Minimizing Economic Cost for Power Systems with Distinct Startup Times and Costs

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Abstract:

There many forms of power plant types in many different configurations that can be implemented in grid design. This paper is meant to show an optimization method for steam, combined cycle and gas turbines to meet the system demands of the ISO (Independent System Operator). Here, the amount the minimum and maximum a unit can produce, cost of fuel, grid demand, and distinct hot, warm, and cold start-up times and cost will be considered. In order to highlight the methods associated with the distinct hot, warm, cold start-up times the network grid losses, and impedance of the system will be neglected.

Introduction:

The characteristic equation to consider in this type of optimization problem is to minimize the total cost of fuel supply. Every turbine has a particular quadratic characteristic input-output characteristic equation for its incremental fuel rate [1].

$$C_{fi}(P_{Gi}) = a_i {P_{Gi}}^2 + b_i P_{Gi} + c_i \qquad 1$$

where C_{fi} is the cost of fuel/hr input into the system, P_{Gi} is the power being produced by a the turbine and $i\epsilon[1,2 \dots n]$ where n is the number of turbines, and$a_i, b_i, and\ c_i$ are constants associated with the i^{th} turbine. Here the performance measure to be solved is formed:

$$min C_{fi}(P_{Gi}) = \sum_{i=1}^{n} C_{fi}(P_{Gi}) \qquad 2$$

Here, constraints on the turbines should also be considered:

$$\sum_{i=1}^{n} P_{Gi} = P_D \qquad 3$$

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \qquad 4$$

where P_{Gimin} , and P_{Gimax} are the generators minimum and maximum production capacity respectively, and P_D is the amount of power dispatched (power demanded from the plant by the ISO).

Methods:

The Lagrange augmented function using constraint (3) and the Lagrange multiplier, λ, can be rewritten from constraint (3) as:

$$L(P_{Gi}, \lambda_i) = C_{fi}(P_{Gi}) - \lambda\left(\sum_{i=1}^{n} P_{Gi} - P_D\right) \quad .$$

Thus, in order to find λ_i, set $\frac{\partial L(P_{Gi}, \lambda_i)}{\partial P_{Gi}} = 0$. Thus,

$$\frac{\partial L(P_{Gi}, \lambda_i)}{\partial P_{Gi}} = \frac{\partial\left(C_f(\boldsymbol{P})\right)}{\partial P_{Gi}} - \lambda \frac{\partial}{\partial P_{Gi}}\left(\sum_{i=1}^{n} P_{Gi} - P_D\right) = 0 \quad .$$

Here, $\frac{\partial}{\partial P_{Gi}}\left(\sum_{i=1}^{n} P_{Gi} - P_D\right) = \frac{\partial P_{Gi}}{\partial P_{Gi}} = 1$. Thus, $\frac{\partial\left(C_f(\boldsymbol{P})\right)}{\partial P_{Gi}} - \lambda = 0$. It follows that,

$$\lambda = \frac{\partial\left(C_f(\boldsymbol{P})\right)}{\partial P_1} = \frac{\partial\left(C_f(\boldsymbol{P})\right)}{\partial P_2} = \cdots = \frac{\partial\left(C_f(\boldsymbol{P})\right)}{\partial P_n} \quad 5$$

This is derived in previous work as *the principle of equal incremental rate* [2].

Taking the partial derivatives of equation (1) as disrobed in equation (5) we obtain:

$$\lambda = a_1 P_{G1} + b_1 = a_2 P_{G2} + b_2 = \cdots = a_n P_{Gn} + b_n \quad 6$$

We can rewrite equation (6) into *n-1* equations. This system of equations combined with the original constraint equation will allow for the solving of optimization of the system neglecting the min/max constraint of the generator. This gives a global optimum without the constraints considered.

There are several ways to approach to accounting for the constraints in the optimization search: Using General Constraints with Lagrange Multipliers, Using Outside Penalty Function, etc. However, none seem as intuitively obvious as maximizing or minimizing the P_{Gi} variable that violated the constraint followed by adjusting the P_D to equate for the difference in demand.

Assume P_j violated constraint (4) where $j\epsilon[1,2 \ldots n]$. Thus, the global maximum is outside the constraint and we let $P_{jnew} = P_{jmax/min}$ closest to optimization point. Letting $\Delta P = P_j - P_{min/max}$ we have $P_{Dnew} = P_D - \Delta P$. Following this procedure equations (3), and (6) to obtain the optimum excluding P_j from the calculations to obtain the optimum can be used a second time.

