Three-Way Group Decision-Making With Personalized Numerical Scale of Comparative Linguistic Expression: An Application to Traditional Chinese Medicine

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***Abstract*—In most real-life decision-making situations, experts tend to utilize linguistic information rather than numerical values to express their preferences or evaluations on alternatives. Considering the complexity of the decision-making problems, it is usually difficult to use single linguistic terms to elicit experts' preferences. Linguistic expressions that are close to cognition of human-being, such as comparative linguistic expression (CLE), are suggested to be applied in such cases. Three-way decision (3WD) has been proved an effective manner to handle multiattribute decision-making (MADM) problems, however there is a lack of 3WD methods dealing with linguistic expressions. By combining CLE and 3WD, a new multiattribute three-way group decision-making (3WGDM) method incorporating CLE (called CLE-3WGDM method) is proposed. A novel personalized numerical scale computation method, based on predecision in 3WGDM, is introduced to manage diverse interpretations of CLE for different individuals. Afterward, the attribute weights are calculated through a novel optimization model, which applies the principle of deviation maximization and minimum entropy of CLE. A novel 3WD-social network method is presented to compute the weights of experts. Comparative analysis with other existing methods have been carried out to verify the feasibility of the proposed one. Finally, the CLE-3WGDM method is applied to a traditional chinese medicine (TCM) decision problem.**

***Index Terms*—Comparative linguistic expression (CLE), computing with words (CWW), linguistic group decision making, personalized numerical scale (PNS), three-way decision (3WD).**

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I. Introduction

THE concept of three-way decision (3WD), proposed by Yao [1], originates from decision-theoretic rough set (DTRS). It involves the process of thinking, problem-solving, and computing in three or through triads [2]. With a tripartite classification principle, it divides a domain into three mutually exclusive parts, related to acceptance, rejection, and noncommitment respectively, in specific decision-making situations. With 3WD theory, "delay action" is added as a strategy to the traditional "accept–reject" decision [3], to address uncertain risks. During the past few decades, 3WD has been successfully applied to various fields, such as medical diagnosis [4], investment appraisal [5], digital optimization [6], [7], conflict analysis [8], clustering analysis [9], and so forth.

In many real-life 3WD situations, it is difficult for experts to determine crisp values as evaluation or loss function. Linguistic variables have been used in 3WD to capture the uncertain information. For instance, Han et al. [10] and Wang et al. [11] combined probabilistic linguistic term sets (PLTS) and 3WD to handle decision-making problems. Using the concept of multi-granularity and 3WD, Pang et al. [12] proposed a data-driven group decision-making (GDM) method that considers risk attitudes in an interval intuitionistic uncertain linguistic environment. Existing linguistic variables concerned in 3WD environment, no matter PLTS or interval intuitionistic uncertain linguistic information, is not close to the cognition of human-being. Rodríguez et al. [13] initially introduced the concept of comparative linguistic expression (CLE) to make the linguistic information expression more aligned with experts' cognition in decision-making. Several extension models of CLE have been constructed after its concept was proposed, such as 2-tuple comparative linguistic expression (2-TCLE) [14], extended comparative linguistic expressions with symbolic translation (ELICIT) information [15], and so on. Therefore, it is necessary to introduce linguistic expression which is closer to human-being cognition (such as CLE) into 3WD, to increase the flexibility of linguistic information expression for experts.

In the computing with words (CWW) paradigm, "words mean different things for different people" [16]. personalized numerical scale (PNS) has outstanding performance to deal with such difference [17], [18]. In recent years, interesting works on PNS are carried out. For instance, Li et al. [19] studied PNS of linguistic terms based on consistency in hesitant linguistic decision-making. Tang et al. [20] studied the PNS of linguistic terms from distribution linguistic preference relations. Li et al. [17] constructed a model to generate the interval-valued numerical scale (NS). In the paradigm of 3WD, a linguistic variable also means different things for different people. Although PNS of single linguistic terms has been explored from various perspectives, it has not received significant attention within the 3WD framework. Furthermore, neither PNS of CLE nor its extension models have been investigated, restricting the management of different meanings for linguistic variables in GDM, limiting the flexibility and rationality of application for CLE in 3WD.

When more than one expert participate in decision, the 3WD turns to a three-way group decision-making (3WGDM), and the weights of experts should be determined to achieve decision result. Considering trust relationships among experts, attempts have been made to utilize social network to determine the experts' weights in 3WGDM. For instance, Liu et al. [21] studied network-based evidential three-way theoretic model to do decision analysis in large-scale GDM. Wang et al. [22] considered the influence of externality in social network and calculated the weights of experts in 3WGDM based on social network. Liang and Duan [23] considered social networks in the large-scale consensus 3WGDM with individual competitive behavior. It is noticed that in all of these social network-based weights determination methods in 3WGDM, the difference in trust degree among experts can not be reflected. To fill this gap, this article integrates the idea of 3WD into a social network to distinguish the trust degree among experts and proposes a novel 3WD-social network to determine the weights of experts in 3WGDM.

If more than one attribute is concerned in 3WGDM, it turns to be multiattribute 3WGDM. In the current research, a new multiattribute 3WGDM approach with new "expert weights" and "attribute weights" determination schemes will be provided, CLE will be applied to increase the flexibility of linguistic information expression, PNS will be adopted to address different meanings of CLE for different people, to make the decision result more reliable. The main motivations to carry out the current research are declared here.

1) While numerous 3WD models have been introduced within a linguistic environment, linguistic expression under the framework of 3WD remains not close to cognition of human beings. In addition, the linguistic information expression in 3WD lacks sufficient flexibility.
2) Linguistic variable has different meanings to different people in 3WD. Previous researches on 3WD fail to distinguish such a difference. Ignorance of PNS results in the loss of information in the CWW process, and limits the rationality of decision result.
3) Under the framework of 3WGDM, previous expert weight determination schemes based on social network can not reflect the difference in trust degree among experts, while existing attribute weight determination schemes usually consider single influence factors. These may lead to unreasonable or unbelievable decision results.
4) With the advantage that the mental characteristics of experts are visible [24], traditional tomada de decisao iterativa multicriterio (TODIM) [25] has been used to compute the conditional probability in 3WGDM [10]. However, the limitation of traditional TODIM has been heated discussed [26], [27]. The application of TODIM within 3WGDM requires updates to incorporate its improved versions, while also adapting to more complex linguistic expression environment.

Therefore, the current research aims to conquer limitations of existing 3WD models in personalized linguistic information processing, and enrich the 3WGDM approaches with PNS of CLE. The innovations of this work are the following ones.

1) A novel 3WD model is proposed, which increases the flexibility of linguistic expression, through the incorporation of CLE. Inspired by the concept of relative loss function proposed by Jia and Liu [28], relative loss function is derived from evaluations on alternatives with related to attributes, in form of CLEs.
2) A new PNS computation model is proposed based on predecision, under the framework of 3WD. It is not the first time predecision is applied in 3WD [29], but it is the first time that predecision in 3WD is used to address the personalized semantic of linguistic variables in 3WD. PNS of CLE is introduced on the basis of PNS of single linguistic terms, and then applied to determine the decision result.
3) Benefit from 3WD thinking mode, new in-degree and out-degree coefficients, which reflect trust disparities, are defined. In this way, a novel social network based expert weight determination method is proposed under the framework of 3WGDM. In addition, a novel scheme for determining attribute weight in multiattribute 3WGDM is proposed, taking into account not only the uncertainty contained in CLE, but also the maximum deviation principle.
4) A CLE-TODIM method is proposed to compute the conditional probability in 3WGDM. This method is not only a simple extension of improved TODIM method [27] under more complex linguistic expression environment, but also an improvement enriched with PNS of CLE.

