```
> restart; read("newlib.m"); with(mylib): with(LinearAlgebra):
```

Результат произвольной замены в исходной системе.

```
> M := zamproc(a1,b1,c1,d1,a2,b2,c2,d2, r1,s1,r2,s2):
```

$$\frac{(-a1\,s2+a2\,s1)\,r1^3+r2\,(-b1\,s2+b2\,s1)\,r1^2+r2^2\,(-c1\,s2+c2\,s1)\,r1+r2^3\,(-d1\,s2+d2\,s1)}{-r1\,s2+r2\,s1},\ \frac{1}{-r1\,s2+r2\,s1}\left((3\,a2\,r1^2+2\,b2\,r1\,r2+c2\,r2^2)\,s1^2-3\left(\left(a1-\frac{b2}{3}\right)r1^2+\frac{2\,r2\,(b1-c2)\,r1}{3}+\frac{r2^2\,(c1-3\,d2)}{3}\right)s2\,s1-3\left(r2^2\,d1+\frac{2}{3}\,r2\,c1\,r1+\frac{1}{3}\,b1\,r1^2\right)s2^2\right),$$

$$\frac{1}{-r1\,s2+r2\,s1}\left((3\,a2\,r1+b2\,r2)\,s1^3-3\,s2\left(\left(a1-\frac{2\,b2}{3}\right)r1+\frac{r2\,(b1-2\,c2)}{3}\right)s1^2-2\left(\left(b1-\frac{c2}{2}\right)r1+r2\left(c1-\frac{3\,d2}{2}\right)\right)s2^2\,s1+(-c1\,r1-3\,d1\,r2)\,s2^3\right),\ \frac{a2\,s1^4-s2\,(a1-b2)\,s1^3-s2^2\,(b1-c2)\,s1^2-s2^3\,(c1-d2)\,s1-d1\,s2^4}{-r1\,s2+r2\,s1}$$

$$\frac{-a2\,r1^4+r2\,(a1-b2)\,r1^3+r2^2\,(b1-c2)\,r1^2+r2^3\,(c1-d2)\,r1+d1\,r2^4}{-r1\,s2+r2\,s1},\qquad\textbf{(1)}$$

$$\frac{1}{-r1\,s2+r2\,s1}\left((-3\,a2\,s1-b2\,s2)\,r1^3+3\left(\left(a1-\frac{2\,b2}{3}\right)s1+\frac{s2\,(b1-2\,c2)}{3}\right)r2\,r1^2+2\,r2^2\left(\left(b1-\frac{c2}{2}\right)s1+s2\left(c1-\frac{3\,d2}{2}\right)\right)r1+r2^3\,(c1\,s1+3\,d1\,s2)\right),\ \frac{1}{-r1\,s2+r2\,s1}\left((-3\,a2\,s1^2-2\,b2\,s1\,s2-c2\,s2^2)\,r1^2+3\left(\left(a1-\frac{b2}{3}\right)s1^2+\frac{2\,s2\,(b1-c2)\,s1}{3}+\frac{s2^2\,(c1-3\,d2)}{3}\right)r2\,r1+r2^2\,(b1\,s1^2+2\,c1\,s1\,s2+3\,d1\,s2^2)\right),$$

$$\frac{(a1\,r2-a2\,r1)\,s1^3+s2\,(r2\,b1-r1\,b2)\,s1^2+s2^2\,(r2\,c1-r1\,c2)\,s1+s2^3\,(d1\,r2-d2\,r1)}{-r1\,s2+r2\,s1}$$

Условие на коэффициенты замены для получения $b_2=0$.

```
> solve((-3*a2*s1-b2*s2)*r1^3+(3*((a1-2*b2*(1/3))*s1+(1/3)*s2*(b1-2*c2)))*r2*r1^2+2*r2^2*((b1-(1/2)*c2)
  *s1+s2*(c1-3*d2*(1/2)))*r1+r2^3*(c1*s1+3*d1*s2), {r1,s1,r2,s2});
```

$$\left\{r1=r1,\ r2=r2,\ s1=-\frac{s2\,(b1\,r1^2\,r2-b2\,r1^3+2\,c1\,r1\,r2^2-2\,c2\,r1^2\,r2+3\,d1\,r2^3-3\,d2\,r1\,r2^2)}{3\,a1\,r1^2\,r2-3\,a2\,r1^3+2\,b1\,r1\,r2^2-2\,b2\,r1^2\,r2+c1\,r2^3-c2\,r1\,r2^2},\ s2=s2\right\}\qquad\textbf{(2)}$$

