

2. Сведение форм с $m \leq 4$ из списка 2.1 к предшествующим (утверждение 3.1).

> restart; read("newlib.m"); with(mylib): with(LinearAlgebra):

Для любой формы из I части списка достаточно искать замены в предшествующие формы из ее класса (1–3) и из части II.

I, класс 1).

$NSF_5^{3,1}$. Результат произвольной замены :

$$\begin{aligned} > M := \text{zamproc}(0, 1, u, 0, 0, 0, 0, 1, r1, s1, r2, s2): \\ & \frac{r2(r1r2s2u+r1^2s2-s1r2^2)}{r1s2-s1r2}, \frac{2s2\left(\frac{s1(u-3)r2^2}{2}+r1(s2u+s1)r2+\frac{r1^2s2}{2}\right)}{r1s2-s1r2}, \frac{\left(r1s2^2u+2s1\left(\left(u-\frac{3}{2}\right)r2+r1\right)s2+r2s1^2\right)s2}{r1s2-s1r2}, \\ & \frac{(u-1)s2+s1)s2^2s1}{r1s2-s1r2} \\ & -\frac{r1((u-1)r2+r1)r2^2}{r1s2-s1r2}, -\frac{2\left(\frac{r2^2s1u}{2}+\left(\left(u-\frac{3}{2}\right)s2+s1\right)r1r2+\frac{r1^2s2}{2}\right)r2}{r1s2-s1r2}, -\frac{(r1(u-3)s2^2+2s1(r2u+r1)s2+r2s1^2)r2}{r1s2-s1r2}, \\ & \frac{s2(-r2s1s2u+r1s2^2-r2s1^2)}{r1s2-s1r2} \end{aligned}$$

Поиск замен к $SF_2^{2,1}$.

> solve([M[1,2], M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], {u, r1, s1, r2, s2});
 $\{r1=0, r2=r2, s1=s1, s2=0, u=0\}, \{r1=-r2, r2=r2, s1=s1, s2=0, u=2\}$

> zamproc(0, 1, 2, 0, 0, 0, 0, 1, r1, s1, -r1, 0):

$$\begin{matrix} r1^2, 0, 0, 0 \\ 0, 0, -r1s1, 0 \end{matrix}$$

$NSF_8^{3,1}$. Результат произвольной замены :

$$\begin{aligned} > M := \text{zamproc}(0, u, 0, 1, 0, 0, 0, 1, r1, s1, r2, s2): \\ & \frac{((-s1+s2)r2^2+r1^2s2u)r2}{r1s2-s1r2}, \frac{((-3s1+3s2)r2^2+2ur1r2s1+r1^2s2u)s2}{r1s2-s1r2}, \frac{s2(2r1s1s2u+r2s1^2u-3r2s1s2+3r2s2^2)}{r1s2-s1r2}, \\ & \frac{s2^2(u s1^2-s1s2+s2^2)}{r1s2-s1r2} \\ & -\frac{r2^2(r1^2u-r1r2+r2^2)}{r1s2-s1r2}, -\frac{\left(3r2^2s2+2r1\left(u s1-\frac{3s2}{2}\right)r2+r1^2s2u\right)r2}{r1s2-s1r2}, -\frac{2r2\left(\left(-\frac{3r1}{2}+\frac{3r2}{2}\right)s2^2+r1s1s2u+\frac{r2s1^2u}{2}\right)}{r1s2-s1r2}, \\ & -\frac{((-r1+r2)s2^2+r2s1^2u)s2}{r1s2-s1r2} \end{aligned}$$

$SF_2^{2,1}$

> solve([M[1,2], M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], {u, r1, s1, r2, s2});
 $\{r1=r2, r2=r2, s1=s1, s2=0, u=0\}$

$SF_5^{3,1}$

> solve([M[1,1], M[1,4], M[2,1], M[2,2], M[2,3]], {u, r1, s1, r2, s2});
 $\left\{r1=r1, r2=0, s1=s1, s2=s2, u=\frac{s2(s1-s2)}{s1^2}\right\}$

> s1 = solve(u = s2*(s1-s2)/s1^2, s1);
 $s1=\left(\frac{(1+\sqrt{1-4u})s2}{2u}, -\frac{(-1+\sqrt{1-4u})s2}{2u}\right)$

> s11 := (1+sqrt(1-4*u))*s2/(2*u):

zamproc(0, u, 0, 1, 0, 0, 0, 1, r1, s11, 0, s2):

$$\begin{matrix} 0, r1s2u, (1+\sqrt{1-4u})s2^2, 0 \\ 0, 0, 0, s2^2 \end{matrix}$$

> s12 := -(-1+sqrt(1-4*u))*s2/(2*u):

zamproc(0, u, 0, 1, 0, 0, 0, 1, r1, s11, 0, s2):

$$\begin{matrix} 0, r1s2u, (1+\sqrt{1-4u})s2^2, 0 \\ 0, 0, 0, s2^2 \end{matrix}$$

I, класс 2).

$NSF_{14}^{3,1}$. Результат произвольной замены :

> M := zamproc(0, u, 0, k, 0, 0, 1, 0, r1, s1, r2, s2):

$$\begin{aligned} & \frac{r2(kr2^2s2+r1^2s2u-r1r2s1)}{r1s2-r2s1}, \frac{(3ks2^2-s1^2)r2^2+2r1s1s2(u-1)r2+ur1^2s2^2}{r1s2-r2s1}, \frac{3s2\left(\frac{r2(u-2)s1^2}{3}+\frac{2s2r1\left(u-\frac{1}{2}\right)s1}{3}+kr2s2^2\right)}{r1s2-r2s1}, \\ & \frac{(u-1)s1^2+ks2^2)s2^2}{r1s2-r2s1} \end{aligned}$$

$$SF_3^{3,1}$$

$$\frac{r2^2 ((u-1) rI^2 + k r2^2)}{rI s2 - r2 sI} - \frac{3 r2 \left(\frac{s2 (u-2) rI^2}{3} + \frac{2 sI \left(u - \frac{1}{2}\right) r2 rI}{3} + k r2^2 s2 \right)}{rI s2 - r2 sI}, \frac{(-3 k s2^2 - sI^2 u) r2^2 - 2 rI sI s2 (u-1) r2 + rI^2 s2^2}{rI s2 - r2 sI},$$

$$\frac{s2 (k r2 s2^2 + r2 sI^2 u - rI sI s2)}{rI s2 - r2 sI}$$

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,k,rI,sI,r2,s2});
```

$$\{k=0, rI=0, r2=r2, sI=sI, s2=0, u=u\}, \{k=k, rI=rI, r2=RootOf(2 _Z^2 k-1) rI, sI=sI, s2=0, u=\frac{1}{2}\}, \{k=0, rI=rI, r2=0, sI=0, s2=s2, u=u\}$$

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> r21 := 2^(-1/2)*rI:
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zamproc(0,1/2,0,1,0,0,1,0, rI,sI,r21,0):
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$$\frac{rI^2 \sqrt{2}}{2}, \frac{sI rI \sqrt{2}}{2}, 0, 0$$

$$0, 0, \frac{sI rI \sqrt{2}}{4}, 0$$

$NSF_7^{4,1}$. Результат произвольной замены :

$$> M := zamproc(u,v,0,0,0,1,1,0, rI,sI,r2,s2);$$

$$\frac{(s2 rI^2 u + r2 (v s2 - sI) rI - sI r2^2) rI}{s2 rI - r2 sI}, \frac{(s2 (3 u - 1) sI + v s2^2) rI^2 + 2 sI r2 (-sI + s2 (v - 1)) rI - r2^2 sI^2}{s2 rI - r2 sI},$$

$$\frac{3 sI \left(-\frac{sI^2 r2}{3} + s2 \left(\left(u - \frac{2}{3} \right) rI + \frac{(v - 2) r2}{3} \right) sI + \frac{2 \left(v - \frac{1}{2} \right) s2^2 rI}{3} \right)}{s2 rI - r2 sI}, \frac{((u - 1) sI + s2 (v - 1)) sI^2 s2}{s2 rI - r2 sI}$$

$$\frac{r2 ((u - 1) rI + r2 (v - 1)) rI^2}{s2 rI - r2 sI}, \frac{3 rI \left(-\frac{s2 rI^2}{3} + r2 \left(\left(u - \frac{2}{3} \right) sI + \frac{(v - 2) s2}{3} \right) rI + \frac{2 \left(v - \frac{1}{2} \right) sI r2^2}{3} \right)}{s2 rI - r2 sI},$$

$$\frac{(2 s2 sI + s2^2) rI^2 - 3 sI \left(\left(u - \frac{1}{3} \right) sI + \frac{2 s2 (v - 1)}{3} \right) r2 rI - v sI^2 r2^2}{s2 rI - r2 sI}, -\frac{sI (u r2 sI^2 + s2 (v r2 - rI) sI - s2^2 rI)}{s2 rI - r2 sI}$$

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {v,rI,sI,r2,s2});
```

$$\{rI=rI, r2=-rI u, sI=0, s2=s2, v=\frac{2 u - 1}{u}\}$$

```
> v1 := (2*u-1)/u:
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```
r21 := -rI*u:
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```
zamproc(u,v1,0,0,0,1,1,0, rI,0,r21,s2):
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$$-(u-1) rI^2, \frac{(2 u - 1) rI s2}{u}, 0, 0, s2 rI, 0$$

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {v,rI,sI,r2,s2});
```

$$\{rI=0, r2=r2, sI=sI, s2=-\frac{1}{2} sI u - \frac{1}{2} sI, v=\frac{2 u}{u+1}\}$$

```
> s21 := -(1/2)*s1*u-(1/2)*s1:
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```
v1 := 2*u/(u+1):
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```
zamproc(u,v1,0,0,0,1,1,0, 0,s1,r2,s21):
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$$0, r2 sI, 0, \frac{sI^3 (u^2 - 1)}{4 r2}$$

$$0, 0, \frac{2 u sI r2}{u + 1}, 0$$

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {v,rI,sI,r2,s2});
```

$$\{rI=rI, r2=-rI u, sI=0, s2=s2, v=\frac{2 u - 1}{u}\}, \{rI=0, r2=r2, sI=sI, s2=-sI u, v=-\frac{-2 u + 1}{u}\}$$

```
> v1 := -(-2*u+1)/u:
```

```
s21 := -u*s1:
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```
zamproc(u,v1,0,0,0,1,1,0, 0,s1,r2,s21):
```

