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Discussion section starts tomorrow!!!

5pm today
ECE ignition
here for all freshman and new transfer students

My office hours this week
Th 1-3 in 2022 ECEB

type: ECE198JL into google
<https://wiki.cites.illinois.edu/wiki/display/ece120>

Alan Turing 1936
universal computation device

Computable
android phone
iPod
Blue waters
Tianlte

Undecidable

Digital convergence
-almost all of you will use digital systems every day

- most solutions are digital
- critical set of skills for every one of you

Why bottom up?

- solid understanding of design and operation
- easier to make effective use and improvements
- our students have been successful based on this model

work load

14 (weekly) homework assignments

(first due next wed)

-->15% of grade

15 (weekly) laboratory assignments (software and hardware) --> 15% of grade

15 (weekly) discussion sections (group work and integration)-->5% of grade

3 midterms

Tuesday 16 September 7-9 pm

Tuesday 14 October 7-9 pm

Tuesday 11 November 7-9 pm

conflict exam 5-7 pm

final exam 25% of grade

Monday 15 December 8-11 am

abstraction in a digital system

an abstraction layer

functions for higher interfaces

black box
many possible
implementations

layers in a computer system
representations
binary digits (bits)
integer representations
Friday (probably some of it anyway)

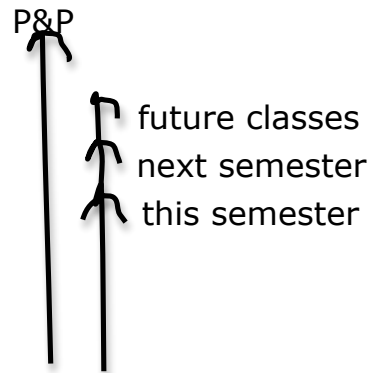
give yourself a hand

Professor Kindratenko's office hours
M: 10am-11am in 2022
W: 10am-11am in 2022
F: 10am-11am in 4034

Lumetta's hours Th 1-3
upstairs at miaZa's (green street)

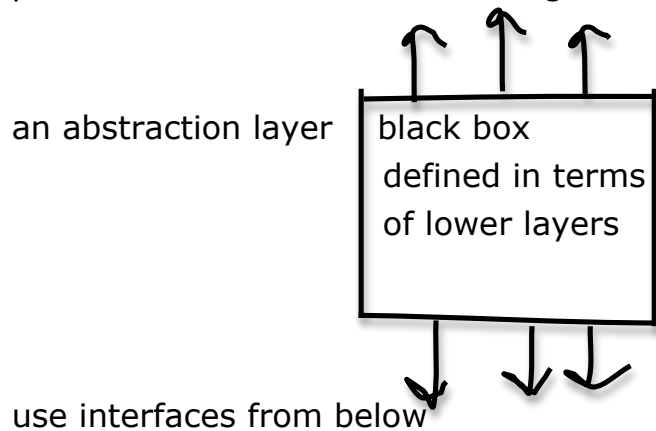
lab 1 is due 8/28 (tomorrow)
see lab document for lab rooms
exams designed as 1 hour (1.5 final); we give 2x time

problems/tasks
algorithms
language
machine/instruction set architecture (ISA)
microarchitecture
circuits
devices



from P&P chapter 1

provide interfaces/functions to higher layers



Problems/tasks/applications

- stated in natural (human) language

- for example: what is the sum of numbers between 1 and 3
6, ∞ , 2, 42

problem: ambiguity in question

time flies like an arrow

a problems can be solved with many algorithms

algorithms is a step by step process to solve a problem

1. definiteness: no ambiguity
2. effective computability: each step simple enough for a computer to execute
3. finiteness: always terminates

an algorithms can be implemented using many languages

- C, C++, Java, Python, and so forth

- 1000s of choices

- we use C to start

 - simple mapping to ISA

 - subset of other language

A program can be executed on many instruction set architectures.

Machine/Instruction set architecture

-interface between software (s/w) and hardware (h/w)

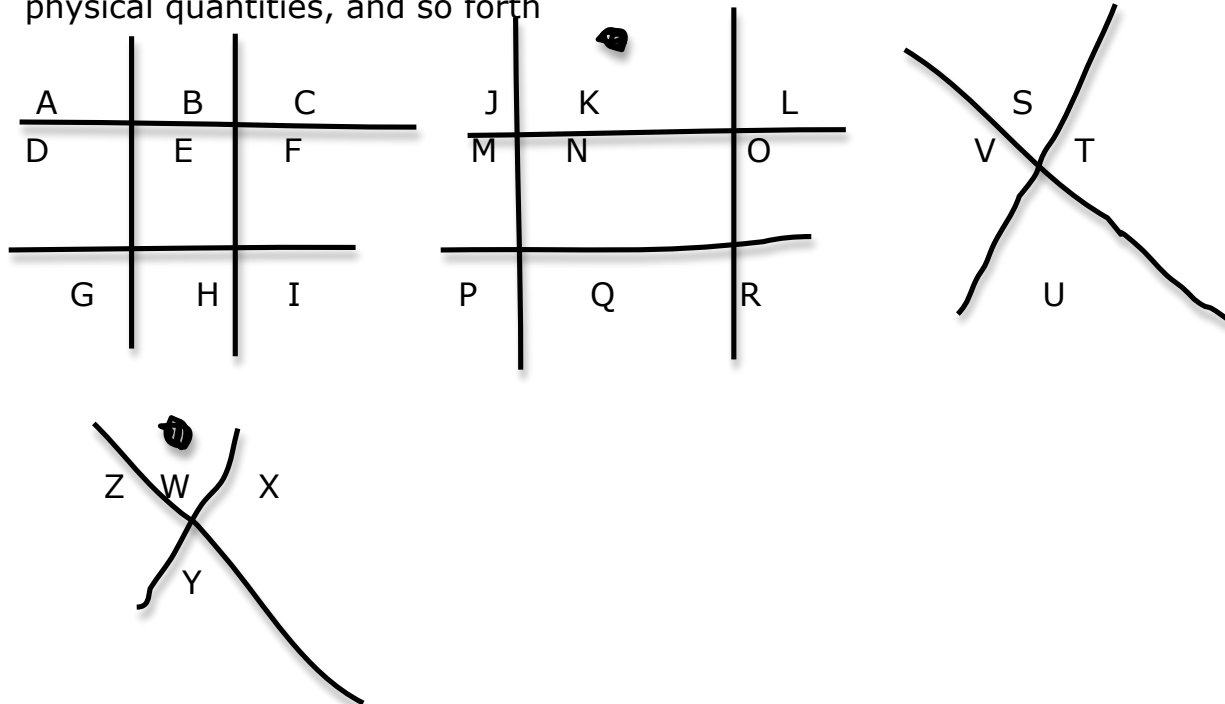
x86-phenom, i5, i7

ARM-cortex, kinetis

PowerPC-many

representations

often useful to represent one type of information with other patterns,
physical quantities, and so forth



what properties does a representation need to be useful?

0 1 2 3 4 5 6 7 8 9

A B C D E ... J

K L M N O ... T

U V W X Y Z

"143"

BED

BOX

VYN

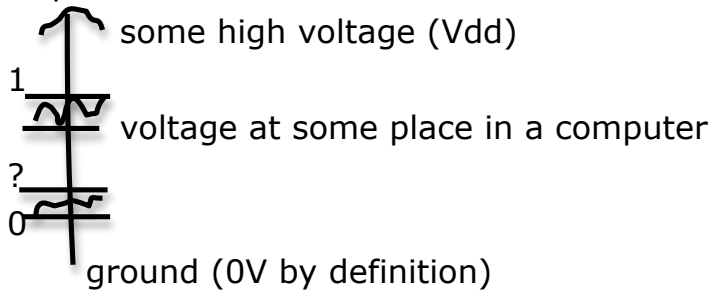
no ambiguity: each pattern represents at most one value

also need the representation to be well defined
all users know representation in advance

BIInary DigiTs (bits)

0 and 1

computers based on electrons



-->one bit

positional/place value

decimal 42

10^2 10^1 10^0
4 2

binary 101010

2^1 2^0
1 0

000000

000001

...

111111

2^6

represent whole #s in range...

0 to 31-->5 bits gives 2^5 patterns

0 to 100? -->7 bits gives $2^7=128$ patterns

Integer representations:

what #s should we represent?

5 bits-->represent which numbers

what makes a representation good?

unsigned representation

"good" representations

2's complement

recall: we can use bits to represent anything

useful examples: integers, real numbers, human language characters
(alphabet, digits, punctuation)

important: computer does not "know" meaning. bits are bits.

-->Lab 1 grades are out

run 'svn update' in your lab1 folder and check grade.txt file

-->Lab 2 is out. it is due next Th.

-->HW 1 is due next W. In lecture

-due at start of lecture at back of room

-legible, stapled, and so forth (see instructions)

-put in your section's envelope

-will disappear ~9:30 (no credit afterwards)

-solutions up in 4034

no lecture Monday. Lab is open...

if access control allows you to enter...

Integer Representations:

What numbers should we represent?

We want to use numbers to do arithmetic.

What if we use 5 bits to represent 100...131?

124+131

100+101

-->represent numbers close to 0

Let's start with a human representation

base 2 from math

17₁₀-->10001₂

42₁₀ → 101010₂

But there is no "blank" bit.

--> use a fixed number of bits

--> add leading 0s.

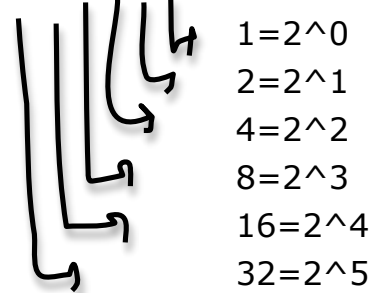
--> unsigned representation

What range of numbers is representable with the N-bit unsigned representation?

$[0, 2^N - 1]$

How do we convert unsigned to decimal?

0 1 1 0 0 1



$$1 + 8 + 16 = 25$$

$a_5 a_4 a_3 a_2 a_1 a_0$

represents: $a_5 \cdot 2^5 + a_4 \cdot 2^4 + a_3 \cdot 2^3 + \dots + a_0 \cdot 2^0$

What about decimal → unsigned conversion?

Start with even number → $a_0 = 0$

odd number → $a_0 = 1$

Subtract out a_0 , divide by two

Repeat until I get 0

$$137 \quad a_0 = 1 \quad (137 - 1) / 2 = 68$$

$$68 \quad a_1 = 0 \quad (68 - 0) / 2 = 34$$

34 $a_2=0$ $(34-0)/2=17$
 17 $a_3=1$ $(17-1)/2=8$
 8 $a_4=0$ $(8-0)/2=4$
 4 $a_5=0$
 2 $a_6=0$
 1 $a_7=1$

What about negative numbers?

Use a minus sign?

$-11000_2 = -24_{10}$

+ $--> 0$

- $--> 1$

$-->$  sign + magnitude (unsigned)

signed-magnitude representation

What does 1000 represent in 4 bit signed magnitude? 0

0000 $--> 0$

Having multiple zero patterns makes hardware (like adders) more complicated?

What makes a representation good?

-efficient, not unary

5 000001111

10 111111111

-effective for use

-use same hardware for more than one representation

unsigned addition

A	B	Sum
0	0	00
0	1	01
1	0	01
1	1	10

01110 (14₁₀)
+00100 (4₁₀)
10010 (18₁₀)

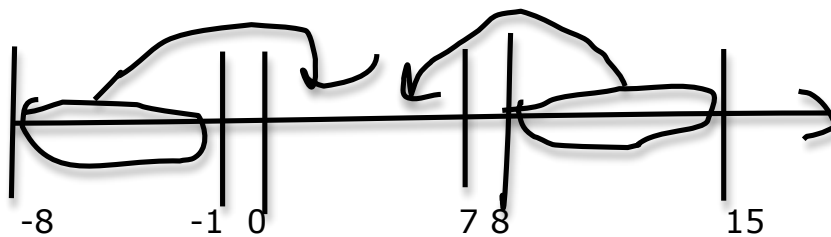
10111 (23₁₀)
+01010 (10₁₀)
100001 (33₁₀)
x00001 (1₁₀)
fixed width representation

overflow (answer; not representable)
 2^N

Answer (sum) is always correct mod 2^N ?

Similar to remainder.

numbers A and B are equivalent mod k if and only if $A=B + pk$ for some integer p



Remember: open lab on Mondays 2022ECEB

Other labs: 440 DCL, 520 DCL, 057 Grainger

turn in labs by computer

next due 7pm Thursday

Today 2-5 pm between Everitt and Engineering Hall, International Engineering Fair.

2's complement

overflow

logic operations

challenge: prove that the expression:

(carry out)XOR(Carry into MSB) is equivalent to the 2's complement

Overflow definition given in lecture (today).

Recall: unsigned addition uses modular arithmetic

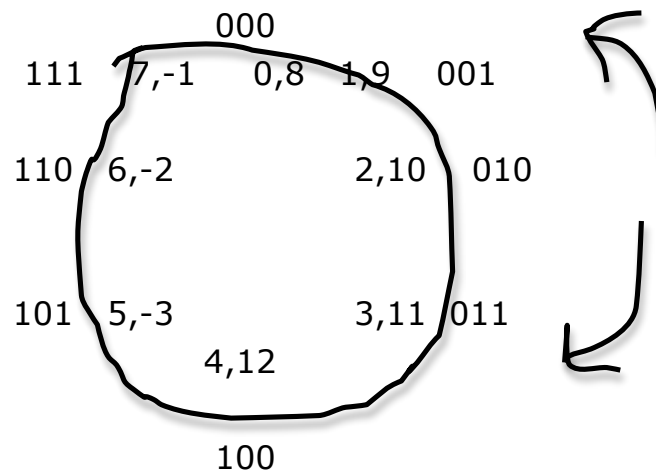
$\text{adder}(A,B) = \{A+B \text{ if } A+B < 2^N\}$

$\{A+B-2^N \text{ if } A+B \geq 2^N\}$

so

$(\text{adder}_N(A, B) = A+B) \bmod 2^N$

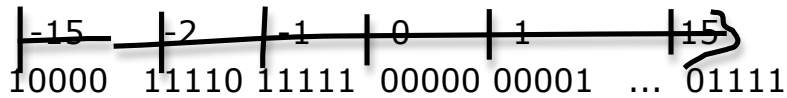
Another way to think about modular arithmetic: draw a circle



subtract to count counter clockwise

add to count clockwise

signed integer representations



$\text{adder_N}(A, B) = (A + B) \bmod 2^N$

given $m \geq 0$, $2^{(N-1)} > k > 0$, can we find $p > 0$ such that

$(-k + m = p + m) \bmod 2^N$

if so, we can use p 's representation to represent $-k$

subtract m from both sides

$(-k = p) \bmod 2^N$

$(2^N = 0) \bmod 2^N$

$(2^N - k = p) \bmod 2^N$

let $p = 2^N - k$, equation is true.

$2^{(N-1)} < p < 2^N$ (all unused patterns!)

$-2^4 + 1 = -15$

How do you calculate $2^N - k$?

100000 ($N=5$)

- k

Instead, notice

$(2^N - 1) + 1 = 2^N$

11111 ($2^N - 1$ for $N=5$)

- k

k complemented (0s become 1s, 1s become 0s)

42

00101010

11010101 + 1 = 11010110

$2^N - 1 - k$

11010001

k

$$-2^8 + 2^7 + 2^6 + 2^4 + 2^0$$

$$-2^7 + \text{rest of bits}$$

$$-2^8 + 2^7 = -2^7$$

Better to define

100000...0

N bits

$$\text{as } + 2^{(N-1)}$$

$$\text{--> OR } -2^{(N-1)}$$

$$10111 \text{ } (-9)$$

$$+10011 \text{ } (-13)$$

$$1 \text{ } 01010 \text{ } (10)$$

$$01000$$

$$+1$$

$$01001$$

$$01000 \text{ } (8)$$

$$+01010 \text{ } (10)$$

$$10010$$

$$01101$$

$$+1$$

$$01110$$

$$A$$

$$+B$$

$$C$$

we have overflow if

1. addends are negative, sum is non-negative

OR 2. addends are non-negative, sum is negative

overflow=

$(A \text{ AND } B \text{ AND } (\text{NOT } C)) \text{ OR } ((\text{NOT } A) \text{ AND } (\text{NOT } B) \text{ AND } C)$

Operators are Boolean logic functions

(true=1, false=0)

AND-the all function

returns value 1 iff all inputs operands are 1

OR-the any function.

returns 1 iff any input operand is 1

NOT-logical complement

(NOT 1) is 0

(NOT 0) is 1

XOR-the odd function

returns 1 iff an odd number of input operands are 1.

Truth table maps input values to output values.

		A*B, AB	A+B
A	B	A and B	A OR B
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

		A' or A_
A		NOT
0		1
1		0

A B $A \oplus B$, AXORB

0 0 0

0 1 1
1 0 1
1 1 0

Logical completeness
fixed and floating point
representation taxonomy
next week C

ASCII
parity
hamming codes
book and notes

Recall:

A
+B
—
C

2s complement

overflow iff $A_{n-1} + A_{n-1}C_{n-1}$

AND=all

NOT=complement

OR=any

XOR=odd

representation taxonomy

bits

unsigned integers	signed integers	real integers	text	vegetables
				ASCII UNICODE
unsigned binary	signed magnitude	2s complement	real numbers	
16 bit unsigned		16 b 32	floating point	
data type			IEEE single prec	
			IEEE double prec	

$A \oplus B \oplus C$

0 0 0 0
 0 0 1 1
 0 1 0 1
 0 1 1 0
 1 0 0 1
 1 0 1 0
 1 1 0 0
 1 1 1 1

$$A \oplus B = B \oplus A$$

Commutative

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

Associative

We can generalize also to sets of bits

$$A = a_3 a_2 a_1 a_0$$

$$B = b_3 b_2 b_1 b_0$$

$$C = AB$$

$$c_3 = a_3 b_3$$

$$c_2 = a_2 b_2$$

...

Logical completeness

How many functions can we define on N bits of input?

$$C = f(A, B)$$

How many choices of f?

$$A \ B \ f(A, B)$$

$$0 \ 0 \ c_0$$

$$0 \ 1 \ c_1$$

$$1 \ 0 \ c_2$$

$$1 \ 1 \ c_3$$

$$2^4 = 2^{2^2} \text{ functions}$$

with 3 bits of input, how many functions?

$$256 = 2^8 = 2^{2^3}$$

With N bits?

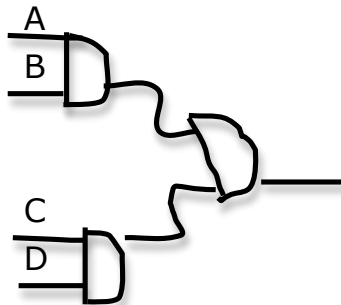
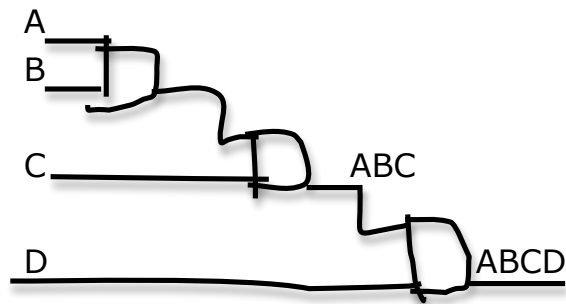
2^{2^N} (big!)

Why do we only talk about AND, OR, NOT?

claim: with enough 2-input ANDs, 2-input ORs, and NOTs, I can compose any logic function on any number of variables.

Proof: by construction

in other words, given a function, I'll show you how to build it.



we can build AND gates with any number of inputs

one 1

A_BC_

one AND gate and up to (# inputs) NOTS

A B C f

0 0 0 0

0 0 1 0

0 1 0 1

0 1 1 0
1 0 0 0
1 0 1 0
1 1 0 0
1 1 1 0

A B f
0 0 0
0 1 1 A_B
1 0 1 AB_
1 1 0

A_, A', (NOT A)

fixed and floating point

3.1415

10^{-1} , 10^{-2} , 10^{-3}

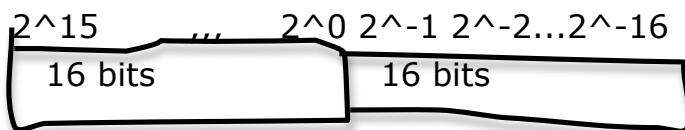
decimal point

11.0010010001

2^{-1} , 2^{-2}

binary point

fixed point



Avogadro's number

6.02252×10^{23}

$10^3 \approx 2^{10}$

so ~ 80 bits will work

Planck's constant

6.626×10^{-27}

so ~ 90 bits

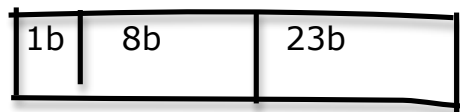
170 bits fixed point!