The startup costs associated with dispatching a turbine needs to also be considered. The cooler a turbine is the longer it will take to come on line and the more fuel will be used in the process. This can be modeled as such:

$$C_{sf}(t) = F\left(1 - e^{-t/\tau}\right) + C_m \quad 7$$

where $C_{sf}(t)$ is the total cost of startup fuel, F is the cost fuel, t is the time the unit has been offline, τ is the thermal time constant derived from the cool down time of the unit, and C_m is the cost of the maintenance of the start-up [4]. This does not address the nature of the discreteness of start-up procedures for three distinct different states: hot start, cold start, and warm start The steam or combined cycle turbines start-up time and procedure is said to be in the hot, warm, or cold start up state if $t\epsilon[t_{hot\,min}, \infty]$, $t\epsilon[t_{warm\,min}, t_{hot\,min}]$, or $t\epsilon[ambiant\ temperature, t_{warm\,min}]$ respectively*. This means that for every turbine the hot start cost, C_{Gih}, Warm start, C_{Giw}, and cold start costs, C_{Gic},constant values for i number turbine exists. These constants can be calculated from equation *1* knowing F, $C_m, t, and\ \tau$. Thus,

$$C_{sfGi}(t) = \left\{\begin{matrix} C_{Gic} & where & t\epsilon[ambiant\ temperature, t_{warm\,min}] \\ C_{Giw} & where & t\epsilon[t_{warm\,min}, t_{hot\,min}] \\ C_{Gih} & where & t\epsilon[t_{hot\,min}, \infty] \end{matrix}\right\} \quad 8$$

where $C_{sfGi}(t)$ is the start-up cost for turbine i.

It should be noted here that there are the hot, warm, and cold start-up procedures for the steam and combined cycle but not GTs. The GTs are made for quick dispatch and can be readily available to run when called. It is for this reason $C_{sfGi}(t) = 0$ for GTs.

Other constraints not considered such as network losses, maintenance being performed on the system, and transformer losses. Many of these add the nodal currents, voltages, transformer taps, season, and many others variables to the picture for optimization of the power grid. All of these variables go into a Smart-Grid system with further complexity. This paper is focused only on the implementation of the discrete costs associated with Hot, Warm, and Cold start-up procedures.

Example of Generator Configuration for a modern Power Plant:

Let the power demand for the next 7 hours be displayed as follows in Table 1:

Suppose the plant under question has 3 Gas Turbines (GTs) where $10MW \leq P_{GT1} \leq 20MW$, $15MW \leq P_{GT2} \leq 30MW, and\ \ 13MW \leq P_{GT3} \leq 25MW$. The plant also contains one steam turbine generator (ST) where $50MW \leq P_{ST1} \leq 522MW$, and one combined cycle (CC) plant where $10MW \leq P_{CC1} \leq 300MW$. The GTs, ST, and CC have a charicterized function for the cost as follows:

$$C_{ST1}(P_{ST1}) = .0082{P_{ST1}}^2 + 18.67P_{ST1} - 500$$
$$C_{CC1}(P_{CC1}) = -.0521{P_{CC1}}^2 + 41.403P_{CC1} + 1848$$
$$C_{GT1}(P_{GT1}) = -1.5{P_{GT1}}^2 + 163P_{GT1} + 429.76$$
$$C_{GT2}(P_{GT2}) = -2.66{P_{GT2}}^2 + 184P_{GT2} + 390$$
$$C_{GT3}(P_{GT3}) = -1{P_{GT3}}^2 + 125P_{GT3} + 809$$

With the current price of gas \$4.48 MCF, the associated costs of different start-up procedures for the cold, warm, and hot starts of the ST and CC and times are as follows in Table 2:

The ST has been offline for 2 hours, while the CC turbine has been off line for 4 hours.

It is assumed in this problem that all turbines are starting from the off position with an entire plant outage taking place. Also, all GTs are assumed to have a 0hr start-up time.