The rest of this article is organized as follows. Section II briefly recalls the concepts of 3WD with DTRSs, CLE and hesitant fuzzy linguistic term set (HFLTS), as well as PNS. Section III-A describes the formulation of the multiattribute group decision-making (MAGDM) problem which needs to be solved. Section III-B presents the optimization model to compute the PNS for single linguistic term, and a method for calculating the PNS for CLE. Section III-C introduces the 3WGDM model based on PNS of CLE, proposes new methods for determining attribute weights and expert weights, and also presents CLE-TODIM method based on PNS for determining the dominance classes. Section III-D presents the multiattribute

TABLE I
LOSS FUNCTION RELATED TO COST OF ACTIONS IN TWO STATES [1]

	J	$\neg J$
a_P	λ_{PP}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}

3WGDM algorithm. Section IV provides an example and the related comparative analysis. Finally, Section V concludes this article and points out the future works.

II. PRELIMINARIES

In this section, some basic concepts that will be applied in the current research are presented.

A. 3WD Theory Based on DTRS

To reduce the decision-making risk, and handle the uncertainty contained in decision-making problems, Yao [1] introduced the concept of DTRS, which considers two states $\{J, \neg J\}$, indicating an alternative belongs to J or not belongs to J, and three actions $\{a_P, a_B, a_N\}$ [30]. With the three actions, it is possible to judge whether x belongs to positive region $POS(J)$, boundary region $BNS(J)$, or negative region $NEG(J)$, corresponding to accept, noncommitment, and reject, respectively. In Table I, λ_{PP}, λ_{BP}, and λ_{NP} denote the loss function with actions a_P, a_B, and a_N when the alternative x belongs to J, and λ_{PN}, λ_{BN}, and λ_{NN} denote the losses caused by actions a_P, a_B, and a_N for the alternative x which belongs to $\neg J$.

The conditional probability of an alternative x belongs to J is denoted by $Pr(J|[x])$, where x is usually expressed by its equivalence class $[x]$. Therefore, for each alternative x, the expected loss $R(a_\bullet|[x])(\bullet = P, B, N)$ can be calculated by [31]

$$R(a_\bullet|[x]) = \lambda_{\bullet P} Pr(J|[x]) + \lambda_{\bullet N} Pr(\neg J|[x]). \quad (1)$$

With the Bayesian decision procedure, the following decision rules are obtained with the minimum decision cost [1], [32].

(P) If $R(a_P|[x]) \leqslant R(a_B|[x])$ and $R(a_P|[x]) \leqslant R(a_N|[x])$, then $x \in POS(J)$.

(B) If $R(a_B|[x]) \leqslant R(a_P|[x])$ and $R(a_B|[x]) \leqslant R(a_N|[x])$, then $x \in BND(J)$.

(N) If $R(a_N|[x]) \leqslant R(a_P|[x])$ and $R(a_N|[x]) \leqslant R(a_B|[x])$, then $x \in NEG(J)$.

The minimum risk decision rules (P)–(N) determine each alternative x is fit for acceptance, noncommitment, or rejection of 3WD. These rules can be further simplified, more details can be found in [1].

B. Comparative Linguistic Expression

Rodríguez et al. [13] defined the context-free grammar, to generate CLE in a formal manner. For the convenience of discussion, it is assumed that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set such that $s_\gamma \leq s_F \iff \gamma \leq F$ $(\gamma, F \in \{0, 1, \ldots, g\})$.

Definition 1 (See [13]): Let G_H be a context-free grammar. $G_H = \{V_N, V_T, I, P\}$ composed of

$$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$$
$$V_T = \{less\ than, greater\ than, between, and, s_0, s_1, \ldots, s_g\}$$
$$I \in V_N.$$
$$The\ production\ rules\ can\ be\ outlined\ as$$
$$P = \{I ::= \langle primary\ term\rangle|\langle composite\ term\rangle$$
$$\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle|\langle binary\ relation\rangle\langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle$$
$$\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g$$
$$\langle unary\ relation\rangle ::= less\ than|greater\ than$$
$$\langle binary\ relation\rangle ::= between$$
$$\langle conjunction\rangle ::= and\}.$$

Based on such a grammar, three types of CLE can be generated, in form of "less than s_γ", "greater than s_γ" and "between s_γ and s_F."

With the concept of CLE, HFLTS was also introduced in [13]. Fuzzy semantic of CLE is usually studied based on the fuzzy representation approaches of HFLTS.

Definition 2 (See [13]): Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. A HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.

With transformation functions, CLE can be converted to HFLTS, to facilitate the CWW process in decision-making.

Definition 3 (See [13]): Let S_{ll} be the set of all CLE generated by G_H based on S, and let E_{G_H} be a function. By applying the function $E_{G_H} : S_{ll} \to H_S$, a CLE can be transformed into a HFLTS.

$$E_{G_H}(s_\gamma) = \{s_\gamma\};$$
$$E_{G_H}(less\ than\ s_\gamma) = \{s_F|s_F \in S\ and\ s_F \leq s_\gamma\};$$
$$E_{G_H}(greater\ than\ s_\gamma) = \{s_F|s_F \in S\ and\ s_F \geq s_\gamma\};$$
$$E_{G_H}(between\ s_\gamma\ and\ s_F) = \{s_v|s_v \in S\ and\ s_\gamma \leq s_v \leq s_F\}.$$

C. Personalized NS

Dong et al. [33] initially introduced the concept of 'NS of linguistic terms.

Definition 4 (See [33]): Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and R be the set of real numbers. A function $NS : S \to R$ is defined as NS of S, and $NS(s_\gamma) \in R$ is called numerical index/NS of $s_\gamma(\gamma \in \{0, 1, \ldots, g\})$.

If $NS(s_{\gamma-1}) < NS(s_\gamma)$ for $\gamma \in \{0, 1, \ldots, g\}$, then NS is called weak ordered on S. NS is also used to extend 2-tuple linguistic representation model. More details about NS of 2-tuple linguistic value can be found in [33].

If more than one expert exists in a GDM, NS of a linguistic term tends to be different for different people. For instance, a linguistic term "good" means 0.3 for an expert, however it may mean 0.6 for another. The function $PNS^k : S \to R$ is defined to describe the PNS for expert e_k. $PNS^k(s_\gamma) \in [0, 1]$ is called the PNS of term s_γ for e_k, and $PNS^k(s_\gamma) + PNS^k(s_{g-\gamma}) = 1$ for $\gamma = 0, 1, \ldots, g$. In [34], we declared the difference between PNS and personalized individual semantic for linguistic variables.

III. CLE-3WGDM Model Based on PNS

After a problem formulation, the approach to compute PNS of CLE in 3WGDM is presented in this section. Afterward, the CLE-3WGDM model, the decision-making steps and the CLE-3WGDM algorithm are provided.

