ECE 198JL Second Midterm Exam
Spring 2014

Tuesday, March 11th, 2014

Name: ____________________________________ NetID: _______________

Discussion Section:

9:00 AM	[ ] JD9	[ ] JDA
10:00 AM	[ ] JD1	
11:00 AM	[ ] JD2	[ ] JDB
12:00 PM	[ ] JD5	
1:00 PM	[ ] JD6	[ ] JD7
2:00 PM	[ ] JD3	
3:00 PM	[ ] JD8	
4:00 PM	[ ] JD4	[ ] JDC

- **Be sure your exam booklet has 13 pages.**
- **Be sure to write your name and lab section on the first page.**
- **Do not tear the exam booklet apart; you can only detach the last page.**
- **We have provided Boolean properties at the back.**
- **Use backs of pages for scratch work if needed.**
- **This is a closed book exam. You may not use a calculator.**
- **You are allowed one handwritten 8.5 x 11" sheet of notes.**
- **Absolutely no interaction between students is allowed.**
- **Be sure to clearly indicate any assumptions that you make.**
- **The questions are not weighted equally. Budget your time accordingly.**
- **Don’t panic, and good luck!**

Problem 1	12 points:	___________
Problem 2	12 points:	___________
Problem 3	11 points:	___________
Problem 4	9 points:	___________
Problem 5	17 points:	___________
Problem 6	17 points:	___________
Problem 7	12 points:	___________
Problem 8	10 points:	___________
Total	100 points:	___________

Problem 1 (12 pts): Boolean algebra

1. **(5 pts)** Which Boolean expressions below are equivalent to the Boolean expression $f(a,b,c,d) = \overline{\bar{a}c + bc + a\bar{c}}$? [More than one answer may be equivalent]

[✓ a) $f(a,b,c,d) = \overline{\bar{a}c + ab + a\bar{c}}$]

[✓ b) $f(a,b,c,d) = \overline{c(b+\bar{a}) + a\bar{c}}$]

[✓ c) $f(a,b,c,d) = (\bar{a}+c)(a+\bar{c})(\bar{b}+\bar{c})$]

d) $f(a,b,c,d) = (b+c)(\bar{a}+c)(a+\bar{c})$

e) $f(a,b,c,d) = a\bar{c} + \bar{a}c + bc$

2. **(5 pts)** Which Boolean expressions below are equivalent to the Boolean expression $f(a,b,c) = a + bc + \bar{c}$? [More than one answer may be equivalent]

[✓ a) $f(a,b,c) = (a+b)(a+c) + \bar{c}$]

b) $f(a,b,c) = a + \bar{c}$

c) $f(a,b,c) = (a+b)a + \bar{c}$

d) $f(a,b,c) = (a+b+\bar{c})(a+c)$

[✓ e) $f(a,b,c) = a + b + \bar{c}$]

3. **(2 pts)** The function $f(w,x,y,z) = m_3 \cdot m_4$, where m_3 and m_4 are minterms. Which expression below is equivalent to $f(w,x,y,z)$?

a) m_7

b) M_8

c) $M_3 + M_4$

[✓ d) 0]

e) 1

Problem 2 (12 pts): Karnaugh Maps

Consider a 4-variable Boolean function `f(w,x,y,z)` given by its K-map (drawn twice):

		yz			
		00	01	11	10
wx	00	0	1	1	1
	01	1	1	0	0
	11	1	1	0	1
	10	0	0	1	1

		yz			
		00	01	11	10
	00	0	1	1	1
wx	01	1	1	0	0
	11	1	1	0	1
	10	0	0	1	1

1. **(2pts)** How many prime implicants are in the K-map?

   a) 4 b) 5 [✓ c) 6] d) 7 e) 8 or more

2. **(2pts)** How many essential prime implicants are in the K-map?

   a) 0 b) 1 [✓ c) 2] d) 3 e) 4

3. **(2pts)** The k-map has how many minimal SOP Boolean expressions?

   a) 1 b) 2 c) 3 [✓ d) 4] e) 5

4. **(2pts)** The minimal SOP Boolean expression has how many product terms?

   a) 1 b) 2 c) 3 [✓ d) 4] e) 5

5. **(2pts)** The minimal SOP Boolean expression has how many literals (include duplicates in your count: x'y + x'z would count as four literals)?

