Minimizing Economic Cost for Power Systems with Distinct Startup Times and Costs

Andrew Grossmann

New York University Tendon School of Engineering
Prof Zhong-Ping Jiang

4/25/17

What is the most optimal configuration of power producing units with a given demand?

- Considered:
  - Constraints of max and min load of turbine
  - Constraints of what is demanded by the grid at that time
  - Start-up cold, warm, and hot costs and times

- Not Considered:
  - System network losses with known nodal currents and voltages
  - Dynamic Programing Method for working backward to figure out best configuration.

Incremental Fuel Rate

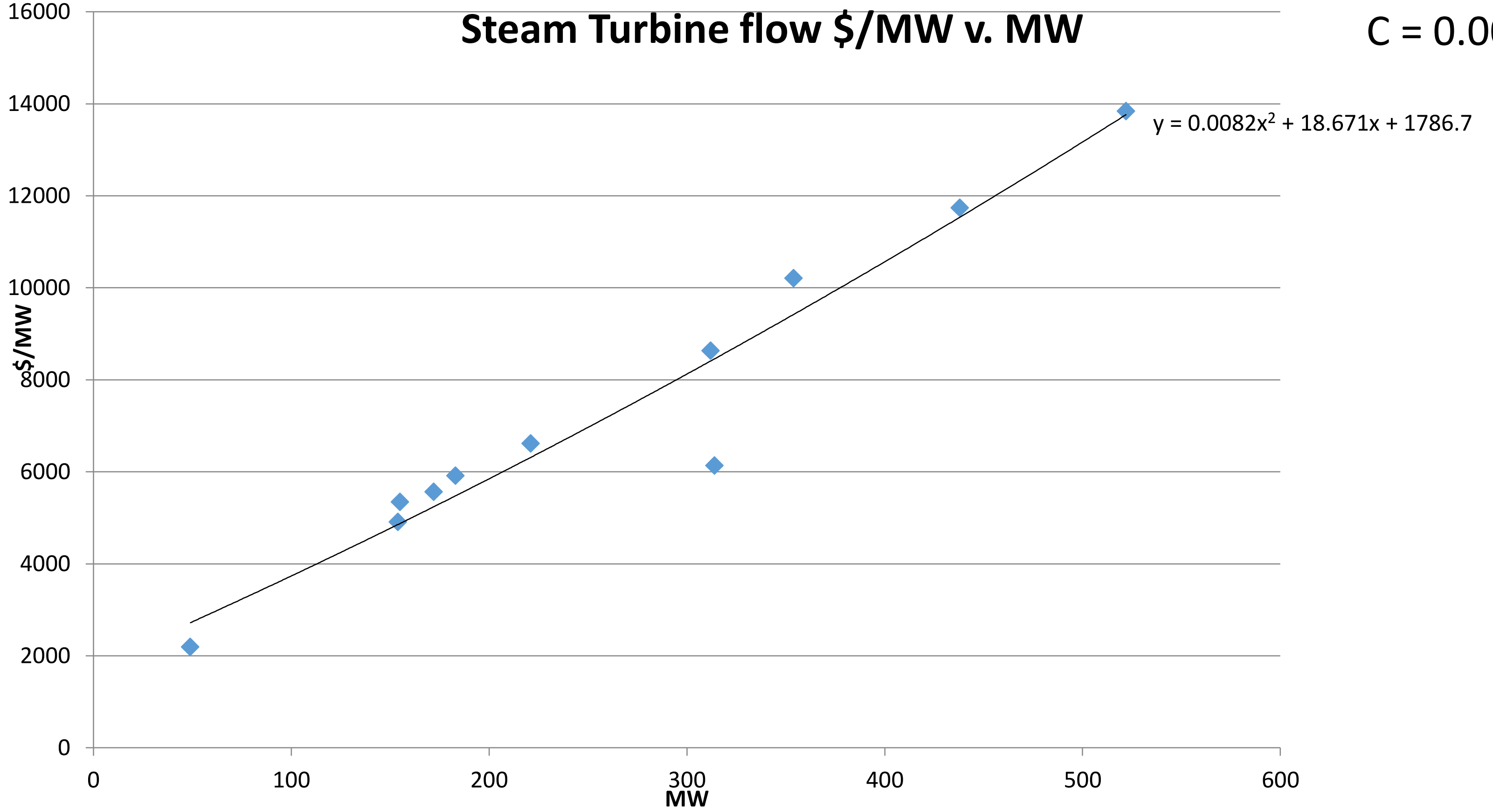
- Every turbine has a characteristic input output equation known as its *Incremental Fuel Rate.*