Avogadro's number

Planck's constant

+6.626*10⁻²⁷

sign + leading digit + fraction + exponent



sign exponent mantissa (fraction)

$-1^{(\text{sign})} \cdot 1.(\text{mantissa}) \cdot 2^{(\text{exponent}-127)}$

EX C Program

```
#include <stdio.h>
```

```
#define PI=3.1416 f
```

```
int main
```

```
{
```

```
/* declare variables */
```

```
float pi=PI;
```

```
/* print inc pi */
```

```
printf("pi=%f\n",pi);
```

```
return 0; /*exit*/
```

```
}
```

```
#include <stdio.h>
#define PI 3.1416f

int main()
{
    /* declare variables */
    float pi = PI;

    /* print message */
    printf("pi=%f\n", pi);

    /* exit */
    return 0;
}
```

a^2+b+c

<type> <name>;

```
{int}
{real}
{text}
```

2's complement

```
{int}
{short int 16 bit}
{long int 32 bit}
{long long 64 bit}
-->add unsigned
```

unsigned int-

```
float -32 IEEE
double -64 IEEE
```


constant

char -8 bits

```
int a=10;
```

```
int b;
```

```
int c;
```

```
b=20;
```

```
c=a+b;
```

```
c+b
```

```
int x, a;
```

```
float y, z;
```

```
z=x+y;
```

```
-->z=(float) x+y;
```

operators

=

```
a+b=c+d-->X
```

```
-->a=c+d
```

* / + - %

~-not

&-and

|-or

^-xor

<<-left shift

>>-right shift

```
unsigned int h,f,g
```

```
h=f&g
```

0-->false

~0=true

!--bit

&&-and

||-or

>-less

>=-less or equal

<-greater

<=greater or equal

==equal

!=not equal

(a>b) &&(a<c)

a	b	a	c
---	---	---	---

a=a+b;

a+=b;

a=a-b;

a-=b;

a^=b

a=a^b

a++

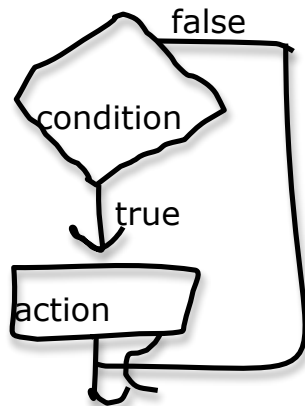
b--

if(condition)

{

action

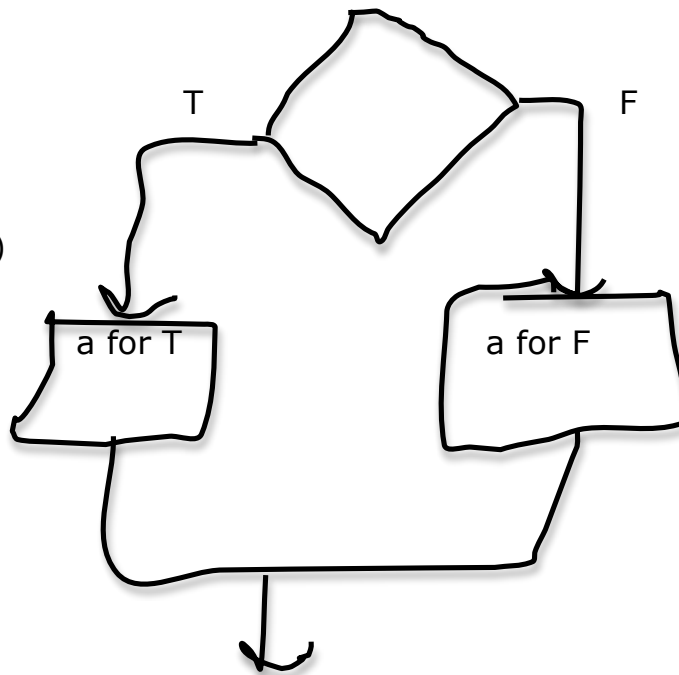
}



$x = |x|;$
 $x = \{x \text{ if } x \geq 0\}$
 $\quad \{-x \text{ if } x < 0\}$

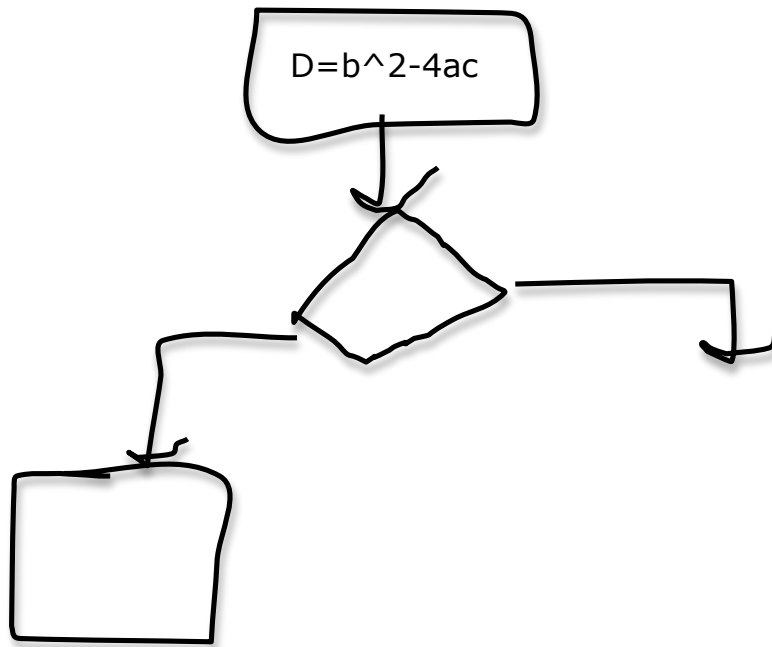
if $((x > 5) \&\& (x < 25))$

if (condition)
 action for true
 else
 action for false
 {
 }



$ax^2 + bx + c = 0$
 $a = b^2 - 4ac$
 if $D > 0$
 $x_{1,2} = (-b \pm \sqrt{D}) / 2a$
 if $D = 0$
 $x = -b / 2a$





```
/* quadratic eq. */
#include <stdio.h>
#include <math.h>

int main()
{
    float a,b,c;
    float D;
    float x1, x2;

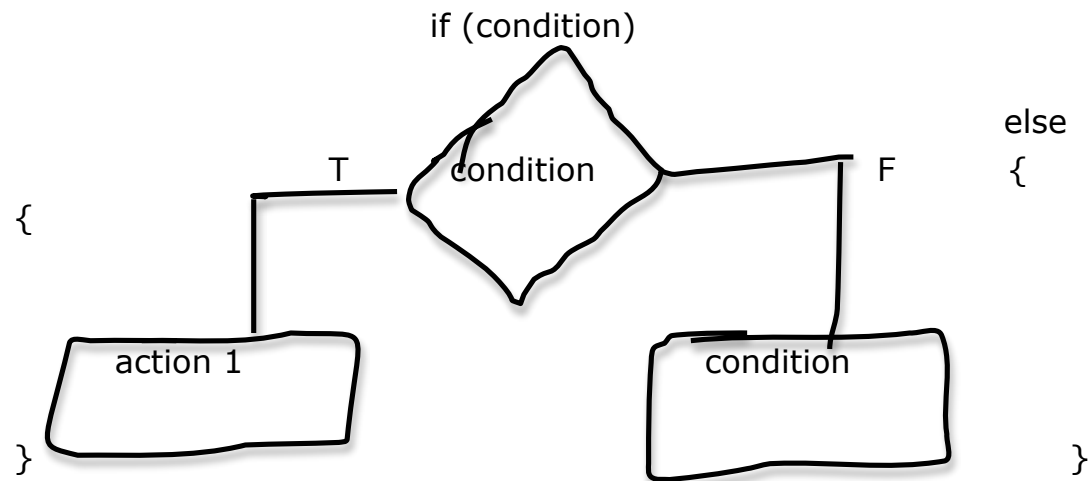
    printf("enter a b c: ");
    scanf("%d %f %f", &a, &b, &c);

    D=b*b-4*a*c;

    if(d>0)
    {
        x1=(-b+sqrt(D))/(2*a);
        x2=(-b-sqrt(D))/(2*a);
    }
```

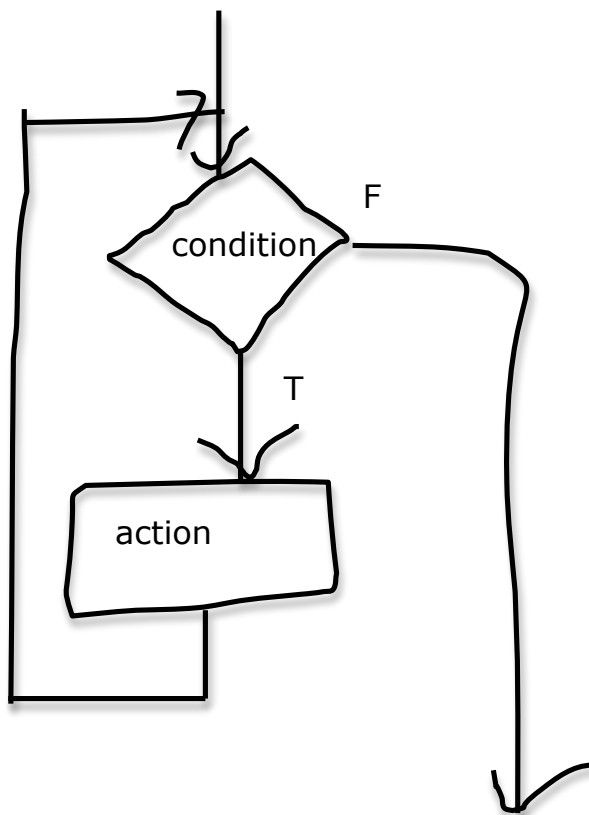
```
else if (D==0)
{
x1=-b/(2*a);
}
```

Sunday 12-2pm Review Session



```

while()
{
  action;
}
  
```



```

1. int a=0;
2. while(a<10)
{
3. print("a=%d\n",a);
4. a=a+1;
}
5.

```

```

a++;
a+=1;

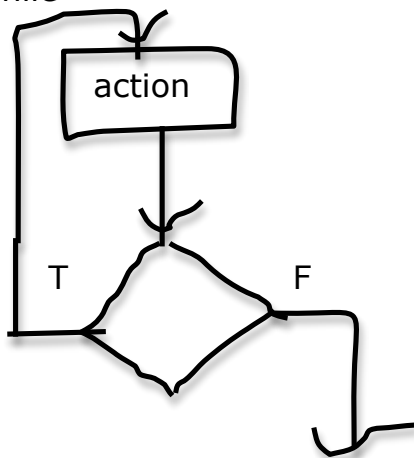
```

```

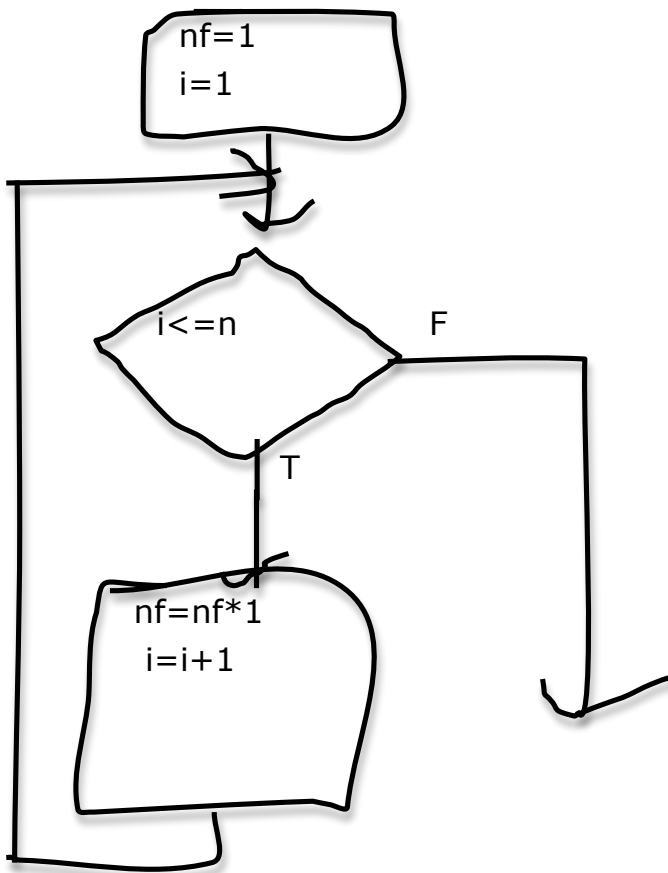
1. a=0
2. 0<10  1<10          a<10
3. print  print
4. a=1    a=2 ...    a=10
5.

```

do while

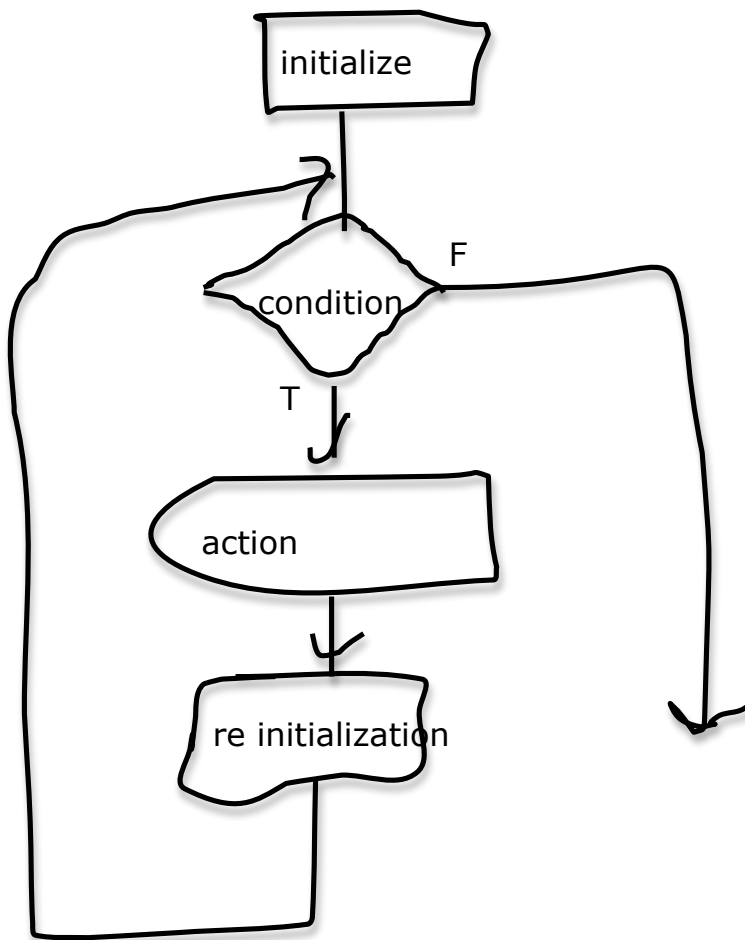


$$n! = 1 * 2 * 3 * \dots * (n-1) * n = \prod_{i=1}^n i$$



```
int n=10;  
int nf, i;  
nf=i=1;
```

```
while(i<=n)  
{  
  nf=nf*i;  
  i=i+1;  
}
```

```
for(initialization; condition; re initialization)
{
  action;
}
```

```
int n = 10;
int i, nf=1;
```

```
for(i=1; i<=n; i+1)
{
  nf=nf*i;
}
```

n

$$x^n = x * x * \dots * x = \prod_{i=1}^n x$$

```
int i;
int x=5;
int xn=1;
for(i=0; i<n; i=i+1)
{
  xn=xn*x;
}
```


$$x^0 = 1$$

```
int i = 1;
for ( ; condition; re initialization)
```

```
for([initialization]; [condition]; [re initialization])
```

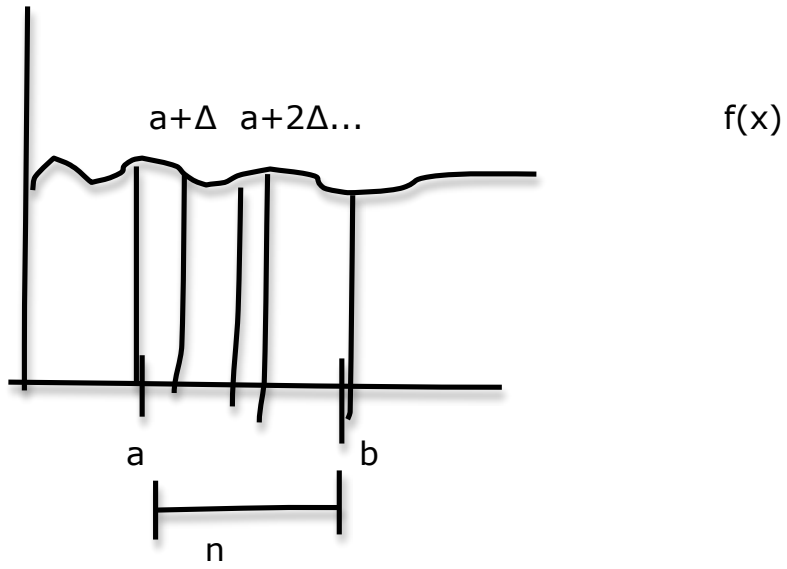
```
if(condition)
  action;
else
  break;
-->don't do this
```

```
for(...)
{
  continue;
...
...
}
```



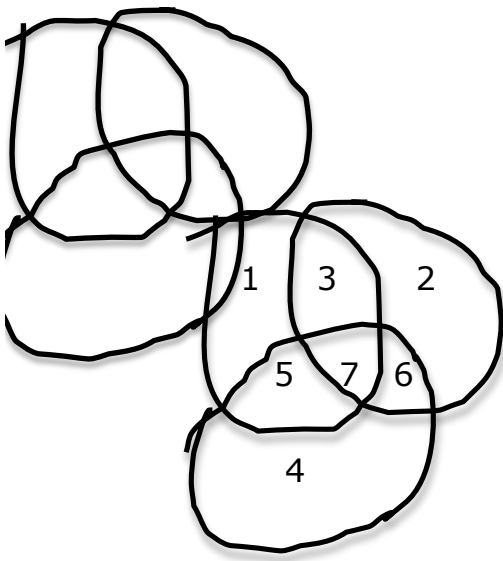
-->don't do this either

$$\int_a^b f(x) dx \approx \sum_{i=0}^n (b-a)/n * f(a + (b-a)/n * i)$$



$$(a-b)/n$$

```
fint=0;
for(i=0;i<n;i++)
{
fint +=...;
}
```



signed

1 1 x x

1 1 x x

1 1 0 x x

0 1 1 1 1 1 1

d d d p d p p

c1 c2

1 1 1

1 1

1

c4

1 1 1 -->7

1 1 1 1 1 1 1

even 0

odd 1

abstraction

levels of transformation

bit representation

hex notation

codes, error detection and correction

binary-->decimal

arithmetic operators

fixed and floating point

Boolean operators

C programming:-variables

-operators

-conditional, iterative constructs

-problem solving with control structures

python-->electrons

1's complement

1111-->0000

1000-->0111

1 1 0 1

1 0 0 0

(1) 0 1 0 1

sign exponent mantissa

$-1^x 2^{(x-127)}$

IEEE 755

$2^{(x-63)}$

$(x-63)$

0001110011

0 0011100 11

$-1^0=1$

$2^{(28-63)}=$

$11=1/2+1/4=3/4$

754

$2^{29}-127$

$1.75=1+3/4$

AND

OR

NOT

XOR=

XOR-->NOR

XNOR:

110

011

010

OR

00 0

01 1

10 1

11 1

AND

00 0

01 0

10 0

11 1

1 AND 1=1

NOT

1 0

0 1

logical

!=

||

&&

!

bitwise

~

while()

{

fool();

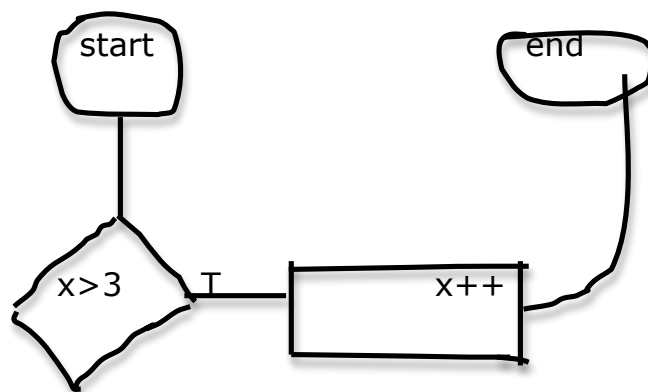
}

do

{

fool();

}while();



if(x > 3){

x++

}

Conflict exams in 4070

Others announced tonight

My office hours: Tu 2-4pm@miaZa's
(THIS WEEK ONLY)