$$0, r2 sI, 0, 0$$

$$0, 0, \frac{(2 u - 1) sI r2}{u}, -(u - 1) sI^2$$

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,rI,sI,r2,s2});
```

$$\left\{ r1 = r1, r2 = -2r1, s1 = s1, s2 = 0, u = 2, v = \frac{3}{2} \right\}, \left\{ r1 = r1, r2 = -\frac{r1}{2}, s1 = 0, s2 = s2, u = \frac{1}{2}, v = 0 \right\}$$

> r21 := -2*r1:

zamproc(2, 3/2, 0, 0, 0, 1, 1, 0, r1, s1, r21, 0):

$$\begin{aligned} & -r1^2, 0, s1^2, 0 \\ & 0, 0, 2s1r1, 2s1^2 \end{aligned}$$

$NSF_{12}^{4,1}$ ($v \neq -u$). Результат произвольной замены :

> M := zamproc(0, u, 0, v, 0, 0, 1, 1, r1, s1, r2, s2):

$$\begin{aligned} & \frac{r2((vs2-s1)r2^2-r2s1r1+s2r1^2u)}{s2r1-r2s1}, \frac{(3vs2^2-s1^2-3s2s1)r2^2+2r1s1s2(u-1)r2+ur1^2s2^2}{s2r1-r2s1}, \\ & \frac{2\left(\frac{r2(u-2)s1^2}{2}+\left(-\frac{3r2}{2}+\left(u-\frac{1}{2}\right)r1\right)s2s1+\frac{3vs2^2r2}{2}\right)s2}{s2r1-r2s1}, \frac{((u-1)s1^2-s2s1+vs2^2)s2^2}{s2r1-r2s1} \\ & -\frac{r2^2((u-1)r1^2-r1r2+vr2^2)}{s2r1-r2s1}, -\frac{\left(s2(u-2)r1^2+2r2\left(-\frac{3s2}{2}+\left(u-\frac{1}{2}\right)s1\right)r1+3vs2r2^2\right)r2}{s2r1-r2s1}, \\ & \frac{(-us1^2-3vs2^2)r2^2-2\left(-\frac{3s2}{2}+(u-1)s1\right)s2r1r2+s2^2r1^2}{s2r1-r2s1}, \frac{s2(-ur2s1^2-vs2^2r2+s2r1s1+s2^2r1)}{s2r1-r2s1} \end{aligned}$$

$SF_3^{3,1}$

> solve([M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], {u, r1, s1, r2, s2});

$$\left\{ r1 = \text{RootOf}(_Z^2 + 2_Z - 2v) r2, r2 = r2, s1 = s1, s2 = 0, u = \frac{1}{2} \right\}$$

> solve(_Z^2+2*_Z-2*v, _Z);

$$-1 + \sqrt{1+2v}, -1 - \sqrt{1+2v}$$

> r11 := (-1+sqrt(1+2*v))*r2:

zamproc(0, 1/2, 0, v, 0, 0, 1, 1, r11, s1, r2, 0):

$$\begin{aligned} & r2^2\sqrt{1+2v}, s1r2, 0, 0 \\ & 0, 0, \frac{s1r2}{2}, 0 \end{aligned}$$

> r12 := (-1-sqrt(1+2*v))*r2:

zamproc(0, 1/2, 0, v, 0, 0, 1, 1, r12, s1, r2, 0):

$$\begin{aligned} & -r2^2\sqrt{1+2v}, s1r2, 0, 0 \\ & 0, 0, \frac{s1r2}{2}, 0 \end{aligned}$$

$SF_{14}^{3,1}$

> solve([M[1,1], M[1,3], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});

$$\left\{ r1 = r1, r2 = 0, s1 = -s2, s2 = s2, u = \frac{1}{2}, v = v \right\}, \left\{ r1 = -r2, r2 = r2, s1 = s1, s2 = 0, u = \frac{1}{2}, v = -\frac{1}{2} \right\}$$

> zamproc(0, 1/2, 0, v, 0, 0, 1, 1, r1, s1, 0, -s1):

$$\begin{aligned} & 0, -\frac{s1r1}{2}, 0, -\frac{s1^3(1+2v)}{2r1} \\ & 0, 0, -s1r1, 0 \end{aligned}$$

$SF_1^{4,1}$

> solve([M[1,3], M[1,4], M[2,1], M[2,2]], {u, r1, s1, r2, s2});

$$\left\{ r1 = r1, r2 = 0, s1 = \text{RootOf}(_Z^2 + 2_Z - 2v) s2, s2 = s2, u = \frac{1}{2} \right\}, \left\{ r1 = \text{RootOf}(_Z^2 + 2_Z - 2v) r2, r2 = r2, s1 = s1, s2 = 0, u = \frac{1}{2} \right\}, \left\{ r1 = \text{RootOf}(_Z^2 + 2_Z - 2v) r2, r2 = r2, s1 = -s2 (\text{RootOf}(_Z^2 + 2_Z - 2v) + 2), s2 = s2, u = \frac{1}{2} \right\}$$

$SF_3^{4,1}$

> solve([M[1,2], M[1,4], M[2,1], M[2,2]], {u, v, r1, s1, r2, s2});

$$\left\{ r1 = 0, r2 = r2, s1 = -3s2, s2 = s2, u = \frac{2}{3}, v = 0 \right\}, \left\{ r1 = r1, r2 = 0, s1 = s1, s2 = s2, u = 0, v = \frac{s1(s1+s2)}{s2^2} \right\}$$

$SF_7^{4,1}$

> solve([M[1,3], M[1,4], M[2,1], M[2,4]], {u, v, r1, s1, r2, s2});

$$\left\{ r1 = r1, r2 = r2, s1 = s1, s2 = 0, u = \frac{-r2^2v+r1^2+r2r1}{r1^2}, v = v \right\}, \left\{ r1 = r1, r2 = 0, s1 = -s2, s2 = s2, u = \frac{1}{2}, v = -\frac{1}{2} \right\}$$

> solve(u = (-r2^2*v+r1^2+r2*r1)/r1^2, r2);

$$\frac{(1+\sqrt{-4vu+4v+1})r1}{2v}, -\frac{(-1+\sqrt{-4vu+4v+1})r1}{2v}$$

> r21 := (1+sqrt(-4*u*v+4*v+1))*r1/(2*v):

zamproc(0, u, 0, v, 0, 0, 1, 1, r1, s1, r21, 0):

$$\frac{(2v+1+\sqrt{1+(-4u+4)v})(1+\sqrt{1+(-4u+4)v})rI^2}{4v^2}, \frac{sI(1+\sqrt{1+(-4u+4)v})rI}{2v}, 0, 0$$

$$0, \frac{(2u-1)(1+\sqrt{1+(-4u+4)v})rI^2}{2v}, \frac{usI(1+\sqrt{1+(-4u+4)v})rI}{2v}, 0$$

> r22 := $-(1+\sqrt{-4u^2+4v^2+1}) * r1 / (2*v) :$
zamproc(0, u, 0, v, 0, 0, 1, 1, r1, s1, r22, 0):

$$\frac{(-2v-1+\sqrt{1+(-4u+4)v})(-1+\sqrt{1+(-4u+4)v})rI^2}{4v^2}, -\frac{sI(-1+\sqrt{1+(-4u+4)v})rI}{2v}, 0, 0$$

$$0, -\frac{(2u-1)(-1+\sqrt{1+(-4u+4)v})rI^2}{2v}, -\frac{usI(-1+\sqrt{1+(-4u+4)v})rI}{2v}, 0$$

$SF_{24}^{4,1}$. Результат произвольной замены :

> M := zamproc(0, u, 1, v, 0, 0, 1, 0, r1, s1, r2, s2):

$$\frac{r2(v s2 r2^2 - r1(s1 - s2)r2 + r1^2 s2 u)}{-s1 r2 + r1 s2}, \frac{(3 s2^2 v - s1^2 + s1 s2)r2^2 + 2(s2 + (u-1)s1)s2 r1 r2 + u r1^2 s2^2}{-s1 r2 + r1 s2},$$

$$\frac{2\left(\left(\frac{3r2v}{2} + \frac{r1}{2}\right)s2^2 + s1\left(r2 + \left(u - \frac{1}{2}\right)r1\right)s2 + \frac{r2(u-2)s1^2}{2}\right)s2}{-s1 r2 + r1 s2}, \frac{(s2^2 v + (u-1)s1^2 + s1 s2)s2^2}{-s1 r2 + r1 s2}$$

$$-\frac{(vr2^2 + (u-1)r1^2 + r1 r2)r2^2}{-s1 r2 + r1 s2}, -\frac{r2\left((3v s2 + s1)r2^2 + 2r1\left(s2 + \left(u - \frac{1}{2}\right)s1\right)r2 + s2(u-2)r1^2\right)}{-s1 r2 + r1 s2},$$

$$\frac{(-s1^2 u - 3s2^2 v - 2s1 s2)r2^2 - 2\left(\frac{s2^2}{2} + (u-1)s1\right)s2 r1 r2 + r1^2 s2^2}{-s1 r2 + r1 s2}, -\frac{s2(v s2^2 r2 - s1(r1 - r2)s2 + u s1^2 r2)}{-s1 r2 + r1 s2}$$

$SF_3^{3,1}$

> solve([M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], {v, r1, s1, r2, s2});

$$\left\{ r1 = -\frac{r2}{2u-1}, r2 = r2, s1 = s1, s2 = 0, v = \frac{u}{4u^2-4u+1} \right\}$$

> r21 := -r1*(2*u-1):
v1 := u/(4*u^2-4*u+1):
zamproc(0, u, 1, v1, 0, 0, 1, 0, r1, s1, r21, 0):

$$(-2u+1)r1^2, (-2u+1)s1r1, 0, 0, -u s1 (2u-1)r1, 0$$

$SF_{14}^{3,1}$

> solve([M[1,1], M[1,3], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});

$SF_1^{4,1}$

> solve([M[1,3], M[1,4], M[2,1], M[2,2]], {v, r1, s1, r2, s2});

$$\left\{ r1 = r1, r2 = 0, s1 = -\frac{s2}{2u-1}, s2 = s2, v = \frac{u}{4u^2-4u+1} \right\}, \left\{ r1 = -\frac{r2}{2u-1}, r2 = r2, s1 = s1, s2 = 0, v = -\frac{(u-1)r2^2}{(2u-1)^2} - \frac{r2^2}{2u-1} \right\}$$

> simplify(-((u-1)*r2^2/(2*u-1)^2 - r2^2/(2*u-1))/r2^2);

$$\frac{u}{(2u-1)^2}$$

$SF_3^{4,1}$

> solve([M[1,2], M[1,4], M[2,1], M[2,2]], {u, v, r1, s1, r2, s2});

$$\left\{ r1 = -3r2, r2 = r2, s1 = 6s2, s2 = s2, u = \frac{2}{3}, v = 6 \right\}, \left\{ r1 = r1, r2 = 0, s1 = s1, s2 = s2, u = 0, v = \frac{s1(s1-s2)}{s2^2} \right\}$$

$SF_7^{4,1}$

> solve([M[1,3], M[1,4], M[2,1], M[2,4]], {r1, s1, r2, s2});

$$\{r1 = RootOf((u-1)_Z^2 + v + _Z)r2, r2 = r2, s1 = s1, s2 = 0\}$$

> solve((u-1)*_Z^2+v+_Z, _Z);

$$\frac{-1 + \sqrt{-4vu + 4v + 1}}{2(u-1)}, -\frac{1 + \sqrt{-4vu + 4v + 1}}{2(u-1)}$$

> u1 := 1/2:

r11 := -(1+sqrt(-4*u1*v+4*v+1))/(2*(u1-1))*r2;
zamproc(0, u1, 1, v, 0, 0, 1, 0, r11, s1, r2, 0):

$$\begin{aligned} r11 &:= (1 + \sqrt{2v+1})r2 \\ &= (1 + \sqrt{2v+1})r2^2, s1r2, 0, 0 \\ &= 0, r2^2, \frac{s1r2^2}{2}, 0 \end{aligned}$$

$SF_{12}^{4,1}$

> solve([M[1,1], M[1,3], M[2,1], M[2,2]], {v, r1, s1, r2, s2});

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$$\left\{ r1 = r1, r2 = 0, s1 = -\frac{s2}{2u-1}, s2 = s2, v = v \right\}$$

> s21 := (1-2*u)*s1:
zamproc(0,u,1,v,0,0,1,0, r1,s1,0,s21):

$$0, -u s1 (2u-1) r1, 0, -\frac{(4u^2v-4vu-u+v) s1^3 (2u-1)}{r1}$$


$$0, 0, (-2u+1) s1 r1, s1^2 (-2u+1)$$


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$SF_{13}^{4,1}$

```

> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2}):

$$\left\{ r1 = -\frac{2r2}{3}, r2 = r2, s1 = \frac{s2}{3}, s2 = s2, u = 2, v = \frac{2}{9} \right\}$$


```

I, класс 3).

$NSF_6^{3,1}$. Результат произвольной замены :

```

> M := zamproc(u,0,1,0,0,1,0, r1,s1,r2,s2):

$$\frac{r1 (s2 r1^2 u - r2^2 (s1 - s2))}{-s1 r2 + s2 r1}, \frac{-s1 (s1 - s2) r2^2 - 2 s2 r1 (s1 - s2) r2 + 3 u r1^2 s1 s2}{-s1 r2 + s2 r1}, \frac{3 s2 \left( \left( r1 u - \frac{2 r2}{3} \right) s1^2 - \frac{s2 (r1 - 2 r2) s1}{3} + \frac{s2^2 r1}{3} \right)}{-s1 r2 + s2 r1},$$


$$\frac{s2 s1 (s1^2 u - s1 s2 + s2^2)}{-s1 r2 + s2 r1}$$


$$-\frac{r2 r1 (r1^2 u - r1 r2 + r2^2)}{-s1 r2 + s2 r1}, -\frac{3 \left( \left( s1 u - \frac{2 s2}{3} \right) r1^2 - \frac{r2 (s1 - 2 s2) r1}{3} + \frac{r2^2 s1}{3} \right) r2}{-s1 r2 + s2 r1}, \frac{r1^2 s2^2 - 3 \left( s1^2 u - \frac{2}{3} s1 s2 + \frac{1}{3} s2^2 \right) r2 r1 - 2 s1 r2^2 s2}{-s1 r2 + s2 r1},$$


$$-\frac{s1 (r2 (s1^2 u + s1 s2) - s2^2 r1)}{-s1 r2 + s2 r1}$$


```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$NSF_{11}^{3,1}$. Результат произвольной замены :

```

> M := zamproc(u,0,k,0,0,1,0,0, r1,s1,r2,s2):

$$\frac{r1 (k r2^2 s2 + r1^2 s2 u - s1 r2 r1)}{s2 r1 - s1 r2}, \frac{3 \left( u - \frac{1}{3} \right) s1 s2 r1^2 + 2 r2 (s2^2 k - s1^2) r1 + s2 s1 r2^2 k}{s2 r1 - s1 r2}, \frac{-r2 s1^3 + 3 \left( u - \frac{2}{3} \right) s2 r1 s1^2 + 2 k s1 r2 s2^2 + k r1 s2^3}{s2 r1 - s1 r2},$$


$$\frac{s1 ((u-1) s1^2 + s2^2) k s2}{s2 r1 - s1 r2}$$


$$-\frac{r2 ((u-1) r1^2 + r2^2 k) r1}{s2 r1 - s1 r2}, \frac{s2 r1^3 - 3 \left( u - \frac{2}{3} \right) s1 r2 r1^2 - 2 s2 r2^2 r1 k - s1 r2^3 k}{s2 r1 - s1 r2}, \frac{2 s2 s1 r1^2 - ((3u-1) s1^2 + s2^2) r2 r1 - 2 s2 s1 r2^2 k}{s2 r1 - s1 r2},$$


$$-\frac{s1 (u r2 s1^2 + s2^2 r2 k - s2 r1 s1)}{s2 r1 - s1 r2}$$


```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,k,r1,s1,r2,s2});
{k=0,r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,k,r1,s1,r2,s2});
{k=0,r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$NSF_{17}^{3,1}$. Результат произвольной замены :

```

> M := zamproc(u,0,1,0,1,0,0,0, r1,s1,r2,s2):

$$\frac{r1 ((s2 u - s1) r1^2 + r2^2 s2)}{-s1 r2 + s2 r1}, \frac{2 r1 r2 s2^2 + (3 r1^2 u + r2^2) s1 s2 - 3 r1^2 s1^2}{-s1 r2 + s2 r1}, \frac{(3 s1^2 s2 u - 3 s1^3 + s2^3) r1 + 2 s1 r2 s2^2}{-s1 r2 + s2 r1}, \frac{s1 (s1^2 s2 u - s1^3 + s2^3)}{-s1 r2 + s2 r1}$$


$$-\frac{r1 (r1^2 r2 u - r1^3 + r2^3)}{-s1 r2 + s2 r1}, \frac{(-3 r1^2 r2 u + 3 r1^3 - r2^3) s1 - 2 r2^2 s2 r1}{-s1 r2 + s2 r1}, \frac{3 r1^2 s1^2 + (-3 s1^2 u - s2^2) r2 r1 - 2 s1 r2^2 s2}{-s1 r2 + s2 r1},$$


$$\frac{s1 (-s1^2 r2 u + s1^2 r1 - r2 s2^2)}{-s1 r2 + s2 r1}$$


```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=r1,r2=0,s1=0,s2=s2,u=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$NSF_{19}^{3,1}$. Результат произвольной замены :

```

> M := zamproc(0,u,0,1,0,1,0,0, r1,s1,r2,s2):

$$\frac{((s2 u - s1) r1^2 + r2^2 s2) r2}{-s1 r2 + s2 r1}, \frac{s2 (s2 u - s1) r1^2 + 2 s1 r2 (s2 u - s1) r1 + 3 r2^2 s2^2}{-s1 r2 + s2 r1}, \frac{-s1^3 r2 + s2 (u r2 - 2 r1) s1^2 + 2 u r1 s1 s2^2 + 3 r2 s2^3}{-s1 r2 + s2 r1},$$


```

$$\frac{s2 (s l^2 s2 u - s l^3 + s2^3)}{-s1 r2 + s2 r1}$$

$$- \frac{r2 (r l^2 r2 u - r l^3 + r2^3)}{-s1 r2 + s2 r1}, \frac{r l^3 s2 - r2 (s2 u - 2 s1) r l^2 - 2 u r1 s1 r2^2 - 3 r2^3 s2}{-s1 r2 + s2 r1}, \frac{(-s l^2 u - 3 s2^2) r2^2 - 2 s1 r1 \left(s2 u - \frac{s l}{2}\right) r2 + 2 r l^2 s1 s2}{-s1 r2 + s2 r1},$$

$$- \frac{s2 (r2 (s l^2 u + s2^2) - s l^2 r l)}{-s1 r2 + s2 r1}$$

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$NSF_{21}^{3,1}$. Результат произвольной замены :

$$> M := zamproc(0,u,1,0,1,0,0,0, r1,s1,r2,s2):
r1 (r l r2 s2 u - r l^2 s1 + r2^2 s2), \frac{(u s2^2 - 3 s l^2) r l^2 + 2 r2 s2 (s1 u + s2) r l + s1 r2^2 s2}{-s1 r2 + s2 r1}, \frac{-3 r1 s l^3 + r2 s l^2 s2 u + 2 s2^2 (r2 + r l u) s1 + r1 s2^3}{-s1 r2 + s2 r1},$$

$$s1 (u s1 s2^2 - s l^3 + s2^3) \frac{-s1 r2 + s2 r1}{-s1 r2 + s2 r1}$$

$$- \frac{r l (r l r2^2 u - r l^3 + r2^3)}{-s1 r2 + s2 r1}, \frac{3 s1 r l^3 - u r l^2 r2 s2 - 2 r2^2 (s1 u + s2) r l - r2^3 s1}{-s1 r2 + s2 r1}, \frac{(-s l^2 u - 2 s1 s2) r2^2 - 2 s2 \left(s1 u + \frac{s2}{2}\right) r l r2 + 3 r l^2 s l^2}{-s1 r2 + s2 r1},$$

$$s1 (-r2 s1 s2 u + s l^2 r l - r2 s2^2) \frac{-s1 r2 + s2 r1}{-s1 r2 + s2 r1}$$

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=r1,r2=0,s1=0,s2=s2,u=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=0,s2=s2,u=2}, {r1=RootOf(_Z^2 - _Z + 1) r2,r2=r2,s1=0,s2=s2,u=2 RootOf(_Z^2 - _Z + 1) - 2}
```

```
> zamproc(0,2,1,0,1,0,0,0, r1,0,-r1,s2):
-r l^2, 0, s2^2, 0
0, 0, r l s2, 0
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=r1,r2=0,s1=0,s2=s2,u=0}
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$NSF_{22}^{3,1}$. Результат произвольной замены :

$$> M := zamproc(0,0,u,1,0,1,0,0, r1,s1,r2,s2):
r2 (r l r2 s2 u - r l^2 s1 + r2^2 s2), \frac{s2 (s1 u + 3 s2) r2^2 + 2 r1 (u s2^2 - s l^2) r2 - r l^2 s1 s2}{-s1 r2 + s2 r1}, \frac{(r1 u + 3 r2) s2^3 + 2 u s1 r2 s2^2 - 2 r1 s l^2 s2 - s l^3 r2}{-s1 r2 + s2 r1},$$

$$s2 (u s1 s2^2 - s l^3 + s2^3) \frac{-s1 r2 + s2 r1}{-s1 r2 + s2 r1}$$

$$- \frac{r2 (r l r2^2 u - r l^3 + r2^3)}{-s1 r2 + s2 r1}, \frac{(-s l u - 3 s2) r2^3 - 2 u r l r2^2 s2 + 2 r l^2 r2 s1 + r l^3 s2}{-s1 r2 + s2 r1}, \frac{(-2 s1 s2 u - 3 s2^2) r2^2 - r l (u s2^2 - s l^2) r2 + 2 r l^2 s1 s2}{-s1 r2 + s2 r1},$$

$$s2 (-r2 s1 s2 u + s l^2 r l - r2 s2^2) \frac{-s1 r2 + s2 r1}{-s1 r2 + s2 r1}$$

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
{rl=0,r2=r2,sl=sl,s2=0,u=0}
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
{rl=rl,r2=0,sl=0,s2=s2,u=0}
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
{rl=0,r2=r2,sl=sl,s2=0,u=0}
```

