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> restart; read("newlib.m"); with(mylib): with(LinearAlgebra):
Произвольная замена в исходной системе.
> M := zamproc(a1,b1,c1,d1,1,0,c2,d2, r1,s1,r2,s2):

$$\frac{(al s2 - sl) rl^3 + s2 b1 r2 rl^2 + r2^2 (cl s2 - c2 sl) rl + r2^3 (dl s2 - d2 sl)}{rl s2 - r2 sl},$$


$$\frac{(bl rl^2 + 2 cl rl r2 + 3 dl r2^2) s2^2 + 3 \left( \left( \frac{cl}{3} - d2 \right) r2^2 + \frac{2 r1 (bl - c2) r2}{3} + al rl^2 \right) sl s2 - sl^2 (c2 r2^2 + 3 rl^2)}{rl s2 - r2 sl},$$


$$\frac{(cl rl + 3 dl r2) s2^3 + 2 \left( \left( bl - \frac{c2}{2} \right) rl + r2 \left( cl - \frac{3 d2}{2} \right) \right) sl s2^2 + 3 \left( al rl + \frac{r2 (bl - 2 c2)}{3} \right) sl^2 s2 - 3 rl sl^3}{rl s2 - r2 sl},$$


$$\frac{-sl^4 + al sl^3 s2 + s2^2 (bl - c2) sl^2 + s2^3 (cl - d2) sl + dl s2^4}{rl s2 - r2 sl}$$


$$\frac{rl^4 - al rl^3 r2 - r2^2 (bl - c2) rl^2 - r2^3 (cl - d2) rl - dl r2^4}{rl s2 - r2 sl}, \quad (1)$$


$$\frac{(-cl sl - 3 dl s2) r2^3 - 2 \left( \left( bl - \frac{c2}{2} \right) sl + s2 \left( cl - \frac{3 d2}{2} \right) \right) rl r2^2 - 3 \left( al sl + \frac{s2 (bl - 2 c2)}{3} \right) rl^2 r2 + 3 rl^3 sl}{rl s2 - r2 sl},$$


$$\frac{(-bl sl^2 - 2 cl sl s2 - 3 dl s2^2) r2^2 - 3 \left( \left( \frac{cl}{3} - d2 \right) s2^2 + \frac{2 sl (bl - c2) s2}{3} + al sl^2 \right) rl r2 + rl^2 (c2 s2^2 + 3 sl^2)}{rl s2 - r2 sl},$$


$$\frac{(-al r2 + rl) sl^3 - bl r2 sl^2 s2 - s2^2 (r2 cl - rl c2) sl - s2^3 (dl r2 - d2 rl)}{rl s2 - r2 sl}$$


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Другая параметризация произвольной замены, используемая при $r_2 \neq 0, s_2 \neq 0$.

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> r11 := eta*r2:
s11 := theta*s2:
N := zamproc(a1,b1,c1,d1,1,0,c2,d2, r11,s11,r2,s2):

$$\frac{((al - \theta) \eta^3 + bl \eta^2 + (-c2 \theta + cl) \eta - d2 \theta + dl) r2^2}{\eta - \theta},$$


$$-\frac{3 \left( \left( -\eta^2 - \frac{c2}{3} \right) \theta^2 + \left( al \eta^2 + \eta \left( \frac{2 bl}{3} - \frac{2 c2}{3} \right) + \frac{cl}{3} - d2 \right) \theta + \frac{bl \eta^2}{3} + \frac{2 cl \eta}{3} + dl \right) s2 r2}{-\eta + \theta},$$


$$\frac{3 \left( -\eta \theta^3 + \left( al \eta + \frac{bl}{3} - \frac{2 c2}{3} \right) \theta^2 + \left( \left( \frac{2 bl}{3} - \frac{c2}{3} \right) \eta + \frac{2 cl}{3} - d2 \right) \theta + \frac{cl \eta}{3} + dl \right) s2^2}{\eta - \theta},$$


$$-\frac{(-\theta^4 + al \theta^3 + \theta^2 (bl - c2) + (cl - d2) \theta + dl) s2^3}{r2 (-\eta + \theta)}, \quad (2)$$


$$\frac{(-\eta^4 + al \eta^3 + (bl - c2) \eta^2 + (cl - d2) \eta + dl) r2^3}{s2 (-\eta + \theta)},$$


$$-\frac{3 \left( -\eta^3 \theta + \left( al \theta + \frac{bl}{3} - \frac{2 c2}{3} \right) \eta^2 + \left( \left( \frac{2 bl}{3} - \frac{c2}{3} \right) \theta + \frac{2 cl}{3} - d2 \right) \eta + \frac{cl \theta}{3} + dl \right) r2^2}{\eta - \theta},$$


$$\frac{3 \left( \left( -\theta^2 - \frac{c2}{3} \right) \eta^2 + \left( al \theta^2 + \left( \frac{2 bl}{3} - \frac{2 c2}{3} \right) \theta + \frac{cl}{3} - d2 \right) \eta + \frac{bl \theta^2}{3} + \frac{2 cl \theta}{3} + dl \right) s2 r2}{-\eta + \theta},$$


$$-\frac{((al - \eta) \theta^3 + bl \theta^2 + (-\eta c2 + cl) \theta - d2 \eta + dl) s2^2}{\eta - \theta}$$


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Поиск замен из исходной системы в каждую из канонических форм.

CF 2-1

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> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,3]],{a1,b1,c1,d1,c2,d2,r1,s1,r2,s2});

$$\left\{ al = \frac{rl s2 + sl r2}{r2 s2}, bl = -\frac{3 rl sl}{r2 s2}, cl = 0, c2 = -\frac{3 rl sl}{r2 s2}, dl = \frac{rl s1 (rl s2 + sl r2)}{s2^2 r2^2}, r1 = rl, r2 = r2, sl = sl, s2 = s2 \right\} \quad (1.1)$$


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```

> solve([N[1,2],N[1,3],N[1,4],N[2,1],N[2,2],N[2,3]],{a1,b1,c1,d1,c2,d2,eta,theta});

$$\left\{ al = \eta + \theta, bl = -3 \theta \eta, cl = 0, c2 = -3 \theta \eta, dl = \eta^2 \theta^2, d2 = \eta^2 \theta + \eta \theta^2, \eta = \theta, \theta = \theta \right\} \quad (1.2)$$


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> solve([a1=eta+theta, b1=-3*theta*eta],[eta,theta]);

$$\left\{ \eta = al - RootOf(3 \_Z^2 - 3 al \_Z - bl), \theta = RootOf(3 \_Z^2 - 3 al \_Z - bl) \right\} \quad (1.3)$$


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```
> solve(3*_Z^2-3*_Z*a1-b1, _Z);
```

$$\begin{aligned}
z1 &:= (1/2)*a1+(1/6)*sqrt(9*a1^2+12*b1); \\
z2 &:= (1/2)*a1-(1/6)*sqrt(9*a1^2+12*b1); \\
&\frac{a1}{2} + \frac{\sqrt{9 a1^2 + 12 b1}}{6}, \frac{a1}{2} - \frac{\sqrt{9 a1^2 + 12 b1}}{6}
\end{aligned} \tag{1.4}$$

$$\begin{aligned}
> a11 &:= \text{eta}+\theta; \\
b11 &:= -3*\theta*\text{eta}; \\
c21 &:= -3*\theta*\text{eta}; \\
d11 &:= \text{eta}^2*\theta^2; \\
d21 &:= \text{eta}^2*\theta^2+\text{eta}*\theta^2; \\
r11 &:= \text{eta}*r2; \\
s11 &:= \theta*s2; \\
\text{zamproc}(a11,b11,0,d11,1,0,c21,d21, r11,s11,r2,s2) &:= \\
&(\eta-\theta)^2 r22 \eta, 0, 0, 0 \\
&0, 0, 0, (\eta-\theta)^2 \theta s22
\end{aligned} \tag{1.5}$$

$$\begin{aligned}
> a11 &:= \text{eta}+\theta; \\
b11 &:= -3*\theta*\text{eta}; \\
c21 &:= -3*\theta*\text{eta}; \\
d11 &:= \text{eta}^2*\theta^2; \\
d21 &:= \text{eta}^2*\theta^2+\text{eta}*\theta^2; \\
r21 &:= 1/((\text{eta}-\theta)*\text{abs}(\text{eta})^{(1/2)}); \\
s21 &:= 1/((\text{eta}-\theta)*\text{abs}(\theta)^{(1/2)}); \\
r11 &:= \text{eta}*r21; \\
s11 &:= \theta*s21; \\
\text{zamproc}(a11,b11,0,d11,1,0,c21,d21, r11,s11,r21,s21) &:= \\
&\frac{\eta}{|\eta|}, 0, 0, 0 \\
&0, 0, 0, \frac{\theta}{|\theta|}
\end{aligned} \tag{1.6}$$

$$\begin{aligned}
> \text{etal} &:= a1-z1; \\
\text{thetal} &:= z1; \\
c21 &:= b1; \\
d11 &:= b1^2/9; \\
d21 &:= -a1*b1/3; \\
r21 &:= 1/((\text{etal}-\text{thetal})*\text{abs}(\text{etal})^{(1/2)}); \\
s21 &:= 1/((\text{etal}-\text{thetal})*\text{abs}(\text{thetal})^{(1/2)}); \\
r11 &:= \text{etal}*r21; \\
s11 &:= \text{thetal}*s21; \\
\text{zamproc}(a1,b1,0,d11,1,0,c21,d21, r11,s11,r21,s21) &:= \\
&\eta := \frac{a1}{2} - \frac{\sqrt{9 a1^2 + 12 b1}}{6} \\
&\theta := \frac{a1}{2} + \frac{\sqrt{9 a1^2 + 12 b1}}{6} \\
r21 &:= -\frac{3 \sqrt{2}}{\sqrt{9 a1^2 + 12 b1} \sqrt{\left| a1 - \frac{\sqrt{9 a1^2 + 12 b1}}{3} \right|}} \\
s21 &:= -\frac{3 \sqrt{2}}{\sqrt{9 a1^2 + 12 b1} \sqrt{\left| a1 + \frac{\sqrt{9 a1^2 + 12 b1}}{3} \right|}} \\
&-\frac{(3 a1^2 + 4 b1)^2 \sqrt{3} (3 a1^2 - a1 \sqrt{9 a1^2 + 12 b1} + 4 b1)}{3 \sqrt{(3 a1^2 + 4 b1)^5} \left| a1 - \frac{\sqrt{9 a1^2 + 12 b1}}{3} \right|}, 0, 0, 0 \\
&0, 0, 0, \frac{(a1 \sqrt{9 a1^2 + 12 b1} + 3 a1^2 + 4 b1) (3 a1^2 + 4 b1)^2 \sqrt{3}}{3 \sqrt{(3 a1^2 + 4 b1)^5} \left| a1 + \frac{\sqrt{9 a1^2 + 12 b1}}{3} \right|}
\end{aligned} \tag{1.7}$$

CF 2-10

$$\begin{aligned}
> \text{solve}([\text{M}[1,1],\text{M}[1,2],\text{M}[1,3],\text{M}[2,2],\text{M}[2,3],\text{M}[2,4]], \{a1,b1,c1,d1,c2,d2,r1,s1,r2,s2\}); \\
\{a1=0, b1=0, c1=0, d1=0, d2=0, r1=r1, r2=0, s1=0, s2=s2\}, \{a1=0, b1=\frac{3 r1 s1}{r2 s2}, c1=-\frac{3 r1 s1 (r1 s2 + s1 r2)}{r2^2 s22}, c2=
\end{aligned} \tag{2.1}$$

$$-\frac{3 r l s l}{r^2 s^2}, d1 = \frac{r l s l (r l^2 s 22 + r l r 2 s 2 s l + r 22 s l^2)}{r^2 s^2}, d2 = \frac{r l s l (r l s 2 + s l r 2)}{r^2 s^2}, r l = r l, r 2 = r 2, s l = s l, s 2 = s 2 \Big\}, \{a l = 0, b l = 0, c l = 0, c 2 = 0, d l = d l, d 2 = 0, r l = 0, r 2 = r 2, s l = s l, s 2 = 0\}$$

> zamproc(0,0,0,d1,1,0,0,0, r1,0,0,s2) :

$$\begin{aligned} & 0, 0, 0, \frac{d l s 23}{r l} \\ & \frac{r l^3}{s 2}, 0, 0, 0 \end{aligned} \quad (2.2)$$

> r11 := abs(d1)^(-1/8) :

s21 := r11^3:

zamproc(0,0,0,d1,1,0,0,0, r11,0,0,s21) :

$$\begin{aligned} & 0, 0, 0, \frac{d l}{|d l|} \\ & 1, 0, 0, 0 \end{aligned} \quad (2.3)$$

$$> solve([N[1,1],N[1,2],N[1,3],N[2,2],N[2,3],N[2,4]],{a1,b1,c1,d1,c2,d2,eta,theta}); \\ \{a l = 0, b l = 3 \theta \eta, c l = -3 \eta^2 \theta - 3 \eta \theta^2, c 2 = -3 \theta \eta, d l = \eta^3 \theta + \eta^2 \theta^2 + \eta \theta^3, d 2 = \eta^2 \theta + \eta \theta^2, \eta = \eta, \theta = \theta\} \quad (2.4)$$

> solve([N[1,1],N[1,2],N[1,3],N[2,2],N[2,3],N[2,4]],{a1,c1,d1,c2,eta,theta});

$$\left\{ a l = 0, c l = -3 d 2, c 2 = -b l, d l = -\frac{b l^3 - 27 d 2^2}{9 b l}, \eta = \text{RootOf}(3 _Z b l - 9 _Z d 2 + b l^2), \theta = \frac{b l^2 (\text{RootOf}(3 _Z b l - 9 _Z d 2 + b l^2) b l^3 - 54 \text{RootOf}(3 _Z b l - 9 _Z d 2 + b l^2) d 2^2 + 6 d 2 b l^2)}{-b l^5 + 27 b l^3 d 2 \text{RootOf}(3 _Z b l - 9 _Z d 2 + b l^2) + 54 b l^2 d 2^2 - 486 d 2^3 \text{RootOf}(3 _Z b l - 9 _Z d 2 + b l^2)} \right\} \quad (2.5)$$

> b11 := 3*theta*eta:

c11 := -3*eta^2*theta-3*eta*theta^2:

c21 := -3*theta*eta:

d11 := eta^3*theta+eta^2*theta^2+eta*theta^3:

d21 := eta^2*theta+eta*theta^2:

r11 := eta*r2:

s11 := theta*s2:

zamproc(0,b11,c11,d11,1,0,c21,d21, r11,s11,r2,s2) :

$$\begin{aligned} & 0, 0, 0, \frac{(\eta - \theta)^2 \theta s 23}{r 2} \\ & \frac{(\eta - \theta)^2 \eta r 23}{s 2}, 0, 0, 0 \end{aligned} \quad (2.6)$$

> b11 := 3*theta*eta:

c11 := -3*eta^2*theta-3*eta*theta^2:

c21 := -3*theta*eta:

d11 := eta^3*theta+eta^2*theta^2+eta*theta^3:

d21 := eta^2*theta+eta*theta^2:

r21 := abs(eta-theta)^(-1)*abs(theta*eta^3)^(-1/8) :

s21 := (eta-theta)^2*eta*r21^3:

r11 := eta*r21:

s11 := theta*s21:

zamproc(0,b11,c11,d11,1,0,c21,d21, r11,s11,r21,s21) :

$$\begin{aligned} & 0, 0, 0, \frac{(\eta - \theta)^8 \theta \eta^3}{|\eta - \theta|^8 |\eta|^3 |\theta|} \\ & 1, 0, 0, 0 \end{aligned} \quad (2.7)$$

> solve([b1=3*theta*eta, d2=eta^2*theta+eta*theta^2], {eta,theta});

$$\left\{ \eta = \text{RootOf}(3 b l _Z - 9 d 2 _Z + b l^2), \theta = \frac{b l}{3 \text{RootOf}(3 b l _Z - 9 d 2 _Z + b l^2)} \right\} \quad (2.8)$$

> solve(3*_Z^2*b1-9*_Z*d2+b1^2, _Z);

z1 := (9*d2+sqrt(-12*b1^3+81*d2^2))/(6*b1):

z2 := (-9*d2+sqrt(-12*b1^3+81*d2^2))/(6*b1):

$$\frac{9 d 2 + \sqrt{-12 b l^3 + 81 d 2^2}}{6 b l}, -\frac{-9 d 2 + \sqrt{-12 b l^3 + 81 d 2^2}}{6 b l} \quad (2.9)$$

> etal := z1;

thetal := evala(b1/(3*z1));

c11 := -3*d2:

c21 := -b1:

d11 := -(b1^3-27*d2^2)/(9*b1):

r21 := evala(abs(etal-thetal)^(-1)*abs(thetal*etal^3)^(-1/8));

s21 := (etal-thetal)^2*etal*r21^3;

r11 := etal*r21:

s11 := theta*etal*s21:

$$\begin{aligned}
\text{zamproc}(0, b1, c11, d11, 1, 0, c21, d2, r11, s11, r21, s21) : \\
\eta l := \frac{9d2 + \sqrt{-12b1^3 + 81d2^2}}{6b1} \\
\theta l := \frac{-\sqrt{-12b1^3 + 81d2^2} + 9d2}{6b1} \\
r21 := \frac{3|b1|\sqrt{2}}{\sqrt{|4b1^3 - 27d2^2|} \left(\frac{|-\sqrt{-12b1^3 + 81d2^2} + 9d2| |9d2 + \sqrt{-12b1^3 + 81d2^2}|^3}{|b1|^4} \right)^{1/8}} \\
s21 := \frac{9 \left(\frac{9d2 + \sqrt{-12b1^3 + 81d2^2}}{6b1} - \frac{-\sqrt{-12b1^3 + 81d2^2} + 9d2}{6b1} \right)^2 (9d2 + \sqrt{-12b1^3 + 81d2^2}) |b1|^3 \sqrt{2}}{b1 |4b1^3 - 27d2^2|^3/2 \left(\frac{|-\sqrt{-12b1^3 + 81d2^2} + 9d2| |9d2 + \sqrt{-12b1^3 + 81d2^2}|^3}{|b1|^4} \right)^{3/8}} \\
0, 0, 0, -\frac{\sqrt{3} (9d2 + \sqrt{-12b1^3 + 81d2^2})^3 (4b1^3 - 27d2^2) (4b1^3 - 27d2^2 + 3d2\sqrt{-12b1^3 + 81d2^2}) |b1|^2}{b1^{12} \sqrt{-(4b1^3 - 27d2^2)^3} |9d2 + \sqrt{-12b1^3 + 81d2^2}|^3 |4b1^3 - 27d2^2|^4 | -9d2 + \sqrt{-12b1^3 + 81d2^2} |} \\
\frac{(4b1^3 - 27d2^2)\sqrt{3} (4b1^3 - 3d2\sqrt{-12b1^3 + 81d2^2} - 27d2^2)}{\sqrt{-(4b1^3 - 27d2^2)^3} (9d2 + \sqrt{-12b1^3 + 81d2^2})}, 0, 0, 0
\end{aligned} \tag{2.10}$$

$$> \text{simplify}(\text{eval}((\text{eta1}-\text{theta1})^2 * \text{eta1})) * \text{r2}^3; \\
\text{s2} = -\frac{(4b1^3 - 27d2^2) (9d2 + \sqrt{-12b1^3 + 81d2^2}) r2^3}{18b1^3} \tag{2.11}$$

CF 3-1

$$\begin{aligned}
> \text{solve}([\text{M}[1, 3], \text{M}[1, 4], \text{M}[2, 1], \text{M}[2, 2], \text{M}[2, 3]], \{\text{a1}, \text{b1}, \text{c1}, \text{d1}, \text{c2}, \text{d2}, \text{r1}, \text{s1}, \text{r2}, \text{s2}\}); \\
\left\{ \begin{array}{l} \text{a1} = \text{a1}, \text{b1} = -\frac{3rl(\text{a1r2}-\text{rl})}{r2^2}, \text{c1} = \frac{3slrl(\text{a1r2s2}-\text{rls2}-\text{r2s1})}{s2^2r2^2}, \text{c2} = \\ -\frac{3(\text{a1rlr2s22}-\text{a1r22s1s2}-\text{rl2s22}+\text{rlr2s1s2}+\text{r22s1s2})}{r2^2s2^2}, \text{d1} \\ = \frac{s1rl(\text{a1rlr2s22}-2\text{a1r22s1s2}-\text{rl2s22}+2\text{rlr2s1s2}+2\text{r22s1s2})}{s2^3r2^3}, \text{d2} \\ = \frac{\text{a1rl2r2s23}+\text{a1rlr22s1s22}-2\text{a1r23s1s2}-\text{rl3s23}-\text{rl2r2s1s22}+2\text{rlr22s1s2}+2\text{r23s1s2}}{r2^3s2^3}, \text{r1} = \text{rl}, \text{r2} = \text{r2}, \text{s1} = \text{s1}, \text{s2} \\ = \text{s2} \end{array} \right\}
\end{aligned} \tag{3.1}$$

$$> \text{solve}([\text{N}[1, 3], \text{N}[1, 4], \text{N}[2, 1], \text{N}[2, 2], \text{N}[2, 3]], \{\text{a1}, \text{b1}, \text{c1}, \text{d1}, \text{c2}, \text{d2}, \text{eta}, \text{theta}\}); \\
\left\{ \begin{array}{l} \text{a1} = \text{a1}, \text{b1} = -3\text{a1}\eta + 3\eta^2, \text{c1} = 3\text{a1}\eta\theta - 3\eta^2\theta - 3\eta\theta^2, \text{c2} = -3\text{a1}\eta + 3\text{a1}\theta + 3\eta^2 - 3\theta\eta - 3\theta^2, \text{d1} = \text{a1}\eta^2\theta - 2\text{a1}\eta\theta^2 - \eta^3\theta \\ + 2\eta^2\theta^2 + 2\eta\theta^3, \text{d2} = \text{a1}\eta\theta - 2\text{a1}\theta^2 - \eta^3 - \eta^2\theta + 2\eta\theta^2 + 2\theta^3, \eta = \eta, \theta = \theta \end{array} \right\}
\end{math>$$

$$\begin{aligned}
> \text{b11} := -3\text{a1}\text{eta} + 3\text{eta}^2; \\
\text{c11} := 3\text{a1}\text{eta}\text{theta} - 3\text{eta}^2\text{theta} - 3\text{eta}\text{theta}^2; \\
\text{c21} := -3\text{a1}\text{eta} + 3\text{a1}\text{theta} + 3\text{eta}^2\text{theta} - 3\text{eta}\text{theta}^2; \\
\text{d11} := \text{a1}\text{eta}^2\text{theta} - 2\text{a1}\text{eta}\text{theta}^2 - \text{eta}^3\text{theta} + 2\text{eta}^2\text{theta}^2 + 2\text{eta}\text{theta}^3; \\
\text{d21} := \text{a1}\text{eta}^2 + \text{a1}\text{eta}\text{theta} - 2\text{a1}\text{theta}^2 - \text{eta}^3\text{theta} + 2\text{eta}^2\text{theta}^2 + 2\text{eta}\text{theta}^3; \\
\text{r11} := \text{eta}\text{r2}; \\
\text{s11} := \text{theta}\text{s2}; \\
\text{zamproc}(\text{a1}, \text{b11}, \text{c11}, \text{d11}, 1, 0, \text{c21}, \text{d21}, \text{r11}, \text{s11}, \text{r21}, \text{s21}) : \\
-2 \left(\text{a1} - \frac{3\eta}{2} - \theta \right) \text{r2}^2 (\eta - \theta)^2, -3(\eta - \theta)^2 (\text{a1} - \eta - \theta) \text{r2} \text{s2}, 0, 0 \\
0, 0, 0, (\eta - \theta)^2 (\text{a1} - \eta) \text{s2}^2
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
> \text{b11} := -3\text{a1}\text{eta} + 3\text{eta}^2; \\
\text{c11} := 3\text{a1}\text{eta}\text{theta} - 3\text{eta}^2\text{theta} - 3\text{eta}\text{theta}^2; \\
\text{c21} := \text{collect}(-3\text{a1}\text{eta} + 3\text{a1}\text{theta} + 3\text{eta}^2\text{theta} - 3\text{eta}\text{theta}^2, \text{theta}, \text{factor}); \\
\text{d11} := \text{factor}(\text{a1}\text{eta}^2\text{theta} - 2\text{a1}\text{eta}\text{theta}^2 - \text{eta}^3\text{theta} + 2\text{eta}^2\text{theta}^2 + 2\text{eta}\text{theta}^3); \\
\text{d21} := \text{collect}(\text{a1}\text{eta}^2 + \text{a1}\text{eta}\text{theta} - 2\text{a1}\text{theta}^2 - \text{eta}^3\text{theta} + 2\text{eta}^2\text{theta}^2 + 2\text{eta}\text{theta}^3, \\
[\text{theta}], \text{factor}); \\
\text{s21} := 1/((\text{eta} - \text{theta}) \text{abs}(\text{a1} - \text{eta})^{1/2}); \\
\text{r21} := -(\text{a1} - \text{eta}) / (3(\text{eta} - \text{theta}) \text{abs}(\text{a1} - \text{eta})^{1/2}); \\
\text{r11} := \text{eta}\text{r21}; \\
\text{s11} := \text{theta}\text{s21}; \\
\text{zamproc}(\text{a1}, \text{b11}, \text{c11}, \text{d11}, 1, 0, \text{c21}, \text{d21}, \text{r11}, \text{s11}, \text{r21}, \text{s21}) : \\
\text{b11} := -3\text{a1}\eta + 3\eta^2 \\
\text{c11} := 3\text{a1}\eta\theta - 3\eta^2\theta - 3\eta\theta^2
\end{aligned}$$

$$\begin{aligned}
c21 &:= -3 \theta^2 + (3 a1 - 3 \eta) \theta - 3 \eta (a1 - \eta) \\
d11 &:= \theta \eta (a1 \eta - 2 a1 \theta - \eta^2 + 2 \theta \eta + 2 \theta^2) \\
d21 &:= 2 \theta^3 + (-2 a1 + 2 \eta) \theta^2 + \eta (a1 - \eta) \theta + \eta^2 (a1 - \eta) \\
s21 &:= \frac{1}{(\eta - \theta) \sqrt{|a1 - \eta|}} \\
r21 &:= -\frac{a1 - \eta}{3 (\eta - \theta) (a1 - \eta - \theta) \sqrt{|a1 - \eta|}} \\
&- \frac{2 (a1 - \eta)^2 \left(a1 - \frac{3 \eta}{2} - \theta \right)}{9 |a1 - \eta| (a1 - \eta - \theta)^2}, \frac{a1 - \eta}{|a1 - \eta|}, 0, 0 \\
&0, 0, 0, \frac{a1 - \eta}{|a1 - \eta|}
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
> u &= -2 * (a1 - \eta) * (a1 - 3 * \eta * (1/2) - \theta) / (9 * (a1 - \eta - \theta)^2); \\
u &= -\frac{2 (a1 - \eta) \left(a1 - \frac{3 \eta}{2} - \theta \right)}{9 (a1 - \eta - \theta)^2}
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
> \text{solve}([b1=-3*a1*eta+3*eta^2, c1=3*a1*eta*theta-3*eta^2*theta-3*eta*theta^2], \{\eta, \theta\}); \\
\{\eta=\text{RootOf}(3 _Z^2 - 3 a1 _Z - b1), \theta=\text{RootOf}(b1 _Z^2 + (\text{RootOf}(3 _Z^2 - 3 a1 _Z - b1) b1 - a1 b1) _Z + \text{RootOf}(3 _Z^2 - 3 a1 _Z - b1) c1 \\
-a1 c1)\}
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
> \text{solve}(3 _Z^2 - 3 _Z * a1 - b1, _Z); \\
z1 &:= (1/2) * a1 + (1/6) * \sqrt{9 a1^2 + 12 b1}; \\
z2 &:= (1/2) * a1 - (1/6) * \sqrt{9 a1^2 + 12 b1}; \\
&\frac{a1}{2} + \frac{\sqrt{9 a1^2 + 12 b1}}{6}, \frac{a1}{2} - \frac{\sqrt{9 a1^2 + 12 b1}}{6}
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
> \text{solve}(b1 _Z^2 + (z1 * b1 - a1 * b1) _Z + z1 * c1 - a1 * c1, _Z); \\
&\frac{b1 \sqrt{9 a1^2 + 12 b1} - 3 a1 b1 - \sqrt{-6 \sqrt{9 a1^2 + 12 b1} a1 b1^2 + 18 a1^2 b1^2 - 24 \sqrt{9 a1^2 + 12 b1} b1 c1 + 72 a1 b1 c1 + 12 b1^3}}{12 b1}, \\
&\frac{b1 \sqrt{9 a1^2 + 12 b1} - 3 a1 b1 + \sqrt{-6 \sqrt{9 a1^2 + 12 b1} a1 b1^2 + 18 a1^2 b1^2 - 24 \sqrt{9 a1^2 + 12 b1} b1 c1 + 72 a1 b1 c1 + 12 b1^3}}{12 b1}
\end{aligned} \tag{3.8}$$

CF 3-2

$$\begin{aligned}
> \text{solve}([M[1,2], M[1,4], M[2,1], M[2,2], M[2,3]], \{a1, b1, c1, d1, c2, d2, r1, s1, r2, s2\}); \\
\left\{ a1 = a1, b1 = -\frac{3 r1 (a1 r2 - r1)}{r22}, c1 = \frac{3 r1 (a1 r1 r2 s22 + a1 r22 s1 s2 - r12 s22 - 2 r1 s1 s2 r2 - r22 s12)}{2 r23 s22}, d2 = \right. \\
\left. -\frac{3 (a1 r1 r2 s22 - a1 r22 s1 s2 - r12 s22 + 2 r1 s1 s2 r2 + r22 s12)}{2 r22 s22}, d1 = \right. \\
\left. -\frac{s1 r1 (a1 r1 r2 s22 + a1 r22 s1 s2 - r12 s22 - 4 r1 s1 s2 r2 - r22 s12)}{2 s23 r23}, \right. \\
\left. = \frac{2 a1 r12 r2 s23 - a1 r1 r22 s1 s22 - a1 r23 s12 s2 - 2 r13 s23 + r12 r2 s1 s22 + 4 r1 r22 s12 s2 + r23 s13}{2 r23 s23}, r1 = r1, r2 = r2, s1 = s1, s2 \right. \\
\left. = s2 \right\}
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
> \text{solve}([N[1,2], N[1,4], N[2,1], N[2,2], N[2,3]], \{a1, b1, c1, d1, c2, d2, eta, theta\}); \\
\left\{ a1 = a1, b1 = -3 a1 \eta + 3 \eta^2, c1 = \frac{3}{2} a1 \eta^2 - \frac{3}{2} \eta^3 + \frac{3}{2} a1 \eta \theta - 3 \eta^2 \theta - \frac{3}{2} \eta \theta^2, c2 = -\frac{3}{2} a1 \eta + \frac{3}{2} a1 \theta + \frac{3}{2} \eta^2 - 3 \theta \eta - \frac{3}{2} \theta^2, d1 = \right. \\
\left. -\frac{1}{2} a1 \eta^2 \theta + \frac{1}{2} \eta^3 \theta + 2 \eta^2 \theta^2 - \frac{1}{2} a1 \eta \theta^2 + \frac{1}{2} \eta \theta^3, d2 = \frac{1}{2} \eta^2 \theta + a1 \eta^2 - \frac{1}{2} a1 \eta \theta - \eta^3 + 2 \eta \theta^2 - \frac{1}{2} a1 \theta^2 + \frac{1}{2} \theta^3, \eta = \eta, \theta \right. \\
\left. = \theta \right\}
\end{aligned} \tag{4.2}$$

