FACULTY OF ECONOMICS AND BUSINESS ADMINISTRATION

DEPARTMENT OF BUSINESS INFORMATICS AND OPERATIONS MANAGEMENT
RESEARCH GROUP OPERATIONS AND PRODUCTION MANAGEMENT

MANUFACTURING PLANNING AND CONTROL

MATERIALS REQUIREMENTS PLANNING (MRP)

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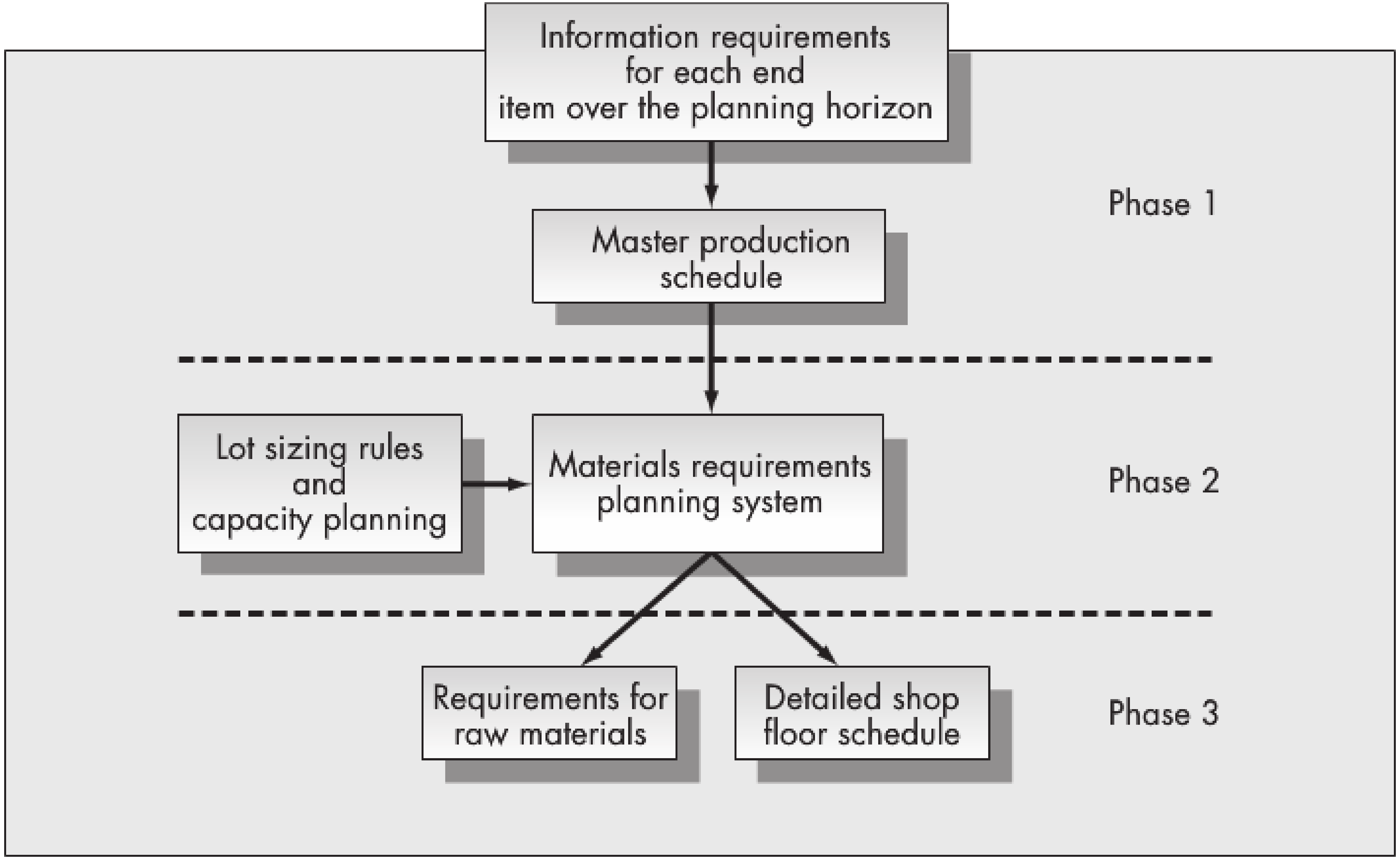
OUTLINE

- Material Requirements Planning (MRP) basics
- Explosion calculus
- Alternative lot-sizing schemes
- Lot sizing with capacity constraints
- Shortcomings of MRP

3 MAJOR CONTROL PHASES



MRP BASICS

- Independent & dependent demand
  - Independent demand (end product) : 10 bikes ordered
  - Dependent demand (parts) : 20 wheels needed
- Before MRP
  - Treat every stock location as ‘independent’
- MRP technique
  - Developed by Orlicky (airplane industry)
  - Regard end product demand & stock as ‘independent’
  - Translate end product demand, using a BOM relation (Bill of Material), towards ‘dependent’ demand for intermediates or parts.

