frac{(36+18\sqrt{2})^{1/3}}{3}, -\frac{(36+18\sqrt{2})^{1/3}}{6} + \frac{1\sqrt{3}(36+18\sqrt{2})^{1/3}}{6}, -\frac{(36+18\sqrt{2})^{1/3}}{6} - \frac{1\sqrt{3}(36+18\sqrt{2})^{1/3}}{6} \end{aligned} \quad (30.23)$$

```
> z1 := (1/3)*(36+18*sqrt(2))^^(1/3);
z2 := (1/3)*(36-18*sqrt(2))^^(1/3);
z1 :=  $\frac{(36+18\sqrt{2})^{1/3}}{3}$ 
z2 :=  $\frac{(36-18\sqrt{2})^{1/3}}{3}$  (30.24)
```

```
> r11 := z1*r2;
s11 := -z1*(3*z1^3-2)/(9*z1^3-4)*s2;
u1 := -z1^4/(9*z1^3-4):
v1 := -(9*z1^6-12*z1^3+4)/((9*z1^3-4)*z1):
zamproc(0,0,1,u1,1,0,v1,0, r11,s11,r2,s2):
- $\frac{(-4+\sqrt{2})r22}{3}, 0, 0, \frac{(-5+3\sqrt{2})s23}{3r2}$ 
0, -(3+\sqrt{2})r22, 0,  $\frac{(-1+2\sqrt{2})s22}{3}$  (30.25)
```

```
> r1 := z1*r2;
s1 = evala((-z1*(3*z1^3-2)/(9*z1^3-4))^3)^(1/3)*s2;
u = evala((-z1^4/(9*z1^3-4))^3)^(1/3);
v = evala((-(-9*z1^6-12*z1^3+4)/((9*z1^3-4)*z1))^3)^(1/3);
r1 =  $\frac{(36+18\sqrt{2})^{1/3}r2}{3}$ 
s1 =  $\left(-1 + \frac{2\sqrt{2}}{3}\right)^{1/3}s2$ 
u =  $\left(\frac{8}{81} - \frac{2\sqrt{2}}{27}\right)^{1/3}$ 
v =  $(3 - 3\sqrt{2})^{1/3}$  (30.26)
```

```
> r12 := z2*r2;
s12 := -z2*(3*z2^3-2)/(9*z2^3-4)*s2;
u2 := -z2^4/(9*z2^3-4):
v2 := -(9*z2^6-12*z2^3+4)/((9*z2^3-4)*z2):
zamproc(0,0,1,u2,1,0,v2,0, r12,s12,r2,s2):
 $\frac{(4+\sqrt{2})r22}{3}, 0, 0, -\frac{(5+3\sqrt{2})s23}{3r2}$ 
0,  $(\sqrt{2}-3)r22, 0, -\frac{2s22\sqrt{2}}{3} - \frac{s22}{3}$  (30.27)
```

```
> r1 := z2*r2;
s1 = evala((-z2*(3*z2^3-2)/(9*z2^3-4))^3)^(1/3)*s2;
u = evala((-z2^4/(9*z2^3-4))^3)^(1/3);
v = evala((-(-9*z2^6-12*z2^3+4)/((9*z2^3-4)*z2))^3)^(1/3);
r1 =  $\frac{(36-18\sqrt{2})^{1/3}r2}{3}$ 
s1 =  $\left(-1 - \frac{2\sqrt{2}}{3}\right)^{1/3}s2$ 
u =  $\left(\frac{8}{81} + \frac{2\sqrt{2}}{27}\right)^{1/3}$ 
v =  $(3 + 3\sqrt{2})^{1/3}$  (30.28)
```

```
> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
(r1=0,r2=r2,s1=s1,s2=0,u=0,v=v)
> 4-21:
solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});
```

(30.29)