$NSF_5^{4,1}$. Результат произвольной замены :

```
> M := zamproc(u,v,1,0,0,0,1,0, r1,sl,r2,s2):  
  
 
$$\frac{(urI^2 s2 + vs2 r2 rI - r2^2 (s1 - s2)) rI}{-s1 r2 + rI s2}, \frac{rI (vrI + 2r2) s2^2 + 3s1 \left( urI^2 + \frac{2r2(v-1)rI}{3} + \frac{r2^2}{3} \right) s2 - sl^2 r2^2}{-s1 r2 + rI s2},$$
  

$$\frac{3 \left( \left( urI + \frac{(v-2)r2}{3} \right) sl^2 + \frac{2 \left( \left( v - \frac{1}{2} \right) rI + r2 \right) s2 sl}{3} + \frac{rl s2^2}{3} \right) s2}{-s1 r2 + rI s2}, \frac{s1 (usl^2 + s2 (v-1) sl + s2^2) s2}{-s1 r2 + rI s2}$$
  

$$-\frac{r2 (urI^2 + r2 (v-1) rI + r2^2) rI}{-s1 r2 + rI s2}, -\frac{3r2 \left( \left( usI + \frac{(v-2)s2}{3} \right) rI^2 + \frac{2 \left( \left( v - \frac{1}{2} \right) sl + s2 \right) r2 rI}{3} + \frac{sl r2^2}{3} \right)}{-s1 r2 + rI s2},$$
  

$$\frac{(-sl^2 v - 2sl s2) r2^2 - 3rl \left( usl^2 + \frac{2s2(v-1)sl}{3} + \frac{s2^2}{3} \right) r2 + rl^2 s2^2}{-s1 r2 + rI s2}, -\frac{s1 ((usl^2 + vs2 sl + s2^2) r2 - rl s2^2)}{-s1 r2 + rI s2}$$

```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,sl,r2,s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,sl,r2,s2});  
{rl=rl,r2=0,sl=0,s2=s2,u=u,v=0}
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,sl,r2,s2});  
{rl=r2,r2=r2,sl=0,s2=s2,u=2,v=-2}
```

```
> zamproc(2,-2,1,0,0,0,1,0, r1,0,rl,sl):
```

$$\begin{matrix} rI^2, 0, s2^2, 0 \\ 0, 2rI^2, 0, 0 \end{matrix}$$

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,sl,r2,s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,sl,r2,s2});  
{rl=0,r2=r2,sl=sl,s2=0,u=0,v=0}
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,sl,r2,s2});  
{rl=r2,r2=r2,sl=0,s2=s2,u=-1,v=0}
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,sl,r2,s2});  
{rl=0,r2=r2,sl=sl,s2=sl,u=1,v=-2}
```

```
> zamproc(1,-2,1,0,0,0,1,0, 0,sl,r2,sl):
```

$$\begin{matrix} 0, 0, 2sl^2, \frac{sl^3}{r2} \\ 0, r2^2, 0, 0 \end{matrix}$$

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,r1,sl,r2,s2});
```

$$\left\{ rl = -\frac{2r2}{v-2}, r2 = r2, sl = sl, s2 = 0, u = -\frac{\left(-\frac{2r2^2(v-1)}{v-2} + r2^2 \right)(v-2)^2}{4r2^2} \right\}, \left\{ rl = rl, r2 = 0, sl = -\frac{2s2}{v-2}, s2 = s2, u = \frac{1}{4}v^2 - \frac{1}{2}v \right\}$$

```
> simplify((-2*x2^2*(v-1)/(v-2)+r2^2)*((v-2)^2/(4*x2^2));
```

$$\frac{v(v-2)}{4}$$

```
> s21 := (2-v)*sl/2;
```

```
u1 := v*(v-2)/4;
```

```
zamproc(u1,v,1,0,0,0,1,0, r1,sl,0,s21);
```

$$\begin{aligned} & \frac{v(v-2)rl^2}{4}, \frac{(v-2)s1rlv}{4}, 0, 0 \\ & 0, 0, -\frac{rl(v-2)s1}{2}, -\frac{s1l^2(v-2)}{2} \end{aligned}$$

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});  
solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});  
 $\left\{ r1=r1, r2=0, s1=s1, s2=s2, u=-\frac{s2(-s2+s1)}{2s1^2}, v=\frac{3(-s2+s1)}{2s1} \right\}, \left\{ r1=\frac{3r2}{2}, r2=r2, s1=s1, s2=-\frac{s1}{3}, u=-\frac{2}{9}, v=\frac{2}{3} \right\}, \{r1=r1, r2=0, s1=0,$   
 $s2=s2, u=u, v=0\}$   
 $\left\{ r1=r1, r2=0, s1=s1, s2=-\frac{2}{3}s1v+s1, u=\frac{2}{9}v^2-\frac{1}{3}v \right\}$ 
```

частный случай ухода в $SF_1^{4,1}$:

```
> r21 := 2*r1/3;  
s21 := -s1/3;  
zamproc(-2/9, 2/3, 1, 0, 0, 0, 1, 0, r1, s1, r21, s21);  
 $\frac{2rl^2}{3}, 0, -\frac{2s1l^2}{3}, 0$   
 $0, 0, -\frac{rls1}{3}, -\frac{s1l^2}{3}$   
> s21 := (1-2*v/3)*s1;  
u1 := v*(2*v-3)/9;  
zamproc(u1, v, 1, 0, 0, 0, 1, 0, r1, s1, 0, s21);  
 $\frac{v(2v-3)rl^2}{9}, 0, -\frac{v(2v-3)s1l^2}{9}, 0$   
 $0, 0, -\frac{rl(2v-3)s1}{3}, -\frac{2}{3}s1l^2v+s1l^2$ 
```

$NSF_{11}^{4,1}$. Результат произвольной замены:

```
> M := zamproc(u, 1, v, 0, 0, 1, 0, 0, r1, s1, r2, s2);  
 $\frac{(vr2^2s2-r1(-s2+s1)r2+ur1^2s2)r1}{s2rl-s1r2}, \frac{3s2\left(\frac{s2}{3}+\left(u-\frac{1}{3}\right)s1\right)rl^2+2r2(vs2^2-s1^2+s2s1)r1+vs2s1r2^2}{s2rl-s1r2},$   
 $\frac{-r2s1^3+3s2\left(\left(u-\frac{2}{3}\right)rl+\frac{r2}{3}\right)s1^2+2s2^2(r2v+r1)s1+vs2^3rl}{s2rl-s1r2}, \frac{s1(vs2^2+(u-1)s1^2+s2s1)s2}{s2rl-s1r2}$   
 $-\frac{(vr2^2+(u-1)r1^2+r2rl)r2r1}{s2rl-s1r2}, \frac{s2r1^3-3r2\left(\left(u-\frac{2}{3}\right)s1+\frac{s2}{3}\right)rl^2-2r2^2(vs2+s1)r1-vs1r2^3}{s2rl-s1r2},$   
 $\frac{(-2vs2s1-s1^2)r2^2-3\left(\left(u-\frac{1}{3}\right)s1^2+\frac{2s2s1}{3}+\frac{vs2^2}{3}\right)rlr2+2s2s1r1^2}{s2rl-s1r2}, -\frac{s1(vs2^2r2-s1(-r2+r1)s2+us1^2r2)}{s2rl-s1r2}$ 
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});  
(r1=0, r2=r2, s1=s1, s2=0, u=0, v=0)
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
(r1=r1, r2=0, s1=0, s2=s2, u=0, v=0)
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
(rl=rl, r2=rl, s1=0, s2=s2, u=-1, v=0)
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});  
(rl=0, r2=r2, s1=s1, s2=s1, u=0, v=0), (rl=rl, r2=rl, s1=0, s2=s2, u=0, v=0)
```

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
```

$$\left\{ r1 = rI, r2 = \frac{rI}{3}, s1 = sI, s2 = -\frac{2sI}{3}, u = \frac{1}{3}, v = 3 \right\}, \{rI = 0, r2 = r2, s1 = sI, s2 = 0, u = u, v = 0\}, \left\{ rI = 0, r2 = r2, s1 = sI, s2 = s2, u = \frac{-s2 + sI}{sI}, v = 0 \right\}$$

```
> r21 := r1/3;
s21 := -2*s1/3;
zamproc(1/3, 1, 3, 0, 0, 1, 0, 0, r1, s1, r21, s21):
```

$$\begin{aligned} & rI^2, 0, -sI^2, 0 \\ & 0, 0, s1 \cdot rI, sI^2 \end{aligned}$$

$SF_5^{4,1}$

```
> solve([M[1,4], M[2,1], M[2,2], M[2,4]], {v, r1, s1, r2, s2});
\left\{ rI = -\frac{r2}{2u-1}, r2 = r2, s1 = 0, s2 = s2, v = \frac{u}{4u^2-4u+1} \right\}
```

```
> r21 := (1-2*u)*r1;
v1 := u^(2*u-1)^(-2);
zamproc(u, 1, v1, 0, 0, 1, 0, 0, r1, 0, r21, s2):
```

$$\begin{aligned} & rI^2, -\frac{s2 \cdot rI}{2u-1}, \frac{u \cdot s2^2}{(2u-1)^2}, 0 \\ & 0, 0, \frac{s2 \cdot rI \cdot u}{2u-1}, 0 \end{aligned}$$

$NSF_{14}^{4,1}$ ($v \neq -u^2$). Результат произвольной замены:

```
> M := zamproc(0, 1, u, 0, 0, 1, 0, v, r1, s1, r2, s2):
r2 ((s2 - s1) rI^2 + r1 r2 s2 u - v r2^2 s1), (2 r2 u rI + rI^2) s2^2 + s1 (-rI^2 + 2 r1 r2 + r2^2 (u - 3 v)) s2 - 2 sI^2 r2 rI,
s2 rI - r2 sI
- r2 sI^3 - 2 s2 \left( rI - \frac{r2}{2} \right) sI^2 + 2 \left( \left( u - \frac{3v}{2} \right) r2 + rI \right) s2^2 sI + u rI s2^3,
s2 rI - r2 sI
s1 s2 ((u - v) s2^2 + s2 sI - sI^2),
s2 rI - r2 sI
- r2 ((u - v) r2^2 + r1 r2 - rI^2) rI,
s2 rI^3 + 2 r2 \left( sI - \frac{s2}{2} \right) rI^2 - 2 r2^2 \left( \left( u - \frac{3v}{2} \right) s2 + sI \right) rI - u sI r2^3,
s2 rI - r2 sI
(-2 s2 sI u - sI^2) r2^2 - (-sI^2 + 2 s2 sI + s2^2 (u - 3 v)) rI r2 + 2 s2 sI rI^2,
s2 rI - r2 sI
- ((r2 - rI) sI^2 + r2 sI s2 u - v rI s2^2) s2
s2 rI - r2 sI
```

$SF_9^{2,1}$

```
> solve([M[1,1], M[1,2], M[1,4], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2], M[1,4], M[2,1], M[2,2], M[2,4]], {u, v, r1, s1, r2, s2});
(rI = 0, r2 = r2, s1 = sI, s2 = 0, u = 0, v = v)
```

$SF_{11}^{3,1}$

```
> solve([M[1,2], M[1,4], M[2,1], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2], M[1,4], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1], M[1,3], M[2,1], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
(rI = rI, r2 = 0, sI = 0, s2 = s2, u = 0, v = 0)
```

$SF_{21}^{3,1}$

```
> solve([M[1,1], M[1,4], M[2,2], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
(rI = r2, r2 = r2, sI = sI, s2 = 0, u = 0, v = -1)
```

$SF_{22}^{3,1}$

```
> solve([M[1,1], M[1,2], M[2,1], M[2,3], M[2,4]], {u, v, r1, s1, r2, s2});
```

$SF_1^{4,1}$

```
> solve([M[1,3], M[1,4], M[2,1], M[2,2]], {v, r1, s1, r2, s2});
```

$$\left\{ rI = -r2 (\text{RootOf}(2 \cdot Z^2 - 2 \cdot Z - u) - 1), r2 = r2, sI = \text{RootOf}(2 \cdot Z^2 - 2 \cdot Z - u) s2, s2 = s2, v = \frac{u}{2} \right\}$$

```
> solve(2 * Z^2 - 2 * Z - u, Z);
```

$$\frac{1}{2} + \frac{\sqrt{2u+1}}{2}, \frac{1}{2} - \frac{\sqrt{2u+1}}{2}$$

```
> r11 := -(1/2 + (1/2) * sqrt(2*u+1) - 1) * r2;
```

```
s11 := (1/2 + (1/2) * sqrt(2*u+1)) * s2;
```

```
v1 := u/2;
```

```
zamproc(0, 1, u, 0, 0, 1, 0, v1, r11, s11, r2, s2):
```

$$\begin{aligned} rII &:= -\left(-\frac{1}{2} + \frac{\sqrt{2u+1}}{2} \right) r2 \\ sII &:= \left(\frac{1}{2} + \frac{\sqrt{2u+1}}{2} \right) s2 \end{aligned}$$

$$\begin{aligned} & \frac{r^2 (\sqrt{2 u+1}-1) \sqrt{2 u+1}}{2}, \frac{r^2 \sqrt{2 u+1} s_2 (\sqrt{2 u+1}-1)}{2}, 0, 0 \\ & 0, 0, \frac{(1+\sqrt{2 u+1}) r^2 \sqrt{2 u+1} s_2}{2}, \frac{(1+\sqrt{2 u+1}) s_2^2 \sqrt{2 u+1}}{2} \end{aligned}$$

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s2, s2=s2, u=0, v=0}, {r1 =  $\frac{3 r^2}{4}$ , r2=r2, s1=0, s2=s2, u=- $\frac{3}{8}$ , v=- $\frac{3}{16}$ }, {r1=0, r2=r2, s1=s1, s2=0, u=0, v=v}
```