Hamming codes

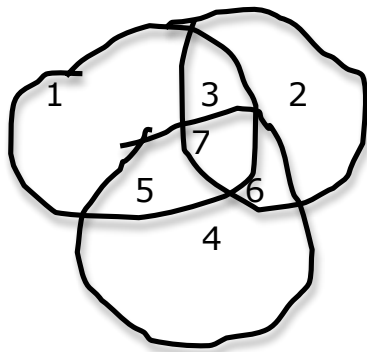
IEEE Floating point

Halting problem

Parity Bits

of 1's in a set of bit's

even parity-->even # of 1 bits



P4 P2 P1

0 0 1

0 1 0

0 1 1

1 0 0

1 0 1

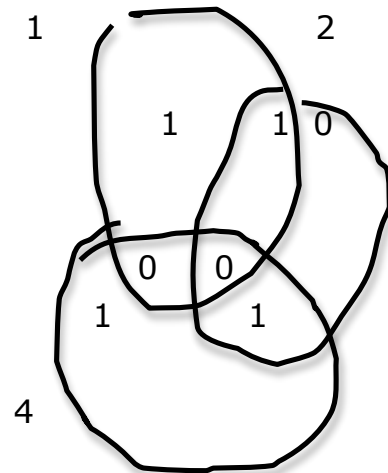
1 1 0

1 1 1

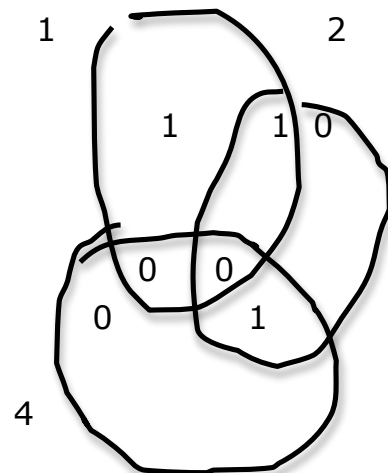
$1, 2^n - 1$

d3 d2 d1 p4 d0 p2 p1

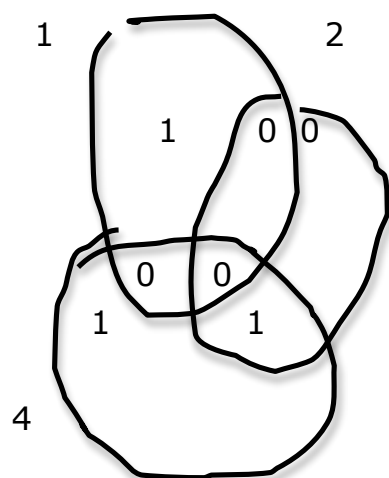
x7 x6 x5 x4 x3 x2 x1



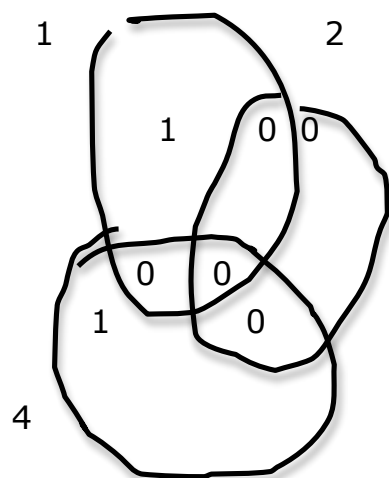
C4 C2 C1
0 0 0



C4 C2 C1
1 0 0



C4 C2 C1
0 1 1



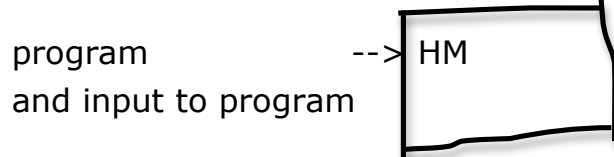
C4 C2 C1
1 0 1

Halting problem
This sentence is false.

$$f(n) = -f(n-1)$$

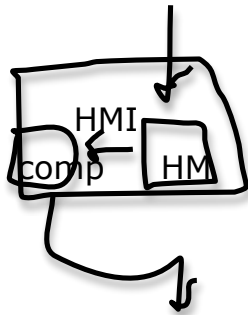
$$\lim f(n) = ?$$

$n \rightarrow \infty$



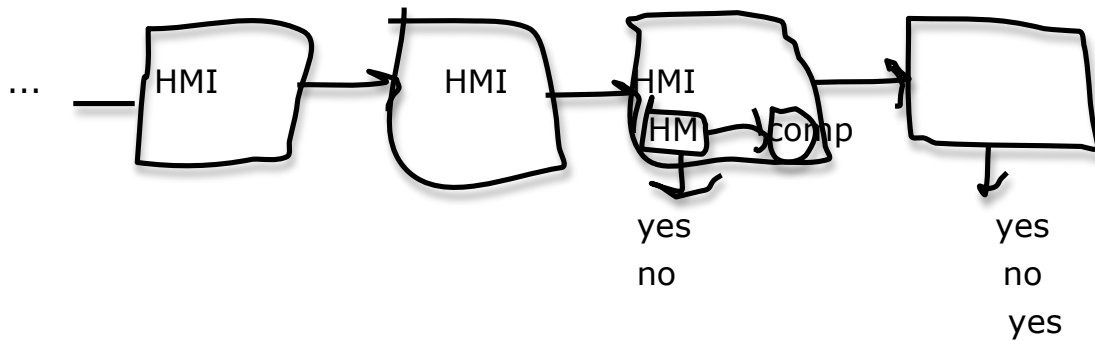
--> halt or not halt

some program and its input



output complement of HM

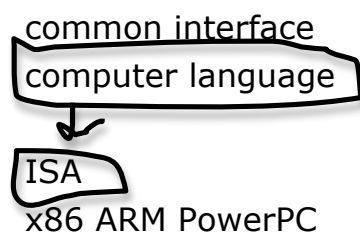
run forever if answer of HM is yes



Writing truth tables

for and while loops

layers of abstractions



-->different implementations

IEEE floating point

1101.11011

1.10111011×2^3

	exponent+127	mantissa
1=neg	8 bits	
0=pos		23 bit
0	10000010	101110110...0
$1 + 1 \times 10^{-9} = 1$		
$-1 + (1 + 1 \times 10^{-9}) = 0$		
$(-1 + 1) + 1 \times 10^{-9} = 1 \times 10^{-9}$		

-3.125

$0.125 \times 2 = 0.250 \rightarrow 0$

$x2 = 0.500 \rightarrow 0$

$x2 = 1.000 \rightarrow 1$

-1=0

1 11.001000...0

$1.10010000 \times 2^1 \rightarrow$ exponent bits

10000000

$(-1)^{(\text{sign})} \times 1.(\text{mantissa}) \times 2^{(\text{exponent}-127)}$

$2^0 \ 2^{-1} \ 2^{-2} \dots$

- HW 2 re grade requests are due next Monday
- pick up your exam after this lecture
- exam re grade requests are due next monday too

transistors

functions on bits (gates)

two voltage levels=1bit

circuits

devices

electrons

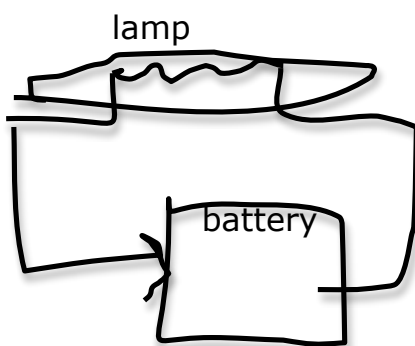
how can we build gates?

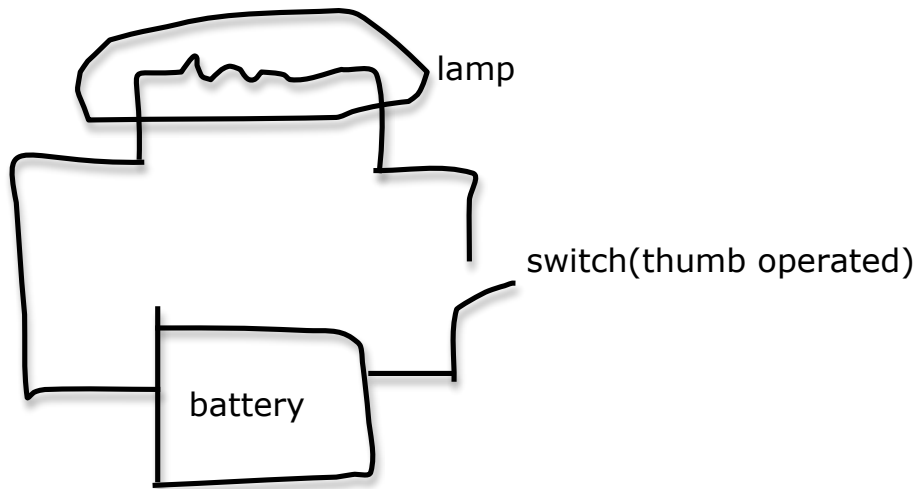
transistors

gates

optimizing Boolean expressions

	F14	F12	F12
	(407 people)	(27 people)	MT2
mean	83	86	69
median	85.5	91	78
st dev	11	14	15
max	100	100	90





transistor=voltage controlled switch
 -->really cool thing!

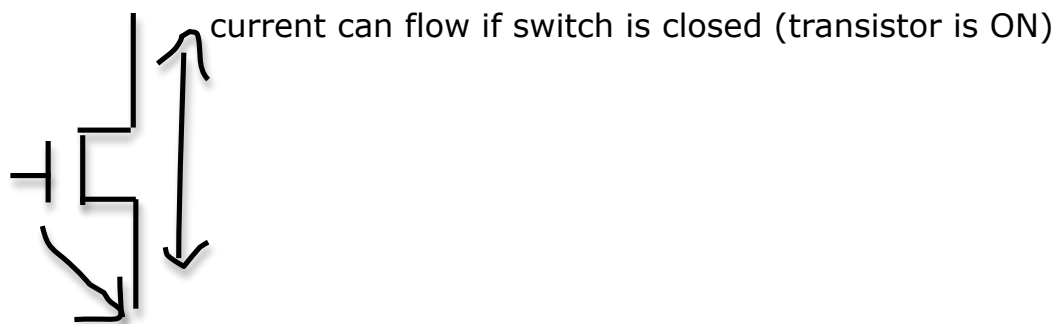
transistors can be used to control other transistors

Bragging break

MOSFETS

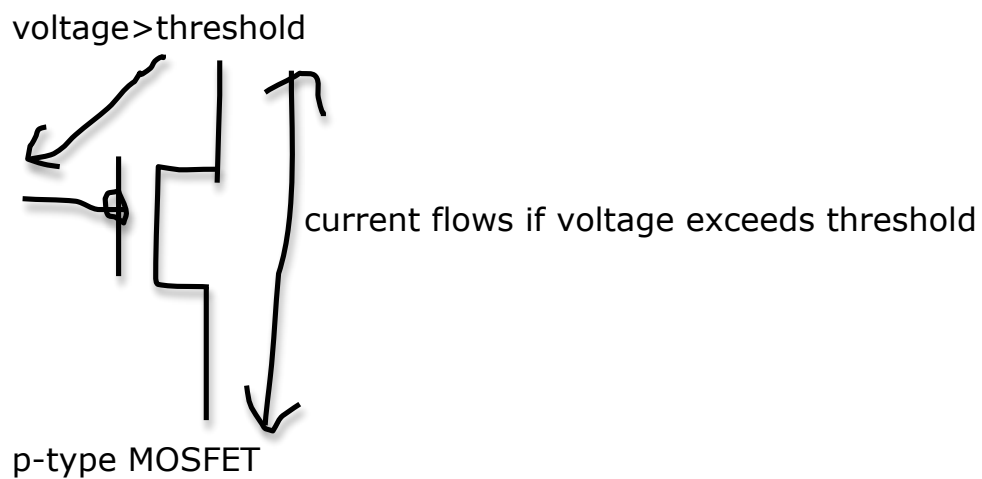
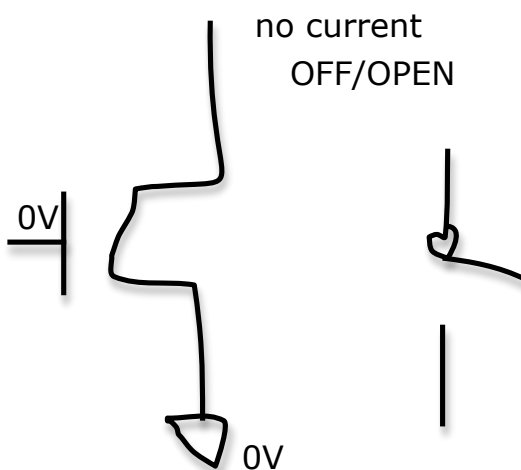
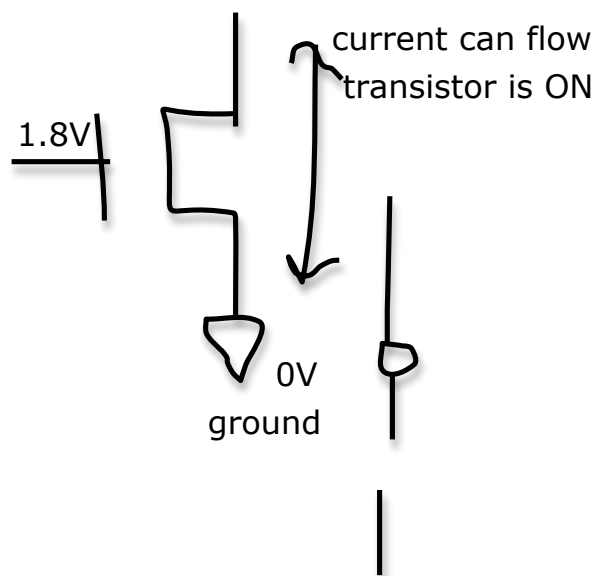
FET-Field effect transistors

MOS-metal oxide semiconductors

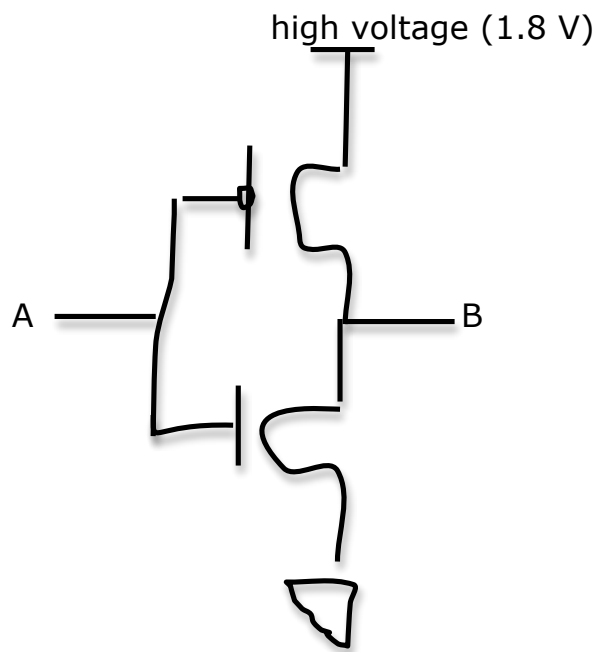


is voltage > threshold

n type MOSFET



Gates



A	B
0 0V	1 1.8 V
1 1.8V	0 0V



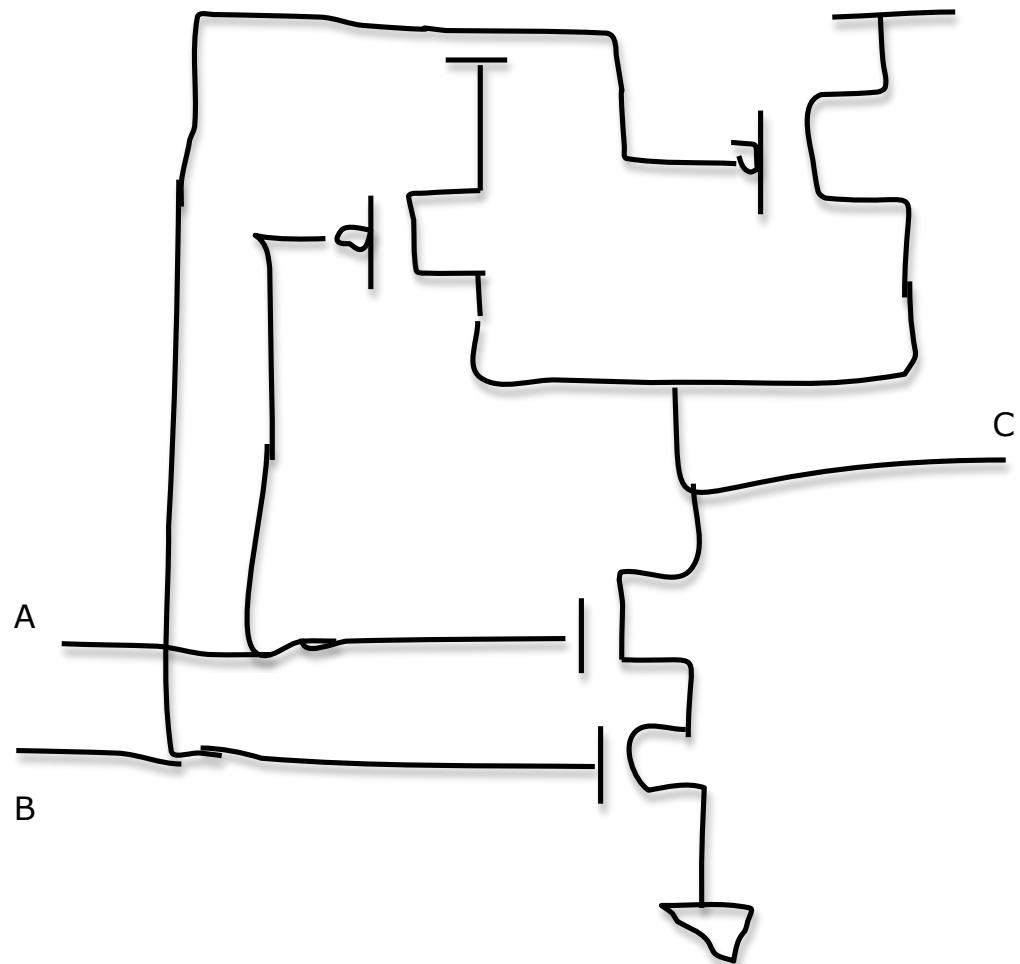


1 1 0

C


$$A \ B \ A+B \ (A+B)_{-} \ AB \ (AB)_{-}$$

0 0 0	1	0	1
0 1 1	0	0	1
1 0 1	0	0	1
1 1 1	0	1	0



A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

A	B	C	F
0	0	0	0
0	0	1	0

0 1 0 0
0 1 1 0
1 0 0 0
1 0 1 1 AB_C
1 1 0 1 ABC_
1 1 1 1 ABC

x $F = AB_C + ABC_ + ABC$

y $F = AB + AC$

z $F = A(B + C)$

the answer (which is best?) depends on our metric?

area/size/cost

performance/speed

power/energy consumption

a heuristic for area

Count literals (A,A_,B,B_,C,C_)

add the number of operations (not including literal complements)

why? each input line-->2 transistors

operators-->2 transistors

form x-->9 literals, 4 operators=13

form y-->4+3=7

form z-->3+2=5

a heuristic for delay

find the maximum number of gates from any input to any output

why? each gate takes time to switch (gate delay)

Finish sample heuristics

terminology

k-maps

two level logic

[Pareto optimality]

recall: area, performance, power

example function

X: $F = ABC + AB_C + ABC$

Y: $F = AB + AC$

Z: $A(B + C)$

area heuristic

-count literals

(variables of complements)

X-->9+4=13

Y-->4+3=7

Z-->3+2=5<--best

delay heuristic

-find maximum number of gates from any input to any output

X-->3 (NOT, AND, OR)

Y-->2 (AND, OR)

Z-->2(OR, AND)

Y, Z best

Terminology

Literal-variable or its complement

Sum-several terms OR'd together

Product-several terms AND's together

Minterm on N inputs-product in which each variable or its complement appears once

Maxterm [on N inputs]-sum in which each variable or its complement appears exactly once

Examples:

$A+B_{-}+C_{-}$

$A+B+C_{-}$

not $AB+C$

A B C $A+B_{-}+C_{-}$

0 0 0 1

0 0 1 1

0 1 0 1

0 1 1 0

1 0 0 1

1 0 1 1

1 1 0 1

1 1 1 1

sum of products (SOP)-sum (OR) of products (AND) of literals

example: $AB+BC$

but not: $A(B+C)+D$

product of sums (POS)-product of sums of literals

Example: $(A+B)(B+C)$

but not: $(A+BC)D$

canonical SOP: sum of minterms

canonical: unique way to write expressions

canonical POS: product of maxterms

implicants

A function G is an implicant of a function F if $G \rightarrow F$.

in other words every row in G's truth table with output 1 also has output 1 in F's truth table

in digital design, implicants must be products of literals

A F G

0 0 1

1 1 1

$F \rightarrow G$

first step in simplification

given an implicant G of F

can we remove any of the literals from G and get another implicant

$F = ABC_{\bar{}} + AB_{\bar{}}C + ABC$

$ABC_{\bar{}}$

implicant of F

A	B	C	F	$BC_{\bar{}}$	$AC_{\bar{}}$	AB
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	0
1	0	1	1	0	0	0
1	1	0	1	1	1	1
1	1	1	0	0	1	1

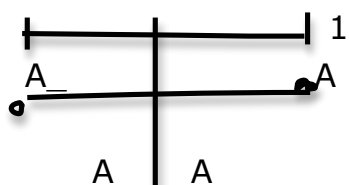
AB

$AC_{\bar{}}$

implicant of F

if we cannot remove any literals from G, G is a prime implicant of F

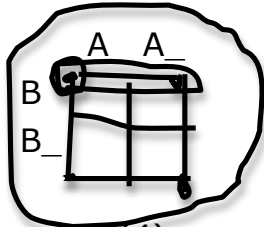
Karnaugh maps (K-maps)



A 1D hypercube

How many implicants on one variable? $A, A_{\bar{}}, 1$

F 0 1



points(4)

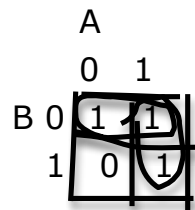
(AB, AB_, A_B, A_B_)

edges(4)

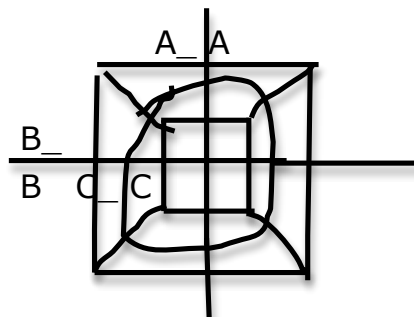
A, A_, B, B_

face(1)

1



$X = A + B_$



$3^3 = 27$

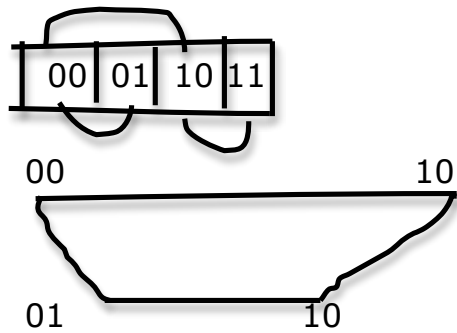
points(8)

edges(12)

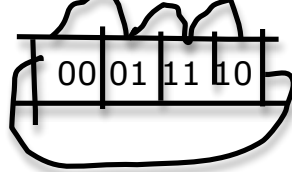
faces(6)

cube(1)

Veitch



Karnaugh map



	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$C_in = A B + B C_in + A C_in$$

	00	01	11	10
0	1	1	0	1
1	0	1	0	0

If some implicant is the only prime implicant covering some input combination, we say that the implicant is essential.

$$F = B_C_ + A_B$$

Exam re grade requests are due NOW.

