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The Dao of three-way decision and three-world thinking

Yiyu Yao

Department of Computer Science, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada

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ABSTRACT

Three-way decision is about thinking, problem-solving, and computing in threes or through triads. Its philosophy and practice of triadic thinking appear everywhere. In particular, there are a number of three-world or triworld models in different fields and disciplines, where a complex system, a complicated issue, or an intricate concept is explained and understood in terms of three interrelated worlds, with each world enclosing a group of elements or representing a particular view. This paper examines various triadic metaphors and structures in general and three-world metaphors and conceptions in particular for building and explaining basic concepts of three-way decision. The three major objectives are (a) to formulate a TAO (Triading-Acting-Optimizing) framework of three-way decision, (b) to discuss the Dao, the Way of three-way decision, and (c) to articulate the Dao, the Way of three-world thinking. Three-world conceptions offer more insights into three-way decision with new viewpoints, methods, and modes. An emphasis of the paper is on the philosophy and the power of three-way decision for building high-level conceptual models. By using three-part structures, by focusing on only three things, or by considering three basic ingredients, we may build a theory, a model, or a method that is simple-to-understand, easy-to-remember, and practical-to-use.

1. Introduction

In their book *Metaphors We Live By*, Lakoff and Johnson [48] state that "metaphor is pervasive in everyday life, not just in language but in thought and action. Our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature." Metaphors "structure how we perceive, how we think, and what we do." Motivated by the power of metaphors, we may take a closer look at metaphors that are relevant to the formulation, development, and explanation of a theory of three-way decision as triadic thinking.

We live in a three-dimensional space and hence tripartite, three-component, and three-part metaphors are abundant. We line on the planet earth and hence, to some degree, our conception of the world, as an abstract idea, is formed around our concrete perception and understandings of the earth. Metaphors of the world as a sphere, a container, a circle, a region, a period, a group of people or things, a domain of studies, a particular view, a school of thought, etc. are everywhere and every time. By combining these two types of metaphors, we have a third type of three-world metaphors that are in wide use in everyday life and in different scientific disciplines.

There are at least three important features of metaphorical thinking. First, a metaphor typically invokes an intuitive understanding of a thing of our concern by drawing its similarities to another more familiar thing. We often use metaphors to help us construct and explain high-level conceptual models or conceptual systems. Second, a metaphor, although very

E-mail address: Yiyu.Yao@uregina.ca.

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powerful for enabling us to quickly grasp and understand the essence of a thing at a high conceptual level, offers a rough, imprecise, and perhaps somewhat misleading understanding in some situations. A metaphorical explanation should not be taken literally but figuratively or metaphorically. It is essential to further develop and articulate a high-level metaphor-based conceptual model more precisely and in details at lower more concrete levels. In other words, metaphorical thinking and explanations are only one of the many steps in model building. Third, a metaphor may make only some aspects explicit and hide some other aspects into the background. It is a common practice to use several metaphors to explain the same thing. A fuller and deeper understanding emerges from the complementary views of different metaphors.

By taking the three types of metaphors and the three features of metaphorical thinking into consideration, in this paper I will make an effort to examine various tripartite metaphors and triadic structures in general and three-world metaphors and conceptions in specific for three-way decision. The main objective is to further explore and develop, from metaphorical thinking perspectives, a wide sense and a more general theory of three-way decision for triadic thinking, problem solving, and computing. I will use metaphors (i.e., various triadic structures) to explain the Dao (道),[1] the Way of three-way decision and three-world thinking. The paper makes three new contributions:

(1) Foundations of three-way decision (Section 2): I examine the cognitive, evolutionary, and cultural bases of three-way decision in an attempt to build a solid foundation of a theory of three-way decision.
(2) The TAO (triading-acting-optimizing) framework of three-way decision and the Dao, the Way of three-way decision (Sections 3 and 4): From the viewpoint of building a model, I restructure and reinterpret the TAO (trisecting-acting-outcome) framework [106] into a new TAO (triading-acting-optimizing) framework. From the viewpoint of forming a philosophy and a paradigm, I relate three-way decision to the concept of Dao in Chinese philosophy, thinking, and culture. The TAO framework and the Dao, the Way of three-way decision together give a blueprint for guiding future research in three-way decision.
(3) The Dao, the Way of three-world thinking (Sections 5 and 6): I use world metaphors to discuss various three-world structures for triadic thinking, that is, thinking through three worlds. I use three exemplary three-world models, from general to specific, to illustrate and explain the Dao, the Way of three-world thinking.

A key point to a new understanding of three-way decision is the construction of a triad, of which a trisection of a whole used in existing studies is an example. Three-way decision is triadic thinking, triadic problem-solving, and triadic computing.

It is generally accepted that, to understand fully a theory or a model, we must have various interpretations and explanations from multiple views and at multiple levels [106]. One possible application of three-way decision is to help us build high-level conceptual models and construct intuitive interpretations by using triadic structures and metaphors. My emphasis of discussion is, therefore, on these particular perspectives. As demonstrated by many examples in the paper, a theory, a model, or a method, with a three-part structure, focusing on only three things, or consisting of three basic ingredients, is simple-to-understand, easy-to-remember, and practical-to-use.

1.1. A historical note on the development of three-way decision

An idea from the Port-Royal Logic of Arnauld and Nicole [3,7] is an understanding of a concept jointly by a pair of an intension (or comprehension) and an extension (or denotation). The intension of a concept consists of all properties or attributes (more generally, some formulas of a language) that are valid for all those objects to which the concept applies. The extension of a concept is the set of objects that are instances of the concept. It is sometimes convenient to equate a concept to a set of objects, namely, its extension.

Forty years ago, Pawlak proposed a theory of rough sets [62,63] for approximating an undefinable concept/set by definable concepts/sets [104]. There are two basic representations of rough set approximations of an undefinable set. One is the pair of lower and upper approximations, and the other is the positive, boundary, and negative three regions. We may use either representation as the primitive notion to define the other representation. The positive region is the same as the lower approximation, the boundary region is the difference of the upper and lower approximations (in the reverse direction, the upper approximation is the union of the positive and boundary regions), and the negative region is the same as the complement of the upper approximation (or equivalently, the lower approximation of the complement of the set). While the lower approximation is a subset of the upper approximation, the three regions are pairwise disjoint. The two representations have led to two interesting and different research directions. The lower and upper approximations connect rough set theory with modal logics, in which the lower and upper approximation operators are interpreted in terms of logical necessity and possibility operators [100]. The three-region representation motivates the introduction of a theory of three-way decision [101,102].

In 1990s, together with Wong and Lingras [114,115], we proposed a probabilistic generalization of Pawlak rough sets known as decision-theoretic rough sets (DTRS). Unlike the rules derived from the Pawlak positive and negative regions,

[1] “Dao” is a fundamental concept in Chinese tradition, culture, and teaching. It is translated as “the Way.” Tao is the traditional spelling. In the new Pinyin system, it is spelled as Dao. In this paper, I use TAO to denote particularly the Triading-Acting-Optimizing (TAO) framework, and use Tao and Dao interchangeably. For an interested reader, I give the Chinese characters and/or Pinyin spellings for some key terms and phrases.

rules from the probabilistic positive and negative regions are no longer definite, but with some tolerable levels of error. Bayesian decision procedure is used to determine the optimal levels of error. In 2009, based on ideas from hypothesis testing and statistical inference [33,96], I introduced the notion of three-way decision (3WD) to explain rules derived from the probabilistic positive, boundary, and negative regions as rules for acceptance, undecided/non-commitment, and rejection [101,102]. The main function of the notion of three-way decision is to provide a semantically sound interpretation of the three types of decision rules in rough sets.

It is possible to use the notion of three-way decision to explain results from two other related studies. In 1993, I proposed the concept of an interval set [99] to represent a partially-known concept with three regions of known instances, known non-instances, and objects with unknown states. In 1998, Pedrycz [64] proposed the concept of a shadowed set to approximate a fuzzy concept with three regions of core members, non-members, and intermediate members. Similar to the case of rough sets, we can make three-way decisions with interval sets and shadowed sets, respectively. These set-theoretic models represent a narrow sense of three-way decision.

Further studies have shown that three-way decision is a much richer concept, with wide-ranging applications. Since 2012, I have been working on a theory of three-way decision, consisting of thinking, problem solving, and computing in threes [103,105,106,108,109]. Three-way decision has fostered new research areas, such as three-way classification, three-way clustering, three-way data analytics, three-way formal concept analysis, three-way approximations of fuzzy sets, three-way conflict analysis, three-way recommendation systems, three-way granular computing, and many others. The field has grown substantially since its inception, with researchers from around the world contributing to a significant number of papers, edited books, journal special issues, workshops, and special sessions on three-way decision. For the current state of research and development of the art, science, and practice of three-way decision, a reader may consult the reports by Yang and Li [97], Wei et al. [94], and Yao [98] based on networks analysis and bibliometrics analysis.

This paper expands a talk delivered at the IRSS President Forum in the 2022 International Joint Conference on Rough Sets [112]. One of the objectives of my talk is to demonstrate that Pawlak's brilliant idea of approximating an undefinable set in terms of three regions has inspired the introduction of the narrow sense of three-way decision. This paper continues with another objective to stimulate further interest and research on the TAO (i.e., triading-acting-optimizing framework) and the Dao, the Way of three-way decision in a wide and general sense. While Section 2 remains unchanged, other sections are either new or extensive revision.

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2. Cognitive, evolutionary, and cultural basis of three-way decision

Thinking in threes (i.e., triads consisting of three things) or triadic thinking is perhaps one of the most common mental models, metaphors, and structures, such as a tripartite scheme, a three-part theory, a three-element structure, a three-pillar framework, a three-word slogan, a three-character story, a three-generation classification, a three-level architecture, a three-version design of a product, a third grey option in addition to commonly used dichotomies (e.g., Yes and No, black and white, good and bad, positive and negative), a third middle point through the balancing and synthesis of the two opposites, and many more [4,6,52,70,92,106,109]. We humans and particularly scientists have an intriguing preference for a ternary patterned theory, model, or explanation of reality [66]. As an illustration, we may give three examples of thinking in threes. The first example is building a model of explanation for explainable artificial intelligence (XAI) based on the What-Why-How triad[2]: What are the results? Why are the results meaningful? How are the results derived? The second example is the MIT Sloan Management Review's short podcast, Three Big Points,[3] in which each episode presents a mold-breaking idea in ten minutes with three useful takeaways. The third example is the effective use of threes in writing a great paper[4]: the three C's of paper structure consisting of the Context for introduction, the Content for results, and the Conclusion for discussion; the ABC (Accurate, Brief, and Clear) of straightforward writing; the DEF (Declarative, Engaging, and Focused) for choosing a title. In particular, advice on straightforward writing is summarized in three sentences: "Never choose a long word when a short one will do. Use simple language to communicate your results. Always aim to distill your message down into the simplest sentence possible." We can find many examples that explore the power of triads for crafting great, powerful, and memorable speeches [31].