A. Decision Making Problem Formulation

In a GDM problem, all experts are requested to provide evaluations on alternatives with respect to different attributes. There are n experts denoted by $E = \{e_1, e_2, \ldots, e_n\}$, m alternatives denoted by $X = \{x_1, x_2, \ldots, x_m\}$, and q attributes denoted by $C = \{c_1, c_2, \ldots, c_q\}$. The experts are allowed to provide assessments in form of CLE. Let $F^k = (f_{ij}^k)_{m\times q}$ be the evaluation matrix provided by expert $e_k (k \in \{1, 2, \ldots, n\})$. By applying the transformation function E_{G_H} on each element, the evaluation matrix F^k can be converted to a matrix in which each element is a HFLTS, which is denoted by $HF^k = (HF_{ij}^k)_{m\times q} = (E_{G_H}(f_{ij}^k))_{m\times q}$. The problem will be solved by applying 3WD method. The predecision $O^k = \{o^k(x_1), o^k(x_2), \ldots, o^k(x_m)\}$, where $o^k(x_i) \in [0, 1] (i \in \{1, 2, \ldots, m\})$ are requested from experts according to their intuition and prior knowledge, to compute the PNS for linguistic information. Preclassification can be obtained from the predecision as $H^k = \{H_1^k, H_2^k, \ldots, H_l^k\}$, where $L = \{1, 2, \ldots, l\}$ denote the index of the preclassification. For each expert e_k, the weighting vector of attributes is denoted by $R_k = \{r_1^k, r_2^k, \ldots, r_q^k\}$, and $r_j^k (j \in \{1, 2, \ldots, q\})$ is the weight of attribute c_j. The weights of experts are determined by the trust relationships among social network, which is denoted by $W = \{w_1, w_2, \ldots, w_n\}$.

The 3WGDM method, which will be proposed in the following sections, is presented as Fig. 1.

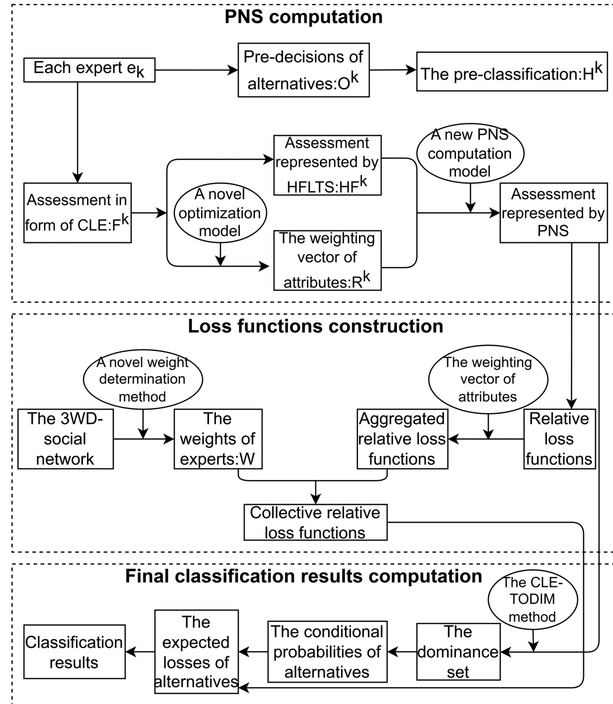

Fig. 1. Process of 3WGDM.

TABLE II
PREDECISION TABLE PROVIDED BY e_k

Alternatives	x_1	x_2	$\ldots$	x_m
Pre-decision	$o^k(x_1)$	$o^k(x_2)$	$\ldots$	$o^k(x_m)$

TABLE III
EVALUATION COLLECTED FROM EXPERT e_k

Alternatives	Attributes			
	c_1	c_2	$\ldots$	c_q
x_1	f_{11}^k	f_{12}^k	$\ldots$	f_{1q}^k
x_2	f_{21}^k	f_{22}^k	$\ldots$	f_{2q}^k
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_m	f_{m1}^k	f_{m2}^k	$\ldots$	f_{mq}^k

B. PNS Computation

1) Preclassification Based on Predecision: In a real-life MAGDM situation, usually it is feasible to request experts to provide preliminary judgments based on their own observation on alternatives. The expert's preliminary judgment has a certain influence on final decisions, but the influence is limited. In [29], the concept of predecision is introduced to rank alternatives in 3WD. Benefit from this idea, we apply the predecisions in 3WD to determine preclassifications of alternatives. The predecision $O^k = \{o^k(x_1), o^k(x_2), \ldots, o^k(x_m)\}$ provided by expert e_k is presented as Table II. According to O^k, the alternatives can be classified into categories $H_1^k, H_2^k, \ldots, H_l^k$. The method to obtain preclassifications of alternatives from predecision is detailed presented in Appendix A of the Supplementary Material.

2) PNS of Single Linguistic Terms: The expert e_k provides evaluations on alternatives with regards to attributes in the form of CLE, which is given in Table III. Based on the predecision and preclassification of alternatives, PNS of single linguistic terms will be computed, afterwards a manner to compute the PNS of CLE is presented.

Let $S^j = \{s_0^j, s_1^j, \ldots, s_{g^j}^j\}$ be the linguistic term set with attribute c_j, $PNS^k(f_{ij}^k)$ is the PNS of f_{ij}^k. Based on the multi-attribute additive model [35], the comprehensive evaluation on $x_i \in H_v^k$ for expert e_k is defined by

$$p_{x_i\{x_i \in H_v^k\}}^k = \sum_{j=1}^{q} r_j^k \times PNS^k(f_{ij}^k) \tag{2}$$

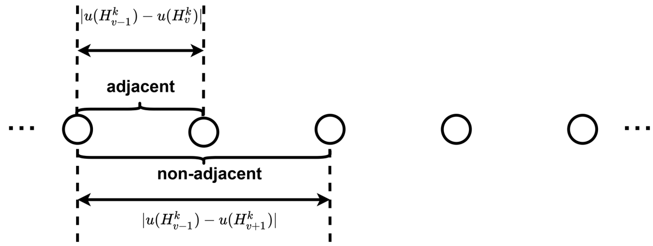

Fig. 2. Distance between two categories.

where r_j^k is the weight of attribute c_j for expert e_k. The category's overall evaluation on all alternatives can be computed by

$$u(H_v^k) = OWA_v^k\left(p^k_{x_{a_1}}, p^k_{x_{a_2}}, \ldots, p^k_{x_{a_{\#H_v^k}}}\right)$$

where $x_{a_1}, x_{a_2}, \ldots, x_{a_{\#H_v^k}} \in H_v^k$ ($\#H_v^k$ denotes the number of alternatives contained in H_v^k) and

$$w_\tau^k = Q\left(\frac{\tau}{\#H_v^k}\right) - Q\left(\frac{\tau - 1}{\#H_v^k}\right), \tau = 1, 2, \ldots, \#H_v^k \quad (3)$$

is the weight for $p^k_{x_\tau}$ in the weighting vector of OWA_v^k. In this equation, $Q(r) = r^\eta (\eta \geq 0)$, and the orness degree of $Q(r)$ can be measured by $orness(Q(r)) = \int r^\eta dr = \frac{1}{\eta+1}$, where η is a parameter to control the orness degree of the aggregation. If $\eta = 1$, the maximum evaluation will be used as the category evaluation. If $\eta = \frac{1}{2}$, the average will be the category evaluation.

The principle to compute the PNS of single linguistic term under the framework of 3WD is as follows.

1) The distance between alternatives in adjacent preclassifications, determined by the predecisions in the 3WD framework, should be shorter than distance between alternatives in nonadjacent preclassifications, which are separated by an intermediate category, as shown in Fig. 2.
2) To minimize the distance among alternatives within the same preclassification in 3WD to enhance their similarity, while to maximize the distance among alternatives across different preclassifications to facilitate clear differentiation [36].