```

> b11 := -3*a1*eta+3*eta^2;
c11 := (3/2)*a1*eta^2-(3/2)*eta^3+(3/2)*a1*eta*theta-3*eta^2*theta-(3/2)*eta*theta^2;
c21 := -(3/2)*a1*eta+(3/2)*a1*theta+(3/2)*eta^2-3*theta*eta-(3/2)*theta^2;
d11 := -(1/2)*a1*eta^2*theta+(1/2)*eta^3*theta+2*eta^2*theta^2-(1/2)*a1*eta*theta^2+(1/2)*eta*theta^3;
d21 := (1/2)*eta^2*theta+a1*eta^2-(1/2)*a1*eta*theta-eta^3+2*eta*theta^2-(1/2)*a1*theta^2+(1/2)*theta^3;
r11 := eta*r2;

```

```

s11 := theta*s2;
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r2,s2):

$$-\frac{(\eta-\theta)^2 (a1-3\eta-\theta) r2^2}{2}, 0, \frac{3(\eta-\theta)^2 (a1-\eta-\theta) s2^2}{2}, 0$$


$$0, 0, 0, (\eta-\theta)^2 (a1-\eta) s2^2$$


```

(4.3)

```

> b11 := -3*a1*eta+3*eta^2;
c11 := factor((3/2)*a1*eta^2-(3/2)*eta^3+(3/2)*a1*eta*theta-3*eta^2*theta-(3/2)*eta*theta^2);
c21 := -(3/2)*a1*eta+(3/2)*a1*theta+(3/2)*eta^2-3*theta*eta-(3/2)*theta^2;
d11 := factor(-(1/2)*a1*eta^2*theta+(1/2)*eta^3*theta+2*eta^2*theta^2-(1/2)*a1*eta*theta^2+(1/2)*eta*theta^3);
d21 := collect((1/2)*eta^2*theta+a1*eta^2-(1/2)*a1*eta*theta-eta^3+2*eta*theta^2-(1/2)*a1*theta^2+(1/2)*theta^3, a1, factor);
s21 := 1/((eta-theta)*abs(a1-eta)^(1/2));
r21 := sqrt(2)/((eta-theta)*abs(a1-3*eta-theta)^(1/2));
r11 := eta*r21;
s11 := theta*s21;
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r21,s21):

$$c11 := \frac{3\eta(\eta+\theta)(-\eta+a1-\theta)}{2}$$


$$d11 := \frac{\eta\theta(a1\eta+a1\theta-\eta^2-4\eta\theta-\theta^2)}{2}$$


$$d21 := \frac{(2\eta+\theta)(\eta-\theta)a1}{2} - \frac{(\eta+\theta)(2\eta^2-3\eta\theta-\theta^2)}{2}$$


$$s21 := \frac{1}{(\eta-\theta)\sqrt{|a1-\eta|}}$$


$$r21 := \frac{\sqrt{2}}{(\eta-\theta)\sqrt{|a1-3\eta-\theta|}}$$


$$\frac{-a1+3\eta+\theta}{|a1-3\eta-\theta|}, 0, \frac{-3\eta+3a1-3\theta}{2|a1-\eta|}, 0$$


$$0, 0, 0, \frac{a1-\eta}{|a1-\eta|}$$


```

(4.4)

```

> solve([b1=-3*a1*eta+3*eta^2, c2=-(3/2)*a1*eta+(3/2)*a1*theta+(3/2)*eta^2-3*eta*theta-(3/2)*theta^2], {eta,theta});

$$\{\eta = \text{RootOf}(3\_Z^2 - 3a1\_Z - b1), \theta = \text{RootOf}(3\_Z^2 + (6\text{RootOf}(3\_Z^2 - 3a1\_Z - b1) - 3a1)\_Z - b1 + 2c2)\}$$


```

(4.5)

```

> solve(3*_Z^2-3*_Z*a1-b1, _Z);
z1 := (1/2)*a1+(1/6)*sqrt(9*a1^2+12*b1):
z2 := (1/2)*a1-(1/6)*sqrt(9*a1^2+12*b1):

$$\frac{a1}{2} + \frac{\sqrt{9a1^2+12b1}}{6}, \frac{a1}{2} - \frac{\sqrt{9a1^2+12b1}}{6}$$


```

(4.6)

```

> solve(3*_Z^2+(6*z1-3*a1)*_Z-b1+2*c2, _Z);

$$-\frac{\sqrt{9a1^2+12b1}}{6} + \frac{\sqrt{9a1^2+24b1-24c2}}{6}, -\frac{\sqrt{9a1^2+12b1}}{6} - \frac{\sqrt{9a1^2+24b1-24c2}}{6}$$


```

(4.7)

CF 3-4

```

> solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]],{a1,b1,c1,c2,d1,d2,r1,s1,r2,s2});
solve([M[1,2],M[1,3],M[2,1],M[2,2],M[2,3]],{a1,b1,c2,d2,r1,s1,r2,s2});
{a1=a1, b1=0, c1=0, c2=0, d1=0, d2=d2, r1=r1, r2=r2, s1=s1, s2=0},  $\left\{ a1 = a1, b1 = -\frac{3r1(a1r2-r1)}{r2^2}, c1 \right.$ 

$$= \frac{3r1^2(a1r2s2-r1s2-r2s1)}{r2^3s2}, c2 = -\frac{3r1s1}{r2s2}, d1 = -\frac{r1^2(a1r1r2s22-r1^2s2^2-r1r2s1s2-r2^2s1^2)}{s2^2r2^4}, d2$$


$$= \left. \frac{r1s1(r1s2+r2s1)}{r2^2s2^2}, r1=r1, r2=r2, s1=s1, s2=s2 \right\}$$


$$\left\{ a1 = \frac{\text{RootOf}(-c1r1r2-3d1r22+3\_Z)c1r23+c1r12r22+3d1r1r23+3\text{RootOf}(-c1r1r2-3d1r22+3\_Z)r1^3}{3r1^2\text{RootOf}(-c1r1r2-3d1r22+3\_Z)r2}, b1 = \right.$$


$$- \frac{\text{RootOf}(-c1r1r2-3d1r22+3\_Z)c1r2+c1r12+3d1r1r2}{r1\text{RootOf}(-c1r1r2-3d1r22+3\_Z)}, c2 = -\frac{c1r1+3d1r2}{\text{RootOf}(-c1r1r2-3d1r22+3\_Z)}, d2$$


$$= \left. \frac{\text{RootOf}(-c1r1r2-3d1r22+3\_Z)c1r1r2+3\text{RootOf}(-c1r1r2-3d1r22+3\_Z)d1r22+c1r1^3+3d1r12r2}{3r1\text{RootOf}(-c1r1r2-3d1r22+3\_Z)r2}, r1=r1, r2 \right\}$$


```

(5.1)

$$= r2, sI = \frac{\text{RootOf}(-c1 r1 r2 - 3 d1 r22 + 3 \underline{Z^2}) s2}{rI}, s2 = s2 \Big\}, \left\{ aI = \frac{cI^4 - 81 dI^3}{27 dI^2 cI}, bI = \frac{cI^2}{3 dI}, c2 = 0, d2 = 0, rI = -\frac{3 dI r2}{cI}, r2 = r2, sI = 0, s2 = s2 \right\}$$

$$\begin{aligned} > \text{zamproc}(a1, 0, 0, 0, 1, 0, 0, d2, 0, s1, r2, 0): \\ & \quad d2 r22, 0, 0, \frac{sI^3}{r2} \\ & \quad 0, 0, 0, aI sI2 \end{aligned} \tag{5.2}$$

$$\begin{aligned} > s11 := \text{abs}(a1)^{-1/2}: \\ & \quad r21 := \text{abs}(a1)^{-1/2}/a1: \\ & \quad \text{zamproc}(a1, 0, 0, 0, 1, 0, 0, d2, 0, s11, r21, 0): \\ & \quad \frac{d2}{|aI| aI^2}, 0, 0, \frac{aI}{|aI|} \\ & \quad 0, 0, 0, \frac{aI}{|aI|} \end{aligned} \tag{5.3}$$

$$\begin{aligned} > a11 := (c1^4 - 81 d1^3) / (27 * d1^2 * c1): \\ & \quad b11 := c1^2 / (3 * d1): \\ & \quad r11 := -3 * d1 * r2 / c1: \\ & \quad \text{zamproc}(a11, b11, c1, d1, 1, 0, 0, 0, r11, 0, r2, s2): \\ & \quad -\frac{27 dI^3 r22}{cI^3}, 0, 0, -\frac{s23 cI}{3 r2} \\ & \quad 0, 0, 0, \frac{s22 cI}{3} \end{aligned} \tag{5.4}$$

$$\begin{aligned} > a11 := (c1^4 - 81 d1^3) / (27 * d1^2 * c1): \\ & \quad b11 := c1^2 / (3 * d1): \\ & \quad s21 := \sqrt{3} / \text{abs}(c1)^{1/2}: \\ & \quad r21 := -\sqrt{3} / \text{abs}(c1)^{1/2}: \\ & \quad r11 := -3 * d1 * r21 / c1: \\ & \quad \text{zamproc}(a11, b11, c1, d1, 1, 0, 0, 0, r11, 0, r21, s21): \\ & \quad -\frac{81 dI^3}{|cI| cI^3}, 0, 0, \frac{cI}{|cI|} \\ & \quad 0, 0, 0, \frac{cI}{|cI|} \end{aligned} \tag{5.5}$$

$$> \text{solve}([N[1, 2], N[1, 3], N[2, 1], N[2, 2], N[2, 3]], \{a1, b1, c1, d1, c2, d2, \eta, \theta\}); \{aI = aI, bI = -3 aI \eta + 3 \eta^2, cI = 3 aI \eta^2 - 3 \eta^3 - 3 \eta^2 \theta, c2 = -3 \theta \eta, dI = -aI \eta^3 + \eta^4 + \eta^3 \theta + \eta^2 \theta^2, d2 = \eta^2 \theta + \eta \theta^2, \eta = \eta, \theta = \theta\} \tag{5.6}$$

$$\begin{aligned} > b11 := -3 * a1 * \eta + 3 * \eta^2: \\ & \quad c11 := 3 * a1 * \eta^2 - 3 * \eta^3 - 3 * \eta^2 * \theta: \\ & \quad c21 := -3 * \theta * \eta: \\ & \quad d11 := -a1 * \eta^3 + \eta^4 + \eta^3 * \theta + \eta^2 * \theta^2: \\ & \quad d21 := \eta^2 * \theta + \eta * \theta^2: \\ & \quad r11 := \eta * \theta^2: \\ & \quad s11 := \theta * \eta^2: \\ & \quad \text{zamproc}(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r2, s2): \\ & \quad (\eta - \theta)^2 r22 \eta, 0, 0, -\frac{(\eta - \theta)^2 (aI - \eta - \theta) s23}{r2} \\ & \quad 0, 0, 0, (\eta - \theta)^2 (aI - \eta) s22 \end{aligned} \tag{5.7}$$

$$\begin{aligned} > b11 := -3 * a1 * \eta + 3 * \eta^2: \\ & \quad c11 := 3 * a1 * \eta^2 - 3 * \eta^3 - 3 * \eta^2 * \theta: \\ & \quad c21 := -3 * \theta * \eta: \\ & \quad d11 := -a1 * \eta^3 + \eta^4 + \eta^3 * \theta + \eta^2 * \theta^2: \\ & \quad d21 := \eta^2 * \theta + \eta * \theta^2: \\ & \quad s21 := (\eta - \theta)^{-1} * \text{abs}(aI - \eta)^{-1/2}: \\ & \quad r21 := -(\eta - \theta)^{-1} * \text{abs}(aI - \eta)^{-1/2} * (aI - \eta - \theta) * (aI - \eta)^{-1}: \\ & \quad r11 := \eta * \theta^2: \\ & \quad s11 := \theta * \eta^2: \\ & \quad \text{zamproc}(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r21, s21): \\ & \quad d11 := -aI \eta^3 + \eta^4 + \eta^3 \theta + \eta^2 \theta^2 \\ & \quad d21 := \eta^2 \theta + \eta \theta^2 \\ & \quad s21 := \frac{1}{(\eta - \theta) \sqrt{|aI - \eta|}} \\ & \quad r21 := \frac{aI - \eta - \theta}{(\eta - \theta) \sqrt{|aI - \eta|} (aI - \eta)} \end{aligned}$$

$$\frac{(al-\eta-\theta)^2 \eta}{|al-\eta| (al-\eta)^2}, 0, 0, \frac{al-\eta}{|al-\eta|} \\ 0, 0, 0, \frac{al-\eta}{|al-\eta|} \quad (5.8)$$

$$> solve([c1=3*a1*eta^2-3*eta^3-3*eta^2*theta, c2=-3*theta*eta], {eta, theta}); \\ \left\{ \eta = RootOf(3 _Z^3 - 3 al _Z^2 - c2 _Z + cl), \theta = - \frac{c2}{3 RootOf(3 _Z^3 - 3 al _Z^2 - c2 _Z + cl)} \right\} \quad (5.9)$$

CF 3-9

$$> solve([M[1,2], M[1,3], M[2,1], M[2,2], M[2,4]], {a1, b1, c1, d1, c2, r1, s1, r2, s2}); \\ \left\{ a1 = 0, b1 = b1, c1 = 0, c2 = 0, d1 = 0, r1 = 0, r2 = r2, s1 = s1, s2 = 0, \begin{aligned} & \left\{ a1 = \frac{c2^2 r24 - 6 d2 r1 r23 + 3 r14}{3 (c2 r1 r22 + d2 r23 + r13) r2}, b1 \right. \\ & = \frac{c2^2 d2 r25 - 2 c2^2 r13 r22 - 3 c2 d2 r12 r23 - 9 d22 r1 r24 + 6 c2 r15 + 18 d2 r14 r2}{(c2 r1 r22 + d2 r23 + r13) (c2 r22 + 3 r12)}, c1 = \\ & - \frac{r1 (c2^2 d2 r24 - 8 c2 r13 r2 - 18 c2 d2 r12 r22 - 18 d22 r1 r23 + 9 d2 r14)}{(c2 r1 r22 + d2 r23 + r13) (c2 r22 + 3 r12)}, c2 = c2, d1 \\ & = \frac{r1 (2 c2^3 r12 r23 + 6 c2^2 d2 r1 r24 + 3 c2 d22 r25 - 6 c2 r14 r2 - 15 c2 d2 r13 r22 - 18 d22 r12 r23 + 9 d2 r15)}{3 r2 (c2 r1 r22 + d2 r23 + r13) (c2 r22 + 3 r12)}, r1 = r1, r2 = r2, s1 \\ & = - \frac{r2 s2 (2 c2 r1 + 3 d2 r2)}{c2 r22 + 3 r12}, s2 = s2 \end{aligned} \right\}, \left\{ a1 = \frac{4 RootOf(2 _Z^3 - d2)^2 s1 s2 - 2 RootOf(2 _Z^3 - d2) s12 - d2 s22}{(RootOf(2 _Z^3 - d2) s2 - s1)^2}, b1 \right. \\ & = \frac{3 (2 RootOf(2 _Z^3 - d2) d2 s23 + 2 RootOf(2 _Z^3 - d2) s13 - 3 d2 s1 s22)}{2 (RootOf(2 _Z^3 - d2) s2 - s1)^2 s2}, c1 = \\ & - \frac{3 (RootOf(2 _Z^3 - d2)^2 d2 s23 + 2 RootOf(2 _Z^3 - d2) s13 - RootOf(2 _Z^3 - d2) d2 s1 s22 - d2 s12 s2)}{(RootOf(2 _Z^3 - d2) s2 - s1)^2 s2}, c2 = \\ & - 3 RootOf(2 _Z^3 - d2)^2, d1 = - \frac{d2 (RootOf(2 _Z^3 - d2)^2 s1 s22 + 4 RootOf(2 _Z^3 - d2) s12 s2 - d2 s23 - 3 s13)}{2 (RootOf(2 _Z^3 - d2) s2 - s1)^2 s2}, r1 = RootOf(2 _Z^3 - d2) r2, r2 = r2, s1 = s1, s2 = s2 \} \\ > zampoc(0, b1, 0, 0, 1, 0, 0, d2, 0, s1, r2, 0); \\ d2 r22, 0, 0, \frac{s13}{r2} \\ 0, 0, b1 s1 r2, 0 \quad (6.2)$$

$$> r21 := abs(d2)^(-1/2); \\ s11 := abs(d2)^(-1/2)*d2/b1; \\ zampoc(0, b1, 0, 0, 1, 0, 0, d2, 0, s11, r21, 0); \\ \frac{d2}{|d2|}, 0, 0, \frac{d23}{b1^3 |d2|} \\ 0, 0, \frac{d2}{|d2|} \quad (6.3)$$

$$> solve([N[1,2], N[1,3], N[2,1], N[2,2], N[2,4]], {a1, b1, c1, d1, c2, d2, eta, theta}); \\ \left\{ a1 = a1, b1 = -2 a1 \eta - a1 \theta + 2 \eta^2 + \theta \eta, c1 = 2 a1 \eta^2 + a1 \eta \theta - 2 \eta^3 - 4 \eta^2 \theta, c2 = a1 \eta - a1 \theta - \eta^2 - 2 \theta \eta, d1 = -\frac{2}{3} a1 \eta^3 + \frac{2}{3} \eta^4 \right. \\ \left. - \frac{2}{3} a1 \eta^2 \theta + \frac{5}{3} \eta^3 \theta + \frac{1}{3} a1 \eta \theta^2 + \frac{2}{3} \eta^2 \theta^2, d2 = \frac{2}{3} \eta^2 \theta - \frac{2}{3} a1 \eta^2 + \frac{1}{3} a1 \eta \theta + \frac{2}{3} \eta^3 + \frac{1}{3} a1 \theta^2 + \frac{2}{3} \eta \theta^2, \eta = \eta, \theta = \theta \right\} \quad (6.4)$$

$$> b11 := -2*a1*eta-a1*theta+2*eta^2+eta*theta; \\ c11 := 2*a1*eta^2+a1*eta*theta-2*eta^3-4*eta^2*theta; \\ c21 := a1*eta-a1*theta-eta^2-2*eta*theta; \\ d11 := -(2/3)*a1*eta^3+(2/3)*eta^4-(2/3)*a1*eta^2*theta+(5/3)*eta^3*theta+(1/3)*a1*eta*theta^2+(2/3)*eta^2*theta^2; \\ d21 := (2/3)*eta^2*theta-(2/3)*a1*eta^2+(1/3)*a1*eta*theta+(2/3)*eta^3+(1/3)*a1*theta^2+(2/3)*eta*theta^2; \\ r11 := eta*r2; \\ s11 := theta*s2; \\ zampoc(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r2, s2); \\ \frac{(\eta-\theta)^2 (a1+2 \eta) r22}{3}, 0, 0, -\frac{2 \left(a1-\eta-\frac{3 \theta}{2}\right) (\eta-\theta)^2 s23}{3 r2} \\ 0, 0, -(\eta-\theta)^2 (a1-\eta) r2 s2, 0 \quad (6.5)$$

$$> b11 := -2*a1*eta-a1*theta+2*eta^2+eta*theta; \\ c11 := collect(2*a1*eta^2+a1*eta*theta-2*eta^3-4*eta^2*theta, theta, factor); \\ c21 := a1*eta-a1*theta-eta^2-2*eta*theta;$$

```

d11 := collect(-(2/3)*a1*eta^3+(2/3)*eta^4-(2/3)*a1*eta^2*theta+(5/3)*eta^3*theta+(1/3)*a1*eta*
theta^2+(2/3)*eta^2*theta^2,theta,factor);
d21 := collect((2/3)*eta^2*theta-(2/3)*a1*eta^2+(1/3)*a1*eta*theta+(2/3)*eta^3+(1/3)*a1*theta^2+
(2/3)*eta*theta^2,theta,factor);
r21 := sqrt(3)*abs(a1+2*eta)^(-1/2)/(eta-theta);
s21 := -(a1+2*eta)*abs(a1+2*eta)^(-1/2)*(eta-theta)^(-1)*(a1-eta)^(-1)/sqrt(3);
r11 := eta*r21;
s11 := theta*s21;
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r21,s21):
c11:=η(a1-4 η) θ+2 η² (a1-η)
d11:= $\frac{\eta(a1+2 \eta) \theta^2}{3}-\frac{\eta^2 (2 a1-5 \eta) \theta}{3}-\frac{2 \eta^3 (a1-\eta)}{3}$ 
d21:= $\left(\frac{a1}{3}+\frac{2 \eta}{3}\right) \theta^2+\frac{\eta (a1+2 \eta) \theta}{3}-\frac{2 \eta^2 (a1-\eta)}{3}$ 
r21:= $\frac{\sqrt{3}}{\sqrt{|a1+2 \eta|} (\eta-\theta)}$ 
s21:=- $\frac{(a1+2 \eta) \sqrt{3}}{3 \sqrt{|a1+2 \eta|} (\eta-\theta) (a1-\eta)}$ 
 $\frac{a1+2 \eta}{|a1+2 \eta|}, 0, 0, \frac{(a1+2 \eta)^3 (2 a1-2 \eta-3 \theta)}{27 |a1+2 \eta| (a1-\eta)^3}$ 
0, 0,  $\frac{a1+2 \eta}{|a1+2 \eta|}, 0$  (6.6)
```