These examples show that we do commonly build an argument, a model, or a theory by thinking in threes. To provide further supporting evidence, it may be more constructive by giving three good reasons why we humans think in threes.

[2] This example will be further examined in the later part of the paper. For an actual application, we may point at the earlier expert system MYCIN that uses the What-Why-How triad, in which an explanation subsystem focuses mainly on Why and How questions to justify the decision of the system or to educate the user [86]. The triad is equally useful for enhancing human intelligence and guiding human behavior [109]. For example, the Golden Circle leadership model, introduced by Sinek [80], is based on the Why-How-What triad, which advises that every organization and everyone of us should know the three most important things: why we do (i.e., purpose and goals), how we do, and what we do. The same Why-How-What triad was used by Clear [14] in his three-level model of behavior change, focusing on what we believe, what we do, and what we get.

[3] https://sloanreview.mit.edu/audio-series/three-big-points/.

[4] https://www.nature.com/articles/d41586-019-01362-9.

The first explanation is the cognitive basis. It has long been recognized that we humans can only hold up a few things in the short-term working memory [18,57]. While there does not exist a general agreement on the exact number, which may range from two to nine, three seems to be a pivoting one. Another related result is our subitizing ability to tell immediately, without counting, the number of items presented to us when the number of items is small, typically fewer than six [44]. This may explain why the very first three Roman numbers are written as one, two, and three vertical lines, respectively, the very first three Chinese numbers are written as one, two, and three horizontal lines, respectively, and the pattern breaks at and after the fourth number. The third result is our natural ability to form patterns in order to make sense of the reality and our experiences. Three seems to be the minimum number of things required to form a meaningful and useful pattern. Drawing from these results of human cognition, thinking in threes comes naturally and may be an innate capacity.

The second explanation is the evolutionary basis. From an evolutionary point of view, we are better at older skills than at newer skills. Counting a few things and thinking about a small number of things, as evidenced by the 'one, two, three, four, many' and 'one, two, many' types of numerical systems [20], may be older skills in the process of human evolution. We, in fact, learned counting and thinking in small numbers at a younger age. Thus, we excel at skills of thinking in small numbers. It may be argued that thinking in threes is one of the products of evolution or early childhood learning.

The third explanation is the cultural basis. The number three plays an essential role across many cultures [22,74]. The number three typically represents completeness, harmony, and perfection, as expressed by the following quotations [74]:

> All good things come in threes. (Folk saying)
> A threefold cord is not quickly broken. (Bible)
> All was divided into three. (Homer)
> A whole is that which has a beginning, middle and end. (Aristotle)
> The Triad is the form of the completion of all things. (Nichomachus of Gerasa)
> Three is the formula of all creation. (Honoré de Balzac)
> The One engenders the Two, the Two engenders the Three and the Three engenders all things. (Daodejing, 道德经)

Using a triad of three things for perceiving, understanding, interpreting, and representing the reality seems to be a universal practice across different cultures. Triads are perhaps one of the most used structures when crafting a story, a speech, a theory, or a worldview. For example, Schneider [74] states, "Whenever there are three, as the three knights, three musketeers, three wise men, or three wishes, there is *throughness*, rebirth, transformation, and success." To a large extent, our cultural immersion experience further re-enforces an inclination and a preference towards thinking in threes.

Given the omnipresence of triadic thinking on the one hand and a lack of a formal theory on the other, a theory of three-way decision has been proposed and received much attention in recent years [103,105,106,109]. The theory is about a systematic study of thinking, problem-solving, and computing in threes. By attaching specific interpretations and meanings to various triads, we can obtain different models and modes of three-way decision. In this paper, I interpret a triad in terms of three worlds, which gives rise to thinking through three worlds.

3. A Triading-Acting-Optimizing (TAO) framework

In this section, I discuss a concrete Triading-Acting-Optimizing (TAO) framework by reinterpreting the original TAO framework [106].

3.1. A new interpretation of TAO

The original TAO framework of three-way decision was introduced in 2018, in which TAO is the abbreviation of the Trisecting-Acting-Outcome triad [106]. A close examination shows that there are a few imperfections. Trisecting represents only a special class of ways to form triads by focusing on division. Although trisecting captures one important aspect of three-way decision, namely, trisect-and-conquer (i.e., a special version of divide-and-conquer), it does not explicitly include other possible ways to construct triads. Outcome does not match well with the first two words that have a clear association to actions. In addition, outcome does not provide a hint on how to obtain an outcome. To resolve these issues, I reinterpret the triad as "Triading-Acting-Optimizing." Specifically, T: Triading is the construction of a triad; A: Acting is the application of strategies to process the three elements of a triad; O: Optimizing is a search of the best combination of triading and acting to achieve a desired outcome. This new explanation of TAO, while keeping the same abbreviation, avoids some shortcomings of the original interpretation. Triading covers various ways to construct a triad and optimizing explicitly indicates the need for a procedure and method for obtaining a desired result. The new interpretation is more general and better captures the essence of three-way decision.

Fig. 1 depicts the three basic components of the TAO model, namely, triading for the construction of a triad to represent the whole, acting for the application of a set of strategies to process or study the triad through processing or studying of the three elements, and optimizing for the search of a combination of triading and acting to produce the best outcome. The triad $\langle A, B, C\rangle$, denoted by the middle triangle, is a representation of the whole. The triad connects two tripods. The upper tripod, consisting of the node 'Whole' and three legs A, B, and C, describes the task of triading. There are two possible ways to interpret the tripod. A top-down reading suggests that the whole is decomposed into three elements; a bottom-up

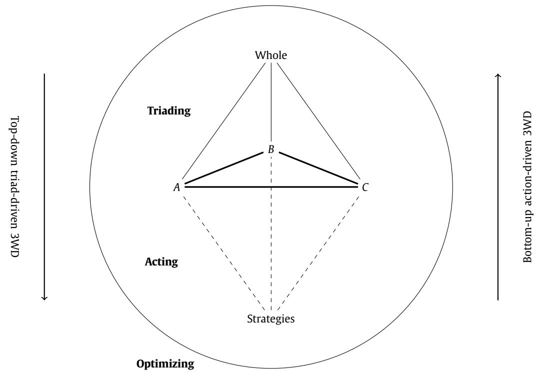


Fig. 1. TAO (Triading-Acting-Optimizing) framework of three-way decision.

reading suggests that the three elements support the whole. The lower inverted tripod, consisting of the node 'Strategies' and three legs A, B, and C, describes the task of acting. A set of strategies is applied to process the triad. There are also two possible ways to interpret the tripod. A top-down reading from A, B, and C to 'Strategies' suggests that a triad of the whole enables the formulation of a set of strategies for processing the triad. We devise a set of strategies based on a triadic structure that represents the whole. A bottom-up reading suggests the use of a pre-defined set of strategies to guide the construction and process of a triad.

By joining the two tripods, we have a triangular bipyramid with the base triangle $\langle A, B, C\rangle$. The bipyramid has a three-level structure, namely, whole-triad-strategies, and provides a basis for trilevel thinking and processing [107], with two basic modes of operation [106]. A top-down triading-acting mode starts with the whole, forms a triad to represent the whole, and searches for the most suitable strategies to process the triad. A bottom-up acting-triading mode starts with a pre-given set of strategies, searches for the most useful triad, and applies the strategies to process the triad to solve the problem of the whole. The circle enclosing the bipyramid represents a combination of triading and acting. We label the circle by optimizing, because a search for the most effective combination of a triad and a set of strategies is usually an optimization process. It often requires multiple iterations of the top-down and bottom-up modes, involving triad-guided redesign of strategies and strategy-guided reconstruction of a triadic structure of the whole.

3.2. T: triading

A triad, a fundamental concept of three-way decision, is a group of three things, items, or entities. We may study the notion of triads from three perspectives, namely, images and structures (i.e., visual representations and geometric shapes of triads), meanings and principles (i.e., interpretations of triads and their metaphorically suggested ideas), and practice and applications (i.e., practical uses of triads for triadic thinking). A triad provides a structural way to understand and represent a whole, focusing on three main components or elements. Depending on different contexts, we may interpret and name the three components or elements of a triad differently. We may consider three basic properties of a triad, namely, threeness-oneness-structuredness. First, a triad is a conceptualization of the whole (or one) through three ingredients, or particularly a division of one into three. The property of the threeness makes it possible to approach a more complex problem by considering three basic ingredients, parts, or elements. Second, a triad is a unification and integration of the three. The property of oneness ensures that the three individual ingredients, parts, or elements jointly describe and reflect the whole or the one. Third, the three elements of a triad are typically connected with each other in a structured way to form a triadic structure [109]. The property of structuredness brings together the threeness and oneness of a triad. In other words, the three elements are both independent in one way and connected in another way. While the threeness emphasizes on the independence, the oneness focuses on connections. According to the three properties, a triad is more precisely a set of three things equipped with a structure. By exploring different types of triadic structures, we can generate a wide range of ways to triadic thinking.

In the TAO framework, triading, a term borrowed from Logan et al. [52], denotes a method, process, or procedure for forming a structured triad. Logan et al. [52] provide a compelling argument that the triad of three people, connected based on their common core values and mutual self-interest, is a powerful and basic structure in any organization. They suggest, in particular, an effective way to build a triad by connecting two unconnected people through their mutual acquaintances. This way of triading, in fact, follows from a principle in social networks known as triadic closure [23,95], that is, if two people have a common friend, then they have a high possibility of being friends or have a high probability of becoming friends in the future. The principle of triadic closure can be used to understand and predict the growth of networks.

Guided by the three properties of a triad, we can study various ways to build a triad. For example, we can trisect a set into three regions, as being done in rough sets theory [63,102]. By considering a triad as a natural extension of a dyad, we can add a third element to a dyad to form a triad. This third elements may be a middle of the two extremes of the dyad, an integration and synthesis of the two, and above the two, in the dyad, or another alternative different from the

(a) Trilevel analysis (b) Six patterns of analysis

Fig. 2. Trilevel seven-element analysis.

two [109,113]. Different methods will produce different triadic structures. For example, in an earlier paper [111], I suggest three classes of methods for generating three types of triad, that is, a class of clustering-based constructive methods for producing a trisection, a class of sequencing-based constructive method for producing a trilevel or a trisegment, and a class of ranking-based constructive methods for producing three most important factors. It is necessary to investigate many other types of ways to triading, in order to meet the need of different types of modes of triadic thinking in different contexts.