Problem statement-->truth table, k-map-->boolean expression(SOP, POS)
-->circuit

		yz			
		00	01	11	10
wx	00	1	1	0	0
	01	1	1	0	0
	11	0	1	1	1
	10	0	0	0	1

		yz			
		11	00		
wx	11	0	0		
	11	0	0		
	01	1	1		
	00	0	1		

$$f(w, x, y, z) = w'y' + wyz' + wxz$$

implicant: all legal loops

prime implicant: all biggest loops

essential prime implicant: prime implicant must be there-biggest loops that do not include cells covered by another loop

min SOP

		yz			
		11	00		
wx	11	0	0		
	11	1	1		
	11	0	1		
	11	0	0		

$$y' + w'x + xz'$$

					yz
wx	0	1	0	0	
	0	1	1	1	
	1	0	1	0	
	0	0	1	0	

					yz
wx	1	0	1	1	
	1	0	0	1	
	1	0	1	0	
	0	0	1	1	

$$A + wxz + wzy'$$

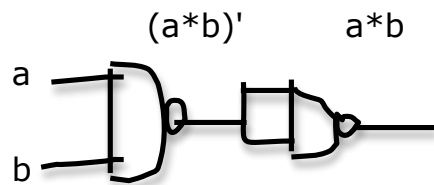
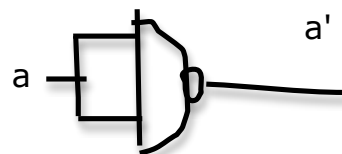
$$A + xy'z' + wxz$$

2-level circuit

{AND, OR, NOT}

{NAND}

$$(a*a)' = a'$$



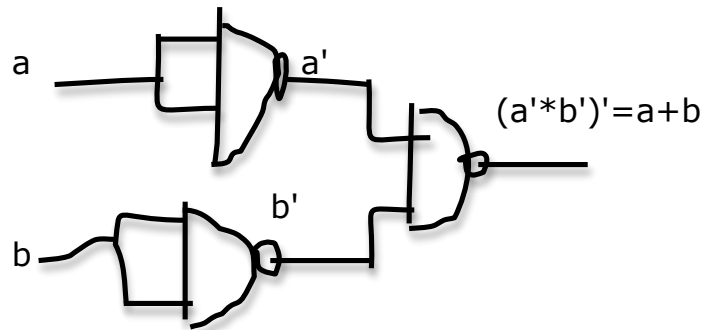
OR

$$(a*b)' = a' + b'$$

a	b	(a*b)'	(a'+b')
0	0	1	1

0	1	1	1
1	0	1	1
1	1	0	0

$$(a'b')' = a + b$$

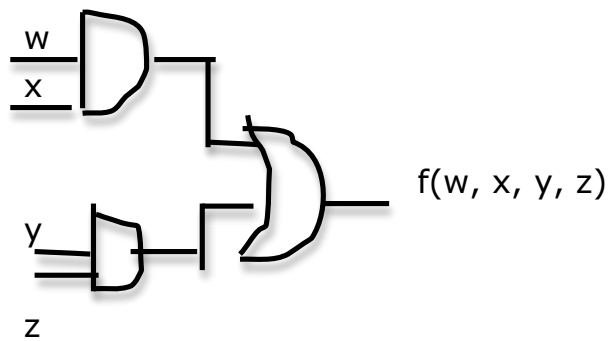


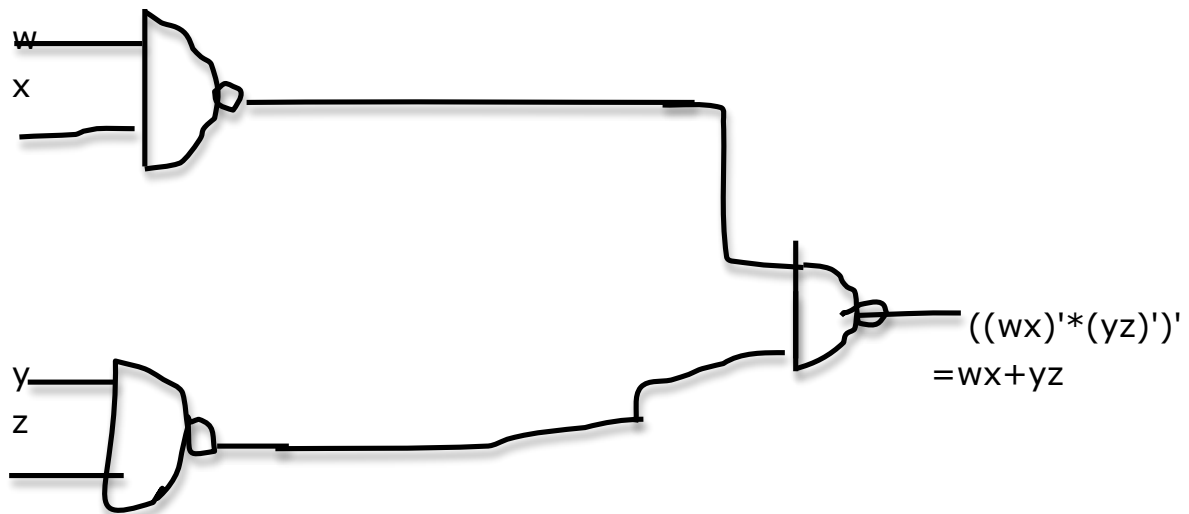
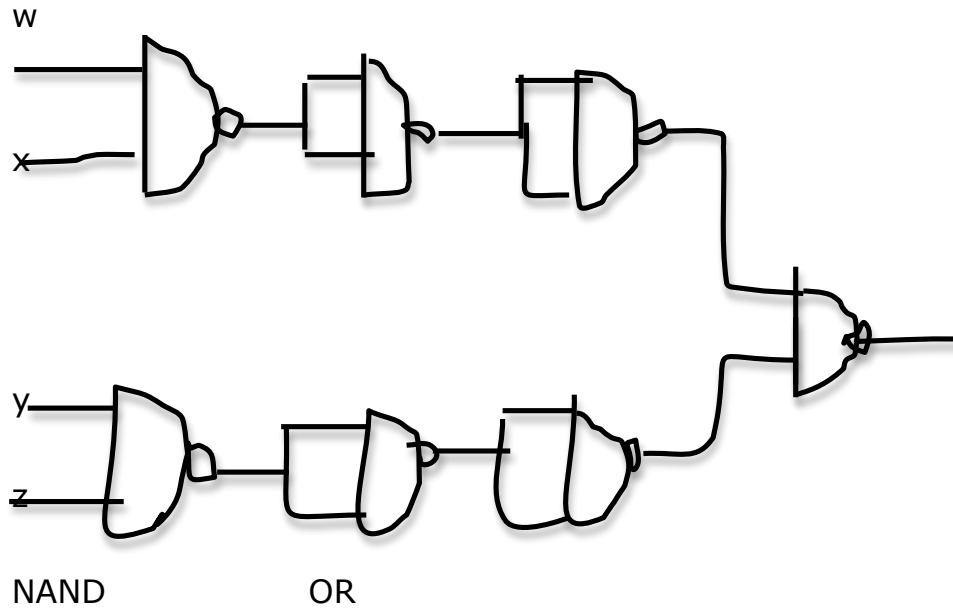
$$f(w, x, y, z)$$

$$= wx + yz$$

$P \downarrow D$

\cup





$$f(w,x,y,z) = (w+x)(y+z)$$

POS

b D

OR-AND

NOR

		yz			
		00	10	11	10
wx	00	1	1	0	0
	01	1	1	1	1
	11	1	1	0	1
	10	1	1	0	0

$$(x + y')(w' + y' + z')$$

$$0 + 1' = 0$$

		yz			
		00	01	11	10
wx	00	1	0	0	1
	01	1	1	0	0
	11	0	1	1	1
	10	1	0	0	1

$$(x + z')(w + x' + y')(w' + x' + y + z)$$

		yz			
		00	01	11	10
wx	00	1	0	0	1
	01	1	1	0	0
	11	0	1	1	1
	10	1	0	0	1

x-->don't care

Lab 5 is due on Friday in lecture. All office hours from 9/25 to 10/3 will be in 4022.

Pick up lab 6 kit on Th, F in office hours

dec digits BCD

0 0000

1 0001

...

9 1001

1010

...

1111

A B C f

0 0 0 0

0 0 1 1

0 1 0 0 1 01 x

0 1 1 0 1 10 x

1 0 0 0

1 0 1 1

1 1 0 0

1 1 1 1

AB

00 01 11 10

C 0 0 1 x x

1 0 1 1 0

0 1 1 0
0 1 1 0

$F = A'B + BC$ 00

$t = A'B + BC + AB'C'$ 01

$F = B$ 10

$F = B + AC'$ 11

A B Sum S Cout

0 0 0 0 0

0 1 1 1 0

1 0 1 1 0

1 1 1 0 1

Cn Cn-1 C2 C1

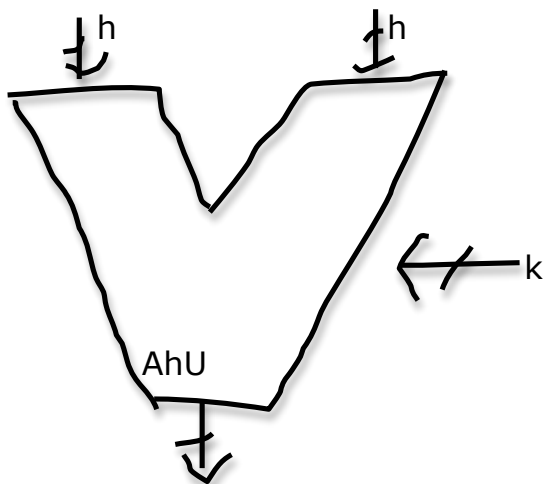
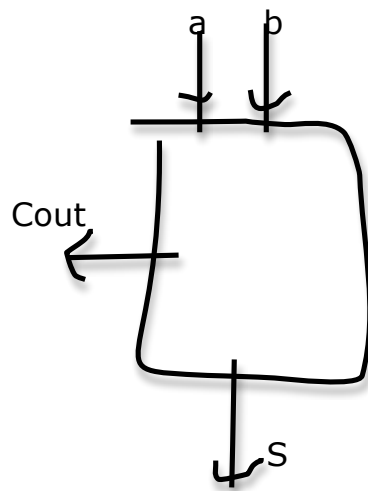
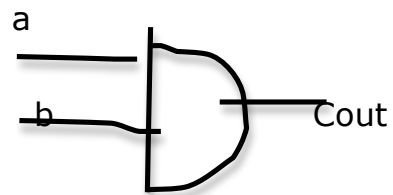
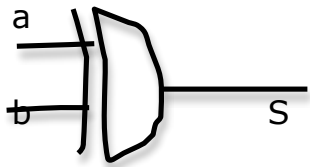
an-1 an-2...a2a1a0

+ bn-1 bn-2 b2b1b0

Sn-1 Sn-2 S1 S0

$S = a' b + a b' = a \oplus b$

$Cout = ab$



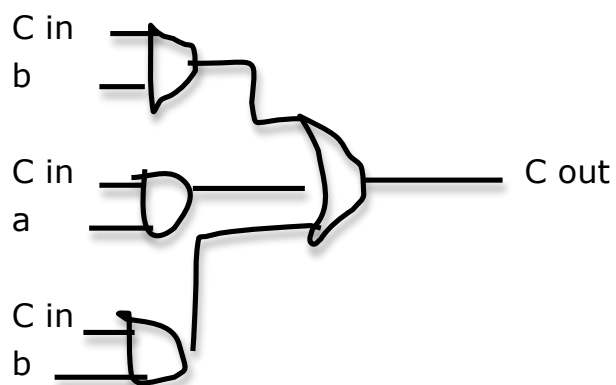
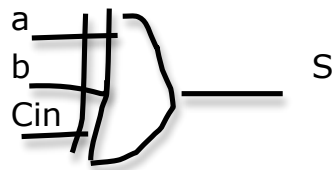
h

ab
00 01 11 10
Cin 0 0 1 0 1
1 1 0 1 0

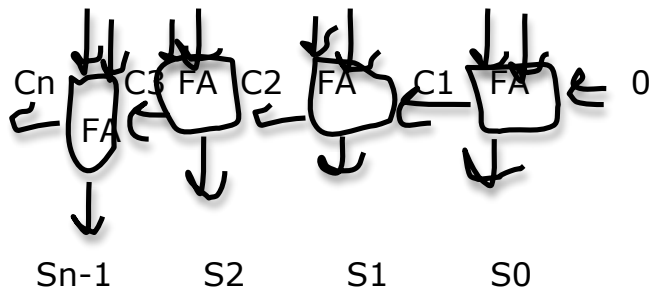
$$S = C_{in} a' b' + C_{in}' a' b + C_{in} a b + C_{in}' a b' = C_{in} \oplus a \oplus b$$

ab
00 01 11 10
Cin 0 0 0 1 0
1 0 1 1 1

$$C_{out} = C_{in} b + C_{in} a + a b$$

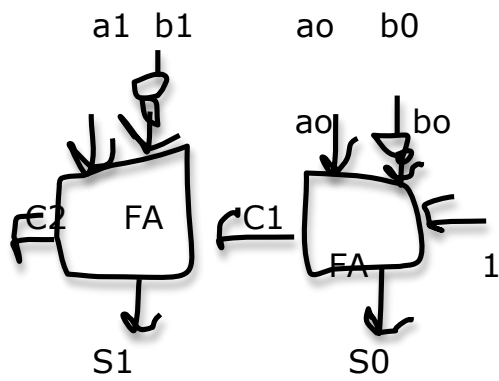


$a_{n-1} b_{n-1} a_2 b_2 \quad a_1 b_1 \quad a_0 b_0$

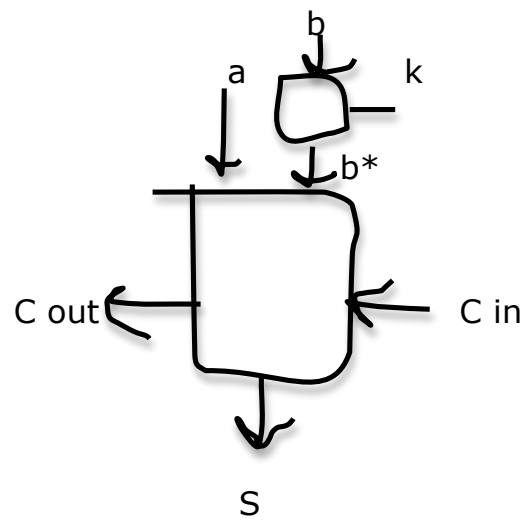


$$A - B = A + (-B)$$

$$\text{NOT}(B) + 1$$



$$S = A - B$$

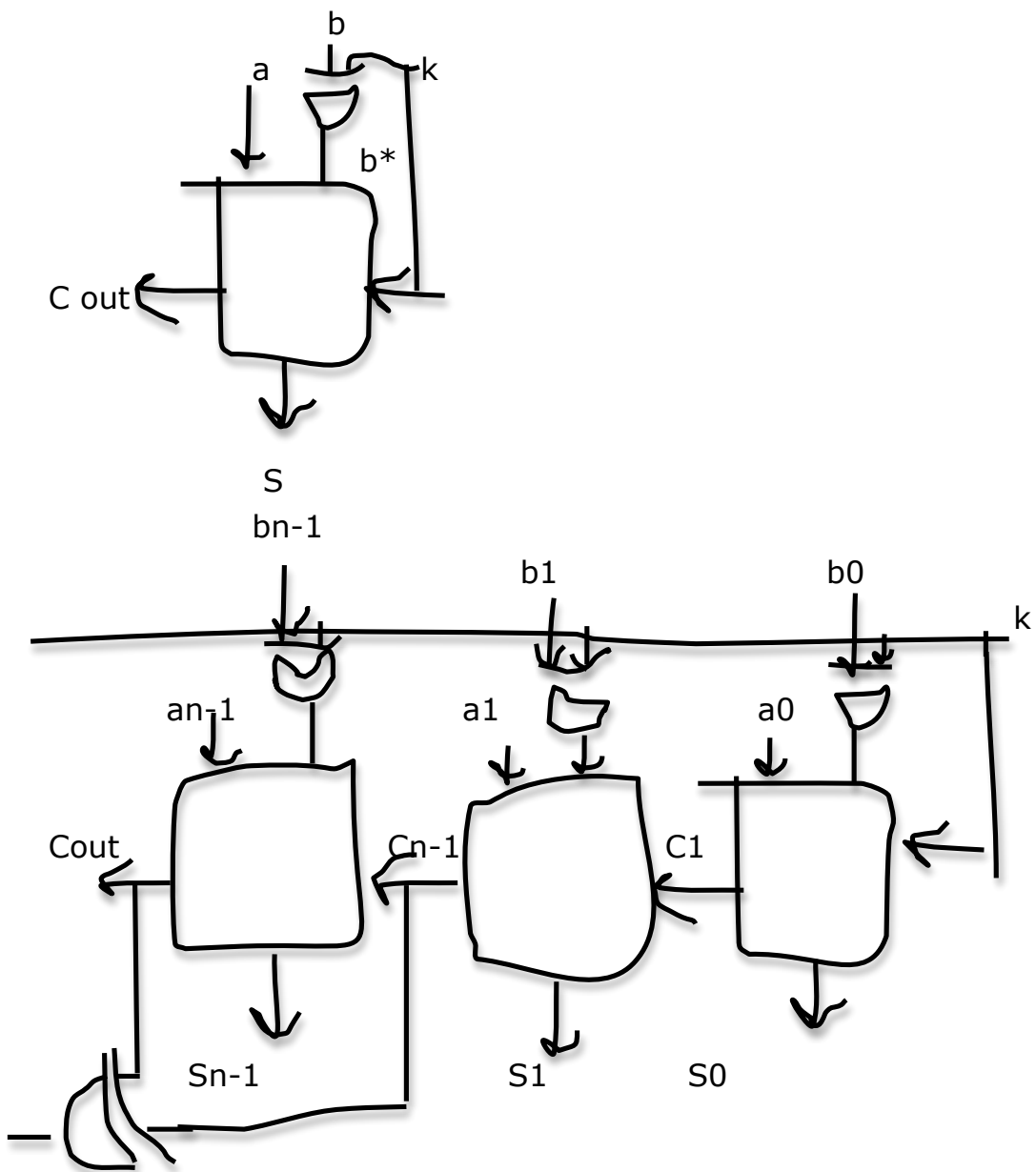


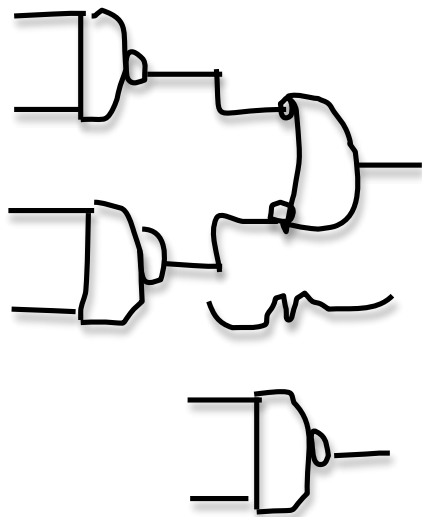
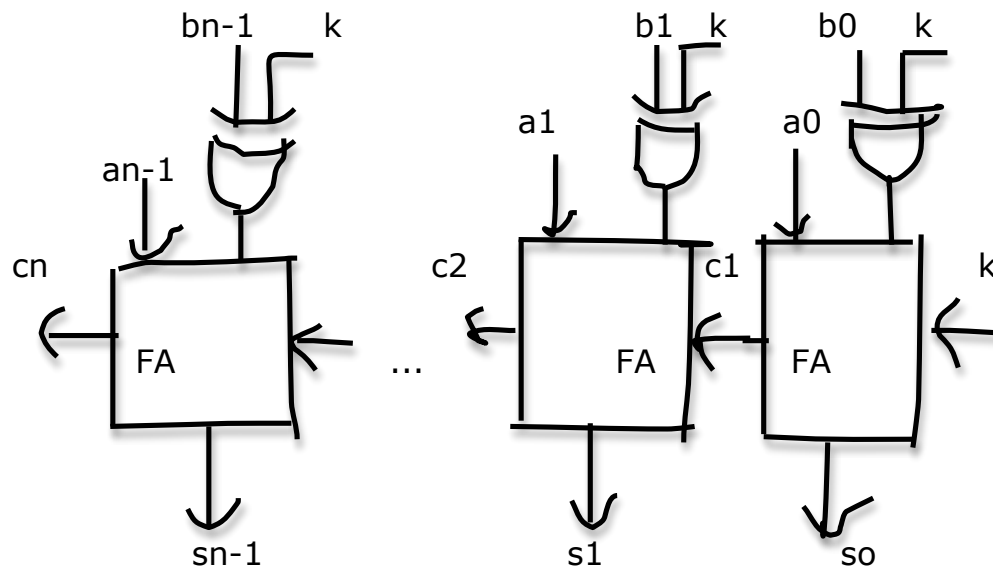
$$\begin{matrix} b & k & b^* \\ 0 & 0 & 0 \end{matrix}$$

