

**3. Сведение форм с  $m = 5$  из списка 2.1 к предшествующим (утверждение 3.2).**

```
> restart; read("newlib.m"); with(mylib): with(LinearAlgebra):
```

$SF_3^{5,1}$  ( $v \neq u$ ). Результат произвольной замены :

$$\begin{aligned} & M := \text{zamproc}(u, v, v-u, 0, 0, 1, 0, -1, r1, s1, r2, s2) : \\ & \frac{(r1+r2)(us2r1^2 - ((u-v)s2+s1)r2r1+r2^2s1)}{r1s2-r2s1}, \\ & \frac{-2r1\left(-\frac{vr1}{2} + r2(u-v)\right)s2^2 + 3s1\left(\left(u-\frac{1}{3}\right)r1^2 + \frac{2vr1r2}{3} - \frac{r2^2(u-v-3)}{3}\right)s2 - 2r1s1^2r2}{r1s2-r2s1}, \\ & \frac{-r1(u-v)s2^3 - 2s1\left(-vr1 + r2\left(u-v-\frac{3}{2}\right)\right)s2^2 + 3\left(\left(u-\frac{2}{3}\right)r1 + \frac{r2v}{3}\right)s1^2s2 - s1^3r2}{r1s2-r2s1}, \frac{(s1+s2)s1((-u+v+1)s2 + (u-1)s1)s2}{r1s2-r2s1}, \\ & \frac{-r1(r1+r2)r2((-u+v+1)r2 + (u-1)r1)}{r1s2-r2s1}, \frac{s1(u-v)r2^3 + 2r1\left(-vs1 + s2\left(u-v-\frac{3}{2}\right)\right)r2^2 - 3\left(\left(u-\frac{2}{3}\right)s1 + \frac{s2v}{3}\right)r1^2r2 + r1^3s2}{r1s2-r2s1}, \\ & \frac{2s1\left(-\frac{vs1}{2} + (u-v)s2\right)r2^2 - 3r1\left(\left(u-\frac{1}{3}\right)s1^2 + \frac{2s1s2v}{3} - \frac{s2^2(u-v-3)}{3}\right)r2 + 2r1^2s1s2}{r1s2-r2s1}, \\ & \frac{(s1+s2)(r2us1^2 - (r2(u-v)+r1)s2s1 + r1s2^2)}{r1s2-r2s1} \end{aligned}$$

$SF_2^{2,1}$

```
> solve([M[1,2], M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});
```

$SF_9^{2,1}$

```
> solve([M[1,1], M[1,2], M[1,4], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});
```

{ $r1=0, r2=r2, s1=-s2, s2=s2, u=2, v=2$ }, { $r1=r2, r2=r2, s1=-s2, s2=s2, u=1, v=0$ }, { $r1=-r2, r2=r2, s1=s2, s2=s2, u=1, v=0$ }

$SF_5^{3,1}$

```
> solve([M[1,1], M[1,4], M[2,1], M[2,2], M[2,3]], {u, v, r1, s1, r2, s2});
```

{ $r1=-r2, r2=r2, s1=s1, s2=0, u=-1, v=-4$ }, { $r1=-r2, r2=r2, s1=0, s2=s2, u=-1, v=-4$ }

```
> zamproc(-1, -4, -3, 0, 0, 1, 0, -1, r1, s1, -r1, 0) :
```

$$\begin{aligned} & 0, 2r1s1, s1^2, 0 \\ & 0, 0, 0, -s1^2 \end{aligned}$$

$SF_6^{3,1}$

```
> solve([M[1,2], M[1,4], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});
```

{ $r1=0, r2=r2, s1=s1, s2=0, u=0, v=0$ }, { $r1=3r2, r2=r2, s1=-s2, s2=s2, u=\frac{1}{3}, v=\frac{4}{3}$ }

$r11 := 3*r2:$

```
zamproc(1/3, 4/3, 1, 0, 0, 1, 0, -1, r11, s1, r2, -s1) :
```

$$\begin{aligned} & 8r2^2, 0, -\frac{8s1^2}{3}, 0 \\ & 0, 0, \frac{16s1r2}{3}, 0 \end{aligned}$$

$SF_8^{3,1}$

```
> solve([M[1,1], M[1,3], M[2,1], M[2,2], M[2,3]], {u, v, r1, s1, r2, s2});
```

{ $r1=-r2, r2=r2, s1=3s2, s2=s2, u=-1, v=-4$ }

$SF_{11}^{3,1}$

```
> solve([M[1,2], M[1,4], M[2,1], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

{ $r1=0, r2=r2, s1=s1, s2=0, u=0, v=0$ }, { $r1=r2, r2=r2, s1=-s2, s2=s2, u=-1, v=0$ }, { $r1=0, r2=r2, s1=-s2, s2=s2, u=-3, v=-6$ }, { $r1=-r2, r2=r2, s1=s2, s2=s2, u=-1, v=0$ }

```
> zamproc(-3, -6, -3, 0, 0, 1, 0, -1, 0, s1, r2, -s1) :
```

$$\begin{aligned} & -r2^2, 0, -2s1^2, 0 \\ & 0, -3r2^2, 0, 0 \end{aligned}$$

$SF_{14}^{3,1}$

```
> solve([M[1,1], M[1,3], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});
```

{ $r1=-r2, r2=r2, s1=s2, s2=s2, u=1, v=0$ }, { $r1=r2, r2=r2, s1=-s2, s2=s2, u=1, v=0$ }

$SF_{17}^{3,1}$

```
> solve([M[1,2], M[1,4], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

```

(r1 = 3*r2, r2 = r2, s1 = -s2, s2 = s2, u = 3, v = 12}, {r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0}

> r11 := 3*r2:
zamproc(3, 12, 9, 0, 0, 1, 0, -1, r11, s1, r2, -s1):

$$\begin{aligned} & 56r2^2, 0, -8s1^2, 0 \\ & \frac{48r2^3}{s1}, 0, 0, 0 \end{aligned}$$

SF193,1

> solve([M[1,1], M[1,3], M[2,1], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});

$$\left\{ r1 = -r2, r2 = r2, s1 = 3s2, s2 = s2, u = -\frac{1}{9}, v = -\frac{4}{9} \right\}$$


> s11 := 3*s2:
zamproc(-1/9, -4/9, -1/3, 0, 0, 1, 0, -1, r1, s11, -r1, s2):

$$\begin{aligned} & 0, \frac{56s2r1}{9}, 0, -\frac{8s2^3}{r1} \\ & 0, -\frac{16r1^2}{9}, 0, 0 \end{aligned}$$

SF213,1

> solve([M[1,1], M[1,4], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
SF223,1

> solve([M[1,1], M[1,2], M[2,1], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
{r1 = r2, r2 = r2, s1 = -s2, s2 = s2, u = -1, v = 0}, {r1 = -r2, r2 = r2, s1 = s2, s2 = s2, u = -1, v = 0},  $\left\{ r1 = -r2, r2 = r2, s1 = -\frac{s2}{2}, s2 = s2, u = -4, v = -9 \right\}$ 

> s21 := -2*s1:
zamproc(-4, -9, -5, 0, 0, 1, 0, -1, r1, s1, -r1, s21):

$$\begin{aligned} & 0, 0, -7s1^2, -\frac{6s1^3}{r1} \\ & 0, -r1^2, 0, 0 \end{aligned}$$

SF14,1

> solve([M[1,3], M[1,4], M[2,1], M[2,2]], {u, v, r1, s1, r2, s2});
{r1 = 0, r2 = r2, s1 = -s2, s2 = s2, u = 2, v = 2}, {r1 = r2, r2 = r2, s1 = -s2, s2 = s2, u = 1, v = 0},  $\left\{ r1 = 2r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{1}{2}, v = \frac{1}{2} \right\}$ , {r1 =
-r2, r2 = r2, s1 = 0, s2 = s2, u = 2, v = 2}, {r1 = -r2, r2 = r2, s1 = s2, s2 = s2, u = 1, v = 0},  $\left\{ r1 = 0, r2 = r2, s1 = 2s2, s2 = s2, u = \frac{1}{2}, v = \frac{1}{2} \right\}$ 
SF34,1

> solve([M[1,2], M[1,4], M[2,1], M[2,2]], {u, v, r1, s1, r2, s2});
{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = v, v = v},  $\left\{ r1 = \text{RootOf}(\underline{Z}^2 - 3) r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{\text{RootOf}(\underline{Z}^2 - 3)^2 + \text{RootOf}(\underline{Z}^2 - 3) - 4}{\text{RootOf}(\underline{Z}^2 - 3) (\text{RootOf}(\underline{Z}^2 - 3) - 1)}, v = \frac{2(\text{RootOf}(\underline{Z}^2 - 3) - 2)}{\text{RootOf}(\underline{Z}^2 - 3)} \right\}$ ,  $\left\{ r1 = 3r2, r2 = r2, s1 = -s2, s2 = s2, u = \frac{1}{3}, v = \frac{4}{3} \right\}$ 

> z1 := sqrt(3):
r11 := z1*r2:
u1 := (z1^2+z1-4)/(z1*(z1-1));
v1 := -2*(z1-2)/z1;
zamproc(u1, v1, v1-u1, 0, 0, 1, 0, -1, r11, 0, r2, s2):

$$\begin{aligned} & u1 := \frac{\sqrt{3}}{3} \\ & v1 := -\frac{2(\sqrt{3} - 2)\sqrt{3}}{3} \\ & 2r2^2, 0, s2^2(\sqrt{3} - 2), 0 \\ & 0, 0, -r2s2(1 + \sqrt{3}), -s2^2 \end{aligned}$$

SF54,1

> solve([M[1,4], M[2,1], M[2,2], M[2,4]], {v, r1, s1, r2, s2});

$$\left\{ r1 = \frac{r2}{u}, r2 = r2, s1 = -s2, s2 = s2, v = -\frac{\frac{(u-1)r2}{u} + (-u+1)r2}{r2} \right\}, \{r1 = 0, r2 = r2, s1 = -s2, s2 = s2, v = u\}$$


> r21 := u*r1: # при u = 1/3 уход в 3-6
v1 := (u-1)^2/u:
zamproc(u, v1, v1-u, 0, 0, 1, 0, -1, r1, s1, r21, -s1):

```

$$\begin{aligned} & (-u^2 + 1) rI^2, \frac{(u+1)(3u-1)rI sI}{u}, \frac{(u^2-1)sI^2}{u}, 0 \\ & 0, 0, (u+1)^2 rI sI, 0 \end{aligned}$$

$SF_7^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {v,r1,s1,r2,s2});  
{r1=r1, r2=0, s1=-s2, s2=s2, v=2 u-2}, {r1=0, r2=r2, s1=-s2, s2=s2, v=2 u-2}
```

> v1 := 2\*(u-1): # при u = -1 уход в 3-5

```
zamproc(u,v1,v1-u,0,0,1,0,-1, 0,s1,r21,-s1):  
-r2I^2, s1 r2I (u+1), 0, 0  
0, (u-2) r2I^2, 2 s1 r2I, 0
```

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=0, r2=r2, s1=-s2, s2=s2, u=  $\frac{v}{2}$ , v=v}, {r1=-r2, r2=r2, s1=s2, s2=s2, u=-1, v=0}, {r1=r1, r2=r2, s1=-s2, s2=s2, u=-1, v=  
 $\frac{2(r1-r2)}{r2}$ }, {r1=0, r2=r2, s1=s2, s2=s2, u=0, v=0}, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}, {r1=-r2, r2=r2, s1=s1, s2=0, u=0, v=  
-1}
```

> v1 := 2\*u: # при u = -3 уход в 3-11

```
zamproc(u,v1,v1-u,0,0,1,0,-1, 0,s1,r21,-s1):  
-r2I^2, s1 r2I (u+3), -2 sI^2, 0  
0, u r2I^2, 0, 0
```

> u1 := -1:

```
r11 := (v/2+1)*r2:  
zamproc(u1,v,v-u1,0,0,1,0,-1, r11,s1,r2,-s1):  
 $\frac{(v+4)vr2^2}{4}, -\frac{(v+4)r2s1v}{2}, -(v+4)sI^2, 0$   
0,  $-\frac{(v+4)^2r2^2}{4}, 0, 0$ 
```

$SF_{12}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {v,r1,s1,r2,s2});  
{r1=-r2, r2=r2, s1=-s2 u+2 s2, s2=s2, v=2 u-2}
```

$SF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});  
solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r2,s2});  
{r1=0, r2=r2, s1=RootOf(_Z^3-3) s2, s2=s2, u=2 RootOf(_Z^3-3)+3, v=2 RootOf(_Z^3-3)}, {r1=r1, r2=0, s1=0, s2=s2, u=0, v=0}, {r1=  
-r2, r2=r2, s1=3 s2, s2=s2, u=3, v=12}  
{r2=-rI, s2=  $\frac{sI}{3}$ , u=3, v=12}, {r2=RootOf(_Z^3+3 _Z^2+_Z+1) rI, s2=-  $\frac{\text{RootOf}(_Z^3+3 _Z^2+_Z+1) sI (\text{RootOf}(_Z^3+3 _Z^2+_Z+1)+2)}{2 \text{RootOf}(_Z^3+3 _Z^2+_Z+1)+1}$ ,  
u=  $-\frac{(2 \text{RootOf}(_Z^3+3 _Z^2+_Z+1)+1) \text{RootOf}(_Z^3+3 _Z^2+_Z+1)}{\text{RootOf}(_Z^3+3 _Z^2+_Z+1)+2}, v=$   
 $-\frac{2 (\text{RootOf}(_Z^3+3 _Z^2+_Z+1)^3-1)}{(\text{RootOf}(_Z^3+3 _Z^2+_Z+1)+2) \text{RootOf}(_Z^3+3 _Z^2+_Z+1)}$  }
```

```
> z1 := sqrt(3):  
u1 := 2*z1+3;  
v1 := 2*z1;  
s11 := z1*s2:  
zamproc(u1,v1,v1-u1,0,0,1,0,-1, 0,s11,r2,s2):
```

$$\begin{aligned} & uI := 2\sqrt{3} + 3 \\ & vI := 2\sqrt{3} \\ & -r2^2, 0, 0, -\frac{2 s2^3 (3\sqrt{3} + 5)}{r2} \\ & 0, -3 r2^2, 0, 6 s2^2 (2 + \sqrt{3}) \end{aligned}$$

> z2 := -sqrt(3):

```
u2 := 2*z2+3;  
v2 := 2*z2;  
s12 := z2*s2:  
zamproc(u2,v2,v2-u2,0,0,1,0,-1, 0,s12,r2,s2):  
u2 := -2\sqrt{3} + 3
```

```

v2 := -2 √3
-r2^2, 0, 0,  $\frac{2 s2^3 (3 \sqrt{3} - 5)}{r2}$ 
0, -3 r2^2, 0, -6 s2^2 ( $\sqrt{3} - 2$ )
> solve(_Z^3+3*_Z^2+_Z+1, _Z);

$$-\frac{(27+3\sqrt{57})^{1/3}}{3} - \frac{2}{(27+3\sqrt{57})^{1/3}} - 1, \frac{(27+3\sqrt{57})^{1/3}}{6} + \frac{1}{(27+3\sqrt{57})^{1/3}} - 1$$


$$+ \frac{1\sqrt{3} \left( -\frac{(27+3\sqrt{57})^{1/3}}{3} + \frac{2}{(27+3\sqrt{57})^{1/3}} \right)}{2}, \frac{(27+3\sqrt{57})^{1/3}}{6} + \frac{1}{(27+3\sqrt{57})^{1/3}} - 1$$


$$- \frac{1\sqrt{3} \left( -\frac{(27+3\sqrt{57})^{1/3}}{3} + \frac{2}{(27+3\sqrt{57})^{1/3}} \right)}{2}$$

> rho := (27+3*sqrt(57))^^(1/3):
z1 := -(rho/3+2*rho^(-1)+1):
r21 := z1*r1:
s21 := -z1*s1*(z1+2)/(2*z1+1):
u1 := -evala((2*z1+1)*z1/(z1+2));
v1 := -evala((2*(z1^3-1))/((z1+2)*z1));
zamproc(u1,v1,v1-u1,0,0,1,0,-1, r1,s1,r21,s21):
u1 :=  $\frac{17}{3} + \frac{7(27+3\sqrt{57})^{1/3}}{3} - \frac{(27+3\sqrt{57})^{1/3}\sqrt{57}}{9} + \frac{11(27+3\sqrt{57})^{2/3}}{9} - \frac{(27+3\sqrt{57})^{2/3}\sqrt{57}}{9}$ 
v1 :=  $\frac{20}{3} + 3(27+3\sqrt{57})^{1/3} - \frac{(27+3\sqrt{57})^{1/3}\sqrt{57}}{9} + \frac{31(27+3\sqrt{57})^{2/3}}{18} - \frac{(27+3\sqrt{57})^{2/3}\sqrt{57}}{6}$ 

$$\frac{((\sqrt{57}-21)(27+3\sqrt{57})^{1/3}-24+(\sqrt{57}-11)(27+3\sqrt{57})^{2/3})rl^2}{18}, 0, 0,$$


$$- \frac{((27+3\sqrt{57})^{2/3}\sqrt{57}-19(27+3\sqrt{57})^{2/3}+5(27+3\sqrt{57})^{1/3}\sqrt{57}-57(27+3\sqrt{57})^{1/3}-84)sl^3}{81rl}$$

0,  $\frac{(4(27+3\sqrt{57})^{2/3}\sqrt{57}-46(27+3\sqrt{57})^{2/3}+5(27+3\sqrt{57})^{1/3}\sqrt{57}-93(27+3\sqrt{57})^{1/3}-192)rl^2}{18}, 0,$ 

$$- \frac{(25(27+3\sqrt{57})^{2/3}\sqrt{57}-277(27+3\sqrt{57})^{2/3}+26(27+3\sqrt{57})^{1/3}\sqrt{57}-534(27+3\sqrt{57})^{1/3}-1344)sl^2}{162}$$

> # проверка выписанного решения
rho := (3*sqrt(57)+1)^^(1/3):
u2 := (8*rho^2+(3*sqrt(57)-1)*rho+68)/12;
v2 := ((sqrt(57)+85)*rho^2+32*(sqrt(57)-1)*rho+640)/96;
r12 := (8*rho^(-1)-rho-1)*r2/3:
s12 := ((11-sqrt(57))*rho^2+4*(sqrt(57)+5)*rho+32)*s2/96:
evala(simplify(u1-u2));
simplify(v1-v2);
zamproc(u2,v2,v2-u2,0,0,1,0,-1, r12,s12,r2,s2):
u2 :=  $\frac{2(3\sqrt{57}+1)^{2/3}}{3} + \frac{(3\sqrt{57}-1)(3\sqrt{57}+1)^{1/3}}{12} + \frac{17}{3}$ 
v2 :=  $\frac{(\sqrt{57}+85)(3\sqrt{57}+1)^{2/3}}{96} + \frac{(\sqrt{57}-1)(3\sqrt{57}+1)^{1/3}}{3} + \frac{20}{3}$ 
0
0

$$- \frac{(\sqrt{57}(3\sqrt{57}+1)^{2/3}-11(3\sqrt{57}+1)^{2/3}-4(3\sqrt{57}+1)^{1/3}\sqrt{57}-20(3\sqrt{57}+1)^{1/3}+256)r2^2}{96}, 0, 0,$$


$$- \frac{(\sqrt{57}(3\sqrt{57}+1)^{2/3}-203(3\sqrt{57}+1)^{2/3}-76(3\sqrt{57}+1)^{1/3}\sqrt{57}+4(3\sqrt{57}+1)^{1/3}-1664)s2^3}{48r2}$$

0, -  $\frac{(\sqrt{57}(3\sqrt{57}+1)^{2/3}+5(3\sqrt{57}+1)^{2/3}+2(3\sqrt{57}+1)^{1/3}\sqrt{57}-22(3\sqrt{57}+1)^{1/3}+128)r2^2}{48}, 0,$ 

$$- \frac{(\sqrt{57}(3\sqrt{57}+1)^{2/3}-427(3\sqrt{57}+1)^{2/3}-160(3\sqrt{57}+1)^{1/3}\sqrt{57}+32(3\sqrt{57}+1)^{1/3}-3328)s2^2}{96}$$