3.3. A: acting

In the TAO framework, acting is the application of a set of strategies to process a triad by considering some or all possible combinations of the three elements of a triad. Acting can be explained by using a trilevel-seven-element hierarchy, as shown in Fig. 2(a). The corresponding dot-line-plane trilevel analysis and processing is described by Fig. 2(b). At the dot-level, acting focuses on analyzing and processing the three individual elements. The pattern of processing is denoted as the (1)-type. At the line-level, acting is about the comparative analysis of the three pairs of elements. The two patterns of processing are denoted as the (2)-type and the (1-1)-type, respectively. A (2)-type analysis considers the two elements together and a (1-1)-type analysis compares the two elements. At the plane-level, acting concerns integrative analysis and processing by considering various groupings of the three elements and the associated relationships between different groups. The three patterns of processing are denoted, respectively, as the (3)-type, (2-1)-type,[5] and (1-1-1)-type. A (3)-type analysis explores the three elements together. A (2-1)-type analysis looks at a combination of two elements versus the third element. A (1-1-1)-type analysis examines three elements as individuals.

As an example to illustrate a (2-1)-type analysis, consider an interview technique known as triading [27,54], which in fact covers one sense of triading as formulating a triad and another sense of triading as analyzing with a triad. By using a group of three things, triading elicits personal constructs that are "bipolar dimensions which each person has created and formed into a system through which they interpret their experiences of the world" [27]. A construct is normally characterized and represented by its two polar opposites. Eliciting personal constructs is, in fact, a (2-1)-type analysis of a triad. Suppose we have a set of examples/stimuli from a domain of discussion. We pick three of the examples to form a triad and ask the participant to explain how two of the three examples are alike on the one hand and are different from the third on the other. The likeness of the two examples gives rise and explains one pole of the construct, and the difference from the third explains the other pole. For example, consider a triad of three persons. If two persons are alike because they are kind, it is possible to understand and obtain the opposite of kind by looking at the third person who differs from the two kind persons [54]. Thus, triading is a (2-1)-type analysis, at the third level in Fig. 2(b), focusing on the likeness of two versus their difference from the third.

The effectiveness of the triading method stems from the fact that, with three elements, the likeness of two and their common difference from the third greatly reduce the number of possible constructs. By requiring both likeness and difference, a construct may also be meaningful and easy to explain. If two elements are used, they may differ in many different ways, and each difference may not necessarily form a meaningful construct. If four or more elements are used, the process of eliciting constructs may be too complicated, although the resulting constructs may be more meaningful. The triading method is, therefore, an example supporting three-way decision as triadic thinking.

The trilevel conception of acting with six types of processing pattern may be related to the Peirce's notions of the firstness, secondness, and thirdness. Houser [40] explains the meaning of the firstness-secondness-thirdness triad and its value as follows: "Peirce speculated that the three elements active in the world are chance, law and **habit**-taking, and he came to associate these three elements with his famous categories: *firstness*, *secondness* and *thirdness*. At the basic level, *firstness* is that which is as it is independently of anything else, *secondness* is that which is as it is in relation to something else, and *thirdness* is that which is as it is as mediate between two others. Peirce regarded these categories as complete and ubiquitous; they provided a useful structure for approaching every discipline or field of study, though in a cascade of forms." Accordingly, we may draw a kind of correspondence between the three levels of analysis in Fig. 2(b) and the

[5] In some situations, we may need to consider directional information by differentiating (2-1)-type and (1-2)-type. For simplicity, I use (2-1)-type to denote both of them.

firstness-secondness-thirdness triad. The bottom level of individual analysis focuses on one element of a triad in dependent of others, namely, the firstness; the middle level of pairwise analysis focuses on one element of a triad in relation to another element, namely, the secondness; the top level of integrative analysis focuses on one element in relation to two other elements, namely, the thirdness. In other words, we have a trilevel understanding of an element of a triad, namely, an element itself, an element in relation to another element, and an element in relation with the other two elements. Although this might be a weak correspondence based on particular interpretations of the related notions, it does capture some essential features of acting in the TAO framework by connecting it with other existing conceptions.

The dot-line-plane trilevel analysis combines the six types of processing pattern, giving rise to the diversity and flexibility of triadic approaches. The three levels of analysis may work in three modes [107]. The top-down control relation suggests that analysis at a higher level guides and directs analysis at a lower level. The bottom-up support relation suggests that results from analysis at a lower level support analysis at a higher level. In the middle-out mode, the middle level is guided by the top level and, at the same time, obtains support from the results of the bottom level. The top-down mode represents a process of decomposing a plane into lines and a line into dots, and the bottom-up mode represents a process of combining dots into a line and lines into a plane. By combining the top-down, bottom-up, and middle-out three modes of analysis iteratively, we may achieve more desirable results of acting. Although the trilevel-seven-element hierarchy gives six types of processing pattern, for a particular application it may be necessary to use only some of them.

3.4. O: optimizing

In the TAO framework, optimizing denotes the search for the most appropriate combination of triading and acting to produce a desirable outcome. Although there are many factors involved, by following the principles of three-way decision, I identify three main criteria for optimization, namely, the Effectiveness, Efficiency, and Explainability. They are referred to as the three Es for optimization. Effectiveness is about the quality and value of the results from triading and acting, efficiency is about the costs of triading and acting, and explainability is about the meaningfulness of a triad and a set of strategies, and the meaningfulness of the methods of triading and acting. In particular, there are three levels of explanations, consisting of explaining the results and the processes for obtaining them, explaining the meaning of the results, and explaining the value of the results. Existing studies concentrate more on the effectiveness by minimizing a cost function or maximizing a benefit function. By bringing in the other two Es, we may gain new insights into the ‘optimizing’ component of the TAO framework. In a nutshell, optimization aims at producing high-quality useful and valuable results efficiently and, moreover, the results can be explained with the aid of the triad, the set of strategies, and the triading-acting processes.

There are two directions of dependency between a triad and a set of strategies. In the triading-acting direction, a triad determines a set of strategies, that is, given a triad produced by triading, we devise a set of strategies for acting. Accordingly, we may first optimize a triading procedure to produce the best triad, and then devise a set of best/optimal strategies to process the triad. In the acting-triading direction, a set of strategies determines a triad, that is, a triading method is formulated to build a triad with respect to the set of strategies. Accordingly, we may design a triading method to produce the best/optimal triad in response to the set of strategies. In general, we need to consider optimization methods that involve both triad/triading and strategies/acting by iteratively exploiting the dependency of a triad and a set of strategies.

Individually, each of the three criteria is important and crucial. Jointly, it might be difficult to achieve the best for all of them simultaneously. We therefore must consider some kind of trade-off. For example, a fast triading method may not produce the most meaningful and useful triad. A complicated triading method may produce a good triad, but the method may be computationally expensive and the process may be difficult to explain. To resolve these issues, it is possible to incorporate multiple-criteria optimization methods into the TAO framework. As a first step, we must study various quantitative measures with respect to the three criteria.

4. The Dao, the Way of three-way decision

The abbreviation TAO is chosen purposely to be the same as the traditional spelling of the concept of Dao that is deeply rooted in Chinese philosophy, thinking, and teaching. Dao literally means road and is usually translated as the Way. Wang [91] describes two basic meanings of the original usage of Dao: (a) “the road or path upon which one walks,” and (b) guidance, namely, “a map for the journey through life.” With the guidance of Dao, “one does not simply walk on the road mindlessly and aimlessly but with a direction and with mindfulness.” The richness, significance, and complexity of the concept of Dao is derived from extending its literal meanings. For example, as explained by Knoblock [45], Dao is “a constant principle” within Nature. It is “the path to something, as to becoming a gentleman, a sage, or a True King, and it is also the path or way that the universe follows in all its processes and movements.” “A *dao* or way consists in the methods, principles, and doctrines that constituted the path to the goal.” Dao “is a cosmic principle that operates according to certain invariable principles that can be grasped by the mind since the mind shares the fundamental qualities of the Dao.”

Many authors have adopted and applied the philosophical positions, universal principles, and practical methodologies of Dao to art, science, and engineering [8,10,87,88], as well as everyday life and living [34,35]. In similar ways, Dao and its multifaceted and multilayered meanings provide a useful lens to study three-way decision. The wide sense of three-way decision is the Dao, the Way of three-way decision. Three-way decision is a worldview and a paradigm of triadic thinking, problem solving, and computing. By making effective uses of triads and triadic structures, a theory of three-way decision consists of a set of triadic methods, principles, and doctrines for building the path for thinking, problem solving, and computing. The Dao, the Way of three-way decision deserves further attention, debate, and elaboration.

The Chinese classic, Yijing (易经, The Book of Changes),[6] has shaped all aspects of Chinese ways of seeing, knowing, and living in all aspects of culture, art, politics, science, etc. throughout the Chinese history. To a large extent, the teaching of Yijing is about the teaching of the Dao with respect to changes. As stated by Huang [41], "The main theme of the I Ching is that everything is in a process of continuous change, rising and falling in a progressive evolutionary advancement." By making use of a tripartite scheme of classifying all situations and circumstances, Huang [41] further states: "The Tao of I also says: In a favorable time and situation, never neglect the unfavorable potential. In an unfavorable time and situation, never act abruptly and blindly. And in adverse circumstances, never become depressed and despair." Many scholars have interpreted and explained, and are continually searching for new interpretations and explanations of, this classic text from many different angles. According to the principle of thinking in threes, it may be useful to divide the studies of Yijing into three basic categories, concerning the (1) images and numbers (象数, xiangshu) [91], (2) meanings and principles (义理, yili) [91], and (3) practice and applications. The three categories together provide a holistic understanding of Yijing through intuitive visual and precise numerical interpretations, verbal description and discussion, and practical applications.

In the spirit of this tripartite scheme, it may be fruitful to organize the studies of the Dao, the Way of three-way decision into the same three categories. First, studies with a focus on images deal with visual representations, visual metaphors, and geometric shapes of triads used in three-way decision [109]; studies with a focus on numbers concern the magical number three, dealing with the optimality, simplicity, and universality of thinking in threes [105]. Second, studies on the meanings and principles of three-way decision are the category of the majority of current research, concerning the art, science, and methodology of three-way decision. Finally, for humans, practice and applications of three-way decision are about living and doing according to the Dao, the Way of three-way decision; for machines, they are about the engineering aspects in implementing intelligent systems by using the principles and methodologies of three-way decision.