   3+3+2+2

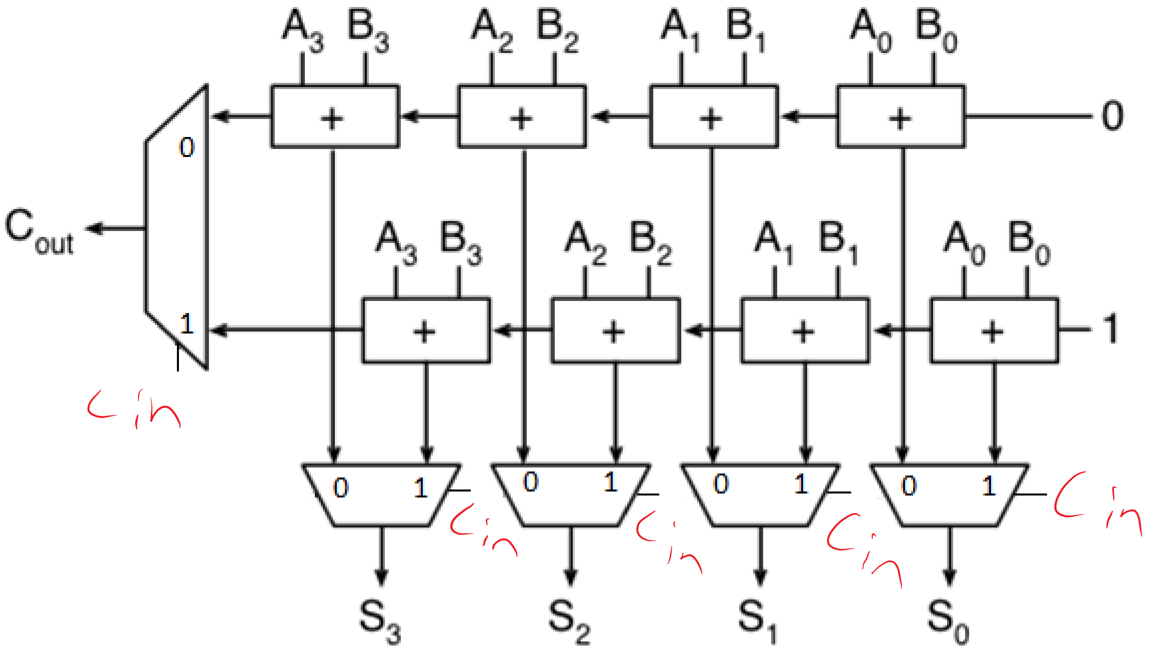
   a) 7 [✓ b) 10] c) 13 d) 14 e) 16

6. **(2pts)** The minimal POS and minimal SOP expressions for this k-map are equivalent.

   [✓ a) True] b) False

Problem 3 (11 pts): Design of Adders

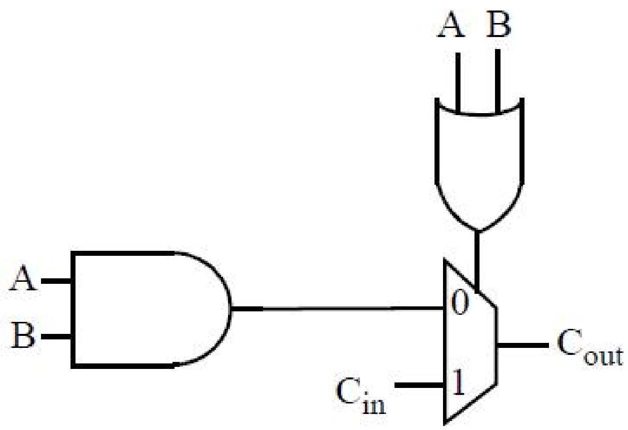
1. **(5 pts)** The circuit below implements a 4-bit, bit-sliced adder. It is composed of eight full adders and five multiplexers. Like the full adders we have seen before, this circuit can be combined in adder arrays to perform n-bit addition. Complete the design of the circuit by adding selection inputs to the MUXes.
   (Note: This is a different way to implement adders than what you have seen in class.)