1 0 1
0 1 1
1 1 0

k b*
0 b
1 b'

$$b^* = b \oplus k$$





$$a' + b' = (a \ b)'$$

k=0, ADD

k=1, SUB

$$F = \begin{cases} \{A \text{ plus } B \\ \{A \text{ plus } B \text{ plus } 1 \\ \{A \\ \{A \text{ minus } B \end{cases}$$

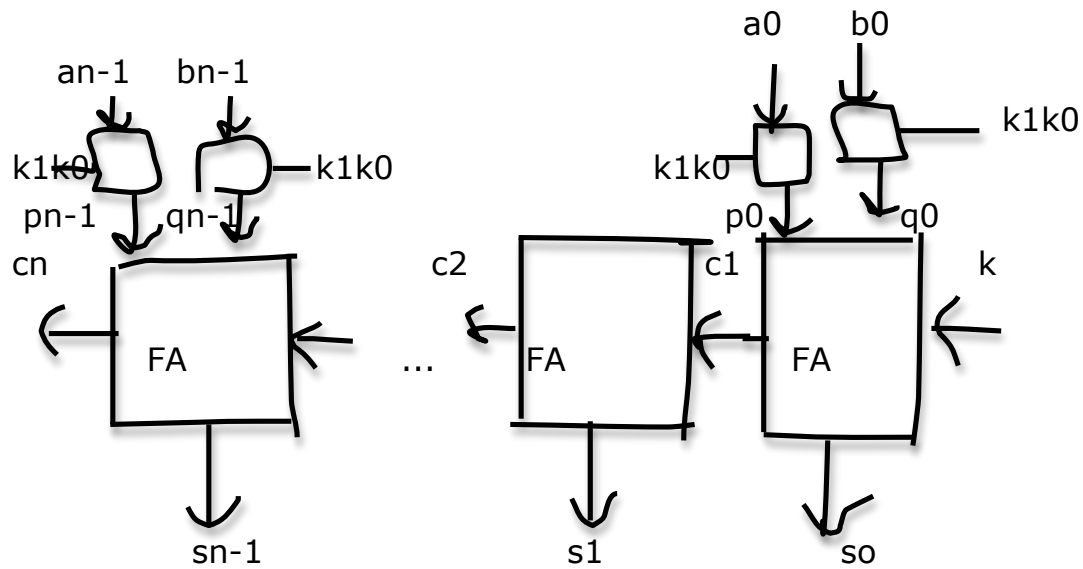
K1 K0

0 0

0 1

1 0

1 1

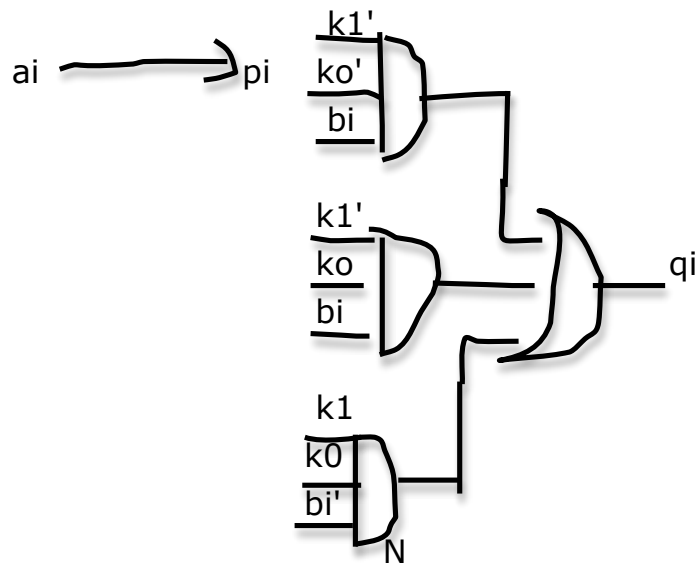


k1	k0		p_i	q_i	c_o
0	0	A+B	a_i	b_i	0
0	1	A+B+1	a_i	b_i	1
1	0	A	a_i	0	0
1	1	A-B	a_i	b_i'	1

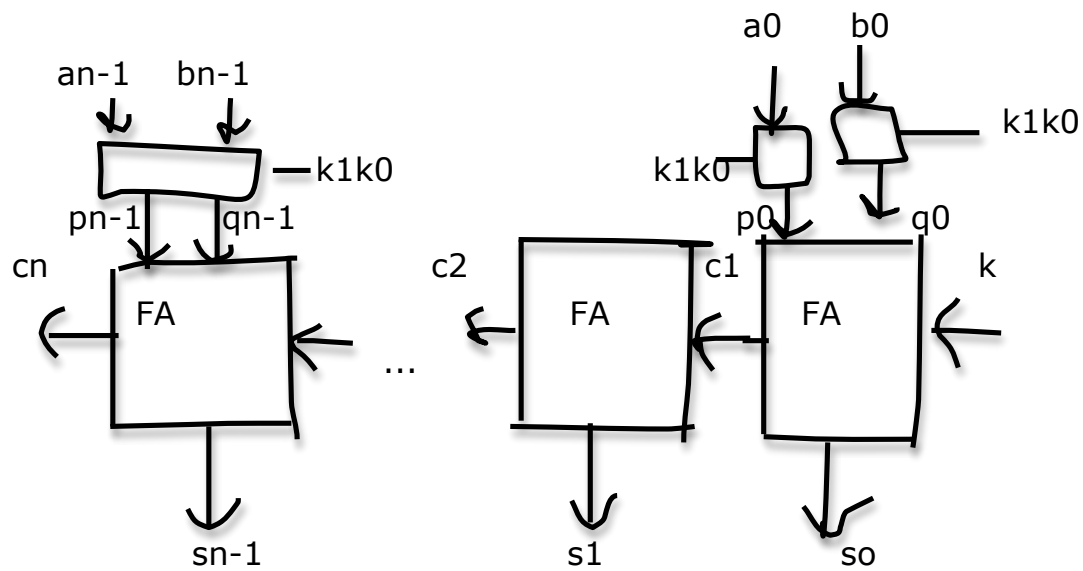
$p_i = a_i$

$q_i = k_1'k_0'b_i + k_1'k_0b_i + k_1k_0b_i'$

$c_o = k_0$



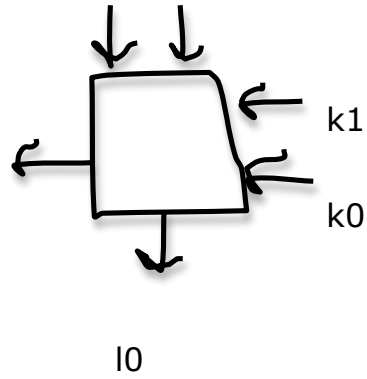
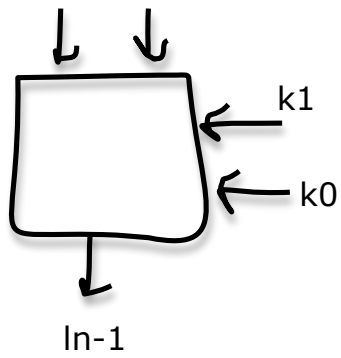
$p(ai, bi, k1, ko)$
 $q(ai, bi, k1, ko)$



$F = \{A \text{ AND } B$
 $\{A \text{ OR } B$
 $\{A \text{ NAND } B$
 $\{0$

$a_{n-1} \quad b_{n-1}$

$a_0 \quad b_0$



$I(a_i, b_i, k_1, k_0)$

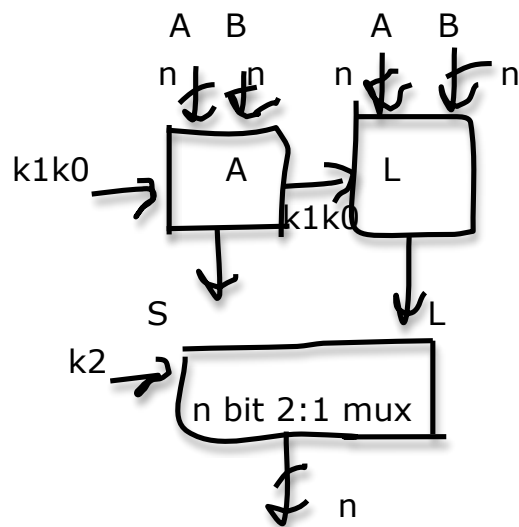
	AND	OR	0	NAND
	00	01	11	10
aibi 00	0	0	0	1
01	0	1	0	1
11	1	1	0	0
10	0	1	0	1

$$I = a_i b_i k_1' + a_i k_1' k_0 + b_i k_1' k_0 + a_i' k_1 k_0' + b_i' k_1 k_0'$$

k2 k1 k0

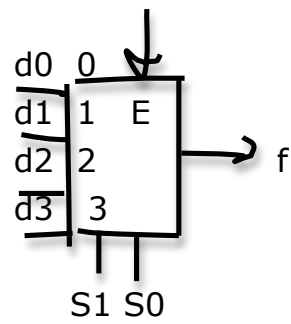
0 arithmetic

1 logical

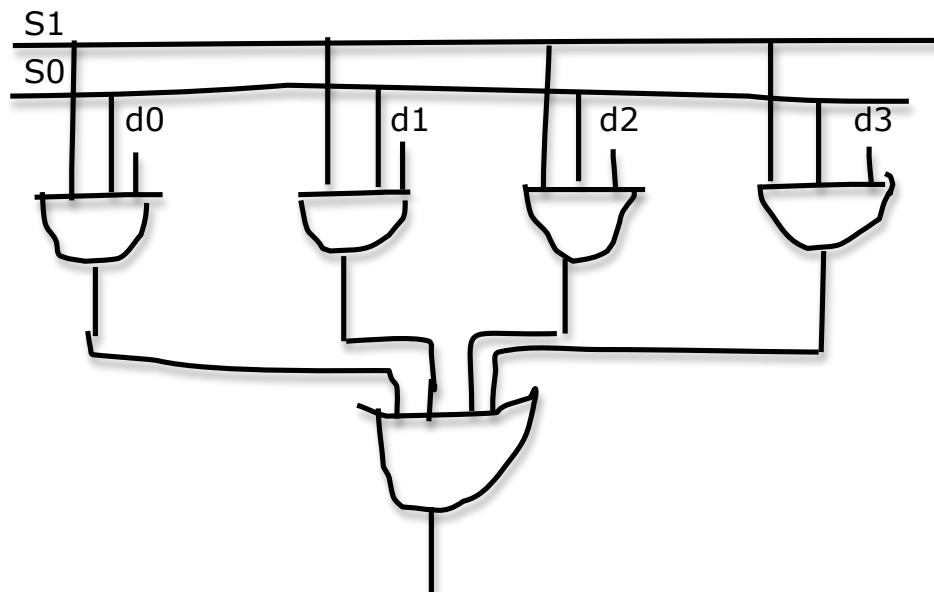


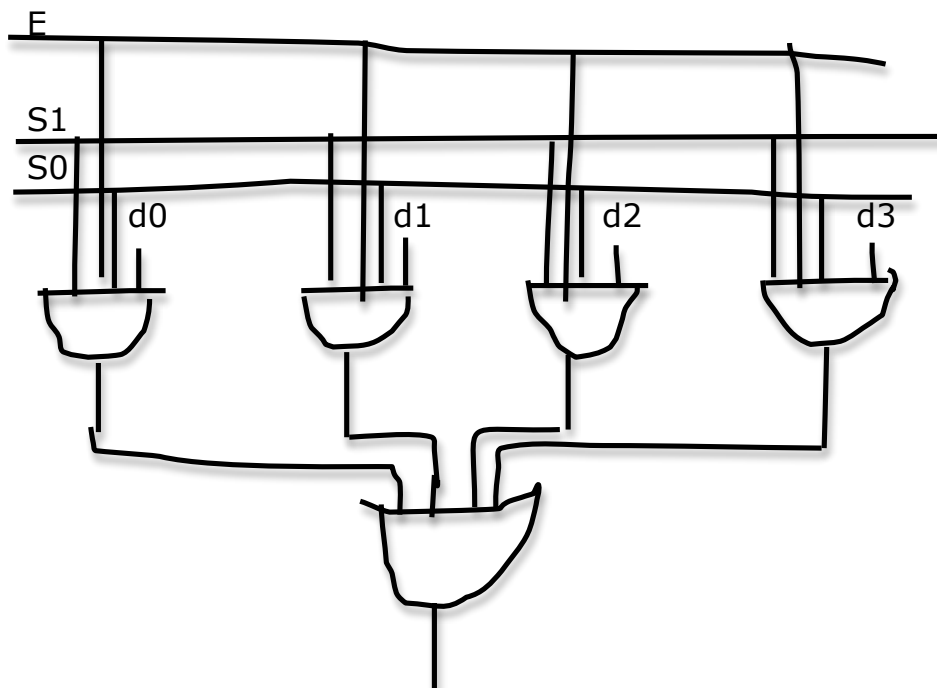
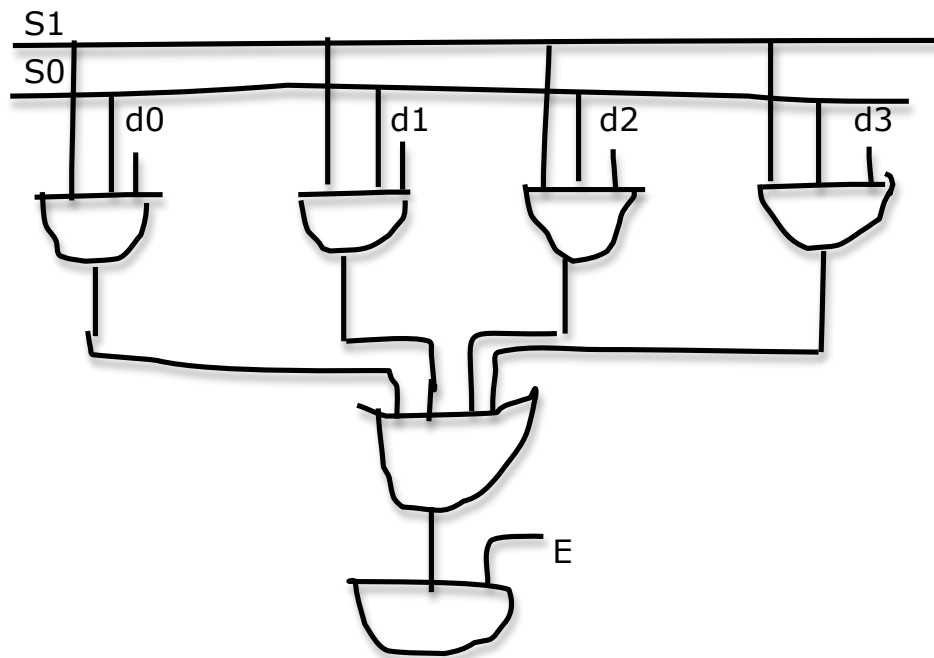
1 bit 4:1 mux

E	s1	s0	f
1	0	0	d0
1	0	1	d1
1	1	0	d2
1	1	1	d3
0	x	x	0

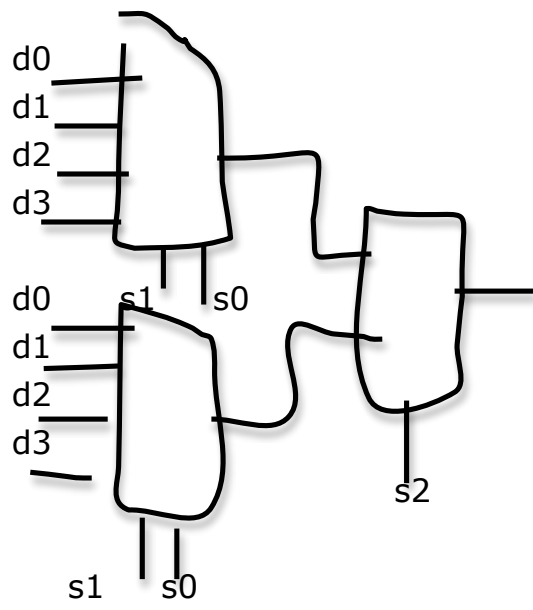


$$f = S1'S0'd0 + s1's0d1 + s1s0'd2 + s1s0d3$$





2X 4:1 mux
 1X 2:1 mux
 -->8:1 mux



This week's TA's office hours are in 4022

Lab6 is due in office hours

Demo your working circuit to a TA

$$x = x_{n-1}x_{n-2}\dots x_2x_1x_0$$

$$y = y_{n-1}y_{n-2}\dots y_2y_1y_0$$

$$x_{n-1} < y_{n-1}: x < y$$

$$x_{n-1} > y_{n-1}: y < x$$

$$x_{n-1} = y_{n-1}:$$

$$x_0 10$$

$$y_0 11$$

$$x ? y$$

$$x_0 = 0$$

$$y_0 = 1$$

$$x_0 < y_0$$

$$x < y$$

$$x = x_{n-1}x_{n-2}\dots x_1x_2$$

$$y = y_{n-1}y_{n-2}\dots y_1y_0$$

$$x_0 = y_0$$

$$x_0 < y_0$$

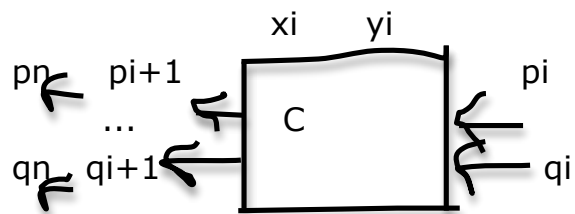
$$x_0 > y_0$$

$$x_1 < y_1$$

$$x > y$$

$$x_1 > y_1$$

$$y > x$$



$p_n \quad q_n$ $p_i \quad q_i$
 0 0
 0 1 $x < y$
 1 0 $y < x$
 1 1 $x = y$

$x_i y_i$
 00 01 11 10
 $p_i q_i$ 00 x x x x
 01 < < < >
 11 = < = >
 10 > < > >

$x < y$
 $< > =$
 $x_i = 0$
 $y_i = 1$

$x_i > y_i$

$x_i y_i$
 00 01 11 10
 $p_i q_i$ 00 x x x x
 01 0 0 0 1
 11 1 0 1 1
 10 1 0 1 1
 p_{i+1}

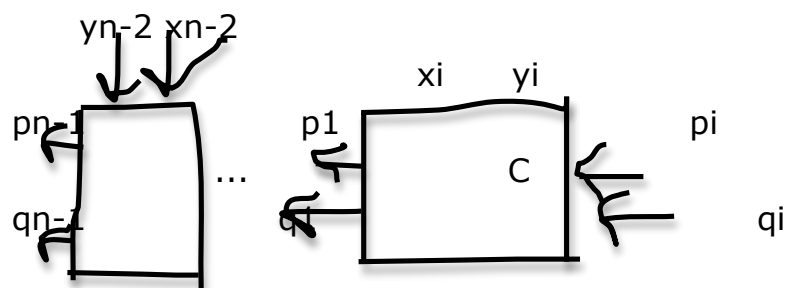
$x_i y_i$
 00 01 11 10
 $p_i q_i$ 00 x x x x
 01 1 1 1 0
 11 1 1 1 0
 10 0 1 0 0
 $q_i + 1$

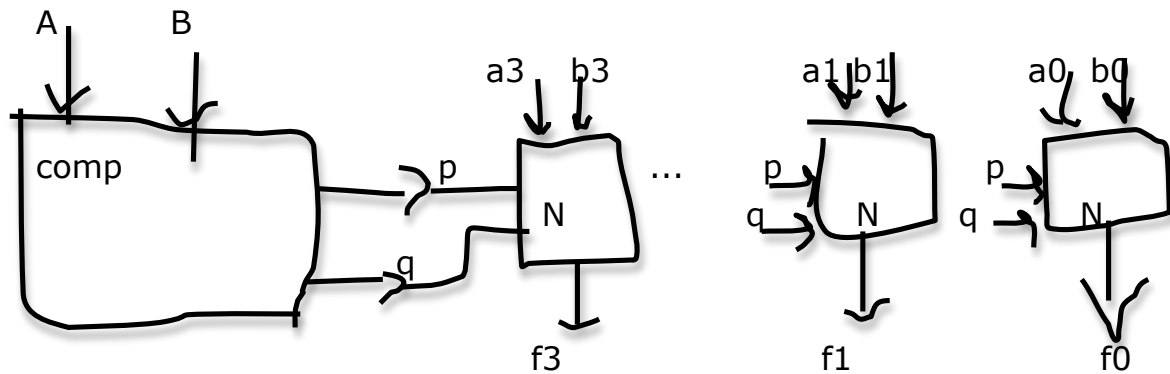
$p_i = 3$ product terms
 $q_i = 3$ product terms