The TAO (triading-acting-optimizing) framework represents a concrete understanding of three-way decision, and the discussion of the Dao of three-way decision moves to a higher level by focusing on the philosophy, principles, and methodology. The two support each other and work together. They offer many research questions and challenges. The triadic scheme of images and numbers, meanings and principles, and practice and applications may provide a blueprint for planning future research. I hope that the TAO model and the Dao of three-way decision may motivate and guide new research on three-way decision, as well as inspiring new ideas.

5. Triadic structures of three worlds

This section examines the concept of worlds and four types of triadic structures, namely, a Venn diagram of three sets, a triangle, a three-dimensional space, and a trilevel hierarchy, for thinking through three worlds.

5.1. The concept of worlds

The concept of "the world" is perhaps one of the most commonly used notions or metaphors for us to describe, view, and understand the reality and our relationships to the reality. The word "world," particularly, "the world," is used in various contexts with multiple meanings [93]. According to Webel [93], "the world" is "a linguistic and historical construction" and "an abstraction, a concept, or idea." It is how the "meaning-creating organisms frame the boundaries of their being-in-this-world." The view of "world as idea" [75,93] provides a starting point for exploring how we use the concept of worlds to understand the reality and to guide our conducts, namely, how to observe the world, how to make sense of the world, and how to change the world.

We may categorize and characterize things into different worlds in many ways, for example, from a temporal, spatial, functional, positional, or contextual consideration. We typically divide various aspects of the reality, for example, a group of geographical regions, a timeline of developments, a discourse of discussion, a family of human activities, etc., into a number of different and interrelated worlds. By restricting to a particular world, we limit our investigation within that world in the context of other worlds. Conceptually, we can talk about the inside, the outside, and the boundary of a world, which offers three interpretations and understandings of the same world. By considering different worlds, we can make comparisons, study their interconnections and influences, and shift our attention by switching between different worlds. While a single world presents a local view, multiple worlds give rise to a global view.

Our extensive living experiences on the planet earth as "the world," our relentless search for a better world, and our constant cultivation of a superior inner world all suggest the value of "world as idea." Conceptualizing the reality in terms of different worlds leads to both intuitive and in-depth understandings. By combining the principles of three-way decision as thinking in threes and the view of "world as idea," we immediately arrive at a paradigm of thinking through three worlds. There are abundant examples of three-world thinking. In the contexts of information processing, knowledge management, problem solving, and human experience, for example, we have: the three-world theory of the reality and knowledge (i.e., physical, mental, and human-created worlds) by Popper [68], the theory of collective human experience (i.e., commonsense,

[6] Yijing is the spelling under the new Pinyin system. It is commonly known as 'I Ching' under the traditional spelling.

religion, and science) by Shaw [77], the classification of three worlds of knowledge (i.e., everyday life lay knowledge, scientific knowledge, and metascientific reflection) by Mouton [58], the theory of three worlds of mathematics (i.e., conceptual embodiment, operational symbolism, and axiomatic formalism) by Tall [84], and the theory of triadic game design (i.e., three worlds of reality, meaning, and play) by Harteveld [38]. Other examples of three-world thinking in more general contexts include various triads, such as the material-intellectual-spiritual three worlds, the three worlds above-below-upon the earth (i.e., heaven, hell, and earth), the three worlds of yours-mine-theirs, etc.

It becomes evident that three-world thinking, with an understanding of "world as idea," offers a new direction for expanding the study of three-way decision as triadic thinking. In the rest of this section, based on a study on the geometry of three-way decision [109], I examine a few commonly used geometrical metaphors and visual representations for interpreting triadic structures.

5.2. Interpretations of three-world structures

Fig. 3 is a summary of four types of triadic structures, denoted by the four rows of three subfigures in each row. Each triadic structure represents a particular understanding of three worlds and their relationships in a particular context. While serving as a metaphor or a visual aid for the purpose of an intuitive and practical understanding, it may not cover all aspects nor convey the precise intended meaning. A variety of these visual representations illustrates the richness of triadic structures as a basic notion of three-way decision.

5.2.1. Three-circle Venn diagram and set-theoretic interpretations

One methodology of the three-world view and analysis is to divide the discourse of discussion into three possibly overlapping and relatively independent worlds. There may exist multiple ways to construct three worlds. Any particular three-world configuration is only one of the many possible simplifications or representations of the reality. According to the different types of configurations of the three worlds, we can systematically investigate various classes of issues in different regions constructed by using set-theoretic operators [110].

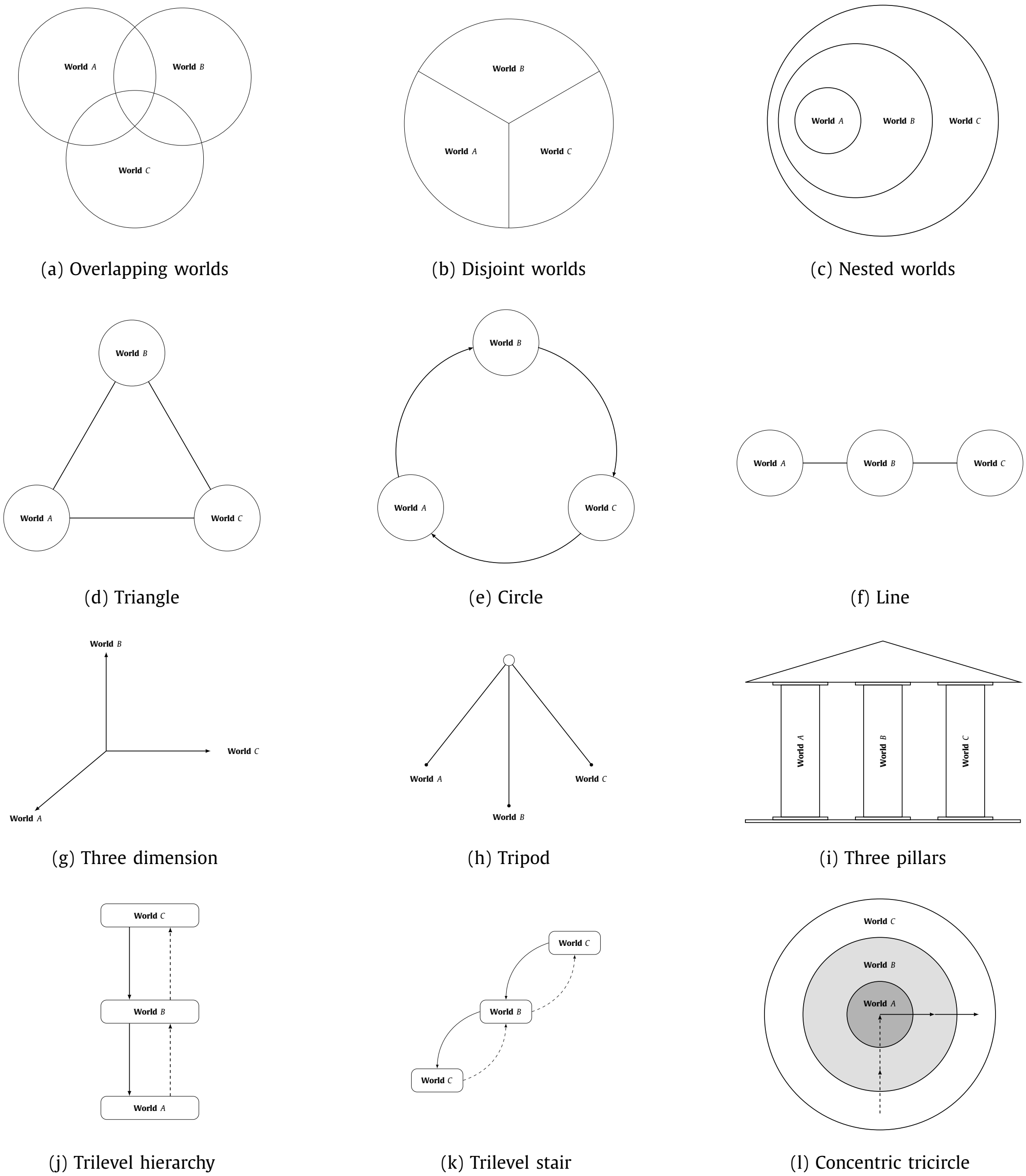
The Venn diagram in Fig. 3(a) depicts three overlapping worlds. Each world represents a particular view and focuses on some particular aspects. While a set covers issues in a world, its complement covers issues not in the world. An intersection of two or three worlds represents their joint issues. With three worlds, the eight disjoint and possibly non-empty regions are, in terms of set intersection, $A \cap B \cap C$, $A \cap B \cap \bar{C}$, $A \cap \bar{B} \cap C$, $A \cap \bar{B} \cap \bar{C}$, $\bar{A} \cap B \cap C$, $\bar{A} \cap B \cap \bar{C}$, $\bar{A} \cap \bar{B} \cap C$, $\bar{A} \cap \bar{B} \cap \bar{C}$, where $\bar{A}$ denotes the set complement of A. If we only consider set intersection, we have seven overlaying and possibly non-empty regions, A, B, C, $A \cap B$, $A \cap C$, $B \cap C$, and $A \cap B \cap C$, corresponding to the seven elements in Fig. 2(a) by interpreting the comma ',' as the set intersection $\cap$. The bottom level of 1-world analysis focuses on each world independently, the middle level of 2-world comparative analysis shifts attention to issues brought by interactions of two worlds, and the top level 3-world integrative analysis looks into more complicated interactions of three worlds. To have a holistic view, it is necessary to have investigations at the three levels, both individually and jointly.

As two special cases, Figs. 3(b) and (c) show, respectively, the triadic structures of three disjoint worlds and three nested worlds. For the case of three disjoint worlds, we can use set union to construct seven regions, A, B, C, $A \cup B$, $A \cup C$, $B \cup C$, and $A \cup B \cup C$, corresponding to the seven elements in Fig. 2(a) by interpreting the comma ',' as the set union $\cup$. For the case of nested three worlds in Fig. 3(c), we may consider the set difference to obtain the following six regions, A, B, C, $B - A$, $C - B$, and $C - A$. In addition, the three regions A, $B - A$ and $C - B$ are pairwise disjoint.

