Peergrade #1

Exercise 1.1

$$\dfrac{\dfrac{\overline{C \to (A \vee B)}^{\,h2} \quad \overline{C}^{\,h3}}{A \vee B}{\scriptstyle \to E} \qquad \dfrac{\dfrac{\overline{\neg A \wedge \neg B}^{\,h1}}{\neg A}{\scriptstyle \wedge E1} \quad \overline{A}^{\,h4}}{F}{\scriptstyle \neg E} \qquad \dfrac{\dfrac{\overline{\neg A \wedge \neg B}^{\,h1}}{\neg B}{\scriptstyle \wedge E2} \quad \overline{B}^{\,h5}}{F}{\scriptstyle \neg E}}{}{\scriptstyle \vee E_{h4,h5}}$$

$$\dfrac{\dfrac{\dfrac{F}{\neg C}{\scriptstyle \neg I_{h3}}}{\big(C \to (A \vee B)\big) \to \neg C}{\scriptstyle \to I_{h2}}}{(\neg A \wedge \neg B) \to \big(C \to (A \vee B)\big) \to \neg C}{\scriptstyle \to I_{h1}}$$

Hypotheses:
$h1: \neg A \wedge \neg B$
$h2: C \to A \vee B$
$h3: C$
$h4: A$
$h5: B$

Exercise 1.2

A	B	C	$\neg A$	$\neg B$	$\neg C$	$\neg A \wedge \neg B$	$A \vee B$	$C \to (A \vee B)$	$(\neg A \wedge \neg B) \to \big(C \to (A \vee B)\big) \to \neg C$
T	T	T	F	F	F	F	T	T	
T	T	F	F	F	T	F	T	T	
T	F	T	F	T	F	F	T	T	
T	F	F	F	T	T	F	T	T	
F	T	T	T	F	F	F	T	T	
F	T	F	T	F	T	F	T	T	
F	F	T	T	T	F	T	F	F	
F	F	F	T	T	T	T	F	T	T

Premises have been highlighted in a pastel green. Conclusions have only been evaluated where all premises are true. QED we have a tautology.

Exercise 2

1. All pets are docile.

$\forall p \in P, D(p)$, where D(x) determines, if x is docile and P is the set of all pets.
$\exists p \in P, \neg D(p)$
There exists a pet which is not docile.

2. For any odd integer, its square is also odd.

$\forall i \in O, Odd(i^2)$ where Odd(x) determines if x is odd and O is the set of all odd integers.
$\exists i \in O, \neg Odd(i^2)$
There exists at least one odd integer, whose square is not odd.

3. There is a country with no neighbours.

$\exists c \in C, \neg HN(c)$ where HN(x) determines if x has at least one neighbour and C is the set of all countries.

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$\forall c \in C, HN(c)$
All countries has at least one neighbouring country.

4. All football games have a winner team.

$\forall g \in FG, HW(g)$ where HW(x) determines if x has a winner and FG is the set of all football games.
$\exists g \in FG, \neg HW(g)$
It is possible for a football game to end without a winner.

5. There is a natural number that is negative.

$\exists n \in \mathbb{N}, Neg(n)$ where Neg(x) determines if a number is negative and $\mathbb{N}$ is the set of all natural numbers.
$\forall n \in \mathbb{N}, \neg Neg(n)$
All natural numbers are not negative.

Exercise 3

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{M(Wo) \to W(Wo)}{\scriptstyle h3} \quad \dfrac{\dfrac{\dfrac{}{\forall z\big(F(z) \to M(z)\big)}{\scriptstyle h2}}{F(Wo) \to M(Wo)}{\scriptstyle \forall E} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\forall x, \forall y\big(F(y) \to S(x,y) \to F(x)\big)}{\scriptstyle h1}}{\forall y\big(F(y) \to S(Wo,y) \to F(Wo)\big)}{\scriptstyle \forall E}}{F(D) \to S(Wo,D) \to F(Wo)}{\scriptstyle \forall E} \quad \dfrac{}{F(D)}{\scriptstyle h4}}{F(Wo)}{\scriptstyle \to E}}{M(Wo)}{\scriptstyle \to E}}{W(Wo)}{\scriptstyle \to E}}{S(Wo,D) \to W(Wo)}{\scriptstyle \to I_{h5}}}{F(D) \to S(Wo,D) \to W(Wo)}{\scriptstyle \to I_{h4}}}{\big(M(Wo) \to W(Wo)\big) \to F(D) \to S(Wo,D) \to W(Wo)}{\scriptstyle \to I_{h3}}}{\forall z, \big(F(z) \to M(z)\big) \to \big(M(Wo) \to W(Wo)\big) \to F(D) \to S(Wo,D) \to W(Wo)}{\scriptstyle \to I_{h2}}}{\forall x, \forall y, \big(F(y) \to S(x,y) \to F(x)\big) \to \forall z, \big(F(z) \to M(z)\big) \to \big(M(Wo) \to W(Wo)\big) \to F(D) \to S(Wo,D) \to W(Wo)}{\scriptstyle \to I_{h1}}
$$

Hypotheses	Abbreviations
$h1: \forall x, \forall y, \big(F(y) \to S(x,y) \to F(x)\big)$ $h2: \forall z\big(F(z) \to M(z)\big)$ $h3: M(Wo) \to W(Wo)$ $h4: F(D)$ $h5: S(Wo, D)$	$Floats(y): F(y)$ $SameWeight(x,y): S(x,y)$ $MadeOfWood(z): M(z)$ $Witch(Wo): W(Wo)$ $Woman: Wo$ $Duck: D$