These principles can be realized by applying different mechanisms. As an initial attempt to realize these principles, we provide a feasible manner as below. Let d^k denote the minimum difference between any two preclassifications, as determined by the predecisions of expert e_k, thus

$$\zeta \frac{1}{l-1} \sum_{v=2}^{l} |u(H_{v-1}^k) - u(H_v^k)| + (1-\zeta)\frac{1}{l-2}\sum_{v=2}^{l-1} |u(H_{v-1}^k) - u(H_{v+1}^k)| \geq d^k \quad (4)$$

where ζ is a parameter that measures the significance of the overall evaluation difference between adjacent categories and the overall evaluation difference between nonadjacent categories. If $\zeta = 1$, then only the difference between adjacent categories makes sense. Otherwise, only the difference between nonadjacent categories makes sense. Without loss of generality, in the current proposal, it is assumed that the alternatives can be equivalently preclassified into three categories, H_1^k, H_2^k, and H_3^k according to each expert's predecisions in the 3WD framework.

Then

$$\zeta \frac{1}{2}(|u(H_2^k) - u(H_1^k)| + |u(H_3^k) - u(H_2^k)|) + (1-\zeta)|u(H_1^k) - u(H_3^k)| \geq d^k. \quad (5)$$

Besides, we apply the following equations to indicate that the distance between nonadjacent categories is larger than the distance between adjacent categories

$$\theta_1^k |u(H_2^k) - u(H_1^k)| \leq |u(H_1^k) - u(H_3^k)|$$
$$\theta_2^k |u(H_3^k) - u(H_2^k)| \leq |u(H_1^k) - u(H_3^k)|.$$

Here, $\theta_i^k, i = 1, 2$ is constrained within the interval [1,2]. When $\theta_i^k \geq 1, i = 1, 2$ signifies that the distance between nonadjacent categories exceeds that of adjacent categories. Conversely, $\theta_i^k \leq 2, i = 1, 2$ ensures that distance between nonadjacent categories is no more than twice the distance between adjacent categories. We expect the value of $\theta_i^k, i = 1, 2$ to be as large as possible, to accentuate the distinction, however we also set a threshold to prevent θ_i^k from becoming too large to overshadow the other objectives in the proposed optimization model. From another point of view, the way we define nonadjacent categories is that there is only one category between the compared two, so the choice of threshold value at 2 is designed to resonate with human cognition patterns.

The first objective (i) is to maximize the difference of alternatives in different categories, which can be represented as follows:

$$min\ (-d^k). \quad (6)$$

Let $d^{\acute{k}}$ denote the difference of the comprehensive values of alternatives within the same category, that is determined by the predecisions of expert e_k, we have

$$d^{\acute{k}} = \sum_{v=1}^{l} \sum_{x_a, x_b \in H_v^k} |\sum_{j=1}^{q} r_j^k \times PNS^k(f_{aj}^k) - \sum_{j=1}^{q} r_j^k \times PNS^k(f_{bj}^k)| = \sum_{v=1}^{l} \sum_{x_a, x_b \in H_v^k} |p^k_{x_a} - p^k_{x_b}|. \quad (7)$$

The second objective (ii) is to minimize the difference of alternatives within the same categories, which can be presented as follows:

$$min\ d^{\acute{k}}. \quad (8)$$

Let λ_1^k and λ_2^k denote the normalization of the objectives (i) and (ii): $\lambda_1^k = \frac{\max(-d^k) - (-d^k)}{\max(-d^k) - \min(-d^k)}$, and $\lambda_2^k = \frac{\max(d^{\acute{k}}) - (d^{\acute{k}})}{\max(d^{\acute{k}}) - \min(d^{\acute{k}})}$. The objective function of the proposed model can be set as:

maximize the distance of alternatives in different categories, minimize the distance of alternatives in the same categories, and maximize the difference between adjacent categories and nonadjacent categories, by $\max \mu\lambda_1^k + \vartheta\lambda_2^k + w \cdot \frac{1}{4}\theta_1^k\theta_2^k$, where μ, ϑ, and w denote the weights of λ_1^k, λ_2^k, and $\frac{1}{4}\theta_1^k\theta_2^k$, satisfying $\mu, \vartheta, w \in [0, 1]$ and $\mu + \vartheta + w = 1$.

Besides, some common conditions of $PNS^k(s_\gamma)$ should also be established [18] as follows.

1) PNSs must be ordered, i.e. $PNS^k(s_{\gamma+1}) > PNS^k(s_\gamma)$ $(\gamma = 0, 1, \ldots, g-1)$. A constraint value λ within (0, 1) is set to limit the distance between $PNS^k(s_{\gamma+1})$ and $PNS^k(s_\gamma)$, i.e. $PNS^k(s_{\gamma+1}) - PNS^k(s_\gamma) \geq \lambda(\gamma = 0, 1, \ldots, g-1)$.
2) The range of $PNS^k(s_\gamma)$ is established as below, $PNS^k(s_\gamma) \in [\frac{\gamma-1}{g}, \frac{\gamma+1}{g}](\gamma = 1, 2, \ldots, g-1; \gamma \neq \frac{g}{2})$, $PNS^k(s_0) = 0$, $PNS^k(\frac{g}{2}) = 0.5$, $PNS^k(s_g) = 1$.

Each expert has his/her own PNS for linguistic terms with regard to each attribute. For different attributes, the linguistic term sets are allowed to be different. For the convenience of discussion, we denote $S^j = \{s_0^j, s_1^j, \ldots, s_{g^j}^j\}$ as the linguistic term set corresponding to attribute $c_j (j = 1, 2, \ldots, q)$, the number of single linguistic terms in S^j is $g^j + 1$. Based on the above considerations, an optimization model is presented as (9), to compute PNS of linguistic terms for each expert

$$\max \mu\lambda_1^k + \vartheta\lambda_2^k + w \cdot \frac{1}{4}\theta_1^k\theta_2^k$$

$$\text{s.t.} \begin{cases} \lambda_1^k = \frac{\max(-d^k) - (-d^k)}{\max(-d^k) - \min(-d^k)} \\ \lambda_2^k = \frac{\max(d^{\acute{k}}) - (d^{\acute{k}})}{\max(d^{\acute{k}}) - \min(d^{\acute{k}})} \\ d^k \leq \zeta \frac{1}{l-1} \sum_{v=2}^{l} |u(H_{v-1}^k) - u(H_v^k)| \\ +(1-\zeta) \frac{1}{l-2} \sum_{v=2}^{l-1} |u(H_{v-1}^k) - u(H_{v+1}^k)| \\ \theta_1^k |u(H_2^k) - u(H_1^k)| \leq |u(H_1^k) - u(H_3^k)| \\ \theta_2^k |u(H_3^k) - u(H_2^k)| \leq |u(H_1^k) - u(H_3^k)| \\ \theta_1^k, \theta_2^k \in [1, 2] \\ d^{\acute{k}} = \sum_{v=1}^{l} \sum_{x_a, x_b \in H_v^k} |p_{x_a}^k - p_{x_b}^k| \\ \text{PNS}^k(f_{ij}^k) = \frac{\sum_{\gamma=\alpha_{ij}^k}^{\beta_{ij}^k} \text{PNS}^k(s_\gamma^j)}{t_{ij}^k} \\ E_{G_H}(f_{ij}^k) = \{s_{\alpha_{ij}^k}^j, \ldots, s_{\beta_{ij}^k}^j\}, and\ t_{ij}^k = \beta_{ij}^k - \alpha_{ij}^k + 1 \\ \text{PNS}^k(s_0^j) = 0 \\ \text{PNS}^k(s_{\frac{g^j}{2}}^j) = 0.5 \\ \text{PNS}^k(s_\gamma^j) \in [(\gamma-1)/g^j, (\gamma+1)/g^j],\ \gamma = 1, \ldots, g^j - 1, \\ and\ \gamma \neq \frac{g^j}{2} \\ \text{PNS}^k(s_{g^j}^j) = 1 \\ \text{PNS}^k(s_{\gamma+1}^j) - PNS^k(s_\gamma^j) \geq \lambda, \quad \gamma = 0, \ldots, g^j - 1 \\ r_j^k \in [0, 1], j = 1, 2, \ldots, q \\ \sum_{j=1}^{q} (r_j^k)^2 = 1 \\ \zeta \in [0, 1]. \end{cases} \tag{9}$$

The output of this optimization model are the PNSs of linguistic terms s_γ^j $(\gamma = 0, \ldots, g^j)$ with respect to attribute $c_j (j = 1, 2, \ldots, q)$ for expert e_k $(k = 1, \ldots, n)$, i.e., $\text{PNS}^k(s_\gamma^j)$, where $s_\gamma^j = s_0^j, \ldots, s_{g^j}^j$. Parameters μ, ϑ, w, and ζ can be set in prior.