$Incremental\ fuel\ rate\ equation$:

$$C_{fi}(P_{Gi}) = a_i {P_{Gi}}^2 + b_i P_{Gi} + c_i \tag{1}$$

where C_{fi} is the cost of fuel/hr input into the system, P_{Gi} is the power being produced by a the turbine and $i \epsilon [1,2 \dots n]$ where n is the number of turbines, and$a_i, b_i, and\ c_i$ are constants associated with the i^{th} turbine.

Data pulled from operational Steam Turbine plant



Incremental Fuel Rate Equation

$C = 0.0082P^2 + 18.671P + 1786.7$

Performance Measure and Constraints

Performance Measure: $minC_{fi}(P_{Gi}) = \sum_{i=1}^{n} C_{fi}(P_{Gi})$

Constraints:

1. Power Demand $\sum_{i=1}^{n} P_{Gi} = P_D$ (3)

   where P_D is the amount of power dispatched (power demanded from the plant by the ISO).

2. Turbine Power Range $P_{Gimin} \leq P_{Gi} \leq P_{Gimax}$ or P_{Gi} = 0 (4)

   where P_{Gimin} , and P_{Gimax} are the generators minimum and maximum production capacity respectively

3. Hot, Warm, and Cold Start-up costs $C_{sfGi}(t) = \begin{cases} C_{Gic} & where \quad t\epsilon[ambiant\ temperature, t_{warm\ min}] \\ C_{Giw} & where \quad t\epsilon[t_{warm\ min}, t_{hot\ min}] \\ C_{Gih} & where \quad t\epsilon[t_{hot\ min}, \infty] \end{cases}$

Where the hot start cost, C_{Gih}, Warm start, C_{Giw}, and cold start costs, C_{Gic},constant values for i number turbine exists are used in a piecewise function to calculate $C_{sfGi}(t)$ the start-up cost for turbine i.

Finding the Global Optimum for the Performance Measure Constrained by Power Demand

Using the Lagrange method:

$$L(P_{Gi}, \lambda_i) = C_{fi}(P_{Gi}) - \lambda(\sum_{i=1}^{n} P_{Gi} - P_D) \quad .$$

Thus, in order to find λ_i, set $\frac{\partial L(P_{Gi}, \lambda_i)}{\partial P_{Gi}} = 0$. Thus,

$$\frac{\partial L(P_{Gi}, \lambda_i)}{\partial P_{Gi}} = \frac{\partial(C_f(\boldsymbol{P}))}{\partial P_{Gi}} - \lambda \frac{\partial}{\partial P_{Gi}} (\sum_{i=1}^{n} P_{Gi} - P_D) = 0 \quad .$$

Here,

$$\frac{\partial}{\partial P_{Gi}} (\sum_{i=1}^{n} P_{Gi} - P_D) = \frac{\partial P_{Gi}}{\partial P_{Gi}} = 1 \Rightarrow \frac{\partial(C_f(\boldsymbol{P}))}{\partial P_{Gi}} - \lambda = 0.$$

It follows that,

$$\lambda = \frac{\partial(C_f(\boldsymbol{P}))}{\partial P_1} = \frac{\partial(C_f(\boldsymbol{P}))}{\partial P_2} = \cdots = \frac{\partial(C_f(\boldsymbol{P}))}{\partial P_n} \qquad 5$$

Taking the partial derivatives of equation (1) as disrobed in equation (5) we obtain:

$$\lambda = a_1 P_{G1} + b_1 = a_2 P_{G2} + b_2 = \cdots = a_n P_{Gn} + b_n \qquad 6$$

Finding the Global Optimum for the Performance Measure Constrained by Power Demand Cont.

-We can rewrite equation (6) into *n-1* equations!

-Combined with the constraint for equation (3) we can make a system of equations of n equations and n variables that can be solved.

-We can use this method to solve any system of power units given an incremental flow rate equation and a demand.