```

$SF_{14}^{4,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {v,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=0,s2=s2,v=u-3}, {r1=-r2,r2=r2,s1=-2 s2
u-1,u-1,s2=s2,v=u+1}, {r1=-r2,r2=r2,s1=s1,s2=0,v=3 u-1}

> v1 := u-3:
zamproc(u,v1,v1-u,0,0,1,0,-1, r1,0,-r1,s2):
0, r1 s2 (u+3), -3 s2^2, 0
0, r1^2 (u+1), 0, -s2^2

> v1 := 3*u-1:
zamproc(u,v1,v1-u,0,0,1,0,-1, r1,s1,-r1,0):
0, 2 s1 r1, s1^2, 0
0, -r1^2 (u+1), 0, u s1^2

> v1 := u+1:
s21 := (1-u)*s1/2:
zamproc(u,v1,v1-u,0,0,1,0,-1, r1,s1,-r1,s21):
0, -(u+1) (u-3) r1 s1
2, -(3 u-1) (u-3) s1^2, 0
0, (u-3) r1^2, 0, -(u+1) (u-3) s1^2
4
```

$SF_{19}^{4,1}$

```
> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {v,r1,s1,r2,s2});
{r1=r2 u, r2=r2, s1=-s2, s2=s2, v=4 u}

> v1 := 4*u:
r11 := u*r2:
zamproc(u,v1,v1-u,0,0,1,0,-1, r11,s1,r2,-s1):
(u+1) (u^2+2 u-1) r2^2, -(u+1) (u-3) r2 s1, -2 (u+1) s1^2, 0
(u+1)^2 u r2^3
s1, 0, 0
```

$SF_{24}^{4,1}$

```
> solve([M[1,1],M[2,1],M[2,2],M[2,4]], {v,r1,s1,r2,s2});
{r1=-r2, r2=r2, s1=s2, v=2 u-2}
```

$SF_{27}^{4,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {v,r1,s1,r2,s2});
{r1=-r2, r2=r2, s1=1/2 s2 u+3/2 s2, s2=s2, v=3 u+3}

> v1 := 3*(u+1):
s11 := (u+3)*s2/2:
zamproc(u,v1,v1-u,0,0,1,0,-1, r1,s11,-r1,s2):
0, 0, (5+u) (u-3) s2^2
4, (5+u) (u+1) (u+3) s2^3
4 r1
0, -(5+u) r1^2, 0, (5+u) (u+4) (u+1) s2^2
4
```

$SF_{28}^{4,1}$

```
> evala([solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s2})]);
{r1=RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3) r2, s2=-1/138 ((-60+31 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^5
-3)+4 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^5+39 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^4
+49 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^3+111 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^2) s1), u
=105
46 + 1297 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)
46 - 176 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^5
23
- 198 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^4
- 839 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^3
- 1005 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^2, v=366
46
+ 1046 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)
- 328 RootOf(4 _Z^6+7 _Z^6+13 _Z^4+18 _Z^3-6 _Z^2-9 _Z-3)^5
23}
```

$$-\frac{438 \operatorname{RootOf}(4 \_Z^6+7 \_Z^5+13 \_Z^4+18 \_Z^3-6 \_Z^2-9 \_Z-3)^4}{23} - \frac{844 \operatorname{RootOf}(4 \_Z^6+7 \_Z^5+13 \_Z^4+18 \_Z^3-6 \_Z^2-9 \_Z-3)^3}{23}$$

$$-\frac{1098 \operatorname{RootOf}(4 \_Z^6+7 \_Z^5+13 \_Z^4+18 \_Z^3-6 \_Z^2-9 \_Z-3)^2}{23}, \left\{ r1 = -r2, s2 = \frac{sl}{3}, u = -1, v = -4 \right\}$$

$SF_{29}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,4]], {v,r1,r2,s2});
  {r1=-r2, r2=r2, s2=RootOf(_Z^2-3_Z+u+1) sl, v=RootOf(_Z^2-3_Z+u+1) u+2 RootOf(_Z^2-3_Z+u+1)-2 u-1
  RootOf(_Z^2-3_Z+u+1)}
```

> solve(\_Z^2-3\*\_Z+u+1, \_Z);

$$\frac{3}{2} + \frac{\sqrt{5-4u}}{2}, \frac{3}{2} - \frac{\sqrt{5-4u}}{2}$$

> z1 := 3/2+(1/2)\*sqrt(5-4\*u):

s21 := z1\*s1:

v1 := evala((z1\*u+2\*z1-2\*u-1)/z1);

zamproc(u,v1,v1-u,0,0,1,0,-1, r1,s1,-r1,s21):

$$v1 := \frac{2u\sqrt{5-4u} + 2u^2 + \sqrt{5-4u} + 1}{2(u+1)}$$

$$0, \frac{s1((u-2)\sqrt{5-4u} + 7u)r1}{2}, 0, \frac{(3 + \sqrt{5-4u})sl^3(3\sqrt{5-4u} - 2u + 5)}{4rl}$$

$$0, \frac{(2u^2 - 2u\sqrt{5-4u} - \sqrt{5-4u} - 5)r1^2}{2u+2}, \frac{(3u\sqrt{5-4u} + 4\sqrt{5-4u} + 13u + 10)slr1}{2u+2}, 0$$

> z2 := 3/2-(1/2)\*sqrt(5-4\*u):

s22 := z2\*s1:

v2 := evala((z2\*u+2\*z2-2\*u-1)/z2);

zamproc(u,v2,v2-u,0,0,1,0,-1, r1,s1,-r1,s22, full=false):

$$v2 := \frac{-2u\sqrt{5-4u} + 2u^2 - \sqrt{5-4u} + 1}{2(u+1)}$$

$$0, -\frac{((u-2)\sqrt{5-4u} - 7u)slr1}{2}, 0, \frac{sl^3(\sqrt{5-4u} - 3)(2u + 3\sqrt{5-4u} - 5)}{4rl}$$

$$0, \frac{(2u\sqrt{5-4u} + 2u^2 + \sqrt{5-4u} - 5)r1^2}{2u+2}, -\frac{(3u\sqrt{5-4u} + 4\sqrt{5-4u} - 13u - 10)slr1}{2u+2}, 0$$

$SF_{30}^{4,1}$

```
> solve([M[1,1],M[2,1],M[2,3],M[2,4]], {v,r1,s1,r2,s2});
  {r1=-r2, r2=r2, sl=RootOf(u_Z^2+1) s2, s2=s2, v=3 RootOf(u_Z^2+1) u - RootOf(u_Z^2+1) - u + 3
  RootOf(u_Z^2+1) - 1}
```

> z1 := sqrt(-1/u):

s11 := z1\*s2:

v1 := evala((3\*z1\*u-z1-u+3)/(z1-1));

zamproc(u,v1,v1-u,0,0,1,0,-1, r1,s11,-r1,s2):

$$v1 := -2u\sqrt{-\frac{1}{u}} + u - 1$$

$$0, \frac{s2\left(3u\sqrt{-\frac{1}{u}} + u - 2\right)(u+1)r1}{u\left(\sqrt{-\frac{1}{u}} + 1\right)}, -\frac{s2^2\left(5u\sqrt{-\frac{1}{u}} + 3u + \sqrt{-\frac{1}{u}} - 1\right)}{u\left(\sqrt{-\frac{1}{u}} + 1\right)}, \frac{s2^3(u+1)}{ur1}$$

$$0, \left(2u\sqrt{-\frac{1}{u}} + u - 1\right)r1^2, 0, 0$$

> z2 := -sqrt(-1/u):

s12 := z2\*s2:

v2 := evala((3\*z2\*u-z2-u+3)/(z2-1));

zamproc(u,v2,v2-u,0,0,1,0,-1, r1,s12,-r1,s2,full=false):

$$v2 := 2u\sqrt{-\frac{1}{u}} + u - 1$$

$$0, \frac{s2\left(3u\sqrt{-\frac{1}{u}} - u + 2\right)(u+1)r1}{u\left(\sqrt{-\frac{1}{u}} - 1\right)}, -\frac{s2^2\left(5u\sqrt{-\frac{1}{u}} - 3u + \sqrt{-\frac{1}{u}} + 1\right)}{u\left(\sqrt{-\frac{1}{u}} - 1\right)}, \frac{s2^3(u+1)}{ur1}$$

$$SF_{32}^{4,1}$$

```
> evala([solve([M[1,1],M[1,2],M[2,2],M[2,3]], {u,v,r1,s1})]);
```

$$\left[ \begin{aligned} & r1 = RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3) \, r2, s1 = \\ & \frac{(\text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3)^3 - \text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3)^2 + 3\,\text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3) + 3)\,s2}{2}, u \\ & = \frac{\text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3)^2}{2} + \frac{3}{2} - \text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3), v = -\frac{3\,\text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3)^3}{2} \\ & + 3\,\text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3)^2 - \frac{11\,\text{RootOf}(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 3\,\_Z + 3)}{2} \end{aligned} \right]$$

$$SF_{33}^{4,1}$$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,4]], {v,r1,s1,r2,s2});
```

$$\left\{ r1 = -r2, r2 = r2, s1 = \frac{s2}{u+2}, s2 = s2, v = \frac{2(u^2+2u+1)}{u+2} \right\}$$

```
> s21 := (u+2)*s1;
v1 := 2*(u+1)^2/(u+2):
zamproc(u,v1,v1-u,0,0,1,0,-1, r1,s1,-r1,s21,full=false):
0, 0, s1^2 (u+3) (u^2+2u-1),  $\frac{s1^3 (u+2) (u+3) (u+1)}{r1}$ 
0,  $-\frac{2 (u+3) r1^2}{u+2}$ ,  $\frac{(u+4) (u+3) (u+1) s1 r1}{u+2}$ , 0
```

$$SF_{36}^{4,1}$$

```
> evala([solve([M[1,1],M[1,2],M[2,2],M[2,4]], {u,v,r1,s1})]);
```

$$\left[ \begin{aligned} & \left\{ r1 = 3\,r2, s1 = -s2, u = -\frac{1}{9}, v = -\frac{4}{9} \right\}, \left\{ r1 = \text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9) \, r2, s1 = \right. \\ & \frac{(2\,\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)^3 + 3\,\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)^2 + 9)\,s2}{6}, u \\ & = \frac{2\,\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)}{3} - \frac{\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)^3}{3} - \frac{\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)^2}{6} \\ & \left. - \frac{5}{2}, v = \frac{8\,\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)}{3} - \frac{4\,\text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)^3}{3} - \text{RootOf}(2\,\_Z^4 + 3\,\_Z^3 - 3\,\_Z^2 + 9\,\_Z + 9)^2 - 7 \right] \end{aligned} \right]$$

$NSF_6^{5,1}$  ( $v \neq u$ ). Результат произвольной замены :

```
> M := zamproc(u,v,0,u-v,0,1,0,-1, r1,s1,r2,s2):
 $\frac{((u-v)s2+s1)r2^2 - r1((u-v)s2+s1)r2 + s2ur1^2)(r2+r1)}{r1s2-s1r2},$ 
 $\frac{(vr1^2+3r2^2(u-v))s2^2+3s1\left(\left(u-\frac{1}{3}\right)r1^2+\frac{2vr1r2}{3}+r2^2\right)s2-2r1s1^2r2}{r1s2-s1r2},$ 
 $\frac{3(u-v)r2s2^3+2\left(r1v+\frac{3r2}{2}\right)s1s2^2+3s1^2\left(\frac{vr2}{3}+r1\left(u-\frac{2}{3}\right)\right)s2-s1^3r2}{r1s2-s1r2}, \frac{(s2+s1)s2((u-v)s2^2-s1(u-v-1)s2+(u-1)s1^2)}{r1s2-s1r2}$ 
 $\frac{r2(r2^2(u-v)-r1(u-v-1)r2+(u-1)r1^2)(r2+r1)}{r1s2-s1r2}, \frac{-3(u-v)r2^3s2-2r1\left(vsl+\frac{3s2}{2}\right)r2^2-3r1^2\left(\frac{s2v}{3}+s1\left(u-\frac{2}{3}\right)\right)r2+r1^3s2}{r1s2-s1r2},$ 
 $\frac{(-vsl^2-3(u-v)s2^2)r2^2-3r1\left(\left(u-\frac{1}{3}\right)s1^2+\frac{2s1s2v}{3}+s2^2\right)r2+2r1^2s1s2}{r1s2-s1r2},$ 
 $\frac{((r2(u-v)+rl)s2^2-(r2(u-v)+rl)s1s2+r2usl^2)(s2+s1)}{r1s2-s1r2}$ 
```

$$SF_2^{2,1}$$

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
```

$$SF_9^{2,1}$$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=2 r2, r2=r2, s1=-s2, s2=s2, u=2/3, v=0}, {r1=0, r2=r2, s1=-s2, s2=s2, u=2, v=2}
```

$SF_5^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});  
{r1=r2, r2=r2, s1=-s2, s2=s2, u=0, v=1}, {r1=r2, r2=r2, s1=s1, s2=0, u=0, v=1}, {r1=-r2, r2=r2, s1=s2, s2=s2, u=0, v=-1}, {r1=-r2, r2=r2, s1=s1, s2=0, u=0, v=-1}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=-3 r2, r2=r2, s1=s2, s2=s2, u=0, v=-3}, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}, {r1=3 r2, r2=r2, s1=-s2, s2=s2, u=0, v=3}
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});  
{r1=-r2, r2=r2, s1=3 s2, s2=s2, u=0, v=-1}, {r1=r2, r2=r2, s1=-3 s2, s2=s2, u=0, v=1}
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=-3 r2, r2=r2, s1=s2, s2=s2, u=0, v=-3}, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}, {r1=r2, r2=r2, s1=-s2, s2=s2, u=-5/3, v=-2}, {r1=3 r2, r2=r2, s1=-s2, s2=s2, u=0, v=3}
```

```
> r21 := 2*r1:
```

```
zamproc(-5/3,-2,0,1/3,0,1,0,-1, r1,s1,r21,-s1):  
-3 r1^2, 0, -3 s1^2, 0  
0, -15 r1^2, 0, 0
```

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=-r2, r2=r2, s1=3 s2, s2=s2, u=0, v=-1}, {r1=r2, r2=r2, s1=-3 s2, s2=s2, u=0, v=1}, {r1=-r2, r2=r2, s1=s2, u=5/3, v=3/2}
```

```
> s21 := 2*s1:
```

```
zamproc(5/3,3/2,0,1/6,0,1,0,-1, r1,s1,-r1,s21):  
0, 6 r1 s1, 0, 6 s1^3/r1  
0, 0, 15 r1 s1/2, 0
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=-3 r2, r2=r2, s1=s2, s2=s2, u=0, v=-3}, {r1=3 r2, r2=r2, s1=-s2, s2=s2, u=0, v=3}, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=-r2, r2=r2, s1=3 s2, s2=s2, u=0, v=-1}, {r1=r2, r2=r2, s1=-3 s2, s2=s2, u=0, v=1}
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=r2, r2=r2, s1=s1, s2=0, u=0, v=1}, {r1=r2, r2=r2, s1=-s2, s2=s2, u=0, v=1}, {r1=-r2, r2=r2, s1=s1, s2=0, u=0, v=-1}, {r1=-r2, r2=r2, s1=s2, s2=s2, u=0, v=-1}
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=-r2, r2=r2, s1=2 s2, s2=s2, u=-2/3, v=1}, {r1=-r2, r2=r2, s1=0, s2=s2, u=-2, v=-3}
```

```
> s11 := 2*s2:
```

```
zamproc(-2/3,1,0,-5/3,0,1,0,-1, r1,s11,-r1,s2):  
0, 0, -3 s2^2, -3 s2^3/r1  
0, -6 r1^2, 0, 0
```

```
> zamproc(-2,-3,0,1,0,1,0,-1, r1,0,-r1,s2):
```

```
0, 0, -3 s2^2, s2^3/r1  
0, -2 r1^2, 0, 0
```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2}) ;
{r1=2 r2, r2=r2, s1=0, s2=s2, u=1/2, v=1/2}, {r1=-r2, r2=r2, s1=2 s2, s2=s2, u=2/3, v=0}, {r1=0, r2=r2, s1=2 s2, s2=s2, u=1/2, v=1/2},
{r1=0, r2=r2, s1=-s2, s2=s2, u=2, v=2}, {r1=2 r2, r2=r2, s1=-s2, s2=s2, u=2/3, v=0}, {r1=-r2, r2=r2, s1=0, s2=s2, u=2, v=2}
```

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2}) ;
{r1=2 r2, r2=r2, s1=-s2, s2=s2, u=5/9, v=1/3}, {r1=-3 r2, r2=r2, s1=s2, s2=s2, u=0, v=-3}, {r1=3 r2, r2=r2, s1=-s2, s2=s2, u=0, v=3},
{r1=0, r2=r2, s1=s1, s2=0, u=v, v=v}
> s21 := -4*s1:
r11 := 2*r2:
zamproc(5/9,1/3,0,2/9,0,1,0,-1, r11,s1,r2,s21):
3 r2^2, 0, -3 s1^2, 0
0, 0, 15 s1 r2, -15 s1^2
```