3) PNS of CLE: The PNS of CLE can be computed based on the PNS of single linguistic terms, which are contained in the HFLTS converted from CLE. Suppose that f_{ij}^k is the CLE provided by expert e_k with respect to alternative x_i and attribute c_j, $E_{G_H}(f_{ij}^k) = \{s_{\alpha_{ij}^k}^j, \ldots, s_{\beta_{ij}^k}^j\}$, and $t_{ij}^k = \beta_{ij}^k - \alpha_{ij}^k + 1$, then the PNS of f_{ij}^k is computed by (10). For e_k, the PNS for all evaluations are given in Table IV

$$\text{PNS}^k(f_{ij}^k) = \frac{\sum_{\gamma=\alpha_{ij}^k}^{\beta_{ij}^k} \text{PNS}^k(s_\gamma^j)}{t_{ij}^k}. \tag{10}$$

TABLE IV
PNS OF EVALUATION VALUES FOR e_k

Alternatives	Attributes			
	c_1	c_2	$\ldots$	c_q
x_1	$PNS^k(f_{11}^k)$	$PNS^k(f_{12}^k)$	$\ldots$	$PNS^k(f_{1q}^k)$
x_2	$PNS^k(f_{21}^k)$	$PNS^k(f_{22}^k)$	$\ldots$	$PNS^k(f_{2q}^k)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_m	$PNS^k(f_{m1}^k)$	$PNS^k(f_{m2}^k)$	$\ldots$	$PNS^k(f_{mq}^k)$

TABLE V
RELATIVE LOSS FUNCTIONS DERIVED FROM f_{ij}^k

	J	$\neg J$
a_P	0	$\text{PNS}_{\max}^k(f_j^k) - \text{PNS}^k(f_{ij}^k)$
a_B	$\sigma(\text{PNS}^k(f_{ij}^k) - PNS_{\min}^k(f_j^k))$	$\sigma(\text{PNS}_{\max}^k(f_j^k) - \text{PNS}^k(f_{ij}^k))$
a_N	$\text{PNS}^k(f_{ij}^k) - PNS_{\min}^k(f_j^k)$	0

C. CLE-3WGDM Model Based on PNS

Traditionally, experts provide loss functions by using accuracy values. However, in real-life, considering the random factors and complexity of the problems, experts may prefer to apply linguistic evaluation rather than crisp values. Traditional manner to catch loss function directly from experts is subjective [10]. To avoid such subjectivity, in this section, a relative loss function with PNS of CLE will be constructed. Afterwards, a novel 3WGDM model based on CLE is presented.

1) 3WGDM Model Construction: The method proposed by Jia and Liu [28] is extended to generate relative loss functions from evaluations with related to attributes. Taking use of evaluations in Table III, the relative loss functions for expert e_k can be presented in Table V.

In Table V, f_{ij}^k is a CLE that denotes the evaluation of the alternative x_i provided by e_k under the attribute c_j. $\text{PNS}_{\max}^k(f_j^k)$ and $\text{PNS}_{\min}^k(f_j^k)$ denote the maximum and minimum values under the attribute c_j, respectively. $\sigma()$ represents risk aversion coefficient, taking values in interval [0,0.5] [28]. After the transformation from evaluation values to relative loss functions, we obtain a 3×2 matrix. Let P, B, and N represent three domains, every alternative will be divided into one of the three domains. The alternatives classified in the P region is related to strategy

TABLE VI
AGGREGATED RELATIVE LOSS FUNCTIONS OF x_i FOR e_k

	J	$\neg J$
a_P	0	$\sum_{j=1}^{q} r_j^k(\text{PNS}_{\max}^k(f_j^k) - \text{PNS}^k(f_{ij}^k))$
a_B	$\sigma(\sum_{j=1}^{q} r_j^k(\text{PNS}^k(f_{ij}^k) - \text{PNS}_{\min}^k(f_j^k)))$	$\sigma(\sum_{j=1}^{q} r_j^k(\text{PNS}_{\max}^k(f_j^k) - \text{PNS}^k(f_{ij}^k)))$
a_N	$\sum_{j=1}^{q} r_j^k(\text{PNS}^k(f_{ij}^k) - \text{PNS}_{\min}^k(f_j^k))$	0

"to accept." The alternatives classified in the B region is related to strategy "noncommitment," and the alternatives classified in the N region is related to strategy "to reject." For e_k, the loss function of x_i in c_j can be denoted by $T(f_{ij}^k)$ and expressed as follows:

$$T(f_{ij}^k) = \begin{pmatrix} l_{PP}^{k,ij} & l_{PN}^{k,ij} \\ l_{BP}^{k,ij} & l_{BN}^{k,ij} \\ l_{NP}^{k,ij} & l_{NN}^{k,ij} \end{pmatrix} \tag{11}$$

where $l_{PP}^{k,ij}$, $l_{BP}^{k,ij}$, and $l_{NP}^{k,ij}$ represent the relative losses of expert e_k, obtained by taking actions a_P, a_B, and a_N under the condition that $x_i \in J$, respectively. Analogously, $l_{PN}^{k,ij}$, $l_{BN}^{k,ij}$, and $l_{NN}^{k,ij}$ represent the relative losses obtained by taking actions a_P, a_B, and a_N under the condition that $x_i \in \neg J$.

For expert e_k, the aggregation of loss functions of x_i with respect to all attributes can be computed by

$$T_i^k = \sum_{j=1}^{q} r_j^k \times T(f_{ij}^k) \tag{12}$$

where r_j^k is the weight for attribute c_j. Equation (12) can also be presented as follows:

$$T_i^k = \begin{pmatrix} \sum_{j=1}^{q} r_j^k \times l_{PP}^{k,ij} & \sum_{j=1}^{q} r_j^k \times l_{PN}^{k,ij} \\ \sum_{j=1}^{q} r_j^k \times l_{BP}^{k,ij} & \sum_{j=1}^{q} r_j^k \times l_{BN}^{k,ij} \\ \sum_{j=1}^{q} r_j^k \times l_{NP}^{k,ij} & \sum_{j=1}^{q} r_j^k \times l_{NN}^{k,ij} \end{pmatrix}. \tag{13}$$

Denote $\sum_{j=1}^{q} r_j^k \times l_{\bullet\circ}^{k,ij}$ by $l_{\bullet\circ}^{k,i}$, where $i = 1, 2, \ldots, m$, $\bullet = P, B, N$, $\circ = P, N$. Based on (13), the aggregated relative loss of each alternative x_i for e_k is obtained as Table VI.

The CLE-TODIM method based on PNS which will be presented in Section III-C4, is used here to order alternatives, and then to define the dominance set for each alternative.