For the first two hours the only available turbines will be the GTs. This is due to the start-up times of the ST and CC turbines. In this problem the ST can be available 3 hours from now and the CC turbine 2 hours from now. We consider the following availability of the turbines in Table 3

We can use equation (6) as previously derived to form the Lagrange multiplier. Thus,

$$\frac{\partial C_{GT1}(P_{GT1})}{\partial P_{GT1}} = -3P_{GT1} + 163$$
$$\frac{\partial C_{GT2}(P_{GT2})}{\partial P_{GT2}} = -5.32P_{GT2} + 184$$
$$\frac{\partial C_{GT3}(P_{GT3})}{\partial P_{GT3}} = -1P_{GT3} + 125$$
$$\frac{\partial C_{ST1}(P_{ST1})}{\partial P_{ST1}} = .0164P_{ST1} + 18.67$$
$$\frac{\partial C_{CC1}(P_{CC1})}{\partial P_{CC1}} = -.1042P_{CC1} + 41.403$$

As derived from the Lagrange multiplier (6) we can consider the system of equations:

$$-3P_{GT1} + 163 = -5.32P_{GT2} + 184 = -1P_{GT3} + 125 \Rightarrow \begin{matrix} -3P_{GT1} + 5.32P_{GT2} = 21 \\ -5.32P_{GT2} + 1P_{GT3} = -59 \end{matrix} \qquad 9$$

Table 1 can be seen as an hourly constraint similar to equation (3). Demanded at start (h=0) is 40 MW. Thus,

$$P_{GT1} + P_{GT2} + P_{GT3} = 40 \qquad 10$$

Equations (9) and (10) can be solved as a system with the result:

$$P_{GT1} = 16.17MW, P_{GT2} = 13.29MW, P_{GT3} = 10.52MW$$

The result in (11) can be seen as the global minimum. We can see that P_{GT2} and P_{GT3} have both violated the constraint by being too low. This means that P_{GT2}, and P_{GT3} will be increased to their minimum points and the maximum power recalculated. Thus, $P_{GT2} = 15MW$, and $P_{GT3} = 13MW$. This results in $P_{GT1} = 12MW$ to meet the demand at h=0.

For 1 hour in the future, equations (9) and (10) are repeated only the constraint in equation (10) is 60. This results in:

$$P_{GT1} = 20.54MW, P_{GT2} = 15.80MW, P_{GT3} = 23.64MW$$

We see here P_{GT1} has violated the constraint. Thus, we reduce P_{GT1} to 20MW and add the difference to be made up by the minimum between P_{GT2} and P_{GT3}. Here we change the demand to 40MW. Thus, we solve the system of equations created by:

$$-5.32P_{GT2} + 184 = -1P_{GT3} + 125 \text{ and } P_{GT2} + P_{GT3} = 40$$

This system solved:

$$P_{GT2} = 15.89MW, P_{GT3} = 24.1MW$$

Thus, after the first hour the optimal configuration is

$$P_{GT1} = 20MW, P_{GT2} = 15.89MW, P_{GT3} = 24.1MW$$

Two hours from the start we must now consider the combined cycle turbine since it can go online within two hours. Here, the cost of start-up is different constants, C_{CCCold}, and C_{CCHot} start-ups. As a result the partial derivatives used in (6) do not reflect these costs. It must be added to the optimum and the choice to start or not to start the turbine must be determined.

Considering two hours from now with only the GTs available we see

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW$$

Checking the performance measure (2) we see:

$$minC_{fi}(P_{Gi}) = \sum_{i=1}^{n} C_{fi}(P_{Gi}) = 3089 + 3516 + 3309 = \$9914$$

Considering all GTs and CC turbine using equation (6) we obtain the following set of equations:

$$-3P_{GT1} + 163 = -5.32P_{GT2} + 184 = -1P_{GT3} + 125 = -.1042P_{CC1} + 41.403$$

With the CC turbine now being considered the optimal is still:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 0\ MW$$

This is due to the low level of efficiency of the CC turbine at lower MW production.

For three hours from now the power being demanded is 300MW. Here, the GTs alone cannot produce the load demanded and the CC turbine will need to be started up to fill the demand. Using equality of the Lagrange multiplier and the system of equations method previously described:

$$P_{GT1} = 45.0MW, P_{GT2} = 29.32MW, P_{GT3} = 97.0MW, P_{CC1} = 128.66\ MW$$

Here, GT1 and GT3 are both in violation of the constraints and need to be maximized. Thus, $P_{GT1} = 20MW, and \quad P_{GT3} = 25MW$. This leaves an optimization of 255MW for turbines P_{GT3} and P_{CC1}. Here,

$$P_{GT1} = 20MW, P_{GT2} = 99.7MW, P_{GT3} = 25MW, P_{CC1} = 155.2MW$$

P_{GT2} violates the constraint and needs to be maximizes to 30MW. This leaves $P_{CC1} = 225MW$ to make up the power demand constraint. This causes the total cost to be