There are abundant examples of three-world thinking with respect to the Venn diagram based set-theoretic interpretations. Figs. 3(a), 3(b), and 3(c) make more explicit three different and particular aspects of the interplay of the three worlds. Fig. 3(a) emphasizes the common parts of two and three worlds, where common issues of different worlds are the focus of attention. For example, Kagan [43] divides the modern academy into the three worlds/cultures of natural sciences, social sciences, and the humanities. Across culture and multidisciplinary studies explore the rich common regions of different worlds and their boundaries. Tall [84] introduces a theory of three worlds of mathematics, consisting of conceptual embodiment, operational symbolism, and axiomatic formalism. While each individual world focuses on a particular type of mathematical methods and skills, a join of two worlds shifts the focus to the integration and combination of the respective methods and skills. Long and Huang [53] discuss a framework of human-machine-environment studies, consisting of the three worlds of human, machine, and environment.[7] In addition, they identify seven areas of research that correspond to the seven regions derived from set intersection operations on the three worlds. As one more example, data science is typically explained and described by considering the interactions and overlaps of three worlds of mathematics/statistics, computer science, and a particular domain [111].

Fig. 3(b) captures ideas of trisect-and-conquer. By dividing a large whole into three relative independent parts, with each part representing a particular type or class of issues, we may reduce the complexity of understanding the whole by investigating independently the three parts. For example, Shaw [77] proposes a theory of collective human experience in terms of the three worlds of commonsense, religion, and science. Mouton [58] classifies human knowledge according to the

[7] Long and Huang [53] use the phrase "man-machine-environment system engineering (MMESE)." It seems that "systems theory of human-machine-environment studies" might be a better choice, which extends the framework to cover topics beyond the scope of engineering.



(a) Overlapping worlds (b) Disjoint worlds (c) Nested worlds

(d) Triangle (e) Circle (f) Line

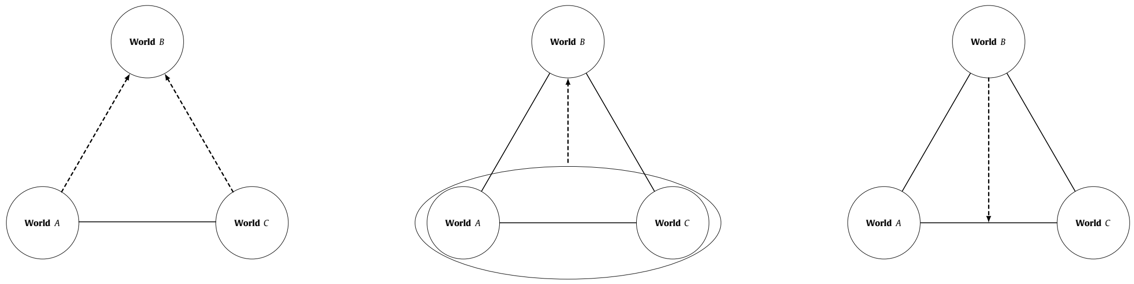
(g) Three dimension (h) Tripod (i) Three pillars

(j) Trilevel hierarchy (k) Trilevel stair (l) Concentric tricircle

Fig. 3. Models of three-world thinking.

three worlds of everyday life (lay knowledge), science (scientific knowledge), and metascientific reflection (metascience). Fig. 3(c) shows progressively enlarging three worlds, in which a larger world contains all issues of a smaller world and at the same time its own issues. An excellent example of the nested three worlds is the notion of circles of control-influence-concern developed by Covey [16]. The largest 'World *C*' of concern consists of all issues or things that we care about. Within the circle of concern, a smaller 'World *B*' of influence consists of issues or things that we have either indirect or direct control. Finally, within the circle of influence, an even smaller 'World *A*' of control consists of issues or things that we have direct control. The circles of control-influence-concern provide a roadmap about where we can direct our attention and spend our time to make possible changes.[8]

[8] Covey [16] explicitly uses two circles, namely, the circle of concern and the circle of influence. Proactive people focus their efforts in the circle of influence, which is positive energy that enlarges the circle of influence. On the other hand, reactive people focus their efforts in the circle of concern, which



(a) A third world (b) (2-1)-type analysis (c) World versus relation

Fig. 4. Triangles of three-worlds.

5.2.2. Triangle and graph-theoretic interpretations

For studying relationships, influences, and transformations of different worlds, we may take graph-theoretic views by treating worlds as nodes and connections of worlds as edges. Fig. 3(d) is a triangle of three worlds, in which each world is linked with the other two worlds. The simple triangle is a basic triadic structure from which a wide range of semantics-added triadic structures can be generated. For example, we can add arrows on edges to represent directional relationships. We can use multiple edges to connect two worlds to represent simultaneously many relationships between the two worlds. We can annotate and color edges to denote different types of information, for example, dependency, transformation, support, and others. By so doing, we can obtain many different types of triangle of three worlds.

In Fig. 1 of the TAO framework, we in fact represent a triad as a triangle. With reference to Fig. 2(b), we examine individual worlds, namely, the corners of the triangle, at the bottom level, relationships between two different worlds, namely, the edges of the triangle, at the middle level, and relationships among three worlds, namely, the triangle, at the top level. An example of (1-1)-type analysis at the middle level may be the transformation of issues in one world into issues in another world. A (2-1)-type analysis at the top level may deal with the joint effects of two worlds on the third worlds. Combining a triangle of three worlds and the trilevel-seven-element architecture of acting offers a great number of triadic approaches.

Fig. 4 gives three examples of triangle-based three-world structures and associated types of analysis. Fig. 4(a) shows that the interaction of two worlds at the bottom results in the third world at the top. For example, for personal relationship management, the bottom 'my world' and 'your world' represent two separated and typically conflicting worlds. In order to resolve conflict, it is meaningful and constructive to search for a third 'our world' that emerges from the other two worlds and arises above the two worlds [17]. The same metaphor may be used to explain human-machine systems [113]. We have two separate 'human world' and 'machine world', representing two different ways of problem solving. By integrating the two worlds, we have a third 'human-machine symbiotic world' for human-machine co-intelligence, which emerges from human and machine intelligence and is above both. Fig. 4(b) suggests that two worlds jointly support the third world. In education, a teacher-student-subject triangle is typically used to describe the relationships between the three. As insightfully stated by Palmer [60], "The teacher, who knows the subject well, must introduce it to students in the way one would introduce a friend." In other words, the world of the teacher is integrated with the world of subject to support the world of students. If we reverse the arrow of the dashed line in Fig. 4(b), we would have a situation where one world supports the integration of two other worlds. Fig. 4(c) suggests that one world supports the link that connects the two other worlds. The early discussed notion of triadic closure in social networks is an example, in which one person makes the connection of two other persons. If we reverse the direction of the arrow of the dashed line in Fig. 4(c), we would have a situation in which the link of two worlds supports the third world. Figs. 4(a)-(c) are similar in the sense that they are special cases of the (2-1)-type pattern for describing a supporting relationship between different worlds and/or their connections, as indicated by the dashed line. Their differences lie in which supports and which is being supported.

Fig. 3(e) redraws a triangle as a circle. A circle does not have a beginning point nor an ending point; starting from any point, one comes back to the same point after a complete circle. A circle thus figuratively gives a sense of completeness and another sense of infinite movement and transformation. We transform 'World A' to 'World B', to 'World C', and then back to 'World A'. Although we come back to 'World A' after a complete circle, our understanding of 'World A' is deepened or is at a higher level. More precisely, the circular movement is more an upward spiral ascending process. An example is the three worlds of cue-routine-reward loop suggested by Duhigg [21] for habit formation. The three-step loop is based on how our brain works: a cue triggers a habit and leads to a physical, mental, or emotional routine, the result of the routine produces a reward, and the loop starts again with a new cue. The continuous many repetitions of the loop give birth to

is negative energy that reduces the circle of influence. The circle of control is added explicitly by many people. This, in fact, is a good example to show that people have a preference for triadic thinking. Given a pair of two elements, people often add a third element in order to form a triad. In different contexts, there are many different interpretations of the three circles, based on which the interpretation given here is constructed.

habits. In human-machine-environment studies, Long and Huang [53] suggest a circular way in which the three worlds of human, machine, and environment correlate with, restrict, and promote each other.

Fig. 3(f) may be interpreted as a degenerated triangle. There are a number of ways to interpret this two-segment line [109]. 'World *A*' and 'World *C*' are the two extremes and 'World *B*' is the moderate middle. Alternatively, the middle 'World *B*' may be considered as a mediator between the two other worlds. The three worlds may represent a sequence of three generations, three stages, or three steps. If we draw a line connecting 'World *A*' and 'World *C*', we can represent a transitivity relationship, namely, 'World *A*' connecting 'World *B*' and 'World *B*' connecting 'World *C*' implies 'World *A*' connecting 'World *C*'.

5.2.3. Three dimensional space and three-component models

Fig. 3(g) uses a three-dimensional (3D) space metaphor to describe a triadic three-world structure. Due to the fact that we live in a 3D space, the 3D metaphor is universally used in different cultures and disciplines. A fundamental result of a three-dimension vector space is that a set of three linearly independent vectors forms a basis such that any point in the space can be produced by a linear combination of the three basis vectors. The 3D space metaphor is about decomposing one into three components and combining three components into one. Accordingly, we have three-component structures by choosing three worlds as a basis to construct and interpret a myriad of things. A good everyday example of using the 3D metaphor is the conception of the triple gunas in the Hindu worldview [51]. The triple gunas, as primary qualities, modes of nature, or driving forces, are sattva (i.e., goodness, purity, light, harmony, superiority, etc.), rajas (i.e., passion, activity, motion, ambivalence, etc.), and tamas (i.e., darkness, decay, inertia, inertness, inactivity, inferiority, etc.) [13,51,85]. It is possible to explain and characterize all things and beings by using the three basic qualities of sattva, rajas, and tamas, in terms of their proportions, dispositions, and compositions. All things have mixtures of these three gunas with differing proportions. A harmony is a right proportion of the three gunas and an unbalanced state of being is a disproportion of the three gunas.

Fig. 3(h) uses a tripod to represent a three-component structure and Fig. 3(i) is another version given by a three-pillar structured house. A tripod or a three-pillar structure is well known for its stability, strength, beauty, and completeness. It is therefore not surprising that three-pillar models are everywhere. Three elements of great communication, according to Aristotle, are ethos (i.e., the credibility of the speaker), pathos (i.e., empathy and emotional connection), and logos (i.e., logic and reasoning) [24]. A good argument is constructed and supported by using three pieces of evidence, three reasons, or three examples [78]. Three pillars of sustainability are composed of social, economic, and environmental sustainability [69], which is related to the triple-bottom-line model of business (i.e., social, economic, and environmental bottom lines), focusing on the three P's of people, profit, and the planet [26].