Definition 5 (See [10]): Let $X = \{x_1, x_2, \ldots, x_m\}$ be a set of alternatives, define the dominance set for each alternative x_i as $[x_i]_d$

$$[x_i]_d = \{x_k | x_k \geq x_i\}. \tag{14}$$

For any alternative within $[x_i]_d$, x_k is always greater than or equal to x_i, indicating that the ranking of x_k, that is obtained by the CLE-TODIM method, is never inferior to that of x_i."

Based on Definition 5, the formula for deriving conditional probabilities can be defined as follows:

$$Pr(J|[x_i]_d) = \frac{|J \cap [x_i]_d|}{[x_i]_d}. \tag{15}$$

The expected losses formula can be improved as follows:

$$R(a_\bullet|[x_i]_d) = l_{\bullet P}^i Pr(J|[x_i]_d) + l_{\bullet N}^i Pr(\neg J|[x_i]_d) \tag{16}$$

where $l_{\bullet\circ}^i = \sum_{k=1}^{n} w_k \times l_{\bullet\circ}^{k,i}, i = 1, 2, \ldots, m$ and $\bullet = P, B, N$, $\circ = P, N$.

Then, the decision rules are as follows:

P1) if $R(a_P|[x_i]_d) \leqslant R(a_B|[x_i]_d)$ and $R(a_P|[x_i]_d) \leqslant R(a_N|[x_i]_d)$, then x_i is assigned to P;

B1) if $R(a_B|[x_i]_d) \leqslant R(a_P|[x_i]_d)$ and $R(a_B|[x_i]_d) \leqslant R(a_N|[x_i]_d)$, then x_i is assigned to B;

N1) if $R(a_N|[x_i]_d) \leqslant R(a_P|[x_i]_d)$ and $R(a_N|[x_i]_d) \leqslant R(a_B|[x_i]_d)$, then x_i is assigned to N.

2) Novel Attributes Weights Determination Model: In MADM problems, the importance of attributes tend to be different. Benefiting from the idea deviation maximization [11], a new optimization model is developed to compute the attribute weights. In this model, the entropy of HFLTS is applied to measure the uncertainty contained in linguistic information, and the distance measure of HFLTS is applied to measure the difference of CLE

$$\begin{cases} \max \sum_{j=1}^{q} r_j^k \sum_{i,z=1}^{m} d(f_{ij}^k, f_{zj}^k) \\ \max \sum_{j=1}^{q} r_j^k \sum_{i=1}^{m} [1 - E(f_{ij}^k)] \\ \text{s.t.} \begin{cases} \sum_{j=1}^{q} (r_j^k)^2 = 1, \\ r_j^k \geq 0,\ j = 1, 2, \ldots, q. \end{cases} \end{cases} \tag{17}$$

The outputs are the weights of attributes. By applying this model, the uncertainty contained in linguistic information is restricted to the minimum degree, while all alternatives can be distinguished by attributes to the maximum degree. In this model, the difference of alternatives under attribute c_j is computed by $\sum_{i,z=1}^{m} d(f_{ij}^k, f_{zj}^k), i, z = 1, 2, \ldots, m, j = 1, 2, \ldots, q$. The total weighted deviation among alternatives considering all attributes is $\sum_{j=1}^{q} r_j^k \sum_{i,z=1}^{m} d(f_{ij}^k, f_{zj}^k), i, z = 1, 2, \ldots, m$, $j = 1, 2, \ldots, q$. $E(f_{ij}^k)$ is the uncertainty contained in f_{ij}^k, which can be measured by comprehensive entropy of the HFLTS[37] converted from the CLE, f_{ij}^k. d is the hausdorff distance measure of HFLTS [38], that is used to measure the difference of CLEs.

3) Novel Expert Weight Determination Method Based on 3WD-Social Network: The social network is denoted by $G(E, V)$, where $E = \{e_1, e_2, \ldots, e_n\}$ is the expert set, and V reflects the trust relationship among experts. It can be graphically presented, in which experts are shown by points, and trust relationships are reflected by edges. Here, we initially use the number of edges to distinguish trust attitudes among experts, where arrows represent the trust direction among experts.

The expert who has a higher in-degree is more important in the social trust network and the expert who has a higher out-degree is less important [22]. Here, the in-degree of an expert e_k is measured by the number of edges pointing from other experts to expert e_k. The out-degree of expert e_k is measured by the number of edges pointing from e_k to other experts. Fig. 3 is an

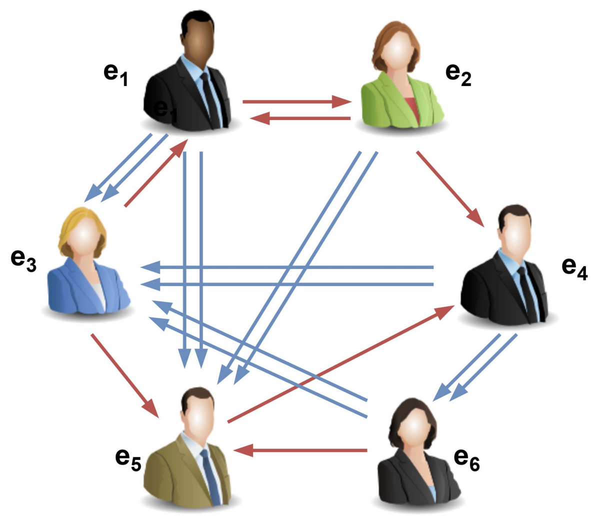

Fig. 3. 3WD-social network of six experts.

example, in which a social network consists of six experts. The adjacent matrix can be constructed as below for a social network that consists of n experts:

$$AD = \begin{array}{c} \\ e_1 \\ e_2 \\ \vdots \\ e_n \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \cdots & e_n \end{array} \\ \begin{pmatrix} - & ad_{12} & \cdots & ad_{1n} \\ ad_{21} & - & \cdots & ad_{2n} \\ \vdots & \vdots & \ddots & \cdots \\ ad_{n1} & ad_{n2} & \cdots & - \end{pmatrix} \end{array}$$

in which

$$ad_{kz} = \begin{cases} 2, & (e_k, e_z) \in A \\ 1, & (e_k, e_z) \in B \\ 0, & (e_k, e_z) \in C. \end{cases} \tag{18}$$

There are three types of trust relationships between two experts.

1) Case A: absolute trust.
2) Case B: not sure trust or not, delay to determine the trust situation.
3) Case C: absolute distrust.

In the graphic representation of a social network, the number of edges point from expert e_k to expert e_z is 2, indicates that expert e_k has an absolute trust in expert e_z, thus $(e_k, e_z) \in A$. The number of edges point from expert e_k to expert e_z is 1, indicates that expert e_k has a "noncommitment" trust in expert e_z, thus $(e_k, e_z) \in B$. When there is no edge point from expert e_k to expert e_z, it means that expert e_k absolutely distrust in expert e_z, thus $(e_k, e_z) \in C$. For instance, in Fig. 3, e_1 absolutely trusts in e_3, while e_2 has a noncommitment trust in e_4, and e_5 absolutely distrusts in e_6. Here, the classification strategy obeys the idea of 3WD, by allowing the existence of a case B. Case A is related to strategy "accept," case C is related to strategy "reject", and case B is related to the "noncommitment" strategy in 3WD. The social network, which allows to use number of edges to reflect the trust situation and considers in-degree and out-degree in the manner of 3WD is called the 3WD-social network.