(30.30)

$$> 4-25: \text{solve}([\mathbf{M}[1,2], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, s2\}); \\ \left\{ r1 = \text{RootOf}(\underline{\mathcal{Z}} - 4) r2, s1 = -\frac{s2 \text{RootOf}(\underline{\mathcal{Z}} - 4)}{2}, s2 = s2, u = \text{RootOf}(\underline{\mathcal{Z}} - 4), v = -\frac{\text{RootOf}(\underline{\mathcal{Z}} - 4)^2}{2} \right\}, \{r1 = 0, s1 = s1, s2 = 0, u = 0, v = 0\} \quad (30.31)$$

$$> r11 := 4^{(1/3)} * r2; \\ s11 := -(1/2) * s2 * 4^{(1/3)}; \\ u1 := 4^{(1/3)}; \\ v1 := -(1/2) * 4^{(2/3)}; \\ \text{zamproc}(0, 0, 1, u1, 1, 0, v1, 0, r11, s11, r2, s2); \\ \begin{aligned} & 2 r2^2, 0, 3 s2^2, \frac{s2^3}{2 r2} \\ & 0, -9 r2^2, 0, 0 \end{aligned} \quad (30.32)$$

$$> 4-26: \text{solve}([\mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 4-28: \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 4-31: \text{solve}([\mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0\} \quad (30.33)$$

$$> 4-32: \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0\} \quad (30.34)$$

$$> 4-34: \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 4-35: \text{solve}([\mathbf{M}[1,1], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0\} \quad (30.35)$$

NSF 4-37

$$> M := \text{zamproc}(0, 0, u, v, 1, 1, 0, 0, r1, s1, r2, s2); \\ \begin{aligned} & \frac{u r22 r1 s2 + v s2 r23 - s1 r13 - s1 r12 r2}{-r2 s1 + r1 s2}, \frac{(2 r1 u r2 + 3 v r22) s22 + s2 (u r22 - r12) s1 - 3 \left(r1 + \frac{2 r2}{3}\right) r1 s12}{-r2 s1 + r1 s2}, \\ & \frac{(-3 r1 - r2) s13 - 2 s12 r1 s2 + 2 s1 r2 s22 u + s23 (r1 u + 3 v r2)}{-r2 s1 + r1 s2}, \frac{s1 s23 u + v s24 - s14 - s13 s2}{-r2 s1 + r1 s2} \\ & \frac{-r1 r23 u - v r24 + r14 + r2 r13}{-r2 s1 + r1 s2}, \frac{(3 s1 + s2) r13 + 2 s1 r12 r2 - 2 u r22 r1 s2 - r23 (s1 u + 3 s2 v)}{-r2 s1 + r1 s2}, \\ & \frac{-r2 s22 (r1 u + 3 v r2) - 2 s2 (u r22 - r12) s1 + 3 r1 s12 \left(r1 + \frac{r2}{3}\right)}{-r2 s1 + r1 s2}, \frac{-s1 r2 s22 u - r2 s23 v + s13 r1 + s12 r1 s2}{-r2 s1 + r1 s2} \end{aligned} \quad (31.1)$$

$$> 2-1: \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 2-10: \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[1,3], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ > 3-1: \text{solve}([\mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ > 3-2: \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ \{r1 = 0, r2 = r2, s1 = -s2, s2 = s2, u = 0, v = 0\}, \left\{ r1 = -\frac{r2}{3}, r2 = r2, s1 = \frac{s2^2}{3}, s2 = s2, u = \frac{1}{9}, v = \frac{1}{81} \right\} \quad (31.2)$$

$$> r21 := -3 * r1; \\ s21 := 3 * s1; \\ u1 := 1/9; \\ v1 := 1/81; \\ \text{zamproc}(0, 0, u1, v1, 1, 1, 0, 0, r1, s1, r21, s21); \\ \begin{aligned} & \frac{2 r1^2}{3}, 0, -2 s12, 0 \\ & 0, 0, 0, \frac{4 s12}{3} \end{aligned} \quad (31.3)$$

```

> 3-4:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 3-9:
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
  {r1=RootOf(3_Z^2+3_Z+1)r2,r2=r2,s1=-s2 RootOf(3_Z^2+3_Z+1)+1
   3 RootOf(3_Z^2+3_Z+1)+2,s2=s2,u=1/3,v=1/9}