$SF_5^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {v,r1,s1,r2,s2}) ;
{r1=2 r2, r2=r2, s1=-s2, s2=s2, v=-3 u+2}, {r1=0, r2=r2, s1=-s2, s2=s2, v=u}
> r11 := 2*r2:
v1 := 2-3*u:
zamproc(u,v1,0,u-v1,0,1,0,-1, r11,s1,r2,-s1):
3 r2^2, 3 s1 r2, (9 u-6) s1^2, 0
0, 0, 9 u r2 s1, 0
```

$SF_7^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {v,r1,s1,r2,s2}) ;
{r1=r1, r2=0, s1=-s2, s2=s2, v=3 u/2 - 1}, {r1=r2 (u-2)/2, r2=r2, s1=-s2, s2=s2, v=3 u/2 - 1}
> v1 := (3*u-2)/2:
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s1,0,-s1):
rl^2 u, 3 r1 s1 u/2, 0, 0
0, rl^2, 2 s1 rl, 0
```

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {v,r1,s1,r2,s2}) ;
{r1=r2/2, r2=r2, s1=-s2, s2=s2, v=3 u/2 + 1/2}
> r21 := 2*r1:
v1 := (3*u+1)/2:
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s1,r21,-s1):
-3 rl^2, 3 s1 rl (3 u+5)/2, -3 s1^2, 0
0, 9 rl^2 u, 0, 0
```

$SF_{12}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {v,r1,s1,r2,s2}) ;
{r1=-r2, r2=r2, s1=-3/2 u s2+3 s2, s2=s2, v=3 u/2 - 1}
```

$SF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2}) ;
{r1=-r2, r2=r2, s1=3 s2, s2=s2, u=0, v=3}, {r1=r1, r2=0, s1=0, s2=s2, u=u, v=0}, {r1=r2, r2=r2, s1=-3 s2, s2=s2, u=0, v=-3},
{r1=-r2/4, r2=r2, s1=2 s2, s2=s2, u=35/3, v=12}
> r21 := -4*r1:
s11 := 2*s2:
zamproc(35/3,12,0,-1/3,0,1,0,-1, r1,s11,r21,s2):
-15 rl^2, 0, 0, 15 s2^3/r1
0, -63 rl^2, 0, 63 s2^2
```

$SF_{14}^{4,1}$

```

> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {v,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=RootOf((u-1)_Z^2-3+(-2*u+4)_Z) s2,s2=s2,v
 =  $\frac{3 \text{RootOf}((u-1)_Z^2-3+(-2*u+4)_Z) u-\text{RootOf}((u-1)_Z^2-3+(-2*u+4)_Z)-3 u+3}{\text{RootOf}((u-1)_Z^2-3+(-2*u+4)_Z)-3}$ }, {r1=-r2,r2=r2,s1=s1,s2=0,v=3 u
 -1}

> solve((u-1)*_Z^2-3+(-2*u+4)*_Z, _Z);
 $\frac{u-2+\sqrt{u^2-u+1}}{u-1}, -\frac{-u+2+\sqrt{u^2-u+1}}{u-1}$ 

> z1 := (u-2+sqrt(u^2-u+1))/(u-1):
s11 := z1*s2:
v1 := evala((3*z1*u-z1-3*u+3)/(z1-3));
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s11,-r1,s2, full=false):
v1 := -u-2*sqrt(u^2-u+1)+1
0,  $\frac{(4 u \sqrt{u^2-u+1}+5 u^2-2 \sqrt{u^2-u+1}-5 u-2) r1 s2}{u-1}-\frac{s2^2 (3 u^2+2 \sqrt{u^2-u+1}-7 u+2)}{(u-1)^2}, 0$ 
0,  $(4 \sqrt{u^2-u+1}+5 u-4) r1^2, 0, \frac{((2 u-4) \sqrt{u^2-u+1}+u^2-3 u+4) s2^2}{(u-1)^2}$ 

> solve(2*u-3+sqrt(u^2-u+1), u);
1

> z2 := (u-2-sqrt(u^2-u+1))/(u-1):
s12 := z2*s2:
v2 := evala((3*z2*u-z2-3*u+3)/(z2-3));
zamproc(u,v2,0,u-v2,0,1,0,-1, r1,s12,-r1,s2, full=false):
v2 := -u+2*sqrt(u^2-u+1)+1
0,  $\frac{(5 u^2-4 u \sqrt{u^2-u+1}-5 u+2 \sqrt{u^2-u+1}-2) r1 s2}{u-1}-\frac{(-3 u^2+2 \sqrt{u^2-u+1}+7 u-2) s2^2}{(u-1)^2}, 0$ 
0,  $(-4 \sqrt{u^2-u+1}+5 u-4) r1^2, 0, \frac{s2^2 ((-2 u+4) \sqrt{u^2-u+1}+u^2-3 u+4)}{(u-1)^2}$ 

> solve(2*u-3-sqrt(u^2-u+1), u);
simplify(subs(u=8/3, v2));
 $\frac{8}{3}$ 

> v1 := 3*u-1:
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s1,-r1,0):
0, 2*r1*s1, s1^2, 0
0, -3*u*r1^2, 0, u*s1^2

```

$SF_{19}^{4,1}$

```

> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1,r2=r2,s1=-s2,s2=s2,u=0,v= $\frac{r1}{r2}$ }, {r1=r1,r2=r2,s1=s1,s2=0,u=0,v= $\frac{r1}{r2}$ }, {r1=r1,r2=r2,s1=s2,s2=s2,u=0,v= $\frac{r1}{r2}$ }

```

$SF_{24}^{4,1}$

```

> solve([M[1,1],M[2,1],M[2,2],M[2,4]], {v,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1= $\frac{s2}{2}$ ,s2=s2,v= $\frac{3 u}{2}-1$ }

```

$SF_{27}^{4,1}$

```

> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {v,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1= $\frac{3}{2} s2 u+3 s2$ ,s2=s2,v=3 u+3}

```

```

> s11 := (3*u+6)*s2/2:
v1 := 3*u+3:
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s11,-r1,s2, full=false):

```

```

0, 0,  $\frac{3 (3 u+8) u s2^2}{4}, \frac{(3 u+8) (3 u+5) u s2^3}{4 r1}$ 
0, (-3 u-8) r1^2, 0,  $\frac{(3 u+2) (u+2) (3 u+8) s2^2}{4}$ 

```

$SF_{28}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2}) ;
{r1=-r2,r2=r2,s1=3 s2,s2=s2,u=0,v=-1}, {r1=r2,r2=r2,s1=-3 s2,s2=s2,u=0,v=1}, {r1=-r2/4,r2=r2,s1=2 s2,s2=s2,u=-35/3,v=-41/4}
> r21 := -4*r1;
s11 := 2*s2;
u1 := -35/3;
v1 := -41/4;
zamproc(u1,v1,0,u-v1,0,1,0,-1, r1,s11,r21,s2) :
0,  $\frac{63 s2 r1}{4}, 0, -\frac{63 s2^3}{4 r1}$ 
 $\frac{60 r1^3}{s2}, 0, 0, -60 s2^2$ 
```

$SF_{29}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,4]], {v,r1,r2,s2}) ;
{r1=-r2,r2=r2,s2=RootOf(3 _Z^2+u+1+(-2 u-4) _Z) s1,v
=(2 RootOf(3 _Z^2+u+1+(-2 u-4) _Z) u^2-RootOf(3 _Z^2+u+1+(-2 u-4) _Z) u-u^2-RootOf(3 _Z^2+u+1+(-2 u-4) _Z)
+3 u+1)/2 RootOf(3 _Z^2+u+1+(-2 u-4) _Z) u+RootOf(3 _Z^2+u+1+(-2 u-4) _Z)-u-1)}
> solve(3*_Z^2+u+1+(-2*u-4)*_Z, _Z) ;
 $\frac{u}{3} + \frac{2}{3} + \frac{\sqrt{u^2+u+1}}{3}, \frac{u}{3} + \frac{2}{3} - \frac{\sqrt{u^2+u+1}}{3}$ 
> z1 := (1/3)*u+2/3+(1/3)*sqrt(u^2+u+1);
s21 := z1*s1;
v1 := evala((2*z1*u^2-z1*u-u^2-z1+3*u+1)/(2*z1*u+z1-u-1));
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s1,r1,s21, full=false):
v1 := -  $\frac{2 u \sqrt{u^2+u+1} + u^2 - \sqrt{u^2+u+1} + 2 u}{u+1}$ 
0,  $\frac{s1 r1 ((u-4) \sqrt{u^2+u+1} + u^2 + 10 u + 4)}{3}, 0, \frac{s1^3 (u+2+\sqrt{u^2+u+1}) (2 u^2 + 2 u \sqrt{u^2+u+1} + 5 u + 4 \sqrt{u^2+u+1} - 4)}{27 r1}$ 
0,  $\frac{(5 u^2 - 4 u \sqrt{u^2+u+1} + 5 u - 2 \sqrt{u^2+u+1} - 2) r1^2}{u+1}, \frac{s1 r1 ((u+2) \sqrt{u^2+u+1} + u^2 + 4 u + 2)}{u+1}, 0$ 
> solve(u+5+sqrt(u^2+u+1), u);
> z2 := (1/3)*u+2/3-(1/3)*sqrt(u^2+u+1);
s22 := z2*s1;
v2 := evala((2*z2*u^2-z2*u-u^2-z2+3*u+1)/(2*z2*u+z2-u-1));
zamproc(u,v2,0,u-v2,0,1,0,-1, r1,s1,r1,s22, full=false):
v2 := -  $\frac{2 u \sqrt{u^2+u+1} + u^2 + \sqrt{u^2+u+1} + 2 u}{u+1}$ 
0,  $\frac{s1 ((-u+4) \sqrt{u^2+u+1} + u^2 + 10 u + 4) r1}{3}, 0, \frac{s1^3 (-u-2 + \sqrt{u^2+u+1}) (-2 u^2 + 2 u \sqrt{u^2+u+1} - 5 u + 4 \sqrt{u^2+u+1} + 4)}{27 r1}$ 
0,  $\frac{(4 u \sqrt{u^2+u+1} + 5 u^2 + 2 \sqrt{u^2+u+1} + 5 u - 2) r1^2}{u+1}, \frac{((-u-2) \sqrt{u^2+u+1} + u^2 + 4 u + 2) s1 r1}{u+1}, 0$ 
```

```
> solve(-u-5+sqrt(u^2+u+1), u);
simplify(subs(u=-8/3, v2));
```

$$\begin{aligned} & -\frac{8}{3} \\ & -5 \end{aligned}$$

$SF_{30}^{4,1}$

```
> solve([M[1,1],M[2,1],M[2,3],M[2,4]], {v,r1,s1,r2,s2}) ;
{r1=-r2,r2=r2,s1=2 s2,s2=s2,v=-3 u-1}, {r1=-r2,r2=r2,s1=0,s2=s2,v=u-1}
> s11 := 2*s2;
v1 := -3*u-1;
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s11,-r1,s2) :
0,  $(9 u + 6) s2 r1, -3 s2^2, -\frac{3 s2^3}{r1}$ 
0,  $9 r1^2 u, 0, 0$ 
```

```

> v1 := u-1:
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,0,-r1,s2):
0, r1 s2 (u+2), -3 s2^2,  $\frac{s2^3}{r1}$ 
0, r1^2 u, 0, 0
SF324,1
> solve([M[1,1],M[1,2],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2}):
{r1=2 r2, r2=r2, s1=- $\frac{s2}{4}$ , s2=s2, u=- $\frac{7}{12}$ , v= $\frac{3}{2}$ }

> r11 := 2*r2:
s21 := -4*s1:
u1 := -7/12:
v1 := 3/2:
zamproc(u1,v1,0,u1-v1,0,1,0,-1, r11,s1,r2,s21):
0, 0, -63 s1^2,  $\frac{63 s1^3}{r2}$ 
- $\frac{3 r2^3}{4 s1}$ , 0, 0,  $\frac{3 s1^2}{4}$ 

SF334,1
> solve([M[1,1],M[1,2],M[2,1],M[2,4]], {v,r1,s1,r2,s2}):
{r1=-r2, r2=r2, s1= $\frac{(u+2)s2}{2(u+1)}$ , s2=s2, v= $\frac{3u^2+4u+2}{2(u+1)}$ }

> s11 := (u+2)*s2/(2*u+2):
v1 := (3*u^2+4*u+2)/(2*(u+1)):
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s11,-r1,s2, full=false):
0, 0,  $\frac{3 s2^2 u (3 u + 4)}{4 u + 4}$ ,  $\frac{(3 u + 4) s2^3 u}{4 r1 (u + 1)^2}$ 
0,  $\frac{(-3 u - 4) r1^2}{u + 1}$ ,  $\frac{(3 u + 2) (3 u + 4) (u + 2) r1 s2}{4 (u + 1)^2}$ , 0

SF364,1
> solve([M[1,1],M[1,2],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2}):
{r1=2 r2, r2=r2, s1=- $\frac{s2}{4}$ , s2=s2, u=- $\frac{5}{9}$ , v= $\frac{17}{12}$ }, {r1=3 r2, r2=r2, s1=-s2, s2=s2, u=0, v=-1}, {r1=-3 r2, r2=r2, s1=s2, s2=s2, u=0, v=1}

> r11 := 2*r2:
s21 := -4*s1:
u1 := -5/9:
v1 := 17/12:
zamproc(u1,v1,0,u1-v1,0,1,0,-1, r11,s1,r2,s21):
0, 0, -60 s1^2,  $\frac{60 s1^3}{r2}$ 
- $\frac{3 r2^3}{4 s1}$ , 0,  $\frac{3 s1 r2}{4}$ , 0

SF35,1
> solve([M[1,4],M[2,1],M[2,3]], {v,r1,s1,r2,s2}):
{r1=r1, r2=0, s1=0, s2=s2, v=u}, {r1=-r2, r2=r2, s1=s1, s2=0, v=3 u - 1}, {r1=RootOf(2 Z^2 - 2 Z - 1) r2, r2=r2, s1=s1, s2=0, v=-RootOf(2 Z^2 - 2 Z - 1) (3 u - 1)}, {r1=-r2, r2=r2, s1= $\frac{3 s2 (\text{RootOf}(Z^2 + (2 u - 2) Z - 3 u^2 + 2 u - 3) - u + 1)}{-3 u + \text{RootOf}(Z^2 + (2 u - 2) Z - 3 u^2 + 2 u - 3) + 1}$ , s2=s2, v=RootOf(-Z^2 + (2 u - 2) Z - 3 u^2 + 2 u - 3)}, {r1= $\frac{r2}{2}$ , r2=r2, s1=-s2, s2=s2, v= $\frac{3 u}{2} + \frac{1}{2}$ }, {r1=- $\frac{r2 (\text{RootOf}((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) + u - 1)}{(u - 1) (\text{RootOf}((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) - 1)}$ , r2=r2, s1=s1, s2=RootOf((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) s1, v=(RootOf((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2)^2 u - RootOf((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) u + RootOf((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) + u - 1) | (RootOf((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) (RootOf((3 u - 3) Z^3 - u^2 + 2 u - 1 + (u^2 - 1) Z + (2 u^2 - 5 u + 5) Z^2) - 1))}

> solve(2*Z^2-2*Z-1, Z);
 $\frac{1}{2} + \frac{\sqrt{3}}{2}$ ,  $\frac{1}{2} - \frac{\sqrt{3}}{2}$ 

```

```

> z1 := 1/2+(1/2)*sqrt(3):
r11 := z1*r2:
v1 := -z1*(3*u-1);
zamproc(u,v1,0,u-v1,0,1,0,-1, r11,s1,r2,0):
  v1:=-(1/2 + sqrt(3)/2) (3 u - 1)
  r2^2*sqrt(3)/2, (1 + sqrt(3)) r2 s1, s1^2, 0
  0, -3/2 (2 + sqrt(3)) r2^2 u, 0, s1^2 u

> z2 := 1/2-(1/2)*sqrt(3):
r12 := z2*r2:
v2 := -z2*(3*u-1);
zamproc(u,v2,0,u-v2,0,1,0,-1, r12,s1,r2,0):
  v2:=-(1/2 - sqrt(3)/2) (3 u - 1)
  -r2^2*sqrt(3)/2, -(sqrt(3) - 1) r2 s1, s1^2, 0
  0, 3/2 (sqrt(3) - 2) r2^2 u, 0, s1^2 u