Benefiting from the basic idea in [39] to compute both the in-degree and out-degree in social network, for an expert e_k, the in-degree centrality index $C^+(e_k)$ and the out-degree centrality index $C^-(e_k)$ can be redefined as follows:

$$C^+(e_k) = \frac{1}{n-1} \sum_{z=1, z\neq k}^{n} ad_{zk} \tag{19}$$

$$C^-(e_k) = \frac{1}{n-1} \sum_{z=1, z\neq k}^{n} ad_{kz}. \tag{20}$$

Generally, a higher value of the in-degree centrality index indicates a higher importance degree of an expert, whereas a higher out-degree centrality index indicates a lower level of importance [22]. The weight of expert e_k is calculated using both the in-degree and out-degree centrality index as follows:

$$w_k = \frac{e^{C^+(e_k) - C^-(e_k)}}{\sum_{d=1}^{n} e^{(C^+(e_d) - C^-(e_d))}}. \tag{21}$$

Then, the dominance set can be calculated in Section III-C4 based on the expert weights. Besides, the collective relative loss for each alternative $x_i, i = 1, 2 \cdots, m$ (see Table VII) can also be calculated based on the expert weights by

$$l^i_{\bullet\circ} = \sum_{k=1}^{n} w_k l^{k,i}_{\bullet\circ} \tag{22}$$

where $i = 1, 2, \ldots, m, \bullet = P, B, N, \circ = P, N$.

4) CLE-TODIM Method Based on PNS: Instead of classical conditional probability formulas, which are subjectively derived from experts [40], the improved TODIM method [27] will be extended to deal with CLE based on PNS, and will be applied to obtain conditional probabilities in the current research.

The dominance degree of two alternatives with related to $c_j (j = 1, 2, \ldots, q)$ is computed by

$$\Phi^k_j(x_i, x_z) = \begin{cases} g_1(r^k_j) f_1(\mathrm{PNS}^k(f^k_{ij}) - \mathrm{PNS}^k(f^k_{zj})) \\ \quad \text{if } \mathrm{PNS}^k(f^k_{ij}) - \mathrm{PNS}^k(f^k_{zj}) > 0 \\ 0, \quad \text{if } \mathrm{PNS}^k(f^k_{ij}) - \mathrm{PNS}^k(f^k_{zj}) = 0 \\ -g_2(r^k_j) f_2(\mathrm{PNS}^k(f^k_{zj}) - \mathrm{PNS}^k(f^k_{ij})) \\ \quad \text{if } \mathrm{PNS}^k(f^k_{ij}) - \mathrm{PNS}^k(f^k_{zj}) < 0 \end{cases} \tag{23}$$

where $g_1(x) = f_1(x) = f_2(x) = \sqrt{x}$ and $g_2(x) = \frac{1}{\theta\sqrt{x}}$, in which $\theta > 0$ represents the effect of loss.[1] To save space, more detailed calculation process of the CLE-TODIM method is presented in Appendix B of the Supplementary Material.

D. Multiattribute 3WGDM Steps and the Algorithm

The steps of the 3WGDM based on PNS of CLE are presented here, the detailed algorithm flow is given as Algorithm 1.

Step 1: Each expert is requested to provide predecision and evaluation in form of CLE. For each expert, pre-classification of alternatives is obtained based on the predecision.[2]

[1]If $\theta > 1$, it reduces loss aversion, while $0 < \theta < 1$, it increases loss aversion. If $\theta = 1$, the loss aversion is neither increased nor reduced.

[2]In this work, alternatives will be equivalently pre-classified to three classes for each expert.

TABLE VII
COLLECTIVE RELATIVE LOSS FUNCTIONS OF x_i

	J	$\neg J$
a_P	0	$\sum_{k=1}^{n} w_k \sum_{j=1}^{q} r_j^k(\text{PNS}^k_{\max}(f_j^k) - \text{PNS}^k(f_{ij}^k))$
a_B	$\sum_{k=1}^{n} w_k(\sigma(\sum_{j=1}^{q} r_j^k(\text{PNS}^k(f_{ij}^k) - \text{PNS}^k_{\min}(f_j^k))))$	$\sum_{k=1}^{n} w_k(\sigma(\sum_{j=1}^{q} r_j^k(\text{PNS}^k_{\max}(f_j^k) - \text{PNS}^k(f_{ij}^k))))$
a_N	$\sum_{k=1}^{n} w_k \sum_{j=1}^{q} r_j^k(\text{PNS}^k(f_{ij}^k) - \text{PNS}^k_{\min}(f_j^k))$	0

Step 2: Based on Definition 3, convert the evaluation in form of CLE into HFLTS by applying the transformation function E_{G_H}.

Step 3: Employ the optimization model (17) to determine the weights for attributes of each expert.

Step 4: Based on the optimization model (9), compute the PNS of linguistic terms of each expert. Compute the PNS of CLE based on the PNS of linguistic terms by (10).

Step 5: According to the (11)–(13), calculate the aggregated relative loss of each alternative for each expert.

Step 6: Calculate the weights of experts based on the 3WD-social network, calculate the overall performance of each alternative by (27), and determine the dominance classes for each alternative by (14).

Step 7: Calculate the collective relative losses with respect to all experts by (22).

Step 8: Calculate conditional probability and expected losses for each alternative based on (15)–(16).

Step 9: The classification results are obtained, according to the decision rules (P1), (B1), and (N1).

IV. CASE STUDY

In this section, we apply the proposed 3WGDM method to a practical case and present its final classification result.

A. Illustrate Example

The manifestations of spleen and stomach disease are diverse, TCM diagnosis and treatment emphasize syndrome differentiation and individualized therapy, which entails tailoring treatment based on the patient's specific symptoms, constitution, and etiology. Not all individuals presenting symptoms of spleen and stomach disease are diagnosed with deficiency-cold of spleen and stomach (DCSS). Furthermore, not every patient diagnosed with DCSS is deemed appropriate for specific treatments, like applying Lizhong Decoction. Deciding on a specific treatment necessitates a comprehensive consideration of various factors, beyond merely simply aggregating the predecision of each expert. The application of 3WGDM can be considered in such a problem, to determine whether patients exhibiting symptoms of spleen and stomach disease are suited for a specific DCSS treatment, forgoing such a DCSS treatment, or deferring the treatment, thereby facilitating a decision that minimizes potential loss. Since the ultimate treatment decision for a patient is derived from the collective assessments of several experts, the interactions among experts can be seen as forming a social network.

Algorithm 1: 3WGDM based on PNS of CLE.

Input: The set of states, pre-decision tables, evaluation tables, the risk aversion coefficient σ.
Output: Classification results of alternatives.

begin
  for $x_i \in X$, $c_j \in C$ **do**
    pre-classify: The classification of alternatives is obtained based on pre-decision of each expert.
  end
  for $f_{ij}^k \in F^k$ **do**
    compute: Convert the evaluation in form of *CLE* into *HFLTS* by using the transformation function E_{G_H}.
  end
  for $c_j \in C$ **do**
    compute: The weights of attributes for each expert r_j^k by Eq.(17).
  end
  for $x_i \in X$, $c_j \in C$ **do**
    compute: The *PNS* of linguistic terms for each expert through Eq.(9).
    obtain: The *PNS* for each item in form of *CLE* in the evaluation matrix.
  end
  for $x_i \in X$, $c_j \in C$ **do**
    compute: The aggregated relative loss of each expert with respect to all attributes by Eqs. (11)-(13).
  end
  for $x_i \in X$ **do**
    compute: The dominance classes $[x_i]_d$ based on Eqs.(23)-(27), where the weights of experts are computed by Eqs.(19)-(21).
  end
  for $x_i \in X$ **do**
    establish: The collective relative loss table with respect to all experts (see Table (VII)).
  end
  for $x_i \in X$ **do**
    compute: The conditional probability $Pr(J|[x_i]_d)$ by Eq.(15).
    compute: The expected loss of x_i based on Eq.(16).
  end
  for $x_i \in X$ **do**
    If $R(a_P|[x_i]_d) \leqslant R(a_B|[x_i]_d)$ and $R(a_P|[x_i]_d) \leqslant R(a_N|[x_i]_d)$, then x_i is assigned to P region;
    If $R(a_B|[x_i]_d) \leqslant R(a_P|[x_i]_d)$ and $R(a_B|[x_i]_d) \leqslant R(a_N|[x_i]_d)$, then x_i is assigned to B region;
    If $R(a_N|[x_i]_d) \leqslant R(a_P|[x_i]_d)$ and $R(a_N|[x_i]_d) \leqslant R(a_B|[x_i]_d)$, then x_i is assigned to N region.
  end
  return The classification results of alternatives.
end