$x_{n-1} y_{n-2} \dots$
 $y_{n-1} y_{n-2} \dots$

$x_{n-1} y_{n-1}$
 0 0 $x_{n-2} \dots x_0 ? y_{n-2} \dots y_0$
 0 1 $x > y$
 1 0 $y > x$
 1 1

$x + 2^{(n-1)} ? y + 2^{(n-1)}$
 if $x + 2^{(n-1)} < y + 2^{(n-1)}$
 $x < y$



$A = a_3a_2a_1a_0$
 $B = b_3b_2b_1b_0$
 $F = \{A \text{ NAND } B \text{ if } A < B$
 $\{0 \quad A > B$
 $\{A \text{ OR } B \quad A = B$


pq

00 A < B

01 A = B

11 A > B

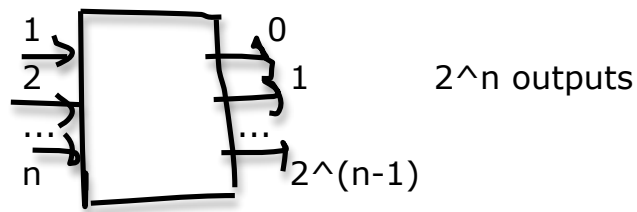
aibi

		00	01	11	10	
pq	00	1	1	0	1	NAND
	01	0	x	1	x	OR
	11	0	0	0	0	
	10	x	x	x	x	

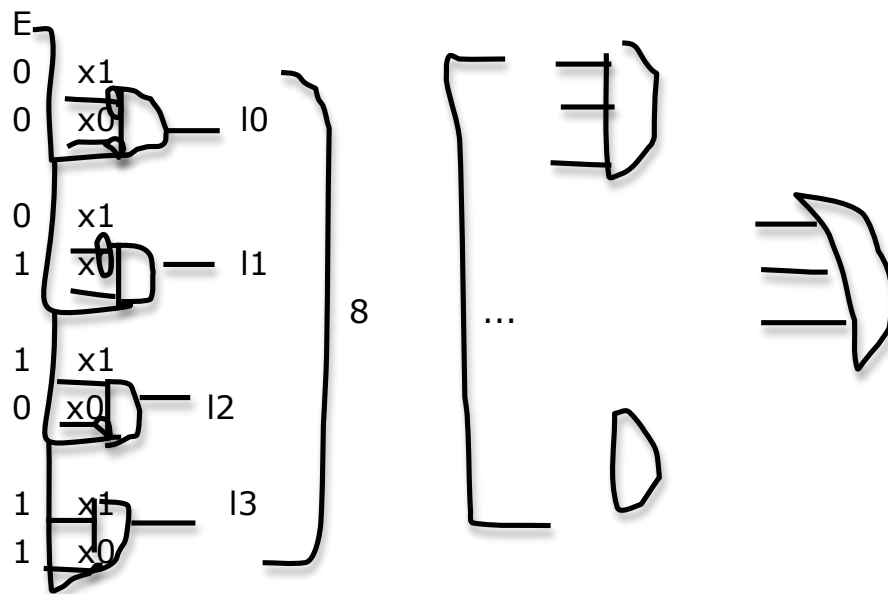
q'b'

x1	x0	l0	l1	l2	l3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

decoder

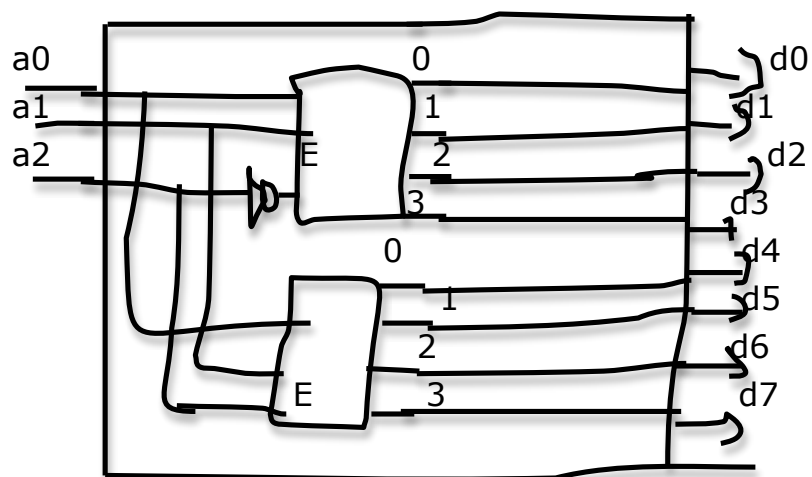


$n=2$



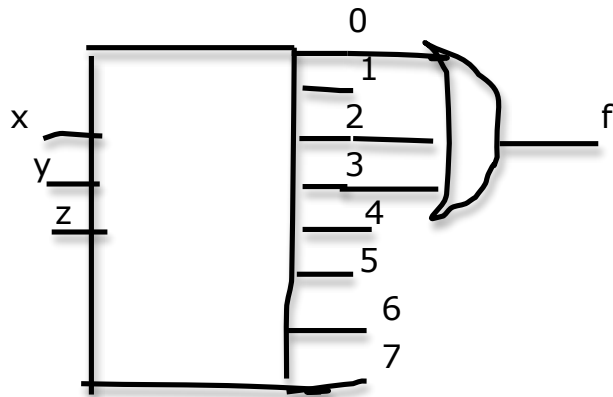
3-8 decoder

2-4



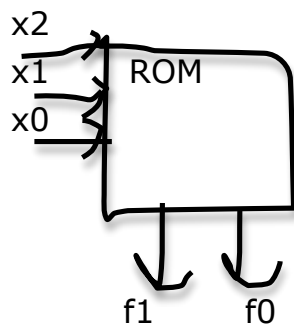
$$f(x,y,z) = x'y'z' + x'yz' + \overline{x'yz}$$

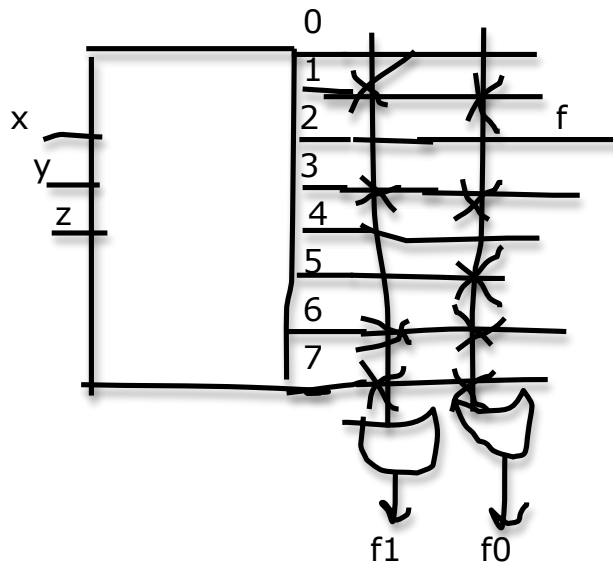
m0 m2 m3



x2	x1	x0	f1	f0		
0	0	0	0	0		
0	0	1	1	1	m1	m1
0	1	0	0	0		
0	1	1	1	1	m3	m3
1	0	0	0	0		
1	0	1	0	1	m5	
1	1	0	1	1	m6	m6
1	1	1	1	1	m7	m7

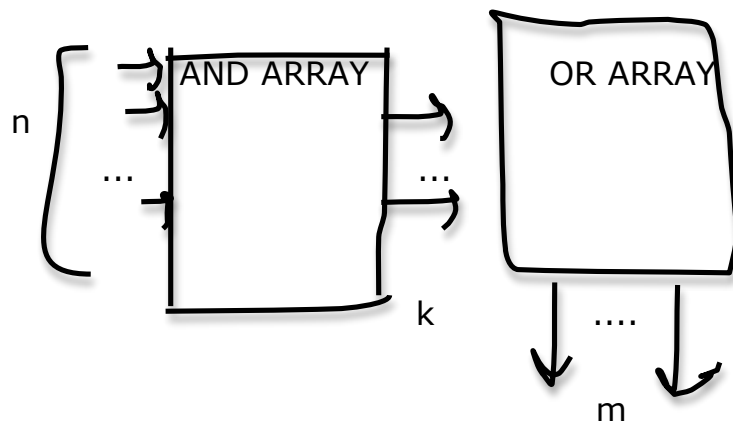
8 2 bit

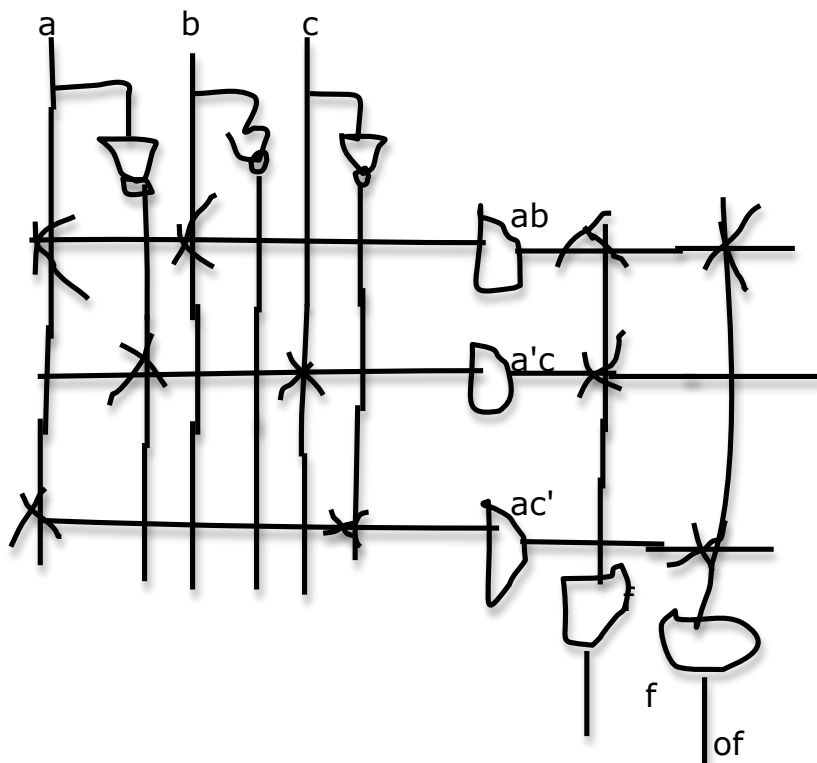




$$f(a, b, c) = m_1 + m_3 + m_6 + m_7 = ab + a'c$$

PLA



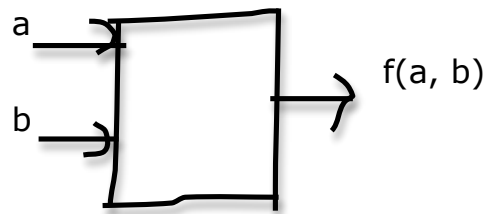


Midterm 2 is on 10/14

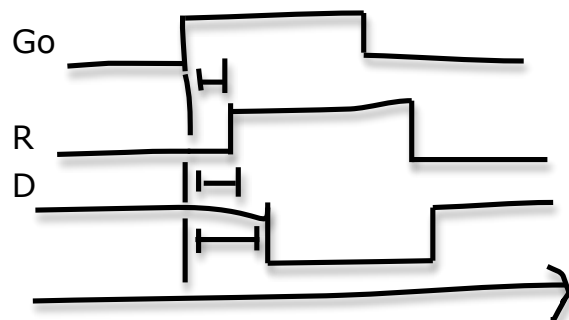
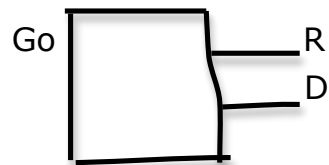
Conflict reporting due day: 10/7

No lecture next Friday :)

7-9 pm

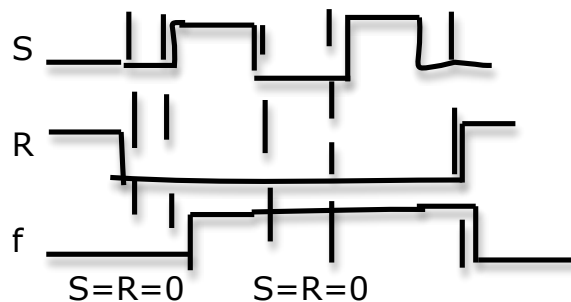


$g(\text{inputs, state})$



inputs: S R

output: $f \{ =1 \text{ iff } S \text{ was 1 more recently than } R$
 $\{ =0 \text{ otherwise}$



$f=0$ $f=1$

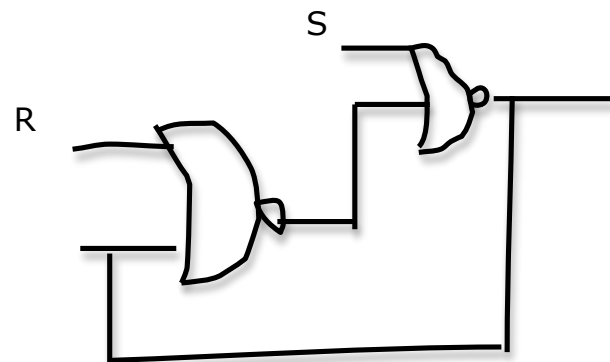
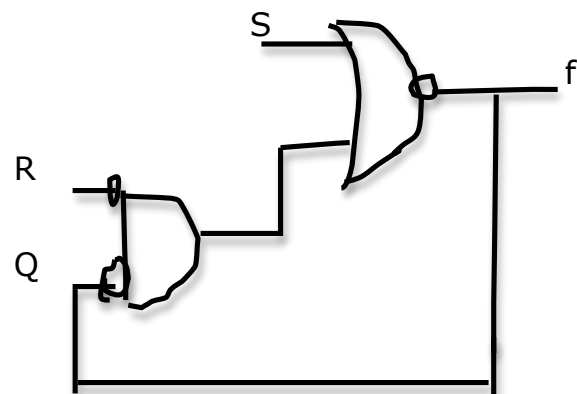
$Q=1$ iff S was 1 more recently than R

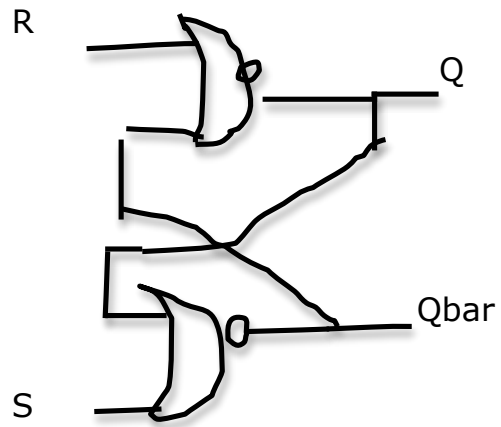
S, R, Q

	S, R			
	00	01	11	10
Q	0	0	x	1
	1	1	0	x

$SR=11 \rightarrow X$

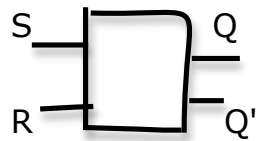
$f = S + QR'$





R	S	Q	Q bar	
0	1	1	0	set
1	0	0	1	reset
0	0	{1	{0	hold
		{0	{1	
1	1	x	x	

SR latch



next state

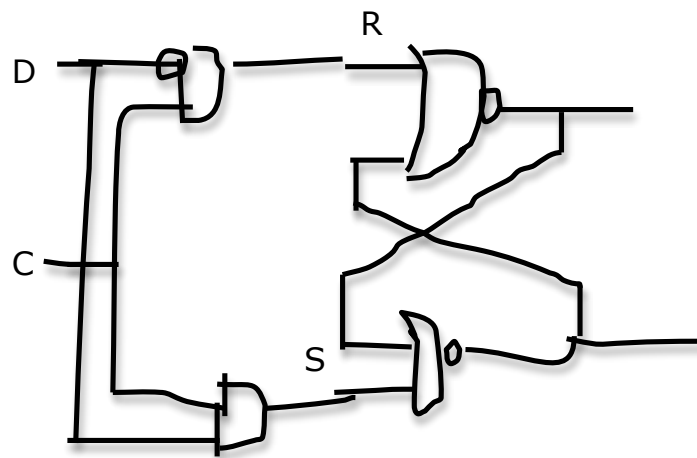
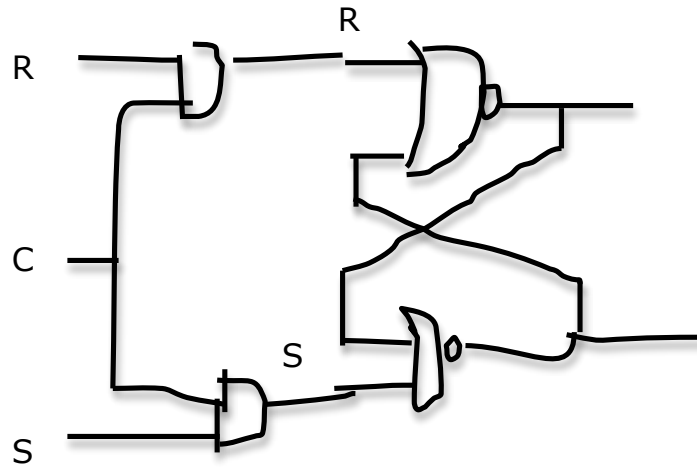
S	R	Q+
0	0	Q
0	1	0 reset
1	0	1 set
1	1	forbidden

11-->00

excitation

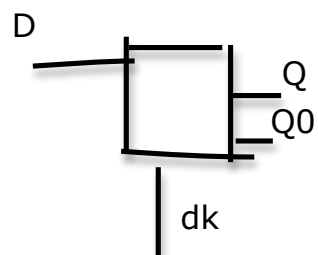
Q-->Q+ S R

0-->0	0 x
0-->1	1 0
1-->0	0 1
1-->1	x 0



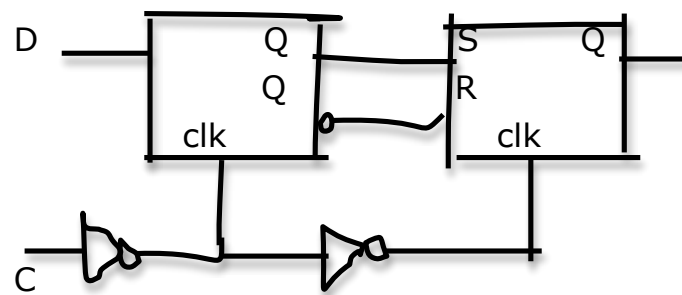
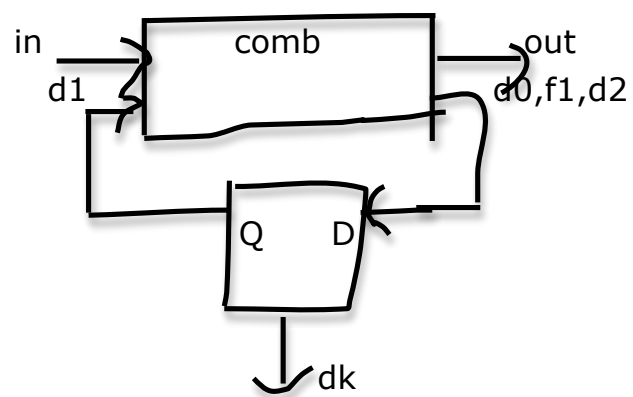
D	R	S	Q
0	1	0	0
1	0	1	1

gated D latch

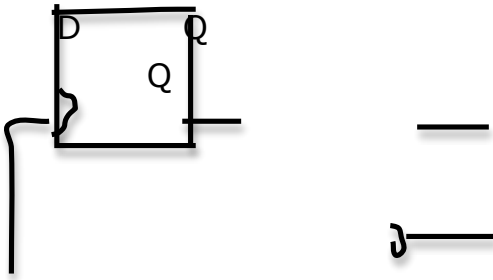


C	D	Q+
1	0	0
1	1	1
0	x	Q

Q	Q+	D
0	0	0
0	1	1
1	0	0
1	1	1



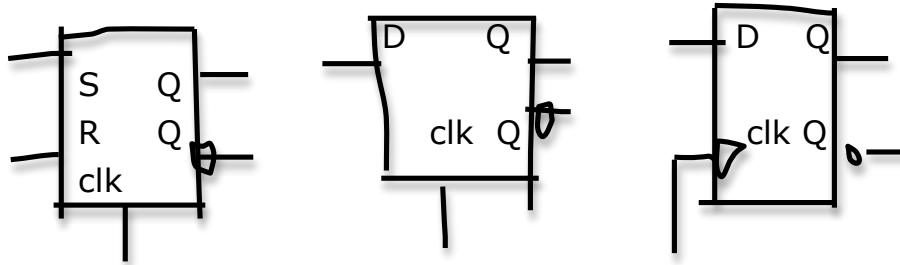
D=flip flop



Conflict exam notification due 10/7

no lecture on Friday

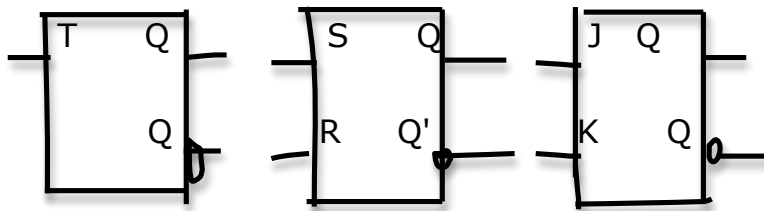
lab 7 is due on Friday by 10am in room 4034



D Q+

0 0

1 1



T Q+

0 Q

1 Q'

S R Q+

0 0 Q

0 1 0

1 0 1

1 1 FORBIDDEN!!!