```

> solve([b1=-2*a1*eta-a1*theta+2*eta^2+eta*theta, c2=a1*eta-a1*theta-eta^2-2*eta*theta], {eta,theta})
;
(6.7)
```

$$\left\{ \eta = \text{RootOf}(3 Z^3 + (-3 a1^2 - 2 b1 - c2) Z - a1 b1 + a1 c2), \theta = \frac{-\text{RootOf}(3 Z^3 + (-3 a1^2 - 2 b1 - c2) Z - a1 b1 + a1 c2)^2 - a1 \text{RootOf}(3 Z^3 + (-3 a1^2 - 2 b1 - c2) Z - a1 b1 + a1 c2) + c2}{a1 + 2 \text{RootOf}(3 Z^3 + (-3 a1^2 - 2 b1 - c2) Z - a1 b1 + a1 c2)} \right\}$$

```

> solve([N[1,2],N[1,3],N[2,1],N[2,2],N[2,4]],{a1,b1,d1,c2,d2,eta,theta,r2,s2});
(6.8)

$$\left\{ a1 = a1, b1 = b1, c2 = c2, d1 = -a1 \theta^3 + \eta \theta^3 - b1 \theta^2 + \frac{1}{3} c2 \eta \theta - c1 \theta - a1 \eta \theta^2 - \frac{2}{3} b1 \eta \theta - \frac{1}{3} c1 \eta, d2 = -a1 \theta^2 - \frac{2}{3} \theta b1 - \frac{2}{3} c2 \theta - \frac{1}{3} c1 \eta, \eta = \eta, r2 = 0, s2 = s2, \theta = \theta \right\}, \left\{ a1 = \frac{2 \eta^3 + 4 \eta^2 \theta + c1 \theta}{\eta (2 \eta + \theta)}, b1 = -\frac{3 \eta^2 \theta + c1 \theta}{\eta}, c2 = \frac{-3 \eta^3 \theta - 6 \eta^2 \theta^2 + c1 \eta - c1 \theta}{\eta (2 \eta + \theta)}, d1 = -\frac{-3 \eta^3 \theta^2 - 6 \eta^2 \theta^3 + 2 c1 \eta^2 + 2 c1 \eta \theta - c1 \theta^2}{3 (2 \eta + \theta)}, d2 = -\frac{-6 \eta^2 \theta^2 + c1 \eta - c1 \theta}{3 \eta}, \eta = \eta, r2 = r2, s2 = s2, \theta = \theta \right\}, \left\{ a1 = a1, b1 = 0, c2 = 3 \text{RootOf}(3 Z^3 + 4 c1) (\text{RootOf}(3 Z^3 + 4 c1) - 2 a1), d1 = \frac{a1 c1}{3}, d2 = \frac{c1}{3}, \eta = -\frac{\text{RootOf}(3 Z^3 + 4 c1)}{2}, r2 = r2, s2 = s2, \theta = \theta = \text{RootOf}(3 Z^3 + 4 c1) \right\}$$


```

```

> z1 := -(4*c1/3)^(1/3);
c21 := 3*z1*(z1-2*a1)*(1/4):
d11 := (1/3)*a1*c1:
d21 := (1/3)*c1:
eta1 := -(1/2)*z1:
theta1 := z1:
r11 := eta1*r2:
s11 := theta1*s2:
zamproc(a1,0,c1,d11,1,0,c21,d21, r11,s11,r2,s2):
z1:=- $\frac{41 \sqrt[3]{3}^{32} \sqrt[3]{c1} \sqrt[3]{3}}{3}$ 
 $\frac{r22 c12 \sqrt[3]{(22 \sqrt[3]{3}^{32} \sqrt[3]{c1}) \sqrt[3]{3} + 3 a1) 21 \sqrt[3]{3}^{31} \sqrt[3]{3}}{6}, 0, 0, -\frac{s2^{23} 31 \sqrt[3]{3} \left(\frac{22 \sqrt[3]{3}^{32} \sqrt[3]{c1} \sqrt[3]{3}}{3} + a1\right) c12 \sqrt[3]{21} \sqrt[3]{3}}{r2}$ 
 $0, 0, -\frac{3 s2 \left(-\frac{22 \sqrt[3]{3}^{32} \sqrt[3]{c1} \sqrt[3]{3}}{6} + a1\right) 31 \sqrt[3]{c12} \sqrt[3]{r2} 21 \sqrt[3]{3}}{2}, 0$  (6.9)
```

```

> z1 := -(4*c1/3)^(1/3);
c21 := 3*z1*(z1-2*a1)*(1/4):
d11 := (1/3)*a1*c1:
d21 := (1/3)*c1:
```

```

etal := -(1/2)*z1;
theta1 := z1;
r21 := 6^(1/3)*abs(2^(2/3)*3^(2/3)*c1^(1/3)+3*a1)^(-1/2)/c1^(1/3);
s21 := 4*(2^(2/3)*3^(2/3)*c1^(1/3)+3*a1)*abs(2^(2/3)*3^(2/3)*c1^(1/3)+3*a1)^(-1/2)*c1^(-1/3)*(2^(2/3)*3^(2/3)*c1^(1/3)-6*a1)^(-1)/6^(2/3);
r11 := etal*r21;
s11 := theta1*s21;
zamproc(a1,0,c1,d11,1,0,c21,d21, r11,s11,r21,s21):
z1:=- $\frac{41\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}}{3}$ 
c21:=- $\frac{41\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}\left(-\frac{41\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}}{3}-2a1\right)}{4}$ 
r21:= $\frac{61\sqrt[3]{3}}{\sqrt[3]{22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1}\sqrt[3]{c1}\sqrt[3]{3}}$ 
s21:= $\frac{2(22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1)61\sqrt[3]{3}}{3\sqrt[3]{22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1}\sqrt[3]{c1}\sqrt[3]{(22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}-6a1)}}$ 
 $\frac{22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1}{22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1}, 0, 0, -\frac{16(22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1)^4}{27(22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1)(22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}-6a1)^3}$ 
 $0, 0, \frac{22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1}{22\sqrt[3]{32}\sqrt[3]{c1}\sqrt[3]{3}+3a1}, 0$  (6.10)

```

CF 3-15

```

> solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]],{a1,b1,c1,d1,c2,d2,r1,s1,r2,s2});
{a1=0, b1=0, c1=c1, c2=0, d1=0, d2=d2, r1=0, r2=r2, s1=s1, s2=0}, {a1=a1, b1=
(7.1)

```

$$\begin{aligned}
& -\frac{a1 r1 r2 s2 + 2 a1 r22 s1 - r12 s2 - 2 r1 r2 s1}{r22 s2}, c1 \\
& = \frac{a1 r12 r2 s22 + a1 r1 r22 s1 s2 + a1 r23 s12 - r13 s22 - 4 r12 r2 s1 s2 - r1 r22 s12}{r23 s22}, c2 \\
& = \frac{a1 r1 r2 s2 - a1 r22 s1 - r12 s2 - 2 r1 r2 s1}{r22 s2}, d1 = \\
& -\frac{r1 (a1 r12 r2 s22 + a1 r1 r22 s1 s2 + a1 r23 s12 - r13 s22 - 4 r12 r2 s1 s2 - 4 r1 r22 s12)}{3 s22 r24}, d2 = \\
& -\frac{a1 r12 r2 s22 + a1 r1 r22 s1 s2 - 2 a1 r23 s12 - r13 s22 - 4 r12 r2 s1 s2 - r1 r22 s12}{3 r23 s22}, r1=r1, r2=r2, s1=s1, s2=s2 \}
\end{aligned}$$

```

> zamproc(0,0,c1,0,1,0,0,d2, 0,s1,r2,0):
d2 r22, 0, 0,  $\frac{s13}{r2}$ 
0, r22 c1, 0, 0 (7.2)

```

```

> r21 := abs(c1)^(-1/2):
s11 := c1^(1/3)*abs(c1)^(-1/2):
zamproc(0,0,c1,0,1,0,0,d2, 0,s11,r21,0):
 $\frac{d2}{|c1|}, 0, 0, \frac{c1}{|c1|}$ 
0,  $\frac{c1}{|c1|}, 0, 0$  (7.3)

```

```

> solve([M[1,2],M[1,3],M[2,1],M[2,3],M[2,4]],{a1,b1,d1,d2,r1,s1,r2,s2});
{a1=(2 RootOf((c2 s23+6 s12 s2) _Z2+c1 s1 s22+c2 s12 s2+(-c1 s23+c2 s1 s22+3 s13) _Z) c23 s26+3 RootOf((c2 s23+6 s12 s2) _Z2+
(7.4)

```

$$\begin{aligned}
& +c1 s1 s22+c2 s12 s2+(-c1 s23+c2 s1 s22+3 s13) _Z) c12 s26+18 RootOf((c2 s23+6 s12 s2) _Z2+c1 s1 s22+c2 s12 s2+(-c1 s23+c2 s1 s22+3 s13) _Z) c22 s12 s24+c1 c22 s26+c23 s1 s25+18 RootOf((c2 s23+6 s12 s2) _Z2+c1 s1 s22+c2 s12 s2+(-c1 s23+c2 s1 s22+3 s13) _Z) c1 s13 s23+27 RootOf((c2 s23+6 s12 s2) _Z2+c1 s1 s22+c2 s12 s2+(-c1 s23+c2 s1 s22+3 s13) _Z) c2 s14 s22-3 c12 s1 s25+3 c1 c2 s12 s24+6 c22 s13 s23-27 RootOf((c2 s23+6 s12 s2) _Z2+c1 s1 s22+c2 s12 s2+(-c1 s23+c2 s1 s22+3 s13) _Z) s16-9 c1 s14 s22-9 c2 s15 s2) /3 (c2 s22+6 s12) (RootOf((c2 s23+6 s12 s2) _Z2+c1 s1 s22
\end{aligned}$$

$+c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c1 s23 - RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c1 s23$
 $+3 s l3) _Z) c2 s1 s22 - 3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) s l3 - c1 s1 s22$
 $- c2 s l2 s2), b1 = -(3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c1 s1 s23$
 $+c1 c2 s24 + c22 s1 s23 + 9 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) s l4$
 $+3 c1 s l2 s22 + 3 c2 s l3 s2) / (c2 s22 + 6 s l2) RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c1 s1 s23$
 $+3 s l3) _Z) s22), d1 = -(RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c l3 c2 s211$
 $- 3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c l2 c22 s1 s210 + 3 RootOf((c2 s23$
 $+6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c l3 s l2 s29 - 15 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22$
 $+c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c l2 c2 s l3 s28 - 9 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23$
 $+c2 s1 s22 + 3 s l3) _Z) c l c22 s l4 s27 - 3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) c23 s l5 s26 - c l3 c2 s1 s210 + c1 c23 s l3 s28 - 9 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23$
 $+c2 s1 s22 + 3 s l3) _Z) c l2 s l5 s26 - 63 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) c l c2 s l6 s25 - 27 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c22 s l7 s24$
 $- 3 c l3 s l3 s28 - 3 c l2 c2 s l4 s27 - 135 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) c l s l8 s23 - 27 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c2 s l9 s22$
 $- 18 c l2 s l6 s25 - 18 c l c2 s l7 s24 + 81 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) s l11$
 $+27 c l s l9 s22 + 27 c2 s l10 s2) / (3 (c2 s22 + 6 s l2)^2 s24) (RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) c l s23 - RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c2 s l s22$
 $- 3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) s l3 - c1 s1 s22 - c2 s l2 s2), d2 =$
 $-(RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c l2 c2 s28 - RootOf((c2 s23$
 $+6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c l c22 s1 s27 - 2 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22$
 $+c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c23 s l2 s26 + 3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23$
 $+c2 s1 s22 + 3 s l3) _Z) c l2 s l2 s26 - 9 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) c l c2 s l3 s25 - 18 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c22 s l4 s24$
 $- c l2 c2 s1 s27 - 2 c l c22 s l2 s26 - c23 s l3 s25 - 36 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) c l s l5 s23 - 27 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c2 s l6 s22$
 $- 3 c l2 s l3 s25 - 9 c l c2 s l4 s24 - 6 c22 s l5 s23 + 27 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22$
 $+3 s l3) _Z) s l8 + 9 c l s l6 s22 + 9 c2 s l7 s2) / (3 s23 (c2 s22 + 6 s l2) (RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23$
 $+c2 s1 s22 + 3 s l3) _Z) c l s23 - RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) c2 s l s22$
 $- 3 RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) s l3 - c1 s1 s22 - c2 s l2 s2), r1$
 $=RootOf((c2 s23 + 6 s l2 s2) _Z^2 + c1 s1 s22 + c2 s l2 s2 + (-c1 s23 + c2 s1 s22 + 3 s l3) _Z) r2, r2 = r2, s l = s l, s2 = s2}, \{a l = \frac{c22}{c l},$
 $b l = 2 c2, d l = 0, d2 = \frac{2 c l}{3}, r1 = 0, r2 = r2, s l = -\frac{c l s2}{c2}, s2 = s2\}, \{a l = \frac{c23 + c l2}{c l c2}, b l = -c2, d l = -\frac{c l2}{3 c2}, d2 = -\frac{c l}{3}, r1 = r1, r2$
 $= \frac{c2 r l}{c l}, s l = 0, s2 = s2\}$

```

> a11 := c2^2/c1;
b11 := 2*c2;
d21 := 2*c1*(1/3);
s11 := -c1*s2/c2;
zamproc(a11,b11,c1,0,1,0,c2,d21, 0,s11,r2,s2):
   $\frac{2 r22 c l}{3}, 0, 0, -\frac{c l s23 (c23 + c l2)}{3 c23 r2}$ 

```

$$0, r22 \text{c}l, 0, 0 \quad (7.5)$$

```

> a11 := c2^2/c1:
b11 := 2*c2:
d21 := 2*c1*(1/3):
r21 := abs(c1)^(-1/2):
s21 := -3^(1/3)*c2*(c2^3+3*c1^2)^(-1/3)*abs(c1)^(-1/2);
s11 := -c1*s21/c2;
zamproc(a11,b11,c1,0,1,0,c2,d21, 0,s11,r21,s21):
  
$$s21 := -\frac{3^{1/3} c2}{(c2^3 + 3 c1^2)^{1/3} \sqrt{|cI|}}$$

  
$$s11 := \frac{c1 3^{1/3}}{(c2^3 + 3 c1^2)^{1/3} \sqrt{|cI|}}$$

  
$$\frac{2 cI}{3 |cI|}, 0, 0, \frac{cI}{|cI|}$$

  
$$0, \frac{cI}{|cI|}, 0, 0$$


```

(7.6)

```

> a11 := (c2^3+c1^2)/(c1*c2):
b11 := -c2:
d11 := -c1^2/(3*c2):
d21 := -(1/3)*c1:
r21 := c2*r1/c1;
zamproc(a11,b11,c1,d11,1,0,c2,d21, r1,0,r21,s2):
  
$$\frac{2 c2^3 rI^2 + 3 rI^2 cI^2}{3 c1 c2}, 0, 0, -\frac{s2^3 cI^2}{3 c2 rI}$$

  
$$0, \frac{c2^2 rI^2}{cI}, 0, 0$$


```

(7.7)

```

> a11 := (c2^3+c1^2)/(c1*c2):
b11 := -c2:
d11 := -c1^2/(3*c2):
d21 := -(1/3)*c1:
r11 := abs(c1)^(1/2)/c2:
r21 := c2*r11/c1:
s21 := -3^(1/3)*abs(c1)^(1/2)/c1;
zamproc(a11,b11,c1,d11,1,0,c2,d21, r11,0,r21,s21):
  
$$s21 := -\frac{3^{1/3} \sqrt{|cI|}}{cI}$$

  
$$\frac{|cI| (2 c2^3 + 3 cI^2)}{3 c2^3 cI}, 0, 0, \frac{|cI|}{cI}$$

  
$$0, \frac{|cI|}{cI}, 0, 0$$


```

(7.8)

```

> solve([N[1,2],N[1,3],N[2,1],N[2,3],N[2,4]],{a1,b1,c1,d1,c2,d2,eta,theta});
{a1=a1, b1=-a1 η-2 a1 θ+η²+2 η θ, c1=a1 η²+a1 θ+η θ²-η³-4 η² θ-η θ², d2=a1 η-a1 θ-η²-2 η θ, dI=- $\frac{1}{3}$  a1 η³
+ $\frac{1}{3}$  η⁴- $\frac{1}{3}$  a1 η² θ+ $\frac{4}{3}$  η³ θ- $\frac{1}{3}$  a1 η θ²+ $\frac{4}{3}$  η² θ², d2=- $\frac{1}{3}$  a1 η²+ $\frac{1}{3}$  η³+ $\frac{4}{3}$  η² θ- $\frac{1}{3}$  a1 η θ+ $\frac{2}{3}$  a1 θ²+ $\frac{1}{3}$  η θ², η=θ, θ
=θ}

```

(7.9)

```

> b11 := -a1*eta-2*a1*theta+eta^2+2*eta*theta:
c11 := a1*eta^2+a1*eta*theta+a1*theta^2-eta^3-4*eta^2*theta-eta*theta^2:
c21 := a1*eta-a1*theta-eta^2-2*eta*theta:
d11 := -(1/3)*a1*eta^3+(1/3)*eta^4-(1/3)*a1*eta^2*theta+(4/3)*eta^3*theta-(1/3)*a1*eta*theta^2+
(4/3)*eta^2*theta^2:
d21 := -(1/3)*a1*eta^2+(1/3)*eta^3+(4/3)*eta^2*theta-(1/3)*a1*eta*theta+(2/3)*a1*theta^2+(1/3)*
eta*theta^2:
r11 := eta*r2:
s11 := theta*s2:
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r2,s2):
  
$$\frac{(\eta-\theta)^2 r2^2 (2 aI + \eta)}{3}, 0, 0, -\frac{(\eta-\theta)^2 (aI - \eta - 3 \theta) s2^3}{3 r2}$$

  
$$0, (\eta-\theta)^2 (aI - \eta) r2^2, 0, 0$$


```

(7.10)

```

> b11 := -a1*eta-2*a1*theta+eta^2+2*eta*theta:
c11 := collect(a1*eta^2+a1*eta*theta+a1*theta^2-eta^3-4*eta^2*theta-eta*theta^2, theta, factor);
c21 := a1*eta-a1*theta-eta^2-2*eta*theta:

```

```

d11 := collect(-(1/3)*a1*eta^3+(1/3)*eta^4-(1/3)*a1*eta^2*theta+(4/3)*eta^3*theta-(1/3)*a1*eta*theta^2+(4/3)*eta^2*theta^2, theta, factor);
d21 := collect(-(1/3)*a1*eta^2+(1/3)*eta^3+(4/3)*eta^2*theta-(1/3)*a1*eta*theta+(2/3)*a1*theta^2+(1/3)*eta*theta^2, theta, factor);
r21 := abs(a1-eta)^(-1/2)*(eta-theta)^(-1);
s21 := -3^(1/3)*(eta-theta)^(-1)*(a1-eta-3*theta)^(-1/3)*abs(a1-eta)^(-1/2)*(a1-eta)^(1/3);
r11 := eta*r21;
s11 := theta*s21;
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r21,s21):
  c11 := (a1-η) θ² + η (a1-4 η) θ + η² (a1-η)
  d11 := - $\frac{\eta (a1-4 \eta) \theta^2}{3} - \frac{\eta^2 (a1-4 \eta) \theta}{3} - \frac{\eta^3 (a1-\eta)}{3}$ 
  d21 :=  $\left( \frac{2 a1}{3} + \frac{\eta}{3} \right) \theta^2 - \frac{\eta (a1-4 \eta) \theta}{3} - \frac{\eta^2 (a1-\eta)}{3}$ 
  r21 :=  $\frac{1}{\sqrt{|a1-\eta|} (\eta-\theta)}$ 
  s21 := - $\frac{3^{1/3} (a1-\eta)^{1/3}}{(\eta-\theta) (a1-\eta-3 \theta)^{1/3} \sqrt{|a1-\eta|}}$ 
         $\frac{2 a1+\eta}{3 |a1-\eta|}, 0, 0, \frac{a1-\eta}{|a1-\eta|}$ 
         $0, \frac{a1-\eta}{|a1-\eta|}, 0, 0$ 

```

(7.11)

```

> solve([b1=-a1*eta-2*a1*theta+eta^2+2*eta*theta, c2=a1*eta-a1*theta-eta^2-2*eta*theta], {eta,theta})
);
 $\left\{ \eta = \text{RootOf}(3 a1 \_Z^2 + (-3 a1^2 - 2 b1 - 2 c2) \_Z - a1 b1 + 2 a1 c2), \theta = -\frac{b1 + c2}{3 a1} \right\}$ 

```

(7.12)

```

> solve(3*a1*_Z^2+(-3*a1^2-2*b1-2*c2)*_Z-a1*b1+2*a1*c2, _Z);
 $\frac{3 a1^2 + 2 b1 + 2 c2 + \sqrt{9 a1^4 + 24 a1^2 b1 - 12 a1^2 c2 + 4 b1^2 + 8 b1 c2 + 4 c2^2}}{6 a1},$ 
 $-\frac{3 a1^2 + \sqrt{9 a1^4 + 24 a1^2 b1 - 12 a1^2 c2 + 4 b1^2 + 8 b1 c2 + 4 c2^2}}{6 a1} - 2 b1 - 2 c2$ 

```

(7.13)

```

> solve([N[1,2],N[1,3],N[2,1],N[2,3],N[2,4]],{a1,b1,c1,d1,d2,eta,theta});
 $\left\{ a1 = a1, b1 = -\frac{3 a1^2 \eta - 3 a1 \eta^2 - 2 a1 c2 + 2 c2 \eta}{a1 + 2 \eta}, c1 = \frac{3 a1^3 \eta^2 - 3 a1^2 \eta^3 - 3 a1 \eta^4 + 3 \eta^5 - 3 a1^2 c2 \eta + 6 a1 c2 \eta^2 + 6 c2 \eta^3 + a1 c22 - c22 \eta}{(a1 + 2 \eta)^2}, d1 = -\frac{\eta (3 a1^3 \eta^2 - 6 a1^2 \eta^3 + 3 a1 \eta^4 - 3 a1^2 c2 \eta + 12 a1 c2 \eta^2 + a1 c22 - 4 c22 \eta)}{3 (a1 + 2 \eta)^2}, d2 = -\frac{3 a1^2 \eta^3 - 6 a1 \eta^4 + 3 \eta^5 + 3 a1^2 c2 \eta + 6 c2 \eta^3 - 2 a1 c22 - c22 \eta}{3 (a1 + 2 \eta)^2}, \eta = \eta, \theta = \frac{a1 \eta - \eta^2 - c2}{a1 + 2 \eta} \right\}, \left\{ a1 = -2 \text{RootOf}(3 \_Z + c2), b1 = 6 \text{RootOf}(3 \_Z + c2) \theta - c2, c1 = -3 \text{RootOf}(3 \_Z + c2) \theta^2 + 2 c2 \theta + c2 \text{RootOf}(3 \_Z + c2), d1 = -\frac{c2 (6 \text{RootOf}(3 \_Z + c2) \theta + 6 \theta^2 - c2)}{9}, d2 = -\text{RootOf}(3 \_Z + c2) \theta^2 - \frac{c2 \text{RootOf}(3 \_Z + c2)}{3} - \frac{2 c2 \theta}{3}, \eta = \text{RootOf}(3 \_Z + c2), \theta = \theta \right\}$ 

```

(7.14)

```

> z1 := sqrt(-c2)/sqrt(3);
z2 := -sqrt(-c2)/sqrt(3);
 $z1 := \frac{\sqrt{-c2} \sqrt{3}}{3}$ 
 $z2 := -\frac{\sqrt{-c2} \sqrt{3}}{3}$ 

```

(7.15)