Assuming there are nine patients $\{x_1, x_2, \ldots, x_9\}$ exhibiting symptoms of spleen and stomach disease. $\{J, \neg J\}$ presents the sets of states, J represents DCSS, while $\neg J$ indicates not DCSS, where $J = \{x_2, x_5, x_6, x_9\}$, $\neg J = \{x_1, x_3, x_4, x_7, x_8\}$. There are five TCM experts $\{e_1, e_2, \ldots, e_5\}$ invited to diagnose the

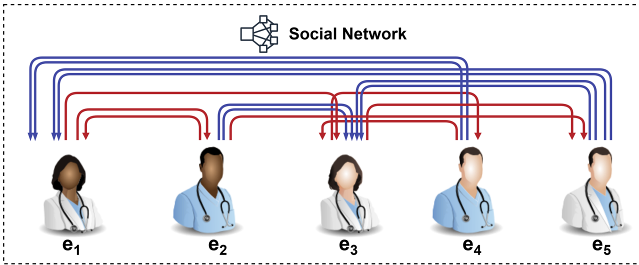

Fig. 4. 3WD-social network of five experts.

TABLE VIII
CLASSIFICATION RESULTS OF EACH PATIENT

Decision rules	classification results
P1	$P = \{x_6, x_8, x_9\}$
B1	$B = \{x_1, x_2\}$
N1	$N = \{x_7, x_3, x_5, x_4\}$

nine patients $\{x_1, x_2, \ldots, x_9\}$ and determine whether a specific DCSS treatment is necessary. Experts provide predecisions on patients with white and greasy tongue coating based on the initial diagnosis, and use CLE as assessments for the patient's conditions with respect to four main attributes: Fatigue, Decreased Appetite, Cold intolerance, Loose Stool (denoted by $c_1, c_2, c_3,$ *and* c_4). This contributes to later determination on whether a treatment is warranted for each patient. Moreover, Fig. 4 depicts the social network among the experts.

The linguistic term sets $S^j (j = 1, 2, 3, 4)$ related to attributes $\{c_1, c_2, c_3, c_4\}$ are presented as: $S^1 = \{s_0^1 = Extreme, s_1^1 = Severe, s_2^1 = Moderate, s_3^1 = Slight, s_4^1 = Faint\}$, $S^2 = \{s_0^2 = Extreme, s_1^2 = Severe, s_2^2 = Heavy, s_3^2 = Moderate, s_4^2 = Mild, s_5^2 = Slight, s_6^2 = Faint\}$, $S^3 = \{s_0^3 = Severe, s_1^3 = Heavy, s_2^3 = Moderate, s_3^3 = Slight, s_4^3 = Faint\}$, $S^4 = \{s_0^4 = Severe, s_1^4 = Heavy, s_2^4 = Moderate, s_3^4 = Slight, s_4^4 = Faint\}$.

The information collected from experts and the computation process are presented in Appendix C of the Supplementary Material. According to the proposed decision steps and Algorithm 1, the classification results are given in Table VIII.

B. Comparative Analysis

From the calculations of attribute weights, expert weights, and conditional probability, we make a comparison between the proposed CLE-3WGDM method and some other methods. The comparison results are given in Table IX.

1) *The approaches to determine attribute weights are distinct:* Among the existing studies on 3WGDM with multiple attributes, part of the studies have assigned the attribute weights by direct assumptions [5], [28], [41], and part of the studies have assigned the attribute weights based on deviation maximization [11], [42] or the information entropy [10], etc. In our study, both the maximum deviation principle and the comprehensive entropy of information are considered, to derive attribute weights in a more comprehensive manner.

TABLE IX
COMPARATIVE ANALYSIS BETWEEN THE CLE-3WGDM METHOD AND OTHER EXISTING METHODS

Methods	Attribute weights	Expert weights	Conditional probability
Jia and Liu [28]	direct assumption	not considered	approximation space
Gao et al. [44]	poisson distribution method	not considered	TOPSIS method
Jiang and Hu [5]	direct assumption	direct assumption	TOPSIS method
Wang et al. [41]	direct assumption	not considered	probability dominance relation
Wang et al. [42]	deviation maximization method	not considered	outranking function and positive ideal solution
Peng et al. [45]	rough entropy theory based on the probabilistic similarity classes	not considered	probabilistic similarity measure
Mandal et al. [43]	not considered	social network	support information system
Han et al. [10]	information entropy	not considered	PL-TODIM method
Wang et al. [11]	deviation maximization method	direct assumption	similarity measure
The CLE-3WGDM method	deviation maximization and comprehensive entropy	3WD-social network	CLE-TODIM method based on PNS

2) *The approaches to determine expert weight are distinct:* Social network has been utilized in 3WGDM to assign weights of experts, however the application does not incorporate the 3WD strategy at the network level, failing to differentiate the trust levels among experts [43]. To ensure the comprehensiveness and effectiveness of the 3WGDM process, we introduce the thinking mode of 3WD to social network, and propose a novel way to derive the expert weight based on the 3WD-social network.

3) *The approaches to calculate conditional probability are distinct:* In different works, the approaches to compute the equivalence classes or dominance classes are various. The advantage of the current proposal is that it can deal with information in form of CLE, and it can consider different semantics of linguistic variables for different people, the application of PNS contributes to a more reasonable computation result.

TABLE X
COMPARATIVE ANALYSIS OF 3WD MODELS

Models		Consider linguistic information more complex than single terms	Consider different semantic of linguistic information to people
classical 3WD	Yao [1]	×	×
	Yao [31]	×	×
utility theory -based 3WD	Zhang et al. [46]	×	×
	Zhan et al. [29]	×	×
prospect theory -based 3WD	Wang et al. [47]	×	×
	Wang et al. [11]	✓	×
regret theory -based 3WD	Wang et al. [48]	×	×
	Zhu et al. [49]	✓	×
the proposed model		✓	✓

Besides, we emphasize the fact that, compared with other existing researches on 3WGDM, the proposed uniquely accommodates personalized semantics, recognizing that the same linguistic variable may carry different meanings for different people (see Table X). Among the 3WGDM models which handle linguistic information more complex than single terms, the proposed model uniquely allows linguistic information closer to the cognition of human-being.

V. CONCLUSION

This article proposes a CLE-3WGDM method based on PNS, which extends the application scope of 3WD to the CLE environment, thereby greatly enhances the flexibility of linguistic information expression in 3WD. A new optimization model based on predecision in 3WD is developed for calculating the PNS of linguistic terms for each expert, a method to compute the PNS of CLE is also presented. A novel model to compute attribute weights based on deviation maximization and comprehensive entropy is developed. Besides, we incorporate the concept of 3WD into social networks and calculate experts' weights based on a 3WD-social network. In the forthcoming research, we will explore more methodologies to manage personalized fuzzy semantic of various linguistic variables within the framework of 3WD, to further increase the flexibility of linguistic expression in 3WD, while simultaneously enhancing the reliability of decision result.

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