2. (6 points)

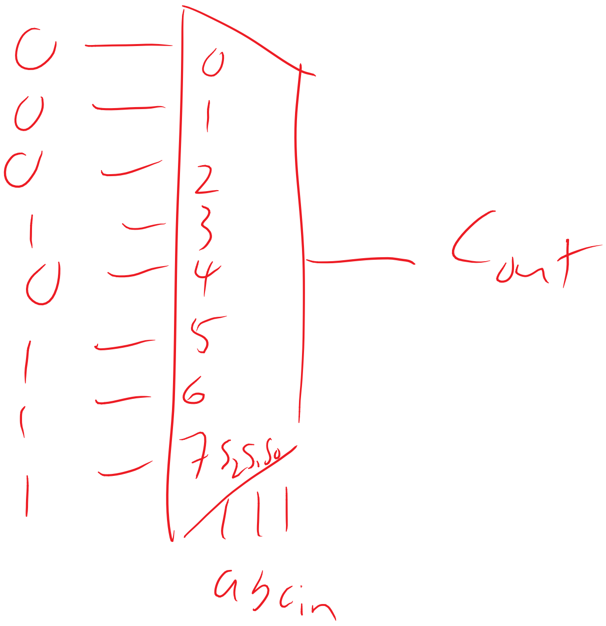
a) The circuit below implements the carry out C_{out} of a full adder (circle one): True / [✓ False]

Justify your answer.



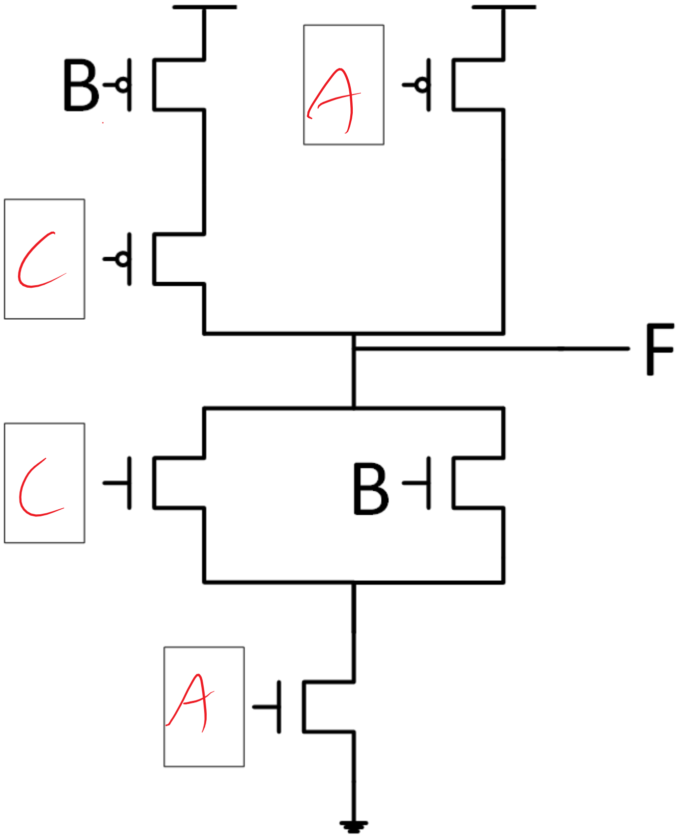
A	B	C_{in}	C_{out}?
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) If your answer to part a) was "False", implement the carry out of a full adder with a multiplexer. If your answer to part a) was "True", derive a different implementation of a carry out of a full adder with multiplexer



Problem 4 (9 pts): CMOS

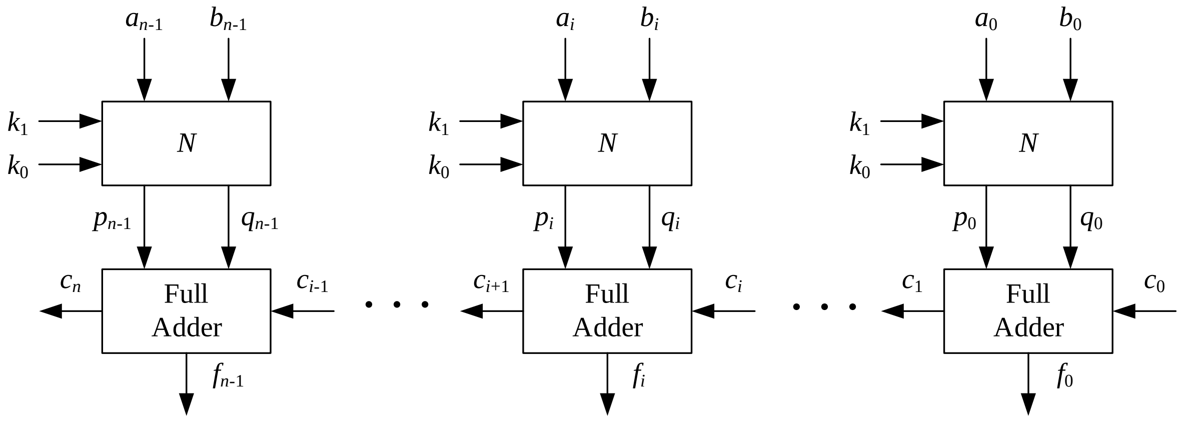
(9 pts) For the CMOS transistor circuit given in the figure below, complete the labeling of the circuit, as well as entries of the truth table that are not filled yet.