Violated turbine’s MW range

Violates Min Constraint:

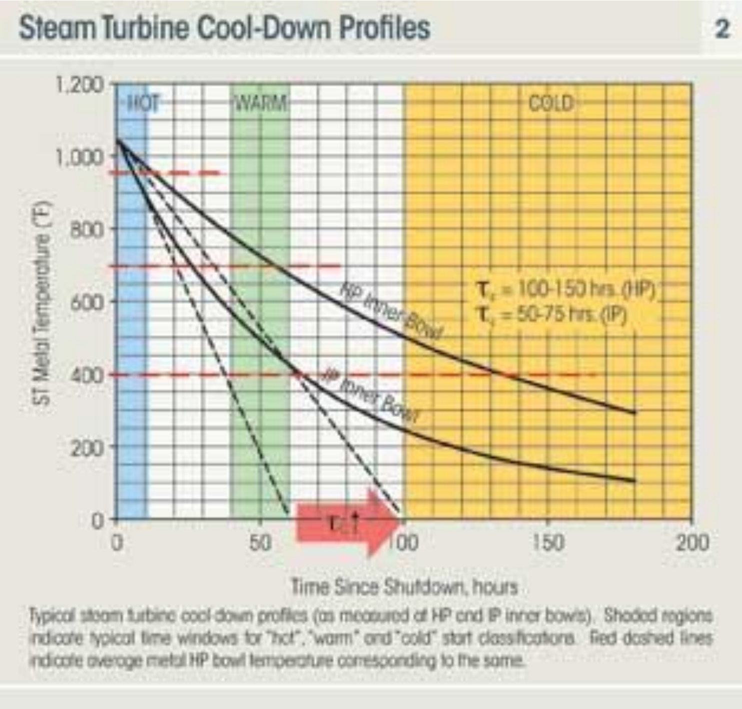
If a min range on equation (4) is violate, bring the turbine to its min or max value range. If turbine is below its minimum range, don’t start up the turbine! Its output will be 0MW if this happens. The optimum can be recalculated with the remaining turbines using the method using equation (6).

Violates Min Constraint:

If the turbine violates the max range of equation (4), bring the turbine that violated the constraint to its maximum range. The optimum for the rest of the turbines can be recalculated using the remaining power demand.

Considering Cost of Start-up

- A cooling function for cost of start-up of a turbine can be formulated as such:



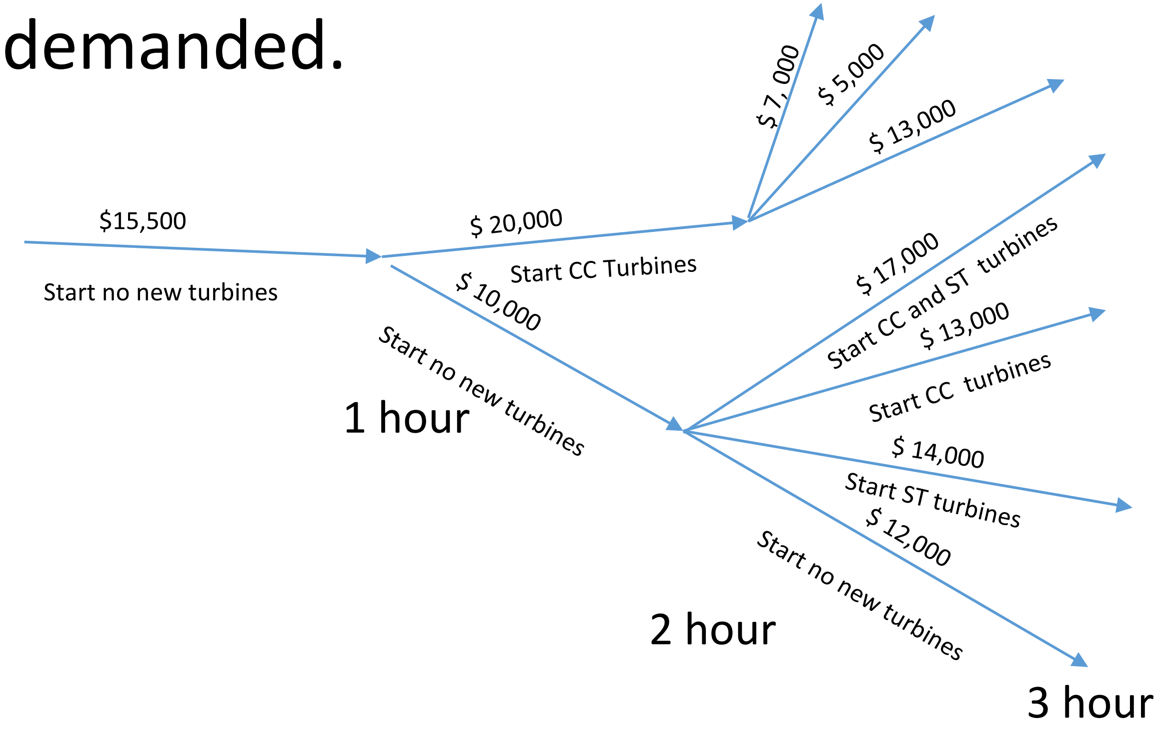
$$C_{sf}(t) = F\left(1 - e^{-t/\tau}\right) + C_m$$

