

Percentage and Percentage Change

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1 Review: Absolute Change (Difference)

Recall from elementary school mathematics how to calculate the difference between two values using subtraction. As illustrated in the examples below, we first identify the **direction** of change (increase or decrease) and then determine the **magnitude** of that change.

Example 1. *The temperature is 18°C today. It is predicted to be 19°C tomorrow and 15°C the day after tomorrow.*

- *Tomorrow is predicted to be 1°C ($19 - 18$) **warmer** than today.*
- *The day after tomorrow is predicted to be 3°C ($18 - 15$) **colder** than today.*

Problem 2. The following table presents demographic data for Arizona. Please use the data to answer the questions below.

Data Table: Arizona Demographics (In thousands)			
Demographics	2023	2000	1990
Foreign Born (Number)	981	656	278
U.S. Born (Number)	6,450	4,474	3,387
% Foreign Born	13.2%	12.8%	7.6%

Table 1: Arizona Demographics Summary¹

- (a) Calculate the change in the number of foreign-born residents between 2000 and 2023. (Did it increase or decrease? By how many?)
- (b) Calculate the change in the number of U.S.-born residents between 2000 and 2023. (Did it increase or decrease? By how many?)
- (c) Which group showed a larger **absolute** change? Based on this, is it reasonable to infer that the U.S.-born population grows faster than the foreign-born population? Why or why not?

We will provide more detailed discussion of Problem 1(c) at the end of this worksheet. Before that, let's explore the concept of **Percentages**.

¹The term “foreign born” refers to people residing in the United States at the time of the population survey who were not U.S. citizens at birth. The foreign-born population includes naturalized U.S. citizens, lawful permanent immigrants (or green-card holders), refugees and asylees, certain legal nonimmigrants (including those on student, work, or some other temporary visas), and persons residing in the country without authorization.

2 Percentages

2.1 Review: Converting Fractions to Decimals

Using long division, we can express any fraction as a decimal number. Please convert the following fractions into their decimal representations.

Problem 3. Answer²

Fraction	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{16}{40}$	$\frac{63}{36}$	$\frac{1}{100}$
Decimal	0._ _	0._ _ _	0._	1._ _	0._ _

2.2 Why Do We Need Percentages (%)?

Applications of decimals and fractions appear everywhere in our daily lives. From calculating tips and sales taxes to understanding the composition of a cup of coffee or the "doneness" of a steak, these concepts are indispensable in the modern world.

However, in daily communication, using *raw* fractions or decimals can be counterintuitive. For example:

- 24/65 of this drink is real juice.
- 0.33 of a human life is spent sleeping on average.
- **Gemini:** Imagine a weather forecast stating, "*The probability of rain today is 17/50,*" or "*your phone showing a battery level of 0.06.*" These values are much harder to interpret quickly than 34% or 6%.

To simplify these expressions, our ancestors introduced the concept of the **percentage**.

(1): 0.25; (2): 0.375; (3) 0.4; (4) 1.75; (5): 0.01²

Definition 4. One percentage, denoted as 1%, is defined as:

$$1\% = \frac{1}{100} = 0.01$$

Why was 1/100 chosen as the standard unit? There are two primary reasons:

1. **Decimal System:** As we live in a base-10 (decimal) civilization, choosing powers of 10 (like 100 or 0.01) as a baseline is natural and intuitive.
2. **The Balance of Precision:** A scale of 100 provides an ideal balance between precision and simplicity. A scale of 10 (0.1) is often too **coarse** for detail, while a scale of 1,000 (0.001) is unnecessarily complex for most daily tasks.³

Example 5. Consider the population of Taiwan. As of late 2025, the combined population of Taipei and New Taipei City is approximately $4,044,831 + 2,439,507 = 6,484,338$. Out of Taiwan's total population of 23,299,132, this group accounts for:

$$\frac{6,484,338}{23,299,132} \approx 0.2783$$

Compare the decimal **0.2783** and the fraction **6,484,338 / 23,299,132** with the percentage **27.8%**. Which one gives you a more immediate "sense" of the population distribution? Why?

³However, smaller units remain crucial in scientific contexts, such as measuring the size of bacteria or the mass of planets.

2.3 Back to math

A common scenario where percentages arise is representing a “**part of a whole.**” In this context, a percentage is simply an alternative representation of a fraction, where $1\% = \frac{1}{100}$.

Since not all fractions can be expressed as finite decimals (recall that $\frac{1}{3} = 0.333\dots$), we typically round the decimal representation to a specific number of decimal places (usually 3 or 4) to determine the corresponding percentage rounded to the nearest tenth/hundredth. (Refer: 2.2 *Example 5*.)