Рассмотрение трёх случаев.

```
> # 1) d1<>0
  r21 := sqrt(3):
  s21 := -c1*r21^3/3;
  s11 := subs([r1=0,r2=sqrt(3)],-s21*(b1*r1^2*r2-b2*r1^3+2*c1*r1*r2^2-2*c2*r1^2*r2+3*d1*r2^3-3*d2*r1*
  r2^2)/(3*a1*r1^2*r2-3*a2*r1^3+2*b1*r1*r2^2-2*b2*r1^2*r2+c1*r2^3-c2*r1*r2^2));
  zamproc(a1,b1,c1,d1,a2,b2,c2,d2, 0,s11,r21,s21):
```

$$s21 := -c1\,\sqrt{3}$$

$$s11 := 3\,d1\,\sqrt{3}$$

$$c1+3\,d2,\ -9\,c1\,d2+9\,c2\,d1,\ -3\,c1^3+9\,c1^2\,d2+9\,d1\,(b1-2\,c2)\,c1+27\,b2\,d1^2,\ 2\,c1^4-3\,c1^3\,d2-9\,d1\,(b1-c2)\,c1^2+27\,d1^2\,(a1-b2)\,c1+81\,a2\,d1^3$$

$$1,\ 0,\ 9\,b1\,d1-3\,c1^2,\ 27\,a1\,d1^2-9\,c1\,b1\,d1+2\,c1^3\qquad\textbf{(3)}$$

```
> # 2) d1=0, a2<>0
  r11 := sqrt(3):
  s21 := a2*r11^3;
  s11 := subs([r2=0,r1=sqrt(3)],-s21*(b1*r1^2*r2-b2*r1^3+2*c1*r1*r2^2-2*c2*r1^2*r2+3*d2*r1*r2^2)/(3*a1*
  r1^2*r2-3*a2*r1^3+2*b1*r1*r2^2-2*b2*r1^2*r2+c1*r2^3-c2*r1*r2^2));
  zamproc(a1,b1,c1,0,a2,b2,c2,d2, r11,s11,0,s21):
```

$$s21 := 3\,a2\,\sqrt{3}$$

$$s11 := -b2\,\sqrt{3}$$

$$3\,a1+b2,\ -9\,a1\,b2+9\,a2\,b1,\ -3\,b2^3+9\,a1\,b2^2-18\,a2\left(b1-\frac{c2}{2}\right)b2+27\,a2^2\,c1,\ -3\,b2\left(-\frac{2\,b2^3}{3}+a1\,b2^2-3\,a2\,(b1-c2)\,b2+9\,a2^2\,(c1-d2)\right)$$

$$1,\ 0,\ 9\,a2\,c2-3\,b2^2,\ 27\,a2^2\,d2-9\,a2\,b2\,c2+2\,b2^3\qquad\textbf{(4)}$$

```
> # 3) a2=0, d1=0
  M := zamproc(a1,b1,c1,0,0,b2,c2,d2, r1,s1,r2,s2):
```

$$\frac{s2\,a1\,r1^3+r2\,(b1\,s2-b2\,s1)\,r1^2+r2^2\,(c1\,s2-c2\,s1)\,r1-d2\,r2^3\,s1}{r1\,s2-s1\,r2},$$

$$\frac{(-2\,b2\,r1\,r2-c2\,r2^2)\,s1^2+3\left(\left(-\frac{b2}{3}+a1\right)r1^2+\frac{2\,r2\,(b1-c2)\,r1}{3}+\frac{r2^2\,(c1-3\,d2)}{3}\right)s2\,s1+s2^2\,r1\,(b1\,r1+2\,c1\,r2)}{r1\,s2-s1\,r2},$$