```

> a11 := -2*z1:
b11 := 6*z1*theta-c2:
c11 := -3*z1*theta^2+2*c2*theta+c2*z1:
d11 := -(1/9)*c2*(6*z1*theta+6*theta^2-c2):
d21 := -z1*theta^2-(1/3)*c2*z1-2*c2*theta*(1/3):
etal := z1:

```

$$\begin{aligned}
r11 &:= \text{etal1} * r2; \\
s11 &:= \text{theta1} * s2; \\
\text{zamproc}(a11, b11, c11, d11, 1, 0, c2, d21, r11, s11, r2, s2) : \\
&\frac{(-3 \theta (-c2)^3 \sqrt[2]{3} - 3 \theta^3 \sqrt{-c2} \sqrt{3} + c2 (-9 \theta^2 + c2)) r2^2}{-3 \sqrt{-c2} \sqrt{3} + 9 \theta}, 0, 0, -\frac{s2^3 (6 \theta^3 \sqrt{-c2} \sqrt{3} - 2 \theta (-c2)^3 \sqrt[2]{3} - 9 \theta^4 + c2^2)}{9 \left(-\frac{\sqrt{-c2} \sqrt{3}}{3} + \theta \right) r2} \\
&0, \frac{r2^2 (9 c2 \theta^2 + 3 \theta (-c2)^3 \sqrt[2]{3} - c2^2 + 3 \theta^3 \sqrt{-c2} \sqrt{3})}{\sqrt{-c2} \sqrt{3} - 3 \theta}, 0, 0
\end{aligned} \tag{7.16}$$

$$> \text{solve}(b1=6*z1*theta-c2, \theta); \\
\frac{(b1+c2)\sqrt{3}}{6\sqrt{-c2}} \tag{7.17}$$

$$\begin{aligned}
a11 &:= -2*z1; \\
\text{theta1} &:= (b1+c2)*\sqrt{3}/(6*\sqrt{-c2}); \\
c11 &:= -3*z1*\text{theta1}^2+2*c2*\text{theta1}+c2*z1; \\
d11 &:= -(1/9)*c2*(6*z1*\text{theta1}+6*\text{theta1}^2-c2); \\
d21 &:= -z1*\text{theta1}^2-(1/3)*c2*z1-2*c2*\text{theta1}*(1/3); \\
\text{etal1} &:= z1; \\
r11 &:= \text{etal1}*r2; \\
s11 &:= \text{theta1}*s2; \\
\text{zamproc}(a11, b1, c11, d11, 1, 0, c2, d21, r11, s11, r2, s2) : \\
&\frac{\sqrt{3} (3 c2 + b1)^2 r2^2}{36 \sqrt{-c2}}, 0, 0, \frac{\sqrt{3} (b1 - c2) (3 c2 + b1)^2 s2^3}{72 (-c2)^3 \sqrt[2]{r2}} \\
&0, -\frac{\sqrt{3} (3 c2 + b1)^2 r2^2}{12 \sqrt{-c2}}, 0, 0
\end{aligned} \tag{7.18}$$

$$\begin{aligned}
a11 &:= -2*z1; \\
\text{theta1} &:= (b1+c2)*\sqrt{3}/(6*\sqrt{-c2}); \\
c11 &:= \text{evala}(-3*z1*\text{theta1}^2+2*c2*\text{theta1}+c2*z1); \\
d11 &:= \text{evala}(-(1/9)*c2*(6*z1*\text{theta1}+6*\text{theta1}^2-c2)); \\
d21 &:= \text{evala}(-z1*\text{theta1}^2-(1/3)*c2*z1-2*c2*\text{theta1}*(1/3)); \\
\text{etal1} &:= z1; \\
r21 &:= 2*3^(1/4)*(-c2)^(1/4)/(3*c2+b1); \\
s21 &:= -\text{simplify}(3^{(-1/12)}*144^(1/3)*(-c2)^(7/12)*(b1-c2)^{(-1/3)}/(3*c2+b1)); \\
r11 &:= \text{etal1}*r21; \\
s11 &:= \text{theta1}*s21; \\
\text{zamproc}(a11, b1, c11, d11, 1, 0, c2, d21, r11, s11, r21, s21) : \\
a11 &:= -\frac{2 \sqrt{-c2} \sqrt{3}}{3} \\
c11 &:= \frac{\sqrt{-c2} \sqrt{3} (b1^2 - 2 b1 c2 + c2^2)}{12 c2} \\
d11 &:= \frac{b1^2}{18} + \frac{c2^2}{18} \\
d21 &:= \frac{\sqrt{-c2} \sqrt{3} (b1^2 + 6 b1 c2 + c2^2)}{36 c2} \\
r21 &:= \frac{2 \sqrt[4]{3} \sqrt[4]{(-c2)^1} \sqrt[4]{4}}{3 c2 + b1} \\
s21 &:= -\frac{2 \sqrt[7]{12} \sqrt[21]{3} \sqrt[3]{(-c2)^7} \sqrt[12]{12}}{(b1 - c2)^{1/3} (3 c2 + b1)} \\
r11 &:= \frac{2 \sqrt[3]{(-c2)^3} \sqrt[4]{3} \sqrt[4]{4}}{3 (3 c2 + b1)} \\
s11 &:= -\frac{(b1 + c2) \sqrt[3]{12} \sqrt[12]{(-c2)^1} \sqrt[12]{21} \sqrt[3]{4}}{(b1 - c2)^{1/3} (3 c2 + b1)} \\
&-\frac{1}{3}, 0, 0, -1 \\
&0, -1, 0, 0
\end{aligned} \tag{7.19}$$

```

> a12 := -2*z2;
b12 := 6*z2*theta-c2;
c12 := -3*z2*theta^2+2*c2*theta+c2*z2;
d12 := -(1/9)*c2*(6*z2*theta+6*theta^2-c2);
d22 := -z2*theta^2-(1/3)*c2*z2-2*c2*theta*(1/3);
eta2 := z2;

```

$$\begin{aligned}
r12 &:= \text{eta2} * r2; \\
s12 &:= \text{theta} * s2; \\
\text{zamproc}(a12, b12, c12, d12, 1, 0, c2, d22, r12, s12, r2, s2) : \\
&\frac{r22 (-9 \theta^2 c2 + c22 + 3 \theta (-c2)^3 \sqrt[2]{3} + 3 \theta^3 \sqrt{-c2} \sqrt{3})}{3 \sqrt{-c2} \sqrt{3} + 9 \theta}, 0, 0, -\frac{s23 (-6 \theta^3 \sqrt{-c2} \sqrt{3} + 2 \theta (-c2)^3 \sqrt[2]{3} - 9 \theta^4 + c22)}{3 r2 (\sqrt{-c2} \sqrt{3} + 3 \theta)}, \\
&0, \frac{r22 (-9 \theta^2 c2 + c22 + 3 \theta (-c2)^3 \sqrt[2]{3} + 3 \theta^3 \sqrt{-c2} \sqrt{3})}{\sqrt{-c2} \sqrt{3} + 3 \theta}, 0, 0
\end{aligned} \tag{7.20}$$

$$> \text{solve}(b1=6*z2*theta-c2, \theta); \\
-\frac{(b1+c2)\sqrt{3}}{6\sqrt{-c2}} \tag{7.21}$$

$$\begin{aligned}
> a12 &:= -2*z2; \\
\text{theta2} &:= -(b1+c2)*\sqrt(3)/(6*\sqrt(-c2)); \\
c12 &:= -3*z2*theta2^2+2*c2*theta2+c2*z2; \\
d12 &:= -(1/9)*c2*(6*z2*theta2+6*theta2^2-c2); \\
d22 &:= -z2*theta2^2-(1/3)*c2*z2-2*c2*theta2*(1/3); \\
\text{eta2} &:= z2; \\
r12 &:= \text{eta2}*r2; \\
s12 &:= \text{theta2}*s2; \\
\text{zamproc}(a12, b1, c12, d12, 1, 0, c2, d22, r12, s12, r2, s2) : \\
&\frac{\sqrt{3} (3 c2+b1)^2 r22}{36 \sqrt{-c2}}, 0, 0, -\frac{\sqrt{3} (b1-c2) (3 c2+b1)^2 s23}{72 (-c2)^3 \sqrt[2]{r2}} \\
&0, \frac{\sqrt{3} (3 c2+b1)^2 r22}{12 \sqrt{-c2}}, 0, 0
\end{aligned} \tag{7.22}$$

$$\begin{aligned}
> a12 &:= -2*z2; \\
\text{theta2} &:= -(b1+c2)*\sqrt(3)/(6*\sqrt(-c2)); \\
c12 &:= \text{evala}(-3*z2*theta2^2+2*c2*theta2+c2*z2); \\
d12 &:= \text{evala}(-(1/9)*c2*(6*z2*theta2+6*theta2^2-c2)); \\
d22 &:= \text{evala}(-z2*theta2^2-(1/3)*c2*z2-2*c2*theta2*(1/3)); \\
\text{eta2} &:= z2; \\
r22 &:= 2*3^(1/4)*(-c2)^(1/4)/(3*c2+b1); \\
s22 &:= \text{simplify}(-3^(-1/12)*144^(1/3)*(-c2)^(7/12)*(b1-c2)^(-1/3)/(3*c2+b1)); \\
r12 &:= \text{eta2}*r22; \\
s12 &:= \text{theta2}*s22; \\
\text{zamproc}(a12, b1, c12, d12, 1, 0, c2, d22, r12, s12, r22, s22) : \\
a12 &:= \frac{2 \sqrt{-c2} \sqrt{3}}{3} \\
c12 &:= -\frac{\sqrt{-c2} \sqrt{3} (b12 - 2 b1 c2 + c22)}{12 c2} \\
d12 &:= \frac{b12}{18} + \frac{c22}{18} \\
d22 &:= -\frac{\sqrt{-c2} \sqrt{3} (b12 + 6 b1 c2 + c22)}{36 c2} \\
r22 &:= \frac{2 \sqrt[4]{3} \sqrt[4]{(-c2)^4}}{3 c2 + b1} \\
s22 &:= -\frac{2 \sqrt[12]{2} \sqrt[3]{(-c2)^7} \sqrt[12]{3}}{(b1 - c2) \sqrt[3]{(3 c2 + b1)}} \\
r12 &:= -\frac{2 \sqrt[4]{(-c2)^3} \sqrt[4]{3} \sqrt[4]{3}}{3 (3 c2 + b1)} \\
s12 &:= \frac{(b1 + c2) \sqrt[12]{(-c2)^1} \sqrt[12]{3}}{(b1 - c2) \sqrt[3]{(3 c2 + b1)}} \\
&\frac{1}{3}, 0, 0, 1 \\
&0, 1, 0, 0
\end{aligned} \tag{7.23}$$

$$\begin{aligned}
> \text{solve}([N[1,2], N[1,3], N[2,1], N[2,3], N[2,4]], \{a1, b1, c1, d1, c2, \text{eta}, \theta\}); \\
\left\{ \begin{array}{l} a1 = -\frac{-\eta^3 - 4\theta\eta^2 - \theta^2\eta + 3d2}{(\eta + 2\theta)(\eta - \theta)}, b1 = \frac{3(-\theta\eta^2 - \theta^2\eta + d2)}{\eta - \theta}, c1 = -\frac{3(-\eta^3\theta^2 - 4\eta^2\theta^3 - \eta\theta^4 + d2\eta^2 + d2\eta\theta + d2\theta^2)}{(\eta + 2\theta)(\eta - \theta)}, c2 = \frac{3(\theta^2\eta + d2)}{\eta + 2\theta}, d1 = \frac{\eta(-3\eta^2\theta^3 - 3\eta\theta^4 + d2\eta^2 + d2\eta\theta + d2\theta^2)}{(\eta + 2\theta)(\eta - \theta)} \end{array} \right\}, \left\{ \begin{array}{l} \eta = \theta, \theta = \theta \\ a1 = a1, b1 = 0, c1 = 0, d1 = 0 \end{array} \right\}
\end{aligned} \tag{7.24}$$

$$= \frac{3 a1 \operatorname{RootOf}(\underline{Z} + 4 d2)^2}{4} - 3 d2, c2 = \frac{3 a1 \operatorname{RootOf}(\underline{Z} + 4 d2)}{2}, d1 = a1 d2, \eta = \operatorname{RootOf}(\underline{Z}^3 + 4 d2), \theta = -\frac{\operatorname{RootOf}(\underline{Z}^3 + 4 d2)}{2} \}$$

```
> z1 := -(4*d2)^(1/3); z1 := -41^(1/3) d2^(1/3) (7.25)
```

```
> c11 := 3*a1*z1^2*(1/4)-3*d2;
c21 := 3*a1*z1*(1/2);
d11 := a1*d2;
etal := z1;
thetal := -z1/2;
r11 := etal*r2;
s11 := thetal*s2;
zamproc(a1,0,c11,d11,1,0,c21,d2, r11,s11,r2,s2):
3 21^(1/3) d22^(1/3) r22 (-22^(1/3) d21^(1/3) + a1), 0, 0, -3 d22^(1/3) s23 (-22^(1/3) d21^(1/3) + a1) 21^(1/3)
0, 9 r22 d22^(1/3) (22^(1/3) d21^(1/3) + a1) 21^(1/3), 0, 0 (7.26)
```

```
> c11 := 3*a1*z1^2*(1/4)-3*d2;
c21 := 3*a1*z1*(1/2);
d11 := a1*d2;
etal := z1;
thetal := -z1/2;
r21 := 2^(1/3)/(3*d2^(1/3)*abs(2^(2/3)*d2^(1/3)+a1)^(1/2));
s21 := 2^(2/3)*(2^(2/3)*d2^(1/3)+a1)^(1/3)/((2^(2/3)*d2^(1/3)-2*a1)^(1/3)*abs(2^(2/3)*d2^(1/3)+a1)^(1/2)*d2^(1/3)*3^(2/3));
r11 := etal*r21;
s11 := thetal*s21;
zamproc(a1,0,c11,d11,1,0,c21,d2, r11,s11,r21,s21):
c11 := 3 a1 42^(1/3) d22^(1/3) / 4 - 3 d2
c21 := -3 a1 41^(1/3) d21^(1/3) / 2
r21 := 21^(1/3)
3 d21^(1/3) sqrt[22^(1/3) d21^(1/3) + a1]
s21 := 22^(1/3) (22^(1/3) d21^(1/3) + a1)^(1/3) 31^(1/3)
3 (22^(1/3) d21^(1/3) - 2 a1)^(1/3) sqrt[22^(1/3) d21^(1/3) + a1] d21^(1/3)
22^(1/3) (a1 21^(1/3) d22^(1/3) - d2), 0, 0, 22^(1/3) (22^(1/3) d21^(1/3) + a1) (a1 21^(1/3) d22^(1/3) - d2)
3 22^(1/3) d21^(1/3) + a1) d22^(1/3), 0, 0, 22^(1/3) (a1 21^(1/3) + 2 d21^(1/3)) / 2 22^(1/3) d21^(1/3) + a1 (7.27)
```

```
> d1 = evala(2^(2/3)*(2^(2/3)*d2^(1/3)+a1)*(a1*2^(1/3)*d2^(2/3)-d2)/(d2^(2/3)*abs(2^(2/3)*d2^(1/3)+a1)*(-2^(2/3)*d2^(1/3)+2*a1)));
d1 = 22^(1/3) d21^(1/3) + a1 / 22^(1/3) d21^(1/3) + a1 (7.28)
```

CF 3-20

```
> solve([M[1,2],M[1,3],M[2,2],M[2,3],M[2,4]],{a1,b1,c1,d1,c2,d2,r1,s1,r2,s2});
{a1=0,b1=0,c1=0,c2=0,d1=d1,d2=d2,r1=0,r2=r2,s1=s1,s2=0}, {a1=-d1 r23 s23 - r13 s1 s22 - s12 s2 r12 r2 - r1 s13 r22, r23 s13}, (8.1)
```

```
b1 = 3 (d1 r23 s22 - r13 s1 s2 - r12 s12 r2) / r23 s12, c1 = -3 (d1 r23 s2 - r13 s1) / r23 s1, c2 = -3 r1 s1 / r2 s2, d1 = d1, d2 = r1 s1 (r1 s2 + r2 s1) / r22 s22, r1 = r1, r2 = r2, s1 = s1, s2 = s2}, {a1=a1, b1=0, c1=0, c2=0, d1=d1, d2=0, r1=r1, r2=0, s1=0, s2=s2}, {a1=a1, b1=0, c1=0, c2=0, d1=d1, d2=0, r1=r1, r2=r2, s1=0, s2=s2}
```

```
> zamproc(a1,0,0,d1,1,0,0,0, r1,0,0,s2):
a1 r12, 0, 0, d1 s23 / r1
r13 / s2, 0, 0, 0 (8.2)
```

```
> r11 := abs(a1)^(-1/2);
s21 := abs(a1)^(-1/2)/a1;
```

$$\begin{aligned} \text{zamproc}(a1, 0, 0, d1, 1, 0, 0, 0, r11, 0, 0, s21) : \\ \frac{a1}{|a1|}, 0, 0, \frac{dl}{|a1| a1^3} \\ \frac{a1}{|a1|}, 0, 0, 0 \end{aligned} \quad (8.3)$$

$$\begin{aligned} > \text{zamproc}(0, 0, 0, d1, 1, 0, 0, d2, 0, s1, r2, 0) : \\ d2 r22, 0, 0, \frac{s1^3}{r2} \\ \frac{dl r2^3}{s1}, 0, 0, 0 \end{aligned} \quad (8.4)$$

$$\begin{aligned} > r21 := \text{abs}(d2)^{-1/2} : \\ s11 := d1 * \text{abs}(d2)^{-1/2} / d2 : \\ \text{zamproc}(0, 0, 0, d1, 1, 0, 0, d2, 0, s11, r21, 0) : \\ \frac{d2}{|d2|}, 0, 0, \frac{dl^3}{|d2| d2^3} \\ \frac{d2}{|d2|}, 0, 0, 0 \end{aligned} \quad (8.5)$$

$$\begin{aligned} > \text{solve}([M[1, 2], M[1, 3], M[2, 2], M[2, 3], M[2, 4]], \{a1, b1, c1, d2, r1, s1, r2, s2\}) ; \\ \left\{ a1 = -\frac{s2 (c2^3 s2^4 - 3 c2^2 s2^2 s1^2 + 9 c2 s1^4 + 27 dl s1^2 s2^2)}{27 s1^5}, b1 = \frac{s2^2 (c2^3 s2^2 - 3 c2^2 s1^2 + 27 dl s1^2)}{9 s1^4}, c1 = -\frac{s2 (c2^3 s2^2 + 27 dl s1^2)}{9 s1^3}, \right. \\ \left. d2 = \frac{c2 (c2 s2^2 - 3 s1^2)}{9 s2 s1}, r1 = -\frac{c2 r2 s2}{3 s1}, r2 = r2, s1 = s1, s2 = s2 \right\} \end{aligned} \quad (8.6)$$

$$> \text{solve}([N[1, 2], N[1, 3], N[2, 2], N[2, 3], N[2, 4]], \{a1, b1, c1, d1, c2, d2, \eta, \theta\}) ; \\ \{a1 = a1, b1 = -3 a1 \theta + 3 \eta, c1 = 3 a1 \theta^2 - 3 \eta^2 \theta - 3 \eta \theta^2, c2 = -3 \eta, d1 = -a1 \theta^3 + \eta^3 \theta + \eta^2 \theta^2 + \eta \theta^3, d2 = \eta^2 \theta + \eta \theta^2, \eta = \eta, \theta = \theta\} \quad (8.7)$$

$$\begin{aligned} > b11 := -3 * a1 * \theta + 3 * \eta : \\ c11 := 3 * a1 * \theta^2 - 3 * \eta^2 \theta - 3 * \eta \theta^2 : \\ c21 := -3 * \theta * \eta : \\ d11 := -a1 * \theta^3 + 3 * \eta^3 \theta + 3 * \eta^2 \theta^2 + 3 * \eta \theta^3 : \\ d21 := \eta^2 \theta + \eta \theta^2 : \\ r11 := \eta * r2 : \\ s11 := \theta * s2 : \\ \text{zamproc}(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r2, s2) : \\ (\eta - \theta)^2 r22 a1, 0, 0, \frac{(\eta - \theta)^2 \theta s2^3}{r2} \\ - \frac{(\eta - \theta)^2 (a1 - \eta) r2^3}{s2}, 0, 0, 0 \end{aligned} \quad (8.8)$$

$$\begin{aligned} > b11 := -3 * a1 * \theta + 3 * \eta : \\ c11 := 3 * a1 * \theta^2 - 3 * \eta^2 \theta - 3 * \eta \theta^2 : \\ c21 := -3 * \theta * \eta : \\ d11 := -a1 * \theta^3 + 3 * \eta^3 \theta + 3 * \eta^2 \theta^2 + 3 * \eta \theta^3 : \\ d21 := \eta^2 \theta + \eta \theta^2 : \\ r21 := \text{abs}(a1)^{-1/2} / (\eta + \theta) : \\ s21 := (-a1 + \eta) / ((\eta - a1) * \text{abs}(a1)^{1/2}) : \\ r11 := \eta * r21 : \\ s11 := \theta * s21 : \\ \text{zamproc}(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r21, s21) : \\ c11 := 3 a1 \theta^2 - 3 \eta^2 \theta - 3 \eta \theta^2 \\ d11 := -a1 \theta^3 + \eta^3 \theta + \eta^2 \theta^2 + \eta \theta^3 \\ d21 := \eta^2 \theta + \eta \theta^2 \\ s21 := \frac{-a1 + \eta}{(\eta - a1) a1 \sqrt{|a1|}} \\ \frac{a1}{|a1|}, 0, 0, -\frac{(a1 - \eta)^3 \theta}{a1^3 |a1|} \\ \frac{a1}{|a1|}, 0, 0, 0 \end{aligned} \quad (8.9)$$

$$> \text{solve}([b1 = -3 * a1 * \theta + 3 * \eta, c2 = -3 * \theta * \eta], \{\eta, \theta\}) ; \\ \left\{ \eta = \frac{c2 a1}{b1 + c2}, \theta = -\frac{b1 + c2}{3 a1} \right\} \quad (8.10)$$

```
> etal := c2 * a1 / (b1 + c2) :
thetal := -(b1 + c2) / (3 * a1) :
c11 := evala(3 * a1 * thetal^2 - 3 * etal^2 * thetal^2 - 3 * etal * thetal^2);
```

```

d11 := -a1*theta1^3+a1^3*theta1+a1^2*theta1^2+a1*theta1^3;
d21 := a1^2*theta1+a1*theta1^2;
r21 := abs(a1)^(-1/2)/(a1-theta1):
s21 := evala((-a1+a1)/((a1-theta1)*a1*abs(a1)^(1/2)));
r11 := a1*r21;
s11 := theta1*s21:
zamproc(a1,b1,c1,d11,1,0,c2,d21, r11,s11,r21,s21):
c11 :=  $\frac{3 c22 a1^2 + b1^3 + 2 b1^2 c2 + b1 c2^2}{3 a1 (b1 + c2)}$ 
d11 :=  $\frac{(b1 + c2)^3}{27 a1^2} - \frac{c23 a1^2}{3 (b1 + c2)^2} + \frac{c22}{9} - \frac{c2 (b1 + c2)^2}{27 a1^2}$ 
d21 :=  $-\frac{c22 a1}{3 (b1 + c2)} + \frac{c2 (b1 + c2)}{9 a1}$ 
s21 :=  $-\frac{3 a1 b1}{(3 c2 a1^2 + b1^2 + 2 b1 c2 + c2^2) \sqrt{|a1|}}$ 
 $\frac{a1}{|a1|}, 0, 0, \frac{b1^3}{3 |a1| a1 (b1 + c2)^2}$ 
 $\frac{a1}{|a1|}, 0, 0, 0$ 

```

(8.11)

```

> solve([N[1,2],N[1,3],N[2,2],N[2,3],N[2,4]],[a1,b1,c1,d1,c2,eta,theta]):
 $\left\{ a1 = a1, b1 = -\frac{3 \operatorname{RootOf}(\mathcal{Z} + 4 d2) (\operatorname{RootOf}(\mathcal{Z} + 4 d2) + 2 a1)}{2}, c1 = 3 a1 \operatorname{RootOf}(\mathcal{Z} + 4 d2)^2 - 3 d2, c2 = \frac{3 \operatorname{RootOf}(\mathcal{Z} + 4 d2)^2}{2} \right\}, \quad (8.12)$ 

```

$$\begin{aligned}
d1 &= \frac{d2 (8 a1 + 3 \operatorname{RootOf}(\mathcal{Z} + 4 d2))}{2}, \eta = -\frac{\operatorname{RootOf}(\mathcal{Z} + 4 d2)}{2}, \theta = \operatorname{RootOf}(\mathcal{Z} + 4 d2), \left\{ a1 = \right. \\
&\left. -\frac{3 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) \theta^2 + 2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) b1 + \theta b1 - 6 d2}{3 \theta (2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) + \theta)}, b1 = b1, c1 = \right. \\
&\left. -\frac{3 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) \theta^3 + 2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) b1 \theta + b1 \theta^2 + 6 d2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) - 3 d2 \theta}{2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) + \theta}, c2 \right. \\
&= \frac{3 (\operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) \theta^2 - 2 d2)}{2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) + \theta}, d1 \\
&= \frac{b1 \theta^4 + 2 b1 \theta^3 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) - 3 d2 \theta^2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) + 6 d2^2}{3 \theta (2 \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2) + \theta)}, \eta = \operatorname{RootOf}(\theta \mathcal{Z} + \theta^2 \mathcal{Z} - d2), \theta \\
&= \theta \left. \right\}
\end{aligned}$$

```

> z1 := -(4*d2)^(1/3);
z1 :=  $-4^{1/3} d2^{1/3}$  (8.13)

```

```

> b11 := -3*z1*(z1+2*a1)*(1/2):
c11 := 3*a1*z1^2-3*d2:
c21 := 3*z1^2*(1/2):
d11 := (1/2)*d2*(8*a1+3*z1):
eta1 := -(1/2)*z1:
theta1 := z1:
r11 := eta1*r2:
s11 := theta1*s2:
zamproc(a1,b11,c11,d11,1,0,c21,d2, r11,s11,r21,s21):
 $\frac{9 r22 a1 d2^2 \sqrt[3]{21}}{2}, 0, 0, -\frac{9 s23 d2}{r2}$ 
 $\frac{9 r23 d2^2 \sqrt[3]{(22 \sqrt[3]{d2})^3 - 2 a1} \sqrt[3]{21}}{4 s2}, 0, 0, 0$ 

```

(8.14)