$SF_5^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {v,r1,s1,r2,s2});
{r1=RootOf(2_Z^2-2_Z-u) r2, r2=r2, s1=s1, s2=0, v= $\frac{u}{2}$ }
```

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{rl=rl, r2=0, s1=0, s2=s2, u=u, v=0}, {rl=r2, r2=r2, s1=0, s2=s2, u=0, v=0}, {rl=r2, r2=r2, s1=s1, s2=0, u=u, v=u}
```

```
> zamproc(0,1,u,0,0,1,0,u, r1,s1,r1,0):
```

$$\begin{aligned} & r l^2 (u+1), 2 r l s l, s l^2, 0 \\ & 0, r l^2 u, 0, 0 \end{aligned}$$

$SF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{rl=0, r2=r2, s1 =  $\frac{s^2}{2}$ , s2=s2, u=- $\frac{1}{4}$ , v=- $\frac{1}{12}$ }
```

```
> s21 := 2*s1:
```

```
zamproc(0,1,-1/4,0,0,1,0,-1/12, 0,s1,r2,s21):
-  $\frac{r^2}{12}$ , 0, 0, -  $\frac{2 s l^3}{3 r_2}$ 
0, -  $\frac{r^2}{4}$ , 0, s l^2
```

$NSF_{14}^{4,1}$. Результат произвольной замены :

```
> M := zamproc(u,v,1,0,1,0,0,0, r1,s1,r2,s2):
 $\frac{(u s_2-s_1) r l^2+v s_2 r l r_2+s_2 r^2) r l}{s_2 r l-r_2 s l}, \frac{(3 s_2 s_1 u+v s_2^2-3 s l^2) r l^2+2 r_2 s_2 (s_1 v+s_2) r l+s_1 s_2 r_2^2}{s_2 r l-r_2 s l},$ 
 $\frac{-3 r_1 s l^3+s_2 (3 r_1 u+r_2 v) s l^2+2 s^2 (r_1 v+r_2) s l+r_1 s^3}{s_2 r l-r_2 s l}, \frac{s_1 (u s_2 s_1)^2+s_2^2 s_1 v-s_1^3+s_2^3}{s_2 r l-r_2 s l}$ 
 $\frac{r_1 (u r_2 r l^2+r^2 r_1 v-r l^3+r^2)}{s_2 r l-r_2 s l}, \frac{3 r l^3 s_1+(-3 s_1 u-v s_2) r_2 r l^2-2 r^2 (s_1 v+s_2) r l-r l^3 s_1}{s_2 r l-r_2 s l},$ 
 $\frac{(-s l^2 v-2 s_2 s_1) r l^2-3 \left(s l^2 u+\frac{2}{3} v s_2 s_1+\frac{1}{3} s^2\right) r l r_2+3 r l^2 s l^2}{s_2 r l-r_2 s l}, \frac{s_1 (-u r_2 s l^2-s_2 r_2 s_1 v-s_2^2 r_2+r_1 s l^2)}{s_2 r l-r_2 s l}$ 
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{rl=rl, r2=0, s1=0, s2=s2, u=0, v=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{rl=rl, r2 =  $-\frac{r l v}{2}$ , s1=0, s2=s2, u= $\frac{v^3-8}{4 v}$ }
```

```
> r21 := -v*r1/2:
```

```
u1 := (v^3-8)/(4*v):
```

```
zamproc(u1,v,1,0,1,0,0,0, r1,0,r21,s2):
```

$$\begin{aligned} & -\frac{2 r l^2}{v}, 0, s^2, 0 \\ & 0, 0, \frac{s_2 r l v}{2}, 0 \end{aligned}$$

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{rl=rl, r2=0, s1=0, s2=s2, u=u, v=0}
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{rl=0, r2=r2, s1=s1, s2 =  $-\frac{s l v}{2}$ , u= $\frac{v^2}{4}$ }
```

```
> s21 := -s1*v/2:
```

```

u1 := v^2/4:
zamproc(u1,v,1,0,1,0,0,0, 0,s1,r2,s2) :

$$0, \frac{r2 s1 v}{2}, 0, \frac{s1^3}{r2}$$


$$0, r2^2, 0, 0$$

SF213,1
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=0, v=v}
SF223,1
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}
SF14,1
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
SF34,1
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});

$$\left\{ r1=r1, r2=-\frac{r1 v}{2}, s1=0, s2=s2, u=-\frac{-v^3+8}{4 v} \right\}$$

SF54,1
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});

$$\left\{ r1=r1, r2=-\frac{r1 v}{2}, s1=0, s2=s2, u=\frac{v^3-8}{4 v} \right\}$$

SF114,1
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
SF134,1
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0}
SF144,1
> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {u,r1,s1,r2,s2});

$$\left\{ r1=0, r2=r2, s1=s1, s2=-\frac{s1 v}{2}, u=\frac{v^3-8}{4 v} \right\}$$

NSF274,1 (v ≠ -u-2). Результат произвольной замены:
> M := zamproc(0,0,u,1,0,1,0,v, r1,s1,r2,s2):

$$\frac{r2 ((-v s1 + s2) r2^2 + u r1 r2 s2 - s1 r1^2)}{r1 s2 - s1 r2}, \frac{(3 s2 + s1 (-3 v + u)) s2 r2^2 + 2 r1 (u s2^2 - s1^2) r2 - r1^2 s1 s2}{r1 s2 - s1 r2},$$


$$\frac{(u r1 + 3 r2) s2^3 + 2 s1 r2 \left(u - \frac{3 v}{2}\right) s2^2 - 2 r1 s1^2 s2 - s1^3 r2}{r1 s2 - s1 r2}, \frac{(s2^3 + s1 (-v + u) s2^2 - s1^3) s2}{r1 s2 - s1 r2}$$


$$-\frac{(r2^3 + r1 (-v + u) r2^2 - r1^3) r2}{r1 s2 - s1 r2}, \frac{(-u s1 - 3 s2) r2^3 - 2 s2 r1 \left(u - \frac{3 v}{2}\right) r2^2 + 2 s1 r1^2 r2 + r1^3 s2}{r1 s2 - s1 r2},$$


$$\frac{(-2 s2 s1 u - 3 s2^2) r2^2 - r1 (s2^2 (-3 v + u) - s1^2) r2 + 2 r1^2 s1 s2}{r1 s2 - s1 r2}, \frac{s2 (v r1 s2^2 - u s1 r2 s2 + r1 s1^2 - r2 s2^2)}{r1 s2 - s1 r2}$$

SF92,1
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}
SF63,1
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});
SF113,1
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
SF173,1
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=v}
SF193,1
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
SF213,1
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}
SF223,1
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=u, v=v}

```

$SF_1^{4,1}$
 $\geq \text{solve}([\text{M}[1,3],\text{M}[1,4],\text{M}[2,1],\text{M}[2,2]], \{u,v,r1,s1,r2,s2\});$
 $SF_3^{4,1}$
 $\geq \text{solve}([\text{M}[1,2],\text{M}[1,4],\text{M}[2,1],\text{M}[2,2]], \{u,v,r1,s1,r2,s2\});$
 $SF_5^{4,1}$
 $\geq \text{solve}([\text{M}[1,4],\text{M}[2,1],\text{M}[2,2],\text{M}[2,4]], \{v,r1,s1,r2,s2\});$

$$\left\{ r1 = \text{RootOf}(2 \cdot Z^2 - u) \cdot r2, r2 = r2, s1 = s1, s2 = 0, v = \frac{\text{RootOf}(2 \cdot Z^2 - u) \cdot u + 2}{2 \cdot \text{RootOf}(2 \cdot Z^2 - u)} \right\}$$

 $\geq r11 := \text{sqrt}(u/2) \cdot r2;$
 $v1 := (\text{sqrt}(u/2) \cdot u + 2) / (2 \cdot \text{sqrt}(u/2));$
 $\text{zamproc}(0,0,u,1,0,1,0,v1, r11,s1,r2,0):$

$$\begin{aligned} & \frac{r2^2(u^3 \cdot 1^2 + \sqrt{2})}{\sqrt{u}}, \sqrt{2} \cdot \sqrt{u} \cdot r2 \cdot s1, s1^2, 0 \\ & 0, 0, -\frac{\sqrt{2} \cdot \sqrt{u} \cdot r2 \cdot s1}{2}, 0 \end{aligned}$$

 $\geq r12 := -\text{sqrt}(u/2) \cdot r2;$
 $v2 := (-\text{sqrt}(u/2) \cdot u + 2) / (-2 \cdot \text{sqrt}(u/2));$
 $\text{zamproc}(0,0,u,1,0,1,0,v2, r12,s1,r2,0):$

$$\begin{aligned} & \frac{-r2^2(-u^3 \cdot 1^2 + \sqrt{2})}{\sqrt{u}}, -\sqrt{2} \cdot \sqrt{u} \cdot r2 \cdot s1, s1^2, 0 \\ & 0, 0, \frac{\sqrt{2} \cdot \sqrt{u} \cdot r2 \cdot s1}{2}, 0 \end{aligned}$$

 $SF_{11}^{4,1}$
 $\geq \text{solve}([\text{M}[1,4],\text{M}[2,1],\text{M}[2,3],\text{M}[2,4]], \{u,v,r1,s1,r2,s2\});$
 $SF_{13}^{4,1}$
 $\geq \text{solve}([\text{M}[1,2],\text{M}[1,3],\text{M}[2,1],\text{M}[2,3]], \{u,v,r1,s1,r2,s2\});$

$$\{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = 0, v = v\}$$

 $\geq u1 := 3^*4^{(-2/3)};$
 $v1 := 4^{(-2/3)};$
 $r11 := 2^{(1/3)} \cdot r2;$
 $s11 := -3^*2^{(-2/3)} \cdot s2;$
 $\text{zamproc}(0,0,u1,1,0,1,0,v1, r11,s11,r2,s2):$

$$\begin{aligned} & \frac{5 \cdot r2^2 \cdot 2^2 \cdot 1^3}{4}, 0, 0, \frac{5 \cdot s2^3 \cdot 2^2 \cdot 1^3}{4 \cdot r2} \\ & 0, -\frac{5 \cdot r2^2 \cdot 2^2 \cdot 1^3}{4}, 0, \frac{5 \cdot s2^2 \cdot 2^2 \cdot 1^3}{4} \end{aligned}$$

 $SF_{14}^{4,1}$
 $\geq \# v = -u^{(-2)} !$
 $\text{solve}([\text{M}[1,1],\text{M}[1,4],\text{M}[2,1],\text{M}[2,3]], \{u,v,r1,s1,r2,s2\});$

$$\left\{ r1 = \text{RootOf}(3 \cdot Z^3 + 4) \cdot r2, r2 = r2, s1 = \frac{2 \cdot s2}{\text{RootOf}(3 \cdot Z^3 + 4)^2}, s2 = s2, u = \frac{3 \cdot \text{RootOf}(3 \cdot Z^3 + 4)^3 + 2}{2 \cdot \text{RootOf}(3 \cdot Z^3 + 4)}, v = \frac{9 \cdot \text{RootOf}(3 \cdot Z^3 + 4)^3 + 20}{6 \cdot \text{RootOf}(3 \cdot Z^3 + 4)} \right\}$$

 $SF_{19}^{4,1}$
 $\geq \text{solve}([\text{M}[1,4],\text{M}[2,2],\text{M}[2,3],\text{M}[2,4]], \{u,v,r1,s1,r2,s2\});$

$$\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = v\}$$

 $NSF_{29}^{4,1} (v \neq -u)$. Результат произвольной замены:
 $\geq M := \text{zamproc}(0,u,0,v,0,1,1,0, r1,s1,r2,s2):$

$$\frac{r2((s2 \cdot u - s1) \cdot r1^2 - r1 \cdot r2 \cdot s1 + r2^2 \cdot s2 \cdot v)}{r1 \cdot s2 - r2 \cdot s1}, \frac{s2(s2 \cdot u - s1) \cdot r1^2 + 2 \cdot r2(-s1 + (u - 1) \cdot s2) \cdot s1 \cdot r1 + r2^2(3 \cdot s2^2 \cdot v - s1^2)}{r1 \cdot s2 - r2 \cdot s1},$$

$$\frac{-s1^3 \cdot r2 + ((u - 2) \cdot r2 - 2 \cdot r1) \cdot s2 \cdot s1^2 + 2 \cdot r1 \left(u - \frac{1}{2} \right) \cdot s2^2 \cdot s1 + 3 \cdot v \cdot r2 \cdot s2^3}{r1 \cdot s2 - r2 \cdot s1}, \frac{(v \cdot s2^3 + s1^2 \cdot (u - 1) \cdot s2 - s1^3) \cdot s2}{r1 \cdot s2 - r2 \cdot s1}$$

$$\frac{-r2(v \cdot r2^3 + r1^2 \cdot (u - 1) \cdot r2 - r1^3)}{r1 \cdot s2 - r2 \cdot s1}, \frac{r1^3 \cdot s2 - r2((u - 2) \cdot s2 - 2 \cdot s1) \cdot r1^2 - 2 \cdot r2^2 \cdot s1 \left(u - \frac{1}{2} \right) \cdot r1 - 3 \cdot v \cdot r2^3 \cdot s2}{r1 \cdot s2 - r2 \cdot s1},$$

$$\frac{(2 \cdot s1 \cdot s2 + s2^2) \cdot r1^2 - 2 \cdot r2 \left(-\frac{s1}{2} + (u - 1) \cdot s2 \right) \cdot s1 \cdot r1 - r2^2(s1^2 \cdot u + 3 \cdot s2^2 \cdot v)}{r1 \cdot s2 - r2 \cdot s1}, \frac{s2(-r2 \cdot s1^2 \cdot u - r2 \cdot s2^2 \cdot v + r1 \cdot s1 \cdot s2)}{r1 \cdot s2 - r2 \cdot s1}$$

 $SF_9^{2,1}$
 $\geq \text{solve}([\text{M}[1,1],\text{M}[1,2],\text{M}[1,4],\text{M}[2,2],\text{M}[2,3],\text{M}[2,4]], \{u,v,r1,s1,r2,s2\});$
 $SF_6^{3,1}$
 $\geq \text{solve}([\text{M}[1,2],\text{M}[1,4],\text{M}[2,1],\text{M}[2,2],\text{M}[2,4]], \{u,v,r1,s1,r2,s2\});$

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$$\left\{ rI = -\frac{r^2}{2}, r2 = r2, s1 = s1, s2 = 0, u = 0, v = \frac{1}{8} \right\}$$

SF113,1
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

$$\left\{ rI = -\frac{r^2}{2}, r2 = r2, s1 = s1, s2 = 0, u = -\frac{1}{2}, v = \frac{1}{4} \right\}$$

> r11 := -r2/2;
zamproc(0,-1/2,0,1/4,0,1,1,0, r11,s1,r2,0):

$$\begin{aligned} & -\frac{r^2}{4}, 0, s1^2, 0 \\ & 0, \frac{r^2}{2}, 0, 0 \end{aligned}$$

SF173,1
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
SF193,1
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

$$\{rI=0, r2=r2, s1=-2s2, s2=s2, u=0, v=0\}$$

SF213,1
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

$$\{rI=0, r2=r2, s1=-s2, s2=s2, u=0, v=0\}, \{rI=0, r2=r2, s1=s1, s2=0, u=0, v=v\}$$