A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Problem 5 (17 pts): Arithmetic unit design

Design circuit *N* such that the *n*-bit unit shown below performs the specified arithmetic operations for n-bit 2's complement inputs $A=a_{n-1}$ … a_0 and $B=b_{n-1}$ … b_0:



1. (9 pts) Complete the table below to show how the outputs of circuit N relate to its inputs. Use only one of the values a_i, b_i, $\bar{a}_i$, $\bar{b}_i$, 0, and 1 in each cell. One row has been completed for you.

Control Inputs		Operation	Full Adder Array Inputs		
k_1	k_0		p_i	q_i	c_0
0	0	2 · *A* (2 TIMES A)	a_i	a_i	0
0	1	A – B (A MINUS B)	a_i	$\bar{b}_i$	1
1	0	A (PASS A)	a_i	0	0
1	1	-B – 1 (NEG B MINUS 1)	0	$\bar{b}_i$	0

2. (8 pts) Draw K-maps for outputs p_iq_i from circuit N. Each cell should have a 0 or 1.

a_ib_i

k_1k_0	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	1

p_i

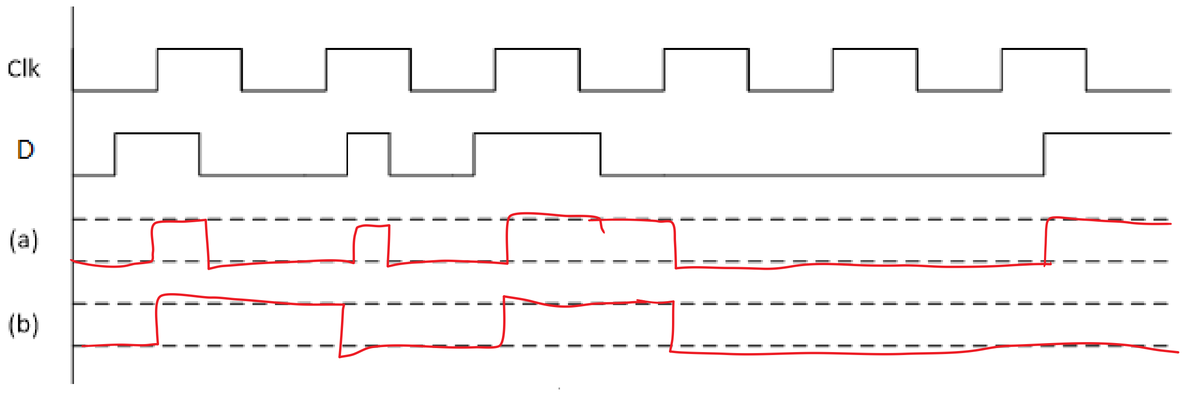
a_ib_i

k_1k_0	00	01	11	10
00	0	0	1	1
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

q_i

Problem 6 (17 pts): Flip flops, latches, and timing diagrams

1. **(6 pts)** Given timing diagram below, indicate the output of a D latch/flip flop assuming
a) It is a clocked, gated, level sensitive latch
b) It is a positive edge triggered flip flop



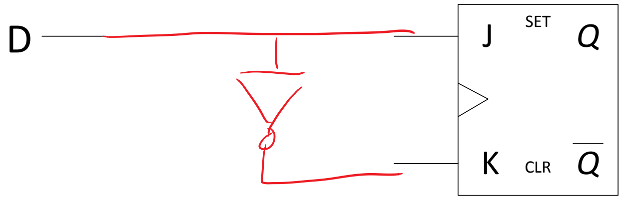
2. **(3 pts)** You have seen a JK flip flop in your discussion. The characteristic equation (expression) for Q^+ of a JK flip flop is given below:

$$Q^+ = QK' + Q'J$$

a) Write the characteristic equation (expression) for Q^+ of a **D flip flop**.