MRP BASICS

- MRP is the basic process of translating a production schedule for an end product (MPS or Master Production Schedule) to a set of requirements for all of the subassemblies and parts needed to make that item.
- The data sources for determining the MPS include:
  - Firm customer orders
  - Forecasts of future demand by item
  - Safety stock requirements
  - Seasonal variations
  - Internal orders from other parts of the organization

MRP BASICS

- The MRP system computes production schedules for all levels based on forecasts of sales of end items.
- It is a PUSH system: once produced, subassemblies are pushed to next level whether needed or not

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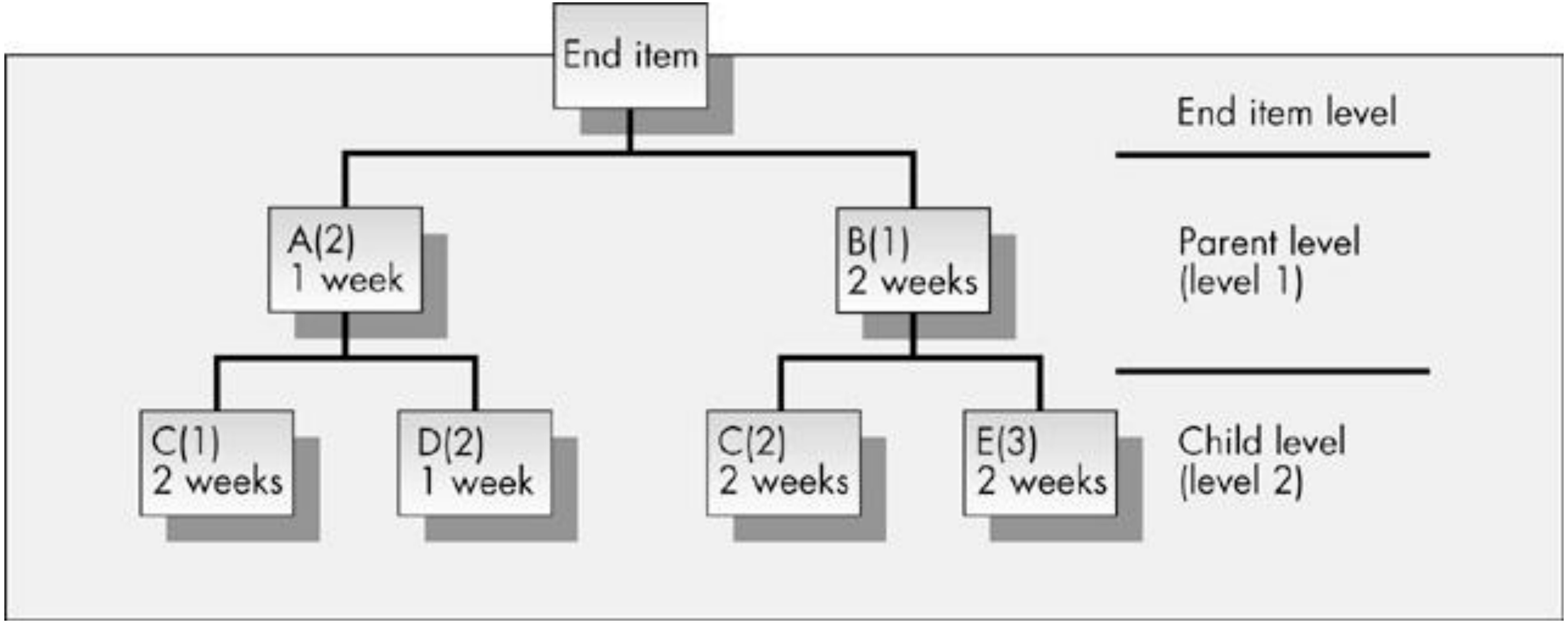
EXPLOSION CALCULUS