SF223,1
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
SF14,1
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
SF34,1
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});

$$\left\{ rI = -\frac{r^2}{2}, r2 = r2, s1 = s1, s2 = 0, u = 0, v = \frac{1}{8} \right\}$$

SF54,1
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {v,r1,s1,r2,s2});

$$\left\{ rI = r2u - \frac{1}{2}r2, r2 = r2, s1 = s1, s2 = 0, v = \frac{1}{2}u^2 - \frac{1}{2}u + \frac{1}{8} \right\}, \{rI = 0, r2 = r2, s1 = s1, s2 = 0, v = 0\}$$

> r11 := (u-1/2)*r2;
v1 := (2*u-1)^2/8;
zamproc(0,u,0,v1,0,1,1,0, r11,s1,r2,0):

$$\begin{aligned} & r2^2u^2 - \frac{1}{4}r2^2, 2r2s1u, s1^2, 0 \\ & 0, 0, \frac{s1r2}{2}, 0 \end{aligned}$$

SF114,1
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {v,r1,s1,r2,s2});

$$\{rI = r2u, r2 = r2, s1 = s1, s2 = 0, v = u^2\}$$

> v1 := u^2;
r11 := u*r2;
zamproc(0,u,0,v1,0,1,1,0, r11,s1,r2,0):

$$\begin{aligned} & r2^2u(u+1), r2s1(2u+1), s1^2, 0 \\ & 0, -r2^2u, 0, 0 \end{aligned}$$

SF134,1
> evala([solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1})]);

$$\left[ \left\{ rI = RootOf(3Z^3 - Z - 3)r2, s1 = -\frac{3(3RootOf(3Z^3 - Z - 3) + 2 + RootOf(3Z^3 - Z - 3)^2)s2}{5}, u = -\frac{6RootOf(3Z^3 - Z - 3)^2}{5} \right. \right. \\ \left. \left. - \frac{3RootOf(3Z^3 - Z - 3)}{5} - \frac{2}{5}, v = \frac{9RootOf(3Z^3 - Z - 3)^2}{5} + \frac{26RootOf(3Z^3 - Z - 3)}{15} + \frac{8}{5} \right\} \right]$$

> solve(3*Z^3-Z-3, Z);

$$\begin{aligned} & \frac{(108 + 20\sqrt{29})^{1/3}}{6} + \frac{2}{3(108 + 20\sqrt{29})^{1/3}}, -\frac{(108 + 20\sqrt{29})^{1/3}}{12} - \frac{1}{3(108 + 20\sqrt{29})^{1/3}} \\ & + \frac{I\sqrt{3}\left(\frac{(108 + 20\sqrt{29})^{1/3}}{6} - \frac{2}{3(108 + 20\sqrt{29})^{1/3}}\right)}{2}, -\frac{(108 + 20\sqrt{29})^{1/3}}{12} - \frac{1}{3(108 + 20\sqrt{29})^{1/3}} \end{aligned}$$


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$$\begin{aligned}
& - \frac{1}{2} \sqrt{3} \left(\frac{\left(108 + 20\sqrt{29}\right)^{1/3}}{6} - \frac{2}{3 \left(108 + 20\sqrt{29}\right)^{1/3}} \right) \\
& z := - \frac{\left(\left(108 + 20\sqrt{29}\right)^{2/3} + 4\right) \left(108 + 20\sqrt{29}\right)^{2/3} \left(-27 + 5\sqrt{29}\right)}{96} \\
> z1 := (r+4*r^(-1))/6: \\
u1 := & \text{simplify}(-6*z1^2*(1/5)-3*z1*(1/5)-2/5); \\
v1 := & \text{simplify}(9*z1^2*(1/5)+26*z1*(1/15)+8/5); \\
r11 := & z1*r2: \\
s11 := & \text{simplify}(-(1/5)*(3*(3*z1+2+z1^2)))*s2; \\
u1 := & \frac{-r^4 - 3r^3 - 20r^2 - 12r - 16}{30r^2} \\
v1 := & \frac{9r^4 + 52r^3 + 360r^2 + 208r + 144}{180r^2} \\
s11 := & \frac{(-r^4 - 18r^3 - 80r^2 - 72r - 16)s2}{60r^2} \\
> ro := (20*sqrt(29)+108)^(1/3): \\
u2 := & ((3*sqrt(29)-17)*ro^2+(4*sqrt(29)-24)*ro-16)/24: \\
v2 := & ((72-13*sqrt(29))*ro^2-(9*sqrt(29)-59)*ro+72)/36: \\
\text{simplify}(u1-u2); \\
\text{simplify}(v1-v2); \\
r12 := & (ro+4*ro^(-1))*r2/6: \\
s12 := & ((9*sqrt(29)-49)*ro^2+(2*sqrt(29)-12)*ro-32)*s2/24: \\
\text{zamproc}(0,u2,0,v2,0,1,1,0, r12,s12,r2,s2): \\
\text{simplify}(r11 - r12); \\
\text{rationalize}(\text{simplify}(s11 - s12)); \\
\end{aligned}$$

$$\begin{aligned}
& \frac{(15 \left(108 + 20\sqrt{29}\right)^{2/3} \sqrt{29} + 10 \left(108 + 20\sqrt{29}\right)^{1/3} \sqrt{29} - 83 \left(108 + 20\sqrt{29}\right)^{2/3} - 66 \left(108 + 20\sqrt{29}\right)^{1/3} - 16) r2^2}{72}, \\
& \frac{\left(\left(108 + 20\sqrt{29}\right)^{2/3} + 3 \left(108 + 20\sqrt{29}\right)^{1/3} + 4\right) r2 s2}{12}, \frac{\left(108 + 20\sqrt{29}\right)^{2/3} s2^2}{16}, \\
& \frac{-\left(30 \left(108 + 20\sqrt{29}\right)^{2/3} \sqrt{29} + 20 \left(108 + 20\sqrt{29}\right)^{1/3} \sqrt{29} - 163 \left(108 + 20\sqrt{29}\right)^{2/3} - 96 \left(108 + 20\sqrt{29}\right)^{1/3} + 16\right) s2^3}{144r2}, \\
0, & \frac{\left(39 \left(108 + 20\sqrt{29}\right)^{2/3} \sqrt{29} + 32 \left(108 + 20\sqrt{29}\right)^{1/3} \sqrt{29} - 217 \left(108 + 20\sqrt{29}\right)^{2/3} - 204 \left(108 + 20\sqrt{29}\right)^{1/3} - 224\right) r2^2}{72}, \\
& \frac{r2 \left(\left(\sqrt{29} - 7\right) \left(108 + 20\sqrt{29}\right)^{2/3} - 4\sqrt{29} - 4 \left(108 + 20\sqrt{29}\right)^{1/3} - 28\right) s2}{24}, \frac{4 \left(\sqrt{29} - 6\right) s2^2 \left(108 + 20\sqrt{29}\right)^{1/3}}{9}, \\
& \frac{-\left(9 \left(108 + 20\sqrt{29}\right)^{2/3} \sqrt{29} + 3 \sqrt{29} - 49 \left(108 + 20\sqrt{29}\right)^{2/3} - 35\right) s2^2}{18}, \\
& \frac{s2 \left(108 + 20\sqrt{29}\right)^{1/3}}{4}
\end{aligned}$$

$SF_{14}^{4,1}$

$$\begin{aligned}
> \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,4], \mathbf{M}[2,1], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\
\{r1=r1, r2=0, s1=-\frac{s2^2}{2}, s2=s2, u=u, v=-\frac{u}{4}+\frac{1}{8}\}, \{r1=-r2, r2=r2, s1=\text{RootOf}(_Z^2-3_Z+1)s2, s2=s2, u=-\frac{\text{RootOf}(_Z^2-3_Z+1)-1}{\text{RootOf}(_Z^2-3_Z+1)-3}, v \\
= \frac{\text{RootOf}(_Z^2-3_Z+1)-1}{\text{RootOf}(_Z^2-3_Z+1)-3}\}, \{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0\}, \{r1=0, r2=r2, s1=-s2, s2=s2, u=0, v=0\}, \{r1=-r2, r2=r2, s1 \\
= s1, s2=0, u=-1, v=1\} \\
> s11 := -s2/2: \\
v1 := & (1-2*u)/8: \\
\text{zamproc}(0,u,0,v1,0,1,1,0, r1,s11,0,s2): \\
& 0, \frac{s2 r1 (2 u + 1)}{2}, -u s2^2, 0 \\
& 0, r1^2, 0, -\frac{s2^2}{4}
\end{aligned}$$

$SF_{19}^{4,1}$

$$\begin{aligned}
> \text{solve}([\mathbf{M}[1,4], \mathbf{M}[2,2], \mathbf{M}[2,3], \mathbf{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\
\{r1=0, r2=r2, s1=s1, s2=0, u=0, v=v\}, \{r1=0, r2=r2, s1=-s2, s2=s2, u=0, v=0\}
\end{aligned}$$

$SF_{27}^{4,1}$

$$\begin{aligned}
> \text{solve}([\mathbf{M}[1,1], \mathbf{M}[1,2], \mathbf{M}[2,1], \mathbf{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\
\{r1=-r2, r2=r2, s1=2 s2, s2=s2, u=1, v=-1\}, \{r1=r1, r2=0, s1=-\frac{s2}{2}, s2=s2, u=-\frac{1}{2}, v=v\}
\end{aligned}$$

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> s11 := -s2/2:
zamproc(0,-1/2,0,v,0,1,1,0, r1,s11,0,s2):
0, 0,  $\frac{s2^2}{2}$ ,  $\frac{s2^3 (4 v - 1)}{4 r l}$ 
0,  $r l^2$ , 0,  $-\frac{s2^2}{4}$ 

 $SF_{28}^{4,1}$ 
> solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=-2 s2, s2=s2, u=0, v=0}

 $NSF_{30}^{4,1}$ . Результат произвольной замены:
> M := zamproc(0,u,v,1,0,1,0,0, r1,s1,r2,s2):
 $\frac{(s2 u - s1) r l^2 + r l r2 s2 v + r2^2 s2) r2}{r l s2 - s1 r2}, \frac{(r l^2 u + 2 r l r2 v + 3 r2^2) s2^2 + 2 \left(u r l r2 + \frac{1}{2} v r2^2 - \frac{1}{2} r l^2\right) s1 s2 - 2 r1 s1^2 r2}{r l s2 - s1 r2},$ 
 $\frac{(r l v + 3 r2) s2^3 + 2 s1 (r l u + r2 v) s2^2 + s1^2 (r2 u - 2 r l) s2 - s1^3 r2}{r l s2 - s1 r2}, \frac{s2 (u s1^2 s2 + v s1 s2^2 - s1^3 + s2^3)}{r l s2 - s1 r2}$ 
 $-\frac{r2 (u r l^2 r2 + v r l r2^2 - r l^3 + r2^3)}{r l s2 - s1 r2}, \frac{(-s1 v - 3 s2) r2^3 - 2 r l (s1 u + s2 v) r2^2 - r l^2 (s2 u - 2 s1) r2 + r l^3 s2}{r l s2 - s1 r2},$ 
 $-\frac{(-s1^2 u - 2 s1 s2 v - 3 s2^2) r2^2 - 2 r l \left(s1 s2 u + \frac{1}{2} s2^2 v - \frac{1}{2} s1^2\right) r2 + 2 r l^2 s1 s2}{r l s2 - s1 r2}, -\frac{((s1^2 u + s1 s2 v + s2^2) r2 - r l s1^2) s2}{r l s2 - s1 r2}$ 

 $SF_9^{2,1}$ 
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}

 $SF_6^{3,1}$ 
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,v,r1,s1,r2,s2});

 $SF_{11}^{3,1}$ 
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});

 $SF_{17}^{3,1}$ 
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}

 $SF_{19}^{3,1}$ 
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=u, v=v}

 $SF_{21}^{3,1}$ 
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=0, r2=r2, s1=s1, s2=0, u=0, v=0}

 $SF_{22}^{3,1}$ 
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
{r1=r1, r2=0, s1=0, s2=s2, u=0, v=v}

 $SF_1^{4,1}$ 
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
 $SF_3^{4,1}$ 
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
 $SF_5^{4,1}$ 
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
 $\left\{r1 = -\frac{2 r2}{v}, r2 = r2, s1 = s1, s2 = 0, u = -\frac{\left(-r2^3 + \frac{8 r2^3}{v^3}\right) v^2}{4 r2^3}\right\}$ 

> u1 := (v^3-8)/(4*v):
r21 := -v*r1/2:
zamproc(0,u1,v,1,0,1,0,0, r1,s1,r21,0):
 $\frac{r l^2, 2 r l s1, s1^2, 0}{0, 0, -\frac{r l s1 v^3}{8}, 0}$ 

 $SF_{11}^{4,1}$ 
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
 $\left\{r1 = -\frac{r2}{v}, r2 = r2, s1 = s1, s2 = 0, u = -\frac{1}{v}\right\}$ 

> u1 := -1/v:
r21 := -r1*v:
zamproc(0,u1,v,1,0,1,0,0, r1,s1,r21,0):

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$rI^2, 2rls1, s1^2, 0$
 $0, v^3 rI^2, 0, 0$

$SF_{13}^{4,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,2], \text{M}[1,3], \text{M}[2,1], \text{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = \text{RootOf}(\underline{Z}^2 + Z + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = -\frac{3}{\text{RootOf}(\underline{Z}^2 + Z + 1) + 1}, v = -\frac{3}{\text{RootOf}(\underline{Z}^2 + Z + 1)} \right\}, \{r1 = r2, r2 = r2, s1 = 0, s2 = s2, u = -3, v = -3\}, \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = 0, v = 0\}, \{r1 = r2, r2 = r2, s1 = -2s2, s2 = s2, u = -1, v = 1\} \end{aligned}$
 $\begin{aligned} > \text{zamproc}(0, 3, -3, 1, 0, 1, 0, 0, r1, 0, r1, s2); \\ & \begin{aligned} & rI^2, 0, 0, \frac{s2^3}{rI} \\ & 0, rI^2, 0, -s2^2 \end{aligned} \end{aligned}$

$SF_{14}^{4,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,4], \text{M}[2,1], \text{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & SF_{19}^{4,1} \end{aligned}$
 $\begin{aligned} > \text{solve}([\text{M}[1,4], \text{M}[2,2], \text{M}[2,3], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = r1, r2 = r2, s1 = s1, s2 = 0, u = \frac{r1}{r2}, v = 0 \right\} \end{aligned}$

$SF_{27}^{4,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,2], \text{M}[2,1], \text{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = 0, v = v\} \end{aligned}$
 $SF_{28}^{4,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,3], \text{M}[2,2], \text{M}[2,3]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1 = -r2, r2 = r2, s1 = 0, s2 = s2, u = 2, v = 3\}, \left\{ r1 = \text{RootOf}(\underline{Z}^2 - Z + 1) r2, r2 = r2, s1 = 0, s2 = s2, u = \frac{2}{\text{RootOf}(\underline{Z}^2 - Z + 1) - 1}, v = -\frac{3}{\text{RootOf}(\underline{Z}^2 - Z + 1)} \right\}, \{r1 = -r2, r2 = r2, s1 = 2s2, s2 = s2, u = 0, v = -1\} \end{aligned}$
 $\begin{aligned} > \text{zamproc}(0, 2, 3, 1, 0, 1, 0, 0, r1, 0, -r1, s2); \\ & \begin{aligned} & 0, -s2 rI, 0, \frac{s2^3}{rI} \\ & -\frac{rI^3}{s2}, 0, 0, s2^2 \end{aligned} \end{aligned}$