The tripod and three-pillar structures are metaphors of supporting one by three, in contrasting of the set-theoretic metaphors of dividing one into three. Using the tripod metaphor, we may argue that data science is supported by three disciplines of mathematics/statistics, computer science, and a specific domain, instead of the overlap of the three [111].

5.2.4. Trilevel and hierarchical models

We may build a three-level hierarchy of three worlds as shown by Fig. 3(j). This gives rise to a particular mode of three-way decision known as trilevel thinking [107]. A hierarchy reflects a top-down control among the three worlds on the one hand and a bottom-up support on the other. A world at a higher level controls its lower level world and is supported by the lower level world. The trilevel-seven-element architecture in Fig. 2 is an example of the trilevel structures of three worlds. The discussions on acting are equally applicable to trilevel thinking. More discussion on the principles and examples of trilevel thinking can be found in another paper [107] and will be given in Section 6.

Fig. 3(k) depicts a trilevel hierarchy of three worlds as a three-step stair. The stair metaphor offers a two-dimensional understanding. Vertically, the three-world stair reflects the control-support relation of levels. Horizontally, the three-world stair reflects the natural sequence of changes from one world to another. By combining the two directions, the stair shows a progressive bottom-up and left-right growth. Johnson [42] uses a three-step stair to describe a structure of personal knowledge, consisting of the three worlds of experiential knowledge (i.e., sensitivity and skill), conceptual knowledge (i.e., understanding and organization), directional knowledge (i.e., identity and motivation) from bottom up. Martin [55] refers to the three levels as stance, tools, and experiences. In their models, it is clear that a higher level guides a lower level, and a lower level informs a higher level.

Fig. 3(l) describes a trilevel structure in terms of three layers produced by three concentric circles. Like an onion, one layer encloses another layer sequentially. A concentric tricircle gives a sense of an inner-outer relationship, or a core-shell relationship, among the three worlds. Typically, an inner world determines an outer world, and the core is more important and serves as a foundation for constructing the outer ones. The inner-outer layered interpretation of a concentric tricircle makes it a commonly used architecture for explanation. For example, in understanding a computer system, the inner kernel represents machine hardware, the middle layer represents system software, and outer layer represents application software. In the Golden Circle leadership model by Sinek [80], the three circles are labeled, respectively, by Why, How, and What. By moving inside-out, a successful leader starts with Why (i.e., purpose and goals) and moves towards What. Similarly, in the model of behavior change by Clear [14], the three circles correspond to Identify, Processes, and Outcome. We build habits by moving inside-out in the identity-directed way. More examples of three-world thinking based on a concentric tricircle can be found in another paper [109].

5.3. Discussions

In this section, I have discussed four types of triadic structures of three worlds in isolation. To make the discussion more accurate and complete, it is necessary to have a few clarifications.

First, the four types are not intended as an exhaustive categorization of all possible triadic structures, but are examples for illustration. Based on the discussion, we can look for and study new types of triadic structures.

Second, these triadic structures are intended as metaphors or visual representation. While each of them brings some important features to the forefront, it unfortunately pushes some other important features to the background.

Third, for each type of structures, I used examples from various fields for illustration. In light of the previous remark, a triadic structure, as a metaphor, may not accurately and completely represent the discussed example. An interested reader may consult the references for more information.

Fourth, in many situations, one triadic structure may not be sufficient. In order to have a fuller understanding and a more complete picture, it may be necessary to use several structures simultaneously for explanation. For example, we can use both Fig. 3(a) of a Venn diagram of three circles and Fig. 3(h) of a tripod to describe data science through three disciplines [111].

Purvis et al. [69] examine the common uses of three structures, namely, Fig. 3(a) of a Venn diagram of three circles, Fig. 3(c) of a Venn diagram of three nested circles, and Fig. 3(i) of a three-pillar house, for explaining the three pillars of sustainability. They comment that “the meaning conveyed by these diagrams and the wider ‘pillar’ conception itself is often unclear, hampering its ability to be coherently operationalised.” Their comments are related to the few remarks made here. To conclude the discussion of this section, it is important to understand both the power of triadic structures for constructing a conceptual model for explanation and the possible limitations and shortcomings of particular triadic structures. The two aspects must be considered at the same time and any one of them cannot be taken as a deny of the other. We must strive for a right balance and trade-off of the two. Normally, simple triadic structures of three worlds are good for a high-level conceptualization and general explanation and their limitations may be removed through lower-level more precise articulations of details.

6. The Tao, the Way of three-world thinking

The opening sentence of the book Daodejing states the dilemma of any articulation or explication of the concept of Dao: “The Dao that can be expressed and told is not the true and eternal Dao.” On the other hand, the entire book itself makes an attempt to explain what is Dao and how to practice Dao. As demonstrated by Daodejing, explanation by examples may be one of the most effective methods. By following the principles of three-way decision, I make an effort to explain the Dao, the Way of three-world thinking by using examples.

6.1. Chinese heaven-Earth-humans triad

The heaven-earth-humans (天-地-人, tian-di-ren) triad is a central element of Chinese thinking and culture. As a multifaceted and all-encompassing notion, it has been associated with many different interpretations and understandings by different schools of thought. Any particular interpretation, while captures and reveals some aspects, inevitably misses or downplays some other important ones. It seems impossible to offer a complete, non-controversial, and universally agreeable definition or description that covers all aspects. Nevertheless, it may be safe to claim that, instead of searching for a precise definition, we can bring out the great value through the power of metaphors given by various triadic structures of the three worlds of heaven, earth, and humans.

Out of many features of heaven, earth, and humans, I consider three classes: Properties and characteristics, meaning and principles, and roles and functions. Heaven, earth, and humans have their own characteristics, follow their own Dao, and play their own roles. They function both individually and together as a whole at the same time. In different contexts, the heaven-earth-humans triad manifests in different ways and forms, offering different metaphors for us to understand and structure the reality and ourselves, as well as guiding our conducts and behaviors. Heaven may represent natural laws, never-ending movement, and constant change. Earth may represent resources, stillness, and stability. By learning the Dao, the Way of heaven and earth, receiving the best from heaven and earth, and being aided by heaven and earth, humans possesses the virtues of heaven and earth, stands firm on principles and, at the same time, is flexible and adaptive to changes.

We may use the triadic structures in Fig. 3 to explain and interpret the triad of heaven-earth-humans by focusing on three distinct and interrelated worlds. This gives rise to many different interpretations, principles, and applications of the notions of the three worlds of heaven, earth, and humans. The Venn diagram of three circles of Fig. 3(a) suggests the importance of the intersections or the common parts of the three worlds of heaven, earth, and humans. Fig. 3(b) explicitly shows that there are three distinct worlds of heaven, earth, and humans. It suggests the dual principles of division of one into three and integration of three into one. The triangle representation of three worlds in Fig. 3(d) stresses the relationships between heaven, earth, and humans. According to the tripod and three-pillar metaphors of Figs. 3(h) and (l), it may be said that Chinese thinking and culture is supported by the three fundamental notions of heaven, earth, and humans. Finally, the trilevel hierarchy of Fig. 3(j) implies the position of heaven, earth, and humans and their respective roles. Heaven is above and plays a guiding role, earth is below and plays a supporting role, and humans are in the middle and play a role of connecting heaven and earth. In similar ways, it is possible to construct other triadic structures for explanation. Each of these metaphorical interpretations and explanations provides partial and possibly misleading understanding. A much deeper understanding emerges from the complementation, integration, and cross-correction of these diversified interpretations, namely, achieving unity through diversity.

Paracka [61] suggests a connection between the heaven-earth-humans triad and three Chinese teachings of Buddhism, Daoism, and Confucianism (三教：佛,道,儒; Sanjiao: Fojiao, Daojiao, Rujiao). In particular, he categorizes each philosophical religious tradition "within the three teachings by its most distinctive attributes: Buddhism with *heaven*, spirituality, compassion, and interdependence; Daoism with *earth*, balance, health, and ecology; and Confucianism with *humanity*, family, community, and social order; thus constituting a triad of interconnected relationships among heaven, earth, and humanity." In Chinese thinking, heaven, earth, and humans work independently as three worlds and integrate together as one. For example, Chinese philosopher Xuzhi claimed: "Heaven has its seasons; Earth its resources; Man his government. This, of course, is why it is said that they 'can form a triad'" [45]. The triad reflects an essential element of Chinese philosophy on the basis of balancing and uniting the forces of heaven, earth and humans [45,61,91], namely, heaven-earth-humans as one (天地人合一). Based on the connection to the three teachings given by Paracka [61], it is related to the idea of three teachings as one (三教合一).

With respect to the metaphors of a three-dimensional space and a tripod of Figs. 3(g) and (h), the heaven-earth-humans triad suggests three basic elements and ingredients for success of any human endeavors. To be successful, we must pay close attention to bring together heavenly timing (天时, tianshi), earthly benefit (地利, dili), and human harmony (人和, renhe) [91]. In general, we may use 'heavenly timing' to denote the natural laws and governing forces, use 'earthly benefit' to denote resources/conditions and supporting forces, and use 'human harmony' to denote the social laws and human forces. In any endeavors, we must combine the three forces into one, namely, follow the natural and social laws, make proper use of resources, and respect the constraints.

Gu and Zhu [36] propose a tripartite WSR (wuli-shili-renli, 物理-事理-人理) model as a basis of a systems methodology of management. To quote from one of their papers [36], "The basic theme of WSR contends that *wuli* (regularities in objective existence), *shili* (ways of seeing and doing), and *renli* (patterns underlying human relations) constitute a differentiated whole that conditions systems projects. WSR suggests that we should design and employ appropriate methods to address and tackle *wuli shili and renli* elements in a theoretically informed and systemic way." In particular, Zhu [123] interprets *li* as representing relations in real world and human society. Thus, *wuli* is about the "relations within the world," *shili* the "relations between the self and the world," and *renli* the "relations between the self and others." The word *li* (理) in Chinese has multiple meanings, including principles, reasons, logics, patterns, order, and managing processes. Thus, the WSR methodology and model is subject to many different interpretations [46].