- A set of rules for converting MPS to a requirements schedule for all subassemblies, components, and raw materials necessary to produce the end item
- There are two basic operations comprising the explosion calculus:
  - **Time phasing**: Requirements for lower level items must be shifted backwards by the lead time required to produce the items
  - **Multiplication**: A multiplicative factor must be applied when more than one subassembly is required for each higher level item

PRODUCT STRUCTURE DIAGRAM

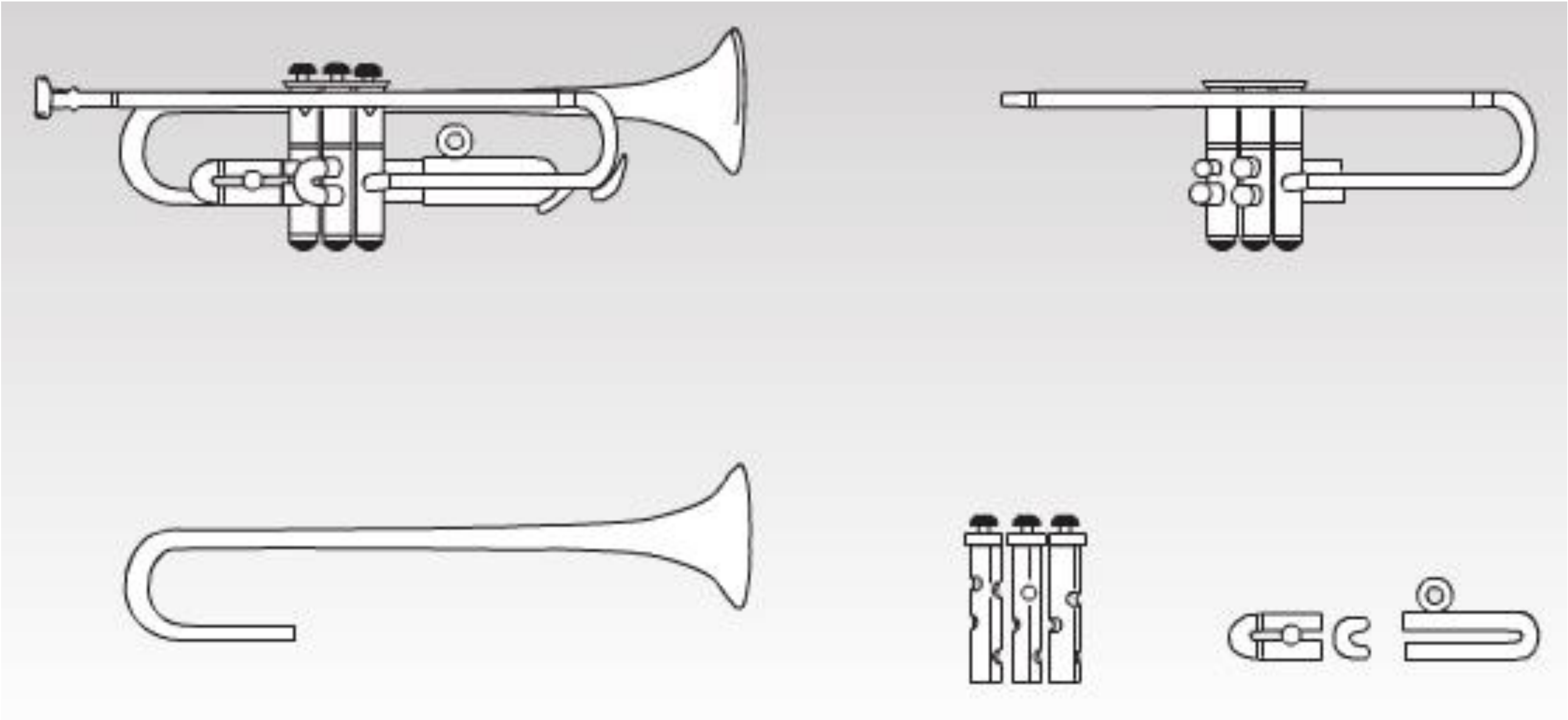
- = Bill of material (BOM)
- Graphical representation of the relationship between the various levels of the productive system
- Necessary to implement the explosion calculus



