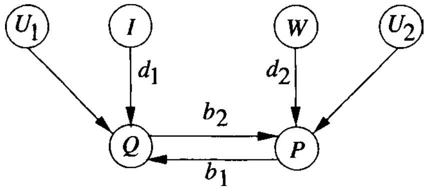

Figure 7.4 Causal diagram illustrating the relationship between price (P) and demand (Q).

Z_{pa_Z}) must obey that relationship as well. We thus obtain a simple graphical representation for any counterfactual variable of the form Z_{pa_Z}, in terms of the so called "error-term" U_Z. Using this representation, we can easily verify from Figure 7.3 that $(U_Y \perp\!\!\!\perp X \mid \{Y^*, Z^*\})_G$ and $(U_Y \perp\!\!\!\perp U_Z \mid \{Y, Z\})_G$ both hold in the twin-network and, therefore,

$$Y_z \perp\!\!\!\perp X \mid \{Y_x, Z_x\} \quad \text{and} \quad Y_z \perp\!\!\!\perp Z_x \mid \{Y, Z\}$$

must hold in the model. Additional considerations involving twin networks, including generalizations to multi-networks (representing counterfactuals under different anticedants) are reported in Shpitser and Pearl (2007). See Sections 11.3.2 and 11.7.3.

7.2 APPLICATIONS AND INTERPRETATION OF STRUCTURAL MODELS

7.2.1 Policy Analysis in Linear Econometric Models: An Example

In Section 1.4 we illustrated the nature of structural equation modeling using the canonical economic problem of demand and price equilibrium (see Figure 7.4). In this chapter, we use this problem to answer policy-related questions.

To recall, this example consists of the two equations

$$q = b_1 p + d_1 i + u_1, \tag{7.9}$$

$$p = b_2 q + d_2 w + u_2, \tag{7.10}$$

where q is the quantity of household demand for a product A, p is the unit price of product A, i is household income, w is the wage rate for producing product A, and u_1 and u_2 represent error terms – omitted factors that affect quantity and price, respectively (Goldberger 1992).

This system of equations constitutes a causal model (Definition 7.1.1) if we define $V = \{Q, P\}$ and $U = \{U_1, U_2, I, W\}$ and assume that each equation represents an autonomous process in the sense of Definition 7.1.3. It is normally assumed that I and W are observed, while U_1 and U_2 are unobservable and independent of I and W. Since the error terms U_1 and U_2 are unobserved, a complete specification of the model must include the distribution of these errors, which is usually taken to be Gaussian with the covariance matrix $\Sigma_{ij} = \text{cov}(u_i, u_j)$. It is well known in economics (dating back to Wright 1928) that the assumptions of linearity, normality, and the independence of $\{I, W\}$ and $\{U_1, U_2\}$ permit consistent estimation of all model parameters, including the covariance matrix Σ_{ij}. However, the focus of this book is not the estimation of parameters but rather their utilization in policy predictions. Accordingly, we will demonstrate how to evaluate the following three queries.

1. What is the expected value of the demand Q if the price is *controlled at* $P = p_0$?
2. What is the expected value of the demand Q if the price is *reported to be* $P = p_0$?
3. Given that the current price is $P = p_0$, what would be the expected value of the demand Q if we *were to control* the price at $P = p_1$?

The reader should recognize these queries as representing (respectively) actions, predictions, and counterfactuals – our three-level hierarchy. The second query, representing prediction, is standard in the literature and can be answered directly from the covariance matrix without reference to causality, structure, or invariance. The first and third queries rest on the structural properties of the equations and, as expected, are not treated in the standard literature of structural equations.[10]