> solve(_Z^2+(2*u-2)*_Z-3*u^2+2*u-3-u+1, _Z);
-u+1+sqrt(4 u^2-3 u+3), -u+1-sqrt(4 u^2-3 u+3)

> z1 := -u+1+sqrt(4*u^2-3*u+3):
evala((-3*u+z1+1)/(3*(z1-u+1)));
s21 := -(2*u*sqrt(4*u^2-3*u+3)+4*u^2-9*u+1)*s1/(3*(5*u-1));
v1 := z1;
zamproc(u,v1,0,u-v1,0,1,0,-1, r1,s1,-r1,s21, full=false):
  -2 u sqrt(4 u^2-3 u+3) + 4 u^2 - 9 u + 1
  3 (5 u - 1)
  s21 := -(2 u sqrt(4 u^2-3 u+3) + 4 u^2 - 9 u + 1) s1
  15 u - 3
  v1 := -u+1+sqrt(4 u^2-3 u+3)

0, -2 (u^2+7 u-1) r1 s1 sqrt(4 u^2-3 u+3) - (4 u^3-41 u^2-19 u+4) r1 s1
  15 u - 3,
  -4 (8 u^3+4 sqrt(4 u^2-3 u+3) u^2-31 u^2-14 u sqrt(4 u^2-3 u+3) +31 u+2 sqrt(4 u^2-3 u+3)-4) u s1^2
  3 (5 u - 1)^2,
  (u-1) (8 u^3+4 sqrt(4 u^2-3 u+3) u^2-31 u^2-14 u sqrt(4 u^2-3 u+3) +31 u+2 sqrt(4 u^2-3 u+3)-4) (2 u sqrt(4 u^2-3 u+3) +4 u^2-9 u +1) s1^3
  27 r1 (5 u - 1)^3
  0, (-2 sqrt(4 u^2-3 u+3) +5 u -4) r1^2, 0, - (8 u^3+4 sqrt(4 u^2-3 u+3) u^2-23 u^2-10 u sqrt(4 u^2-3 u+3) -17 u+2 sqrt(4 u^2-3 u+3)+4) s1^2
  45 u - 9

> solve(u^2+(1/2)*u*sqrt(4*u^2-3*u+3)-6*u+1, u);
simplify(subs(u=(14+4*sqrt(10))/9, v1));
  1/5, 14/9 + 4*sqrt(10)/9
  4/3 + 2*sqrt(10)/3

> z2 := -u+1-sqrt(4*u^2-3*u+3):
evala((-3*u+z2+1)/(3*(z2-u+1)));
s22 := -(-2*u*sqrt(4*u^2-3*u+3)+4*u^2-9*u+1)*s1/(3*(5*u-1));
v2 := z2;
zamproc(u,v2,0,u-v2,0,1,0,-1, r1,s1,-r1,s22, full=false):
  -2 u sqrt(4 u^2-3 u+3) + 4 u^2 - 9 u + 1
  3 (5 u - 1)
  s22 := -(-2 u sqrt(4 u^2-3 u+3) +4 u^2 - 9 u + 1) s1
  15 u - 3
  v2 := -u+1-sqrt(4 u^2-3 u+3)

```

```

0,  $\frac{2(u^2+7u-1)r1s1\sqrt{4u^2-3u+3}-(4u^3-41u^2-19u+4)r1s1}{15u-3},$ 
 $\frac{4(-8u^3+4\sqrt{4u^2-3u+3})u^2+31u^2-14u\sqrt{4u^2-3u+3}-31u+2\sqrt{4u^2-3u+3}+4)s1l^2}{3(5u-1)^2},$ 
 $\frac{1}{27r1(5u-1)^3}((u-1)(-8u^3+4\sqrt{4u^2-3u+3})u^2+31u^2-14u\sqrt{4u^2-3u+3}-31u+2\sqrt{4u^2-3u+3}$ 
 $+4)(2u\sqrt{4u^2-3u+3}-4u^2+9u-1)s1l^3)$ 
 $0, \frac{(2\sqrt{4u^2-3u+3}+5u-4)r1l^2, 0, \frac{(-8u^3+4\sqrt{4u^2-3u+3})u^2+23u^2-10u\sqrt{4u^2-3u+3}+17u+2\sqrt{4u^2-3u+3}-4)s1l^2}{45u-9}}$ 

> solve(u^2-(1/2)*u*sqrt(4*u^2-3*u+3)-6*u+1, u);
simplify(subs(u=(14-4*sqrt(10))/9, v2));
 $\frac{14}{9}-\frac{4\sqrt{10}}{9}$ 
 $\frac{4}{3}-\frac{2\sqrt{10}}{3}$ 

> solve((3*u-3)*_Z^3-u^2+2*u-1+(u^2-1)*_Z+(2*u^2-5*u+5)*_Z^2, _Z);
z1 := (-8*u^6+87*u^5+27*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)*u^3-183*u^4-54*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)*u^2+47*u^1+3+27*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)+231*u^2-246*u+64)^(1/3)/(9*(u-1))+(4*u^4-29*u^3+54*u^2-41*u+16)/((9*(u-1))*(-8*u^6+87*u^5+27*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)*u^3-183*u^2+47*u^1+3+27*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)+231*u^2-246*u+64)^(1/3)-(2*u^2-5*u+5)/(9*(u-1)));
 $\frac{1}{9(u-1)}(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3$ 
 $+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}+(4u^4-29u^3+54u^2-41u+16)/(9(u-1)(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-\frac{2u^2-5u+5}{9(u-1)}, -\frac{1}{18(u-1)}(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}-(4u^4-29u^3$ 
 $+54u^2-41u+16)/(18(u-1)(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3})$ 
 $-\frac{2u^2-5u+5}{9(u-1)}+\frac{1}{2}\left(1\sqrt{3}\left(\frac{1}{9(u-1)}(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}-(4u^4-29u^3$ 
 $+54u^2-41u+16)/(9(u-1)(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}))$ 
 $-\frac{1}{18(u-1)}(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3$ 
 $+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}-(4u^4-29u^3+54u^2-41u+16)/(18(u-1)(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-\frac{2u^2-5u+5}{9(u-1)}-\frac{1}{2}\left(1\sqrt{3}\left(\frac{1}{9(u-1)}(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}-(4u^4-29u^3$ 
 $+54u^2-41u+16)/(9(u-1)(-8u^6+87u^5+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u^3-183u^4$ 
 $-54\sqrt{-3u^4+31u^3-81u^2+93u-44}u^2+47u^3+27\sqrt{-3u^4+31u^3-81u^2+93u-44}u+231u^2-246u+64)^{1/3}))$ 

> r11 := -r2*(z1+u-1)/((u-1)*(z1-1));
s21 := z1*s1;
v1 := (z1^2*u-z1*u+z1+u-1)/(z1*(z1-1));
#zamproc(u,v1,0,u-v1,0,1,0,-1, r11,s1,r2,s21);
> # проверка записанного решения
solve(2*(u-1)*_Z^3-(5*u-7)*_Z^2+2*(u-2)*_Z-1, _Z);
z2 := ((35*u^3+18*u*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44))-75*u^2-18*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)
+69*u-37)^(1/3)/(6*(u-1))+(13*u^2-34*u+25)/((6*(u-1))*(35*u^3+18*u*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44))
-75*u^2-18*sqrt(-3*u^4+31*u^3-81*u^2+93*u-44)+69*u-37)^(1/3)+(5*u-7)/(6*(u-1));

```

$$\begin{aligned}
& \frac{(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}}{6(u-1)} \\
& + \frac{13 u^2 - 34 u + 25}{6(u-1)(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}} + \frac{5 u - 7}{6(u-1)}, \\
& - \frac{(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}}{12(u-1)} \\
& - \frac{13 u^2 - 34 u + 25}{12(u-1)(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}} + \frac{5 u - 7}{12(u-1)} \\
& + \frac{1}{2} \left( i\sqrt{3} \left( \frac{(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}}{6(u-1)} \right. \right. \\
& - \frac{13 u^2 - 34 u + 25}{6(u-1)(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}} \Bigg) \\
& - \frac{(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}}{12(u-1)} \\
& - \frac{13 u^2 - 34 u + 25}{12(u-1)(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}} + \frac{5 u - 7}{12(u-1)} \\
& - \frac{1}{2} \left( i\sqrt{3} \left( \frac{(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}}{6(u-1)} \right. \right. \\
& - \frac{13 u^2 - 34 u + 25}{6(u-1)(35 u^3 + 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} u - 75 u^2 - 18 \sqrt{-3 u^4 + 31 u^3 - 81 u^2 + 93 u - 44} + 69 u - 37)^{1/3}} \Bigg)
\end{aligned}$$

```

> r12 := z2*r2:
s22 := (2*z2^2-2*z2-1)*s1/(3*z2):
v2 := (2*z2^3-4*z2^2+4*z2+1)/(z2-2)*(2*z2-1)*z2):
#zamproc(u,v2,0,u-v2,0,1,0,-1, r12,s1,r2,s22):
NSF75,1 (v ≠ u). Результат произвольной замены:

> M := zamproc(u,v,v-u,0,1,0,0,1, r1,s1,r2,s2):

$$\frac{((s2 u - s1) r1^2 - ((u - v) s2 - s1) r2 r1 - r2^2 s1) (r1 + r2)}{r1 s2 - s1 r2},$$


$$\frac{-2 \left( -\frac{v r1}{2} + r2 (u - v) \right) r1 s2^2 + 3 s1 \left( u r1^2 + \frac{2 v r1 r2}{3} - \frac{r2^2 (u - v + 3)}{3} \right) s2 - 3 r1^2 s1^2}{r1 s2 - s1 r2},$$


$$\frac{-r1 (u - v) s2^3 - 2 s1 \left( -v r1 + r2 \left( u - v + \frac{3}{2} \right) \right) s2^2 + s1^2 (3 r1 u + r2 v) s2 - 3 r1 s1^3}{r1 s2 - s1 r2}, \frac{(s1 + s2) s1 ((-u + v - 1) s2^2 + s1 (u + 1) s2 - s1^2)}{r1 s2 - s1 r2}$$


$$- \frac{((-u + v - 1) r2^2 + r1 (u + 1) r2 - r1^2) r1 (r1 + r2)}{r1 s2 - s1 r2}, \frac{s1 (u - v) r2^3 + 2 r1 \left( -s1 v + s2 \left( u - v + \frac{3}{2} \right) \right) r2^2 + (-3 s1 u - s2 v) r1^2 r2 + 3 r1^3 s1}{r1 s2 - s1 r2},$$


$$\frac{2 s1 \left( -\frac{s1 v}{2} + (u - v) s2 \right) r2^2 - 3 \left( u s1^2 + \frac{2 v s1 s2}{3} - \frac{s2^2 (u - v + 3)}{3} \right) r1 r2 + 3 r1^2 s1^2}{r1 s2 - s1 r2},$$


$$- \frac{(s1 + s2) ((r2 u - r1) s1^2 - s2 (r2 (u - v) - r1) s1 - r1 s2^2)}{r1 s2 - s1 r2}$$


```

SF<sub>2</sub><sup>2,1</sup>

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
```

SF<sub>9</sub><sup>2,1</sup>

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

SF<sub>3</sub><sup>3,1</sup>

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
```

$$\{r1=0, r2=r2, s1=s1, s2=-s1, u=-3, v=-3\}, \{r1=r1, r2=2 r1, s1=s1, s2=-s1, u=-\frac{3}{2}, v=0\}$$

SF<sub>5</sub><sup>3,1</sup>

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

$$\left\{ r1=r1, r2=-r1, s1=s1, s2=\frac{s1}{2}, u=0, v=3 \right\}, \{r1=r1, r2=-r1, s1=0, s2=s2, u=0, v=3\}$$

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=r1, s1=s1, s2=-s1, u=0, v=1}
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=-r1, s1=s2, s2=s2, u=0, v=3}
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=-s1, u=3, v=6}, {r1=r1, r2=2*r1, s1=s1, s2=-s1, u=3/2, v=3/2}
```

```
> zamproc(3,6,3,0,1,0,0,1, 0,s1,r2,-s1):
r2^2, 0, 3 s1^2, 0
0, 3 r2^2, 0, 0
```

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=-r1, s1=s2*(RootOf(_Z^2+2*_Z+9)+3)/6, s2=s2, u=RootOf(_Z^2+2*_Z+9)-3/2, v=RootOf(_Z^2+2*_Z+9)}
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r2, r2=r2, s1=s1, s2=-s1, u=0, v=-3}
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=-r1, s1=s2, s2=s2, u=0, v=-1}
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r2/2, r2=r2, s1=s1, s2=-s1, u=0, v=-3/2}
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1});
{r1=-r2, s1=RootOf(3_Z^2-_Z-1)s2, u=-3 RootOf(3_Z^2-_Z-1)+3, v=-9 RootOf(3_Z^2-_Z-1)+6}
```

```
> solve(3*_Z^2-_Z-1, _Z);
 $\frac{1}{6} + \frac{\sqrt{13}}{6}, \frac{1}{6} - \frac{\sqrt{13}}{6}$ 
```

```
> # частный случай ухода в 4-27
z1 := 1/6+(1/6)*sqrt(13):
s11 := z1*s2:
u1 := -3*z1+3;
v1 := -9*z1+6;
zamproc(u1,v1,v1-u1,0,1,0,0,1, r1,s11,-r1,s2):
u1 :=  $\frac{5}{2} - \frac{\sqrt{13}}{2}$ 
v1 :=  $\frac{9}{2} - \frac{3\sqrt{13}}{2}$ 
0, 0, -  $\frac{(-11+\sqrt{13})s2^2}{6}, - \frac{2(16+\sqrt{13})s2^3}{27r1}$ 
0,  $\frac{(7+\sqrt{13})r1^2}{2}, 0, 0$ 
```

```
> z2 := 1/6-(1/6)*sqrt(13):
s12 := z2*s2:
u2 := -3*z2+3:
v2 := -9*z2+6:
zamproc(u2,v2,v2-u2,0,1,0,0,1, r1,s12,-r1,s2):
0, 0,  $\frac{(11+\sqrt{13})s2^2}{6}, \frac{2(-16+\sqrt{13})s2^3}{27r1}$ 
0, -  $\frac{(-7+\sqrt{13})r1^2}{2}, 0, 0$ 
```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=2 r1, s1=s1, s2=-s1, u=- $\frac{3}{2}$ , v=0}, {r1=0, r2=r2, s1=s1, s2=-s1, u=-3, v=-3}, {r1=r1, r2= $\frac{rl}{2}$ , s1=0, s2=s2, u= $\frac{3}{2}$ , v= $\frac{3}{2}$ },
{r1=0, r2=r2, s1=s1, s2= $\frac{sl}{2}$ , u= $\frac{3}{2}$ , v= $\frac{3}{2}$ }, {r1=r1, r2=-r1, s1=s1, s2=2 sl, u=- $\frac{3}{2}$ , v=0}, {r1=r1, r2=-r1, s1=0, s2=s2, u=-3, v=-3})
```

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=r1, s1=s1, s2=-s1, u=0, v=1}
```

$SF_5^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=-s1, u=v}, {r1=r1, r2=-r1 (v-2), s1=s1, s2=-s1, u=- $\frac{-v^2+4 v-3}{v-2}$ }
```

```
> r21 := (2-v)*r1:
```

```
u1 := (v-1)*(v-3)/(v-2):
```

```
zamproc(u1,v,v-u1,0,1,0,0,1, r1,s1,r21,-s1):
```

$$\frac{(v-3)(v^2-3v+3)rl^2}{v-2}, \frac{3(v-1)(v-3)rlsl}{v-2}, \frac{(v-3)sl^2v}{v-2}, 0$$

$$0, 0, -s1rl(v-3)^2, 0$$

$SF_7^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=-s1, v=2 u+3}, {r1=r1, r2= $\frac{rl}{u+2}$ , s1=s1, s2=-s1, v=2 u+3}
```

```
> v1 := 2*u+3:
```

```
zamproc(u,v1,v1-u,0,1,0,0,1, 0,s1,r2,-s1):
```

$$r2^2, r2slu, 0, 0$$

$$0, (u+3)r2^2, -3r2sl, 0$$

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=-s1, v=2 u}, {r1=r1, r2= $\frac{rl}{u-1}$ , s1=s1, s2=-s1, v=-u+3}
```

```
> v1 := 2*u:
```

```
zamproc(u,v1,v1-u,0,1,0,0,1, 0,s1,r2,-s1):
```

$$r2^2, r2sl(u-3), 3sl^2, 0$$

$$0, r2^2u, 0, 0$$

```
> v1 := 3-u:
```

```
r11 := (u-1)*r2:
```

```
zamproc(u,v1,v1-u,0,1,0,0,1, r11,s1,r2,-s1):
```

$$r2^2u(u^2-3u+3), 2r2slu(2u-3), 3sl^2u, 0$$

$$0, r2^2u^2, 0, 0$$

$SF_{12}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {v,r1,s1,r2,s2});
{r1=r1, r2=-r1, s1= $\frac{1}{3}$  s2u+s2, s2=s2, v=2 u+3}
```

$SF_{13}^{4,1}$

```
> evala([solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1})]);
{r1=-r2, s1=s2, u=0, v=-3}, {r1=RootOf(_Z^3+12_Z+4) r2, s1= $\frac{(RootOf(_Z^3+12_Z+4)^2-RootOf(_Z^3+12_Z+4)+10)s2}{6}$ , u=
-RootOf(_Z^3+12_Z+4)^2+RootOf(_Z^3+12_Z+4)-13, v=-RootOf(_Z^3+12_Z+4)^2+RootOf(_Z^3+12_Z+4)-16}]
```

```
> solve(_Z^3+12*_Z+4, _Z);
```

$$-(2+2\sqrt{17})^{1/3} + \frac{4}{(2+2\sqrt{17})^{1/3}}, \frac{(2+2\sqrt{17})^{1/3}}{2} - \frac{2}{(2+2\sqrt{17})^{1/3}} + \frac{i\sqrt{3}\left(-(2+2\sqrt{17})^{1/3} - \frac{4}{(2+2\sqrt{17})^{1/3}}\right)}{2},$$