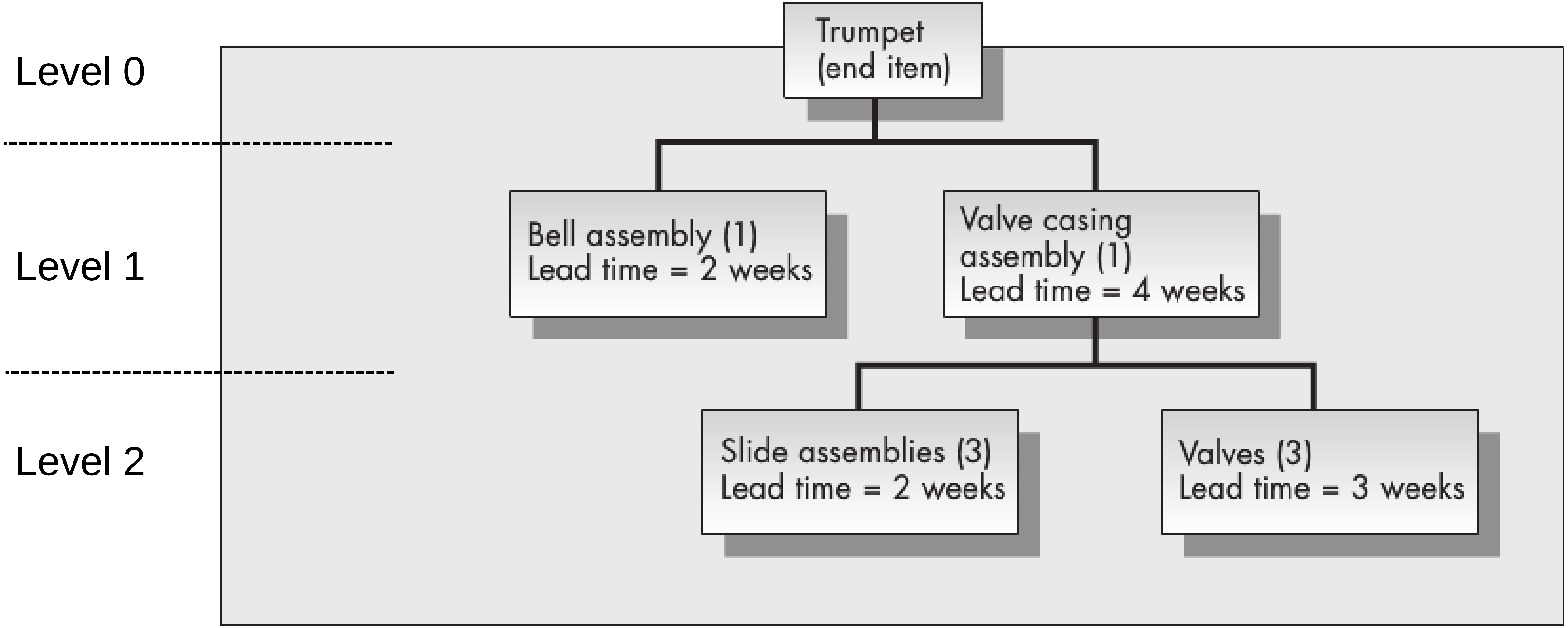
EXPLOSION CALCULUS – AN EXAMPLE

- Also: Bill-of-materials explosion
- Harmon Music Company produces a variety of wind instruments
- One of the instruments: model 85C trumpet

Trumpet and subassemblies



BILL OF MATERIAL – MODEL 85C TRUMPET



- It takes 7 weeks to produce a trumpet

DETERMINING THE MPS

- Forecasts for demands

Week	8	9	10	11	12	13	14	15	16	17
Demand	77	42	38	21	26	112	45	14	76	38

- Anticipated returns

Week	8	9	10	11
Scheduled receipts	12		6	9

- Expected on-hand inventory at the end of week 7 = 23 trumpets

- MPS for trumpets 85C (lot-for-lot)

Week	8	9	10	11	12	13	14	15	16	17

MRP CALCULATIONS FOR THE BELL ASSEMBLY

- From the BOM
  - 1 bell assembly per trumpet
  - Lead time = 2 weeks
- Assume lot-for-lot production
- No on-hand inventory or scheduled receipts

Week	
Gross requirements	
Net requirements	
Time-phased net requirements	
Planned order release (lot for lot)	