In order to answer the first query, we replace (7.10) with $p = p_0$, leaving

$$p = b_1 p + d_1 i + u_1, \tag{7.11}$$

$$p = p_0, \tag{7.12}$$

with the statistics of U_1 and I unaltered. The controlled demand is then $q = b_1 p_0 + d_1 i + u_1$, and its expected value (conditional on $I = i$) is given by

$$E[Q \mid do(P = p_0), i] = b_1 p_0 + d_1 i + E(U_1 \mid i). \tag{7.13}$$

Since U_1 is independent of I, the last term evaluates to

$$E(U_1 \mid i) = E(U_1) = E(Q) - b_1 E(P) - d_1 E(I)$$

and, substituted into (7.13), yields

$$E[Q \mid do(P = p_0), i] = E(Q) + b_1(p_0 - E(P)) + d_i(i - E(I)).$$

The answer to the second query is obtained by conditioning (7.9) on the current observation $\{P = p_0, I = i, W = w\}$ and taking the expectation,

$$E(Q \mid p_0, i, w) = b_1 p_0 + d_1 i + E(U_1 \mid p_0, i, w). \tag{7.14}$$

The computation of $E[U_1 \mid p_0, i, w]$ is a standard procedure once $\sum_{ij}$ is given (Whittaker 1990, p. 163). Note that, although U_1 was assumed to be independent of I and W, this independence no longer holds once $P = p_0$ is observed. Note also that (7.9) and (7.10)

[10] I have presented this example to well over a hundred econometrics students and faculty across the United States. Respondents had no problem answering question 2, one person was able to solve question 1, and none managed to answer question 3. Chapter 5 (Section 5.1) suggests an explanation, and Section 11.5.4 a more recent assessment based on Heckman and Vytlacil (2007).

both participate in the solution and that the observed value p_0 will affect the expected demand Q (through $E(U_1 \mid p_0, i, w)$) even when $b_1 = 0$, which is not the case in query 1.

The third query requires the expectation of the counterfactual quantity $Q_{p=p_1}$, conditional on the current observations $\{P = p_0, I = i, W = w\}$ (see Section 11.7.1). According to Definition 7.1.5, $Q_{p=p_1}$ is governed by the submodel

$$q = b_1 p + d_1 i + u_1, \tag{7.15}$$

$$p = p_1; \tag{7.16}$$

the density of u_1 should be conditioned on the observations $\{P = p_0, I = i, W = \omega\}$. We therefore obtain

$$E(Q_{p=p_1} \mid p_0, i, w) = b_1 p_1 + d_1 i + E(U_1 \mid p_0, i, w). \tag{7.17}$$

The expected value $E(U_1 \mid p_0, i, w)$ is the same as in the solution to the second query; the latter differs only in the term $b_1 p_1$. A general matrix method for evaluating counterfactual queries in linear Gaussian models is described in Balke and Pearl (1995a).

At this point, it is worth emphasizing that the problem of computing counterfactual expectations is not an academic exercise; it represents in fact the typical case in almost every decision-making situation. Whenever we undertake to predict the effect of policy, two considerations apply. First, the policy variables (e.g., price and interest rates in economics, pressure and temperature in process control) are rarely exogenous. Policy variables are endogenous when we observe a system under operation; they become exogenous in the planning phase, when we contemplate actions and changes. Second, policies are rarely evaluated in the abstract; rather, they are brought into focus by certain eventualities that demand remedial correction. In troubleshooting, for example, we observe undesirable effects e that are influenced by other conditions $X = x$ and wish to predict whether an action that brings about a change in X would remedy the situation. The information provided by e is extremely valuable, and it must be processed (using abduction) before we can predict the effect of any action. This step of abduction endows practical queries about actions with a counterfactual character, as we have seen in the evaluation of the third query (7.17).

The current price p_0 reflects economic conditions (e.g., Q) that prevail at the time of decision, and these conditions are presumed to be changeable by the policies considered. Thus, the price P represents an endogenous decision variable (as shown in Figure 7.4) that becomes exogenous in deliberation, as dictated by the submodel $M_{p=p_1}$. The hypothetical mood of query 3 translates into a practical problem of policy analysis: "Given that the current price is $P = p_0$, find the expected value of the demand (Q) if we change the price *today* to $P = p_1$." The reasons for using hypothetical phrases in practical decision-making situations are discussed in the next section, as well as 11.7.2.

7.2.2 The Empirical Content of Counterfactuals

The word "counterfactual" is a misnomer, since it connotes a statement that stands contrary to facts or, at the very least, a statement that escapes empirical verification. Counterfactuals are in neither category; they are fundamental to scientific thought and carry as clear an empirical message as any scientific law.