```


$$\frac{(2+2\sqrt{17})^{1/3}}{2} - \frac{2}{(2+2\sqrt{17})^{1/3}} - \frac{i\sqrt{3} \left( -(2+2\sqrt{17})^{1/3} - \frac{4}{(2+2\sqrt{17})^{1/3}} \right)}{2}$$


> z1 := (- (2+2*sqrt(17))^(1/3) + 4 / (2+2*sqrt(17))^(1/3)) :
r11 := z1*r2:
s11 := (1/6)*(z1^2-z1+10)*s2:
u1 := -z1^2+z1-13:
v1 := -z1^2+z1-16:
zamproc(u1,v1,v1-u1,0,1,0,0,1, r11,s11,r2,s2):

$$- \frac{3r2^2((\sqrt{17}-1)(2+2\sqrt{17})^{2/3}-8(2+2\sqrt{17})^{1/3}+2)}{2}, 0, 0,$$


$$- \frac{(11(2+2\sqrt{17})^{2/3}\sqrt{17}-267(2+2\sqrt{17})^{2/3}-128(2+2\sqrt{17})^{1/3}\sqrt{17}+40(2+2\sqrt{17})^{1/3}-1064)s2^3}{48r2}$$


$$0, - \frac{r2^2((\sqrt{17}+7)(2+2\sqrt{17})^{1/3}-26+(-\sqrt{17}+3)(2+2\sqrt{17})^{2/3})}{2}, 0,$$


$$\frac{(5(2+2\sqrt{17})^{2/3}\sqrt{17}-123(2+2\sqrt{17})^{2/3}-59(2+2\sqrt{17})^{1/3}\sqrt{17}+19(2+2\sqrt{17})^{1/3}-464)s2^2}{24}$$


> # проверка записанного решения
rho := (2+2*sqrt(17))^(1/3):
u2 := ((sqrt(17)-9)*rho^2-4*(sqrt(17)+1)*rho-40)/8:
v2 := u2-3:
r12 := (-rho+4*rho^(-1))*r2:
s12 := -(u2+3)*s2/6:
zamproc(u2,v2,v2-u2,0,1,0,0,1, r12,s12,r2,s2):

$$- \frac{3r2^2((\sqrt{17}-1)(2+2\sqrt{17})^{2/3}-8(2+2\sqrt{17})^{1/3}+2)}{2}, 0, 0,$$


$$- \frac{(11(2+2\sqrt{17})^{2/3}\sqrt{17}-267(2+2\sqrt{17})^{2/3}-128(2+2\sqrt{17})^{1/3}\sqrt{17}+40(2+2\sqrt{17})^{1/3}-1064)s2^3}{48r2}$$


$$0, - \frac{r2^2((\sqrt{17}+7)(2+2\sqrt{17})^{1/3}-26+(-\sqrt{17}+3)(2+2\sqrt{17})^{2/3})}{2}, 0,$$


$$\frac{(5(2+2\sqrt{17})^{2/3}\sqrt{17}-123(2+2\sqrt{17})^{2/3}-59(2+2\sqrt{17})^{1/3}\sqrt{17}+19(2+2\sqrt{17})^{1/3}-464)s2^2}{24}$$


> evala(u1-u2);
evala(v1-v2);
0
0

SF144,1

> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {v,r1,s1,r2});
{r1=r1, r2=-r1, s1=RootOf(_Z2+2+(-u-3)_Z) s2, v=2 RootOf(_Z2+2+(-u-3)_Z)+u-1}, {rI=r1, r2=-r1, sI=0, v=u+3}

> v1 := u+3:
zamproc(u,v1,v1-u,0,1,0,0,1, r1,0,-r1,s2):

$$0, r1 s2 (u-3), 3 s2^2, 0$$


$$0, r1^2 u, 0, s2^2$$


> solve(_Z2+2+(-u-3)*_Z, _Z);

$$\frac{u}{2} + \frac{3}{2} + \frac{\sqrt{u^2+6u+1}}{2}, \frac{u}{2} + \frac{3}{2} - \frac{\sqrt{u^2+6u+1}}{2}$$


> z1 := (1/2)*u+3/2+(1/2)*sqrt(u^2+6*u+1):
s11 := z1*s2:
v1 := 2*z1+u-1:
zamproc(u,v1,v1-u,0,1,0,0,1, r1,s11,-r1,s2, full=false):

$$v1 := 2u+2+\sqrt{u^2+6u+1}$$


$$0, - \frac{(5\sqrt{u^2+6u+1}+3u+13)r1s2}{2}, - \frac{3s2^2((u+3)\sqrt{u^2+6u+1}+u^2+6u+3)}{2}, 0$$


$$0, -(\sqrt{u^2+6u+1}-1)r1^2, 0, \frac{((u^2+6u+7)\sqrt{u^2+6u+1}+u^3+9u^2+21u+11)s2^2}{2}$$


> solve(u+5+sqrt(u^2+6*u+1), u);
subs(u=-6, v1);

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$$\begin{aligned} & \text{z2 := } (1/2)*u+3/2-(1/2)*\sqrt{u^2+6*u+1}: \\ & \text{s12 := z2*s2:} \\ & \text{v2 := } 2*z2+u-1; \\ & \text{zamproc(u,v2,v2-u,0,1,0,0,1, r1,s12,-r1,s2, full=false):} \\ & \quad v2 := 2 u + 2 - \sqrt{u^2 + 6 u + 1} \\ & \quad 0, \frac{(5 \sqrt{u^2 + 6 u + 1} - 3 u - 13) r1 s2}{2}, \frac{-3 ((-u - 3) \sqrt{u^2 + 6 u + 1} + u^2 + 6 u + 3) s2^2}{2}, 0 \\ & \quad 0, (\sqrt{u^2 + 6 u + 1} + 1) r1^2, 0, \frac{s2^2 ((-u^2 - 6 u - 7) \sqrt{u^2 + 6 u + 1} + u^3 + 9 u^2 + 21 u + 11)}{2} \\ > \text{solve}(-u-5+\sqrt{u^2+6*u+1}, u); \\ & SF_{19}^{4,1} \\ > \text{solve}([\text{M}[1,4],\text{M}[2,2],\text{M}[2,3],\text{M}[2,4]], \{u,v,r1,s1,r2,s2\}); \\ & \left\{ r1 = 0, r2 = r2, s1 = s1, s2 = \text{RootOf}(\underline{Z^2 - Z + 1}) s1, u = 0, v = 0 \right\}, \left\{ r1 = r1, r2 = r2, s1 = s1, s2 = -s1, u = 0, v = -\frac{3 r1}{r2} \right\} \\ & SF_{24}^{4,1} \\ > \text{solve}([\text{M}[1,1],\text{M}[2,1],\text{M}[2,2],\text{M}[2,4]], \{v,r1,s1,r2,s2\}); \\ & \left\{ r1 = r1, r2 = -r1, s1 = -\frac{s2}{u + 1}, s2 = s2, v = 2 u + 3 \right\} \\ & SF_{27}^{4,1} \\ > \text{solve}([\text{M}[1,1],\text{M}[1,2],\text{M}[2,1],\text{M}[2,3]], \{v,r1,s1,r2,s2\}); \\ & \left\{ r1 = r1, r2 = -r1, s1 = -\frac{1}{3} s2 u + s2, s2 = s2, v = 3 u - 3 \right\} \\ > \text{v1 := } 3*u-3: \\ & \text{s11 := } (3-u)*s2/3: \\ & \text{zamproc(u,v1,v1-u,0,1,0,0,1, r1,s11,-r1,s2, full=false):} \\ & \quad 0, 0, -\frac{(u - 6) u s2^2}{3}, \frac{4 s2^3 \left(u - \frac{3}{2}\right) (u - 3) (u - 6)}{27 r1} \\ & \quad 0, -(u - 6) r1^2, 0, \frac{(u - 6) (u^2 - 5 u + 3) s2^2}{9} \\ & SF_{28}^{4,1} \\ > \text{evala}([\text{solve}([\text{M}[1,1],\text{M}[1,3],\text{M}[2,2],\text{M}[2,3]], \{u,v,r1,s2\})]); \\ & \left[ \left\{ r1 = \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1) r2, s2 = \frac{1}{15} ((39 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^3 + 37 \text{RootOf}(9 \underline{Z^6} \right. \\ & \quad \left. + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^2 + 2 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1) + 9 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} \right. \\ & \quad \left. + 3 \underline{Z} + 1)^4 + 7) s1), u = 25 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^2 - 18 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^4 \right. \\ & \quad \left. - 24 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^3 - 16 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1) + 4, v = \right. \\ & \quad \left. - \frac{148 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)}{5} - \frac{456 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^3}{5} \right. \\ & \quad \left. + \frac{187 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^2}{5} - \frac{261 \text{RootOf}(9 \underline{Z^6} + 21 \underline{Z^4} + 4 \underline{Z^3} + 3 \underline{Z^2} + 3 \underline{Z} + 1)^4}{5} - \frac{23}{5} \right\}, \{r1 = -r2, s2 \right. \\ & \quad \left. = s1, u = 0, v = 3\} \right] \\ & SF_{29}^{4,1} \\ > \text{solve}([\text{M}[1,1],\text{M}[1,3],\text{M}[2,1],\text{M}[2,4]], \{v,r2,s2\}); \\ & \left\{ r2 = -r1, s2 = \text{RootOf}(\underline{Z^2 - 3 Z - u + 2}) s1, v = \frac{\text{RootOf}(\underline{Z^2 - 3 Z - u + 2}) u - 2 \text{RootOf}(\underline{Z^2 - 3 Z - u + 2}) - 2 u + 1}{\text{RootOf}(\underline{Z^2 - 3 Z - u + 2})} \right\} \\ > \text{solve}(\underline{Z^2 - 3 Z - u + 2}, \underline{Z}); \\ & \frac{3}{2} + \frac{\sqrt{1+4 u}}{2}, \frac{3}{2} - \frac{\sqrt{1+4 u}}{2} \\ > \text{z1 := } 3/2+(1/2)*\sqrt{1+4*u}: \\ & \text{s21 := z1*s1:} \\ & \text{v1 := evala((z1*u-2*z1-2*u+1)/z1);} \end{aligned}$$


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zamproc(u,v1,v1-u,0,1,0,0,1, r1,s1,-r1,s21, full=false):
     $v1 := \frac{-2 u \sqrt{1+4 u} + 2 u^2 + \sqrt{1+4 u} - 2 u + 5}{2 (u - 2)}$ 
     $0, \frac{s1 r1 ((u+2) \sqrt{1+4 u} + 7 u - 2)}{2}, 0, -\frac{s1^3 ((u+7) \sqrt{1+4 u} + 9 u + 11)}{2 r1}$ 
     $0, \frac{(2 u \sqrt{1+4 u} + 2 u^2 - \sqrt{1+4 u} - 17) r1^2}{2 u - 4}, -\frac{3 ((u-3) \sqrt{1+4 u} + 3 u - 3) s1 r1}{2 u - 4}, 0$ 
> z2 := 3/2-(1/2)*sqrt(1+4*u):
s22 := z2*s1:
v2 := evala((z2*u-2*z2-2*u+1)/z2);
zamproc(u,v2,v2-u,0,1,0,0,1, r1,s1,-r1,s22, full=false):
     $v2 := \frac{2 u \sqrt{1+4 u} + 2 u^2 - \sqrt{1+4 u} - 2 u + 5}{2 (u - 2)}$ 
     $0, -\frac{s1 ((u+2) \sqrt{1+4 u} - 7 u + 2) r1}{2}, 0, \frac{s1^3 ((u+7) \sqrt{1+4 u} - 9 u - 11)}{2 r1}$ 
     $0, \frac{(2 u^2 - 2 u \sqrt{1+4 u} + \sqrt{1+4 u} - 17) r1^2}{2 u - 4}, \frac{3 s1 ((u-3) \sqrt{1+4 u} - 3 u + 3) r1}{2 u - 4}, 0$ 
> solve(-5+sqrt(1+4*u), u);
simplify(subs(u=6, v2));
 $\frac{6}{15}$ 
SF304,1
> solve([M[1,1],M[2,1],M[2,3],M[2,4]], {v,r1,r2,s2});
 $\left\{ r1 = r1, r2 = -r1, s2 = RootOf(\sqrt{Z^2 - u - 1}) \right. u + \frac{RootOf(\sqrt{Z^2 - u - 1}) - 2 u - 2}{RootOf(\sqrt{Z^2 - u - 1})} \left. \right\}$ 
> solve(_Z^2-u-1, _Z);
 $\sqrt{u+1}, -\sqrt{u+1}$ 
> z1 := sqrt(u+1):
s21 := z1*s1:
v1 := evala((z1*u+z1-2*u-2)/z1);
zamproc(u,v1,v1-u,0,1,0,0,1, r1,s1,-r1,s21, full=false):
     $v1 := u - 2 \sqrt{u + 1} + 1$ 
     $0, \frac{s1 ((3 u - 2) \sqrt{u + 1} + u^2 + 2 u - 2) r1}{\sqrt{u + 1} + 1}, 3 s1^2 u, -\frac{(u + 2) \sqrt{u + 1} + u^2 + 2 u + 2}{r1 (\sqrt{u + 1} + 1)} s1^3$ 
     $0, (2 \sqrt{u + 1} + u + 2) r1^2, 0, 0$ 
> z2 := -sqrt(u+1):
s22 := z2*s1:
v2 := evala((z2*u+z2-2*u-2)/z2);
zamproc(u,v2,v2-u,0,1,0,0,1, r1,s1,-r1,s22, full=false):
     $v2 := u + 2 \sqrt{u + 1} + 1$ 
     $0, -\frac{s1 r1 ((-3 u + 2) \sqrt{u + 1} + u^2 + 2 u - 2)}{\sqrt{u + 1} - 1}, 3 s1^2 u, \frac{s1^3 ((-u - 2) \sqrt{u + 1} + u^2 + 2 u + 2)}{r1 (\sqrt{u + 1} - 1)}$ 
     $0, (-2 \sqrt{u + 1} + u + 2) r1^2, 0, 0$ 
SF324,1
> evala([solve([M[1,1],M[1,2],M[2,2],M[2,3]], {u,v,r1,s1})]);
[ {r1=RootOf(3_Z^3-Z-1) r2,s1=(3 RootOf(3_Z^3-Z-1)^2-3 RootOf(3_Z^3-Z-1)-1) s2,u=9 RootOf(3_Z^3-Z-1)^2-6,v=9 RootOf(3_Z^3-Z-1)-9} ]
> solve(3*_Z^3-_Z-1, _Z);
 $\frac{(36 + 4 \sqrt{77})^{1/3}}{6} + \frac{2}{3 (36 + 4 \sqrt{77})^{1/3}}, -\frac{(36 + 4 \sqrt{77})^{1/3}}{12} - \frac{1}{3 (36 + 4 \sqrt{77})^{1/3}} + \frac{i \sqrt{3} \left( \frac{(36 + 4 \sqrt{77})^{1/3}}{6} - \frac{2}{3 (36 + 4 \sqrt{77})^{1/3}} \right)}{2},$ 
 $-\frac{(36 + 4 \sqrt{77})^{1/3}}{12} - \frac{1}{3 (36 + 4 \sqrt{77})^{1/3}} - \frac{i \sqrt{3} \left( \frac{(36 + 4 \sqrt{77})^{1/3}}{6} - \frac{2}{3 (36 + 4 \sqrt{77})^{1/3}} \right)}{2}$ 

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> z1 := (1/6)*(36+4*sqrt(77))^^(1/3)+2/(3*(36+4*sqrt(77))^^(1/3)) :
r11 := z1*r2:
s11 := evala(3*z1^2-3*z1-1)*s2;
u1 := evala(9*z1^2-6);
v1 := evala(9*z1-9);
zamproc(u1,v1,v1-u1,0,1,0,0,1, r11,s11,r2,s2):
s11:=
$$\left( -\frac{1}{3} + \frac{(36+4\sqrt{77})^{1/3}}{4} - \frac{(36+4\sqrt{77})^{1/3}\sqrt{77}}{12} - \frac{25(36+4\sqrt{77})^{2/3}}{24} + \frac{(36+4\sqrt{77})^{2/3}\sqrt{77}}{8} \right) s2$$

u1:=-4+
$$\frac{9(36+4\sqrt{77})^{1/3}}{4} - \frac{(36+4\sqrt{77})^{1/3}\sqrt{77}}{4} + \frac{(36+4\sqrt{77})^{2/3}}{4}$$

v1:=
$$\frac{3(36+4\sqrt{77})^{1/3}}{2} + \frac{27(36+4\sqrt{77})^{2/3}}{8} - \frac{3(36+4\sqrt{77})^{2/3}\sqrt{77}}{8} - 9$$

0,0,
$$\frac{(3(36+4\sqrt{77})^{2/3}\sqrt{77} - 25(36+4\sqrt{77})^{1/3}\sqrt{77} + 6(36+4\sqrt{77})^{1/3} + 88)s2^2}{8},$$


$$\frac{3((36+4\sqrt{77})^{2/3}\sqrt{77} - 6(36+4\sqrt{77})^{1/3}\sqrt{77} + 23(36+4\sqrt{77})^{1/3} - 36)s2^3}{4r2}$$


$$-\frac{r2^3((\sqrt{77}-9)(36+4\sqrt{77})^{2/3} - 4(36+4\sqrt{77})^{1/3} - 96)}{72s2}, 0,0,$$


$$-\frac{(33(36+4\sqrt{77})^{2/3}\sqrt{77} - 227(36+4\sqrt{77})^{1/3} - 70(36+4\sqrt{77})^{1/3}\sqrt{77} + 498(36+4\sqrt{77})^{1/3} - 592)s2^2}{24}$$