MRP FOR THE VALVE CASING ASSEMBLY

- From the BOM
  - 1 valve casing assembly per trumpet
  - Lead time = 4 weeks
- Assume lot-for-lot production
- No on-hand inventory or scheduled receipts

Week	
Gross requirements	
Net requirements	
Time-phased net requirements	
Planned order release (lot for lot)	

MRP CALCULATIONS FOR THE VALVES

- From the BOM
  - 3 valves per valve casing assembly
  - Lead time = 3 weeks
- Assume lot-for-lot production
- On-hand inventory at the end of week 3 = 186 valves, and scheduled receipts at the start of week 5 = 96 valves

Week	
Gross requirements	
Scheduled receipts	
On-hand inventory	
Net requirements	
Time-phased net requirements	
Planned order release (lot for lot)	

LOT-SIZING SCHEME

- We assumed lot-for-lot
- A lot-for-lot policy may be:
  - ➢ Suboptimal
  - ➢ And even infeasible

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THE LOT SIZING PROBLEM

- What production quantities will minimize total holding and setup costs over the planning horizon?
- Assume there is a known **set of requirements** $(r_1, r_2, \ldots, r_n)$, *i.e.* time-varying demands, over an n period planning horizon. Both the **set up cost,** K, and the **holding cost,** h, are given. The objective is to determine **production quantities** $(y_1, y_2, \ldots, y_n)$ to meet the requirements at minimum cost.
- The feasibility condition to assure there are no stock-outs in any period is:

$$\sum_{i=1}^{j} y_i \geq \sum_{i=1}^{j} r_i \quad for \quad 1 \leq j \leq n$$

ALTERNATIVE LOT-SIZING SCHEMES

Some heuristics:

1. EOQ lot sizing
2. Silver-Meal heuristic
3. Least unit cost
4. Part period balancing

1 EOQ LOT SIZING

Valve casing assembly

- Setup: 2 workers ($22/hour) for three hours
  → Setup cost = $2 \times 3 \times 22 = \$132$
- Annual interest rate: $I = 22\%$,
- Cost of materials and value added for labor: $c = \$141.82$
- Convention: holding cost is charged against inventory each week

EOQ LOT SIZING – VALVE CASING ASSEMBLY

Week	
Net requirements	
Time-phased net requirements	
Planned order release (EOQ)	
Planned deliveries	
Ending inventory	

COMPARE LOT-FOR-LOT WITH EOQ LOT SIZING

Valve casing assembly

- Total holding and setup cost for EOQ:

$$(132)(4) + (0.6)(653) = \$919.80$$

- Total holding and setup cost for lot-for-lot:
  - 10 lots of varying sizes (one in every period)
  - Cumulative ending inventory: $0 + 0 + \cdots + 0 = 0$
  - Total holding and setup cost: $(132)(10) + (0.6)(0) = \$1320$

- Remark: cost savings of EOQ over lot-for-lot do not consider the cost impact that lot sizing at this level may have upon lower levels in the product tree

LOOKING BACK AT CHAPTERS ON INVENTORY CONTROL

- EOQ assumes known and constant demand
  - Dependent demand is known, but it is certainly not constant, but spiky
- Stochastic inventory models assume random demand
  - Dependent demand is variable but not random; it is predictable

2 SILVER-MEAL HEURISTIC

- Named for Harlan Meal and Edward Silver
- $C(T)$ = average holding and setup cost **per period** if the current order spans the next T periods
- A forward method that requires determining $C(T)$ for $(T = 1, 2, \ldots, \mathrm{n})$, and stopping the computation when this function first increases
- Let $(r_1, r_2, \ldots, r_n)$ be the requirements over the n-period horizon

$$C(1) = K$$
$$C(2) = (K + hr_2)/2$$
$$C(3) = (K + hr_2 + 2hr_3)/3$$
$$C(j) = \left(K + hr_2 + 2hr_3 + \cdots + (j-1)hr_j\right)/j$$

- Once $C(j) > C(j-1)$, we stop and set $y_1 = r_1 + r_2 \ldots + r_{j-1}$ (the production quantity in period 1), and begin the process again starting at period j

SILVER-MEAL – VALVE CASING ASSEMBLY

h = 0.60   K = 132.00   r = (42, 42, 32, 12, 26, 112, 45, 14, 76, 38)

SILVER-MEAL – VALVE CASING ASSEMBLY

Week	
Net requirements	
Time-phased net requirements	
Planned order release (S–M)	
Planned deliveries	
Ending inventory	