Demographics	2023	2000	1990
Foreign Born (Number)	981	656	278
U.S. Born (Number)	6,450	4,474	3,387
% Foreign Born	13.2%	12.8%	7.6%

Problem 6. Refer to the Arizona demographics table (Table 1). Please use a calculator to perform the following tasks:

- (a) Calculate the “Whole” (total population) for the years 2000 and 2023 by summing the foreign-born and U.S.-born populations.
- (b) For both 2000 and 2023, calculate the **percentage** of the population that is foreign-born.

$$\frac{\text{Foreign Born}_{2000}}{\text{Whole}_{2000}} = \underline{\hspace{2cm}} \approx 0.\underline{\hspace{3cm}} \approx \underline{\hspace{2cm}}.\underline{\hspace{2cm}} \%$$

$$\frac{\text{Foreign Born}_{2023}}{\text{Whole}_{2023}} = \underline{\hspace{2cm}} \approx 0.\underline{\hspace{3cm}} \approx \underline{\hspace{2cm}}.\underline{\hspace{2cm}} %$$

Does your result match the values provided in the third row of the table? (Hint: Round your answers to the nearest tenth of a percent.)

- (c) Calculate the difference between the percentage in 2023 and the percentage in 2000. This result is known as the **change in percentage points** of the group of the foreign-born.

3 Percentage Change (Relative change)

Thinking Box: Is \$100 always the same?

Does an extra \$100 in income mean the same thing to a billionaire as it does to a struggling tutor, Ding?

While the **absolute amount** is identical, the impact on their lives is vastly different. This discrepancy arises because our brains naturally perceive change *relative* to our current state.

3.1 Why We Need Relative Change

The difference in the example above stems from the different **baselines** (or initial states). To account for the effect of the initial magnitude and provide a more meaningful description of “change,” we divide the absolute difference by the initial value. This is known as **Relative Change**.

The formula for relative change is:

$$\text{Relative Change} = \frac{\text{New Value} - \text{Initial Value}}{\text{Initial Value}}$$

In daily contexts, we still follow a “**direction-magnitude**” procedure (As the review section shows). The direction (increase/decrease) matches that of the absolute change. The magnitude, however, is now expressed as a ratio that indicates how large the change is compared to where we started.

3.2 Percentage Change

While relative change is often calculated as a fraction or decimal, most news reports and academic articles prefer to express it as a percentage. This is called the **Percentage Change**.

Problem 7. Return once more to our Arizona demographics data. Please use your calculator to answer the following. Note: You must show your calculation steps (i.e., exactly what you entered into the calculator).

Demographics	2023	2000	1990
Foreign Born (Number)	981	656	278
U.S. Born (Number)	6,450	4,474	3,387
% Foreign Born	13.2%	12.8%	7.6%

(a) Calculate the **percentage change** from 2000 to 2023 for:

- The Foreign-born population.
- The U.S.-born population.

(Note: Round to the fourth decimal place, for example, 0.13458... → 0.1346 → 13.46%)

- (b) Based on your results in (a), which group is growing **faster**? Does this conclusion align with what the **absolute change** indicated in Section 1?
- (c) T|F: The “**change in the percentage**” (**percentage points**) of the foreign-born population is the same as the “**percentage change**” of the foreign-born population between 2000 and 2023. Explain your reasoning.

4 Answer Key

- **Problem 2 (a):** $981 - 656 = 325$. There was an increase of 325 thousand foreign-born residents.
- **Problem 2 (b):** $6450 - 4474 = 1976$. There was an increase of 1,976 thousand U.S.-born residents.
- **Problem 2 (c):** No. A larger initial base naturally leads to a greater absolute increase. Absolute change does not indicate growth speed.

- **Problem 3:** (1) 0.25; (2) 0.375; (3) 0.4; (4) 1.75; (5) 0.01.
- **Problem 6 (a):** 2000: $656 + 4474 = 5,130$; 2023: $981 + 6450 = 7,431$.
- **Problem 6 (b):** Yes.

$$\frac{\text{Foreign Born}_{2000}}{\text{Whole}_{2000}} = \frac{656}{5130} \approx 0.1278 \approx 12.8\%$$

$$\frac{\text{Foreign Born}_{2023}}{\text{Whole}_{2023}} = \frac{981}{7431} \approx 0.1320 \approx 13.2\%$$

- **Problem 6 (c):** $(13.2 - 12.8)\% = 0.4$ percentage points.

- **Problem 7 (a):**

$$\text{Foreign Born : } \frac{981 - 656}{656} = \frac{325}{656} \approx 0.4954 \approx 49.5\%$$

$$\text{U.S. Born : } \frac{6450 - 4474}{4474} = \frac{1976}{4474} \approx 0.4416 \approx 44.2\%$$

- **Problem 7 (b):** Clearly, the foreign-born group grows faster, which aligns with our reasoning in 1(c): **absolute change does not indicate growth speed.**

Additionally, since the growth rate of the foreign-born population is higher than that of the U.S.-born, the percentage of the total population that is foreign-born increased from 12.8% to 13.2%.

- **Problem 7 (c):** No, they are entirely different concepts. As stated above, the “**change in the percentage**” reflects information regarding the whole population, taking both U.S.-born and foreign-born residents into account.

On the other hand, “**percentage change**” compares only the specific numbers of the foreign-born population between two periods.