$$\frac{-b2\,s1^3\,r2+3\,s2\left(\left(a1-\frac{2\,b2}{3}\right)r1+\frac{r2\,(b1-2\,c2)}{3}\right)s1^2+2\,s2^2\left(\left(b1-\frac{c2}{2}\right)r1+r2\left(c1-\frac{3\,d2}{2}\right)\right)s1+c1\,r1\,s2^3}{r1\,s2-s1\,r2},$$

$$\frac{((a1-b2)\,s1^2+s2\,(b1-c2)\,s1+s2^2\,(c1-d2))\,s2\,s1}{r1\,s2-s1\,r2}$$

$$-\frac{r2\,r1\,((a1-b2)\,r1^2+r2\,(b1-c2)\,r1+r2^2\,(c1-d2))}{r1\,s2-s1\,r2},$$

$$\frac{b2\,r1^3\,s2-3\left(\left(a1-\frac{2\,b2}{3}\right)s1+\frac{s2\,(b1-2\,c2)}{3}\right)r2\,r1^2-2\,r2^2\left(\left(b1-\frac{c2}{2}\right)s1+s2\left(c1-\frac{3\,d2}{2}\right)\right)r1-c1\,s1\,r2^3}{r1\,s2-s1\,r2},$$

$$\frac{s2\,(2\,b2\,s1+s2\,c2)\,r1^2-3\,r2\left(\left(-\frac{b2}{3}+a1\right)s1^2+\frac{2\,s2\,(b1-c2)\,s1}{3}+\frac{s2^2\,(c1-3\,d2)}{3}\right)r1+(-b1\,s1^2-2\,c1\,s1\,s2)\,r2^2}{r1\,s2-s1\,r2},$$

$$\frac{-a1\,r2\,s1^3-s2\,(r2\,b1-b2\,r1)\,s1^2-s2^2\,(c1\,r2-c2\,r1)\,s1+d2\,r1\,s2^3}{r1\,s2-s1\,r2}$$

(5)

Условие на коэффициенты замены для получения $b_2=0$.

```
> solve(b2*r1^3*s2-(3*(s1*(a1-2*b2*(1/3))+(1/3)*s2*(b1-2*c2)))*r2*r1^2-(2*((c1-3*d2*(1/2))*s2+s1*(b1-
  (1/2)*c2)))*r1*r2^2-c1*r2^3*s1, {r1,s1,r2,s2});
```

$$\left\{r1=r1,\ r2=r2,\ s1=-\frac{r1\,s2\,(r2\,r1\,b1-b2\,r1^2+2\,c1\,r2^2-2\,r2\,r1\,c2-3\,r2^2\,d2)}{r2\,(3\,a1\,r1^2+2\,r2\,r1\,b1-2\,b2\,r1^2+c1\,r2^2-r2\,r1\,c2)},\ s2=s2\right\}$$

(6)

```
> s11 := -r1*s2*(b1*r1*r2-b2*r1^2+2*c1*r2^2-2*c2*r1*r2-3*d2*r2^2)/(r2*(3*a1*r1^2+2*b1*r1*r2-2*b2*r1^2+
  c1*r2^2-c2*r1*r2)):
  zamproc(a1,b1,c1,0,0,b2,c2,d2, r1,s11,r2,s2):
```

$$\frac{r1^2\,(3\,a1+b2)}{3}+\frac{2\,r2\,(b1+c2)\,r1}{3}+\frac{r2^2\,(c1+3\,d2)}{3},$$

$$\frac{\left(a1\,b2\,r1^4+2\,a1\,c2\,r1^3\,r2+3\,r2^2\left(a1\,d2+\frac{b1\,c2}{3}-\frac{b2\,c1}{3}\right)r1^2+2\,b1\,d2\,r1\,r2^3+c1\,d2\,r2^4\right)s2}{r2\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+\frac{2\,r2\left(b1-\frac{c2}{2}\right)r1}{3}+\frac{c1\,r2^2}{3}\right)},$$