```

> b11 := -3*z1*(z1+2*a1)*(1/2):
c11 := 3*a1*z1^2-3*d2;
c21 := 3*z1^2*(1/2):
d11 := (1/2)*d2*(8*a1+3*z1);
eta1 := -(1/2)*z1;
theta1 := z1:
r21 := 2^(1/3)*abs(a1)^(-1/2)*d2^(-1/3)/3;
s21 := (-a1*2^(1/3)*d2^(-1/3)+1)*abs(a1)^(-1/2)/(3*a1);
r11 := eta1*r21;
s11 := theta1*s21:
zamproc(a1,b11,c11,d11,1,0,c21,d2, r11,s11,r21,s21):
b11 :=  $\frac{3 4^{1/3} d2^{1/3} (-4^{1/3} d2^{1/3} + 2 a1)}{2}$ 

```

$$\begin{aligned}
c11 &:= 3 \cdot a1 \cdot 42 \cdot 3 \cdot d22 \cdot 3 - 3 \cdot d2 \\
c21 &:= \frac{3 \cdot 42 \cdot 3 \cdot d22 \cdot 3}{2} \\
d11 &:= \frac{d2 \cdot (8 \cdot a1 - 3 \cdot 41 \cdot 3 \cdot d21 \cdot 3)}{2} \\
\eta l &:= \frac{41 \cdot 3 \cdot d21 \cdot 3}{2} \\
r21 &:= \frac{21 \cdot 3}{3 \sqrt{|a1|} \cdot d21 \cdot 3} \\
s21 &:= \frac{-\frac{a1 \cdot 21 \cdot 3}{d21 \cdot 3} + 1}{3 \sqrt{|a1|} \cdot a1} \\
\frac{a1}{|a1|}, 0, 0, & \frac{d21 \cdot 3 \cdot 22 \cdot 3 \cdot (a1 \cdot 21 \cdot 3 - d21 \cdot 3)^3}{2 \cdot a1^3 \cdot |a1|} \\
& \frac{(a1 \cdot 21 \cdot 3 \cdot d22 \cdot 3 - d2) \cdot a1}{|a1| \cdot d22 \cdot 3 \cdot (a1 \cdot 21 \cdot 3 - d21 \cdot 3)}, 0, 0, 0
\end{aligned} \tag{8.15}$$

$$\begin{aligned}
> \text{solve}([N[1,2], N[1,3], N[2,2], N[2,3], N[2,4]], \{a1, c1, c2, d2, eta, theta\}); \\
\left\{ \begin{aligned}
a1 &= \frac{3 \eta \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) - b1}{3 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1)}, c1 = \\
& - \frac{3 \eta^3 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) + 2 \cdot b1 \cdot \eta \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) + 3 \cdot d1}{2 \eta + \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1)}, c2 = \\
& - \frac{3 \eta \cdot (3 \eta^3 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) + 2 \cdot b1 \cdot \eta \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) + 3 \cdot d1)}{(3 \eta^2 + b1) \cdot (2 \eta + \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1))}, d2 = \\
& = \frac{1}{(3 \eta^2 + b1)^2 \cdot (2 \eta + \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1))} \cdot (\eta \cdot (3 \cdot b1 \cdot \eta^4 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) \\
& + 2 \cdot b1^2 \cdot \eta^2 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) + 18 \cdot d1 \cdot \eta^3 + 9 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) \cdot d1 \cdot \eta^2 + 9 \cdot b1 \cdot d1 \cdot \eta \\
& + 3 \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1) \cdot b1 \cdot d1)), \eta = \eta, \theta = \text{RootOf}((3 \eta^2 + b1) \cdot Z + 3 \eta^3 \cdot Z - 3 \cdot d1)\}, \{a1 = \\
& - \frac{3 \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)^2 + 2 \cdot b1}{6 \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)}, c1 = \frac{b1 \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)^2 - 6 \cdot d1}{\text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)}, c2 = \\
& - \frac{4 \cdot (b1 \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)^2 - 3 \cdot d1)}{\text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)^2}, d2 = \frac{2 \cdot (b1 \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)^2 - 3 \cdot d1)}{3 \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)}, \eta = \\
& - \frac{\text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)}{2}, \theta = \text{RootOf}(3 \cdot Z^4 + 8 \cdot b1 \cdot Z^2 - 24 \cdot d1)\}
\end{aligned} \right. \tag{8.16}
\end{aligned}$$

$$> \text{solve}(3 \cdot Z^4 + 8 \cdot Z^2 \cdot b1 - 24 \cdot d1, Z); \\
\frac{\sqrt{6 \sqrt{4 \cdot b1^2 + 18 \cdot d1} - 12 \cdot b1}}{3}, -\frac{\sqrt{6 \sqrt{4 \cdot b1^2 + 18 \cdot d1} - 12 \cdot b1}}{3}, \frac{\sqrt{-6 \sqrt{4 \cdot b1^2 + 18 \cdot d1} - 12 \cdot b1}}{3}, -\frac{\sqrt{-6 \sqrt{4 \cdot b1^2 + 18 \cdot d1} - 12 \cdot b1}}{3} \tag{8.17}$$

$$\begin{aligned}
> \text{a11} := -(3 \cdot z1^2 + 2 \cdot b1) / (6 \cdot z1); \\
\text{c11} := (\text{b1} \cdot z1^2 - 6 \cdot d1) / z1; \\
\text{c21} := -(4 \cdot (\text{b1} \cdot z1^2 - 3 \cdot d1)) / z1^2; \\
\text{d21} := (2 \cdot (\text{b1} \cdot z1^2 - 3 \cdot d1)) / (3 \cdot z1); \\
\text{etal} := -(1/2) \cdot z1; \\
\text{theta1} := z1; \\
\text{r11} := \text{etal} \cdot \text{r2}; \\
\text{s11} := \text{theta1} \cdot \text{s2}; \\
\text{zamproc}(\text{a11}, \text{b1}, \text{c11}, \text{d1}, 1, 0, \text{c21}, \text{d21}, \text{r11}, \text{s11}, \text{r2}, \text{s2}): \\
\frac{r22 \cdot (-3 \cdot z1^4 + 46 \cdot b1 \cdot z1^2 - 192 \cdot d1)}{24 \cdot z1}, \frac{r2 \cdot s2 \cdot (3 \cdot z1^4 + 8 \cdot b1 \cdot z1^2 - 24 \cdot d1)}{4 \cdot z1}, -\frac{4 \cdot s2^2 \left(\frac{3}{8} \cdot z1^4 + b1 \cdot z1^2 - 3 \cdot d1 \right)}{z1}, \\
-\frac{10 \left(-\frac{3}{10} \cdot z1^4 + b1 \cdot z1^2 - 3 \cdot d1 \right) \cdot s2^3}{3 \cdot z1 \cdot r2}, \\
\frac{3 \cdot r2^3 \cdot z1 \cdot b1}{4 \cdot s2}, 0, 0, 0
\end{aligned} \tag{8.18}$$

```

> unassign('z1');
a11 := -(3*z1^2+2*b1)/(6*z1);
c11 := (b1*z1^2-6*d1)/z1;
c21 := -(4*(b1*z1^2-3*d1))/z1^2;
d21 := (2*(b1*z1^2-3*d1))/(3*z1);
etal := -(1/2)*z1;

```

```

theta1 := z1;
r21 := simplify(sqrt(24)*abs(z1)^(1/2)*abs(-3*z1^4+46*b1*z1^2-192*d1+3*z1^4+8*b1*z1^2-24*d1)^(1/2));
);
s21 := 34992*z1^2*b1*abs(z1)^(1/2)*abs(b1*z1^2-4*d1)^(1/2)*(b1*z1^2-4*d1);
r11 := eta1*r21;
s11 := theta1*s21;
zamproc(a11,b1,c11,d1,1,0,c21,d21, r11,s11,r21,s21):
a11 := - $\frac{3zI^2+2bI}{6zI}$ 
c11 :=  $\frac{bIzI^2-6dI}{zI}$ 
c21 := - $\frac{4(bIzI^2-3dI)}{zI^2}$ 
d21 :=  $\frac{2(bIzI^2-3dI)}{3zI}$ 
r21 :=  $36\sqrt{|zI|}\sqrt{|bIzI^2-4dI|}$ 
s21 :=  $34992zI^2bI\sqrt{|zI|}\sqrt{|bIzI^2-4dI|}(bIzI^2-4dI)$ 
 $\frac{(-162zI^4+2484bIzI^2-10368dI)\sqrt{|zI|}(bIzI^2-4dI)}{zI}, 2519424zIbI\left(\frac{3}{8}zI^4+bIzI^2-3dI\right)(bIzI^2-4dI)\sqrt{|zI|}(bIzI^2-4dI),$ 
 $-4897760256zI^3bI^2\left(\frac{3}{8}zI^4+bIzI^2-3dI\right)(bIzI^2-4dI)^2\sqrt{|zI|}(bIzI^2-4dI), -3967185807360\left(-\frac{3}{10}zI^4+bIzI^2$ 
 $-3dI\right)zI^5bI^3(bIzI^2-4dI)^3\sqrt{|zI|}(bIzI^2-4dI)$ 
 $\frac{\sqrt{|zI|}(bIzI^2-4dI)}{zI(bIzI^2-4dI)}, 0, 0, 0$ 

```

(8.19)

```

> 34992/36;
ifactor(3967185807360);
simplify(-3*z1^4+10*b1*z1^2-30*d1+3*z1^4+8*b1*z1^2-24*d1);
 $\frac{972}{18bIzI^2-54dI}$ 

```

(8.20)

```

> z1 := (1/3)*sqrt(6*sqrt(4*b1^2+18*d1)-12*b1);
z1 :=  $\frac{\sqrt{6\sqrt{4bI^2+18dI}-12bI}}{3}$ 

```

(8.21)

```

> a11 := -(3*z1^2+2*b1)/(6*z1):
c11 := (b1*z1^2-6*d1)/z1:
c21 := -(4*(b1*z1^2-3*d1))/z1^2:
d21 := (2*(b1*z1^2-3*d1))/(3*z1):
eta1 := -(1/2)*z1:
theta1 := z1:
r21 := simplify(sqrt(24)*abs(z1)^(1/2)*abs(-3*z1^4+46*b1*z1^2-192*d1+3*z1^4+8*b1*z1^2-24*d1)^(1/2));
);
s21 := 34992*z1^2*b1*abs(z1)^(1/2)*abs(b1*z1^2-4*d1)^(1/2)*(b1*z1^2-4*d1);
r11 := eta1*r21;
s11 := theta1*s21;
zamproc(a11,b1,c11,d1,1,0,c21,d21, r11,s11,r21,s21):
r21 :=  $12\sqrt[4]{4bI^2+18dI}|- \sqrt{4bI^2+18dI} + 2bI|^{\frac{1}{4}}\sqrt{|-bI\sqrt{4bI^2+18dI} + 2bI^2 + 6dI|}$ 
s21 :=  $1296(6\sqrt{4bI^2+18dI}$ 
 $-12bI)$ 
 $bI\sqrt{3}\sqrt{\sqrt{6}\sqrt{|-\sqrt{4bI^2+18dI} + 2bI|}}\sqrt{\left|-\frac{bI(6\sqrt{4bI^2+18dI} - 12bI)}{9} + 4dI\right|}\left(\frac{bI(6\sqrt{4bI^2+18dI} - 12bI)}{9}\right.$ 
 $\left.-4dI\right)$ 
 $\frac{1}{(\sqrt{4bI^2+18dI} - 2bI)\sqrt{6\sqrt{4bI^2+18dI} - 12bI}}$ 
 $(2592\sqrt{3}\sqrt{2}| - bI\sqrt{4bI^2+18dI} + 2bI^2$ 
 $+ 6dI|\sqrt{|-\sqrt{4bI^2+18dI} + 2bI|}(-4bI^3 + 2\sqrt{4bI^2+18dI}bI^2 - 15bIdI + 3\sqrt{4bI^2+18dI}dI)), 0, 0,$ 

```

$$\begin{aligned}
& \left(82170781731127296 \sqrt{|-\sqrt{4 b l^2 + 18 d l} + 2 b l|} \sqrt{2} \right) \left| -\frac{b l \sqrt{4 b l^2 + 18 d l}}{3} + \frac{2 b l^2}{3} + 2 d l \right|^3 \sqrt{2} \\
& - \frac{\left(b l^8 + \frac{39}{4} b l^6 d l + \frac{1953}{64} b l^4 d l^2 + \frac{16605}{512} b l^2 d l^3 + \frac{2187}{256} d l^4 \right) \left(b l^2 + \frac{21 d l}{8} \right) b l \sqrt{4 b l^2 + 18 d l}}{2} + \left(b l^4 + \frac{39}{8} b l^2 d l \right. \\
& \left. + \frac{27}{8} d l^2 \right) \left(b l^8 + \frac{39}{4} b l^6 d l + \frac{1953}{64} b l^4 d l^2 + \frac{16119}{512} b l^2 d l^3 + \frac{729}{256} d l^4 \right) b l^3 \Bigg) \\
& \left(\sqrt{|-b l \sqrt{4 b l^2 + 18 d l} + 2 b l^2 + 6 d l|} \sqrt{6 \sqrt{4 b l^2 + 18 d l} - 12 b l} (-\sqrt{4 b l^2 + 18 d l} + 2 b l) \right) \\
& \frac{|-b l \sqrt{4 b l^2 + 18 d l} + 2 b l^2 + 6 d l|^3 \sqrt{|-\sqrt{4 b l^2 + 18 d l} + 2 b l|} \sqrt{2}}{, 0, 0, 0}, \quad (8.22)
\end{aligned}$$

```

> solve([N[1,2],N[1,3],N[2,2],N[2,3],N[2,4]],{b1,c1,c2,d2,eta,theta});
{b1 = -3 a1 θ + 3 RootOf(_Z^3 + θ^2 _Z + θ^3 _Z - a1 θ^3 - d1) θ, c1 = - $\frac{1}{2 \operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1) + \theta}$  (3 (a1 θ^3 - 2 a1 RootOf(_Z θ + _Z θ^2 + _Z θ^3 - a1 θ^3 - d1) θ^2 - θ^3 RootOf(_Z θ + _Z θ^2 + _Z θ^3 - a1 θ^3 - d1) + RootOf(_Z θ + _Z θ^2 + _Z θ^3 - a1 θ^3 - d1)^2 θ^2 + 2 d1)), c2 =  $\frac{3 (\operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1)^2 \theta^2 + 2 \theta^3 \operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1) - 2 a1 \theta^3 - 2 d1)}{(2 \operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1) + \theta) \operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1)}$ , d2 =  $\frac{\theta^3 \operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1) - a1 \theta^3 - d1}{\operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1)}, \eta = \operatorname{RootOf}(_Z \theta + _Z \theta^2 + _Z \theta^3 - a1 \theta^3 - d1), \theta = \theta}, {b1 =  $\frac{3 \operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1) (2 a1 + \operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1))}{2}, c1 = \frac{a1 \operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)^3 - 2 d1}{\operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)}$ , c2 =  $\frac{4 (a1 \operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)^3 + d1)}{\operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)^2}, d2 = \frac{2 (a1 \operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)^3 + d1)}{3 \operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)}$ , η =  $\operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)$ , θ =  $\operatorname{RootOf}(3 Z^4 + 8 _Z a1 + 8 d1)$ , b1 =  $\frac{3 d1}{\operatorname{RootOf}(_Z a1 + d1)^2}$ , c1 =  $\frac{3 d1}{\operatorname{RootOf}(_Z a1 + d1)}$ , c2 = 0, d2 = 0, η = 0, θ =  $\operatorname{RootOf}(_Z a1 + d1)$ } } ;$ 
```

```

> b11 := 3*d1/z1^2:
c11 := -3*d1/z1:
theta1 := z1:
s11 := theta1*s2:
zamproc(a1,b11,c11,d1,1,0,0,0, 0,s11,r2,s2):
  
$$\frac{\frac{dl\ r22}{(dl)^{1/3}}, 0, 0, -\frac{s23\ dl}{al\ r2}}{(dl)^{1/3}}$$

  
$$-\frac{dl\ r23}{(dl)^{1/3}\ s2}, 0, 0, 0$$


```

(8.25)

```

> b11 := 3*d1/z1^2;
c11 := -3*d1/z1;
theta1 := z1;
r21 := d1^(-1/3)*abs(a1)^(-1/6);
s21 := -d1^(-1/3)*abs(a1)^(-1/6);
s11 := theta1*s21;
zamproc(a1,b11,c11,d1,1,0,0,0, 0,s11,r21,s21):

$$b11 := \frac{3 \frac{dl}{al}}{\left(\frac{dl}{al}\right)^2 \sqrt[3]{}}$$


$$c11 := \frac{3 \frac{dl}{al}}{\left(\frac{dl}{al}\right)^1 \sqrt[3]{}}$$


```

$$\begin{aligned}
r21 &:= \frac{1}{dI^{1/3} |aI|^{1/6}} \\
s21 &:= -\frac{1}{dI^{1/3} |aI|^{1/6}} \\
s11 &:= \frac{\left(\frac{dI}{aI}\right)^{1/3}}{dI^{1/3} |aI|^{1/6}} \\
\frac{dI^{1/3}}{|aI|^{1/3} \left(\frac{dI}{aI}\right)^{1/3}}, 0, 0, \frac{dI^{1/3}}{|aI|^{1/3} aI} \\
\frac{dI^{1/3}}{|aI|^{1/3} \left(\frac{dI}{aI}\right)^{1/3}}, 0, 0, 0
\end{aligned} \tag{8.26}$$

CF 3-23

$$> \text{solve}([\text{M}[1,1], \text{M}[1,3], \text{M}[2,2], \text{M}[2,3], \text{M}[2,4]], \{a1, b1, c1, d1, c2, d2, r1, s1, r2, s2\}); \\
\{a1=0, b1=b1, c1=0, c2=0, d1=d1, d2=0, r1=r1, r2=0, s1=0, s2=s2\}, \left\{ a1 = \frac{2(d2 r2^2 s2^2 - r1^2 s1 s2 - r1 r2 s1^2)}{(r1 s2 + 2 r2 s1)(r1 s2 - r2 s1)}, b1 = \right. \tag{9.1}$$

$$-\frac{3 s1 (2 d2 r2^3 s2 - r1^3 s2 - 3 r1^2 r2 s1)}{(r1 s2 + 2 r2 s1)(r1 s2 - r2 s1) r2}, c1 = -\frac{3 (d2 r1^2 r2 s2^3 - d2 r1 r2^2 s1 s2^2 - 2 d2 r2^3 s1^2 s2 + 2 r1^3 s1^2 s2 + 2 r1^2 r2 s1^3)}{s2 r2 (r1 s2 - r2 s1)(r1 s2 + 2 r2 s1)}, \\
c2 = -\frac{3 (d2 r2 s2^2 + r1 s1^2)}{s2 (r1 s2 + 2 r2 s1)}, d1 = \frac{d2 r1^3 s2^4 + d2 r1^2 r2 s1 s2^3 - 2 d2 r1 r2^2 s1^2 s2^2 - 2 d2 r2^3 s1^3 s2 + r1^3 s1^3 s2 + 3 r1^2 r2 s1^4}{r2 s2^2 (r1 s2 + 2 r2 s1)(r1 s2 - r2 s1)}, d2$$

$$=d2, r1=r1, r2=r2, s1=s1, s2=s2\}, \left\{ a1 = -\frac{d1 s2^4 + 6 s1^4}{s1^3 s2}, b1 = \frac{3 (d1 s2^4 + 4 s1^4)}{s1^2 s2^2}, c1 = \frac{6 (d1 s2^4 + 5 s1^4)}{s1 s2^3}, c2 = \right. \tag{9.1}$$

$$-\frac{3 (3 d1 s2^4 + 14 s1^4)}{2 s1^2 s2^2}, d1 = d1, d2 = \frac{2 s1^3}{s2^3}, r1=r1, r2 = -\frac{r1 s2}{2 s1}, s1=s1, s2=s2\}, \{a1=0, b1=0, c1=0, c2=c2, d1=d1, d2=0, \\
r1=0, r2=r2, s1=s1, s2=0\}, \left\{ a1 = -\frac{d1 s2^3}{s1^3}, b1 = \frac{3 d1 s2^2}{s1^2}, c1 = -\frac{3 d1 s2}{s1}, c2 = -\frac{3 d1 s2^2}{2 s1^2}, d1 = d1, d2 = \frac{d1 s2}{s1}, r1=0, r2=r2, \right. \tag{9.1}$$

$$s1=s1, s2=s2\}$$

$$> \# 1) \\
\text{zamproc}(0, b1, 0, d1, 1, 0, 0, 0, r1, 0, 0, s2): \\
0, b1 r1 s2, 0, \frac{d1 s2^3}{r1} \\
\frac{r1^3}{s2}, 0, 0, 0 \tag{9.2}$$

$$> r11 := \text{abs}(d1)^{-1/8}: \\
s21 := r11^{3/8}: \\
\text{zamproc}(0, b1, 0, d1, 1, 0, 0, 0, r11, 0, 0, s21): \\
0, \frac{b1}{\sqrt{|d1|}}, 0, \frac{d1}{|d1|} \\
1, 0, 0, 0 \tag{9.3}$$

$$> \# 2) \\
\text{zamproc}(0, 0, 0, d1, 1, 0, c2, 0, 0, s1, r2, 0): \\
0, c2 r2 s1, 0, \frac{s1^3}{r2} \\
\frac{d1 r2^3}{s1}, 0, 0, 0 \tag{9.4}$$

$$> r21 := \text{abs}(d1)^{-3/8}: \\
s11 := d1 * r21^{3/8}; \\
\text{zamproc}(0, 0, 0, d1, 1, 0, c2, 0, 0, s11, r21, 0): \\
s11 := \frac{d1}{|d1|^{\beta/8}} \\
0, \frac{c2 d1}{|d1|^{\beta/2}}, 0, \frac{d1^3}{|d1|^{\beta}} \\
1, 0, 0, 0 \tag{9.5}$$

$$> \# 3) \\
s11 := (d2/2)^{(1/3)*s2}: \\
a11 := \text{evala}(-(d1*s2^4+6*s11^4)/(s11^3*s2)); \\
b11 := \text{evala}((3*(d1*s2^4+4*s11^4))/(s11^2*s2^2)); \\
c11 := \text{evala}((6*(d1*s2^4+5*s11^4))/(s11*s2^3));$$

```

c21 := evala(-(3*(3*d1*s2^4+14*s11^4))/(2*s11^2*s2^2));
r21 := -r1*s2/(2*s11);
zamproc(a11,b11,c11,d1,1,0,c21,d2, r1,s11,r21,s2):
  a11 := - $\frac{3 \sqrt[3]{d2^4} + 2 d1}{d2}$ 
  b11 :=  $\frac{3 (\sqrt[3]{d2^4} + d1) \sqrt[3]{d2}}{d2^2 \sqrt[3]{d2}}$ 
  c11 :=  $\frac{3 (5 \sqrt[3]{d2^4} + 4 d1) \sqrt[3]{d2}}{2 d2^1 \sqrt[3]{d2}}$ 
  c21 := - $\frac{3 (7 \sqrt[3]{d2^4} + 6 d1) \sqrt[3]{d2}}{4 d2^2 \sqrt[3]{d2}}$ 
  r21 := - $\frac{r1 \sqrt[3]{d2}}{2 d2^1 \sqrt[3]{d2}}$ 
  0, - $\frac{27 s2 r1 \sqrt[3]{d2} (3 d2^4 + 21 \sqrt[3]{d2})}{4 d2^2 \sqrt[3]{d2}}, 0, \frac{9 s2^3 (22 \sqrt[3]{d2^4} + d1)}{r1}$ 
  - $\frac{9 r1^3 (21 \sqrt[3]{d2} + 2 d2^4 \sqrt[3]{d2})}{4 d2^4 \sqrt[3]{s2}}, 0, 0, 0$ 

```

(9.6)

```

> s21 := -9*r1^3*(2^(1/3)*d1+2*d2^(4/3))/(4*d2^(4/3));
s11 := (d2/2)^(1/3)*s21;
a11 := evala(-(d1*s21^4+6*s11^4)/(s11^3*s21));
b11 := evala((3*(d1*s21^4+4*s11^4))/(s11^2*s21^2));
c11 := evala((6*(d1*s21^4+5*s11^4))/(s11*s21^3));
c21 := evala(-(3*(3*d1*s21^4+14*s11^4))/(2*s11^2*s21^2));
r21 := -r1*s21/(2*s11);
zamproc(a11,b11,c11,d1,1,0,c21,d2, r1,s11,r21,s21):
  s21 := - $\frac{9 r1^3 (21 \sqrt[3]{d2} + 2 d2^4 \sqrt[3]{d2})}{4 d2^4 \sqrt[3]{d2}}$ 
  a11 := - $\frac{3 \sqrt[3]{d2^4} + 2 d1}{d2}$ 
  b11 :=  $\frac{3 (\sqrt[3]{d2^4} + d1) \sqrt[3]{d2}}{d2^2 \sqrt[3]{d2}}$ 
  c11 :=  $\frac{3 (5 \sqrt[3]{d2^4} + 4 d1) \sqrt[3]{d2}}{2 d2^1 \sqrt[3]{d2}}$ 
  c21 := - $\frac{3 (7 \sqrt[3]{d2^4} + 6 d1) \sqrt[3]{d2}}{4 d2^2 \sqrt[3]{d2}}$ 
  r21 := - $\frac{r1 \sqrt[3]{d2}}{2 d2^1 \sqrt[3]{d2}}$ 
  0,  $\frac{243 (3 d2^4 + 21 \sqrt[3]{d2}) (21 \sqrt[3]{d2} + 2 d2^4 \sqrt[3]{d2}) 21 \sqrt[3]{d2} r1^4}{16 d2^2}, 0, -\frac{6561 (22 \sqrt[3]{d2^4} + d1) (21 \sqrt[3]{d2} + 2 d2^4 \sqrt[3]{d2})^3 r1^8}{64 d2^4}$ 
  1, 0, 0, 0

```

(9.7)

```

> ifactor(6561);
(3)^8

```

(9.8)

```

> r11 := 2^(5/8)*abs(d2)^(1/2)*abs(2^(2/3)*d2^(4/3)+d1)^(-1/2)/3;
s21 := -9*r11^3*(2^(1/3)*d1+2*d2^(4/3))/(4*d2^(4/3));
s11 := (d2/2)^(1/3)*s21;
a11 := evala(-(d1*s21^4+6*s11^4)/(s11^3*s21));
b11 := evala((3*(d1*s21^4+4*s11^4))/(s11^2*s21^2));
c11 := evala((6*(d1*s21^4+5*s11^4))/(s11*s21^3));
c21 := evala(-(3*(3*d1*s21^4+14*s11^4))/(2*s11^2*s21^2));
r21 := -r11*s21/(2*s11);
zamproc(a11,b11,c11,d1,1,0,c21,d2, r11,s11,r21,s21):
  r11 :=  $\frac{25 \sqrt[8]{d2}}{3 \sqrt[3]{22 \sqrt[3]{d2^4} + d1}}$ 
  s21 :=  $\frac{27 \sqrt[8]{d2} \sqrt[2]{(21 \sqrt[3]{d2} + 2 d2^4 \sqrt[3]{d2})}}{6 \sqrt[3]{22 \sqrt[3]{d2^4} + d1} \sqrt[2]{d2^4 \sqrt[3]{d2}}}$ 
  a11 := - $\frac{(d1 \sqrt[3]{d2} + 3 d2^2) \sqrt[3]{d2}}{d2^5 \sqrt[3]{d2}}$ 
  b11 :=  $\frac{3 (d1 \sqrt[3]{d2} + 2 d2^2) \sqrt[3]{d2}}{d2^4 \sqrt[3]{d2}}$ 
  c11 :=  $\frac{3 (2 d1 \sqrt[3]{d2} + 5 d2^2)}{d2}$ 
  c21 := - $\frac{3 (3 d1 \sqrt[3]{d2} + 7 d2^2) \sqrt[3]{d2}}{2 d2^4 \sqrt[3]{d2}}$ 

```

$$r21 := -\frac{2^{23} \sqrt{d2}^4}{6 \sqrt{|2^2 \sqrt{3} d2^4 \sqrt{3} + dI|} d2^1 \sqrt{3}} \\ 0, \frac{3 \cdot 2^5 \cdot 6 (2^1 \sqrt{3} dI + 3 d2^4 \sqrt{3}) (2^1 \sqrt{3} dI + 2 d2^4 \sqrt{3}) |d2|^2}{4 d2^2 |2^2 \sqrt{3} d2^4 \sqrt{3} + dI|^2}, 0, -\frac{2^1 \sqrt{8} (2 \cdot 2^1 \sqrt{3} d2^2 + 27 \sqrt{8} d2^2 \sqrt{3} dI) (2^1 \sqrt{3} dI + 2 d2^4 \sqrt{3})^3 |d2|^4}{4 d2^4 \sqrt{3} |2^2 \sqrt{3} d2^4 \sqrt{3} + dI|^4} \\ \frac{(2^5 \sqrt{6} dI d2^2 \sqrt{3} + 2 \sqrt{2} d2^2) \sqrt{2}}{d2^2 \sqrt{3} (2 \cdot 2^1 \sqrt{3} dI + 4 d2^4 \sqrt{3})}, 0, 0, 0 \quad (9.9)$$