This cool down process can be divided into distinct start-up procedures for turbines for cold, warm, and hot start ups:

$$C_{sfGi}(t) = \begin{cases} C_{Gic} \quad where \quad t\epsilon[ambiant\ temperature, t_{warm\ min}] \\ C_{Giw} \quad where \quad t\epsilon[t_{warm\ min}, t_{hot\ min}] \\ C_{Gih} \quad where \quad t\epsilon[t_{hot\ min}, \infty] \end{cases}$$

Considering Cost of Start-up

- In order to determine the minimum (optimum) operating costs we must consider all combinations of start ups meeting the power demanded.



The model on the left shows how the problem can be broken down into a dynamic programming problem working backwards from hour 3 to create the least combination

Simulated Power 5 Turbine Plant Real-Life Problem

Hours From Now	Power Demand (MW)
0	40
1	60
2	75
3	300
4	375
5	475
6	350

Table 1

Turbine	Min Load (MW)	Max Load (MW)	Incremental Fuel Rate Equation
(Gas Turbine) GT1	10	20	$C_{GT1}(P_{GT1}) = -1.5P_{GT1}{}^2 + 163P_{GT1} + 429.76$
GT2	15	30	$C_{GT2}(P_{GT2}) = -2.66P_{GT2}{}^2 + 184P_{GT2} + 390$
GT3	13	25	$C_{GT3}(P_{GT3}) = -1P_{GT3}{}^2 + 125P_{GT3} + 809$
(Steam Turbine) ST1	50	522	$C_{ST1}(P_{ST1}) = .0082P_{ST1}{}^2 + 18.67P_{ST1} - 500$
(Combined Cycle) CC1	10	300	$.C_{CC1}(P_{CC1}) = -.0521P_{CC1}{}^2 + 41.403P_{CC1} + 1848$

Table 2

Generator Unit	Cost ($)	Time to Start-up (hr)	Time From Coming Offline (hr)
C_{STCold}	12,000	4	8
C_{STWarm}	8,000	3	4
C_{STHot}	5,500	2	1
C_{CCCold}	6,000	2	2
C_{CCHot}	4,500	1	1

Table 3

*Note: For GT1 and the Steam Turbine real data was pulled off of a turbine was pulled off of a Statistical Collection and Analysis tool (SCADA) of running turbines between 3 start-ups. Also, a gas price was used of $4.58/MCFH (per million cubic feet of gas times hours). We will see with the numbers this may have been a bit too high of choice.

Finding Partial Derivatives

- By finding the partials for each incremental fuel rate and setting them equal we can form a system of equations to solve for the configuration of MW on generators for minimum cost.

$$\frac{\partial C_{GT1}(P_{GT1})}{\partial P_{GT1}} = -3P_{GT1} + 163$$

$$\frac{\partial C_{GT2}(P_{GT2})}{\partial P_{GT2}} = -5.32P_{GT2} + 184$$

$$\frac{\partial C_{GT3}(P_{GT3})}{\partial P_{GT3}} = -1P_{GT3} + 125$$

$$\frac{\partial C_{ST1}(P_{ST1})}{\partial P_{ST1}} = .0164P_{ST1} + 18.67$$

$$\frac{\partial C_{CC1}(P_{CC1})}{\partial P_{CC1}} = -.1042P_{CC1} + 41.403$$

Availability of Turbines

It is assumed in this problem that GTs can start up almost instantaneously… (In reality is takes about 5-10 minutes).

