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ECE 198 JL Worksheet 5: Karnaugh maps

Before you begin today's discussion, you should have read Lecture Notes Section 2.1. Be sure that you are familiar with terminology, such as literals, minterms, maxterms, canonical forms, implicants, prime implicants, and K-maps. Also be sure that you know how to find canonical SOP and POS forms for Boolean functions, and that you know how to use K-maps. To check these skills, you can make up a truth table at random, find a Boolean expression for the function, then check your result by writing a truth table for your expression. Finally, you should know how to convert the two-level logic circuits that one obtains from SOP and POS expressions into NAND and NOR circuits, respectively. As you may recall, the CMOS technology used predominantly in today's semiconductor electronics industry can provide simple implementations of NAND and NOR gates, so being able to map easily into these gates generally helps us to produce smaller designs.

Use the area heuristic—number of literals plus the number of operators, not including complemented literals—to find minimal expressions in today's problems. Choosing a minimal number of prime implicants from a K-map will give you minimal solutions in this sense.

Truth table for Problem 1

A	*B*	*C*	*D*	*V*
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

K-map for Problem 1

V		*AB*			
		00	**01**	**11**	**10**
	00				
	01				
CD	**11**				
	10				

1. From Problem Statement to Digital Circuit

You may have noticed that the carry out of a one-bit adder is a majority function of the two addends and the carry in. In other words, the carry out is a 1 whenever two or more of the inputs are 1s.

Let's design a majority voting unit to calculate the outcome of a vote by a committee of four members, *A*, *B*, *C*, and *D*. For each member, a value of 1 means a yes vote, and a value of 0 means a no vote. Let's use the same convention for the vote result *V(A,B,C,D)*.

Sadly, a committee size of four can result in a tied vote, so let's say that, in such cases, the vote *V* goes with *A*'s vote.

a. Can you write down *V* as a function of *A*, *B*, *C*, and *D* by just thinking the problem through? It's ok if your answer is, "No," but think about the question.

b. Fill in the truth table on the previous page, then look at the truth table to answer these questions:
   i. How many product terms appear in the canonical SOP form of *V*?
      (Note: you do **not** need to write down the canonical SOP form.)

   ii. How many sum terms appear in the canonical POS form of *V*?
      (Note: Again, you do **not** need to write down the canonical POS form.)

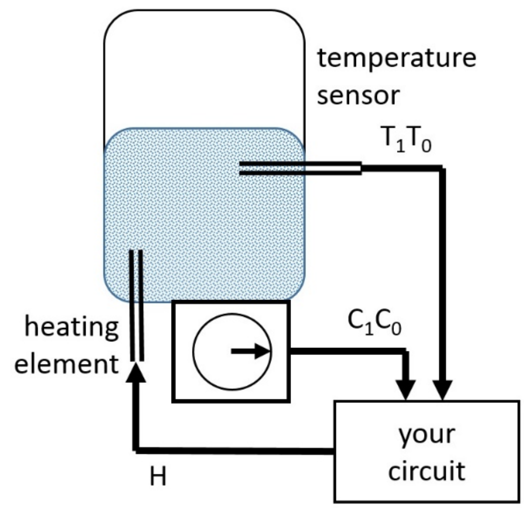
c. Copy the bits from the truth table to the K-map. Be careful with ordering. Then use the K-map to find a minimal SOP expression for *V*.

d. Draw the circuit using AND and OR gates. Then draw the same circuit using NAND gates. For each drawing, you should only need five gates.

2. A Simple Thermal Controller

Prof. Lumetta needs some help with the water heater in his home. The design is as follows. The unit has a control dial that allows the user to set the desired temperature using a two-bit unsigned value C_1C_0. A temperature sensor reads the current water temperature, also as a two-bit unsigned value T_1T_0. A digital circuit, which you must design, then decides whether to turn on a heating element or not. The heating element is controlled by logic signal H, which should be set as follows:

$$H = \begin{cases} 0 & \text{if } T \geq C \\ 1 & \text{if } T < C \end{cases}$$



a. Fill in the truth table below for function H.

C_1	C_0	T_1	T_0	H
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

H

T_1T_0 \ C_1C_0	**00**	**01**	**11**	**10**
00				
01				
11				
10				

b. Copy the bits from the truth table to the K-map above. Be careful with ordering.

c. Use the K-map to find a minimal SOP expression for H.

3. Using K-maps for POS Expressions

a. Fill in the K-maps below for the functions $X = A + C$, $Y = B + D$, and $Z = (A + C)(B + D)$.

X		***AB***			
		00	**01**	**11**	**10**
	00				
	01				
CD	**11**				
	10				

Y		***AB***			
		00	**01**	**11**	**10**
	00				
	01				
CD	**11**				
	10				

Z		***AB***			
		00	**01**	**11**	**10**
	00				
	01				
CD	**11**				
	10				

b. Notice that a sum corresponds to a group of 0s in a K-map, and that by making a product-of-sums (a POS expression), the function we obtain is the union of the groups of 0s in the individual factors in the product (that is, the sums).

Be careful when you use this approach: notice that the literals are complemented from the values that they would have when writing an implicant for an SOP expression with a K-map. For example, by DeMorgan's law, we have $A + C = (A'C')'$. The group of 0s formed by the sum $A + C$ are in the same location as the group of 1s formed by the product $A'C'$.

Use the K-map below to calculate both a minimal SOP expression and a minimal POS expression for *F*.

F		***AB***			
		00	**01**	**11**	**10**
	00	0	0	0	0
	01	1	1	0	0
CD	**11**	0	1	0	0
	10	0	0	0	0

4. Two-bit Thermal Control

Prof. Lumetta has issues. Apparently, the thermal control circuit that you designed for Part 2 today doesn't quite solve the problem. His water heater radiates too much heat. So we need to use a two-bit unsigned output H_1H_0 and keep the heat on at a low level even when the temperature reaches the desired level.

Remember that the inputs provided by the system are a control dial that allows the user to set the desired temperature using a two-bit unsigned value C_1C_0, and a temperature sensor reads the current water temperature, also as a two-bit unsigned value T_1T_0.

Your function *H* should then be given by:

$$H = \begin{cases} \min(3, C - T + 1) & \text{if } T \leq C \\ 0 & \text{if } T > C \end{cases}$$

The use of the minimum function ensures that we do not try to set the heat to 4 when C is 3 and T is 0.

Fill in the K-maps below, then derive minimal expressions for H_1 and H_0. *Hints: H_1 should be quite easy. For H_0, consider both SOP and POS expressions, and use the better of the two.*

H_1

T_1T_0 \ C_1C_0	00	01	11	10
00				
01				
11				
10				

H_0

T_1T_0 \ C_1C_0	00	01	11	10
00				
01				
11				
10				