```
> # 4)
s21 := s1*d2/d1;
a11 := -d1*s21^3/s1^3;
b11 := 3*d1*s21^2/s1^2;
c11 := -3*d1*s21/s1;
c21 := -3*d1*s21^2/(2*s1^2);
zamproc(a11,b11,c11,d1,1,0,c21,d2, 0,s1,r2,s21):
0,  $\frac{3 d22 s1 r2}{2 dI}$ , 0,  $\frac{s1^3 (-d24 + 2 dI^3)}{2 dI^3 r2}$ 
 $\frac{dI r2^3}{s1}$ , 0, 0, 0 \quad (9.10)
```

```
> r21 := 2^(1/8)*abs(2*d1^3-d2^4)^(-1/8);
s11 := d1*r21^3;
s21 := s11*d2/d1;
a11 := -d1*s21^3/s11^3;
b11 := 3*d1*s21^2/s11^2;
c11 := -3*d1*s21/s11;
c21 := -3*d1*s21^2/(2*s11^2);
zamproc(a11,b11,c11,d1,1,0,c21,d2, 0,s11,r21,s21):
r21 :=  $\frac{2^1 \sqrt{8}}{|-d24 + 2 dI^3| \sqrt{8}}$ 
a11 :=  $-\frac{d23}{dI^2}$ 
b11 :=  $\frac{3 d22}{dI}$ 
c11 := -3 d2
c21 :=  $-\frac{3 d22}{2 dI}$ 
0,  $\frac{3 d22 \sqrt{2}}{2 \sqrt{|-d24 + 2 dI^3|}}, 0, \frac{-d24 + 2 dI^3}{|-d24 + 2 dI^3|}$ 
1, 0, 0, 0 \quad (9.11)
```

```
> solve([N[1,1],N[1,3],N[2,2],N[2,3],N[2,4]],{a1,b1,c1,d1,c2,d2,eta,theta});
{a1=a1, b1=-3 a1 θ+3 ηθ, c1=  $\frac{3}{2}$  a1 ηθ-3 η²θ+3 a1 θ²-3 ηθ²-  $\frac{3}{2}$  a1 η², c2=-  $\frac{3}{2}$  a1 η+  $\frac{3}{2}$  a1 θ-3 ηθ, d1=  $\frac{1}{2}$  a1 η³+η³θ  $\theta=\eta, \theta=0$  (9.12)
+  $\frac{1}{2}$  a1 η²θ-a1 ηθ²+η²θ²-a1 θ³+ηθ³, d2=  $\frac{1}{2}$  a1 η²+  $\frac{1}{2}$  a1 ηθ+η²θ-a1 θ²+ηθ², η=η, θ=0}
```

```
> # 5)
b11 := -3*a1*theta+3*eta*theta;
c11 := (3/2)*a1*eta*theta^3*eta^2*theta+3*a1*theta^2-3*eta*theta^2-(3/2)*a1*eta^2;
c21 := -(3/2)*a1*eta+(3/2)*a1*theta-3*eta*theta;
d11 := (1/2)*a1*eta^3+eta^3*theta+(1/2)*a1*eta^2*theta-a1*eta*theta^2+eta^2*theta^2-a1*theta^3+eta*theta^3;
d21 := (1/2)*a1*eta^2+(1/2)*a1*eta*theta+eta^2*theta-a1*theta^2+eta*theta^2;
r11 := eta*r2;
s11 := theta*s2;
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r2,s2):
0, -  $\frac{3 (\eta-\theta)^2 a1 r2 s2}{2}$ , 0,  $\frac{(\eta-\theta)^2 (a1+2 \theta) s2^3}{2 r2}$ 
-  $\frac{(\eta-\theta)^2 (a1-\eta) r2^3}{s2}$ , 0, 0, 0 \quad (9.13)
```

```
> b11 := -3*a1*theta+3*eta*theta;
c11 := (3/2)*a1*eta*theta-3*eta^2*theta+3*a1*theta^2-3*eta*theta^2-(3/2)*a1*eta^2;
c21 := -(3/2)*a1*eta+(3/2)*a1*theta-3*eta*theta;
d11 := collect((1/2)*a1*eta^3+eta^3*theta+(1/2)*a1*eta^2*theta-a1*eta*theta^2+eta^2*theta^2-a1*theta^3+eta*theta^3, theta, factor);
d21 := collect((1/2)*a1*eta^2+(1/2)*a1*eta*theta+eta^2*theta-a1*theta^2+eta*theta^2, theta,
```

```

factor);
r21 := 2^(1/8)*abs(a1+2*theta)^(-1/8)*(eta-theta)^(-1)*abs(a1-eta)^(-3/8);
s21 := -(eta-theta)^2*(a1-eta)*r21^3;
r11 := eta*r21;
s11 := theta*s21;
zamproc(a1,b11,c11,d11,1,0,c21,d21, r11,s11,r21,s21):

$$c21 := -\frac{3}{2} a1 \eta + \frac{3}{2} a1 \theta - 3 \eta \theta$$


$$d11 := (-a1 + \eta) \theta^3 - \eta (a1 - \eta) \theta^2 + \frac{\eta^2 (a1 + 2 \eta) \theta}{2} + \frac{a1 \eta^3}{2}$$


$$d21 := (-a1 + \eta) \theta^2 + \frac{\eta (a1 + 2 \eta) \theta}{2} + \frac{a1 \eta^2}{2}$$


$$r21 := \frac{21^{1/8}}{|a1 + 2 \theta|^{1/8} (\eta - \theta) |a1 - \eta|^{3/8}}$$


$$s21 := -\frac{(a1 - \eta) 23^{1/8}}{(\eta - \theta) |a1 + 2 \theta|^{3/8} |a1 - \eta|^9/8}$$


$$0, \frac{3 (a1 - \eta) a1 \sqrt{2}}{2 |a1 - \eta|^3 \sqrt[12]{|a1 + 2 \theta|}}, 0, -\frac{(a1 - \eta)^3 (a1 + 2 \theta)}{|a1 - \eta|^3 |a1 + 2 \theta|}$$

1, 0, 0, 0

```

(9.14)

```

> solve([b1=-3*a1*theta+3*eta*theta, c1=(3/2)*a1*eta*theta-3*eta^2*theta+3*a1*theta^2-3*eta*theta^2-3*a1*eta^2], {eta,theta});

$$\left\{ \eta = \frac{3 a1 \text{RootOf}(9 a1^2 + 6 b1) \text{Z}^3 + (9 a1^3 + 9 a1 b1 + 6 c1) \text{Z}^2 + (6 a1^2 b1 + 2 b1^2) \text{Z} + a1 b1^2 + b1}{3 \text{RootOf}(9 a1^2 + 6 b1) \text{Z}^3 + (9 a1^3 + 9 a1 b1 + 6 c1) \text{Z}^2 + (6 a1^2 b1 + 2 b1^2) \text{Z} + a1 b1^2}, \theta = \text{RootOf}(9 a1^2 + 6 b1) \text{Z} \right. \\ \left. + (9 a1^3 + 9 a1 b1 + 6 c1) \text{Z}^2 + (6 a1^2 b1 + 2 b1^2) \text{Z} + a1 b1^2 \right\}$$


```

(9.15)

```

> solve([N[1,1],N[1,3],N[2,2],N[2,3],N[2,4]],{a1,b1,d1,c2,d2,eta,theta});

$$\left\{ a1 = -\frac{2 (3 \eta^2 \theta + 3 \eta \theta^2 + c1)}{3 (\eta + \theta) (\eta - 2 \theta)}, b1 = \frac{\theta (3 \eta^3 + 3 \eta^2 \theta + 2 c1)}{(\eta + \theta) (\eta - 2 \theta)}, c2 = \frac{3 \eta^2 \theta^2 + 3 \eta \theta^3 + c1 \eta - c1 \theta}{(\eta + \theta) (\eta - 2 \theta)}, d1 = \right. \\ \left. -\frac{6 \eta^3 \theta^2 - 3 \eta^2 \theta^3 + c1 \eta^2 - 2 c1 \theta^2}{3 (\eta - 2 \theta)}, d2 = -\frac{6 \eta^3 \theta^2 + 6 \eta^2 \theta^3 + c1 \eta^2 + c1 \eta \theta - 2 c1 \theta^2}{3 (\eta + \theta) (\eta - 2 \theta)}, \eta = \eta, \theta = \theta \right\}, \left\{ a1 = a1, b1 = \right. \\ \left. -3 a1 \text{RootOf}(18 \text{Z}^3 + c1) + 6 \text{RootOf}(18 \text{Z}^3 + c1)^2, c2 = -\frac{3 \text{RootOf}(18 \text{Z}^3 + c1) (4 \text{RootOf}(18 \text{Z}^3 + c1) + a1)}{2}, d1 = \right. \\ \left. -\frac{c1 (3 a1 + 14 \text{RootOf}(18 \text{Z}^3 + c1))}{18}, d2 = 2 a1 \text{RootOf}(18 \text{Z}^3 + c1)^2 - \frac{c1}{3}, \eta = 2 \text{RootOf}(18 \text{Z}^3 + c1), \theta = \text{RootOf}(18 \text{Z}^3 + c1) \right\}$$


```

(9.16)

```

> # 6)
z1 := -(c1/18)^(1/3):
b11 := -3*a1*z1+6*z1^2:
c21 := -3*z1*(4*z1+a1)*(1/2):
d11 := -(1/18)*c1*(3*a1+14*z1):
d21 := 2*a1*z1^2-(1/3)*c1:
etal1 := 2*z1:
thetal1 := z1:
r11 := etal1*r2:
s11 := thetal1*s2:
zamproc(a1,b11,c1,d11,1,0,c21,d21, r11,s11,r2,s2):

$$0, -\frac{r2 s2 a1 c1^2 \sqrt[3]{181}^3}{12}, 0, -\frac{s2^3 c1^2 \sqrt[3]{182}^3 c1 \sqrt[3]{-9 a1} 18^1 \sqrt[3]{3}}{324 r2}$$


$$-\frac{r2^3 18^1 \sqrt[3]{3} \left(\frac{182 \sqrt[3]{c1} \sqrt[3]{3}}{9} + a1\right) c1^2 \sqrt[3]{3}}{18 s2}, 0, 0, 0$$


```

(9.17)

```

> b11 := -3*a1*z1+6*z1^2:
c21 := -3*z1*(4*z1+a1)*(1/2):
d11 := -(1/18)*c1*(3*a1+14*z1):
d21 := 2*a1*z1^2-(1/3)*c1:
etal1 := 2*z1:
thetal1 := z1:
s21 := -c1^(2/3)*18^(1/3)*((1/9)*18^(2/3)*c1^(1/3)+a1)*r2^3/18;
r11 := etal1*r2:
s11 := thetal1*s21:
zamproc(a1,b11,c1,d11,1,0,c21,d21, r11,s11,r2,s21):

```

$$s21 := -\frac{cI^2 \sqrt[3]{18} \sqrt[3]{cI}^3}{18} + aI \\ 0, \frac{aI \sqrt[3]{182} \sqrt[3]{cI^4} \sqrt[3]{(182 \sqrt[3]{cI}^3 + 9 aI) r^{24}}}{1944}, 0, -\frac{\left(18 \sqrt[3]{aI^4 cI^2} \sqrt[3]{3} + 4 cI aI^3 - \frac{8 \sqrt[3]{18} \sqrt[3]{cI^5} \sqrt[3]{aI}}{9} - \frac{8 cI^2}{9}\right) r^{28} cI^2}{11664} \\ 1, 0, 0, 0 \quad (9.18)$$

$$> \text{ifactor}(11664); \quad (2)^4 (3)^6 \quad (9.19)$$

$$\begin{aligned} &> b11 := -3*a1*z1+6*z1^2; \\ &c21 := -3*z1*(4*z1+a1)*(1/2); \\ &d11 := -(1/18)*c1*(3*a1+14*z1); \\ &d21 := 2*a1*z1^2-(1/3)*c1; \\ &\text{eta1} := 2*z1; \\ &\text{theta1} := z1; \\ &r21 := 2^{(1/2)}*3*abs(c1)^{-1/4}*abs(9*18^{(1/3)}*a1^4*c1^{(2/3)}+36*c1*a1^3-8*18^{(1/3)}*c1^{(5/3)}*a1-8*c1^2)^{-1/8}; \\ &s21 := -c1^{(2/3)}*18^{(1/3)}*((1/9)*18^{(2/3)}*c1^{(1/3)}+a1)*r21^3/18; \\ &r11 := \text{eta1}*r21; \\ &s11 := \text{theta1}*s21; \\ &\text{zamproc}(a1,b11,c1,d11,1,0,c21,d21, r11,s11,r21,s21): \\ &\quad b11 := \frac{aI \sqrt[3]{182} \sqrt[3]{cI}^3}{6} + \frac{18 \sqrt[3]{cI^2} \sqrt[3]{3}}{3} \\ &\quad c21 := \frac{182 \sqrt[3]{cI}^3 \left(-\frac{2 \sqrt[3]{182} \sqrt[3]{cI}^3}{9} + aI\right)}{12} \\ &\quad d11 := -\frac{cI \left(3 aI - \frac{7 \sqrt[3]{182} \sqrt[3]{cI}^3}{9}\right)}{18} \\ &\quad d21 := \frac{aI \sqrt[3]{18} \sqrt[3]{cI^2} \sqrt[3]{3}}{9} - \frac{cI}{3} \\ &\quad r21 := \frac{3 \sqrt{2}}{|cI|^4 |9 \sqrt[3]{18} \sqrt[3]{cI^4 cI^2} \sqrt[3]{3} - 8 \sqrt[3]{18} \sqrt[3]{cI^5} \sqrt[3]{aI} + 36 cI aI^3 - 8 cI^2|^{1/8}} \\ &0, \frac{aI \sqrt[3]{182} \sqrt[3]{cI^4} \sqrt[3]{(182 \sqrt[3]{cI}^3 + 9 aI) \sqrt[3]{9 \sqrt[3]{18} \sqrt[3]{cI^4 cI^2} \sqrt[3]{3} - 8 \sqrt[3]{18} \sqrt[3]{cI^5} \sqrt[3]{aI} + 36 cI aI^3 - 8 cI^2}}}{9}, 0, \\ &\quad \frac{54 \left(\left(18 \sqrt[3]{aI^4 cI^2} \sqrt[3]{3} + 4 cI aI^3 - \frac{8 \sqrt[3]{18} \sqrt[3]{cI^5} \sqrt[3]{aI}}{9} - \frac{8 cI^2}{9}\right) cI\right)}{|cI|^2 |-9 \sqrt[3]{18} \sqrt[3]{aI^4 cI^2} \sqrt[3]{3} + 8 \sqrt[3]{18} \sqrt[3]{cI^5} \sqrt[3]{aI} - 36 cI aI^3 + 8 cI^2|} \\ &\quad - \frac{9 \left(\frac{182 \sqrt[3]{cI}^3}{9} + aI\right)^3 (aI \sqrt[3]{cI^2} \sqrt[3]{3} - 2 cI) cI}{|cI|^2 |-9 \sqrt[3]{18} \sqrt[3]{aI^4 cI^2} \sqrt[3]{3} + 8 \sqrt[3]{18} \sqrt[3]{cI^5} \sqrt[3]{aI} - 36 cI aI^3 + 8 cI^2|} \\ &\quad 1, 0, 0, 0 \quad (9.20) \end{aligned}$$

$$> \text{simplify}(\text{expand}(9*((1/9)*18^{(2/3)}*c1^{(1/3)}+a1)^3*(a1*18^{(1/3)}*c1^{(2/3)}-2*c1))); \quad (9.21)$$

$$\begin{aligned} &> \text{solve}([\text{N}[1,1], \text{N}[1,3], \text{N}[2,2], \text{N}[2,3], \text{N}[2,4]], \{a1, b1, c1, c2, d2, \text{eta}, \text{theta}\}); \\ &\left\{aI = \frac{2 (-\eta^3 \theta - \eta^2 \theta^2 - \eta \theta^3 + dI)}{(\eta + \theta) (\eta^2 - 2 \theta^2)}, bI = -\frac{3 \theta (-\eta^4 - 3 \eta^3 \theta + 2 dI)}{(\eta + \theta) (\eta^2 - 2 \theta^2)}, cI = -\frac{3 (2 \theta^2 \eta^3 - \eta^2 \theta^3 + dI \eta - 2 dI \theta)}{\eta^2 - 2 \theta^2}, c2 = \right. \\ &\quad \left. -\frac{3 (\theta^2 \eta^3 - 2 \eta^2 \theta^3 - \eta \theta^4 + dI \eta - dI \theta)}{(\eta + \theta) (\eta^2 - 2 \theta^2)}, d2 = \frac{-\eta^3 \theta^3 - 3 \theta^4 \eta^2 + dI \eta^2 + dI \eta \theta - 2 dI \theta^2}{(\eta + \theta) (\eta^2 - 2 \theta^2)}, \eta = \eta, \theta = \theta\right\}, \left\{aI = \frac{\text{RootOf}(_Z^4 + dI)^2 d2}{dI}, bI = -\frac{3 (\text{RootOf}(_Z^4 + dI) d2 + dI) \text{RootOf}(_Z^4 + dI)^2}{dI}, cI = 0, c2 = \right. \\ &\quad \left. = \frac{3 (\text{RootOf}(_Z^4 + dI) d2 + dI) \text{RootOf}(_Z^4 + dI)^2}{dI}, d2 = d2, \eta = -\text{RootOf}(_Z^4 + dI), \theta = \text{RootOf}(_Z^4 + dI)\right\}, \left\{aI = \right. \\ &\quad \left. \left(\frac{1000003615456565844 \text{RootOf}(14 _Z^8 + 4 dI _Z^4 - dI^2)^5 dI^5 d2}{2401} + \frac{1828433735348598648 \text{RootOf}(14 _Z^8 + 4 dI _Z^4 - dI^2)^4 dI^6}{16807} \right. \right. \\ &\quad \left. \left. - \frac{160189199663901075 \text{RootOf}(14 _Z^8 + 4 dI _Z^4 - dI^2) dI^6 d2}{2401} - \frac{292894277755274721 dI^7}{16807}\right) \middle/ \right. \end{aligned} \quad (9.22)$$

$$\begin{aligned}
& - \frac{707109337701291123 d1^5 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^7}{2401} + \frac{113270869354793949 d1^6 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^3}{2401}, b1 \\
& = \left(\frac{2522695806 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^5 d1^4 d2}{343} + \frac{1783812483 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^4 d1^5}{343} \right. \\
& \quad \left. - \frac{404105949 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2) d1^5 d2}{343} - \frac{285747318 d1^6}{343} \right) \Big/ \left(\frac{594604161 d1^4 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^6}{343} \right. \\
& \quad \left. - \frac{95249106 d1^5 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^2}{343} \right), c1 = \left(- \frac{170559 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^5 d1^3 d2}{49} \right. \\
& \quad \left. - \frac{622998 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^4 d1^4}{343} + \frac{54567 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2) d1^4 d2}{98} + \frac{100089 d1^5}{343} \right) \Big/ \left(\right. \\
& \quad \left. - \frac{56853 d1^3 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^5}{49} + \frac{18189 d1^4 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)}{98} \right), c2 = \left(\right. \\
& \quad \left. - \frac{3996 d1^2 d2 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^5}{49} - \frac{17550 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^4 d1^3}{343} \right. \\
& \quad \left. + \frac{621 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2) d1^3 d2}{49} + \frac{2781 d1^4}{343} \right) \Big/ \left(\frac{810 d1^2 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^6}{49} \right. \\
& \quad \left. - \frac{108 d1^3 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^2}{49} \right), d2 = d2, \eta = - \frac{6 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^5}{2 RootOf(14 _Z^8 + 4 d1 _Z^4 - d1^2)^4 - d1}, \theta = RootOf(14 _Z^8 \\
& \quad + 4 d1 _Z^4 - d1^2) \Big\} \\
> \# 7) \\
z1 := (-d1)^(1/4); \\
z2 := -(-d1)^(1/4); \\
z1 := (-d1)^(1/4) \\
z2 := -(-d1)^(1/4) \\
\end{aligned} \tag{9.23}$$

$$\begin{aligned}
& a11 := z1^2 * d2 / d1: \\
& b11 := -(3 * (z1 * d2 + d1)) * z1^2 / d1: \\
& c21 := (3 * (z1 * d2 + d1)) * z1^2 / d1: \\
& etal := -z1: \\
& thetal := z1: \\
& r11 := etal * r2: \\
& s11 := thetal * s2: \\
& zamproc(a11, b11, 0, d1, 1, 0, c21, d2, r11, s11, r2, s2): \\
& \quad 0, 6 r2 d2 s2, 0, - \frac{4 s2^3 \left(\frac{d2 (-d1)^(1/4)}{2} + d1 \right)}{(-d1)^(1/4) r2} \\
& \quad \frac{4 r2^3 (d2 (-d1)^(1/4) + d1)}{(-d1)^(1/4) s2}, 0, 0, 0
\end{aligned} \tag{9.24}$$

$$\begin{aligned}
& a11 := z1^2 * d2 / d1; \\
& b11 := -(3 * (z1 * d2 + d1)) * z1^2 / d1; \\
& c21 := (3 * (z1 * d2 + d1)) * z1^2 / d1; \\
& etal := -z1; \\
& thetal := z1; \\
& r21 := (-d1)^(1/8) * abs(d2 * (-d1)^(1/4) + 2 * d1)^(1/8) * abs(d2 * (-d1)^(1/4) + d1)^(1/8) * 2^(7/8); \\
& s21 := 4 * (d2 * (-d1)^(1/4) + d1) * r21^3 * (-d1)^(1/4); \\
& r11 := etal * r21; \\
& s11 := thetal * s21; \\
& zamproc(a11, b11, 0, d1, 1, 0, c21, d2, r11, s11, r21, s21): \\
& \quad a11 := \frac{\sqrt{-d1} d2}{d1} \\
& \quad b11 := - \frac{3 (d2 (-d1)^(1/4) + d1) \sqrt{-d1}}{d1}
\end{aligned}$$

$$\begin{aligned}
c21 &:= \frac{3(d2(-dI)^{1/4} + dI)\sqrt{-dI}}{dI} \\
r21 &:= \frac{(-dI)^{1/8} 2^{1/8}}{2|d2(-dI)^{1/4} + 2dI|^{1/8}|d2(-dI)^{1/4} + dI|^3|^{1/8}} \\
s21 &:= \frac{(d2(-dI)^{1/4} + dI)(-dI)^{1/8} 2^{3/8}}{2|d2(-dI)^{1/4} + 2dI|^3|^{1/8}|d2(-dI)^{1/4} + dI|^9|^{1/8}} \\
0, \frac{3\sqrt{2}(-dI)^{1/4}d2(d2(-dI)^{1/4} + dI)}{2|d2(-dI)^{1/4} + dI|^3|^{1/2}\sqrt{|d2(-dI)^{1/4} + 2dI|}}, 0, -\frac{(d2(-dI)^{1/4} + dI)^3(d2(-dI)^{1/4} + 2dI)}{|d2(-dI)^{1/4} + dI|^3|d2(-dI)^{1/4} + 2dI|} \\
1, 0, 0, 0
\end{aligned} \tag{9.25}$$

```

> a12 := z2^2*d2/d1;
b12 := -(3*(z2*d2+d1))*z2^2/d1;
c22 := (3*(z2*d2+d1))*z2^2/d1;
eta2 := -z2;
theta2 := z2;
r12 := eta2*r2;
s12 := theta2*s2;
zamproc(a12,b12,0,d1,1,0,c22,d2, r12,s12,r2,s2):
0, 6 d2 r2 s2, 0,  $\frac{4 \left( -\frac{d2(-dI)^{1/4}}{2} + dI \right) s2}{(-dI)^{1/4} r2}$ 
-  $\frac{4 r2^{3/2} (-d2(-dI)^{1/4} + dI)}{(-dI)^{1/4} s2}, 0, 0, 0$  \tag{9.26}

```

```

> a12 := z2^2*d2/d1;
b12 := -(3*(z2*d2+d1))*z2^2/d1;
c22 := (3*(z2*d2+d1))*z2^2/d1;
eta2 := -z2;
theta2 := z2;
r22 := (-d1)^(1/8)*abs(d2*(-d1)^(1/4)-2*d1)^(-1/8)*abs(d2*(-d1)^(1/4)-d1)^(-3/8)*2^(-7/8);
s22 := 4*(d2*(-d1)^(1/4)-d1)*r22^3*(-d1)^(-1/4);
r12 := eta2*r22;
s12 := theta2*s22;
zamproc(a12,b12,0,d1,1,0,c22,d2, r12,s12,r22,s22):
a12 :=  $\frac{\sqrt{-dI} d2}{dI}$ 
b12 :=  $-\frac{3 (-d2(-dI)^{1/4} + dI)\sqrt{-dI}}{dI}$ 
c22 :=  $\frac{3 (-d2(-dI)^{1/4} + dI)\sqrt{-dI}}{dI}$ 
r22 :=  $\frac{(-dI)^{1/8} 2^{1/8}}{2|-d2(-dI)^{1/4} + 2dI|^{1/8}|-d2(-dI)^{1/4} + dI|^3|^{1/8}}$ 
s22 :=  $\frac{(d2(-dI)^{1/4} - dI)(-dI)^{1/8} 2^{3/8}}{2|-d2(-dI)^{1/4} + 2dI|^3|^{1/8}|-d2(-dI)^{1/4} + dI|^9|^{1/8}}$ 
0, -  $\frac{3\sqrt{2}(-dI)^{1/4}d2(-d2(-dI)^{1/4} + dI)}{2\sqrt{|-d2(-dI)^{1/4} + 2dI|}|d2(-dI)^{1/4} + dI|^3|^{1/2}}, 0, -\frac{2(-d2(-dI)^{1/4} + dI)^3\left(-\frac{d2(-dI)^{1/4}}{2} + dI\right)}{|-d2(-dI)^{1/4} + dI|^3|d2(-dI)^{1/4} - 2dI|}$ 
1, 0, 0, 0 \tag{9.27}

```