$SF_{29}^{4,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,3], \text{M}[2,1], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = u, v = 0\} \end{aligned}$
 $NSF_{33}^{4,1}$ ($v \neq u$). Результат произвольной замены:
 $\begin{aligned} > M := \text{zamproc}(0, 0, u, v, 0, 1, 1, 0, r1, s1, r2, s2); \\ & \frac{r2(r2^2 s2 v + r1(u s2 - s1)r2 - s1 r1^2)}{-s1 r2 + s2 r1}, \frac{(s1 s2 u + 3 s2^2 v - s1^2)r2^2 + 2 r1(s2^2 u - s1^2 - s1 s2)r2 - s2 r1^2 s1}{-s1 r2 + s2 r1}, \\ & \frac{(u r1 + 3 v r2)s2^3 + 2 \left(u r2 - \frac{r1}{2} \right) s1 s2^2 - 2 s1^2(r2 + r1)s2 - r2 s1^3}{-s1 r2 + s2 r1}, \frac{s2(s1 s2^2 u + s2^3 v - s1^3 - s1^2 s2)}{-s1 r2 + s2 r1} \\ & \frac{r2(-u r1 r2^2 - r2^3 v + r1^3 + r2 r1^2)}{-s1 r2 + s2 r1}, \frac{(-s1 u - 3 s2 v)r2^3 - 2 r1 \left(u s2 - \frac{s1}{2} \right) r2^2 + 2 r1^2(s2 + s1)r2 + s2 r1^3}{-s1 r2 + s2 r1}, \\ & \frac{(-u r1 r2 - 3 v r2^2 + r1^2)s2^2 - 2 s1(u r2^2 - r1^2 - r1 r2)s2 + r2 r1 s1^2}{-s1 r2 + s2 r1}, \frac{(r2 s2^2 v + s1(u r2 - r1)s2 - r1 s1^2)s2}{-s1 r2 + s2 r1} \end{aligned}$

$SF_9^{2,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,2], \text{M}[1,4], \text{M}[2,2], \text{M}[2,3], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & SF_6^{3,1} \end{aligned}$
 $\begin{aligned} > \text{solve}([\text{M}[1,2], \text{M}[1,4], \text{M}[2,1], \text{M}[2,2], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \left\{ r1 = -\frac{r2}{2}, r2 = r2, s1 = s1, s2 = 0, u = 0, v = \frac{1}{8} \right\} \end{aligned}$

$SF_{11}^{3,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,2], \text{M}[1,4], \text{M}[2,1], \text{M}[2,3], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & SF_{17}^{3,1} \end{aligned}$
 $\begin{aligned} > \text{solve}([\text{M}[1,2], \text{M}[1,4], \text{M}[2,2], \text{M}[2,3], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & SF_{19}^{3,1} \end{aligned}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,3], \text{M}[2,1], \text{M}[2,3], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1 = 0, r2 = r2, s1 = -2s2, s2 = s2, u = 0, v = 0\} \end{aligned}$
 $SF_{21}^{3,1}$
 $\begin{aligned} > \text{solve}([\text{M}[1,1], \text{M}[1,4], \text{M}[2,2], \text{M}[2,3], \text{M}[2,4]], \{u, v, r1, s1, r2, s2\}); \\ & \{r1 = 0, r2 = r2, s1 = -s2, s2 = s2, u = 0, v = 0\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0, v = v\} \end{aligned}$

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
```

$$\left\{ \begin{array}{l} r1 = -\frac{\text{RootOf}(\sqrt{Z} + Z + 1) r2}{2 (\text{RootOf}(\sqrt{Z} + Z + 1) + 1)}, r2 = r2, s1 = \frac{\text{RootOf}(\sqrt{Z} + Z + 1) s2}{2}, s2 = s2, u = -\frac{\text{RootOf}(\sqrt{Z} + Z + 1)}{2 (\text{RootOf}(\sqrt{Z} + Z + 1) + 1)^2}, v \\ = \frac{\text{RootOf}(\sqrt{Z} + Z + 1)^3}{8 (\text{RootOf}(\sqrt{Z} + Z + 1) + 1)^3} \end{array} \right.$$

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,v,r1,s1,r2,s2});
```

$$\left\{ \begin{array}{l} r1 = -\frac{r2^2}{2}, r2 = r2, s1 = s1, s2 = 0, u = 0, v = \frac{1}{8} \end{array} \right.$$

$SF_5^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {v,r1,s1,r2,s2});
```

$$\left\{ \begin{array}{l} r1 = \text{RootOf}(2 \sqrt{Z} + Z - u) r2, r2 = r2, s1 = s1, s2 = 0, v = -\frac{\text{RootOf}(2 \sqrt{Z} + Z - u) u}{2} - \frac{\text{RootOf}(2 \sqrt{Z} + Z - u)}{4} + \frac{u}{4} \end{array} \right.$$

```
> solve(2 * Z^2 + Z - u, Z);
```

$$-\frac{1}{4} + \frac{\sqrt{1+8u}}{4}, -\frac{1}{4} - \frac{\sqrt{1+8u}}{4}$$

```
> z1 := -1/4 + (1/4)*sqrt(1+8*u);
```

```
v1 := simplify(-(1/2)*z1*u - (1/4)*z1 + (1/4)*u);
```

```
r11 := z1*r2;
```

```
zamproc(0,0,u,v1,0,1,1,0, r11,s1,r2,0):
```

$$\begin{aligned} v1 &:= \frac{(-2u - 1)\sqrt{1+8u}}{16} + \frac{3u}{8} + \frac{1}{16} \\ &\quad \frac{(-1 + \sqrt{1+8u})r2^2(3 + \sqrt{1+8u})}{16}, \frac{s1r2(1 + \sqrt{1+8u})}{2}, s1^2, 0 \\ &\quad 0, 0, -\frac{(-1 + \sqrt{1+8u})r2s1}{4}, 0 \end{aligned}$$

```
> z2 := -1/4 - (1/4)*sqrt(1+8*u);
```

```
v2 := simplify(-(1/2)*z2*u - (1/4)*z2 + (1/4)*u);
```

```
r12 := z2*r2;
```

```
zamproc(0,0,u,v2,0,1,1,0, r12,s1,r2,0):
```

$$\begin{aligned} v2 &:= \frac{(2u + 1)\sqrt{1+8u}}{16} + \frac{3u}{8} + \frac{1}{16} \\ &\quad \frac{(1 + \sqrt{1+8u})r2^2(-3 + \sqrt{1+8u})}{16}, -\frac{(-1 + \sqrt{1+8u})r2s1}{2}, s1^2, 0 \\ &\quad 0, 0, \frac{s1r2(1 + \sqrt{1+8u})}{4}, 0 \end{aligned}$$

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$$\{r1=0, r2=r2, s1=s1, s2=0, u=u, v=0\}, \{r1=0, r2=r2, s1=-s2, s2=s2, u=0, v=0\}$$

$SF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
```

$SF_{14}^{4,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
```

$$\left\{ \begin{array}{l} r1 = r1, r2 = 0, s1 = -\frac{s2}{2}, s2 = s2, u = u, v = \frac{u}{2} + \frac{1}{8} \end{array} \right\}, \{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = u, v = 0\}, \{r1 = 0, r2 = r2, s1 = -s2, s2 = s2, u = 0, v = 0\}, \{r1 = -r2, r2 = r2, s1 = \frac{s2}{2}, s2 = s2, u = \frac{1}{4}, v = \frac{1}{4} \}$$

```
> v1 := (4*u+1)/8;
```

```
s21 := -2*s1;
```

```
zamproc(0,0,u,v1,0,1,1,0, r1,s1,0,s21):
```

$$\begin{aligned} &0, -s1r1, 4us1^2, 0 \\ &0, r1^2, 0, -s1^2 \end{aligned}$$

$SF_{19}^{4,1}$

```
> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});
```

$$\{r1=0, r2=r2, s1=-s2, s2=s2, u=0, v=0\}, \{r1=0, r2=r2, s1=s1, s2=0, u=0, v=v\}$$

$SF_{27}^{4,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {u,v,r1,s1,r2,s2});
```

$SF_{28}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});  
{r1=0,r2=r2,s1=-2 s2,s2=s2,u=0,v=0}
```

$SF_{29}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=r1,r2=0,s1=-s2,s2=s2,u=1,v=v}, {r1=r1,r2=0,s1=0,s2=s2,u=0,v=v}, {r1=-r2,r2=r2,s1=0,s2=s2,u=0,v=0}, {r1=0,r2=r2,  
s1=-2 s2,s2=s2,u=0,v=0}
```

```
> zamproc(0,0,1,v,0,1,1,0, r1,s1,0,-s1):
```

$$\begin{aligned} & 0, -s1 r1, 0, \frac{-s1^3 (v-1)}{r1} \\ & 0, r1^2, s1 r1, 0 \end{aligned}$$

$SF_{30}^{4,1}$

```
> solve([M[1,1],M[2,1],M[2,3],M[2,4]], {u,v,r1,s1,r2,s2});  
{r1=0,r2=r2,s1=s1,s2=s2,u=0,v=0}, {r1=0,r2=r2,s1=s1,s2=0,u=u,v=0}
```

$SF_{32}^{4,1}$

```
> solve([M[1,1],M[1,2],M[2,2],M[2,3]], {u,v,r1,s1,r2,s2});
```

II

$NSF_1^{4,1}$. Результат произвольной замены :

```
> M := zamproc(u,u,0,0,0,0,1,1, r1,s1,r2,s2):  
_ (r1+r2) (s2 r1^2 u - s1 r2^2),  $\frac{-s1^2 r2^2 + 3 \left( u r1^2 + \frac{2 r2 (u-1) r1}{3} - r2^2 \right) s2 s1 + u s2^2 r1^2}{-s1 r2 + s2 r1},$   
_  $\frac{3 \left( \left( u r1 + \frac{r2 (u-2)}{3} \right) s1 + \frac{2 \left( \left( u - \frac{1}{2} \right) r1 - \frac{3 r2^2}{2} \right) s2}{3} \right) s1 s2}{-s1 r2 + s2 r1}, \frac{s1 s2 (s1 + s2) (s1 u - s2)}{-s1 r2 + s2 r1}$   
_  $\frac{-r1 r2 (r1 + r2) (u r1 - r2)}{-s1 r2 + s2 r1}, \frac{3 r2 \left( \left( \left( \frac{2 u}{3} - \frac{1}{3} \right) s1 - s2 \right) r2 + \left( s1 u + \frac{(u-2) s2}{3} \right) r1 \right) r1}{-s1 r2 + s2 r1},$   
_  $\frac{s2^2 r1^2 - 3 r2 \left( s1^2 u + \frac{2 s2 (u-1) s1}{3} - s2^2 \right) r1 - u r2^2 s1^2}{-s1 r2 + s2 r1}, \frac{(s1 + s2) (-u r2 s1^2 + s2^2 r1)}{-s1 r2 + s2 r1}$ 
```

$SF_2^{2,1}$

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

{r1=0,r2=r2,s1=s1,s2=0,u=0}, {r1=0,r2=r2,s1=-s2,s2=s2,u=-1}, {r1=r1,r2=0,s1=-s2,s2=s2,u=-1}

```
> zamproc(-1,-1,0,0,0,0,1,1, r1,s1,0,-s1):
```

$$\begin{aligned} & -r1^2, -2 s1 r1, 0, 0, 0, -s1 r1, 0 \end{aligned}$$

$SF_5^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});  
(r1=r1,r2=0,s1=-s2,s2=s2,u=0)
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
(r1=r2,r2=r2,s1=-s2,s2=s2,u=1)
```

```
> zamproc(1,1,0,0,0,0,1,1, r1,s1,r1,-s1):
```

$$\begin{aligned} & 2 r1^2, 0, 2 s1^2, 0 \\ & 0, 4 r1^2, 0, 0 \end{aligned}$$

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});  
(r1=-r2,r2=r2,s1=s2,s2=s2,u=-1)
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
          {r1=-r2,r2=r2,s1=s1,s2=0,u=0}
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$NSF_3^{4,1}$. Результат произвольной замены:

```
> M := zamproc(u,0,-u,0,0,0,1,1, r1,s1,r2,s2):  
  
$$\frac{(rl+r2)(s2rl^2u-u s2 rl r2-s1rl^2)}{-s1 r2+s2 r1}, \frac{-s1(s1+s2(u+3))r2^2-2s2rl(u s2+s1)r2+3url^2s1s2}{-s1 r2+s2 r1},$$
  
  
$$\frac{3s2\left(\left(url-\frac{2r2}{3}\right)s1^2-\frac{2\left(\frac{rl}{2}+\left(u+\frac{3}{2}\right)r2\right)s2s1}{3}-\frac{rls2^2u}{3}\right)}{-s1 r2+s2 r1}, \frac{(s1+s2)((-u-1)s2+s1u)s1s2}{-s1 r2+s2 r1}$$
  
  
$$-\frac{r2rl(rl+r2)((-u-1)r2+url)}{-s1 r2+s2 r1}, -\frac{3\left(\left(s1u-\frac{2s2}{3}\right)rl^2-\frac{2r2\left(\frac{s1}{2}+\left(u+\frac{3}{2}\right)s2\right)r1}{3}-\frac{r2^2s1u}{3}\right)r2}{-s1 r2+s2 r1},$$
  
  
$$\frac{s2^2rl^2-3r2\left(\left(-\frac{u}{3}-1\right)s2^2-\frac{2s1s2}{3}+s1^2u\right)rl+2us1s2r2^2}{-s1 r2+s2 r1}, \frac{(s1+s2)(-ur2s1^2+ur2s1s2+s2^2rl)}{-s1 r2+s2 r1}$$

```

$SF_2^{2,1}$

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$$\left\{ rl=rl, r2=0, s1=-s2, s2=s2, u=-\frac{1}{2} \right\}, \left\{ rl=0, r2=r2, s1=s1, s2=0, u=0 \right\}$$

```
> zamproc(-1/2,0,1/2,0,0,0,1,1, r1,s1,0,-s1):
```

$$\begin{aligned} & -\frac{rl^2}{2}, -\frac{3s1rl}{2}, 0, 0 \\ & 0, 0, -s1rl, 0 \end{aligned}$$

$SF_5^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});  
          {rl=rl,r2=0,s1=-s2,s2=s2,u=0}
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
          {rl=-r2,r2=r2,s1=s1,s2=0,u=0}
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
```

$$\left\{ rl=rl, r2=0, s1=-s2, s2=s2, u=-\frac{1}{2} \right\}, \left\{ rl=rl, r2=0, s1=\frac{s2}{2}, s2=s2, u=-2 \right\}, \left\{ rl=\frac{r2}{2}, r2=r2, s1=s1, s2=0, u=-2 \right\}, \left\{ rl=-r2, r2=r2, s1=s1, s2=0, u=-2 \right\}$$

$= s1, s2 = 0, u = -\frac{1}{2} \}, \{ r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0 \}, \{ r1 = r1, r2 = 0, s1 = 0, s2 = s2, u = 0 \}$
 $\Rightarrow s21 := 2*s1:$
 $\text{zamproc}(-2, 0, 2, 0, 0, 0, 1, 1, r1, s1, 0, s21):$

$$\begin{aligned} & -2 r1^2, -6 s1 r1, 0, 0 \\ & 0, 0, 2 s1 r1, 6 s1^2 \end{aligned}$$