SF334,1
> solve([M[1,1],M[1,2],M[2,1],M[2,4]], {v,r1,s1,r2,s2});

$$\left\{ r1=r1, r2=-r1, s1=-\frac{s2}{u-2}, s2=s2, v=\frac{2u^2-4u+3}{u-2} \right\}$$

> s21 := (2-u)*s1:
v1 := (2*u^2-4*u+3)/(u-2);
zamproc(u,v1,v1-u,0,1,0,0,1, r1,s1,-r1,s21, full=false):
v1:=
$$\frac{2u^2-4u+3}{u-2}$$

0,0,u s12 (u-2) (u-3), 
$$\frac{s1^3 (u-3) (u^2-3 u+3)}{r1}$$

0, 
$$\frac{3 (u-3) r1^2}{u-2}, -\frac{(u-3) (u^2-5 u+3) s1 r1}{u-2}, 0$$

SF364,1
> evala([solve([M[1,1],M[1,2],M[2,2],M[2,4]], {u,v,r1,s2})]);
[ {r1=RootOf(_Z4+_Z3+_Z2-_Z-1)r2,s2=-RootOf(_Z4+_Z3+_Z2-_Z-1)2s1,u=-RootOf(_Z4+_Z3+_Z2-_Z-1)3-RootOf(_Z4+_Z3+_Z2-_Z-1)+2,v=-6RootOf(_Z4+_Z3+_Z2-_Z-1)3-5RootOf(_Z4+_Z3+_Z2-_Z-1)-2RootOf(_Z4+_Z3+_Z2-_Z-1)2+8}, {r1=r2,s2=-s1,u=0,v=-1} ]
SF35,1
> evala([solve([M[1,4],M[2,1],M[2,3]], {u,r1,r2,s2})]);
[ {r1=0,r2=r2,s2=-s1,u= $\frac{v}{2}$ }, {r1=-vr2+2r2,r2=r2,s2=-s1,u=-v+3}, {r1=r2(-2RootOf(2_Z3-3+(2v+2)_Z+(-v-2)_Z2)2+RootOf(2_Z3-3+(2v+2)_Z+(-v-2)_Z2)v-4RootOf(2_Z3-3+(2v+2)_Z+(-v-2)_Z2)2/3+2RootOf(2_Z3-3+(2v+2)_Z+(-v-2)_Z2)v/3-2RootOf(2_Z3-3+(2v+2)_Z+(-v-2)_Z2)/3+2v/3-7/3), {r1=0,r2=r2,s2=RootOf(2_Z2-v+2+(-v-2)_Z)s1,u}, {r1=-r2,r2=r2,s2=RootOf(2_Z2+3+(-v-4)_Z)s1,u=v/3+4RootOf(2_Z2+3+(-v-4)_Z)/3-5/3} ]
> solve(2*_Z^3-3+(2*v+2)*_Z+(-v-2)*_Z^2, _Z);

$$\frac{(-12v^2-42v+134+v^3+6\sqrt{-3v^4+39v^3-3v^2-270v+513})^{1/3}}{6} - \frac{6\left(\frac{2}{9}v+\frac{2}{9}-\frac{1}{36}v^2\right)}{(-12v^2-42v+134+v^3+6\sqrt{-3v^4+39v^3-3v^2-270v+513})^{1/3}}$$


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$$\begin{aligned}
& + \frac{v}{6} + \frac{1}{3}, - \frac{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}}{12} \\
& + \frac{3\left(\frac{2}{9}v + \frac{2}{9} - \frac{1}{36}v^2\right)}{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}} + \frac{v}{6} + \frac{1}{3} \\
& + \frac{1}{2}\left(1\sqrt{3}\left(\frac{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}}{6}\right.\right. \\
& \left.\left. + \frac{6\left(\frac{2}{9}v + \frac{2}{9} - \frac{1}{36}v^2\right)}{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}}\right)\right), \\
& - \frac{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}}{12} \\
& + \frac{3\left(\frac{2}{9}v + \frac{2}{9} - \frac{1}{36}v^2\right)}{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}} + \frac{v}{6} + \frac{1}{3} \\
& - \frac{1}{2}\left(1\sqrt{3}\left(\frac{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}}{6}\right.\right. \\
& \left.\left. + \frac{6\left(\frac{2}{9}v + \frac{2}{9} - \frac{1}{36}v^2\right)}{(-12v^2 - 42v + 134 + v^3 + 6\sqrt{-3v^4 + 39v^3 - 3v^2 - 270v + 513})^{1/3}}\right)\right) \\
> z1 := (1/6)*(-12*v^2-42*v+134+v^3+6*sqrt(-3*v^4+39*v^3-3*v^2-270*v+513))^(1/3)-(6*((2/9)*v+2/9-(1/36)*v^2))/(-12*v^2-42*v+134+v^3+6*sqrt(-3*v^4+39*v^3-3*v^2-270*v+513))^(1/3)+(1/6)*v+1/3: \\
r11 := r2*(-2*z1^2+z1*v-2): \\
s21 := z1*s1: \\
u1 := -4*z1^2*(1/3)+2*z1*v*(1/3)-2*z1*(1/3)+2*v*(1/3)-7/3: \\
#zamproc(u1,v,v-u1,0,1,0,0,1, r11,s1,r2,s21, full=false): \\
> solve(2*_Z^2-v+2+(-v-2)*_Z, _Z); \\
\frac{v}{4} + \frac{1}{2} + \frac{\sqrt{v^2 + 12v - 12}}{4}, \frac{v}{4} + \frac{1}{2} - \frac{\sqrt{v^2 + 12v - 12}}{4} \\
> z1 := (1/4)*v+1/2+(1/4)*sqrt(v^2+12*v-12): \\
s21 := z1*s1: \\
u1 := (2*z1+v-6)*v/(2*(v-2)); \\
zamproc(u1,v,v-u1,0,1,0,0,1, 0,s1,r2,s21): \\
u1 := \frac{\left(\frac{3v}{2} - 5 + \frac{\sqrt{v^2 + 12v - 12}}{2}\right)v}{2v - 4} \\
r2^2, -\frac{s1r2(v+2+\sqrt{v^2 + 12v - 12})(v^2 - v\sqrt{v^2 + 12v - 12} - 10v + 24)}{16v - 32}, \frac{3(v+2+\sqrt{v^2 + 12v - 12})^2sl^2}{16}, 0 \\
0, \frac{v(v+2-\sqrt{v^2 + 12v - 12})r2^2}{4v - 8}, 0, \frac{vsl^2(v^2 + v\sqrt{v^2 + 12v - 12} + 6v - 24)}{8v - 16} \\
> z2 := (1/4)*v+1/2-(1/4)*sqrt(v^2+12*v-12): \\
s22 := z2*s1: \\
u2 := (2*z2+v-6)*v/(2*(v-2)); \\
zamproc(u2,v,v-u2,0,1,0,0,1, 0,s1,r2,s22): \\
u2 := \frac{\left(\frac{3v}{2} - 5 - \frac{\sqrt{v^2 + 12v - 12}}{2}\right)v}{2v - 4} \\
r2^2, \frac{s1r2(-v-2+\sqrt{v^2 + 12v - 12})(v\sqrt{v^2 + 12v - 12} + v^2 - 10v + 24)}{16v - 32}, \frac{3(-v-2+\sqrt{v^2 + 12v - 12})^2sl^2}{16}, 0 \\
0, \frac{v(v+2+\sqrt{v^2 + 12v - 12})r2^2}{4v - 8}, 0, \frac{vsl^2(v^2 - v\sqrt{v^2 + 12v - 12} + 6v - 24)}{8v - 16} \\
> \# уход в 4-14 \\
solve(2*_Z^2+3+(-v-4)*_Z, _Z);
\end{aligned}$$

$$\frac{v}{4} + 1 + \frac{\sqrt{v^2 + 8v - 8}}{4}, \frac{v}{4} + 1 - \frac{\sqrt{v^2 + 8v - 8}}{4}$$

```
> z1 := (1/4)*v+1+(1/4)*sqrt(v^2+8*v-8):
s21 := z1*s1:
u1 := (1/3)*v+4*z1*(1/3)-5/3;
solve(u = (1/3)*v+4*z1*(1/3)-5/3, v);
zamproc(u1,v,v-u1,0,1,0,0,1, r1,s1,-r1,s21):
```

$$u1 := \frac{2v}{3} - \frac{1}{3} + \frac{\sqrt{v^2 + 8v - 8}}{3}$$

$$2u + 2 + \sqrt{u^2 + 6u + 1}, 2u + 2 - \sqrt{u^2 + 6u + 1}$$

$$0, \frac{s1((v+2)\sqrt{v^2 + 8v - 8} + v^2 + 6v - 20)r1}{4}, \frac{3s1^2((v+4)\sqrt{v^2 + 8v - 8} + v^2 + 8v - 4)}{8}, 0$$

$$0, \frac{(2\sqrt{v^2 + 8v - 8} + v + 7)r1^2}{3}, 0, \frac{(3v\sqrt{v^2 + 8v - 8} + 3v^2 + 8\sqrt{v^2 + 8v - 8} + 28v + 28)s1^2}{24}$$

$SF_6^{5,1}$

```
> ([solve([M[1,3],M[2,1],M[2,3]], {u,r1,s2})]);
{r1=-r2, s2=s1, u=0}, {r1=RootOf(_Z^4+v^2+(3v^2-6v+4)_Z+(v^2-2v-3)_Z^2+(-2v+3)_Z^3) r2, s2=(s1(2RootOf(_Z^4+v^2+(3v^2-6v+4)_Z+(v^2-2v-3)_Z^2+(-2v+3)_Z^3)+v))|
```

$$(RootOf(_Z^4+v^2+(3v^2-6v+4)_Z+(v^2-2v-3)_Z^2+(-2v+3)_Z^3)^2-RootOf(_Z^4+v^2+(3v^2-6v+4)_Z+(v^2-2v-3)_Z^2+(-2v+3)_Z^3)-2), u=(RootOf(_Z^4+v^2+(3v^2-6v+4)_Z+(v^2-2v-3)_Z^2+(-2v+3)_Z^3)-v+1)|$$

$$(RootOf(_Z^4+v^2+(3v^2-6v+4)_Z+(v^2-2v-3)_Z^2+(-2v+3)_Z^3)-1)}$$

$NSF_8^{5,1}$  ( $w \neq v-u$ ). Результат произвольной замены :

```
> M := zamproc(u,v,w,0,0,1,1,0, r1,s1,r2,s2):
r1(r1^2s2u+r2(s2v-s1)r1+r2^2(ws2-s1)), (s2(3u-1)s1+ws2^2)r1^2+2r2(-s1^2+s2(v-1)s1+ws2^2)r1+r2^2s1(ws2-s1),
r1s2-r2s1
-r2s1^3+3((u-\frac{2}{3})r1+\frac{r2(v-2)}{3})s2s1^2+2((v-\frac{1}{2})r1+wr2)s2^2s1+wr1s2^3,
((u-1)s1^2+s2(v-1)s1+ws2^2)s2s1
r2r1((u-1)r1^2+r2(v-1)r1+r2^2w),
r1^3s2-3r2((u-\frac{2}{3})s1+\frac{s2(v-2)}{3})r1^2-2r2^2((v-\frac{1}{2})s1+ws2)r1-r2^3s1w,
(2s1s2+ws2^2)r1^2-3r2((u-\frac{1}{3})s1^2+\frac{2s2(v-1)s1}{3}+\frac{ws2^2}{3})r1-s1r2^2(s1v+2ws2),
(r2s1^2u+s2(r2v-r1)s1+ws2^2(wr2-r1))s1
r1s2-r2s1
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,w,r1,s1,r2,s2});
{r1=r1, r2=-\frac{r1}{2}, s1=0, s2=s2, u=-\frac{1}{2}, v=-2, w=-2}
```

```
> r11 := -2*r2:
zamproc(-1/2,-2,-2,0,0,1,1,0, r11,0,r2,s2):
```

$$0, 0, -2s2^2, 0$$

$$\frac{2r2^3}{s2}, 0, 0, 0$$

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,w,r1,s1,r2,s2});
{r1=-\frac{r2}{2}, r2=r2, s1=s1, s2=0, u=0, v=-\frac{1}{2}, w=-\frac{1}{2}}, {r1=r2, r2=r2, s1=s1, s2=-s1, u=0, v=1, w=1}, {r1=-2r2, r2=r2, s1=0, s2=s2, u=\frac{v}{4}+\frac{1}{2}, v=v, w=v}
```

```
> r11 := -2*r2:
u1 := (v+2)/4:
zamproc(u1,v,v,0,0,1,1,0, r11,0,r2,s2):
```

$$SF_{11}^{3,1}$$
  

$$\begin{aligned} & 2 r^2, 0, s2^2 v, 0 \\ & 0, 0, -s2 r2 (v+2), 0 \end{aligned}$$
  

$$\text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{v, w, r1, s1, r2, s2\});$$
  

$$\left\{ r1 = \frac{2 r2}{u-1}, r2 = r2, s1 = 0, s2 = s2, v = -2, w = \frac{2}{u-1} \right\}$$
  

$$\begin{aligned} & w1 := 2/(u-1); \\ & r21 := (u-1)*r1/2; \\ & \text{zamproc}(u, -2, w1, 0, 0, 1, 1, 0, r1, 0, r21, s2); \\ & \quad \frac{r1^2 (u+1)}{2}, 0, \frac{2 s2^2}{u-1}, 0 \\ & \quad 0, r1^2 u, 0, 0 \end{aligned}$$
  

$$SF_{17}^{3,1}$$
  

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, w, r1, s1, r2, s2\}); \\ & \{r1 = -2 r2, r2 = r2, s1 = 0, s2 = s2, u = u, v = -2, w = -2\}, \{r1 = r2, r2 = r2, s1 = s1, s2 = -s1, u = 0, v = 1, w = 1\}, \left\{ r1 = -\frac{r2}{2}, r2 = r2, s1 = s1, s2 = 0, u \right. \\ & \left. = 0, v = -\frac{1}{2}, w = -\frac{1}{2} \right\} \end{aligned}$$
  

$$\begin{aligned} & r11 := -2 * r2; \\ & \text{zamproc}(u, -2, -2, 0, 0, 1, 1, 0, r11, 0, r2, s2); \\ & \quad \frac{(4 u+2) r2^2}{s2}, 0, -2 s2^2, 0 \\ & \quad -\frac{4 r2^3 u}{s2}, 0, 0, 0 \end{aligned}$$
  

$$SF_{19}^{3,1}$$
  

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,3], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, w, r1, s1, r2, s2\}); \\ & \left\{ r1 = 0, r2 = r2, s1 = s1, s2 = -\frac{s1}{2}, u = \frac{v}{4}, v = v, w = v \right\}, \{r1 = r1, r2 = -r1, s1 = s1, s2 = s1, u = 0, v = -1, w = -1\} \end{aligned}$$
  

$$\begin{aligned} & s11 := -2 * s2; \\ & u1 := v/4; \\ & \text{zamproc}(u1, v, v, 0, 0, 1, 1, 0, 0, s11, r2, s2); \\ & \quad 0, -s2 r2 (v+2), 0, \frac{2 s2^3}{r2} \\ & \quad 0, v r2^2, 0, 0 \end{aligned}$$
  

$$SF_{21}^{3,1}$$
  

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,4], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, w, r1, s1, r2, s2\}); \\ & \{r1 = 0, r2 = r2, s1 = s1, s2 = -s1, u = 0, v = 0, w = 0\}, \{r1 = r1, r2 = -r1, s1 = s1, s2 = 0, u = 0, v = -1, w = -1\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = 0, \\ & w = 0\} \end{aligned}$$
  

$$SF_{22}^{3,1}$$
  

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[2,1], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{v, w, r1, s1, r2, s2\}); \\ & \left\{ r1 = 0, r2 = r2, s1 = s1, s2 = s1 u, v = -2, w = \frac{1}{u} \right\} \end{aligned}$$
  

$$\begin{aligned} & s21 := u * s1; \\ & w1 := 1/u; \\ & \text{zamproc}(u, -2, w1, 0, 0, 1, 1, 0, 0, s1, r2, s21); \\ & \quad 0, 0, s1^2 (2 u+1), \frac{u s1^3 (u+1)}{r2} \\ & \quad 0, \frac{r2^2}{u}, 0, 0 \end{aligned}$$
  

$$SF_1^{4,1}$$
  

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,3], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2]], \{u, v, w, r1, s1, r2, s2\}); \\ & \left\{ r1 = 0, r2 = r2, s1 = -\frac{s2}{u}, s2 = s2, u = u, v = \frac{2 u-1}{u}, w = 0 \right\}, \left\{ r1 = -\frac{r2}{u}, r2 = r2, s1 = 0, s2 = s2, u = u, v = \frac{2 u-1}{u}, w = 0 \right\}, \left\{ r1 = r1, r2 = r2, s1 = s1, s2 = \right. \\ & \left. -\frac{r2 s1}{r1+2 r2}, u = \frac{1}{2}, v = 0, w = \frac{r1 (r1+2 r2)}{2 r2^2} \right\} \end{aligned}$$
  

$$SF_3^{4,1}$$
  

$$\begin{aligned} & \text{solve}([\mathbf{M}[1,2], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,2]], \{v, w, r1, s1, r2, s2\}); \\ & \{r1 = -2 r2, r2 = r2, s1 = 0, s2 = s2, v = 4 u - 2, w = 4 u - 2\}, \left\{ r1 = -\frac{r2}{2}, r2 = r2, s1 = s1, s2 = 0, v = u - \frac{1}{2}, w = \frac{u}{4} - \frac{1}{2} \right\}, \left\{ r1 = -\frac{r2}{3 u-1}, r2 = r2, s1 \right. \\ & \left. = 0, v = u - \frac{1}{3}, w = \frac{u}{3} - \frac{1}{3} \right\} \end{aligned}$$