One possibility is to interpret the WSR model based on a triangle of three worlds [112]: W represents the natural world (physical things, domains of natural sciences), R represents the human world (human and human society, domains of psychology, social sciences, humanities, etc.), and S represents the applied world (affairs of concerns, pragmatic problem-solving, human conduct, domains of management science, engineering, operational research, etc.). The interpretation of world S is an extension of the "ways of seeing and doing" suggested by Gu and Zhu [36]. Theories and knowledge discovered in the two worlds W and R are used to guide human conduct in world S, which may change both worlds W and R. To be a better problem-solver, one must integrate the three worlds into a "differentiable whole" [123].

Given the importance of the heaven-earth-humans triad in Chinese thinking in general and the three indispensable ingredients, i.e., heavenly timing, earthly benefit, and human harmony, for success in specific, it might be useful to search for a corresponding interpretation for the WSR model. Worlds W and R match, to a large degree, with the worlds of heaven and humans, but world S does not match well with the world of earth. According to an online Chinese dictionary,[9] '*shi*' (事) means "phenomena and activities in nature and society (自然界和社会中的现象和活动)." Thus, *shili* (事理) may have two meanings, one is the inherent nature, principles, patterns, structures, and order of the phenomena and activities, and the other is human perception of the phenomena and activities and the associated response of actions. The latter is exemplified by interpreting *shili* as the "ways of seeing and doing" or the "relations between the self and the world." If we take the first meaning, we may relate *shili* to earth. Earth generates its benefit and advantage by providing resources that support all living things on the planet. Having the necessary resources is a fundamental constraint and condition for the existence of any "phenomena and activities in nature and society." By extending this way of reasoning, it may not be far-fetched to suggest that *shili* can be viewed as the internal principles, the Dao, the Way of "phenomena and activities" on earth, connecting the world S and earth. Therefore, we have a new understanding of the WSR methodology and model. WSR is inspired by the Chinese three teachings and heaven-earth-humans triadic thinking. It promotes a holistic Chinese management style that follows the physical laws of nature (i.e., *wuli*, 物理, the world of heaven), obeys social laws of human society (i.e., *renli*, 人理, the world of humans), and accords with the internal principles, such as the resources, constraints, and conditions, of the concerned affairs (i.e., *shili*, 事理, the world of earth). Moreover, it aims at balancing and uniting the three to form a wholistic one, in correspondence to the thinking of heaven-earth-humans as one and Buddhism-Daoism-Confucianism as one.

[9] https://zd.hwxnet.com/.

Table 1
Main ingredients of the triadic conception of CPSS of Wang [90].

3 world	Physical world World 1	Mental world World 2	Artificial world World 3
3 thinking	Being	Becoming	Believing
3 philosophy	Phenomenographical Philosophy of Being	Process philosophy of Becoming	Parallel philosophy of Believing
3 knowledge	Descriptive knowledge	Predictive knowledge	Prescriptive knowledge

6.2. *Popper's physical-mental-artificial three worlds*

Popper [67,68] introduces a three-world model for studying the reality, knowledge, and human knowing. The model consists of three interconnected and interacting worlds: World 1 of physical bodies (e.g., non-living physical objects and biological objects), world 2 of mental/psychological states and processes, perception, thought, and subjective experiences, etc., and world 3 of whatever produced by the human mind (e.g., languages, theories, scientific conjectures, etc.). This model has been adopted and adapted by many authors from different domains [65,71,81,89].

We may explain the three worlds and their interactions by using triadic structures in Fig. 3, as well as their variations or combinations. For example, many authors suggest various explanations by using a Venn diagram of three circles, i.e., Fig. 3(a) [30], a disjoint trisection, i.e., Fig. 3(b) [56,118], a triangle with arrowed edges to form a circle, i.e., a triangle version of Fig. 3(e) [25,65], a variation of three-node line, i.e., Fig. 3(f) [32], a combination of Venn diagram and circle, i.e., Figs. 3(b) and (e) [90], a trilevel hierarchy, i.e., Fig. 3(j) [47], and a concentric tricircle, i.e., Fig. 3(l) [113]. I briefly describe a few interpretations and explanations.

The numbering of the three worlds gives a sense of circular dependency or transformation of three worlds as shown by Fig. 3(e). Physical world 1 exists first; through world 2 of mental activities and processes, humans observe and make sense of world 1; the results are human-created things that exist in world 3 as abstract ideas and/or in world 1 as physical objects. The things in world 3, created by world 2, may be used to change world 1. Humans are constantly searching for a better world by exploring the three worlds and their relationships [68]. The inner-middle-outer architecture of Fig. 3(l) gives another sense of human understanding and human relationship to the physical world [113]. We have simultaneously three aspects of the same reality, namely, the inner mental world 2, the outer physical world 1, and the middle conceptual world 3 of knowledge that connects the other two worlds. In this case, world 3 serves as the interface between our inner mental world and external physical world [32].

By adapting Popper's three-world model, Penrose [65] introduces a three-world model, consisting of the physical, the mental, and the Platonic mathematical worlds, which is illustrated by a triangular physical-mental-Platonic circle. Stern [83], without referring to Popper's model, suggests another triadic conception of the reality, in which the reality is conceived and represented as "unified and wholistic as well as differentiated" three worlds: physical world of matter/energy, theoretical world of meaning, and phenomenological world of experience. Furthermore, Stern gives a diagram by enclosing the triangle of the three worlds in a circle that represents the unity and wholeness.

Inspired by Popper's three-world model, Wang [89,90,119] initiates a new field of study known as the cyber-physical-social systems (CPSS) [11,120–122] for intelligent enterprises and industries. Two conceptual constructs used by Wang are the notions of three worlds and two spaces. The three worlds are the physical world, mental world, and artificial world, which gives rise to the physical space (or physicspace) and cyberspace. Wang adds the mental world to a two-world framework of cyber-physical (or cyberphysical) systems (CPS) [5], in order to show explicitly the role of humans in such systems.[10] There is a circular physical-mental-artificial worlds causality that can be depicted by combining Figs. 3(b) and (e). Humans and machines work together collaboratively in the parallel physical space and cyberspace. Table 1, taken from Wang [90], summaries the main ingredients of this triadic conception of CPSS. The parallel physically embodied intelligence in the physicspace and the computationally embodied intelligence in the cyberspace are supported by the prescriptive intelligence, predictive intelligence, and descriptive intelligence.

The discussions of Wang [89,90] suggest that the physicspace concerns physical implementations and physical systems, and cyberspace concerns digital implementations (or simulations) and artificial systems. Machines and humans work together in the two spaces. Wang argues that the two spaces emerge from three worlds, instead of corresponding to the three world. On the other hand, it seems that physicspace corresponds more to the physical worlds than to the other two worlds, and cyberspace more to the artificial world. This leaves none for the mental world. By following the principles of three-way decision, to make a triad of three spaces, I may suggest that we create a mentalspace corresponding to the mental world. In other words, we can work out mental implementations or systems in our head. A piece of evidence supporting this creation is the methods and concept of thought experiments [28,82], denoting the play of various ideas and logical arguments in our head. With three spaces, we have three realizations or systems that work in parallel, as well as the corresponding

[10] This is, in fact, another example showing a preference for a triadic patterned conception, namely, adding a third element to two existing elements to form a triad.

physical, mental, and conceptual models. Systems and models in the three parallel spaces cross-validate, cross-confirm, and cross-correct each other. This may increase the probability of success and decrease chances of failure. By mirroring the metaphorical saying, “A threefold cord is not quickly broken,” we may say that “a three-space model is not quickly broken.”

One can easily observe both similarities and differences of these interpretations and variations of Popper’s three-world model. Although they may name or interpret the three worlds differently and use different triadic structures, they do agree on a triadic understanding. It is true that the contents and insights of Popper’s three-way model play a crucial role for its widely acceptance and adaptation. At the same time, it may be said that its triadic form and view is another important factor. This claim is supported by Google searches of “m-world model”, where m takes the value of ‘one’, ‘two’, ‘three’, ‘four’, etc. Among the many multi-world models, three-world models are one of the most popular ones.[11]

6.3. Symbols-meaning-value space

The concept of a symbols-meaning-value (SMV) space provides a trilevel hierarchy of three worlds [111]. It weaves together three ideas, namely, Weaver trilevel categorization of communication problems [76], the perception-cognition-action triad in psychology and cognitive science [39,59], and the data-knowledge-wisdom hierarchy in information and management sciences [1,29,72].

To put Shannon’s information theory into a proper context, Weaver [76] insightfully divides communication problems into a trilevel hierarchy of three categories:

- Technical level problems: Transmitting the symbols.
- Semantic level problems: Conveying the meaning of the symbols.
- Affective level problems: Affecting the conduct of the receiver.

The three levels focus on different types of problems and answer different types of questions, from easier ones to more difficult ones.[12] In the case of human communication through speaking and writing, we may interpret the three levels by the Words-Meaning-Impact triad.

We can draw a correspondence between Weaver trilevel and the perception-cognition-action triad, or the cognition-affection-conation triad, in psychology and cognitive science [39,59]. More specifically, perception is related to symbols, cognition is related to meaning, and action is related to value. These triads serve as a basis for building communication models that are closely related to Weaver trilevel hierarchy. For example, based on the perception-cognition-action triad, Carliner [9] describes a trilevel framework for information design, consisting of (a) physical design for helping users find information, (b) cognitive design for helping users understand information, and (c) affective design for motivating users to perform. Based on the cognition-affection-conation triad, Lavidge and Steiner [49] introduce a six-stage marketing model that progressively focuses on the three major functions of advertising. Effective advertising consists of the cognitive phase for providing information, affective phase for changing attitudes, and conative phase for stimulating desires.

We can further extend the correspondence between Weaver trilevel hierarchy and the perception-cognition-action triad to the DKW (Data-Knowledge-Wisdom) hierarchy [111]. Perception concerns symbols-level processes, which produces data; cognition concerns semantic-level processes, which produces knowledge; action concerns wise use of knowledge at affective level, which produces values and shows wisdom. By labeling the three levels as the symbols, meaning, and value, we immediately have the concept of an SMV (Symbols-Meaning-Value) space. An SMV space consists of three worlds, namely, world S of symbols, world M of meaning, and world V of value. By combining ideas from the three fields of communication, psychology and cognitive science, and information and managements sciences, the concept of an SMV space is applicable to a much broader context and provides a structure for trilevel thinking in many other fields.