Because of the times for start-up as seen in table 3, we can derive the following table

Turbine\Hours From Now	0	1	2	3	4	5	6
GT1							
GT2							
GT3							
CC1							
ST1							

Table 4

Generator Unit	Cost ($)	Time to Start-up (hr)	Time From Coming Offline (hr)
C_{STCold}	12,000	4	8
C_{STWarm}	8,000	3	4
C_{STHot}	5,500	2	1
C_{CCCold}	6,000	2	2
C_{CCHot}	4,500	1	1

	GT Available
	Turbine Not Available
	Warm Start Available
	Cold Start Available

0 Hours From Now

We have only the GTs available using table 4. Using equation (6):

$$-3P_{GT1} + 163 = -5.32P_{GT2} + 184 = -1P_{GT3} + 125 \Longrightarrow \begin{matrix} -3P_{GT1} + 5.32P_{GT2} = 21 \\ \\ -5.32P_{GT2} + 1P_{GT3} = -59 \end{matrix}$$

Also, for 0 hours from now

$$P_{GT1} + P_{GT2} + P_{GT3} = 40\ .$$

Using these three equations we can form the augmented matrix:

$$\begin{bmatrix} -3 & 5.23 & 0 & \vdots & 21 \\ 0 & -5.23 & 1 & \vdots & -59 \\ 1 & 1 & 1 & & 40 \end{bmatrix} \Rightarrow \text{Rref} \begin{bmatrix} 1 & 0 & 0 & & 16.17 \\ 0 & 1 & 0 & \vdots & 13.29 \\ 0 & 0 & 1 & & 10.5\ 2 \end{bmatrix} \Rightarrow \begin{matrix} P_{GT1} = 16.17MW, P_{GT2} = 13.29MW, \\ P_{GT3} = 10.52MW \end{matrix}$$

*None of the powers given violate the min and max values given in table 3. Thus, this is the optimum value for zero hours from now.

1 hour from now

Per table 4 we only have the GTs available again. As from before we derive the same method to derive a system of equations with the demand equal to 60MW…

$$P_{GT1} = 20.54MW, P_{GT2} = 15.80MW, P_{GT3} = 23.64MW$$

Here GT1 violates the maximum constraint and should be reduced to 20MW. Thus, 20MW can be taken off total demand for the hour and the new demand, 40MW, needs to be optimized between GT2 and GT3.

P_{GT2} + P_{GT3}=40.

Using the method from the previous slide:

$$P_{GT2} = 15.89MW,\ P_{GT3} = 24.1MW \text{ and } P_{GT1} = 20MW$$

2 hours From Now

Here, we must consider two cases:

Case 1 The GTs only cover the power demand

Case 2 The CC is used and started up now.

2 hours From Now (Case 1)

Case 1:

With only the GTs, all GTs operating at max load will produce the needed 75 MW. Thus,

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW$$

Calculating cost,

$$minC_{fi}(P_{Gi}) = \sum_{i=1}^{n} C_{fi}(P_{Gi}) = 3089 + 3516 + 3309 = \$9914$$

2 hours From Now (Case 2)

Case 2:

With only the GTs and the CC turbine started up we can use the previous method to derive a system of equations such that:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 0MW$$

Solving a system of equations brings a negative value to P_{CC1}. This is due to the low level of efficiency of the turbine. Here it is taken out of the picture and replaced by 0MW.

3 hours From Now (Case 1 and 2)

With the GTs unable to make more than 75MW the power demand of 300MW will be achieved by either starting the Steam Turbine or Combined Cycle Turbine.