$$\frac{1}{r2^2\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+\frac{2\,r2\left(b1-\frac{c2}{2}\right)r1}{3}+\frac{c1\,r2^2}{3}\right)^2}\left(\left(\left(\frac{\left(-\frac{b2}{3}+a1\right)b2^2\,r1^6}{3}+b2\,c2\,r2\left(-\frac{b2}{3}+a1\right)r1^5+r2^2\left(\left(-\frac{a1}{3}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+\frac{b2}{3}\right)b1^2+\frac{a1\,b1\,c2}{3}+\frac{b2^2\,c1}{3}+\left(-\frac{1}{3}\,c2^2-\frac{5}{3}\,a1\,c1+a1\,d2\right)b2+a1^2\,c1+\frac{2\,a1\,c2^2}{3}\right)r1^4\right.\right.$$

$$\left.+\frac{\left(-\frac{2\,b1^3}{3}+b1^2\,c2+(a1\,c1+3\,a1\,d2-2\,b2\,d2+c2^2)\,b1-2\left(a1\,c1-\frac{3}{2}\,a1\,d2-\frac{1}{2}\,b2\,d2+\frac{1}{3}\,c2^2\right)c2\right)r2^3\,r1^3}{3}+r2^4\left(\left(-\frac{c1}{3}\right.\right.\right.$$

$$\left.\left.\left.+\frac{2\,d2}{3}\right)b1^2+\frac{b1\,c2\,d2}{3}+\left(\frac{1}{3}\,c1^2+d2^2-\frac{5}{3}\,c1\,d2\right)b2+\left(a1\,d2+\frac{c2^2}{3}\right)c1-\frac{c2^2\,d2}{3}\right)r1^2-\frac{b1\,c1\,r2^5\,(c1-3\,d2)\,r1}{3}\right.$$

$$\left.-\frac{c1^2\,r2^6\,(c1-3\,d2)}{9}\right)s2^2\Bigg),\ -\frac{1}{3\,r2^3\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+\frac{2\,r2\left(b1-\frac{c2}{2}\right)r1}{3}+\frac{c1\,r2^2}{3}\right)^2}\left(\left(\frac{c1^2\,r2^4}{9}+\frac{c1\,r1\,(b1+c2)\,r2^3}{9}\right.\right.$$

$$+\left(\left(-\frac{7\,c1}{9}+d2\right)b2+a1\,c1-a1\,d2-\frac{2\,b1^2}{9}+\frac{5\,b1\,c2}{9}-\frac{2\,c2^2}{9}\right)r1^2\,r2^2+\frac{b2\,r1^3\,(b1+c2)\,r2}{9}+\frac{b2^2\,r1^4}{9}\Bigg)\Big((2\,c1-3\,d2)\,r2^2$$

$$\left.+r1\,(b1-2\,c2)\,r2-b2\,r1^2)\,s2^3\right)$$

$$-\frac{\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+\frac{2\,r2\left(b1-\frac{c2}{2}\right)r1}{3}+\frac{c1\,r2^2}{3}\right)r2}{s2},\ 0,\ -\frac{1}{r2\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+\frac{2\,r2\left(b1-\frac{c2}{2}\right)r1}{3}+\frac{c1\,r2^2}{3}\right)}\left(\left(-\frac{b2^2\,r1^4}{3}\right.\right. \tag{7}$$

$$-r2\left(-\frac{2\,b1\,b2}{3}+c2\left(a1+\frac{b2}{3}\right)\right)r1^3+\left(-\frac{b1^2}{3}+\frac{b1\,c2}{3}+\left(\frac{c1}{3}+d2\right)b2+a1\,c1-3\,a1\,d2-\frac{c2^2}{3}\right)r2^2\,r1^2$$

