1 Port-Hamiltonian system

$$\underbrace{\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \\ \mathbf{w} \\ \mathbf{y} \end{pmatrix}}_{\mathbf{b}} = \underbrace{\Big(\mathbf{J} - \mathbf{R}\Big)}_{\mathbf{M}} \underbrace{\begin{pmatrix} \nabla \mathrm{H}(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}}_{\mathbf{a}} \tag{1}$$

with $\mathrm{P} = \mathbf{u}^\intercal\,\mathbf{y}$ the power received *by* the sources *from* the system.

2 Split in linear and nonlinear parts

The state is split according to $\mathbf{x} = (\mathbf{x}_{\mathtt{l}}^\intercal,\, \mathbf{x}_{\mathtt{nl}}^\intercal)^\intercal$ with

$\mathbf{x}_{\mathtt{l}} = (x_1, \cdots, x_{n_{\mathtt{xl}}})^\intercal$ the states associated with the quadratic components of the Hamilotnian $\mathrm{H}_{\mathtt{l}}(\mathbf{x}_{\mathtt{l}}) = \mathbf{x}_{\mathtt{l}}^\intercal\,\mathbf{Q}\,\mathbf{x}_{\mathtt{l}}/2$

$\mathbf{x}_{\mathtt{nl}} = (x_{n_{\mathtt{xl}}+1}, \cdots, x_{n_{\mathtt{x}}})^\intercal$ the states associated with the non-quadratic components of the Hamiltonian with $n_{\mathtt{x}} = n_{\mathtt{xl}} + n_{\mathtt{xnl}}$.

The set of dissipative variables is split according to $\mathbf{w} = (\mathbf{x}_{\mathtt{l}}^\intercal,\, \mathbf{w}_{\mathtt{nl}}^\intercal)^\intercal$ with

$\mathbf{w}_{\mathtt{l}} = (w_1, \cdots, w_{n_{\mathtt{wl}}})^\intercal$ the variables associated with the linear components of the dissipative relation $\mathbf{z}_{\mathtt{l}}(\mathbf{w}_{\mathtt{l}}) = \mathbf{Z}_{\mathtt{l}}\,\mathbf{w}_{\mathtt{l}}$

$\mathbf{w}_{\mathtt{nl}} = (w_{n_{\mathtt{wl}}+1}, \cdots, w_{n_{\mathtt{w}}})^\intercal$ the variables associated with the nonlinear components of the dissipative relation $\mathbf{z}_{\mathtt{nl}}(\mathbf{w}_{\mathtt{nl}})$ with $n_{\mathtt{w}} = n_{\mathtt{wl}} + n_{\mathtt{wnl}}$.

$$\underbrace{\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}_{\mathtt{l}}}{\mathrm{d}t} \\ \frac{\mathrm{d}\mathbf{x}_{\mathtt{nl}}}{\mathrm{d}t} \\ \mathbf{w}_{\mathtt{l}} \\ \mathbf{w}_{\mathtt{nl}} \\ \mathbf{y} \end{pmatrix}}_{\mathbf{b}} = \underbrace{\left(\begin{array}{cc|cc|c} \mathbf{M}_{\mathtt{xlxl}} & \mathbf{M}_{\mathtt{xlxnl}} & \mathbf{M}_{\mathtt{xlwl}} & \mathbf{M}_{\mathtt{xlwnl}} & \mathbf{M}_{\mathtt{xly}} \\ \mathbf{M}_{\mathtt{xnlxl}} & \mathbf{M}_{\mathtt{xnlxnl}} & \mathbf{M}_{\mathtt{xnlwl}} & \mathbf{M}_{\mathtt{xnlwnl}} & \mathbf{M}_{\mathtt{xnly}} \\ \hline \mathbf{M}_{\mathtt{wlxl}} & \mathbf{M}_{\mathtt{wlxnl}} & \mathbf{M}_{\mathtt{wlwl}} & \mathbf{M}_{\mathtt{wlwnl}} & \mathbf{M}_{\mathtt{wly}} \\ \mathbf{M}_{\mathtt{wnlwl}} & \mathbf{M}_{\mathtt{wnlxnl}} & \mathbf{M}_{\mathtt{wnlwl}} & \mathbf{M}_{\mathtt{wnlwnl}} & \mathbf{M}_{\mathtt{wnly}} \\ \hline \mathbf{M}_{\mathtt{yxl}} & \mathbf{M}_{\mathtt{yxnl}} & \mathbf{M}_{\mathtt{ywl}} & \mathbf{M}_{\mathtt{ywnl}} & \mathbf{M}_{\mathtt{yy}} \end{array}\right)}_{\mathbf{M}} \underbrace{\begin{pmatrix} \nabla \mathrm{H}_{\mathtt{l}} \\ \nabla \mathrm{H}_{\mathtt{nl}} \\ \mathbf{Z}_{\mathtt{l}}\,\mathbf{w}_{\mathtt{l}} \\ \mathbf{z}_{\mathtt{nl}} \\ \mathbf{u} \end{pmatrix}}_{\mathbf{a}} \tag{2}$$