Case 1 Starting the CC turbine:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 225MW$$

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) + C_{CCCold} = 16317 + 6000 = 22{,}317$$

Case 2 Starting the ST:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 225MW$$

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) + \boldsymbol{C_{STWarm}} = 16092 + 8000 = 24{,}092$$

3 hours From Now (Case 3)

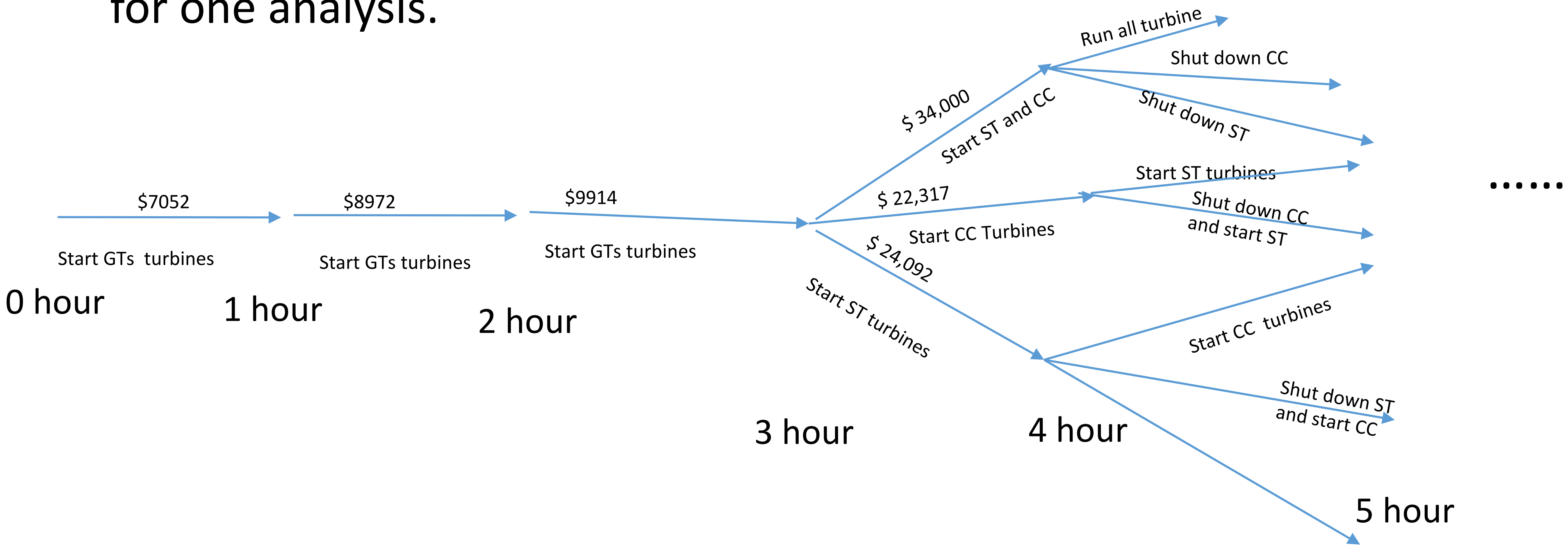
Case 3 The cost of starting up both the ST and CC is not considered.   It will cost more here but should be to find the minimum over the 6 hours.

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{ST1} = 8.48MW, P_{CC1} = 216MW$$

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) + \boldsymbol{C_{STWarm}} + C_{CCCold} = 20220 + 14000 = 34{,}220$$

Note on use of Dynamic programing Backward Bias

We must now consider each additional scenario if in 3 hours from now 3 cases occurred… One can see how the calculations can grow exponentially in this matter. To somewhat shorten the calculation a forward bias with **minimal starting and stopping of turbines** was taken to get to the answer rather than a backward bias for one analysis.



4 hours From Now

For 4 hours from now we have the CC running and it is now cheaper to run the CC because of start-up costs. Thus, in order to meet demand:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{CC1} = 300\ MW$$

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) = 17146.66$$

5 hours From Now

Here, all turbines will be left to run and we will be left with no option but to optimize all turbines:

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{ST1} = 215.83\ MW, P_{CC1} = 184.16MW$$

And for cost

$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) + \boldsymbol{C_{STCold}} = 29470$$