To demonstrate the usefulness of the concept of an SMV space, I made an attempt to use it to construct a conceptual model of data science. In particular, world S is about data (i.e., raw symbols), world M is about knowledge (i.e., meaning of data), and world V is about wisdom (i.e., value from wise use of knowledge). The three-level structure reflects the dependency and transformation between data, knowledge, and wisdom. A conceptual model of data science needs to consider the issues in the three worlds of the data, the knowledge hidden in the data, and the value of the knowledge, as well as the issues arisen from the interactions of the three worlds. Broadly speaking, three goals of data science are (a) to make data a kind of resources through data collection, storage, retrieval, etc., (b) to make data meaningful through data analysis and knowledge discovery, and (c) to make data valuable through practical application of the knowledge in data for making wise decisions and taking the right actions.

In another attempt to build a conceptual model for explaining human-machine co-intelligence [113], I viewed the SMV space as an architectural system or a metaphorical structure used by an intelligent being to understand and organize itself,

[11] Google searches using exact phrase “m-world model” on March 22, 2023 show the following numbers of results: one, 162,000; two, 110,000; three, 36,600; four, 11,500; five, 6,700; six, 1,560; seven, 6.

[12] According to Shannon [76], “semantic aspects of communication are irrelevant to the engineering problem” of transmitting symbols. The success of Shannon’s formulation of the information theory may possibly be explained by the simplification obtained from this separation of the symbols of a message from the meaning and effect of the message.

its environments, and others,[13] as well as their relationships with each. Human-machine co-intelligence emerges from human-machine symbiosis in the SMV space. There are three fundamental principles of human-machine co-intelligence. The principle of unified oneness: Human-machine co-intelligence is the third intelligence that is based on human intelligence and machine intelligence on the one hand and is above both on the other hand, as illustrated by Fig. 4(a). Human-machine co-intelligence is not possessed by either humans or machines, but through their seamless unification and integration. The principle of division of labor: Human-machine co-intelligence combines the computational power of machines and the cognitive power of humans through proper division of labor. Moving from world S, to world M, and to world V, humans are doing more work and the machines are doing less. The principle of coevolution: Humans and machines mutually adapt to each other, learn from each other, and work with each other as equal partners. Human-machine co-intelligence exploits a mutualism symbiosis in which both humans and machines benefit and, at the same time, avoids a parasitism symbiosis in which one hurts the other. In this respect, in addition to their own SMV spaces, humans and machines share a common SMV space.[14] The notion of SMV space is a structure and a starting point for explaining human-machine co-intelligence. As a possible future research direction, we may study human-machine co-intelligence with a human-machine-environment triad by explicitly bringing in a third element of environment. This will establish a connection to human-machine-environment studies [53].

6.4. A 3×3 architecture for explainable AI

The concept of an SMV space suggests that we need to explain a theory or model at three levels: the content of the theory, the meaning of the theory, and the utility of the theory.[15] By following this argument, I turn my attention to the possibility of applying the SMV space to explainable Artificial Intelligence (XAI) [2,37]. Several authors have applied three-way decision for building interpretable intelligent systems, for example, interpretable classification and recommendation systems [50,79,116,117]. Independent of any particular types of intelligent system, I introduce a 3×3 architecture as a blueprint for designing an easy-to-explain intelligent system, as a methodology for constructing an explanation, and as a template of an explanation.

The concept of SMV spaces suggests a plausible trilevel scheme for constructing an easy-to-understand explanation in XAI. The notion of SMV spaces provides a trilevel hierarchy. Following the principles of three-way decision, one can ask three questions at each level. The combination of the three levels and the three questions at each level results in a 3×3 architecture. Depending on particular interpretations of the three levels and the three questions at each level, there are at least three possible uses of the 3×3 architecture for XAI.

First, the 3×3 architecture may facilitate the design and implementation of an intelligent system that can be easily explained. The 3×3 architecture requires a trilevel understanding of an intelligent system, with the three levels representing the data, knowledge, and decision/action of the system. This offers a data-driven, knowledge-based, and action-oriented simple view of an intelligent system. In other words, an intelligent system turns the input data into the right and wise decision and action, through the use of knowledge. The three questions at each level may concern particular tasks and functions of the system at that level. By viewing data as the input, action as the output, and data-to-action transformation as the process of an intelligent system, it might be possible to construct an easy-to-understand explanation of the system. This view is basically used in developing the notion of human-machine co-intelligence [113]. The model of human intellectual functioning proposed by Costa [15] uses the triad of input-processing-output to represent "intake of data through the senses," "making sense out of the data," and "applying and evaluating," which closely corresponds to the use of SMV spaces for explaining human intelligence and machine intelligence.

Second, the 3×3 architecture offers a systematic approach for constructing an explanation. One may ask different questions sequentially at the data/input, knowledge/process, and action/output levels. While a data-driven explanation is constructed in one direction, an action-guided explanation is constructed in the other direction, as guided by the earlier discussed principles of trilevel thinking. Finally, the 3×3 architecture provides a format and template of an explanation. In other words, an easy-to-understand explanation of an intelligent system may be presented in a 3×3 format. The three uses of the 3×3 architecture support each other. A system following the 3×3 design easily offers a 3×3 method of explanation

[13] This tripartite characterization corresponds to the three varieties of knowledge suggested by Davidson [19]. My use of the knowledge of self, environments, and others easily follows from Davidson's words [19]: "I know, for the most part, what I think, want, and intend, and what my sensations are. In addition, I know a great deal about the world around me. I also sometimes know what goes on in other people's minds. Each of these three kinds of empirical knowledge has its distinctive characteristics."

[14] A few important issues regarding AI and human-machine relations are relevant to the discussion here, such as alignment and control. Christian [12] argues that artificial intelligence systems, in particular machine learning, need to be aligned with human values. Russell [73] points out that advances in AI may pose a potential risk to the human race by out of control superhuman AI. Future AI research must ensure that machines remain beneficial to humans and we humans must retain "absolute power over machines that are more powerful than us." By living together in the three worlds of SMV, namely, symbols/data, meaning/knowledge, value/wisdom, humans and machines may coexist in harmony.

[15] The notion of SMV space may also shed a new light on the earlier discussed tripartite categorization of the interpretations, explanations, and applications of the Chinese classic Yijing at the three levels: (1) images and numbers at the S (Symbols) level, (2) meanings and principles at the M (Meaning) level, and (3) practice (living) and applications, according to the meanings and principles, at the V (Value) level. Although this organization may not be hundred percent appropriate or accurate, it does provide a good enough approximation in terms of the text itself, the meaning of the text, and the value of the text.

Table 2
3×3 architecture of explanations.

SMV	Explanation level	Questions
Value	Value	What is the value of the results? Why are the results valuable? How to use the results?
Meaning	Meaning	What is the meaning of the results? Why are the results meaningful? How to interpret the results?
Symbols	Results	What are the results? Why are certain input/conditions required? How does the system derive the results?

construction, which in turn naturally produces a 3×3 explanation. In the reverse direction, a 3×3 explanation calls for a 3×3 construction method, which in turn calls for 3×3 design principles for intelligent systems.

Consider now the formulation of a particular 3×3 explanation. The SMV triad may be viewed as a trilevel results-meaning-value (RMV) framework of explanation. Like data, the results from a system may be considered as the raw materials that need, or can be used to construct, an explanation. An intelligent system explains its results, outcome, or output (e.g., recommendations, actions, behaviors, etc.), the meaning of the results, and the value of the results at three separate levels. Moreover, at each level, it is possible to apply the ideas of the Venn diagram or the triangle configurations of three worlds to focus on three related questions characterized by the What-Why-How triad. Table 2 summarizes the main features of this 3×3 architecture of explanations. A 'What' question is about the existence, a 'Why' question is about the reasons/motivations, and a 'How' question is about the processes/applications. By focusing on three fundamental questions of What, Why, and How at each of the three levels, an explanation follows a clearly defined logic, is easy-to-understand, and covers three important aspects.

A trilevel explanation with three basic questions at each level reflects the principles of triadic thinking. Generally speaking, at a given time, it is possible to focus on the discussion at each level without much interference from the other two levels. In other words, we may need to consider only three questions at a particular level, instead of nine questions at all three levels simultaneously. The labels of the three levels and the three questions at each level in Table 2 may be interpreted more liberally. Depending on different applications, it is possible to use other labels and to ask other types of questions. Nevertheless, the essential components and the structure of the 3×3 architecture remain unchanged. The 3×3 architecture provides a general framework. In some situations, it may be only necessary to consider some of the nine issues when constructing an explanation. This is particularly true if the results from a system are simple and/or self-explanatory.

7. Conclusion

Three-way decision and three-world conception mutually support each other. In one direction, three-world models enrich the studies of three-way decision by offering new views, models, and methods. In the other direction, the fundamental philosophy and principles of three-way decision may find new applications in three-world models. In this paper, I explored the connections of three-way decision and three-world conceptions. I examined many triadic structures, metaphors, and three-world conceptions for the purpose of building and explaining the basic concepts, a concrete TAO (triading-acting-optimizing) framework, and the Dao, the Way of three-way decision. I used many examples taken from different disciplines to support a claim that thinking through three worlds offers the necessary simplicity and flexibility for building a theory, a model, an argument, etc. A high-level conceptual model, with a three-part structure, focusing on only three things, or consisting of three basic ingredients, is simple-to-understand, easy-to-remember, and practical-to-use.

To articulate the Dao, the Way of three-way decision, I examined three theories of three-world thinking: the Chinese heaven-earth-humans triad, the three-world model of Popper [67,68], and the concept of symbols-meaning-value (SMV) spaces [111,113]. These three theories are rich in contents, high in impact, and significant in value. Each of them involves complex, deep, and multifaceted concepts and ideas subject to many different interpretations in different contexts. I have just scratched the surface. In particular, an SMV space is a new concept introduced recently and remains largely unexplored. A more systematic investigation on the contributions and implications of these three-world conceptions to a theory of three-way decision may be a worthwhile future research direction.

I motivated this study by stating that three-way decision is a human-inspired theory. Since humans frequently and naturally think in threes, theories, models, or methods are easy to grasp and understand if they are constructed based on a tripartite architecture. Therefore, explanations of any intelligent systems may be built in a human-friendly way by following a tripartite scheme. It may be fruitful to apply the principles and ideas of three-way decision and three-world thinking to address the issues of the quality and effectiveness of explanations in explainable artificial intelligence (XAI). In the paper, I only presented a proposal for an important research direction, which may be called "three-way decision for explainable AI." Although I gave an outline of a trilevel framework for building explanations based on the notion of an SMV space, many fundamental questions remain unanswered. Based on the discussion in the paper, we can explore the new territory of three-way decision and three-world thinking for XAI.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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