3 LEAST UNIT COST

- Similar to Silver-Meal
- Instead of dividing the cost over j periods by the number of periods j, we divide it by the total number of units demanded through period j, $r_1 + r_2 + \cdots + r_j$
- Choose the order horizon that minimizes the cost per unit of demand

$$C(1) = K/r_1$$

$$C(2) = (K + hr_2)/(r_1 + r_2)$$

$$C(3) = (K + hr_2 + 2hr_3)/(r_1 + r_2 + r_3)$$

$$C(j) = \left(K + hr_2 + 2hr_3 + \cdots + (j-1)hr_j\right)/\left(r_1 + r_2 + \cdots + r_j\right)$$

- Once $C(j) > C(j-1)$, we stop and set $y_1 = r_1 + r_2 \ldots + r_{j-1}$ (the production quantity in period 1), and begin the process again starting at period j

LEAST UNIT COST – VALVE CASING ASSEMBLY

h = 0.60 K = 132.00 r = (42, 42, 32, 12, 26, 112, 45, 14, 76, 38)

LEAST UNIT COST – VALVE CASING ASSEMBLY

Week
Net requirements
Time-phased net requirements
Planned order release (LUC)
Planned deliveries
Ending inventory

4 PART PERIOD BALANCING

- Set the order horizon equal to the number of periods that *most closely matches* the total holding cost with the setup cost over that period
- Origin of the name of the method = the order horizon that exactly *equates* holding and setup costs will rarely be an integer number of periods

PART PERIOD BALANCING – VALVE CASING ASSEMBLY

h = 0.60 K = 132.00 r = (42, 42, 32, 12, 26, 112, 45, 14, 76, 38)

PART PERIOD BALANCING – VALVE CASING ASSEMBLY

Week	
Net requirements	
Time-phased net requirements	
Planned order release (PPB)	
Planned deliveries	
Ending inventory	

COMPARISON OF LOT-SIZING SCHEMES

- What production quantities will minimize total holding and setup costs over the planning horizon?
- We covered some heuristics: they do not necessarily give the optimal solution
- An optimal solution method also exists: Wagner-Whitin algorithm
  - ➢ See Appendix 7-A for more information (not part of the course)
  - ➢ The optimal solution for the valve casing assembly problem turns out to be: y_4 = 154, y_9 = 171, y_{12} = 114 with a total cost of $\$610.20$
- Summarized for the valve casing assembly example:

LFL: $\$1320$; EOQ: $\$919.80$; LUC: $\$781.80$; PPB: $\$724.20$; S-M: $\$650.40$; W-W: $\$610.20$

COMPARISON OF LOT-SIZING SCHEMES

- Property of the optimal solution: every optimal solution orders exact requirements: that is,

$$y_1 = r_1 \ or \ y_1 = r_1 + r_2 \ or \ \ldots or \ \ldots y_1 = r_1 + r_2 + \cdots + r_n$$

  - LFL, S-M, LUC, PPB and of course W-W are lot sizing schemes that satisfy this property; EOQ does not
- Experimental evidence seems to favor the Silver Meal Heuristic among the four discussed as the most cost efficient (S-M performs better in a greater number of cases)

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LOT SIZING WITH CAPACITY CONSTRAINTS

- An infeasible problem: **r**$= (52, 87, 23, 56)$ and **c**$= (60, 60\ 60, 60)$
- Even though $(52 + 87 + 23 + 56) = 218 < (60 + 60 + 60 + 60) = 240$, the problem is infeasible because:
- $(52 + 87) = 139 > (60 + 60) = 120$

LOT SIZING WITH CAPACITY CONSTRAINTS

Finding a feasible solution

- A feasible problem: **r**$= (20, 40, 100, 35, 80, 75, 25)$ and **c**$= (60, 60\ 60, 60, 60, 60\ 60)$
- The feasibility condition can be checked
- A feasible solution exists, but how to find it?
- Lot-for-lot is not possible (capacity constraints period 3, 5 and 6)
- **Approximate lot-shifting technique**: back-shift demand from periods, in which demand exceeds capacity, to prior periods, in which there is excess capacity
- Construct a new requirements schedule, **r'**, in which lot for lot is feasible

LOT SIZING WITH CAPACITY CONSTRAINTS

Finding a feasible solution

Example: A feasible problem: **r**$= (20, 40, 100, 35, 80, 75, 25)$ and **c**$= (60, 60\ 60, 60, 60, 60\ 60)$