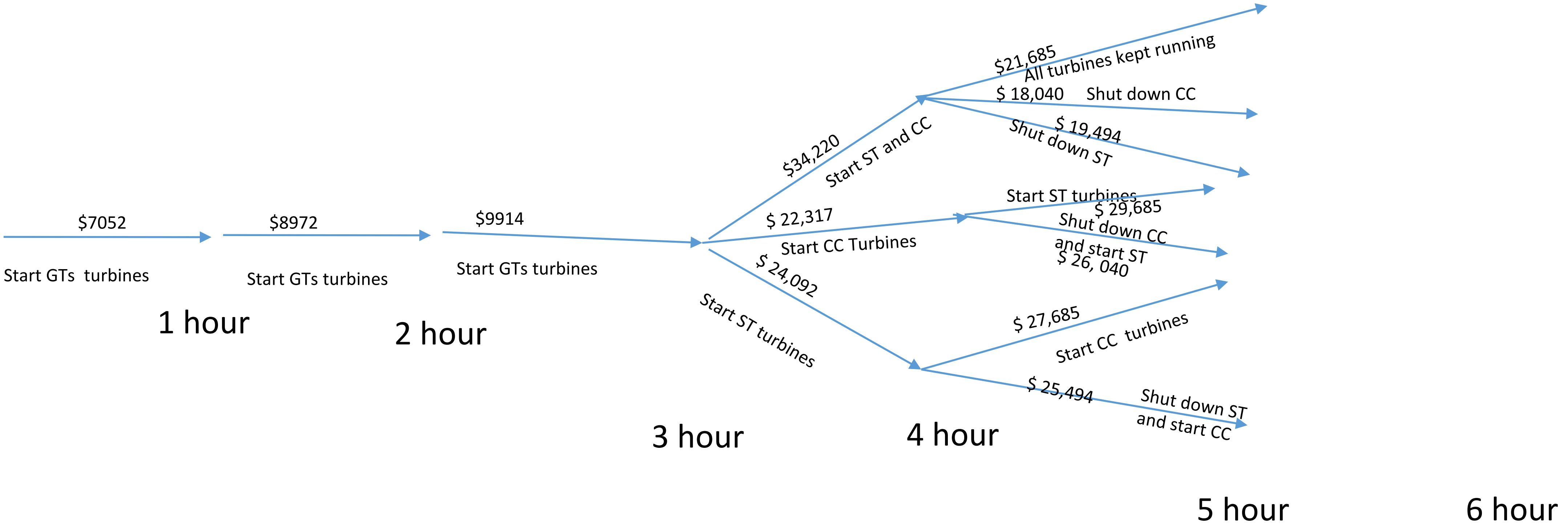
6 hours From Now

Here we consider if any turbines should be shut down.

$$P_{GT1} = 20MW, P_{GT2} = 30MW, P_{GT3} = 25MW, P_{ST1} = 67.48\ MW, P_{CC1} = 207.51MW$$

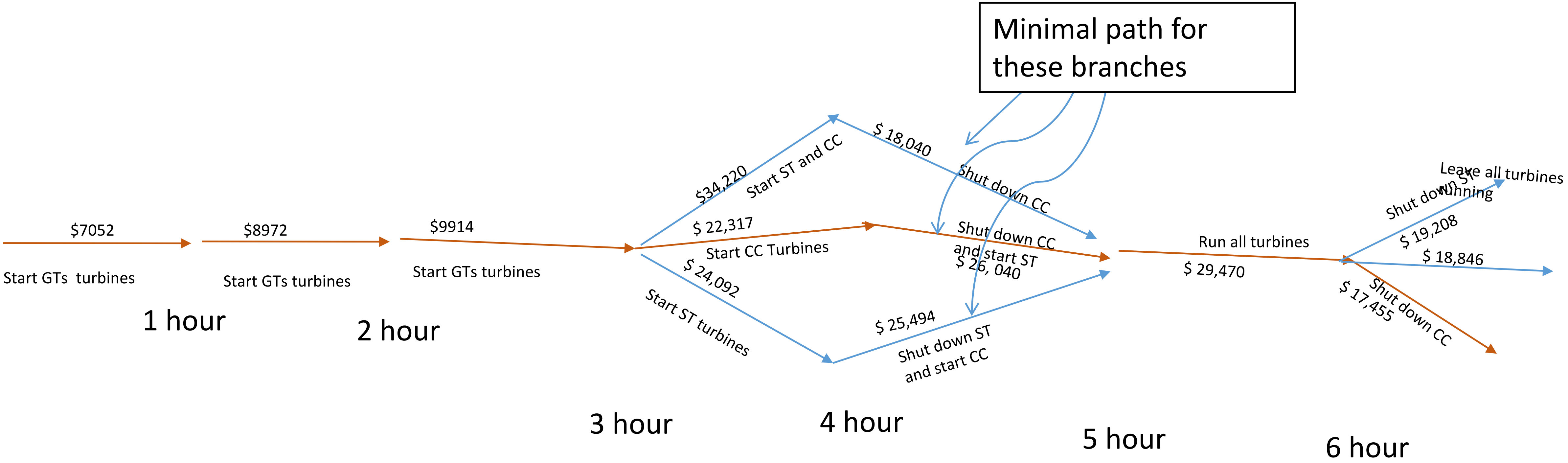
$$\sum_{i=1}^{n} C_{3i}(P_{Gi}) = 18846$$