$NSF_{13}^{4,1}$. Результат произвольной замены:
 $\Rightarrow M := \text{zamproc}(u, 0, 0, u, 0, 1, 0, -1, r1, s1, r2, s2):$

$$\begin{aligned} & \frac{((u s2 + s1) r2^2 - r1 (u s2 + s1) r2 + u s2 r1^2) (r1 + r2)}{-s1 r2 + s2 r1}, \frac{-2 s1^2 r1 r2 + 3 \left(\left(u - \frac{1}{3} \right) r1^2 + r2^2 \right) s2 s1 + 3 u s2^2 r2^2}{-s1 r2 + s2 r1}, \\ & \frac{-s1^3 r2 + 3 \left(u - \frac{2}{3} \right) r1 s2 s1^2 + 3 s1 s2^2 r2 + 3 u s2^3 r2}{-s1 r2 + s2 r1}, \frac{((u - 1) s1^2 - s2 (u - 1) s1 + u s2^2) (s1 + s2) s2}{-s1 r2 + s2 r1} \\ & - \frac{r2 ((u - 1) r1^2 - r2 (u - 1) r1 + u r2^2) (r1 + r2)}{-s1 r2 + s2 r1}, \frac{s2 r1^3 - 3 r2 \left(u - \frac{2}{3} \right) s1 r1^2 - 3 s2 r1 r2^2 - 3 u s2 r2^3}{-s1 r2 + s2 r1}, \\ & \frac{2 s1 s2 r1^2 - 3 r2 \left(\left(u - \frac{1}{3} \right) s1^2 + s2^2 \right) r1 - 3 u s2^2 r2^2}{-s1 r2 + s2 r1}, - \frac{(s1 + s2) ((u r2 + r1) s2^2 - s1 (u r2 + r1) s2 + u r2 s1^2)}{-s1 r2 + s2 r1} \end{aligned}$$

$SF_2^{2,1}$
 $\Rightarrow \text{solve}([M[1,2], M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], \{u, r1, s1, r2, s2\});$
 $SF_9^{2,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,2], M[1,4], M[2,2], M[2,3], M[2,4]], \{u, r1, s1, r2, s2\});$
 $SF_3^{3,1}$
 $\Rightarrow \text{solve}([M[1,3], M[1,4], M[2,1], M[2,2], M[2,4]], \{u, r1, s1, r2, s2\});$

$$\{r1 = 2 r2, r2 = r2, s1 = -s2, s2 = s2, u = \frac{2}{3}\}$$

 $\Rightarrow r11 := 2 * r2:$
 $\text{zamproc}(2/3, 0, 0, 2/3, 0, 1, 0, -1, r11, s1, r2, -s1):$

$$\begin{aligned} & 3 r2^2, 3 s1 r2, 0, 0 \\ & 0, 0, 6 s1 r2, 0 \end{aligned}$$

$SF_5^{3,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,4], M[2,1], M[2,2], M[2,3]], \{u, r1, s1, r2, s2\});$
 $SF_6^{3,1}$
 $\Rightarrow \text{solve}([M[1,2], M[1,4], M[2,1], M[2,2], M[2,4]], \{u, r1, s1, r2, s2\});$

$$\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0\}$$

 $SF_8^{3,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,3], M[2,1], M[2,2], M[2,3]], \{u, r1, s1, r2, s2\});$
 $SF_{11}^{3,1}$
 $\Rightarrow \text{solve}([M[1,2], M[1,4], M[2,1], M[2,3], M[2,4]], \{u, r1, s1, r2, s2\});$

$$\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0\}$$

 $SF_{14}^{3,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,3], M[2,1], M[2,2], M[2,4]], \{u, r1, s1, r2, s2\});$
 $SF_{17}^{3,1}$
 $\Rightarrow \text{solve}([M[1,2], M[1,4], M[2,2], M[2,3], M[2,4]], \{u, r1, s1, r2, s2\});$

$$\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0\}$$

 $SF_{19}^{3,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,3], M[2,1], M[2,3], M[2,4]], \{u, r1, s1, r2, s2\});$
 $SF_{21}^{3,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,4], M[2,2], M[2,3], M[2,4]], \{u, r1, s1, r2, s2\});$
 $SF_{22}^{3,1}$
 $\Rightarrow \text{solve}([M[1,1], M[1,2], M[2,1], M[2,3], M[2,4]], \{u, r1, s1, r2, s2\});$
 $SF_1^{4,1}$
 $\Rightarrow \text{solve}([M[1,3], M[1,4], M[2,1], M[2,2]], \{u, r1, s1, r2, s2\});$

$$\{r1 = 2 r2, r2 = r2, s1 = -s2, s2 = s2, u = \frac{2}{3}\}, \{r1 = -r2, r2 = r2, s1 = 2 s2, s2 = s2, u = \frac{2}{3}\}$$

 $SF_3^{4,1}$
 $\Rightarrow \text{solve}([M[1,2], M[1,4], M[2,1], M[2,2]], \{u, r1, s1, r2, s2\});$

$$\{r1 = 0, r2 = r2, s1 = s1, s2 = 0, u = 0\}$$

$SF_5^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=-s2,s2=s2,u=0}, {r1=0,r2=r2,s1=s1,s2=0,u=0}, {r1=0,r2=r2,s1=s2,s2=s2,u=0}, {r1=2 r2,r2=r2,s1=-s2,s2=s2,u=0}
=s2,u= $\frac{2}{3}$  }
```

$SF_7^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {u,r1,s1,r2,s2});
{r1=r1,r2=0,s1=-s2,s2=s2,u= $\frac{2}{3}$ }, {r1=2 r2,r2=r2,s1=-s2,s2=s2,u= $\frac{2}{3}$ }
```

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1= $\frac{r2}{2}$ ,r2=r2,s1=-s2,s2=s2,u=- $\frac{1}{3}$ }, {r1=0,r2=r2,s1=-s2,s2=s2,u=0}, {r1=0,r2=r2,s1=s2,s2=s2,u=0}, {r1=0,r2=r2,s1=s1,s2=0,u=0}
> r21 := 2*r1;
zamproc(-1/3,0,0,-1/3,0,1,0,-1, r1,s1,r21,-s1):
-3 r1^2, 6 s1 r1, -3 s1^2, 0
0, -3 r1^2, 0, 0
```

$SF_{12}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=2 s2,s2=s2,u= $\frac{2}{3}$ }
```

$NSF_{28}^{4,1}$. Результат произвольной замены :

```
> M := zamproc(0,u,0,-u,1,0,0,1, r1,s1,r2,s2):
(( -u s2 - s1) r2^2 + r1 (u s2 + s1) r2 - s1 r1^2) (r1 + r2),  $\frac{u (r1^2 - 3 r2^2) s2^2 + 2 r2 s1 \left(u r1 - \frac{3 r2}{2}\right) s2 - 3 s1^2 r1^2}{-s1 r2 + s2 r1},$ 
-3 s1^3 r1 + u s1^2 s2 r2 + 2 \left(u r1 - \frac{3 r2}{2}\right) s2^2 s1 - 3 u s2^3 r2,  $\frac{(s1 + s2) (-s1^3 + s1^2 s2 + s2^2 (u - 1) s1 - u s2^3)}{-s1 r2 + s2 r1}$ 
- ( -r1^3 + r1^2 r2 + r2^2 (u - 1) r1 - u r2^3) (r1 + r2),  $\frac{3 s1 r1^3 - u s2 r1^2 r2 - 2 r2^2 \left(u s1 - \frac{3 s2}{2}\right) r1 + 3 u s2 r2^3}{-s1 r2 + s2 r1},$ 
- u (s1^2 - 3 s2^2) r2^2 - 2 \left(u s1 - \frac{3 s2}{2}\right) r1 s2 r2 + 3 s1^2 r1^2,  $\frac{(s1 + s2) ((-u r2 - r1) s2^2 + s1 (u r2 + r1) s2 - s1^2 r1)}{-s1 r2 + s2 r1}$ 
```

$SF_2^{2,1}$

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_5^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=2 s2,s2=s2,u=-3}
> s11 := 2*s2:
zamproc(0,-3,0,3,1,0,0,1, r1,s11,-r1,s2):

$$0, 0, -9 s2^2, -\frac{9 s2^3}{r1}$$


$$0, 9 r1^2, 0, 0$$

```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});

$$SF_3^{4,1}$$

> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});

$$SF_5^{4,1}$$

> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=-s2,s2=s2,u=0}, {r1=0,r2=r2,s1=RootOf(_Z^2-_Z+1) s2,s2=s2,u=0}, {r1=2 r2,r2=r2,s1=-s2,s2=s2,u=6}
> r11 := 2*r2:
zamproc(0,6,0,-6,1,0,0,1, r11,s1,r2,-s1):

$$9 r2^2, 9 s1 r2, -9 s1^2, 0$$


$$0, 0, -18 s1 r2, 0$$

```

$SF_7^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {u,r1,s1,r2,s2});

$$\left\{ r1=RootOf(2 \_Z^2-4 \_Z+3) r2, r2=r2, s1=-s2, s2=s2, u=\frac{3}{2} \right\}$$

```

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=\frac{r2^2}{2},r2=r2,s1=-s2,s2=s2,u=-\frac{3}{4}}, {r1=0,r2=r2,s1=RootOf(_Z^2-_Z+1) s2,s2=s2,u=0}, {r1=0,r2=r2,s1=-s2,s2=s2,u=0}
> r21 := 2*r1:
zamproc(0,-3/4,0,3/4,1,0,0,1, r1,s1,r21,-s1):

$$\frac{9 r1^2}{2}, -\frac{27 r1 s1}{4}, \frac{9 s1^2}{2}, 0$$


$$0, -\frac{9 r1^2}{2}, 0, 0$$

```

$SF_{12}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {u,r1,s1,r2,s2});

$$\left\{ r1=-r2, r2=r2, s1=\frac{3 s2}{2}, s2=s2, u=\frac{3}{2} \right\}$$

> s11 := 3*s2/2:
zamproc(0,3/2,0,-3/2,1,0,0,1, r1,s11,-r1,s2):

$$0, -\frac{15 s2 r1}{2}, 0, -\frac{15 s2^3}{8 r1}$$


$$0, 0, \frac{15 s2 r1}{4}, \frac{5 s2^2}{2}$$

```

$SF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$SF_{14}^{4,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {u,r1,s1,r2,s2});

$$\left\{ r1=-r2, r2=r2, s1=RootOf(_Z^3-5 \_Z^2+6 \_Z-3) s2, s2=s2, u=\frac{3 (RootOf(_Z^3-5 \_Z^2+6 \_Z-3)-1)}{RootOf(_Z^3-5 \_Z^2+6 \_Z-3)-3} \right\}$$

> solve(_Z^3-5*_Z^2+6*_Z-3, _Z);

$$\frac{(244+36\sqrt{29})^{1/3}}{6} + \frac{14}{3(244+36\sqrt{29})^{1/3}} + \frac{5}{3}, -\frac{(244+36\sqrt{29})^{1/3}}{12} - \frac{7}{3(244+36\sqrt{29})^{1/3}} + \frac{5}{3}$$


$$+\frac{I\sqrt{3}\left(\frac{(244+36\sqrt{29})^{1/3}}{6}-\frac{14}{3(244+36\sqrt{29})^{1/3}}\right)}{2}, -\frac{(244+36\sqrt{29})^{1/3}}{12} - \frac{7}{3(244+36\sqrt{29})^{1/3}} + \frac{5}{3}$$


$$-\frac{I\sqrt{3}\left(\frac{(244+36\sqrt{29})^{1/3}}{6}-\frac{14}{3(244+36\sqrt{29})^{1/3}}\right)}{2}$$

> solve(3*_Z^3-6*_Z^2+5*_Z-1, _Z);
```

$$\begin{aligned}
& -\frac{(20+4\sqrt{29})^{1/3}}{6} + \frac{2}{3(20+4\sqrt{29})^{1/3}} + \frac{2}{3}, \frac{(20+4\sqrt{29})^{1/3}}{12} - \frac{1}{3(20+4\sqrt{29})^{1/3}} + \frac{2}{3} \\
& + \frac{I\sqrt{3} \left(-\frac{(20+4\sqrt{29})^{1/3}}{6} - \frac{2}{3(20+4\sqrt{29})^{1/3}} \right)}{2}, \frac{(20+4\sqrt{29})^{1/3}}{12} - \frac{1}{3(20+4\sqrt{29})^{1/3}} + \frac{2}{3} \\
& - \frac{I\sqrt{3} \left(-\frac{(20+4\sqrt{29})^{1/3}}{6} - \frac{2}{3(20+4\sqrt{29})^{1/3}} \right)}{2}
\end{aligned}$$

> r := (244+36*sqrt(29))^(1/3);
z := (r+28*r^(-1)+10)/6;
u1 := simplify(expand(rationalize(simplify((3*(z-1))/(z-3))));
s11 := z*s2;
zamproc(0,u1,0,-u1,1,0,0,1, r1,s11,-r1,s2):
z := $\frac{(244+36\sqrt{29})^{1/3}}{6} + \frac{14}{3(244+36\sqrt{29})^{1/3}} + \frac{5}{3}$
u1 := $\frac{(-7\sqrt{29}+91)(244+36\sqrt{29})^{1/3}}{98} + 5 + \frac{(-2\sqrt{29}+19)(244+36\sqrt{29})^{2/3}}{98}$
0, $\frac{(-5880-924(244+36\sqrt{29})^{1/3}+56(244+36\sqrt{29})^{1/3}\sqrt{29}+25(244+36\sqrt{29})^{2/3}\sqrt{29}-213(244+36\sqrt{29})^{2/3})s2r1}{392}$,
 $\frac{(15(244+36\sqrt{29})^{2/3}\sqrt{29}+21(244+36\sqrt{29})^{1/3}\sqrt{29}-118(244+36\sqrt{29})^{2/3}-469(244+36\sqrt{29})^{1/3}-1960)s2^2}{196}$, 0
0, $\frac{(2(244+36\sqrt{29})^{2/3}\sqrt{29}+7(244+36\sqrt{29})^{1/3}\sqrt{29}-19(244+36\sqrt{29})^{2/3}-91(244+36\sqrt{29})^{1/3}-343)rI^2}{588}$,
 $\frac{(48(244+36\sqrt{29})^{2/3}\sqrt{29}+105(244+36\sqrt{29})^{1/3}\sqrt{29}-407(244+36\sqrt{29})^{2/3}-1757(244+36\sqrt{29})^{1/3}-9212)s2^2}{588}$

проверка записанного решения

rho := (4*sqrt(29)+92)^(1/3);
u2 := rho+20*rho^(-1)+5;
s12 := ((sqrt(29)+27)*rho^2-(10*sqrt(29)-130)*rho+1000)*s2/600;
simplify(u1-u2);
zamproc(0,u2,0,-u2,1,0,0,1, r1,s12,-r1,s2):
0, $\frac{(3(4\sqrt{29}+92)^2\sqrt{29}-119(4\sqrt{29}+92)^2\sqrt{29}-530(4\sqrt{29}+92)^1\sqrt{29}+10(4\sqrt{29}+92)^1\sqrt{29}-3000)rIs2}{200}$,
 $\frac{((4\sqrt{29}+92)^2\sqrt{29}+20(4\sqrt{29}+92)^1\sqrt{29}-123(4\sqrt{29}+92)^2\sqrt{29}-560(4\sqrt{29}+92)^1\sqrt{29}-2000)s2^2}{200}$, 0
0, $\frac{rI^2((-23+\sqrt{29})(4\sqrt{29}+92)^2\sqrt{29}-100(4\sqrt{29}+92)^1\sqrt{29}-350)}{50}$, 0,
 $\frac{-(11(4\sqrt{29}+92)^2\sqrt{29}+40(4\sqrt{29}+92)^1\sqrt{29}-453(4\sqrt{29}+92)^2\sqrt{29}-2020(4\sqrt{29}+92)^1\sqrt{29}-9400)s2^2}{600}$