$Q^+ = D$

b) In the figure below, implement the functionality of a D-flip flop using a JK flip flop. Comparing the characteristic equations of a D and JK flip-flop may be helpful.



3. (8 pts) Consider a 3-bit shift register that has the functionality specified in the table to the right. Answer the following questions about the behavior and implementation of the shift register.

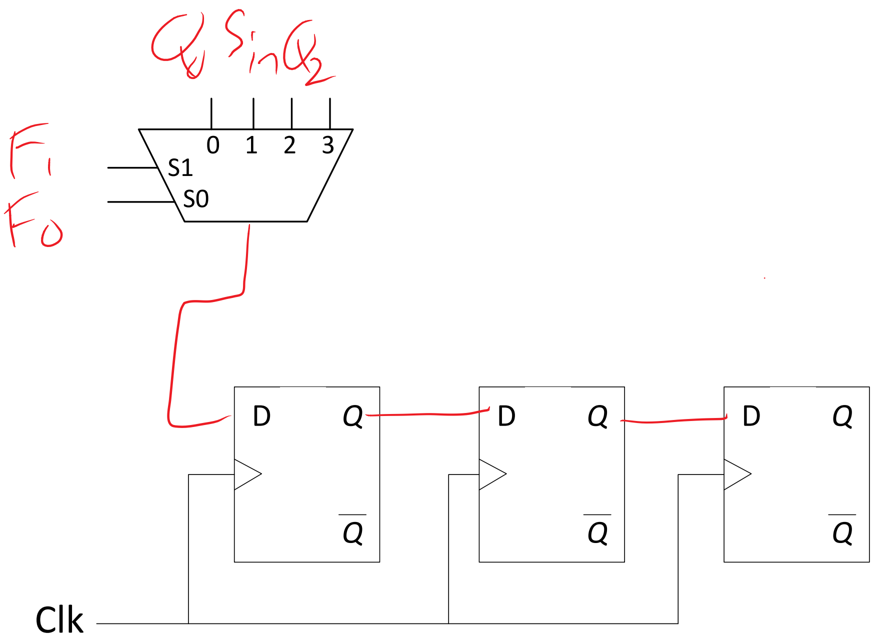
F_1	F_0	Operation
0	0	Circular shift right
0	1	Logical shift right
1	0	Arithmetic shift right
1	1	Unused

a) The shift register initially stores 110, what is stored in the register after one clock pulse and

$F_1F_0 = 00?$ 011

$F_1F_0 = 10?$ 111 (Assume again that 110 is stored before the operation)

b) Complete the design of a 3-bit register that performs the operations listed in the table. Serial input is labeled as S_{in}.



Problem 7 (12 pts): Combinational logic design

Design a 2-bit comparator circuit that compares two 2-bit unsigned binary numbers A = a_1a_0 and B = b_1b_0. The circuit should output 1 if and only if A ≥ B.