- Approximate lot-shifting technique:

	$50 \qquad\qquad 60$
$40, 60, 60, 55, 60$	$\cancel{40}, 60, 60, \cancel{55}, 60, 60$
r'$= (\cancel{20}, \cancel{40}, \cancel{100}, \cancel{35}, \cancel{80}, 75, 25)$	**r'**$= (\cancel{20}, \cancel{40}, \cancel{100}, \cancel{35}, \cancel{80}, 75, 25)$
c$= (60, 60\ 60, 60, 60, 60\ 60)$	**c**$= (60, 60\ 60, 60, 60, 60\ 60)$

- Setting **y**$=$**r'**$= (50, 60, 60, 60, 60, 60, 25)$ gives a feasible solution
- This feasible solution can be improved by shifting production to prior periods if the additional holding cost is less than the setup cost (which is eliminated)

LOT SIZING WITH CAPACITY CONSTRAINTS

Improving a feasible solution

- **Improvement step**: starting from the last period and working backward to the beginning, determine whether it is cheaper to shift a lot to prior periods with excess capacity

Example: An infeasible lot-for lot solution **r**$= (100, 79, 230, 105, 3, 10, 99, 126, 40)$ and **c**$= (120, 200, 200, 400, 300, 50, 120, 50,30)$

- **First finding a feasible solution**: Approximate lot-shifting technique

- Assume K=$450 and h=$2

LOT SIZING WITH CAPACITY CONSTRAINTS

Improving a feasible solution (example contd)

	1	2	3	4	5	6	7	8	9
r′	100	109	200	105	28	50	120	50	30
c	120	200	200	400	300	50	120	50	30
y	100	109	200	105	108 ~~58~~ ~~28~~	50	120	0 ~~50~~	0 ~~30~~
Excess capacity	20	91	0	295	192 ~~242~~ ~~272~~	0	0	0	0

- $2 \times 30 \times 4 = \$240 < \$450$
- $2 \times 50 \times 3 = \$300 < \$450$

LOT SIZING WITH CAPACITY CONSTRAINTS

Improving a feasible solution (example contd)

LOT SIZING WITH CAPACITY CONSTRAINTS

- Cost comparison

Initial requirements: **r**= $(100, 79, 230, 105, 3, 10, 99, 126, 40)$

Feasible solution: **y=r'**= $(100, 109, 200, 105, 28{,}50, 120, 50{,}30)$

$Setup\ cost + \ Holding\ cost$

$= 9 \times 450 + 2(0 + 30 + 0 + 0 + 25 + 65 + 86 + 10 + 0)$

$= 4050 + 432 = \$4482$

Improved solution: **y=r'**= $(100, 109, 200, 263, 0{,}0, 120, 0{,}0)$

$Setup\ cost + \ Holding\ cost$

$= \ 5 \times 450 + 2(0 + 30 + 0 + 158 + 155 + 145 + 166 + 40 + 0)$

$= 2250 + 1388 = \$3638$

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SHORTCOMINGS OF MRP

1. ***Uncertainty:*** MRP ignores demand uncertainty, supply uncertainty, and internal uncertainties that arise in a manufacturing process
2. ***Capacity Planning:*** Basic MRP does not take capacity constraints into account.
3. ***Rolling Horizons and System Nervousness:*** MRP is treated as a static system with a fixed horizon of n periods. The choice of n is arbitrary and can affect the results.
4. ***Lead Times Dependent on Lot Sizes:*** In MRP lead times are assumed fixed, but they clearly depend on the size of the lot required.
5. ***Quality Problems:*** Defective items can destroy the linking of the levels in an MRP system.
6. ***Data Integrity:*** Real MRP systems are big (perhaps more than 20 levels deep) and the integrity of the data can be a serious problem.
7. ***Order Pegging:*** A single component may be used in multiple end items, and each lot must then be pegged to the appropriate item.