```

=sl,s2=  $\frac{(3 u^2 - 4 u + 1) sl}{2 u - 1}, v = \frac{2 u - 1}{3 u - 1}, w = -\frac{2 u - 1}{(3 u - 1)^2} \right\}$ 
> v1 := u-1/2:
w1 := (1/4)*u-1/2:
r21 := -2*r1:
zamproc(u,v1,w1,0,0,1,1,0, r1,sl,r21,0):
                                         -rl^2,0,sl^2,0
                                         0,0,rl*sl*u,sl^2*u

> v1 := (2*u-1)/(3*u-1):
w1 := -(2*u-1)/(3*u-1)^2:
r21 := -(3*u-1)*r1:
s21 := (3*u^2-4*u+1)*sl/(2*u-1):
zamproc(u,v1,w1,0,0,1,1,0, r1,sl,r21,s21):
                                         (-3 u + 2) rl^2, 0, (3 u - 2) sl^2, 0
                                         0, 0,  $\frac{sl \cdot rl \cdot u \cdot (3 u - 2)}{2 u - 1}, \frac{sl^2 \cdot u \cdot (3 u - 2)}{2 u - 1}$ 

SF54,1
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {w,r1,sl,r2,s2});
                                          $\left\{ r1 = rl, r2 = -\frac{rl \cdot (2 u - 1)}{v}, sl = 0, s2 = s2, w = \frac{(v u - 2 u + 1) v}{(2 u - 1)^2} \right\}$ 

> r21 := -r1*(2*u-1)/v:
w1 := (u*v-2*u+1)*v/(2*u-1)^2:
zamproc(u,v,w1,0,0,1,1,0, r1,0,r21,s2):
                                          $\frac{(-2 u + v - 1) rl^2}{v}, \frac{rl \cdot s2 \cdot (4 u - v - 2)}{2 u - 1}, \frac{s2^2 \cdot v \cdot (1 + (v - 2) u)}{(2 u - 1)^2}, 0$ 
                                         0, 0,  $\frac{rl \cdot s2 \cdot v \cdot u}{2 u - 1}, 0$ 

SF114,1
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {w,sl,r2,s2});
                                          $\left\{ r2 = -\frac{rl \cdot (u - 1)}{v}, sl = 0, s2 = s2, w = -\frac{v}{u - 1} \right\}$ 

> r21 := -r1*(u-1)/v:
w1 := -v/(u-1):
zamproc(u,v,w1,0,0,1,1,0, r1,0,r21,s2):
                                          $-\frac{rl^2 \cdot (u - v - 1)}{v}, rl \cdot s2 \cdot (v + 2), -\frac{v \cdot s2^2}{u - 1}, 0$ 
                                         0, rl^2*u, 0, 0

SF134,1
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,sl,r2});
                                          $\left\{ r1 = rl, r2 = 0, sl = -\frac{s2}{2}, u = \frac{2}{3} + \frac{4 w}{3}, v = \frac{1}{2} + 2 w \right\}, \left\{ r1 = rl, r2 = -\frac{rl \cdot RootOf(\_Z^2 - w) + 2 w}{w \cdot (RootOf(\_Z^2 - w) + 2)}, sl = RootOf(\_Z^2 - w) \cdot s2, u = -\frac{-w^2 + RootOf(\_Z^2 - w)}{w \cdot (4 \cdot RootOf(\_Z^2 - w) + w + 4)}, v = \frac{2 w + 1}{RootOf(\_Z^2 - w) + 2} \right\}$ 

> u1 := 2/3+4*w^(1/3):
v1 := 1/2+2*w:
s21 := -2*s1:
zamproc(u1,v1,w,0,0,1,1,0, r1,sl,0,s21):
                                          $\frac{4}{3} rl^2 \cdot w + \frac{2}{3} rl^2, 0, 0, \frac{2 sI^3 \cdot (2 w + 1)}{3 rl}$ 
                                         0, rl^2, 0, -sl^2

> z1 := sqrt(w):
u1 := -(-w^2+z1)/(w*(4*z1+w+4)):
v1 := (2*w+1)/(z1+2):
r21 := -r1*(z1+2*w)/(w*(z1+2)):
s11 := z1*s2:
zamproc(u1,v1,w,0,0,1,1,0, r1,s11,r21,s2):
                                          $\frac{rl^2 \cdot (w^3 + 5 w^5 \cdot \sqrt{w} + 7 w^2 - w^3 \cdot \sqrt{w}^2 - 8 w - 4 \sqrt{w})}{\sqrt{w} \cdot (w + \sqrt{w}) \cdot (\sqrt{w} + 2) \cdot (4 \sqrt{w} + w + 4)}, 0, 0, \frac{w^3 \cdot \sqrt{w} \cdot s2^3 \cdot (w^2 + 3 w^3 \cdot \sqrt{w} + w - 3 \sqrt{w} - 2)}{(4 \sqrt{w} + w + 4) \cdot rl \cdot (w + \sqrt{w})}$ 
                                         0,  $-\frac{3 rl^2 \cdot (w^3 + 7 w^5 \cdot \sqrt{w} + 20 w^2 + 31 w^3 \cdot \sqrt{w} + 29 w + 16 \sqrt{w} + 4)}{(w + \sqrt{w}) \cdot (4 \sqrt{w} + w + 4) \cdot (\sqrt{w} + 2)^2}, 0, \frac{3 s2^2 \cdot \sqrt{w} \cdot (w^3 + 5 w^5 \cdot \sqrt{w} + 10 w^2 + 11 w^3 \cdot \sqrt{w} + 7 w + 2 \sqrt{w})}{(w + \sqrt{w}) \cdot (\sqrt{w} + 2) \cdot (4 \sqrt{w} + w + 4)}$ 

```

```

> z2 := -sqrt(w):
u2 := (-w^2+z2)/(w*(4*z2+w+4)):
v2 := (2*w+1)/(z2+2):
r22 := -r1*(z2+2*w)/(w*(z2+2)):
s12 := z2*s2:
zamproc(u2,v2,w,0,0,1,1,0, r1,s12,r22,s2):

$$\frac{rl^2 (w^3 - 5 w^{5/2} + 7 w^2 + w^{3/2} - 8 w + 4 \sqrt{w})}{\sqrt{w} (w - \sqrt{w}) (\sqrt{w} - 2) (-4 \sqrt{w} + w + 4)}, 0, 0, -\frac{(w^2 - 3 w^{3/2} + w + 3 \sqrt{w} - 2) w^{3/2} s2^3}{(-4 \sqrt{w} + w + 4) rl (w - \sqrt{w})}$$


$$0, -\frac{3 rl^2 (w^3 - 7 w^{5/2} + 20 w^2 - 31 w^{3/2} + 29 w - 16 \sqrt{w} + 4)}{(w - \sqrt{w}) (-4 \sqrt{w} + w + 4) (\sqrt{w} - 2)^2}, 0, \frac{3 s2^2 \sqrt{w} (w^3 - 5 w^{5/2} + 10 w^2 - 11 w^{3/2} + 7 w - 2 \sqrt{w})}{(w - \sqrt{w}) (\sqrt{w} - 2) (-4 \sqrt{w} + w + 4)}$$


```

$SF_{14}^{4,1}$

```

> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {w,r1,s1,r2,s2});

$$\left\{ rl = 0, r2 = r2, s1 = -\frac{(v-2) s2}{2 (u-1)}, s2 = s2, w = \frac{v (v-2)}{4 (u-1)} \right\}$$


```

```

> w1 := v*(v-2)/(4*(u-1)):
s11 := -(v-2)*s2/(2*(u-1)):
zamproc(u,v,w1,0,0,1,1,0, 0,s11,r2,s2):

$$0, \frac{(-v^2 + 4) s2 r2}{4 u - 4}, -\frac{(-v - 2 + 4 u) s2^2 (v - 2)}{4 (u - 1)^2}, 0$$


$$0, \frac{v (v - 2) r2^2}{4 u - 4}, 0, -\frac{(v - 2) \left(u - \frac{v}{2}\right) s2^2}{2 (u - 1)^2}$$


```

$SF_{19}^{4,1}$

```

> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,w,r1,s1,r2,s2});

$$\{rl = wr2, r2 = r2, s1 = 0, s2 = s2, u = u, v = w, w = w\}, \{rl = wr2, r2 = r2, s1 = s1, s2 = 0, u = 0, v = w, w = w\}, \{rl = wr2, r2 = r2, s1 = s1, s2 = -s1, u = 0, v = w, w = w\}$$


```

```

> r11 := v*r2:
w1 := v:
zamproc(u,v,w1,0,0,1,1,0, r11,0,r2,s2):

$$vr2^2 (u v + v + 1), vr2 s2 (v + 2), vs2^2, 0$$


$$-\frac{r2^3 u v^2}{s2}, 0, 0, 0$$


```

$SF_{27}^{4,1}$

```

> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {v,w,r1,s1,r2,s2});

$$\{rl = 0, r2 = r2, s1 = ws2, s2 = s2, v = -2, w = w\}, \{rl = rl, r2 = -rl, s1 = s2 u + s2, s2 = s2, v = 3 u + 1, w = 2 u + 1\}$$


```

```

> s11 := w*s2:
zamproc(u,-2,w,0,0,1,1,0, 0,s11,r2,s2):

$$0, 0, ws2^2 (w + 2), -\frac{ws2^3 (-2 + (u - 1) w)}{r2}$$


$$0, r2^2 w, 0, ws2^2 (u w - 1)$$


```

$SF_{28}^{4,1}$

```

> evala([solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2})]);

$$\left[ \begin{aligned} & \{rl = rl, r2 = \frac{(6 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z)^2 + 2 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z) w + 2 w + 3) rl}{w (w + 6)}, s1 = \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z) s2, \\ & s2 = s2, u = -\frac{1}{w (w^2 + 12 w + 36)} (16 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z)^2 w + 4 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z) w^2 + w^3 \\ & + 18 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z)^2 - 2 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z) w + 14 w^2 + 30 w + 9), v \\ & = \frac{\operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z)^2 w - 2 \operatorname{RootOf}(2 \_Z^3 + w + (2 w + 1) \_Z) w + w^2 + 4 w - 3}{w + 6} \} \end{aligned} \right]$$


```

```

> solve(2*_Z^3+w+(2*w+1)*_Z, _Z);

$$\frac{(-54 w + 6 \sqrt{48 w^3 + 153 w^2 + 36 w + 6})^{1/3}}{6} - \frac{6 \left(\frac{w}{3} + \frac{1}{6}\right)}{(-54 w + 6 \sqrt{48 w^3 + 153 w^2 + 36 w + 6})^{1/3}}, -\frac{(-54 w + 6 \sqrt{48 w^3 + 153 w^2 + 36 w + 6})^{1/3}}{12}$$


$$+\frac{3 \left(\frac{w}{3} + \frac{1}{6}\right)}{(-54 w + 6 \sqrt{48 w^3 + 153 w^2 + 36 w + 6})^{1/3}}$$


```

$$\begin{aligned}
& + \frac{I\sqrt{3} \left( \frac{(-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}}{6} + \frac{6\left(\frac{w}{3}+\frac{1}{6}\right)}{(-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}} \right)}{2}, \\
& - \frac{\frac{(-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}}{12} + \frac{3\left(\frac{w}{3}+\frac{1}{6}\right)}{(-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}}}{2} \\
& - I\sqrt{3} \left( \frac{(-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}}{6} + \frac{6\left(\frac{w}{3}+\frac{1}{6}\right)}{(-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}} \right)
\end{aligned}$$

$\text{>} z1 := (1/6) * (-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3} - (6 * ((1/3)*w+1/6)) / (-54w+6\sqrt{48w^3+153w^2+36w+6})^{1/3}$   
 $\text{r21} := (6*z1^{1/3}+2*z1*w+2*w+3)*r1/(w*(w+6)):$   
 $\text{s11} := z1*s2:$   
 $\text{u1} := -(16*z1^{1/3}*w+4*z1*w^{1/3}+w^{1/3}+18*z1^{1/3}-2*z1*w+14*w^{1/3}+30*w+9)/(w*(w^{1/3}+12*w+36)):$   
 $\text{v1} := (z1^{1/3}*w-2*z1*w+w^{1/3}+2+4*w-3)/(w+6):$   
 $\text{#zamproc(u1,v1,w,0,0,1,1,0, r1,s11,r21,s2):}$   
 $SF_{29}^{4,1}$   
 $\text{>} \text{solve}([\text{M}[1,1],\text{M}[1,3],\text{M}[2,1],\text{M}[2,4]], \{w,r1,s1,r2,s2\});$   
 $\left\{ r1=0, r2=r2, s1=-\frac{s2(v+2)}{2u+1}, s2=s2, w=\frac{(v+2)(vu-2u+v)}{(2u+1)^2} \right\}$   
 $\text{>} s11 := -s2*(v+2)/(2*u+1):$   
 $\text{w1} := (v+2)*(u*v-2*u+v)/(2*u+1)^2:$   
 $\text{zamproc(u,v,w1,0,0,1,1,0, 0,s11,r2,s2):}$   
 $0, -\frac{(uv+v+1)r2(v+2)s2}{(2u+1)^2}, 0, -\frac{(2u-v-1)(v+2)s2^3}{r2(2u+1)^2}$   
 $0, \frac{((v-2)u+v)r2^2(v+2)}{(2u+1)^2}, -\frac{4r2(v+2)s2\left(u-\frac{v}{4}\right)}{(2u+1)^2}, 0$   
 $SF_{30}^{4,1}$   
 $\text{>} \text{solve}([\text{M}[1,1],\text{M}[2,1],\text{M}[2,3],\text{M}[2,4]], \{w,r1,s1,r2,s2\});$   
 $\left\{ r1=0, r2=r2, s1=-\frac{vs2}{2u}, s2=s2, w=\frac{v^2}{4u} \right\}$   
 $\text{>} s11 := -v*s2/(2*u):$   
 $\text{w1} := v^{1/2}/(4*u):$   
 $\text{zamproc(u,v,w1,0,0,1,1,0, 0,s11,r2,s2):}$   
 $0, -\frac{vs2r2(v+2)}{4u}, -\frac{vs2^2(4u-v)}{4u^2}, -\frac{vs2^3(-v+2u)}{4u^2r2}$   
 $0, \frac{v^2r2^2}{4u}, 0, 0$   
 $SF_{32}^{4,1}$   
 $\text{>} \text{evala}([\text{solve}([\text{M}[1,1],\text{M}[1,2],\text{M}[2,2],\text{M}[2,3]], \{u,w,r1,s1,s2\})])$   
 $\left[ \left\{ r1=\text{RootOf}(\text{Z}^2-2\text{Z}v-v+2) r2, s1=\frac{(-2\text{RootOf}(\text{Z}^2-2\text{Z}v-v+2)+v-2)s2}{3}, s2=s2, u \right. \right.$   
 $= \frac{-\text{RootOf}(\text{Z}^2-2\text{Z}v-v+2)v+2v^2-\text{RootOf}(\text{Z}^2-2\text{Z}v-v+2)+v+2}{3(v-2)}, w=-\text{RootOf}(\text{Z}^2-2\text{Z}v-v+2)v-\text{RootOf}(\text{Z}^2-2\text{Z}v-v+2) \Bigg]$   
 $\text{>} \text{solve}(\text{Z}^2-2\text{Z}v-v+2, \text{Z});$   
 $v+\sqrt{v^2+v-2}, v-\sqrt{v^2+v-2}$   
 $\text{>} z1 := v+\sqrt{v^2+v-2}:$   
 $r11 := z1*r2:$   
 $s11 := (1/3)*(-2*z1+v-2)*s2;$   
 $\text{u1} := (-z1*v+2*v^2-z1+v+2)/(3*(v-2));$   
 $w1 := -z1*v-z1;$   
 $\text{zamproc(u1,v,w1,0,0,1,1,0, r11,s11,r2,s2):}$   
 $s11 := \frac{(-v-2\sqrt{v^2+v-2}-2)s2}{3}$