2.1 Linear subsystem

$$\underbrace{\big(\mathbf{I_d} - \mathbf{M}_{\mathtt{wlwl}}\,\mathbf{Z}_{\mathtt{l}}\big)}_{\mathbf{iD}_{\mathtt{wl}}}\,\mathbf{w}_{\mathtt{l}} = \underbrace{\left(\begin{array}{cc|c|c} \mathbf{M}_{\mathtt{wlxl}} & \mathbf{M}_{\mathtt{wlxnl}} & \mathbf{M}_{\mathtt{wlwnl}} & \mathbf{M}_{\mathtt{wly}} \end{array}\right)}_{\mathbf{M}_{\mathtt{wlnl}}} \begin{pmatrix} \nabla \mathrm{H}_{\mathtt{l}} \\ \nabla \mathrm{H}_{\mathtt{nl}} \\ \mathbf{z}_{\mathtt{nl}} \\ \mathbf{u} \end{pmatrix} \tag{3}$$

2.2 Nonlinear subsystem

$$\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}_{\mathtt{l}}}{\mathrm{d}t} \\ \frac{\mathrm{d}\mathbf{x}_{\mathtt{nl}}}{\mathrm{d}t} \\ \mathbf{w}_{\mathtt{nl}} \\ \mathbf{y} \end{pmatrix} = \underbrace{\left(\begin{array}{c} \mathbf{M}_{\mathtt{xlwl}} \\ \mathbf{M}_{\mathtt{xnlwl}} \\ \hline \mathbf{M}_{\mathtt{wnlwl}} \\ \hline \mathbf{M}_{\mathtt{ywl}} \end{array}\right)}_{\mathbf{M}_{\mathtt{nlwl}}} \mathbf{Z}_{\mathtt{l}}\,\mathbf{w}_{\mathtt{l}} + \underbrace{\left(\begin{array}{cc|c|c} \mathbf{M}_{\mathtt{xlxl}} & \mathbf{M}_{\mathtt{xlxnl}} & \mathbf{M}_{\mathtt{xlwnl}} & \mathbf{M}_{\mathtt{xly}} \\ \mathbf{M}_{\mathtt{xnlxl}} & \mathbf{M}_{\mathtt{xnlxnl}} & \mathbf{M}_{\mathtt{xnlwnl}} & \mathbf{M}_{\mathtt{xnly}} \\ \hline \mathbf{M}_{\mathtt{wnlwl}} & \mathbf{M}_{\mathtt{wnlxnl}} & \mathbf{M}_{\mathtt{wnlwnl}} & \mathbf{M}_{\mathtt{wnly}} \\ \hline \mathbf{M}_{\mathtt{yxl}} & \mathbf{M}_{\mathtt{yxnl}} & \mathbf{M}_{\mathtt{ywnl}} & \mathbf{M}_{\mathtt{yy}} \end{array}\right)}_{\mathbf{M}_{\mathtt{nl}}} \begin{pmatrix} \nabla \mathrm{H}_{\mathtt{l}} \\ \nabla \mathrm{H}_{\mathtt{nl}} \\ \mathbf{z}_{\mathtt{nl}} \\ \mathbf{u} \end{pmatrix} \tag{4}$$

3 Presolve linear part

$$\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}_{\mathtt{l}}}{\mathrm{d}t} \\ \frac{\mathrm{d}\mathbf{x}_{\mathtt{nl}}}{\mathrm{d}t} \\ \mathbf{w}_{\mathtt{nl}} \\ \mathbf{y} \end{pmatrix} = \underbrace{\left(\mathbf{M}_{\mathtt{nlwl}}\,\mathbf{Z}_{\mathtt{l}}\,\mathbf{iD}_{\mathtt{wl}}\,\mathbf{M}_{\mathtt{wlnl}} + \mathbf{M}_{\mathtt{nl}}\right)}_{\overline{\mathbf{M}}} \begin{pmatrix} \nabla \mathrm{H}_{\mathtt{l}} \\ \nabla \mathrm{H}_{\mathtt{nl}} \\ \mathbf{z}_{\mathtt{nl}} \\ \mathbf{u} \end{pmatrix} \tag{5}$$