```

> solve([N[1,1],N[1,3],N[2,2],N[2,3],N[2,4]],{a1,b1,c1,d1,c2,eta,theta});
 $\begin{cases} a1 = \frac{2(-\eta^2\theta - \eta\theta^2 + d2)}{(\eta + 2\theta)(\eta - \theta)}, b1 = -\frac{3\theta(-\eta^3 - 3\eta^2\theta + 2d2)}{(\eta + 2\theta)(\eta - \theta)}, c1 = -\frac{3(2\eta^3\theta^2 + 2\eta^2\theta^3 + d2\eta^2 - d2\eta\theta - 2d2\theta^2)}{(\eta + 2\theta)(\eta - \theta)}, c2 = -\frac{3(\eta\theta^2 + d2)}{\eta + 2\theta}, dI = \frac{\eta^3\theta^3 + 3\eta^2\theta^4 + d2\eta^3 + d2\eta^2\theta - 2d2\eta\theta^2 - 2d2\theta^3}{(\eta + 2\theta)(\eta - \theta)}, \eta = \eta, \theta = \theta \end{cases}, \{a1 = a1, b1 = -3 a1 \text{RootOf}(2_Z^3 - d2) - 6 \text{RootOf}(2_Z^3 - d2)^2, c1 = -6 a1 \text{RootOf}(2_Z^3 - d2)^2 - 3 d2, c2 = \frac{3 \text{RootOf}(2_Z^3 - d2) (4 \text{RootOf}(2_Z^3 - d2) + 3 a1)}{2}, dI = -\frac{d2 (a1 + 6 \text{RootOf}(2_Z^3 - d2))}{2}, \eta = -2 \text{RootOf}(2_Z^3 - d2), \theta = \text{RootOf}(2_Z^3 - d2)\} \tag{9.28}$ 

```

```

> # 8)
z1 := (d2/2)^(1/3);
zI :=  $\frac{22\sqrt[3]{d2}\sqrt[3]{1}}{2} \tag{9.29}$ 

```

```
> b11 := -3*a1*z1-6*z1^2;
```

```

c11 := -6*a1*z1^2-3*d2;
c21 := 3*z1*(4*z1+3*a1)*(1/2):
d11 := -(1/2)*d2*(a1+6*z1):
etal := -2*z1:
thetal := z1:
r11 := etal*r2:
s11 := thetal*s2:
zamproc(a1,b11,c11,d11,1,0,c21,d2, r11,s11,r2,s2):
0, -  $\frac{27 r2 s2 a1 d22 \sqrt[3]{21}^3}{4}, 0, \frac{9 s23 (a1 \sqrt[3]{21} \sqrt[3]{d22})^3 + 2 d2}{4 r2}$ 
-  $\frac{9 r23 d22 \sqrt[3]{(22 \sqrt[3]{d21})^3 + a1} \sqrt[3]{21}}{2 s2}, 0, 0, 0$ 

```

(9.30)

```

> b11 := -3*a1*z1-6*z1^2:
c11 := -6*a1*z1^2-3*d2:
c21 := 3*z1*(4*z1+3*a1)*(1/2):
d11 := -(1/2)*d2*(a1+6*z1):
etal := -2*z1:
thetal := z1:
s21 := -9*r2^3*d2^(2/3)*(2^(2/3)*d2^(1/3)+a1)*2^(1/3)/2:
r11 := etal*r2:
s11 := thetal*s21:
zamproc(a1,b11,c11,d11,1,0,c21,d2, r11,s11,r2,s21):
0,  $\frac{243 a1 \sqrt[3]{22} \sqrt[3]{d24} \sqrt[3]{(22 \sqrt[3]{d21})^3 + a1} r24}{8}, 0,$ 
-  $\frac{6561 d22 (21 \sqrt[3]{a1^4} d22 \sqrt[3]{3} + 12 22 \sqrt[3]{a1^2} d24 \sqrt[3]{3} + 16 21 \sqrt[3]{a1} d25 \sqrt[3]{3} + 8 a1^3 d2 + 8 d22) r28}{16}, 1, 0, 0, 0$ 

```

(9.31)

```
> ifactor(6561);

$$(3)^8$$


```

(9.32)

```

> evala(2^(1/3)*a1^4*d2^(2/3)+12*2^(2/3)*a1^2*d2^(4/3)+16*2^(1/3)*a1*d2^(5/3)+8*a1^3*d2+8*d2^2);
evala(expand(2^(1/3)*d2^(2/3)*(2^(2/3)*d2^(1/3)+a1)^4));

$$21 \sqrt[3]{d22} \sqrt[3]{(4 22 \sqrt[3]{d21} \sqrt[3]{a1^3} + 12 21 \sqrt[3]{d22} \sqrt[3]{a1^2} + 4 22 \sqrt[3]{d24} \sqrt[3]{a1^4} + 16 a1 d2)} \\ 21 \sqrt[3]{d22} \sqrt[3]{(4 22 \sqrt[3]{d21} \sqrt[3]{a1^3} + 12 21 \sqrt[3]{d22} \sqrt[3]{a1^2} + 4 22 \sqrt[3]{d24} \sqrt[3]{a1^4} + 16 a1 d2)}$$


```

(9.33)

```

> b11 := evala(-3*a1*z1-6*z1^2);
c11 := -6*a1*z1^2-3*d2;
c21 := 3*z1*(4*z1+3*a1)*(1/2);
d11 := -(1/2)*d2*(a1+6*z1);
etal := -2*z1;
thetal := z1;
r21 := 2^(11/24)*d2^(-1/3)*abs(2^(2/3)*d2^(1/3)+a1)^(-1/2)/3;
s21 := -9*r21^3*d2^(2/3)*(2^(2/3)*d2^(1/3)+a1)*2^(1/3)/2;
r11 := etal*r21;
s11 := thetal*s21:
zamproc(a1,b11,c11,d11,1,0,c21,d2, r11,s11,r21,s21):
b11 :=  $\frac{3 \sqrt[3]{22} \sqrt[3]{d21} \sqrt[3]{(22 \sqrt[3]{d21})^3 + a1}}{2}$ 
c11 :=  $-3 a1 \sqrt[3]{21} \sqrt[3]{d22} \sqrt[3]{3} - 3 d2$ 
c21 :=  $\frac{3 \sqrt[3]{22} \sqrt[3]{d21} \sqrt[3]{(22 \sqrt[3]{d21})^3 + 3 a1}}{4}$ 
d11 :=  $-\frac{d2 (a1 + 3 \sqrt[3]{22} \sqrt[3]{d21} \sqrt[3]{3})}{2}$ 
r21 :=  $\frac{211 \sqrt[24]{3 d21 \sqrt[3]{(22 \sqrt[3]{d21})^3 + a1}}}{3 d21 \sqrt[3]{22 \sqrt[3]{d21} \sqrt[3]{3 + a1} \sqrt[3]{2}}}$ 
s21 :=  $-\frac{217 \sqrt[24]{(22 \sqrt[3]{d21})^3 + a1}}{3 d21 \sqrt[3]{22 \sqrt[3]{d21} \sqrt[3]{3 + a1} \sqrt[3]{2}}}$ 
0,  $\frac{3 \sqrt[3]{2} a1 (22 \sqrt[3]{d21} \sqrt[3]{3 + a1})}{2 |22 \sqrt[3]{d21} \sqrt[3]{3 + a1}|^2}, 0, -\frac{21 \sqrt[6]{(25 \sqrt[6]{d22} \sqrt[3]{a1} + 2 \sqrt[2]{d2}) (22 \sqrt[3]{d21} \sqrt[3]{3 + a1})^3}}{2 d22 \sqrt[3]{22 \sqrt[3]{d21} \sqrt[3]{3 + a1}}^4}$ 

$$\frac{(25 \sqrt[6]{d22} \sqrt[3]{a1} + 2 \sqrt[2]{d2}) \sqrt[6]{21}}{d22 \sqrt[3]{(22 \sqrt[3]{d21} \sqrt[3]{3 + 2 a1})}}, 0, 0, 0$$


```

(9.34)

```

> solve([N[1,1],N[1,3],N[2,2],N[2,3],N[2,4]],{a1,b1,c1,d1,d2,eta,theta});

$$\left\{ a1 = a1, b1 = -\frac{(3 a1 \eta + 2 c2) (a1 - \eta)}{a1 - 2 \eta}, c1 = \frac{9 a1^3 \eta^2 - 9 a1^2 \eta^3 + 15 a1^2 c2 \eta - 24 a1 c2 \eta^2 + 12 c2 \eta^3 + 4 a1 c22 - 4 c22 \eta}{3 (a1 - 2 \eta)^2}, d1 = \right.$$


```

(9.35)

```


$$\begin{aligned}
& -\frac{1}{27(aI-2\eta)^3} (27aI^4\eta^3 - 54aI^2\eta^5 + 81aI^3c2\eta^2 - 144aI^2c2\eta^3 + 108aIc2\eta^4 - 72c2\eta^5 + 48aI^2c22\eta - 72aIc22\eta^2 \\
& + 24c22\eta^3 + 8aIc23 - 8c23\eta), d2 = -\frac{9aI^2\eta^3 + 9aI^2c2\eta - 12aIc2\eta^2 + 12c2\eta^3 + 4aIc22 - 4c22\eta}{9(aI-2\eta)^2}, \eta=\eta, \theta \\
& = \frac{3aI\eta + 2c2}{3(aI-2\eta)}, \left\{ aI = 2\text{RootOf}(3_Z^2+c2), bI = -3\text{RootOf}(3_Z^2+c2)\theta, cI = 3\theta^2\text{RootOf}(3_Z^2+c2) + \text{RootOf}(3_Z^2+c2)c2, \right. \\
& dI = -\theta^3\text{RootOf}(3_Z^2+c2) + \frac{c2\theta^2}{3} - \frac{2c2\text{RootOf}(3_Z^2+c2)\theta}{3} + \frac{c22}{9}, d2 = -\theta^2\text{RootOf}(3_Z^2+c2) - \frac{2c2\theta}{3} \\
& \left. - \frac{\text{RootOf}(3_Z^2+c2)c2}{3}, \eta = \text{RootOf}(3_Z^2+c2), \theta = \theta \right\}
\end{aligned}$$


> # 9)



$z1 := (-c2/3)^{(1/2)};$



$z2 := -(-c2/3)^{(1/2)};$


$$\begin{aligned}
z1 &:= \sqrt{-\frac{c2}{3}} \\
z2 &:= -\sqrt{-\frac{c2}{3}}
\end{aligned} \tag{9.36}$$


> a11 := 2*z1;



$b11 := -3*z1*\theta;$



$c11 := 3*\theta^2 z1 + z1*c2;$



$d11 := -\theta^3 z1 + (1/3)*c2*\theta^2 - 2*c2*z1*\theta + (1/9)*c2^2;$



$d21 := -\theta^2 z1 - 2*c2*\theta + (1/3) - (1/3)*z1*c2;$



$\eta := z1;$



$r11 := \eta*\theta;$



$s11 := \theta*s2;$



$\text{zamproc}(a11, b11, c11, d11, 1, 0, c2, d21, r11, s11, r2, s2) :$


$$\begin{aligned}
& 0, \frac{(3\theta^3\sqrt{3}\sqrt{-c2} + 3\sqrt{3}(-c2)^{3/2}\theta + 9c2\theta^2 - c22)r2s2}{\sqrt{3}\sqrt{-c2} - 3\theta}, 0, -\frac{s23(6\theta^3\sqrt{3}\sqrt{-c2} - 9\theta^4 - 2\sqrt{3}(-c2)^{3/2}\theta + c22)}{9\left(\frac{\sqrt{3}\sqrt{-c2}}{3} + \theta\right)r2} \\
& \frac{r23(-3\theta^3\sqrt{3}\sqrt{-c2} - 3\sqrt{3}(-c2)^{3/2}\theta - 9c2\theta^2 + c22)}{-3s2\sqrt{3}\sqrt{-c2} + 9\theta s2}, 0, 0, 0
\end{aligned} \tag{9.37}$$


> solve(b1=-3*z1*\theta, theta);


$$-\frac{b1}{\sqrt{-3c2}} \tag{9.38}$$


> thetal := -b1/sqrt(-3*c2);



$a11 := 2*z1;$



$c11 := 3*\theta^2 z1 + z1*c2;$



$d11 := -\theta^3 z1 + (1/3)*c2*\theta^2 - 2*c2*z1*\theta + (1/9)*c2^2;$



$d21 := -\theta^2 z1 - 2*c2*\theta + (1/3) - (1/3)*z1*c2;$



$\eta := z1;$



$r11 := \eta*\theta;$



$s11 := \theta*s2;$



$\text{zamproc}(a11, b1, c11, d11, 1, 0, c2, d21, r11, s11, r2, s2) :$


$$\begin{aligned}
& 0, -\frac{(b1 - c2)^2 r2 s2 \sqrt{3}}{3 \sqrt{-c2}}, 0, -\frac{(b1 + c2)(b1 - c2)^2 s23 \sqrt{3}}{9 (-c2)^{3/2} r2} \\
& -\frac{(b1 - c2)^2 \sqrt{3} r23}{9 \sqrt{-c2} s2}, 0, 0, 0
\end{aligned} \tag{9.39}$$


> thetal := -b1/sqrt(-3*c2);



$a11 := 2*z1;$



$c11 := \text{eval}(3*\theta^2 z1 + z1*c2);$



$d11 := \text{simplify}(\text{eval}(-\theta^3 z1 + (1/3)*c2*\theta^2 - 2*c2*z1*\theta + (1/9)*c2^2));$



$d21 := \text{simplify}(\text{eval}(-\theta^2 z1 - 2*c2*\theta + (1/3) - (1/3)*z1*c2));$



$\eta := z1;$



$r11 := 3^{(3/4)}*|\text{abs}(c2)|^{(3/8)}*|\text{abs}(b1+c2)|^{(-1/8)}/(b1-c2);$



$s11 := -(b1-c2)^2*\sqrt{3}*r21^{3/2}/(9*\sqrt{-c2});$



$r11 := \eta*\theta;$



$s11 := \theta*\eta;$



$\text{zamproc}(a11, b1, c11, d11, 1, 0, c2, d21, r11, s11, r21, s21) :$


```

$$\begin{aligned}
a11 &:= 2 \sqrt{-\frac{c2}{3}} \\
c11 &:= -\sqrt{-\frac{c2}{3}} \frac{(bI2 - c2^2)}{c2} \\
d11 &:= \frac{-bI3 - bI2 c2 + 2 bI c2^2 + c2^3}{9 c2} \\
d21 &:= -\frac{(bI2 - 2 bI c2 - c2^2) \sqrt{3}}{9 \sqrt{-c2}} \\
r21 &:= \frac{3^{3/4} |c2|^3 |8|}{|bI + c2|^1 |8| (bI - c2)} \\
0, -\frac{3 |c2|^3 |2|}{c2 \sqrt{|bI + c2|}}, 0, -\frac{(bI + c2) |c2|^3}{c2^3 |bI + c2|} \\
&\quad 1, 0, 0, 0
\end{aligned} \tag{9.40}$$

$$> \text{solve}(b1=-3*z2*theta, \theta); \\
\frac{b1}{\sqrt{-3 c2}} \tag{9.41}$$

$$\begin{aligned}
> \theta := b1/\sqrt{-3*c2}: \\
a12 &:= 2*z2; \\
c12 &:= 3*theta^2*z2+z2*c2; \\
d12 &:= -theta2^3*z2+(1/3)*c2*theta2^2-2*c2*z2*theta2*(1/3)+(1/9)*c2^2; \\
d22 &:= -theta2^2*z2-2*c2*theta2*(1/3)-(1/3)*z2*c2; \\
\eta &:= z2; \\
r12 &:= \eta * r2; \\
s12 &:= \theta * s2; \\
\text{zamproc}(a12, b1, c12, d12, 1, 0, c2, d22, r12, s12, r2, s2): \\
0, \frac{(bI - c2)^2 r2 s2 \sqrt{3}}{3 \sqrt{-c2}}, 0, \frac{(bI + c2) (bI - c2)^2 s2^3 \sqrt{3}}{9 (-c2)^3 |2| r2} \\
\frac{(bI - c2)^2 \sqrt{3} r2^3}{9 \sqrt{-c2} s2}, 0, 0, 0
\end{aligned} \tag{9.42}$$

$$\begin{aligned}
> \theta := b1/\sqrt{-3*c2}: \\
a12 &:= 2*z2; \\
c12 &:= \text{eval}(3*\theta^2*z2+z2*c2); \\
d12 &:= \text{simplify}(\text{eval}(-\theta^3*z2+(1/3)*c2*\theta^2-2*c2*z2*\theta*(1/3)+(1/9)*c2^2)); \\
d21 &:= \text{simplify}(\text{eval}(-\theta^2*z2-2*c2*\theta*(1/3)-(1/3)*z2*c2)); \\
\eta &:= z2; \\
r22 &:= 3^(3/4)*abs(c2)^(3/8)*abs(b1+c2)^(-1/8)/(b1-c2); \\
s22 &:= (b1-c2)^2*sqrt(3)*r22^3/(9*sqrt(-c2)); \\
r12 &:= \eta * r22; \\
s12 &:= \theta * s22; \\
\text{zamproc}(a12, b1, c12, d12, 1, 0, c2, d22, r12, s12, r22, s22): \\
a12 &:= -2 \sqrt{-\frac{c2}{3}} \\
c12 &:= \sqrt{-\frac{c2}{3}} \frac{(bI2 - c2^2)}{c2} \\
d12 &:= \frac{-bI3 - bI2 c2 + 2 c2^2 bI + c2^3}{9 c2} \\
d21 &:= \frac{(bI2 - 2 c2 bI - c2^2) \sqrt{3}}{9 \sqrt{-c2}} \\
r22 &:= \frac{3^{3/4} |c2|^3 |8|}{|bI + c2|^1 |8| (bI - c2)} \\
0, -\frac{3 |c2|^3 |2|}{c2 \sqrt{|bI + c2|}}, 0, -\frac{(bI + c2) |c2|^3}{c2^3 |bI + c2|} \\
&\quad 1, 0, 0, 0
\end{aligned} \tag{9.43}$$

$$> \text{solve}([N[1,1], N[1,3], N[2,2], N[2,3], N[2,4]], \{a1, b1, c1, d2, \eta, \theta\}); \\
\left\{ a1 = \frac{2 c2}{3 \text{Root}(2 \text{Z} c2 + 3 d1)}, b1 = -2 c2, c1 = -\frac{3 d1}{\text{Root}(2 \text{Z} c2 + 3 d1)}, d2 = \frac{d1}{\text{Root}(2 \text{Z} c2 + 3 d1)}, \eta = 0, \theta = \text{Root}(2 \text{Z} c2) \right\} \tag{9.44}$$

$$\begin{aligned}
& -2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) + _Z^3)^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) - 6 d1 \operatorname{RootOf}(_Z^4 \\
& + _Z^2 c2 + d1)^2 \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) + _Z^3) \\
& + 6 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 d1 - 2 \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 \\
& + _Z^2 c2 + d1) + _Z^3) c2 d1 + 3 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) c2 d1), d2 \\
& = \frac{1}{3 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) (c2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 + d1)} (-2 c2^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 \\
& + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) + _Z^3) + 3 c2^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 \\
& + d1)^2 \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) + _Z^3)^2 \\
& + c2^3 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 - 3 d1 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 \\
& - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) + _Z^3)^2 - c2 d1 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 \\
& - 2 c2 d1 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 \\
& + _Z^2 c2 + d1) + _Z^3) + 3 c2 d1 \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 \\
& + _Z^2 c2 + d1) + _Z^3)^2 + c2^2 d1), \eta = \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1), \theta = \operatorname{RootOf}(\operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1)^3 - 2 \operatorname{RootOf}(_Z^4 \\
& + _Z^2 c2 + d1)^2 Z + _Z^2 \operatorname{RootOf}(_Z^4 + _Z^2 c2 + d1) + _Z^3) \}
\end{aligned}$$

```

> # 10)
solve(_Z^2*c2+3*d1, _Z);
z1 := sqrt(-3*c2*d1)/c2;
z2 := -sqrt(-3*c2*d1)/c2;

$$\frac{\sqrt{-3 c2 d1}}{c2}, -\frac{\sqrt{-3 c2 d1}}{c2} \quad (9.45)$$


```

```

> a11 := -2*c2/(3*z1):
c11 := -3*d1/z1;
d21 := d1/z1;
etal1 := z1;
r11 := etal1*r2;
zamproc(a11,0,c11,d1,1,0,c2,d21, r11,0,r2,s2):

$$0, -\frac{d1 r2 s2 c2 \sqrt{3}}{\sqrt{-c2 d1}}, 0, \frac{d1 s2^3 c2 \sqrt{3}}{3 \sqrt{-c2 d1} r2} - \frac{2 d1 \sqrt{3} r2^3 \left(c2^2 - \frac{9 d1}{2}\right)}{3 \sqrt{-c2 d1} c2 s2} \quad (9.46)$$


```

```

> a11 := -2*c2/(3*z1);
c11 := -3*d1/z1;
d21 := d1/z1;
etal1 := z1;
r21 := 3^(1/4)*abs(c2)^(1/2)*abs(d1)^(-1/4)*abs(2*c2^2-9*d1)^(-3/8);
s21 := -2*d1*sqrt(3)*r21^3*(c2^2-9*d1*(1/2))/(3*sqrt(-c2*d1)*c2):
r11 := etal1*r21;
zamproc(a11,0,c11,d1,1,0,c2,d21, r11,0,r21,s21):

$$a11 := -\frac{2 c2}{3 \sqrt{-3 c2 d1}}$$


$$c11 := -\frac{3 d1 c2}{\sqrt{-3 c2 d1}}$$


$$d21 := \frac{d1 c2}{\sqrt{-3 c2 d1}}$$


$$r21 := \frac{3^{1/4} \sqrt{|c2|}}{|d1|^1/4 |2 c2^2 - 9 d1|^3/8}$$


$$0, -\frac{6 |c2|^2 \left(c2^2 - \frac{9 d1}{2}\right) d1}{|2 c2^2 - 9 d1|^2 |d1| c2}, 0, -\frac{d1^2 |c2|^4 (2 c2^2 - 9 d1)^3}{|d1|^2 |2 c2^2 - 9 d1|^3 c2^4}$$


$$1, 0, 0 \quad (9.47)$$


```

```

> a12 := -2*c2/(3*z2):
c12 := -3*d1/z2;
d22 := d1/z2;
etal2 := z2;

```

```

r12 := eta2*r2;
zamproc(a12,0,c12,d1,1,0,c2,d22, r12,0,r2,s2):
0,  $\frac{dl r2 s2 c2 \sqrt{3}}{\sqrt{-c2 dl}}, 0, -\frac{dl s2^3 c2 \sqrt{3}}{3 \sqrt{-c2 dl} r2}$ 
 $\frac{r2^3 dl (2 c2 - 9 dl) \sqrt{3}}{3 c2 \sqrt{-c2 dl} s2}, 0, 0, 0$ 

```

(9.48)

```

> a12 := -2*c2/(3*z2);
c12 := -3*d1/z2;
d22 := d1/z2;
eta2 := z2;
r22 := 3^(1/4)*abs(c2)^(1/2)*abs(d1)^(-1/4)*abs(2*c2^2-9*d1)^(-3/8);
s22 := 2*d1*sqrt(3)*r22^3*(c2^2-9*d1^(1/2))/(3*sqrt(-c2*d1)*c2):
r12 := eta2*r22;
zamproc(a12,0,c12,d1,1,0,c2,d22, r12,0,r22,s22):
a12 :=  $\frac{2 c2^2}{3 \sqrt{-3 c2 dl}}$ 
c12 :=  $\frac{3 dl c2}{\sqrt{-3 c2 dl}}$ 
d22 :=  $-\frac{dl c2}{\sqrt{-3 c2 dl}}$ 
r22 :=  $\frac{3^{1/4} \sqrt{|c2|}}{|dl|^{1/4} |2 c2 - 9 dl|^{3/8}}$ 
0, -  $\frac{6 |c2|^2 \left(c2 - \frac{9 dl}{2}\right) dl}{|2 c2 - 9 dl|^{3/2} |dl| c2}, 0, -\frac{dl^2 |c2|^4 (2 c2 - 9 dl)^3}{|dl|^2 |2 c2 - 9 dl|^3 c2^4}$ 
1, 0, 0

```

(9.49)

CF 3-24

```

> solve([M[1,1],M[1,2],M[2,2],M[2,3],M[2,4]],{a1,b1,c1,d1,c2,d2,r1,s1,r2,s2});
{a1=0,b1=0,c1=c1,c2=0,d1=d1,d2=0,r1=r1,r2=0,s1=0,s2=s2}, {a1=- $\frac{c2 s2 r2 + 3 r1 s1}{3 (r1 s2 - r2 s1)}, b1 = \frac{s1 (c2 r2^2 + 3 r12)}{r2 (r1 s2 - r2 s1)}, c1 = \frac{c2 r12 s22 - c2 r1 r2 s1 s2 - c2 r22 s12 - 3 r12 s12}{r2 s2 (r1 s2 - r2 s1)}, d1 = c2, d2 = -\frac{2 c2 r13 s23 r2 - c2 r12 r22 s1 s22 - c2 r23 s12 r1 s2 - c2 r24 s13 + 3 r14 s1 s22 - 3 r13 s12 r2 s2 - 3 r12 r22 s13}{3 r23 s22 (r1 s2 - r2 s1)}, r1 = r1, r2 = r2, s1 = s1, s2 = s2}, {a1 = - $\frac{dl s2^3}{s1^3}, b1 = \frac{3 dl s22}{s1^2}, c1 = -\frac{3 dl s2}{s1}, c2 = -\frac{3 dl s22}{s1^2}, d1 = dl, d2 = \frac{dl s2}{s1}, r1 = 0, r2 = r2, s1 = s1, s2 = s2}, {a1 =  $\frac{r1}{r2}, b1 = 0, c1 = -\frac{3 r13}{r2^3}, c2 = -\frac{3 r12}{r2^2}, d1 = dl, d2 = \frac{2 r13}{r2^3}, r1 = r1, r2 = r2, s1 = s1, s2 = 0$$$ 
```

(10.1)