1 UNCERTAINTY

- Two key sources of uncertainty:
  - ➢ forecasts for future sales of the end item
  - ➢ estimation of the production lead times from one level to another

- Remember: safety stock to protect against the uncertainty of demand
  - ➢ Build suitable safety levels into the forecasts for the end item
  - ➢ Compute safety stock based on a service level criterion (Type I or II) or by using a stock-out cost model
- Determine safety lead times by using a safety factor, e.g., a safety factor of 1.5 means all lead times will be multiplied by 1.5 before starting the explosion calculus

1 UNCERTAINTY

- Safety stock to protect against the uncertainty of demand

  Example: Model 85C trumpets

  Type I service level = 90% → z=1.28

Week	8	9	10	11	12	13	14	15	16	17
Predicted demand (μ)	77	42	38	21	26	112	45	14	76	38
Standard deviation (σ)	23.1	12.6	11.4	6.3	7.8	33.6	13.5	4.2	22.8	11.4
Mean demand plus safety stock ($\mu + \sigma_z$)	107	58	53	29	36	155	62	19	105	53

2 CAPACITY PLANNING

- We discussed lot sizing with capacity constraints
- However, we only dealt with production capacities at one level of the system
- A feasible production schedule at one level may result in an infeasible requirements schedule at a lower level
- Capacity requirements planning (CRP) computes the capacity requirements placed on a work center by using the output of the MRP planned order releases
- In case of infeasibility:
  - Schedule overtime at the bottleneck locations
  - Revise the MPS to be able to work with the current system capacity: trial-and-error

3 ROLLING HORIZONS AND SYSTEM NERVOUSNESS

- MRP is a static system ↔ a production planning environment is dynamic
- Rolling horizons:
  - Only the first-period decision of an N-period problem is implemented
  - The full N-period problem is rerun each period
  - The planning horizon should be long enough to guarantee that the first-period decision does not change
- **Example:** optimal solutions are shown

$\mathbf{r} = (190, 210, 190, 210, 190)$ $\quad$ $\mathbf{r} = (190, 210, 190, 210, 190, 210)$

$\mathbf{y} = (190, 400, 0, 400, 0)$ $\quad$ $\mathbf{y} = (400, 0, 400, 0, 400, 0)$

3 ROLLING HORIZONS AND SYSTEM NERVOUSNESS

- Nervousness:
  - Changes that can occur in the schedule when the horizon is moved forward one period
  - Causes of nervousness: unanticipated changes in MPS, late deliveries of raw materials, failure of key equipment, absenteeism, unpredictable yields, …
  - Nervousness = if a revised schedule requires a setup in a period in which the prior schedule did not
- Reducing system nervousness:
  - Introduce an additional cost of ν if the new schedule $\boldsymbol{y}$ calls for a setup in a period that the old schedule $\hat{\boldsymbol{y}}$ did not
  - Increase the setup cost from K to $\mathrm{K} + \nu$ if $\hat{y}_k = 0$ prior to determining the new schedule $\boldsymbol{y}$
  - Resolving the problem with modified setup costs will results in fewer revisions

4 LEAD TIMES DEPENDENT ON LOT SIZES

- A constant production lead time is assumed, independent of the size of the lot
- One would expect ↗ lead time if ↗ lot size
- Difficult to include this into the explosion calculus

5 QUALITY PROBLEMS

- MRP implicitly assumes that no defective items are produced
- Defective products → yields $< 100\%$
- If yields are stable: incorporating yield losses is obvious
- If yields are random and variances are large: difficult to avoid stock-outs
- The costs of producing too many and too few should be balanced in a way

6 DATA INTEGRITY

- Incorrect data is possible and common:
  - A shipment is not recorded or recorded incorrectly
  - Items entering inventory from rework are not included
  - Scrap rates are higher than anticipated
  - …
- Technologies such as scanning, RFID, etc. can help reducing mistakes
- Cycle counting to reduce mistakes: counting a small amount of inventory each day, with the intent of cycling through the entire inventory on an ongoing basis.

7 ORDER PEGGING

- A single component may be used in multiple end items
- The gross requirements schedule for this component comes from several sources
- Each lot must then be pegged to the appropriate item
- Pegging adds considerable complexity to information storage requirements of the system

FROM MRP I TO MRP II TO ERP

- MRP I: Material Requirements Planning

  Material control system planning the availability of components, parts, intermediates and raw materials

- MRP II: Manufacturing **Resource** Planning

  Integrated material control system incorporating next to MRP I functionality the availability of other resources (finances, marketing, …)

  - MPS not treated as input but as a decision variable
  - Respecting the availability of machine capacity and labor

- ERP: Enterprise Resource Planning

  Integrated company control system, with extended functionality in logistics, distribution and manufacturing planning, maintenance, finance, etc ...

QUESTIONS/REMARKS