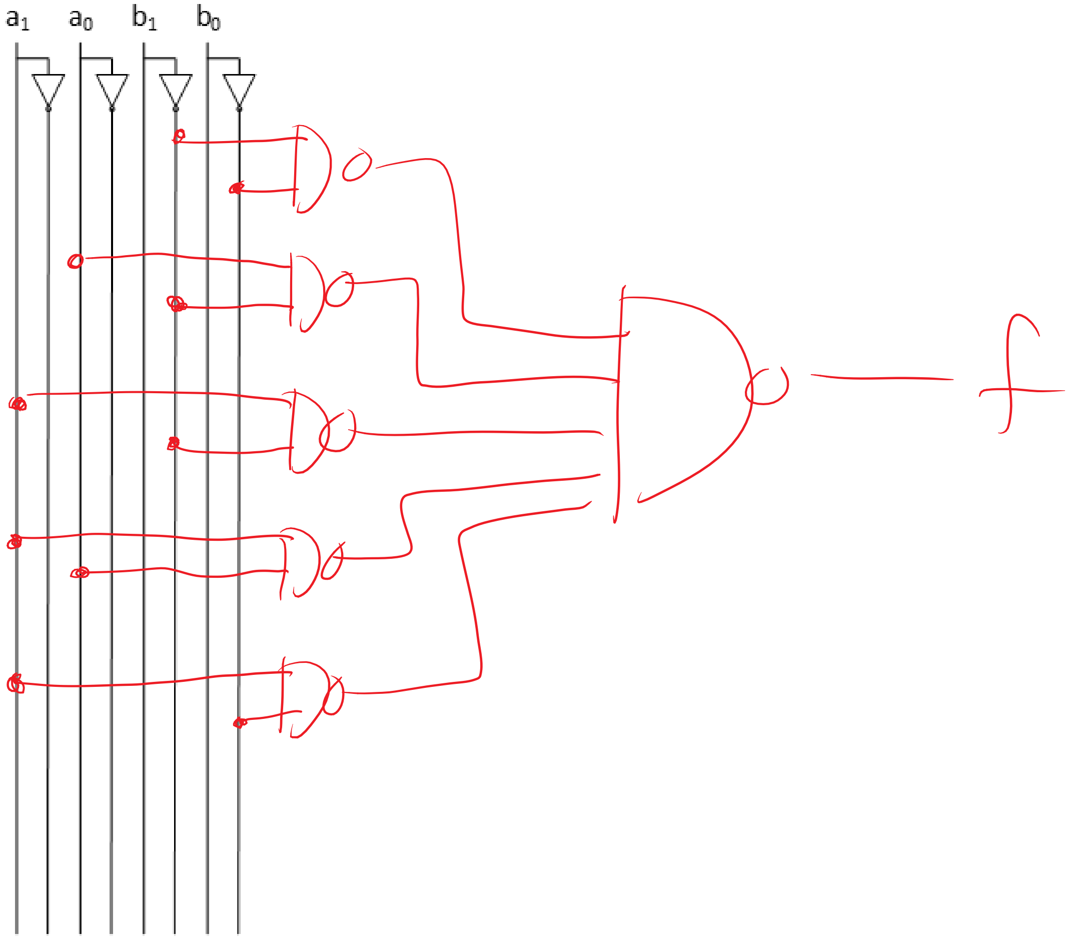
1. **(4 pts)** Use the kmap below, to specify how your quality control circuit should be built

a_1a_0 \ b_1b_0	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	1	1	1	1
10	1	1	0	1

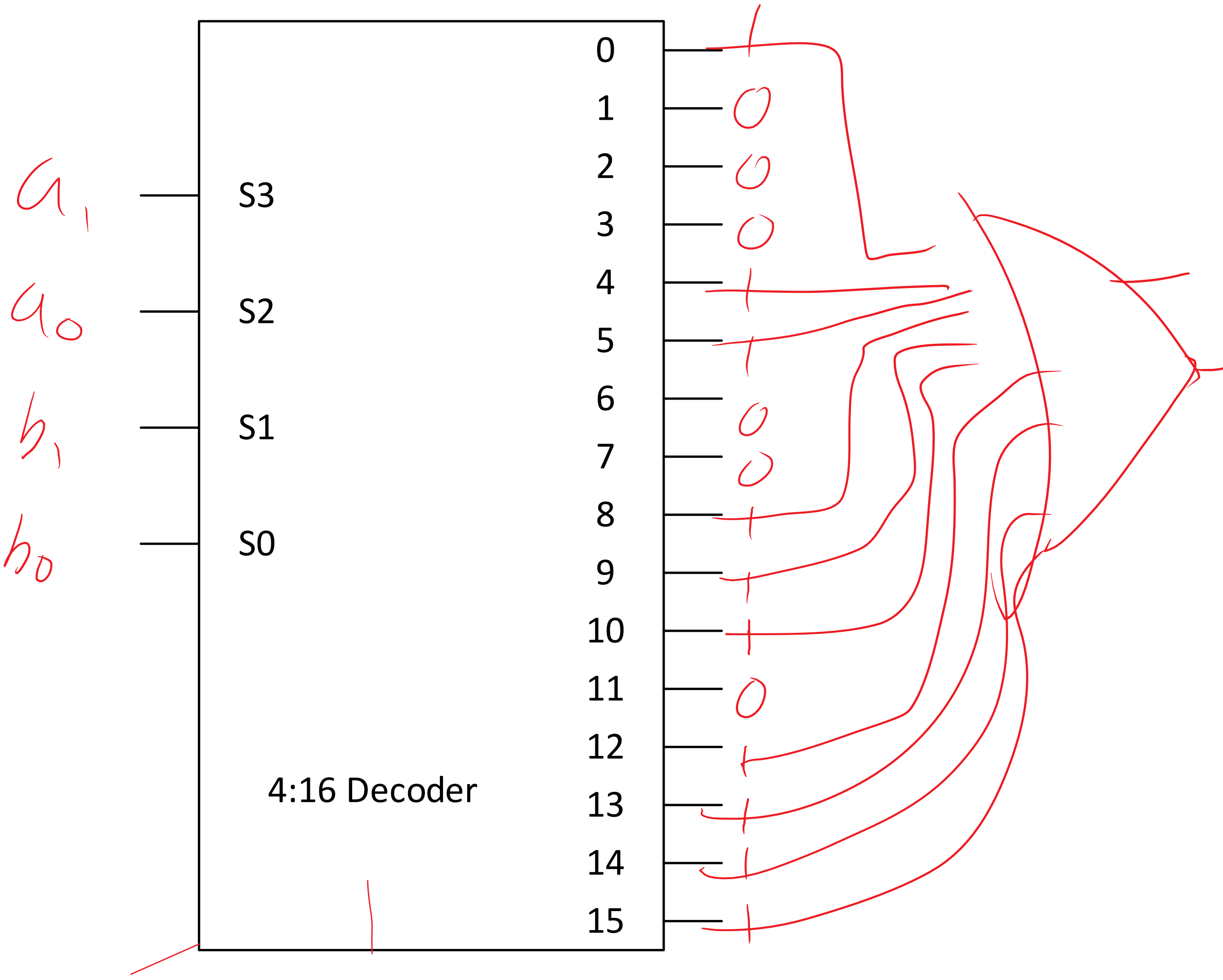
2. **(2 pts)** Derive a minimal SOP expression from your kmap.