```

(31.4)

```

> solve(3*_Z^2+3*_Z+1, _Z);
  -1/2 + I\sqrt{3}/6, -1/2 - I\sqrt{3}/6

```

(31.5)

```

> 3-15:
solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
  {r1=-r2,r2=r2,s1=0,s2=s2,u=0,v=0}

```

(31.6)

```

> 3-20:
solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

```

```

> 3-23:
solve([M[1,1],M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
  {r1=r2^2/3,r2=r2,s1=-s2^2/3,s2=s2,u=-1/9,v=-1/81}

```

(31.7)

```

> r21 := 3*r1;
s21 := -3*s1;
u1 := -1/9;
v1 := -1/81;
zamproc(0,0,u1,v1,1,1,0,0, r1,s1,r21,s21):
  0, 2 r1 s1, 0, -2 s1^3
  3 r1
  - 4 r1^3
  3 s1, 0, 0, 0

```

(31.8)

```

> 3-24:
solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
  {(r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0), (r1=0,r2=r2,s1=s1,s2=0,u=0,v=v)}

```

(31.9)

```

> 4-1:
solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
  {r1=RootOf(3_Z^2+3_Z+1)r2,r2=r2,s1=-s2 (3 RootOf(3_Z^2+3_Z+1)+1)
   3 (2 RootOf(3_Z^2+3_Z+1)+1),s2=s2,u=1/3,v=1/9}, {r1
   = RootOf(_Z^2+_Z-1)r2
   3 ,r2=r2,s1=-s2 (RootOf(_Z^2+_Z-1)+1)
   3 ,s2=s2,u=1/9,v=-1/81}

```

(31.10)

```

> solve(_Z^2+_Z-1, _Z);
z1 := (1/2)*sqrt(5)-1/2;
z2 := -1/2-(1/2)*sqrt(5);
  \sqrt{5}/2 - 1/2, -1/2 - \sqrt{5}/2
  z1 := \sqrt{5}/2 - 1/2
  z2 := -1/2 - \sqrt{5}/2

```

(31.11)

```

> r11 := z1*r2/3;
s11 := -(1/3)*s2*(z1+1);
u1 := 1/9;
v1 := -1/81;
zamproc(0,0,u1,v1,1,1,0,0, r11,s11,r2,s2):
  r11 := \left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right) r2
  3
  s11 := -\frac{s2 \left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right)}{3}
  - \frac{(-5 + \sqrt{5}) r2^2}{54}, -\frac{s2 r2 \sqrt{5}}{9}, 0, 0
  0, 0, \frac{s2 r2 \sqrt{5}}{9}, \frac{(5 + \sqrt{5}) s2^2}{54}

```

(31.12)

```

> r12 := z2*r2/3;
s12 := -(1/3)*s2*(z2+1);
u1 := 1/9;
v1 := -1/81;
zamproc(0,0,u1,v1,1,1,0,0, r12,s12,r2,s2):

$$r12 := \frac{\left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right) r2}{3}$$


$$s12 := -\frac{s2 \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)}{3}$$


$$\frac{(5 + \sqrt{5}) r2^2}{54}, \frac{s2 r2 \sqrt{5}}{9}, 0, 0$$


$$0, 0, -\frac{s2 r2 \sqrt{5}}{9}, -\frac{(-5 + \sqrt{5}) s2^2}{54} \quad (31.13)$$


```

```

> 4-2:
solve([M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1 = - $\frac{r2}{3}$ , r2 = r2, s1 =  $\frac{\text{RootOf}(\underline{Z}+3-\underline{Z}+1)s2}{3}$ , s2 = s2, u =  $\frac{1}{9}$ , v =  $\frac{1}{81}$ }, {r1 = 0, r2 = r2, s1 = -s2, s2 = s2, u = 0, v = 0}, {r1 =
 $-\frac{r2}{3}$ , r2 = r2, s1 =  $\frac{s2}{3}$ , s2 = s2, u =  $\frac{1}{9}$ , v =  $\frac{1}{81}$ } \quad (31.14)