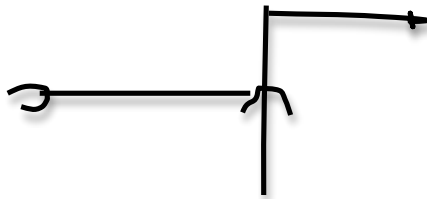
J K Q+

0 0 Q

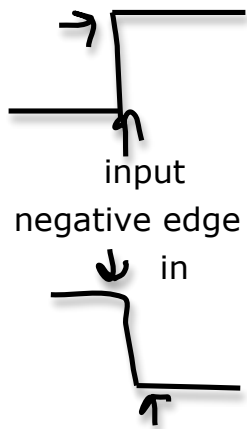
0 1 0

1 0 1

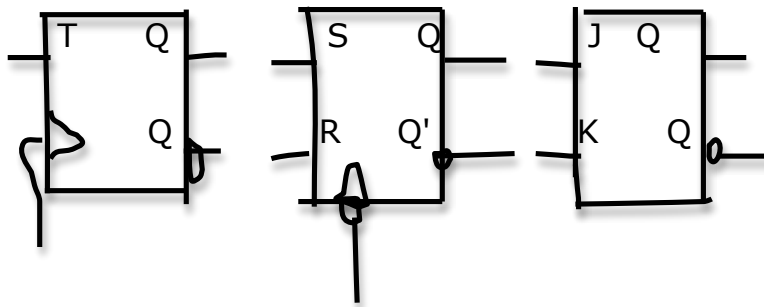
1 1 Q'



positive-edge triggered flip-flops



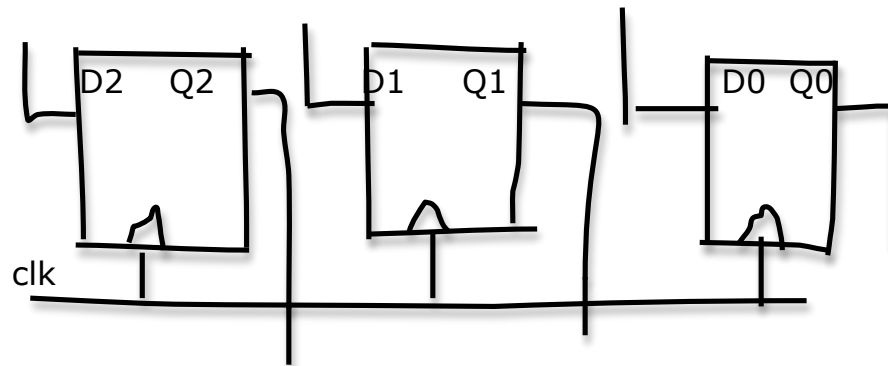
1 1



active high vs active low

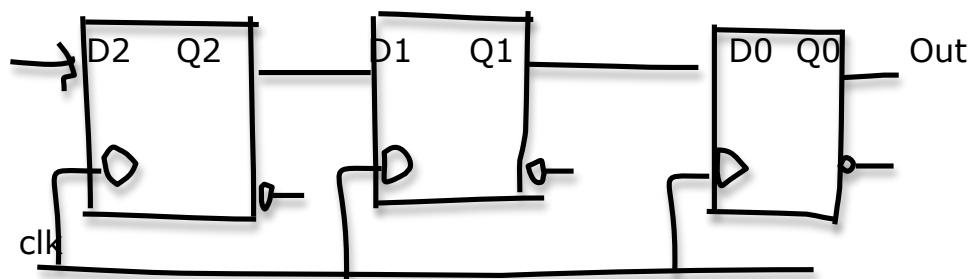


LSI

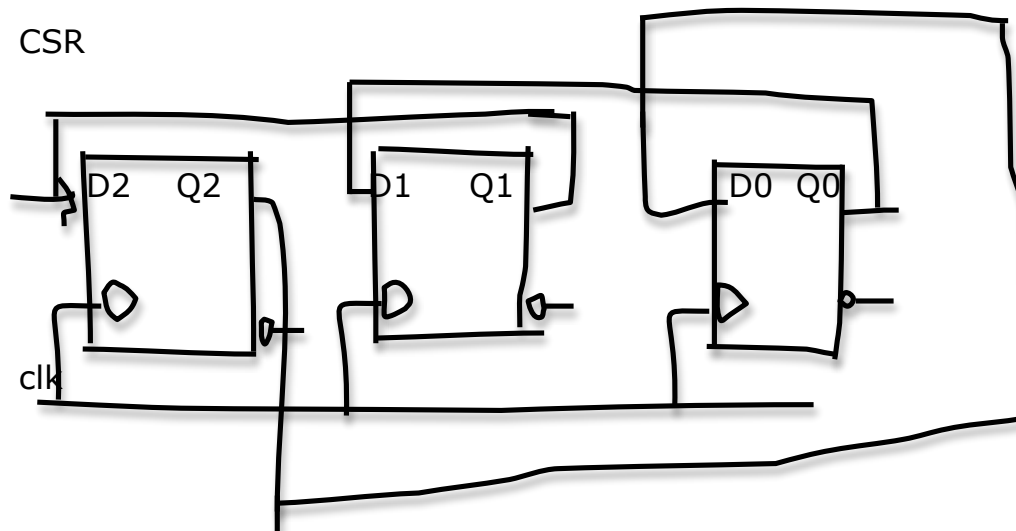


register:
shift

LSR

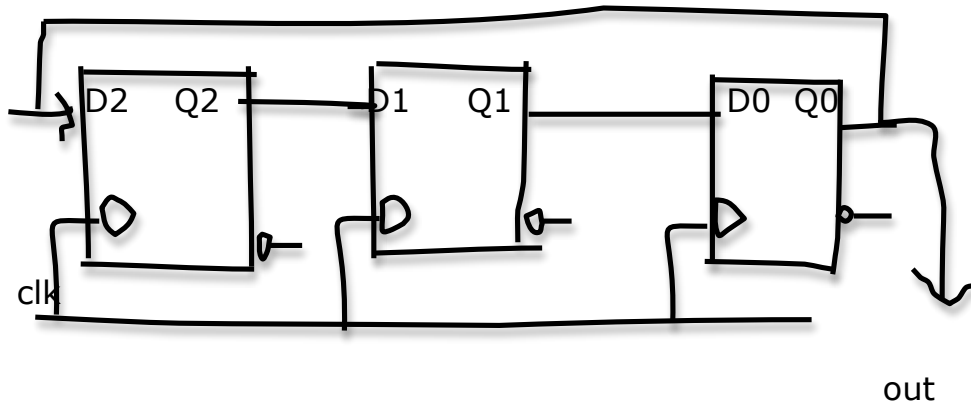


CSR

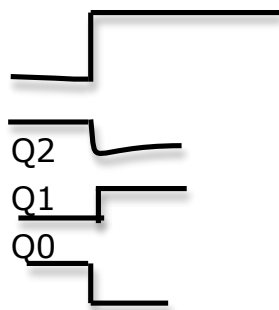
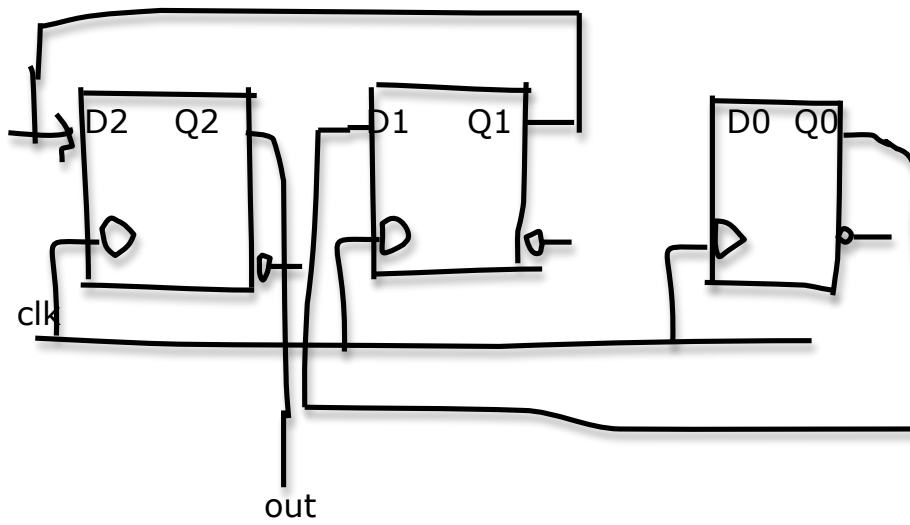


Out

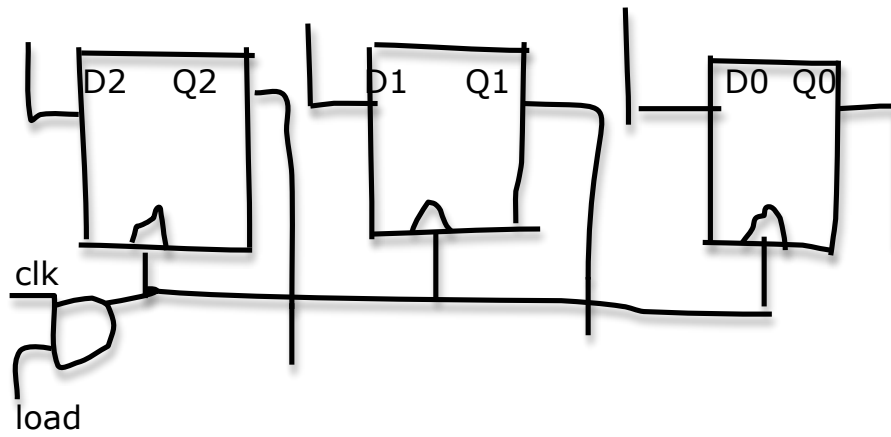
CSL



ASL



LSI

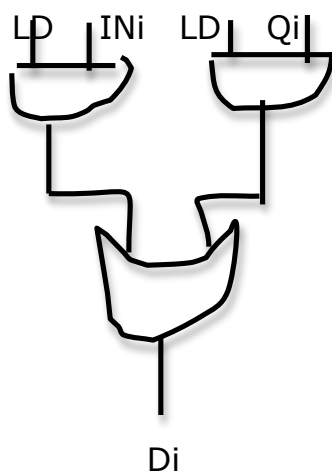


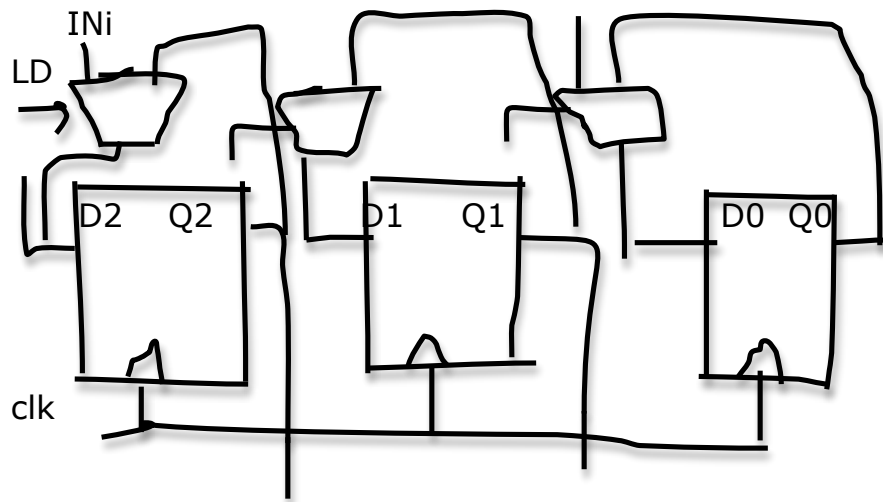
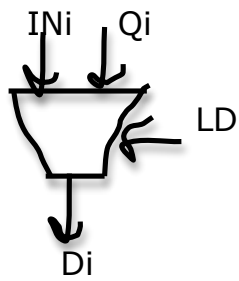
load=1: store

load=0: read

clk	Load	OP
0	0	Read
0	1	Read
1	0	Read
1	1	Write

$$LD \cdot IN + LD' \cdot Q$$



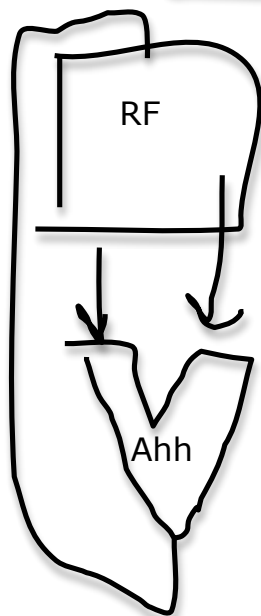
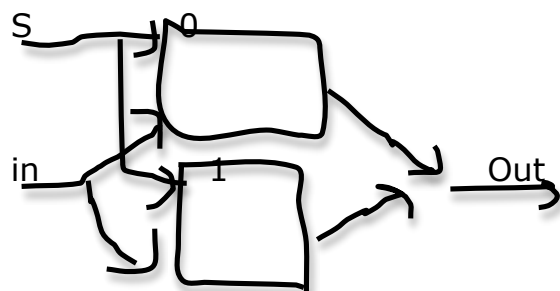
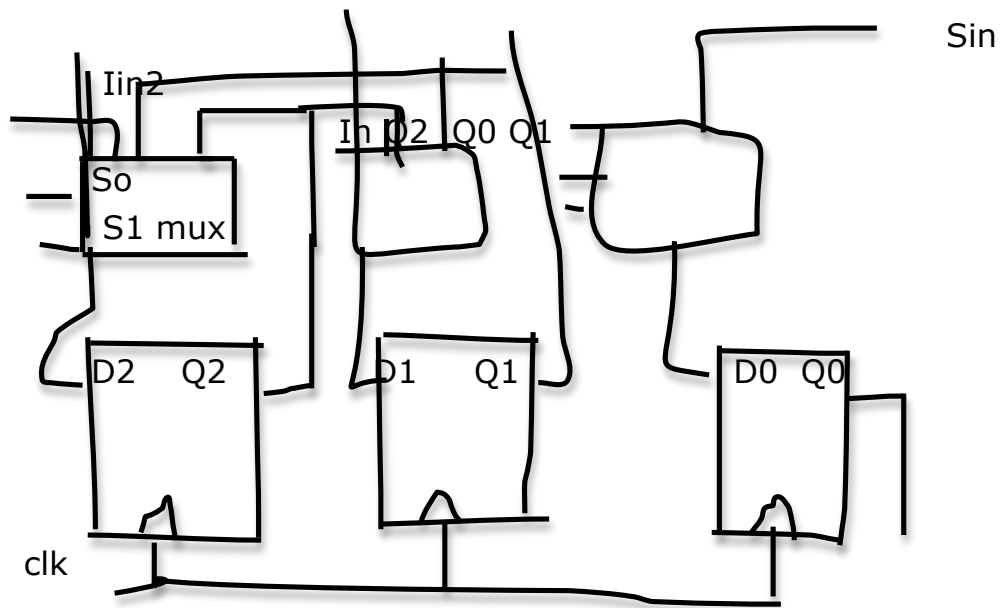


load

74194

So S1

0 0	Hold
0 1	Shift + left
1 0	shift + right
1 1	parallel load



No lecture on Friday

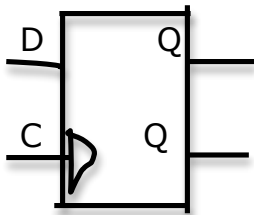
Lab 7 is due on Fr by 10am in 4034

check grades in compass

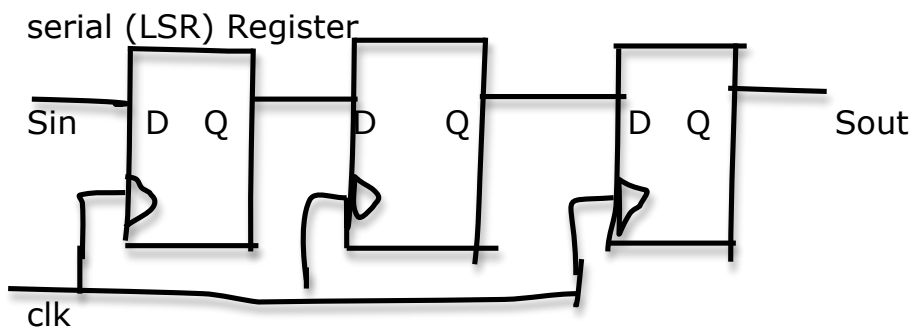
Exam review sessions on Sunday

HW7 is due on Monday

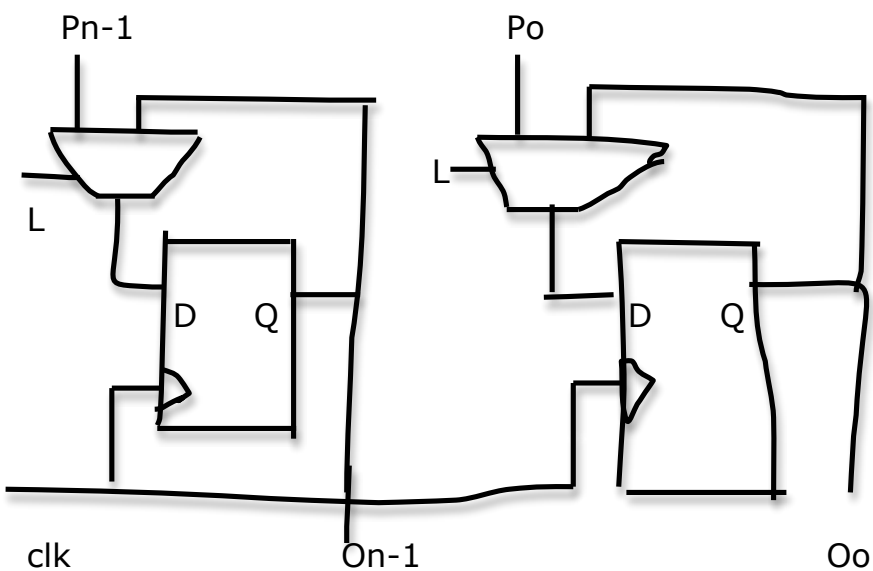
Lecture on Monday is exam review

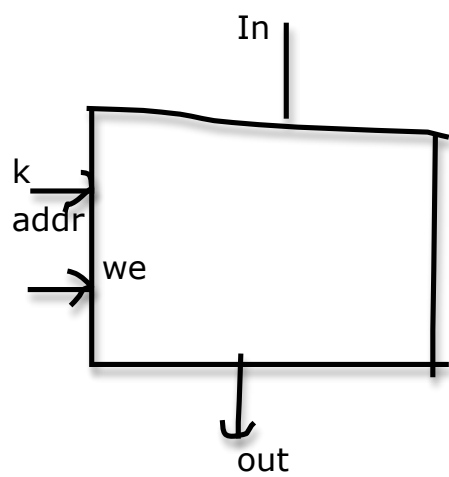
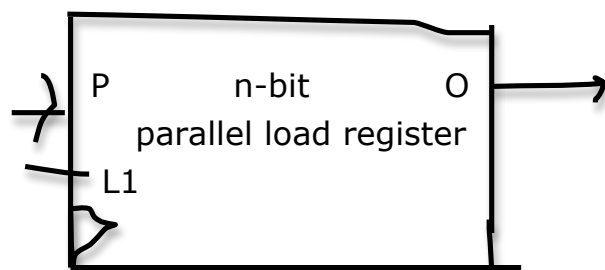
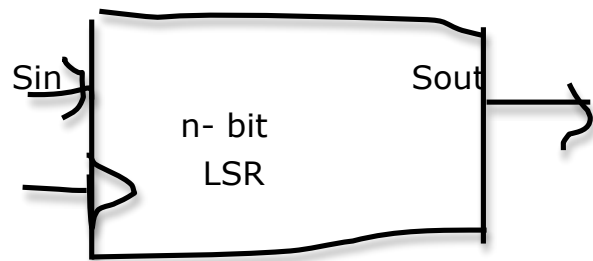