```


$$uI := \frac{2v^2 - v(v + \sqrt{v^2 + v - 2}) - \sqrt{v^2 + v - 2} + 2}{3v - 6}$$


$$wI := -v(v + \sqrt{v^2 + v - 2}) - v - \sqrt{v^2 + v - 2}$$


$$0, 0, -\frac{(122v^3 + 121\sqrt{(v+2)(v-1)}v^2 + 183v^2 + 121v\sqrt{(v+2)(v-1)} - 111v - 26\sqrt{(v+2)(v-1)} - 86)s2^2}{36v + 45\sqrt{(v+2)(v-1)} + 18},$$


$$-\frac{s2^3((-\nu^2 - 64\nu - 43)\sqrt{(v+2)(v-1)} + \nu^3 - 57\nu^2 - 78\nu + 26)(v + 2\sqrt{(v+2)(v-1)} + 2)}{27(4\nu + 5\sqrt{(v+2)(v-1)} + 2)r2}$$


$$\frac{(9\nu^2 + 9\nu\sqrt{(v+2)(v-1)} + 11\nu + 7\sqrt{(v+2)(v-1)} - 8)(v + \sqrt{(v+2)(v-1)})r2^3}{(4\nu + 5\sqrt{(v+2)(v-1)} + 2)s2}, 0, 0,$$


$$\frac{(v+2)s2^2(v + 2\sqrt{(v+2)(v-1)} + 2)((-\nu - 23)\sqrt{(v+2)(v-1)} + \nu^2 - 17\nu - 20)}{108\nu + 135\sqrt{(v+2)(v-1)} + 54}$$


$$> z2 := v-sqrt(v^2+v-2):$$


$$r12 := z2*r2:$$


$$s12 := (1/3)*(-2*z2+v-2)*s2;$$


$$u2 := (-z2*v+2*v^2-z2+v+2)/(3*(v-2));$$


$$w2 := -z2*v-z2;$$


$$\text{zamproc}(u2, v, w2, 0, 0, 1, 1, 0, r12, s12, r2, s2):$$


$$s12 := \frac{(-v + 2\sqrt{v^2 + v - 2} - 2)s2}{3}$$


$$u2 := \frac{2v^2 - v(v - \sqrt{v^2 + v - 2}) + \sqrt{v^2 + v - 2} + 2}{3v - 6}$$


$$w2 := -v(v - \sqrt{v^2 + v - 2}) - v + \sqrt{v^2 + v - 2}$$


$$0, 0, -\frac{(-122v^3 + 121\sqrt{(v+2)(v-1)}v^2 - 183v^2 + 121v\sqrt{(v+2)(v-1)} + 111v - 26\sqrt{(v+2)(v-1)} + 86)s2^2}{-36v + 45\sqrt{(v+2)(v-1)} - 18},$$


$$-\frac{(v - 2\sqrt{(v+2)(v-1)} + 2)((\nu^2 + 64\nu + 43)\sqrt{(v+2)(v-1)} + \nu^3 - 57\nu^2 - 78\nu + 26)s2^3}{27(4\nu - 5\sqrt{(v+2)(v-1)} + 2)r2}$$


$$\frac{(9\nu^2 - 9\nu\sqrt{(v+2)(v-1)} + 11\nu - 7\sqrt{(v+2)(v-1)} - 8)(v - \sqrt{(v+2)(v-1)})r2^3}{(4\nu - 5\sqrt{(v+2)(v-1)} + 2)s2}, 0, 0,$$


$$\frac{(v - 2\sqrt{(v+2)(v-1)} + 2)((\nu + 23)\sqrt{(v+2)(v-1)} + \nu^2 - 17\nu - 20)(v + 2)s2^2}{108\nu - 135\sqrt{(v+2)(v-1)} + 54}$$


$$SF_{33}^{4,1}$$


$$> \text{solve}([\text{M}[1,1], \text{M}[1,2], \text{M}[2,1], \text{M}[2,4]], \{w, r1, s1, r2, s2\});$$


$$\left\{ r1=0, r2=r2, s1=-\frac{(\nu+1)s2}{u}, s2=s2, w=-\frac{\nu+1}{u} \right\}$$


$$> s11 := -(v+1)*s2/u;$$


$$w1 := -(v+1)/u;$$


$$\text{zamproc}(u, v, w1, 0, 0, 1, 1, 0, 0, s11, r2, s2):$$


$$0, 0, \frac{(u\nu + \nu + 1)s2^2(\nu + 1)}{u^2}, -\frac{(u - \nu - 1)s2^3(\nu + 1)}{u^2 r2}$$


$$0, -\frac{(\nu + 1)r2^2}{u}, -\frac{(\nu + 2)r2s2(\nu + 1)}{u}, 0$$


$$SF_{36}^{4,1}$$


$$> \text{solve}([\text{M}[1,1], \text{M}[1,2], \text{M}[2,2], \text{M}[2,4]], \{v, w, r1, s1, r2, s2\});$$


$$\left\{ r1 = -2r2, r2 = r2, s1 = 0, s2 = s2, v = 4u, w = 4u \right\}, \left\{ r1 = \frac{r2}{3u + 1}, r2 = r2, s1 = s1, s2 = -\frac{(3u^2 + 4u + 1)s1}{2u + 1}, v = -\frac{2u^2 + 4u + 1}{3u^2 + 4u + 1}, w = -\frac{5u^2 + 4u + 1}{(3u^2 + 4u + 1)(3u + 1)} \right\}$$


$$> r21 := (3*u+1)*r1;$$


$$s11 := -(2*u+1)*s2/(3*u^2+4*u+1);$$


$$v1 := -(2*u^2+4*u+1)/(3*u^2+4*u+1);$$


$$w1 := -(5*u^2+4*u+1)/((3*u^2+4*u+1)*(3*u+1));$$


$$\text{zamproc}(u, v1, w1, 0, 0, 1, 1, 0, r1, s11, r21, s2):$$


$$0, 0, \frac{(3u + 2)us2^2}{(3u + 1)^2(u + 1)}, -\frac{(2u + 1)s2^3u(3u + 2)}{(3u + 1)^3(u + 1)^2r1}$$


```

$\frac{rI^3 (3u+1)(3u+2)}{s2}, 0, -\frac{s2(3u+2)rI(2u+1)^2}{(3u+1)(u+1)^2}, 0$

$SF_3^{5,1}$   
 $> \text{solve}([\text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 3]], \{w, r1, s1, r2, s2\});$   
 $\left\{ r1=0, r2=r2, s1=-\frac{(v-2)s2}{2(u-1)}, s2=s2, w=\frac{v(v-2)}{4(u-1)} \right\}, \left\{ r1=-\frac{vr2}{u-1}, r2=r2, s1=0, s2=s2, w=-\frac{v}{u-1} \right\}, \left\{ r1=r1, r2=0, s1=s1, s2=-2s1, w=-\frac{u}{4}+\frac{v}{2}-\frac{1}{4} \right\}, \left\{ r1=r1, r2=-\frac{rI(3u-1)}{v}, s1=s1, s2=0, w=\frac{(2uv-3u+1)v}{(3u-1)^2} \right\}, \left\{ r1=\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)+v-1, s2=\frac{s2(\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)\_u-\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)\_v-1)}{u-1}, s2=w=\frac{1}{2u}(\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)\_u\text{v}-\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)\_u-\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)\_v+v^2+\text{RootOf}((2u^2-2u)\_Z^2+v^2-v+(3uv-3u-v+1)\_Z)-v) \right\}$   
 $> w1 := (2*v-u-1)/4;$   
 $s21 := -2*s1;$   
 $\text{zamproc}(u, v, w1, 0, 0, 1, 1, 0, r1, s1, 0, s21):$   
 $\quad rI^2 u, rI s1 (3u-2v-1), sI^2 (2u-2v-1), 0$   
 $\quad 0, rI^2, 0, -sI^2$   
 $> w1 := (2*u*v-3*u+1)*v/(3*u-1)^2;$   
 $r21 := -r1*(3*u-1)/v;$   
 $\text{zamproc}(u, v, w1, 0, 0, 1, 1, 0, r1, s1, r21, 0):$   
 $\quad \frac{(-3u+v+1)rI^2}{v}, \frac{(2v-3u+1)sI rI}{v}, sI^2, 0$   
 $\quad 0, -rI^2 u, 0, sI^2 u$   
 $> \text{solve}([\text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 3]], \{v, r1, s1, r2, s2\});$   
 $\left\{ r1=0, r2=r2, s1=s1, s2=0, v=0 \right\}, \left\{ r1=0, r2=r2, s1=s1, s2=\text{RootOf}(w\_Z^2+_Z-u+1) s1, v=-2w\text{RootOf}(w\_Z^2+_Z-u+1) \right\}, \left\{ r1=w r2, r2=r2, s1=0, s2=s2, v=-w u+w \right\}, \left\{ r1=r1, r2=0, s1=s1, s2=-2s1, v=\frac{u}{2}+2w+\frac{1}{2} \right\}, \left\{ r1=\text{RootOf}(2\_Z^2 u+_Z-w) r2, r2=r2, s1=s1, s2=0, v=\frac{3\text{RootOf}(2\_Z^2 u+_Z-w) u-3w u-\text{RootOf}(2\_Z^2 u+_Z-w)+w}{2u\text{RootOf}(2\_Z^2 u+_Z-w)} \right\}, \left\{ r1=r1, r2=\text{RootOf}(w^2\_Z^3-u+1+(-w u-w)\_Z-w\_Z^2) rI, s1=s1, s2=\frac{s1(u-1)}{w\text{RootOf}(w^2\_Z^3-u+1+(-w u-w)\_Z-w\_Z^2)}, v=\frac{\text{RootOf}(w^2\_Z^3-u+1+(-w u-w)\_Z-w\_Z^2)^2 w-\text{RootOf}(w^2\_Z^3-u+1+(-w u-w)\_Z-w\_Z^2)+u-1}{\text{RootOf}(w^2\_Z^3-u+1+(-w u-w)\_Z-w\_Z^2)} \right\}$   
 $> \# \text{solve}(w^2 Z^3-u+1+(-u*w-w)*_Z-w*_Z^2, _Z);$   
 $> z1 := (3*sqrt(-(3*(u^3*w^2+3*u^2*w^2-11*u^2*w+3*u*w^2+14*u*w+w^2-u-2*w+1))/w)*w+18*w*u-9*w+1)^(1/3)/(3*w)+(3*u*w+3*w+1)/(3*w*sqrt(-(3*(u^3*w^2+3*u^2*w^2-11*u^2*w+3*u*w^2+14*u*w+w^2-u-2*w+1))/w)*w+18*w*u-9*w+1)^(1/3))^(1/3);$   
 $r21 := z1*r1;$   
 $s21 := s1*(u-1)/(w*z1);$   
 $v1 := -(z1^2*w-z1+u-1)/z1;$   
 $\# \text{zamproc}(u, v1, w, 0, 0, 1, 1, 0, r1, s1, r21, s21);$   

$SF_6^{5,1}$   
 $> ([\text{solve}([\text{M}[1, 3], \text{M}[2, 1], \text{M}[2, 3]], \{w, r1, r2, s2\})]);$   
 $\left\{ r1=0, r2=r2, s2=-\frac{s1}{2}, w=v \right\}, \left\{ r1=r1, r2=0, s2=-2s1, w=-\frac{3u}{4}+v \right\}, \left\{ r1=r1, r2=\text{RootOf}(v^2\_Z^3+5u^2-6u+1+(-3u^2+6uv-2u-2v+1)\_Z+(2uv+v^2-2v)\_Z^2) rI, s2=-(\text{RootOf}(v^2\_Z^3+5u^2-6u+1+(-3u^2+6uv-2u-2v+1)\_Z+(2uv+v^2-2v)\_Z^2) v+3u-1) \right\}, \left\{ r1=r1, r2=\text{RootOf}(v^2\_Z^3+5u^2-6u+1+(-3u^2+6uv-2u-2v+1)\_Z+(2uv+v^2-2v)\_Z^2) +u-1 \right\}, \left\{ r1=r1, r2=\text{RootOf}(v^2\_Z^3+5u^2-6u+1+(-3u^2+6uv-2u-2v+1)\_Z+(2uv+v^2-2v)\_Z^2) \right\}$   
 $> s21 := -2*s1;$

```

w1 := -3*u*(1/4)+v:
zamproc(u,v,w1,0,0,1,1,0, r1,s1,0,s21):
    
$$rI^2 u, rI sI (3 u - 2 v - 1), 0, \frac{(-2 u + 2 v + 1) sI^3}{rI}$$

    0, rI^2, 0, -sI^2

> #solve(v^2*_Z^3+5*u^2-6*u+1+(-3*u^2+6*u*v-2*u-2*v+1)*_Z+(2*u*v+v^2-2*v)*_Z^2, _Z);
> z1 := (-35*u^3-39*u^2*v+21*u*v^2-v^3+6*u*sqrt(-27*u^4+273*u^3*v-225*u^2*v^2+63*u*v^3-3*v^4-36*u^3
-207*u^2*v+90*u*v^2-9*v^3+6*u^2+87*u*v-12*v^2+12*u-9*v-3)+33*u^2+24*u*v-3*v^2+3*u-3*v-1)^(1/3)/(3*v)+
(13*u^2-14*u*v+v^2-2*u+2*v+1)/(3*v*(-35*u^3-39*u^2*v+21*u*v^2-v^3+6*u*sqrt(-27*u^4+273*u^3*v-225*u^2*v^2+63*u*v^3-3*v^4-36*u^3
-207*u^2*v+90*u*v^2-9*v^3+6*u^2+87*u*v-12*v^2+12*u-9*v-3)+33*u^2+24*u*v-3*v^2+3*u-3*v-1)^(1/3))-
(2*u+v-2)/(3*v):
r21 := z1*r1:
s21 := -z1*s1*(z1*v+3*u-1)/(z1*v+u-1):
w1 := -(z1*v-z1+u-1)/z1^2:
#zamproc(u,v,w1,0,0,1,1,0, r1,s1,r21,s21):
SF5,1
> solve([M[1,4],M[2,2],M[2,3]], {w,r1,s2}):
{r1=- $\frac{r^2 v}{3 u - 1}, s2 = 0, w = \frac{(3 u v - 3 u + 1) v}{9 u^2 - 6 u + 1}$ } , {r1=(r2(RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+v^2-9 u-5 v+9) _Z^2) v-2 RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+9) _Z^2)+2 u-2)) / RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+9) _Z^2)), s2=RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+9) _Z^2) sI, w=-(RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+9) _Z^2) v-RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+9) _Z^2)+u-1) / RootOf((v^2-3 v+3) _Z^3+8 u^2-12 u+4+(6 u^2+6 u v-20 u-2 v+10) _Z+(5 u v+v^2-9 u-5 v+9) _Z^2)^2}
> r11 := -r2*v/(3*u-1):
w1 := (3*u*v-3*u+1)*v/(3*u-1)^2:
zamproc(u,v,w1,0,0,1,1,0, r11,s1,r2,0):
    
$$-\frac{r^2 v (3 u - v - 1)}{(3 u - 1)^2}, \frac{r2 sI (3 u - 2 v - 1)}{3 u - 1}, sI^2, 0$$

    
$$-\frac{r2^3 v^3 u}{(3 u - 1)^3 sI}, 0, 0, sI^2 u$$


> solve([M[1,4],M[2,2],M[2,3]], {u,r1,s2}):
{r1=RootOf(_Z^2-w+(-v+1) _Z) r2,s2=0,u= $\frac{\text{RootOf}(\ _Z^2-w+(-v+1) \ _Z)-v}{3 \text{RootOf}(\ _Z^2-w+(-v+1) \ _Z)}$ } , {r1=RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2) r2,s2=- $\frac{1}{3 \text{RootOf}(\ _Z^3+2 w^2+(w v-w) \ _Z+(2 v-4 w-1) \ _Z^2) w}$  (sI (RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2) v-RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2)+w)), u= $\frac{1}{9 \text{RootOf}(\ _Z^3+2 w^2+(w v-w) \ _Z+(2 v-4 w-1) \ _Z^2)^2} (13 \text{RootOf}(\ _Z^3+2 w^2+(w v-w) \ _Z+(2 v-4 w-1) \ _Z^2)^2 v$ 
-16 RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2)^2 w-RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2) v^2+4 RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2) v w-6 RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2)^2+2 RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2) v-2 RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2) w-2 w v+8 w^2-3 RootOf(_Z^3+2 w^2+(w v-w) _Z+(2 v-4 w-1) _Z^2)+3 w)}
> #solve(_Z^3+2*w^2+(v*w-w)*_Z+(2*v-v-4*w-1)*_Z^2, _Z);
> z1 := (1/6)*(456*v^2*w-912*v*w^2-492*w*v+312*w^2+132*w-64*v^3+512*w^3+96*v^2-48*v+8+12*sqrt(-12*v^4*w^4+260*v^3*w^3-48*v^2*w^4+228*v^3*w^2-1524*v^2*w^3+2832*v*w^4-1536*w^5-327*v^2*w^2+1608*v*w^3-1308*w^4+1622*v*w^2-432*w^3-27*w^2))^(1/3)- (6*((19/9)*w*v-(11/9)*w-(4/9)*v^2-(16/9)*w^2+(4/9)*v-1/9))/ (456*v^2*w-912*v*w^2-492*w*v+312*w^2+132*w-64*v^3+512*w^3+96*v^2-48*v+8+12*sqrt(-12*v^4*w^4+260*v^3*w^3-48*w^4+228*v^3*w^2-1524*v^2*w^3+2832*v*w^4-1536*w^5-327*v^2*w^2+1608*v*w^3-1308*w^4+1622*v*w^2-432*w^3-27*w^2))^(1/3)-2*v*(1/3)+4*w*(1/3)+1/3:
r11 := z1*r2:
s21 := -s1*(z1^2+2*z1*v-z1+w)/(3*z1*w):
u1 := (13*z1^2*v-16*z1^2*w-z1*v^2+4*z1*v*w-6*z1^2+2*z1*v-2*z1*w-2*w*v+8*w^2-3*z1+3*w)/(9*z1^2):
#zamproc(u1,v,w,0,0,1,1,0, r11,s1,r2,s21):

```

```

> solve( [M[1,4],M[2,2],M[2,3]], {u,v,w,r1,s1,r2,s2}) ;
{r1=w r2, r2=r2, s1=0, s2=s2, u=u, v=w, w=w}, {r1=r1, r2=r2, s1=s1, s2=0, u=u, v=-r1 (3 u-1)/r2, w=r1 (3 r1 u+r2)/r2^2}, {r1=r1, r2=r2, s1=s1, s2=-s1 (-r2^2 w+3 r1^2+r1 r2)/r1 (w r2+2 r1), u=-r2^5 w^3+3 r1^2 r2^3 w^2-3 r1^4 r2 w-3 r1^3 r2^2 w+3 r1^5+r1^3 r2^2/r1^2 (w r2+2 r1)^2 r2}, {v=-2 r2^3 w^2-4 r1^2 r2 w-r1 r2^2 w+r1^3-r1^2 r2/r1 (w r2+2 r1) r2, w=w}, {r1=0, r2=r2, s1=s1, s2=s2, u=s1+s2/s1, v=0, w=0}, {r1=-r2, r2=r2, s1=s1, s2=s2, u=s1^2-s1 s2+s2^2/s1^2, v=2 s1-3 s2/s1, w=2}

> solve(v = (2*s1-3*s2)/s1, s2)
                                         - (v-2) s1
                                         3

> simplify(subs(s2 = -(1/3)*(v-2)*s1, (s1^2-s1*s2+s2^2)/s1^2));
                                         7
                                         - 1
                                         9 v + 1
                                         9 v^2

> s21 := -(1/3)*(v-2)*s1;
u1 := 7/9-(1/9)*v+(1/9)*v^2;
zamproc(u1,v,2,0,0,1,1,0, r1,s1,-r1,s21):
                                         (v-5) r1^2 (v-2)
                                         9 , - r1 s1 (v-5)
                                         3 , - (v+1) (v-5) s1^2
                                         9 , 0
                                         - (v-5) r1^3
                                         3 s1 , 0, 0, - (v-5) s1^2
                                         3

```