```

```

> 4-3:
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,s2});
{r1 = - $\frac{r2}{3}$ , s1 =  $\frac{s2}{3}$ , s2 = s2, u =  $\frac{1}{9}$ , v =  $\frac{1}{81}$ }, {r1 =  $\frac{r2 (\text{RootOf}(\underline{Z}-8\underline{Z}+25)-10)}{9}$ , s1 = - $\frac{2 s2 \text{RootOf}(\underline{Z}-8\underline{Z}+25)}{9} + \frac{5 s2}{9}$ , s2 = s2, u =  $\frac{\text{RootOf}(\underline{Z}-8\underline{Z}+25)}{9}$ , v =  $\frac{5 \text{RootOf}(\underline{Z}-8\underline{Z}+25)}{81}$ }, {r1 = 0, s1 = -s2, s2 = s2, u = 0, v = 0} \quad (31.15)

```

```

> solve(_Z^2-8*_Z+25, _Z); 4+3I, 4-3I \quad (31.16)

```

```

> 4-6:
solve([M[1,2],M[1,3],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1 = RootOf(3_Z^2+3_Z+1)r2, r2 = r2, s1 = - $\frac{s2 \text{RootOf}(3_Z^2+3_Z+1)}{3 \text{RootOf}(3_Z^2+3_Z+1)+2}$ , s2 = s2, u =  $\frac{1}{3}$ , v =  $\frac{1}{9}$ } \quad (31.17)

```

```

> 4-8:
solve([M[1,2],M[1,4],M[2,1],M[2,3]], {v,r1,s1,r2,s2});
{r1 = RootOf(_Z-u)r2, r2 = r2, s1 = - $\frac{s2 u (3 \text{RootOf}(_Z-u)+2)}{2 \text{RootOf}(_Z-u)+3 u}$ , s2 = s2, v = u^2} \quad (31.18)

```

```

> z1 := sqrt(u):
r11 := z1*r2:
s11 := -s2*evala(u^(3*z1+2)/(2*z1+3*u));
v1 := u^2:
zamproc(0,0,u,v1,1,1,0,0, r11,s11,r2,s2):

$$s11 := -s2 \sqrt{u}$$


$$r22 \sqrt{u} (u + \sqrt{u}), 0, s22 \sqrt{u} (3 u - \sqrt{u}), 0$$


$$0, -(3 u + \sqrt{u}) r22 \sqrt{u}, 0, (-u + \sqrt{u}) s22 \sqrt{u} \quad (31.19)$$


```

```

> z2 := -sqrt(u):
r12 := z2*r2:
s12 := -s2*evala(u^(3*z2+2)/(2*z2+3*u));
v1 := u^2:
zamproc(0,0,u,v1,1,1,0,0, r11,s11,r2,s2):

$$s12 := s2 \sqrt{u}$$


$$r22 \sqrt{u} (u + \sqrt{u}), 0, s22 \sqrt{u} (3 u - \sqrt{u}), 0$$


$$0, -(3 u + \sqrt{u}) r22 \sqrt{u}, 0, (-u + \sqrt{u}) s22 \sqrt{u} \quad (31.20)$$


```

```

> 4-10:
solve([M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1 = -r2, r2 = r2, s1 = s2, s2 = s2, u = -1, v = -1}, {r1 = RootOf(3_Z^2+3_Z+1)r2, r2 = r2, s1 = -s2 (2 RootOf(3_Z^2+3_Z+1) + 1), s2 = s2, u =  $\frac{1}{3}$ , v =  $\frac{1}{9}$ } \quad (31.21)