```

> # 1)
zamproc(0,0,c1,d1,1,0,0,0, r1,0,0,s2):
0,0,s22 c1,  $\frac{dl s2^3}{r1}$ 
 $\frac{rl^3}{s2}, 0, 0, 0$ 

```

(10.2)

```

> s21 := abs(c1)^(-1/2):
r11 := abs(c1)^(-1/2)*c1^(1/3):
zamproc(0,0,c1,d1,1,0,0,0, r11,0,0,s21):
0,0, $\frac{c1}{|c1|}, \frac{dl}{|c1| c1^{1/3}}$ 
 $\frac{c1}{|c1|}, 0, 0, 0$ 

```

(10.3)

```

> # 2)
s11 := d1*s2/d2:
a11 := -d1*s2^3/s11^3:
b11 := 3*d1*s2^2/s11^2:
c11 := -3*d1*s2/s11:

```

$$\begin{aligned}
c21 &:= -3*d1*s2^2/s11^2; \\
zamproc(a11, b11, c11, d1, 1, 0, c21, d2, 0, s11, r2, s2) : \\
&\quad 0, 0, -3 s2^2 d2, \frac{s2^3 (-2 d2^4 + d1^3)}{d2^3 r2} \\
&\quad \frac{r2^3 d2}{s2}, 0, 0, 0
\end{aligned} \tag{10.4}$$

$$\begin{aligned}
> s21 &:= \text{abs}(d2)^{-1/2}/\sqrt{3}; \\
r21 &:= -\text{abs}(d2)^{-1/2}*3^{-1/6}; \\
s11 &:= d1*s21/d2; \\
a11 &:= -d1*s21^3/s11^3; \\
b11 &:= 3*d1*s21^2/s11^2; \\
c11 &:= -3*d1*s21/s11; \\
c21 &:= -3*d1*s21^2/s11^2; \\
zamproc(a11, b11, c11, d1, 1, 0, c21, d2, 0, s11, r21, s21) : \\
&\quad s21 := \frac{\sqrt{3}}{3 \sqrt{|d2|}} \\
&\quad r21 := -\frac{3^{5/6}}{3 \sqrt{|d2|}} \\
&\quad a11 := -\frac{d2^3}{d1^2} \\
&\quad b11 := \frac{3 d2^2}{d1} \\
&\quad c11 := -3 d2 \\
&\quad c21 := -\frac{3 d2^2}{d1} \\
&\quad 0, 0, -\frac{d2}{|d2|}, -\frac{32 \sqrt{3} (-2 d2^4 + d1^3)}{9 |d2| d2^3} \\
&\quad -\frac{d2}{|d2|}, 0, 0, 0
\end{aligned} \tag{10.5}$$

$$\begin{aligned}
> \# 3) \\
r11 &:= a1*r2; \\
c11 &:= -3*r11^3/r2^3; \\
c21 &:= -3*r11^2/r2^2; \\
d21 &:= 2*r11^3/r2^3; \\
zamproc(a1, 0, c11, d1, 1, 0, c21, d21, r11, s1, r2, 0) : \\
&\quad 0, 0, 3 a1 s12, \frac{s1^3}{r2} \\
&\quad \frac{-2 a1^4 r2^3 + d1 r2^3}{s1}, 0, 0, 0
\end{aligned} \tag{10.6}$$

$$\begin{aligned}
> s11 &:= \text{abs}(a1)^{-1/2}/\sqrt{3}; \\
r21 &:= 3^{-1/6}*\text{abs}(a1)^{-1/2}*(d1-2*a1^4)^{-1/3}*a1^{1/3}; \\
r11 &:= a1*r21; \\
c11 &:= -3*r11^3/r21^3; \\
c21 &:= -3*r11^2/r21^2; \\
d21 &:= 2*r11^3/r21^3; \\
zamproc(a1, 0, c11, d1, 1, 0, c21, d21, r11, s11, r21, 0) : \\
&\quad s11 := \frac{\sqrt{3}}{3 \sqrt{|a1|}} \\
&\quad r21 := \frac{3^{5/6} a1^{1/3}}{3 \sqrt{|a1|} (-2 a1^4 + d1)^{1/3}} \\
&\quad c11 := -3 a1^3 \\
&\quad c21 := -3 a1^2 \\
&\quad d21 := 2 a1^3 \\
&\quad 0, 0, \frac{a1}{|a1|}, \frac{32 \sqrt{3} (-2 a1^4 + d1)^{1/3}}{9 |a1| a1^{1/3}} \\
&\quad \frac{a1}{|a1|}, 0, 0, 0
\end{aligned} \tag{10.7}$$

$$\begin{aligned}
> \text{solve}([M[1,1], M[1,2], M[2,1], M[2,2], M[2,3], M[2,4]], \{a1, b1, c1, d1, c2, r1, s1, r2, s2\}); \\
\left\{ \begin{aligned}
a1 &= \frac{d2 r2^2 s2^2 - r1 s2 s1 s2 - r1 r2 s1^2}{(2 r1 s2 + r2 s1) (r1 s2 - r2 s1)}, \\
b1 &= -\frac{3 s1 s2 (d2 r2^3 - 2 r1^3)}{r2 (r1 s2 - r2 s1) (2 r1 s2 + r2 s1)}, \\
c1 &= -\frac{3 (d2 r1^2 r2^2 s2^2 - d2 r2^3 s1 s2 r1 - d2 r2^4 s1^2 s2 + r1^3 s12 r2)}{(r1 s2 - r2 s1) r2^2 (2 r1 s2 + r2 s1)}, \\
c2 &= -\frac{3 (d2 r2^2 s2 + r1^2 s1)}{r2 (2 r1 s2 + r2 s1)}, \\
d1 &=
\end{aligned} \right. \tag{10.8}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2 d2 r1^3 s2^3 - d2 r1^2 r2 s1 s2^2 - d2 r1 r2^2 s1^2 s2 - d2 r2^3 s1^3 + 2 r1^3 s1^3}{r2 s2 (2 r1 s2 + r2 s1) (r1 s2 - r2 s1)}, \{r1 = r1, r2 = r2, s1 = s1, s2 = s2\}, \{a1 = \\
&- \frac{3 \text{RootOf}(\mathcal{Z} + 4 d2) d2 - 2 d1}{8 d2}, b1 = \frac{3 (\text{RootOf}(\mathcal{Z} + 4 d2) d2 + 2 d1)}{2 \text{RootOf}(\mathcal{Z} + 4 d2)^2}, c1 = -\frac{3 (5 \text{RootOf}(\mathcal{Z} + 4 d2) d2 + 2 d1)}{8 \text{RootOf}(\mathcal{Z} + 4 d2)}, c2 \\
&= \frac{3 (\text{RootOf}(\mathcal{Z} + 4 d2) d2 - 6 d1)}{4 \text{RootOf}(\mathcal{Z} + 4 d2)^2}, d1 = d1, r1 = r1, r2 = -\frac{2 r1}{\text{RootOf}(\mathcal{Z} + 4 d2)}, s1 = \text{RootOf}(\mathcal{Z} + 4 d2) s2, s2 = s2\}, \{a1 = \\
&-\frac{d2^3}{d1^2}, b1 = \frac{3 d2^2}{d1}, c1 = -3 d2, c2 = -\frac{3 d2^2}{d1}, d1 = d1, r1 = 0, r2 = r2, s1 = \frac{d1 s2}{d2}, s2 = s2\}, \{a1 = \text{RootOf}(2 \mathcal{Z} - d2), b1 = 0, c1 = \\
&-\frac{3 d2}{2}, c2 = -3 \text{RootOf}(2 \mathcal{Z} - d2)^2, d1 = d1, r1 = \text{RootOf}(2 \mathcal{Z} - d2) r2, r2 = r2, s1 = s1, s2 = 0\}
\end{aligned}$$

```

> # 4)
z1 := -(4*d2)^(1/3);
zI := -41\3 d21\3

```

(10.9)

```

> a11 := -(3*z1*d2-2*d1)/(8*d2):
b11 := (3*(z1*d2+2*d1))/(2*z1^2):
c11 := -(3*(5*z1*d2+2*d1))/(8*z1):
c21 := (3*(z1*d2-6*d1))/(4*z1^2):
r21 := -2*r1/z1:
s11 := z1*s2:
zamproc(a11,b11,c11,d1,1,0,c21,d2, r1,s11,r21,s2):
0,0, \frac{27 s2^2 (3 d24\3 + 21\3 d1)}{8 d21\3}, -\frac{9 s2^3 (d24\3 22\3 - 2 d1)}{8 r1}
\frac{9 r1^3 (-21\3 d1 + d24\3)}{4 d24\3 s2}, 0,0,0

```

(10.10)

```

> s21 := 2^(3/2)*3^(-3/2)*abs(d2)^(1/6)*abs(3*d2^(4/3)+2^(1/3)*d1)^(-1/2);
r11 := 2^(7/6)*3^(-7/6)*abs(d2)^(1/6)*d2^(1/3)*(3*d2^(4/3)+2^(1/3)*d1)^(1/3)*abs(3*d2^(4/3)+2^(1/3)*d1)^(-1/2)*(-2^(1/3)*d1+d2^(4/3))^(-1/3);
a11 := -(3*z1*d2-2*d1)/(8*d2);
b11 := (3*(z1*d2+2*d1))/(2*z1^2);
c11 := -(3*(5*z1*d2+2*d1))/(8*z1);
c21 := (3*(z1*d2-6*d1))/(4*z1^2);
r21 := -2*r11/z1:
s11 := z1*s21:
zamproc(a11,b11,c11,d1,1,0,c21,d2, r11,s11,r21,s21):
s21 := \frac{2 \sqrt{2} \sqrt{3} |d2|^{\frac{1}{6}}}{9 \sqrt{3} d24\3 + 21\3 d1}
r11 := \frac{2 21\3 6 35\6 |d2|^{\frac{1}{6}} d21\3 (3 d24\3 + 21\3 d1)^{\frac{1}{3}}}{9 \sqrt{3} d24\3 + 21\3 d1 (-21\3 d1 + d24\3)^{\frac{1}{3}}}
a11 := -\frac{-3 d24\3 41\3 - 2 d1}{8 d2}
b11 := \frac{3 (-d24\3 41\3 + 2 d1) 41\3}{8 d22\3}
c11 := \frac{3 (-5 d24\3 41\3 + 2 d1) 42\3}{32 d21\3}
c21 := \frac{3 (-d24\3 41\3 - 6 d1) 41\3}{16 d22\3}
0,0, \frac{|d2|^{\frac{1}{3}} (3 d24\3 22\3 + 2 d1) 21\3}{2 d21\3 |3 d24\3 + 21\3 d1|}, -\frac{|d2|^{\frac{1}{3}} (d24\3 22\3 - 2 d1) 21\3 32\3 (-21\3 d1 + d24\3)^{\frac{1}{3}}}{9 d21\3 |3 d24\3 + 21\3 d1| (3 d24\3 + 21\3 d1)^{\frac{1}{3}}}
\frac{21\3 (d24\3 22\3 - 2 d1) (3 d24\3 + 21\3 d1) |d2|^{\frac{1}{3}}}{2 d21\3 |3 d24\3 + 21\3 d1| (-21\3 d1 + d24\3)}, 0,0,0

```

(10.11)

```

> zamproc(a1,b1,c1,d1,1,0,c2,d2, r1,s1,r2,s2):
> solve([N[1,1],N[1,2],N[2,2],N[2,3],N[2,4]],{a1,b1,c1,d1,c2,d2,eta,theta});
{a1=a1,b1=-3 a1 \theta + 3 \eta \theta,c1=-3 a1 \eta^2 + 3 a1 \eta \theta + 3 a1 \theta^2 - 3 \eta^2 \theta - 3 \eta \theta^2,c2=-3 a1 \eta + 3 a1 \theta - 3 \eta \theta,d1=2 a1 \eta^3 - \eta^2 a1 \theta - \eta a1 \theta^2 - a1 \theta^3 + \eta^3 \theta + \theta^2 \eta^2 + \eta \theta^3,d2=2 a1 \eta^2 - a1 \eta \theta - a1 \theta^2 + \eta^2 \theta + \eta \theta^2,\eta=\eta,\theta=\theta}

```

(10.12)

```

> # 5)
b11 := -3*a1*theta+3*eta*theta:
c11 := -3*a1*eta^2+3*a1*eta*theta+3*a1*theta^2-3*eta^2*theta-3*eta*theta^2:
c21 := -3*a1*eta+3*a1*theta-3*eta*theta:
d11 := 2*a1*eta^3-a1*eta^2*a1*theta-a1*eta*theta^2+a1*theta^3+eta^3*theta+eta^2*theta^2+eta*theta^3:
d21 := 2*a1*eta^2-a1*eta*theta-a1*theta^2+eta^2*theta+eta*theta^2:
r11 := eta*r2:
s11 := theta*s2:

```

$$\begin{aligned}
& \text{zamproc}(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r2, s2) : \\
& \quad 0, 0, 3 (\eta - \theta)^2 a1 s22, \frac{(\eta - \theta)^2 (2 a1 + \theta) s23}{r2} \\
& \quad - \frac{(\eta - \theta)^2 (a1 - \eta) r23}{s2}, 0, 0, 0
\end{aligned} \tag{10.13}$$

$$\begin{aligned}
> b11 &:= \text{collect}(-3*a1*theta+3*eta*theta, theta, factor); \\
c11 &:= \text{collect}(-3*a1*eta^2+3*a1*eta*theta+3*a1*theta^2-3*eta^2*theta-3*eta*theta^2, theta, factor); \\
c21 &:= \text{collect}(-3*a1*eta+3*a1*theta-3*eta*theta, theta, factor); \\
d11 &:= \text{collect}(2*a1*eta^3-a1*eta^2*theta-a1*eta*theta^2-a1*theta^3+eta^3*theta+eta^2*theta^2+eta*theta^3, theta, factor); \\
d21 &:= \text{collect}(2*a1*eta^2-a1*eta*theta-a1*theta^2+eta^2*theta+eta*theta^2, theta, factor); \\
s21 &:= \text{abs}(a1)^{(-1/2)} * (\eta - theta)^{(-1)} / \sqrt{3}; \\
r21 &:= 3^{(-1/6)} * a1^{(1/3)} * \text{abs}(a1)^{(-1/2)} * (\eta - theta)^{(-1)} * (\eta - a1)^{(-1/3)}; \\
r11 &:= \eta * r21; \\
s11 &:= theta * s21; \\
\text{zamproc}(a1, b11, c11, d11, 1, 0, c21, d21, r11, s11, r21, s21) : \\
c11 &:= (3 a1 - 3 \eta) \theta^2 + 3 \eta (a1 - \eta) \theta - 3 a1 \eta^2 \\
d11 &:= (-a1 + \eta) \theta^3 - (a1 - \eta) \theta^2 \eta - (a1 - \eta) \theta \eta^2 + 2 a1 \eta^3 \\
d21 &:= (-a1 + \eta) \theta^2 - \eta (a1 - \eta) \theta + 2 a1 \eta^2 \\
s21 &:= \frac{\sqrt{3}}{3 \sqrt{|a1|} (\eta - \theta)} \\
r21 &:= \frac{35 \sqrt[6]{a1^3}}{3 \sqrt{|a1|} (\eta - \theta) (-a1 + \eta)^{1/3}} \\
0, 0, \frac{a1}{|a1|}, \frac{(-a1 + \eta)^{1/3} (2 a1 + \theta) 32^{1/3}}{9 |a1| a1^{1/3}} \\
\frac{a1}{|a1|}, 0, 0, 0
\end{aligned} \tag{10.14}$$

$$> \text{solve}([b1=-3*a1*theta+3*eta*theta, c2=-3*a1*eta+3*a1*theta-3*eta*theta], \{eta, theta\}); \\
\left\{ \eta = -\frac{b1 + c2}{3 a1}, \theta = -\frac{a1 b1}{3 a1^2 + b1 + c2} \right\} \tag{10.15}$$

$$\begin{aligned}
> \text{solve}([N[1,1], N[1,2], N[2,2], N[2,3], N[2,4]], \{c1, d1, c2, d2, eta, theta\}); \\
\left\{ c1 = -\frac{9 a1^3 \theta^2 + 6 a1^2 b1 \theta + 3 a1 b1 \theta^2 + 3 \theta^3 b1 + a1 b1^2 + b1^2 \theta}{3 \theta^2}, c2 = -\frac{3 a1^2 \theta + a1 b1 + \theta b1}{\theta}, d1 \right. \\
= \frac{54 a1^4 \theta^3 + 54 a1^3 b1 \theta^2 + 9 a1^2 b1 \theta^3 + 9 a1 b1 \theta^4 + 9 b1 \theta^5 + 18 a1^2 b1^2 \theta + 6 a1 b1^2 \theta^2 + 3 b1^2 \theta^3 + 2 a1 b1^3 + b1^3 \theta}{27 \theta^3}, d2 \\
= \left. \frac{18 a1^3 \theta^2 + 12 a1^2 b1 \theta + 3 a1 b1 \theta^2 + 3 \theta^3 b1 + 2 a1 b1^2 + b1^2 \theta}{9 \theta^2}, \eta = \frac{3 a1 \theta + b1}{3 \theta}, \theta = \theta \right\} \tag{10.16}
\end{aligned}$$

$$\begin{aligned}
> \text{solve}([N[1,1], N[1,2], N[2,2], N[2,3], N[2,4]], \{a1, b1, d1, c2, d2, eta, theta\}); \\
\left\{ a1 = -\frac{3 \eta^2 \theta + 3 \eta \theta^2 + c1}{3 (\eta^2 - \eta \theta - \theta^2)}, b1 = \frac{\theta (3 \eta^3 + c1)}{\eta^2 - \eta \theta - \theta^2}, c2 = \frac{3 \eta^2 \theta^2 + c1 \eta - c1 \theta}{\eta^2 - \eta \theta - \theta^2}, d1 = \right. \\
-\frac{3 \eta^5 \theta + 3 \eta^4 \theta^2 - 3 \eta^3 \theta^3 + 2 c1 \eta^3 - c1 \eta^2 \theta - c1 \eta \theta^2 - c1 \theta^3}{3 (\eta^2 - \eta \theta - \theta^2)}, d2 = -\frac{3 \eta^4 \theta + 3 \eta^3 \theta^2 + 2 c1 \eta^2 \theta - c1 \eta \theta - c1 \theta^2}{3 (\eta^2 - \eta \theta - \theta^2)}, \eta = \eta, \theta = \theta \Big\}, \\
\left. \begin{aligned}
a1 &= ((-3 \text{RootOf}(3 _Z^3 + c1)^2 \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2)) + 2 \text{RootOf}(3 _Z^3 + c1) b1 \\
&+ \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2) b1 + c1) \text{RootOf}(3 _Z^3 + c1)^2 \Big/ c1 (\text{RootOf}(3 _Z^3 + c1) + \text{RootOf}(\\
&- \text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2))), b1 = b1, c2 = \\
&- \frac{3 (\text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2) b1 + c1) \text{RootOf}(3 _Z^3 + c1)^2}{c1}, d1 = \\
&-\frac{2 c1 (-3 \text{RootOf}(3 _Z^3 + c1)^2 - 3 \text{RootOf}(3 _Z^3 + c1) \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2) + b1)}{9 (\text{RootOf}(3 _Z^3 + c1) + \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2))}, d2 = \\
&-(2 b1 \text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2) b1) \text{RootOf}(3 _Z^3 + c1) \\
&+ 2 c1 \text{RootOf}(3 _Z^3 + c1) + 2 c1 \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2)) \Big/ (3 (\text{RootOf}(3 _Z^3 + c1) \\
&+ \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2 + \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2))), \eta = \text{RootOf}(3 _Z^3 + c1), \theta = \text{RootOf}(-\text{RootOf}(3 _Z^3 + c1)^2) \\
&+ \text{RootOf}(3 _Z^3 + c1) _Z + _Z^2) \Big\}, \left. \begin{aligned}
a1 &= -\frac{c1}{3 \theta^2}, b1 = \frac{-3 \theta^3 + c1}{\theta}, c2 = -\frac{-3 \theta^3 + 2 c1}{\theta}, d1 = -\theta^4 + c1 \theta, d2 = -\frac{2 c1}{3}, \eta = -\theta, \theta = \theta \Big\}
\end{aligned} \right\} \tag{10.17}
\end{aligned}$$

```

> thetal := sqrt(-c1/(3*a1)):
b11 := (-3*thetal^3+c1)/thetal:
c21 := -(-3*thetal^3+2*c1)/thetal:
d11 := -thetal^4+c1*thetal:
d21 := -2*c1*(1/3):
etal := -thetal:
r11 := etal*r2:
s11 := thetal*s2:
zamproc(a1,b11,c1,d11,1,0,c21,d21, r11,s11,r2,s2):

$$0, 0, -4 s22 cl, -\frac{4 s23 cl \left(\sqrt{3} \sqrt{-\frac{cl}{al}} + 6 al\right)}{9 al r2}$$


$$\frac{4 r23 cl \left(\sqrt{3} \sqrt{-\frac{cl}{al}} + 3 al\right)}{9 al s2}, 0, 0, 0$$


```

(10.18)

```

> thetal := sqrt(-c1/(3*a1)):
b11 := evala((-3*thetal^3+c1)/thetal);
c21 := evala(-(-3*thetal^3+2*c1)/thetal);
d11 := evala(-thetal^4+c1*thetal);
d21 := -2*c1*(1/3);
etal := -thetal;
s21 := abs(c1)^(-1/2)/2;
r21 := -3^(2/3)*a1^(1/3)*abs(c1)^(-1/2)*(sqrt(3)*sqrt(-c1/a1)+3*a1)^(-1/3)/2;
r11 := etal*r21;
s11 := thetal*s21:
zamproc(a1,b11,c1,d11,1,0,c21,d21, r11,s11,r21,s21):

$$b11 := \frac{cl}{al} - \sqrt{-\frac{3 cl}{al}} al$$


$$c21 := 2 \sqrt{-\frac{3 cl}{al}} al - \frac{cl}{al}$$


$$d11 := \frac{cl \left(3 \sqrt{-\frac{3 cl}{al}} al^2 - cl\right)}{9 al^2}$$


$$d21 := -\frac{2 cl}{3}$$


$$s21 := \frac{1}{2 \sqrt{|cl|}}$$


$$r21 := -\frac{32 \sqrt[3]{al^1} \sqrt[3]{3}}{2 \sqrt{|cl|} \left(\sqrt{3} \sqrt{-\frac{cl}{al}} + 3 al\right)^{1/3}}$$


$$0, 0, -\frac{cl}{|cl|}, \frac{cl \left(2 \sqrt{3} \sqrt{-\frac{cl}{al}} al^2 - cl\right) 35 \sqrt[6]{\left(\sqrt{3} \sqrt{-\frac{cl}{al}} + 3 al\right)^{1/3}}}{27 |cl| al^7 \sqrt[3]{-\frac{cl}{al}}}$$


$$-\frac{\left(\sqrt{3} \sqrt{-\frac{cl}{al}} al^2 - cl\right) cl \sqrt{3}}{al |cl| \left(\sqrt{3} \sqrt{-\frac{cl}{al}} + 3 al\right) \sqrt{-\frac{cl}{al}}}, 0, 0, 0$$


```

(10.19)

```

> theta2 := -sqrt(-c1/(3*a1)):
b12 := (-3*theta2^3+c1)/theta2:
c22 := -(-3*theta2^3+2*c1)/theta2:
d12 := -theta2^4+c1*theta2:
d22 := -2*c1*(1/3):
eta2 := -theta2:
r12 := eta2*r2:
s12 := theta2*s2:
zamproc(a1,b12,c1,d12,1,0,c22,d22, r12,s12,r2,s2):

$$0, 0, -4 s22 cl, \frac{4 s23 cl \left(\sqrt{3} \sqrt{-\frac{cl}{al}} - 6 al\right)}{9 al r2}$$


```

$$-\frac{4 r^{23} c l \left(\sqrt{3} \sqrt{-\frac{c l}{a l}}-3 a l\right)}{9 a l s^2}, 0, 0, 0 \quad (10.20)$$

```
> theta2 := -sqrt(-c1/(3*a1)):
b12 := evala((-3*theta2^3+c1)/theta2);
c22 := evala((-(-3*theta2^3+2*c1)/theta2));
d12 := evala(-theta2^4+c1*theta2);
d22 := -2*c1*(1/3);
eta2 := -theta2;
s22 := abs(c1)^(-1/2)/2;
r22 := 3^(2/3)*a1^(1/3)*abs(c1)^(-1/2)*(sqrt(3)*sqrt(-c1/a1)-3*a1)^(-1/3)/2;
r12 := eta2*r22;
s12 := theta2*s22;
zamproc(a1,b12,c1,d12,1,0,c22,d22, r12,s12,r22,s22):
```

$$\begin{aligned} b12 &:= \sqrt{-\frac{3 c l}{a l}} a l + \frac{c l}{a l} \\ c22 &:= -\frac{c l}{a l} - 2 \sqrt{-\frac{3 c l}{a l}} a l \\ d12 &:= -\frac{c l \left(3 \sqrt{-\frac{3 c l}{a l}} a l^2 + c l\right)}{9 a l^2} \\ d22 &:= -\frac{2 c l}{3} \\ s22 &:= \frac{1}{2 \sqrt{|c l|}} \\ r22 &:= \frac{32 \sqrt[3]{a l^1} \sqrt[3]{3}}{2 \sqrt{|c l|} \left(\sqrt{3} \sqrt{-\frac{c l}{a l}} - 3 a l\right)^{1/3}} \\ &\quad 0, 0, -\frac{c l}{|c l|}, -\frac{c l \left(2 \sqrt{3} \sqrt{-\frac{c l}{a l}} a l^2 + c l\right) 35 \sqrt[6]{6} \left(\sqrt{3} \sqrt{-\frac{c l}{a l}} - 3 a l\right)^{1/3}}{27 |c l| a l^7 \sqrt[3]{-\frac{c l}{a l}}} \\ &\quad \frac{c l \left(\sqrt{3} \sqrt{-\frac{c l}{a l}} a l^2 + c l\right) \sqrt{3}}{|c l| a l \left(\sqrt{3} \sqrt{-\frac{c l}{a l}} - 3 a l\right) \sqrt{-\frac{c l}{a l}}}, 0, 0, 0 \end{aligned} \quad (10.21)$$