$SF_{19}^{4,1}$

> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0, r2=r2, s1=-s2, s2=s2, u=0}, {r1=0, r2=r2, s1=RootOf(_Z^2-_Z+1) s2, s2=s2, u=0}

$SF_{24}^{4,1}$

> solve([M[1,1],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=-r2, r2=r2, s1=s2, s2=s2, u=3/2}

$SF_{27}^{4,1}$

> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {u,r1,s1,r2,s2});
{r1=-r2, r2=r2, s1=2s2, s2=s2, u=-3}

$NSF_{32}^{4,1}$. Результат произвольной замены:

> M := zamproc(0,0,u,u,1,0,0,1, r1,s1,r2,s2):
 $\frac{(us2-s1)r2^2+s1r1r2-s1r1^2)(r1+r2)}{-s1r2+s2r1}, \frac{(3us2+s1(u-3))s2r2^2+2us2^2r1r2-3s1^2r1^2}{-s1r2+s2r1}$,
 $\frac{u(r1+3r2)s2^3+2r2\left(u-\frac{3}{2}\right)s1s2^2-3s1^3r1}{-s1r2+s2r1}, \frac{(s1+s2)(us2^3-s1^3+s1^2s2-s1s2^2)}{-s1r2+s2r1}$,
 $\frac{(r1+r2)(-ur2^3+r1^3-r1^2r2+r1r2^2)}{-s1r2+s2r1}, \frac{-u(s1+3s2)r2^3-2\left(u-\frac{3}{2}\right)r1s2r2^2+3s1r1^3}{-s1r2+s2r1}$,

$$\frac{-2 u \left(s I + \frac{3 s^2}{2}\right) s_2 r^2 - r I s^2 (u - 3) r^2 + 3 s I^2 r I^2}{-s I r^2 + s_2 r I}, \frac{(s I + s_2) (-u r^2 s^2 + s I^2 r I - r I s I s_2 + s^2 r I)}{-s I r^2 + s_2 r I}$$

$SF_2^{2,1}$

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_5^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_{11}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{14}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
    (rI = -r2, r2 = r2, sI = 2 s2, s2 = s2, u = -3)
```

$s11 := 2*s2:$

```
zamproc(0,0,-3,-3,1,0,0,1, r1,s11,-r1,s2):
    0, -9 s2 rI, 0, - $\frac{9 s^2 r^3}{r I}$ 
    0, 0, 9 s2 rI, 0
```

$SF_{17}^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{19}^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{21}^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_{22}^{3,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_1^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
```

$SF_3^{4,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
```

$SF_5^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
    \{rI =  $\frac{r^2}{2}$ , r2 = r2, sI = -s2, s2 = s2, u =  $\frac{3}{8}$ \}, \{rI = 0, r2 = r2, sI = RootOf(_Z^2 - _Z + 1) s2, s2 = s2, u = 0\}, \{rI = 0, r2 = r2, sI = -s2, s2 = s2, u = 0\}
```

$r21 := 2*r1:$

```
zamproc(0,0,3/8,3/8,1,0,0,1, r1,s1,r21,-s1):
     $\frac{9 r I^2}{2}, -\frac{9 s I r I}{2}, \frac{27 s I^2}{8}, 0$ 
    0, 0, - $\frac{9 s I r I}{4}, 0$ 
```

$SF_7^{4,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {u,r1,s1,r2,s2});
    \{rI = RootOf(_Z^2 - 2 _Z + 3) r2, r2 = r2, sI = -s2, s2 = s2, u = -3\}
```

$SF_{11}^{4,1}$

```
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
    \{rI = 0, r2 = r2, sI = -s2, s2 = s2, u = 0\}, \{rI = 2 r2, r2 = r2, sI = -s2, s2 = s2, u = 6\}, \{rI = 0, r2 = r2, sI = RootOf(_Z^2 - _Z + 1) s2, s2 = s2, u = 0\}
```

$r11 := 2*r2:$

```
zamproc(0,0,6,6,1,0,0,1, r11,s1,r2,-s1):
    9 r2^2, -9 r2 sI, 9 sI^2, 0
    0, -18 r2^2, 0, 0
```

$SF_{12}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {u,r1,s1,r2,s2});  
{r1=-r2,r2=r2,s1=2 s2,s2=s2,u=-3}
```

$SF_{13}^{4,1}$

```
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,r1,s1,r2,s2});  
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

$SF_{14}^{4,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {u,r1,s1,r2,s2});  
{r1=-r2,r2=r2,s1=s2 RootOf(2 _Z^2+1)+s2,s2=s2,u=3 RootOf(2 _Z^2+1) / 2}
```

$SF_{19}^{4,1}$

```
> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
{r1=0,r2=r2,s1=-s2,s2=s2,u=0},{r1=0,r2=r2,s1=RootOf(_Z^2-Z+1)s2,s2=s2,u=0}
```

$SF_{24}^{4,1}$

```
> solve([M[1,1],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});  
{r1=-r2,r2=r2,s1=2 s2,s2=s2,u=-3}
```

$SF_{27}^{4,1}$

```
> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {u,r1,s1,r2,s2});
```

$SF_{28}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,r1,s1,r2,s2});  
{r1=r1,r2=0,s1=0,s2=s2,u=0}
```

$SF_{29}^{4,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,4]], {u,r1,s1,r2,s2});  
{r1=-r2,r2=r2,s1=2 s2,s2=s2,u=-3}
```

$SF_{30}^{4,1}$

```
> solve([M[1,1],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});  
{r1=-r2,r2=r2,s1=s2,u=-3}
```

$s21 := 2*s1:$
 $\text{zamproc}(0,0,-3/4,-3/4,1,0,0,1, r1,s1,-r1,s21):$
$$0, -\frac{9 s1 r1}{2}, 9 s1^2, -\frac{9 s1^3}{r1}$$

$$0, \frac{9 r1^2}{4}, 0, 0$$

$NSF_{36}^{4,1}$. Результат произвольной замены :

```
> M := zamproc(0,0,u,u,1,0,-1,0, r1,s1,r2,s2):  

$$\frac{(r1+r2)(ur2^2s2-s1r1^2+s1r1r2)}{-s1r2+r1s2}, \frac{(us1s2+3us2^2+s1^2)r2^2+2s2r1(us2+s1)r2-3s1^2r1^2}{-s1r2+r1s2},$$
  

$$\frac{u(r1+3r2)s2^3+2s1\left(ur2+\frac{r1}{2}\right)s2^2+2s1^2s2r2-3s1^3r1}{-s1r2+r1s2}, \frac{(s1+s2)(us2^3-s1^3+s2s1^2)}{-s1r2+r1s2}$$
  

$$\frac{(r1+r2)(-ur2^3+r1^3-r1^2r2)}{-s1r2+r1s2}, \frac{-u(s1+3s2)r2^3-2\left(us2+\frac{s1}{2}\right)r1r2^2-2s2r1^2r2+3s1r1^3}{-s1r2+r1s2},$$
  

$$\frac{(-ur1r2-3ur2^2-r1^2)s2^2-2s1r2(ur2+r1)s2+3s1^2r1^2}{-s1r2+r1s2}, \frac{(s1+s2)(-ur2s2^2+s1^2r1-r1s1s2)}{-s1r2+r1s2}$$

```

$SF_2^{2,1}$

```
> solve([M[1,2],M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_9^{2,1}$

```
> solve([M[1,1],M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_3^{3,1}$

```
> solve([M[1,3],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_5^{3,1}$

```
> solve([M[1,1],M[1,4],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});  
{r1=0,r2=r2,s1=-s2,s2=s2,u=0},{r1=0,r2=r2,s1=s2,s2=s2,u=0}
```

$SF_6^{3,1}$

```
> solve([M[1,2],M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
```

$SF_8^{3,1}$

```
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,3]], {u,r1,s1,r2,s2});  
{r1=0,r2=r2,s1=s1,s2=0,u=0}
```

```

 $SF_{11}^{3,1}$ 
> solve([M[1,2],M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
 $SF_{14}^{3,1}$ 
> solve([M[1,1],M[1,3],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
 $SF_{17}^{3,1}$ 
> solve([M[1,2],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
 $SF_{19}^{3,1}$ 
> solve([M[1,1],M[1,3],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{rl=0,r2=r2,s1=s1,s2=0,u=0}
 $SF_{21}^{3,1}$ 
> solve([M[1,1],M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{rl=0,r2=r2,s1=-s2,s2=s2,u=0}, {rl=0,r2=r2,s1=s2,s2=s2,u=0}
 $SF_{22}^{3,1}$ 
> solve([M[1,1],M[1,2],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
 $SF_1^{4,1}$ 
> solve([M[1,3],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
 $SF_3^{4,1}$ 
> solve([M[1,2],M[1,4],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
 $SF_5^{4,1}$ 
> solve([M[1,4],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{rl=0,r2=r2,s1=s2,s2=s2,u=0}, {rl=0,r2=r2,s1=-s2,s2=s2,u=0}, {rl =  $\frac{r^2}{2}$ , r2=r2,s1=-s2,s2=s2,u = - $\frac{1}{8}$ }
> r21 := 2*r1:
zamproc(0,0,-1/8,-1/8,1,0,-1,0, r1,s1,r21,-s1):

$$-\frac{3rl^2}{2}, \frac{3s1rl}{2}, \frac{15s1^2}{8}, 0$$


$$0, 0, -\frac{9s1rl}{4}, 0$$

 $SF_7^{4,1}$ 
> solve([M[1,3],M[1,4],M[2,1],M[2,4]], {u,r1,s1,r2,s2});
{rl=r2,r2=r2,s1=0,s2=s2,u=0}, {rl=-r2,r2=r2,s1=0,s2=s2,u=0}, {rl=RootOf(_Z^2-2_Z+2) r2,r2=r2,s1=-s2,s2=s2,u=-2}
 $SF_{11}^{4,1}$ 
> solve([M[1,4],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{rl=0,r2=r2,s1=-s2,s2=s2,u=0}, {rl=0,r2=r2,s1=s2,s2=s2,u=0}, {rl=2 r2,r2=r2,s1=-s2,s2=s2,u=4}
> r11 := 2*r2:
zamproc(0,0,4,4,1,0,-1,0, r11,s1,r2,-s1):

$$6r2^2, -3r2s1, 6s1^2, 0$$


$$0, -18r2^2, 0, 0$$

 $SF_{12}^{4,1}$ 
> solve([M[1,1],M[1,3],M[2,1],M[2,2]], {u,r1,s1,r2,s2});
{rl=0,r2=r2,s1=s1,s2=0,u=0}, {rl = -r2, r2=r2, s1 =  $\frac{4s2}{3}$ , s2=s2, u = -2}
> s11 := 4*s2/3:
zamproc(0,0,-2,-2,1,0,-1,0, r1,s11,-r1,s2):

$$0, -\frac{14s2rl}{3}, 0, -\frac{70s2^3}{27rl}$$


$$0, 0, 7s2rl, -\frac{14s2^2}{9}$$

 $SF_{13}^{4,1}$ 
> solve([M[1,2],M[1,3],M[2,1],M[2,3]], {u,r1,s1,r2,s2});
 $SF_{14}^{4,1}$ 
> solve([M[1,1],M[1,4],M[2,1],M[2,3]], {u,r1,s1,r2,s2});
{rl = -r2, r2=r2, s1 =  $\frac{s2(\text{RootOf}(2\ Z^2-8\ Z-1)+1)}{3}$ , s2=s2, u =  $\frac{\text{RootOf}(2\ Z^2-8\ Z-1)}{2}$ }, {rl=0,r2=r2,s1=s2,s2=s2,u=0}, {rl=0,r2=r2,s1=-s2,s2=s2,u=0}
> solve(2*_Z^2-8*_Z-1, _Z);

$$2 + \frac{3\sqrt{2}}{2}, 2 - \frac{3\sqrt{2}}{2}$$

> z1 := 2+3*sqrt(2)*(1/2):
s11 := simplify((1/3)*s2*(z1+1));

```

```

u1 := z1/2:
zamproc(0,0,u1,u1,1,0,-1,0, r1,s11,-r1,s2):

$$s11 := \frac{s2(2+\sqrt{2})}{2}$$


$$0, -\frac{s2 r l (4+\sqrt{2})}{4}, -4 s2^2 \sqrt{2} - \frac{11 s2^2}{2}, 0$$


$$0, \frac{3 (4+\sqrt{2}) r l^2}{4}, 0, \frac{(6+5 \sqrt{2}) s2^2}{4}$$


> z2 := 2-3*sqrt(2)*(1/2):
s12 := simplify((1/3)*s2*(z2+1));
u2 := z2/2:
zamproc(0,0,u2,u2,1,0,-1,0, r1,s12,-r1,s2):

$$s12 := -\frac{s2(-2+\sqrt{2})}{2}$$


$$0, \frac{s2 r l (-4+\sqrt{2})}{4}, \frac{(-11+8 \sqrt{2}) s2^2}{2}, 0$$


$$0, -\frac{3 (-4+\sqrt{2}) r l^2}{4}, 0, -\frac{5 s2^2 \sqrt{2}}{4} + \frac{3 s2^2}{2}$$


SF194,1
> solve([M[1,4],M[2,2],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=-s2,s2=s2,u=0}, {r1=0,r2=r2,s1=s2,s2=s2,u=0}
SF244,1
> solve([M[1,1],M[2,1],M[2,2],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=s2,u=0}, {r1=-r2,r2=r2,s1=2 s2,s2=s2,u=-2}
SF274,1
> solve([M[1,1],M[1,2],M[2,1],M[2,3]], {u,r1,s1,r2,s2});
SF284,1
> solve([M[1,1],M[1,3],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}
SF294,1
> solve([M[1,1],M[1,3],M[2,1],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=0,u=0}, {r1=r2,r2=r2,s1=0,s2=s2,u=0}, {r1=-r2,r2=r2,s1=0,s2=s2,u=0}
SF304,1
> solve([M[1,1],M[2,1],M[2,3],M[2,4]], {u,r1,s1,r2,s2});
{r1=0,r2=r2,s1=s1,s2=s2,u=0}, {r1=-r2,r2=r2,s1=0,s2=s2,u=0}
> s21 := 2*s1:
zamproc(0,0,1/4,1/4,1,0,-1,0, r1,s1,-r1,s21):

$$0, -\frac{3 s1 r l}{2}, -3 s l^2, \frac{3 s l^3}{r l}$$


$$0, \frac{9 r l^2}{4}, 0, 0$$


SF324,1
> solve([M[1,1],M[1,2],M[2,2],M[2,3]], {u,r1,s1,r2,s2});
SF334,1
> solve([M[1,1],M[1,2],M[2,1],M[2,4]], {u,r1,s1,r2,s2});
{r1=-r2,r2=r2,s1=0,s2=s2,u=0}, {r1=r2,r2=r2,s1=0,s2=s2,u=0}

```