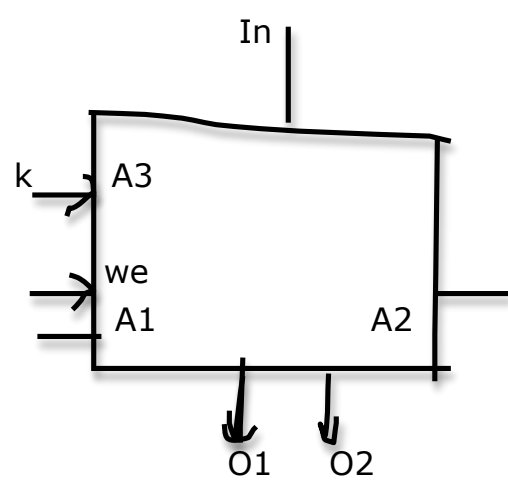
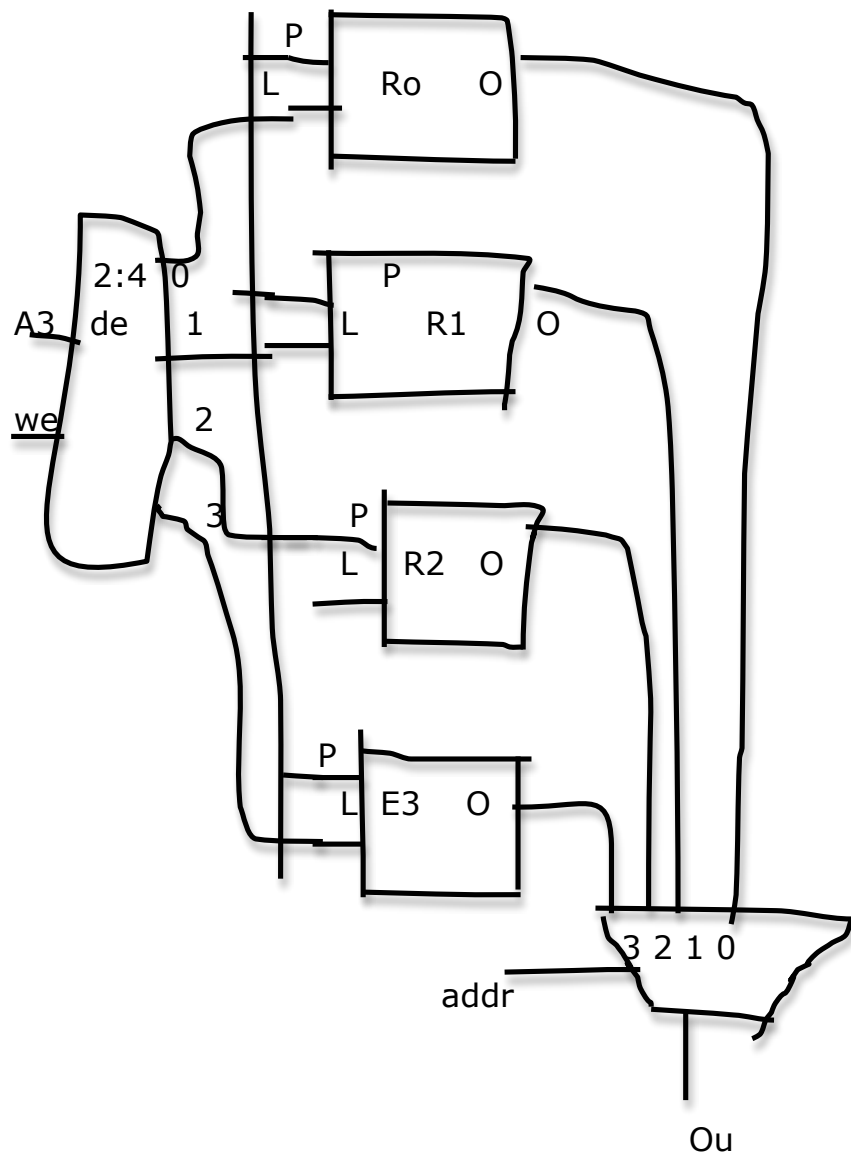
D flip flop

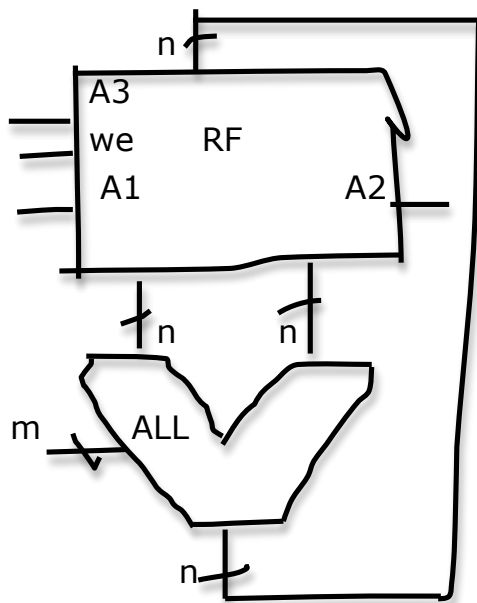


parallel load register



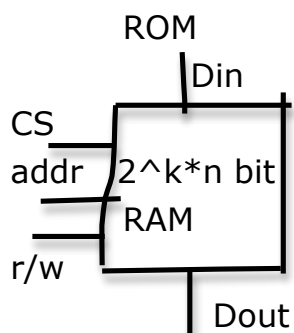
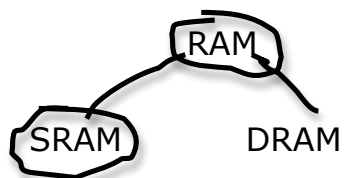






$R4 \leftarrow R2 + R3$

ROM



CS R/w' OP

0	x	no operation
1	0	write
1	1	read

read

CS-->1

r/w'-->1

adder-->address

Dout

write

CS-->1

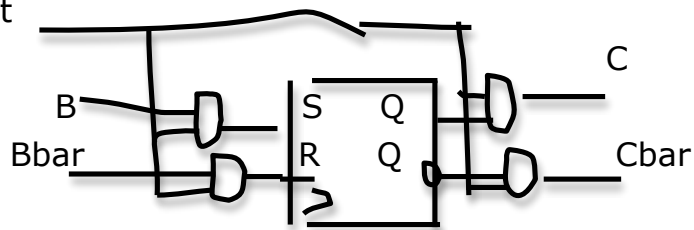
r/w'-->0

addr-->address

Din-->value

SRAM cell

select



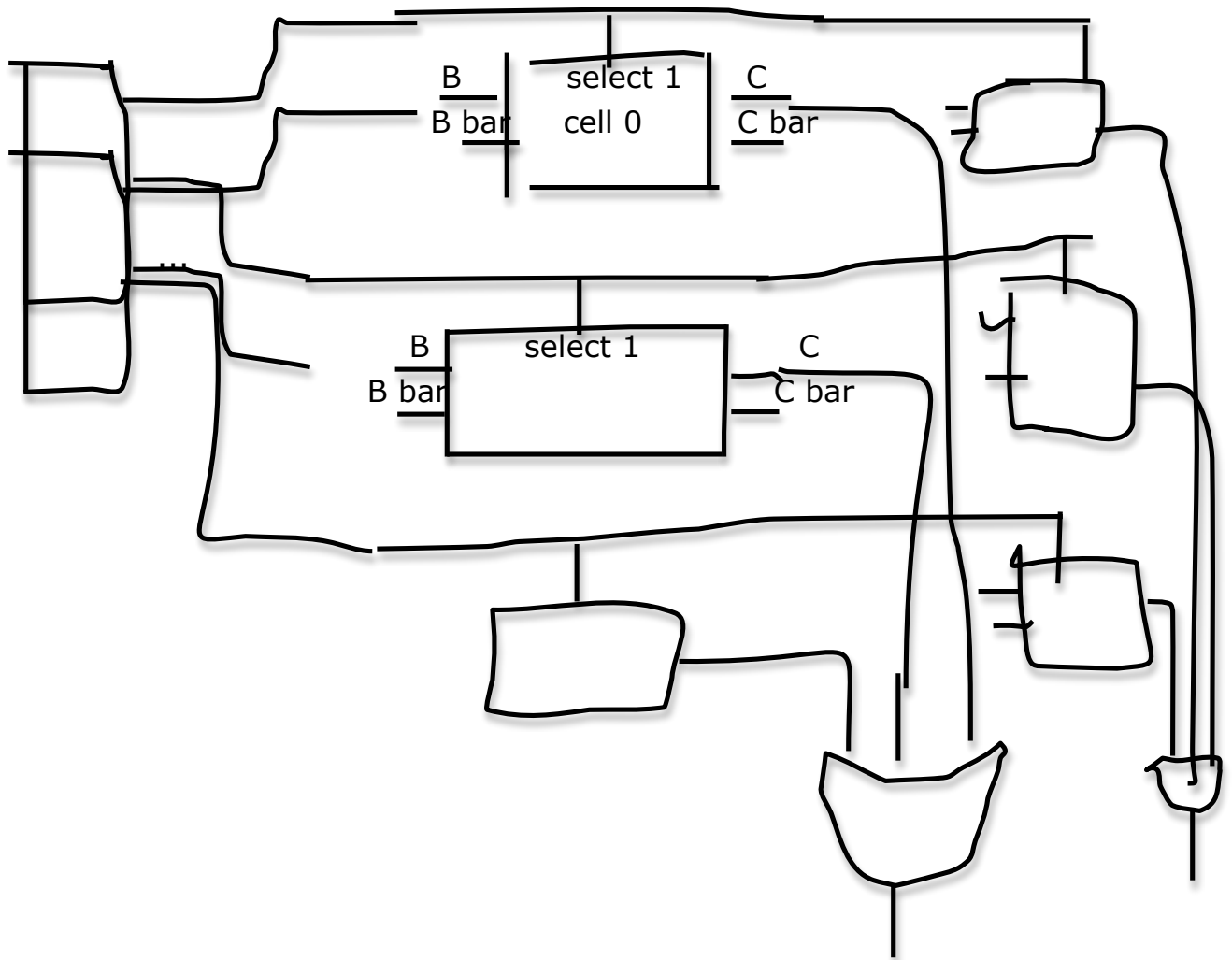
S R Q

0 0 hold

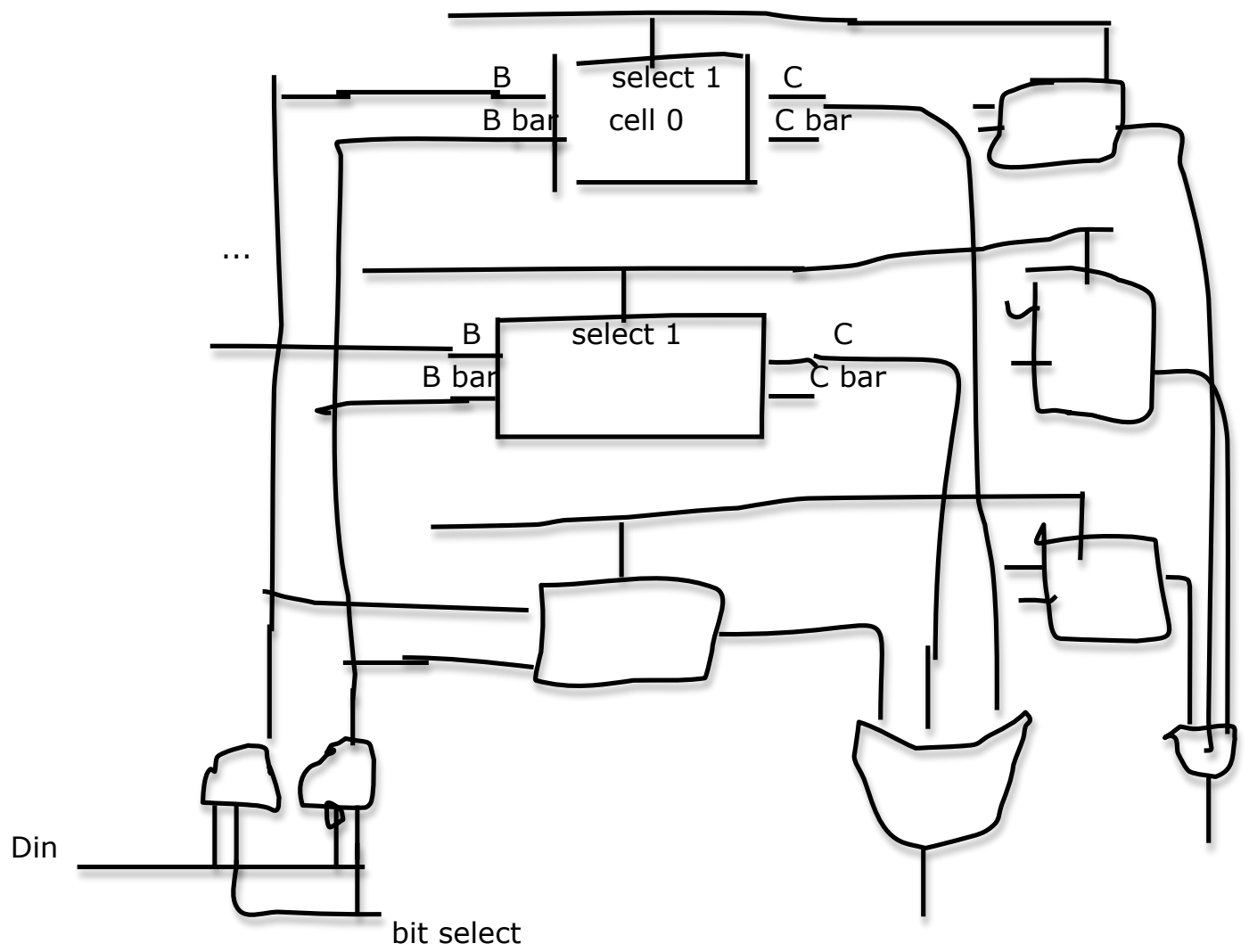
0 1 0

1 0 1

1 1 FORBIDDEN!!!!



nx2 memory



Comparators

Parallel and series load registers

simplify boolean algebra

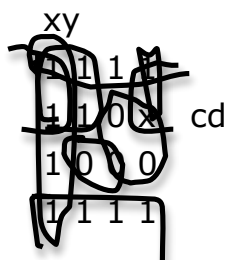
essential and prime implicants

adders

addressibilities

master slave design

perfect induction

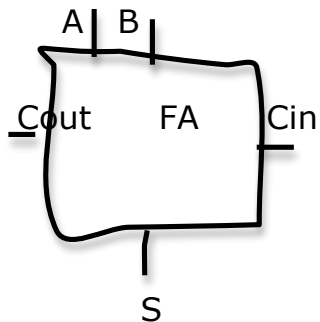


EPI: $a'b'$, d' , $a'c$

Min SOP = $a'b' + d' + a'c$

canonical POS

$= (a' + b' + c + d')(a + b' + c' + d')(a' + b' + c' + d')(a' + b + c' + d')$

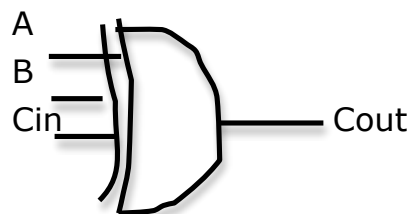
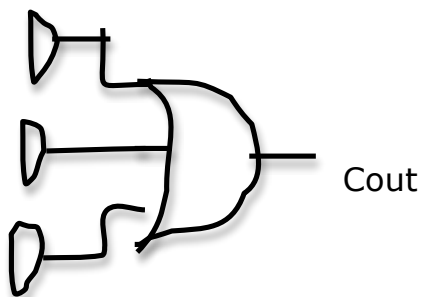
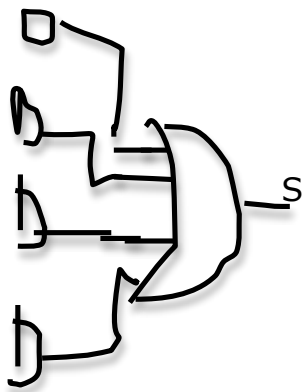


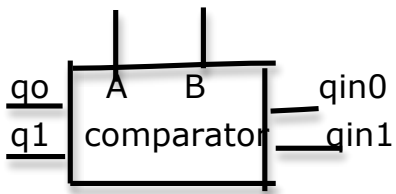
AB

00 01 11 10

S	C	0	0	1	0	1
	1	0	1	0		

AB						
		00	01	11	10	
Cout	C	0	0	0	1	0
	1	0	1	1	1	1





q0q1 00 equal
 01 A<B
 11 don't care
 10 A>B

q0

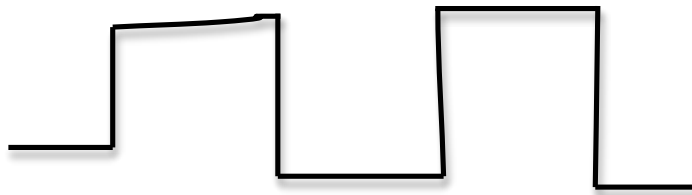
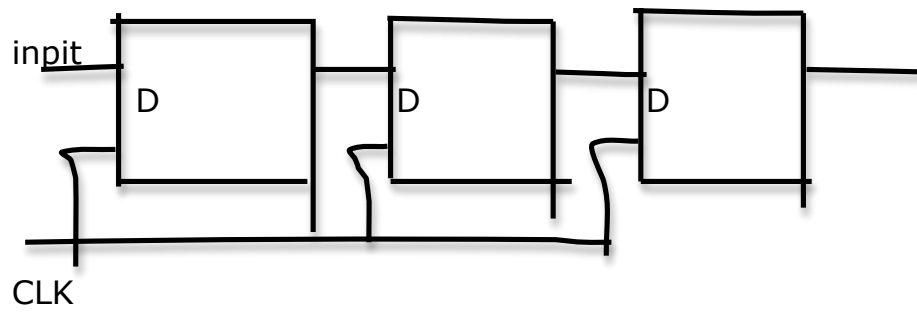
	AB				
	00	01	11	10	
qin01	00	0	0	0	1
	01	0	0	0	1
	11	x	x	x	x
	10	1	0	1	1

q1

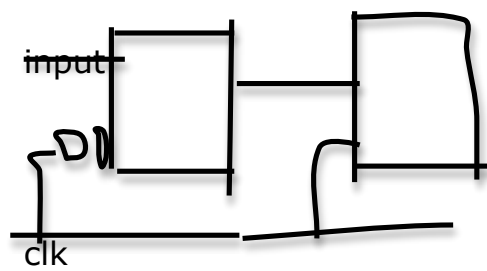
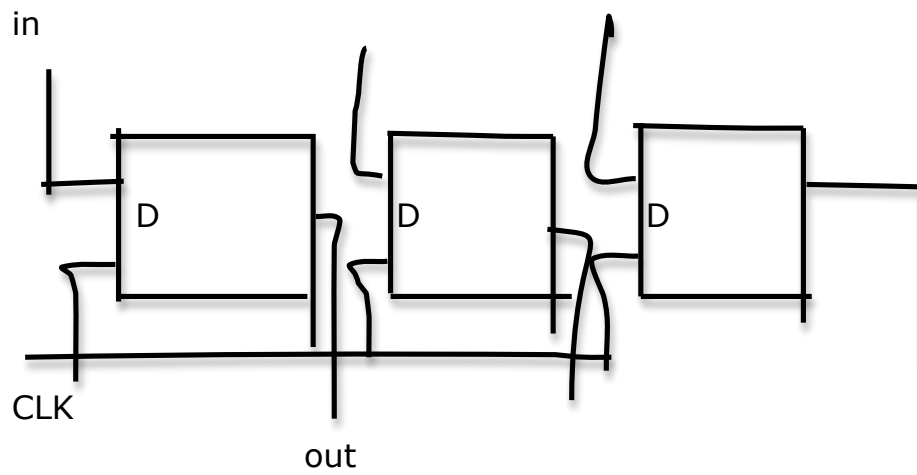
	AB				
	00	01	11	10	
qin01	00	0	1	0	1
	01	1	1	1	0
	11	x	x	x	x
	10	0	1	0	0

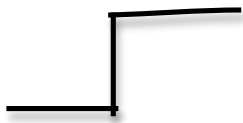
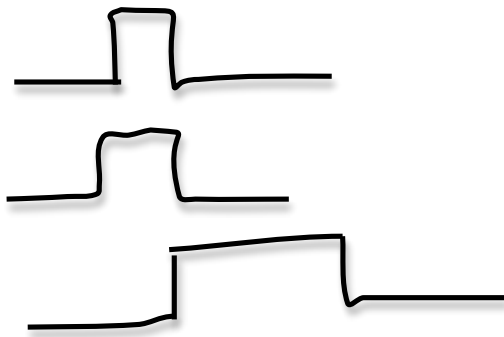
00 equ
 01 a<b
 11 dont care
 10 a>b

Series



Parallel



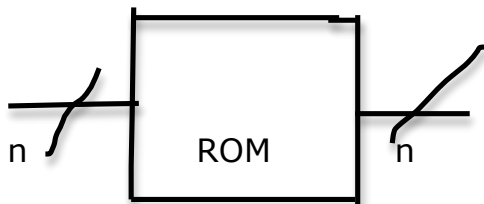


Next State for D latch

Current CLK

Q 0

Q 1



Word input

out

word: n =adressibility
for n bit input

adress space=adressibility times number of adresses

$$y'(x'z + y'z)'$$

$$=(y + x' z + y' z)'$$

$$=(y + z + x' z)'$$

$$=(y+z)'$$

$$=y'z'$$

2012

1.

D.

$$(a' \text{ xor } b) + c'd' + ac' + ad' + bcd$$

$$\rightarrow a'b' + ab + c'd' + ac' + ad' + bcd$$

$$=AB' + CD +$$