```

```

> 4-13:
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=0,s2=s2,u=0,v=0}, {r1 =  $\frac{r^2}{3}$ , r2=r2, s1 = - $\frac{s^2}{3}$ , s2=s2, u =  $\frac{1}{9}$ , v =  $\frac{1}{81}$ } (31.22)

> 4-16:
solve([M[1,2],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=RootOf(3_Z+3_Z+1)r2,r2=r2,s1=- $\frac{s^2 \text{RootOf}(3_Z+3_Z+1)}{3 \text{RootOf}(3_Z+3_Z+1)+2}$ ,s2=s2,u= $\frac{1}{3}$ ,v= $\frac{1}{9}$ },{r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0} (31.23)

> 4-17:
solve([M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,r2,s2});
{r1=RootOf(3_Z+1)r2,r2=r2,s2=- $\frac{2s1}{\text{RootOf}(3_Z+1)+1}$ ,u=- $\frac{(\text{RootOf}(3_Z+1)+1)^2}{4}$ ,v= $\frac{\text{RootOf}(3_Z+1)^3}{4}$ 
 + $\frac{\text{RootOf}(3_Z+1)^2}{2}$ - $\frac{\text{RootOf}(3_Z+1)}{12}$ + $\frac{1}{9}$ } (31.24)

> 4-21:
solve([M[1,2],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
> 4-22:
solve([M[1,2],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2});
{r1=-r2,r2=r2,s1=0,u=0,v=0},{r1=RootOf(3_Z+3_Z+1)r2,r2=r2,s1=- $\frac{s^2 \text{RootOf}(3_Z+3_Z+1)}{3 \text{RootOf}(3_Z+3_Z+1)+2}$ ,u=
 - $\frac{\text{RootOf}(3_Z+3_Z+1)^2}{(3 \text{RootOf}(3_Z+3_Z+1)+2)^2}$ ,v= $\frac{\text{RootOf}(3_Z+3_Z+1)^3 (2 \text{RootOf}(3_Z+3_Z+1)+1)}{(3 \text{RootOf}(3_Z+3_Z+1)+2)^3}$ } (31.25)

> 4-25:
solve([M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0},{r1=-r2,r2=r2,s1=0,s2=s2,u=0,v=0},{r1=r2,r2=r2,s1=-s2,s2=s2,u=1,v=1},{r1=0,r2=r2,s1=s1,s2=0,u=u,v=0} (31.26)

> 4-26:
solve([M[1,3],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1 =  $\frac{r^2}{3}$ , r2=r2, s1 = - $\frac{s^2}{3}$ , s2=s2, u = - $\frac{1}{9}$ , v = - $\frac{1}{81}$ } (31.27)

> 4-28:
solve([M[1,1],M[1,3],M[2,2],M[2,3]], {v,r1,s1,s2});
{r1=RootOf(_Z+u)r2,s1= $\frac{2s2u}{\text{RootOf}(_Z+u)-3u}$ ,s2=s2,v= $\frac{u^2 (\text{RootOf}(_Z+u)-1)}{\text{RootOf}(_Z+u)-3u}$ } (31.28)

> z1 := sqrt(-u):
r11 := z1*r2:
s11 := 2*s2*u/(z1-3*u);
v1 := evala(u^2*(z1-1)/(z1-3*u));
zamproc(0,0,u,v1,1,1,0,0, r11,s11,r2,s2):

$$s11 := \frac{2s2u}{\sqrt{-u}-3u}$$


$$v1 := -\frac{(3\sqrt{-u}u-\sqrt{-u}-4u)u}{9u+1}$$


$$0, \frac{3((-u)^3\sqrt{2}-2u+\sqrt{-u})s2r2u}{(\sqrt{-u}+1)(\sqrt{-u}-3u)}, 0, \frac{3(3(-u)^5\sqrt{2}+6u^2+4(-u)^3\sqrt{2}-2u+\sqrt{-u})u^2s2^3}{(\sqrt{-u}+1)r2(\sqrt{-u}-3u)^3}$$


$$\frac{r2^3(3u^2+5(-u)^3\sqrt{2}-u-\sqrt{-u})(-\sqrt{-u}+3u)}{(\sqrt{-u}+1)s2(9u+1)}, 0, 0, \frac{(-3(-u)^3\sqrt{2}+5u-\sqrt{-u}+1)u^2s2^2}{(\sqrt{-u}+1)(-\sqrt{-u}+3u)^2} (31.29)$$


> z2 := -sqrt(-u):
r12 := z2*r2:
s12 := 2*s2*u/(z2-3*u);
v2 := evala(u^2*(z2-1)/(z2-3*u));
zamproc(0,0,u,v2,1,1,0,0, r12,s12,r2,s2):

$$s12 := \frac{2s2u}{-\sqrt{-u}-3u}$$


$$v2 := \frac{(4u+3\sqrt{-u}u-\sqrt{-u})u}{9u+1}$$


$$0, -\frac{3((-u)^3\sqrt{2}+2u+\sqrt{-u})s2r2u}{(\sqrt{-u}-1)(\sqrt{-u}+3u)}, 0, -\frac{3(3(-u)^5\sqrt{2}-6u^2+4(-u)^3\sqrt{2}+2u+\sqrt{-u})u^2s2^3}{(\sqrt{-u}-1)r2(\sqrt{-u}+3u)^3}$$