f(a,b,c_{in},c_{out}) = $\bar{b}_1\bar{b}_0 + a_0\bar{b}_1 + a_1\bar{b}_1 + a_1a_0 + a_1\bar{b}_0$

3. **(3 pts)** Show a 2-level NAND only implementation of your circuit below.



4. **(3 pts)** Using the figure below, implement the 2-bit comparator circuit using a decoder and as few additional logic gates as possible.



Problem 8 (10 pts): Boolean algebra in C

1. **(8 pts)** Implement a program in C that prints canonical POS representation for function $g(a,b,c) = \overline{(ab + \bar{b}c)}$ using Maxterm notation. The program is partially implemented, you only need to complete some parts of it. **Use bit-wise and arithmetic operators only.**

```
#include <stdio.h>

int main()
{
   unsigned int a,b,c;
   int g;
   int i;
   int first=1;

   for ( a=0 ; a <= 1; a++ )
   {
      for ( b = 0; b <= 1; b++ )
      {
         for ( c = 0; c<=1 ; c = c + 1 )
         {
            g = ~((a&b)|((~b)&c))&1 ; /* function g */
            if (g == 0 )
            {
               i = 4*a+2*b + c ; /* maxterm index */
               if(first){
                  printf("g(a,b,c)=M%d", i );
                  first = 0;
               }
               else
               printf("M%d", i);
            }
         }
      }
   }

   printf("\n");

   return 0;
}
```

2. **(2 pts)** Write down EXACTLY the formatted text that will be printed on the terminal screen by the program.

g(a,b,c) = M1m5m6m7

Boolean algebra properties

Commutativity	$x \cdot y = y \cdot x$	$x + y = y + x$
Associativity	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	$(x + y) + z = x + (y + z)$
Distributivity	$x \cdot (y + z) = x \cdot y + x \cdot z$	$x + y \cdot z = (x + y) \cdot (x + z)$
Idempotence	$x \cdot x = x$	$x + x = x$
Identity	$x \cdot 1 = x$	$x + 0 = x$
Null	$x \cdot 0 = 0$	$x + 1 = 1$
Complementarity	$x \cdot x' = 0$	$x + x' = 1$
Involution	$(x')' = x$	
DeMorgan's	$(x \cdot y)' = x' + y'$	$(x + y)' = x' \cdot y'$
Absorption	$x \cdot (x + y) = x$	$x + x \cdot y = x$
No-Name	$x \cdot (x' + y) = x \cdot y$	$x + x' \cdot y = x + y$
Consensus	$(x+y) \cdot (y+z) \cdot (x'+z) = (x+y) \cdot (x'+z)$	$x \cdot y + y \cdot z + x' \cdot z = x \cdot y + x' \cdot z$