$$\left.-\frac{r2^3\left((c1+3\,d2)\,b1-2\,c1\,c2\right)r1}{3}-\frac{c1^2\,r2^4}{3}\right)s2\Big),\ \frac{1}{r2^2\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+\frac{2\,r2\left(b1-\frac{c2}{2}\right)r1}{3}+\frac{c1\,r2^2}{3}\right)^2}\left(\left(\left(\frac{2\,b2^3\,r1^6}{27}\right.\right.\right.$$

$$+\frac{r2\left(-\frac{2\,b1\,b2}{3}+c2\left(a1+\frac{b2}{3}\right)\right)b2\,r1^5}{3}+\left(\frac{2\,b1^2\,b2}{9}-\frac{c2\left(\frac{2\,b2}{3}+a1\right)b1}{3}+\left(-\frac{c1}{9}+\frac{2\,d2}{3}\right)b2^2+\left(-\frac{1}{9}\,c2^2-\frac{1}{3}\,a1\,c1\right.\right.$$

$$\left.\left.-a1\,d2\right)b2+\frac{2\,a1\,c2^2}{3}+a1^2\,d2\right)r2^2\,r1^4$$

$$+\frac{1}{3}\left(r2^3\left(-\frac{2\,b1^3}{9}+\frac{b1^2\,c2}{3}+\left(\left(\frac{2\,c1}{3}-4\,d2\right)b2+\frac{c2^2}{3}+a1\,c1+3\,a1\,d2\right)b1-4\left(\left(-\frac{c1}{6}-\frac{d2}{4}\right)b2+a1\,c1-\frac{3\,a1\,d2}{4}\right.\right.\right.$$

$$\left.\left.\left.+\frac{c2^2}{18}\right)c2\right)r1^3\right)$$

$$+\frac{2\,r2^4\left(\left(-\frac{c1}{6}+d2\right)b1^2-\frac{\left(c1+\frac{3\,d2}{2}\right)c2\,b1}{3}-\frac{c1\,(c1+3\,d2)\,b2}{6}+a1\,c1^2+\left(-\frac{3\,a1\,d2}{2}+\frac{c2^2}{3}\right)c1+\frac{3\,a1\,d2^2}{2}\right)r1^2}{3}$$

$$\left.\left.+\frac{c1\,r2^5\left((c1+3\,d2)\,b1-2\,c1\,c2\right)r1}{9}+\frac{2\,c1^3\,r2^6}{27}\right)s2^2\right)$$

```
> s21 := ((-3*a1+2*b2)*r1^2+(-2*b1+c2)*r1-c1)/3:
  s11 := subs(r2=1,-r1*s21*(b1*r1*r2-b2*r1^2+2*c1*r2^2-2*c2*r1*r2-3*d2*r2^2)/(r2*(3*a1*r1^2+2*b1*r1*r2
  -2*b2*r1^2+c1*r2^2-c2*r1*r2))):
  zamproc(a1,b1,c1,0,0,b2,c2,d2, r1,s11,1,s21,full):
```

Initial system:

$$a1,\ b1,\ c1,\ 0$$

$$0,\ b2,\ c2,\ d2$$

substitution: $r1,\ \frac{r1\,(-b2\,r1^2+r1\,(b1-2\,c2)+2\,c1-3\,d2)}{3};\ 1,\ \frac{(-3\,a1+2\,b2)\,r1^2}{3}+\frac{(-2\,b1+c2)\,r1}{3}-\frac{c1}{3}$

det: $-((a1-b2)\,r1^2+(b1-c2)\,r1+c1-d2)\,r1$

system after substitution:

$$\frac{r1^2\,(3\,a1+b2)}{3}+\frac{(2\,b1+2\,c2)\,r1}{3}+\frac{c1}{3}+d2,\ -a1\,b2\,r1^4-2\,a1\,c2\,r1^3+(-3\,a1\,d2-b1\,c2+b2\,c1)\,r1^2-2\,b1\,d2\,r1-c1\,d2,$$

$$\frac{(3\,a1\,b2^2-b2^3)\,r1^6}{9}+b2\,c2\left(-\frac{b2}{3}+a1\right)r1^5$$

$$+\frac{((-3\,a1+3\,b2)\,b1^2+3\,a1\,b1\,c2+3\,b2^2\,c1+(-15\,a1\,c1+9\,a1\,d2-3\,c2^2)\,b2+9\,a1^2\,c1+6\,a1\,c2^2)\,r1^4}{9}$$