Analyzing All Possible Combinations with Dynamic Programing



Analyzing All Possible Combinations

Optimal Path is Given in Orange. **Notice all paths converge at hour 5 to a common point due to the need of starting all turbines because of power demand!** This reduces the problem a great deal!



Dynamic Programing v. Turbine Stop/Start Minimizing

Dynamic Programing for the decision tree in the last slide will provide a different configuration for hour 6 than from the first analysis of trying to keep turbines that are running and not start any more unneeded. We may see in the future that shutting down a turbine here is a poor choice.

End result

This gives us the table below:

Hour	GT1	GT2	GT3	ST1	CC1	Operating Costs	MW Production	MWh Market	Production Value	Profit
0	16.1	13.3	10.5	0	0	7052.13	40	300.5	12020	4967
1	20	15.9	24.1	0	0	8972.58	60	110	6600	-2372
2	20	30	25	0	0	9914	75	100	7500	-2414
3	20	30	25	0	225	22092	300	110	33000	10908
4	20	30	25	0	300	17146.66	375	50	18750	1603.3
5	20	30	25	215.8	184.1	29470	475	75	33750	4280
6	20	30	25	67.48	207.5	18846	350	85	29750	10904

Table 5

	Turbine under start-up

Notice the choice to leave all turbines running in hour 6 is due to an unknown future and trying to minimize start-up costs when they run again

The market value of a MW and the profit of the turbine running is also calculated here. Notice that there is no profit at times but the turbine is often still run when ordered to do so by the ISO.

Procedure Summery

The method described here can be summed up in the following steps:

1. Use the equality of the Lagrange multipliers to find a system of equations for the equality of the partial derivatives of the fuel characteristic equations, and the power demand formula. Do this on only available units.
2. Check if any units violated the constraint. Maximize or minimize these units based on which way (greater than/less than) they violated the constraint.
3. Disperse the power over the other units that did not violate the constraint and repeat step 1 to calculate the optimized power units left over.
4. If there is cold, warm, or hot starts to consider take them as separate scenarios of each starting up and calculate total costs including start-up costs.
5. Take the minimized cost as the optimum scenario.

Note the dynamic programing methods of backward bias can be used for a more precise result. However, this involves more and more computation the further and further one goes in the future.

Procedure Summery

The method described here can be summed up in the following steps:

1. Use the equality of the Lagrange multipliers to find a system of equations for the equality of the partial derivatives of the fuel characteristic equations, and the power demand formula. Do this on only available units.
2. Check if any units violated the constraint. Maximize or minimize these units based on which way (greater than/less than) they violated the constraint.
3. Disperse the power over the other units that did not violate the constraint and repeat step 1 to calculate the optimized power units left over.
4. If there is cold, warm, or hot starts to consider take them as separate scenarios of each starting up and calculate total costs including start-up costs.
5. Take the minimized cost as the optimum scenario.

Note the dynamic programing methods of backward bias can be used for a more precise result. However, this generally involves more and more computation the further and further one goes in the future.

Resources

[1] L.K. Kirchamayer , *Economic Operation of Power Systems* , New York : Wiley , 1958 .

[2] J.Z. Zhu , *"Power System Optimal Operation*, Tutorial of Chongqing University, 1990 .

[3] N.V. Ramana, *"Power System Operation & Control"* Pearson Education India, 2010.

[4]S.C. Gulen, *Gas Turbine Combined Cycle Fast Start: The Physics Behind the Concept* Power engineering, 6/12/2013. Retrieved 4/10/17 from http://www.power-eng.com/articles/print/volume-117/issue-6/features/gas-turbine-combined-cycle-fast-start-the-physics-behind-the-con.html

[5] D. A. Pierre, *Optimization Theory with Applications* Montana State Engineering, 1986.