```

$$\frac{(-3 u^2 + 5 (-u)^3 \sqrt{2} + u - \sqrt{-u}) (\sqrt{-u} + 3 u) r_{23}}{(\sqrt{-u} - 1) s_2 (9 u + 1)} , 0, 0, - \frac{(3 (-u)^3 \sqrt{2} + 5 u + \sqrt{-u} + 1) u^2 s_{22}}{(\sqrt{-u} - 1) (\sqrt{-u} + 3 u)^2} \quad (31.30)$$

```
> 4-31:
solve([M[1,2],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} \quad (31.31)
```

```
> 4-32:
solve([M[1,1],M[1,2],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} \quad (31.32)
```

```
> 4-34:
solve([M[1,1],M[1,3],M[2,2],M[2,4]], {v,r1,s1,r2,s2});
{r1=RootOf(_Z^2+u)r2,r2=r2,s1=-RootOf(_Z^2+u)s2,s2=s2,v=RootOf(_Z^2+u)^2u} \quad (31.33)
```

```
> z1 := sqrt(-u):
r11 := z1*r2:
s11 := -z1*s2:
v1 := -u^2:
zamproc(0,0,u,v1,1,1,0,0, r11,s11,r2,s2):
0, s2 r2 (-\sqrt{-u} + 3 u) \sqrt{-u}, 0, \frac{(\sqrt{-u} + u) s_{23} \sqrt{-u}}{r2}
\frac{r_{23} (\sqrt{-u} - u) \sqrt{-u}}{s2}, 0, -s2 r2 (\sqrt{-u} + 3 u) \sqrt{-u}, 0 \quad (31.34)
```

```
> z2 := -sqrt(-u):
r12 := z2*r2:
s12 := -z2*s2:
v1 := -u^2:
zamproc(0,0,u,v1,1,1,0,0, r12,s12,r2,s2):
0, -s2 r2 (\sqrt{-u} + 3 u) \sqrt{-u}, 0, \frac{s_{23} (\sqrt{-u} - u) \sqrt{-u}}{r2}
\frac{(\sqrt{-u} + u) r_{23} \sqrt{-u}}{s2}, 0, s2 r2 (-\sqrt{-u} + 3 u) \sqrt{-u}, 0 \quad (31.35)
```

```
> 4-35:
solve([M[1,1],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0,v=v}, \left\{ r1 = \frac{r2}{3}, r2 = r2, s1 = -\frac{s2}{3}, s2 = s2, u = -\frac{1}{9}, v = -\frac{1}{81} \right\}, {r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0} \quad (31.36)
```

```
> 4-36:
solve([M[1,1],M[1,2],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0}, \left\{ r1 = -\frac{r2}{3}, r2 = r2, s1 = \frac{s2}{3}, s2 = s2, u = -\frac{1}{9}, v = -\frac{1}{81} \right\}, {r1=0,r2=r2,s1=s1,s2=0,u=0,v=v} \quad (31.37)
```