$$+\frac{(-2\,b1^3+3\,b1^2\,c2+(3\,a1\,c1+9\,a1\,d2-6\,b2\,d2+3\,c2^2)\,b1-6\,a1\,c1\,c2+9\,a1\,c2\,d2+3\,c2\,b2\,d2-2\,c2^3)\,r1^3}{9}$$

$$+\frac{((-3\,c1+6\,d2)\,b1^2+3\,b1\,c2\,d2+(3\,c1^2-15\,c1\,d2+9\,d2^2)\,b2+(9\,a1\,d2+3\,c2^2)\,c1-3\,c2^2\,d2)\,r1^2}{9}-\frac{b1\,c1\,(c1-3\,d2)\,r1}{3}$$

$$-\frac{c1^2\,(c1-3\,d2)}{9},\frac{1}{3}\left((-b2\,r1^2+r1\,(b1-2\,c2)+2\,c1-3\,d2)\left(\left(a1-\frac{2\,b2}{3}\right)r1^2+r1\left(\frac{2\,b1}{3}-\frac{c2}{3}\right)+\frac{c1}{3}\right)\left(\frac{b2^2\,r1^4}{9}\right.\right.$$

$$\left.\left.+\frac{b2\,r1^3\,(b1+c2)}{9}+\left(\left(-\frac{7\,c1}{9}+d2\right)b2+a1\,c1-a1\,d2-\frac{2\,b1^2}{9}+\frac{5\,b1\,c2}{9}-\frac{2\,c2^2}{9}\right)r1^2+\frac{c1\,r1\,(b1+c2)}{9}+\frac{c1^2}{9}\right)\right)$$

$$1,0,-\frac{b2^2\,r1^4}{3}+\frac{\left(2\,b1\,b2-3\,c2\left(a1+\frac{b2}{3}\right)\right)r1^3}{3}+\frac{(-b1^2+b1\,c2+(c1+3\,d2)\,b2+3\,a1\,c1-9\,a1\,d2-c2^2)\,r1^2}{3} \qquad \textbf{(8)}$$

$$+\frac{((-c1-3\,d2)\,b1+2\,c1\,c2)\,r1}{3}-\frac{c1^2}{3},\frac{2\,b2^3\,r1^6}{27}+\frac{\left(-6\,b1\,b2^2+9\left(a1+\frac{b2}{3}\right)b2\,c2\right)r1^5}{27}$$

$$+\frac{\left(6\,b1^2\,b2-9\,c2\left(\frac{2\,b2}{3}+a1\right)\right)b1+(-3\,c1+18\,d2)\,b2^2+(-9\,a1\,c1-27\,a1\,d2-3\,c2^2)\,b2+27\,a1^2\,d2+18\,a1\,c2^2\Big)\,r1^4}{27}$$

$$+\frac{1}{27}\left(\left(-2\,b1^3+3\,b1^2\,c2+((6\,c1-36\,d2)\,b2+9\,a1\,c1+27\,a1\,d2+3\,c2^2)\,b1-36\left(\left(-\frac{c1}{6}-\frac{d2}{4}\right)b2+a1\,c1-\frac{3\,a1\,d2}{4}\right.\right.\right.$$

$$\left.\left.\left.+\frac{c2^2}{18}\right)c2\right)r1^3\right)$$

$$+\frac{\left((-3\,c1+18\,d2)\,b1^2-6\left(c1+\frac{3\,d2}{2}\right)c2\,b1-3\,c1\,(c1+3\,d2)\,b2+18\,a1\,c1^2+(-27\,a1\,d2+6\,c2^2)\,c1+27\,a1\,d2^2\right)r1^2}{27}$$

$$+\frac{c1\,((c1+3\,d2)\,b1-2\,c1\,c2)\,r1}{9}+\frac{2\,c1^3}{27}$$