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) + C_{CCCold} = 16317 + 6000 = 22{,}317$$

We must consider the case of a warm-start up on the ST (Note: a cold start on CC and a warm-start on the ST both is not considered because of the start-up cost being great enough to exceeding the two situations of the CC start-up or the ST warm start-up. Using previous methods:

$$P_{GT1} = 47.5MW, P_{GT2} = 30.7MW, P_{GT3} = 104.4MW, P_{ST1} = 117.4\ MW$$

Modified with this leaves $P_{ST1} = 225MW$. This causes the total cost to be

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) + \boldsymbol{C_{CCCold}} = 16092 + 8000 = 24{,}092$$

Thus, the optimal start-up for 3 hours from now is:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 225\ MW$$

For 4 hours from now we have the CC running and it is now cheaper to run the CC because of start-up costs. Thus, in order to meet demand:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 300\ MW$$

For hour 5 the Steam Turbine must be run to meet the demand of 600MW. However, the turbine can only be used in a cold start since it has been offline for more than 4 hours. Using the Lagrange multiplier technique previously described:

$$P_{GT1} = 47.63MW, P_{GT2} = 30.80MW, P_{GT3} = 104.89MW, , P_{ST1} = 87.25\ MW, P_{CC1} = 204.4\ MW$$

Here, all GTs have violated the maximum constraint and $P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW$ where the CC and ST turbines have to make up the difference. With this consideration the new power demand for the ST and CC is 400MW. Using this as the demand between the two:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{ST1} = 215.83\ MW, P_{CC1} = 184.16MW$$

Starting the ST adds significantly to the cost which we will see later.

Finally, for 6 hours from now evaluated using previous methods we have:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{ST1} = 67.48\ MW, P_{CC1} = 207.51MW$$

It should be noted that shutting off a turbine once it is started up is not advised. The cost of starting a turbine is significant and can severely increase the operational cost. We see the hourly operational costs and power in table 4

It can be seen that the profit is negative for the 1 hour and 2 hour. This means it would be more profitable to not run the GTs. However, the plant could be under capacity payments and ordered to run by the ISO with a compensation plan worked out for the distribution system.

Conclusion:

The method described here can be summed up in the following steps:

1. Use the equality of the Lagrange multipliers to find a system of equations for the equality of the partial derivatives of the fuel characteristic equations, and the power demand formula. Do this on only available units.
2. Check if any units violated the constraint. Maximize or minimize these units based on which way (greater than/less than) they violated the constraint.
3. Disperse the power over the other units that did not violate the constraint and repeat step 1 to calculate the optimized power units left over.

4. If there is cold, warm, or hot starts to consider take them as separate scenarios of each starting up and calculate total costs including start-up costs.
5. Take the minimized cost as the optimum scenario.

This is a feasible method to which an algorithm can be written for and incorporated in grid technologies. Much of the work using *the principle of equal incremental rate* proved by equal Lagrange multipliers has been previously done, with the addition of the start-up cost assessment in this paper. Different weightings to equation (5) can be further added to compensate for network losses[]. However, it is necessary to know the nodal current and voltages here.

Figures and Tables

Table 1

Hours From Now	Power Demand (MW)
0	40
1	60
2	75
3	300
4	375
5	475
6	350

Table 2

Generator Unit	Cost ($)	Time to Start-up (hr)	Time From Coming Offline (hr)
C_{STCold}	12,000	4	8
C_{STWarm}	8,000	3	4
C_{STHot}	5,500	2	1
C_{CCCold}	6,000	2	2
C_{CCHot}	4,500	1	1

Table 3

Turbine\Hours From Now	0	1	2	3	4	5	6
GT1							
GT2							
GT3							
CC1							
ST1							

Key

	GT Available
	Turbine Not Available
	Warm Start Available
	Cold Start Available

Table 4

Hour	GT1	GT2	GT3	ST1	CC1	Operating Costs	MW Production	MWh Market	Production Value	Profit
0	16.1	13.3	10.5	0	0	7052.13	40	300.5	12020	4967
1	20	15.9	24.1	0	0	8972.58	60	110	6600	-2372
2	20	30	25	0	0	9914	75	100	7500	-2414
3	20	30	25	0	225	22092	300	110	33000	10908
4	20	30	25	0	300	17146.66	375	50	18750	1603.3
5	20	30	25	215.8	184.1	29470	450	75	33750	4280
6	20	30	25	67.48	207.5	18846	350	85	29750	10904

	Turbine under start-up

Resources

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