So since textboxes were not a thing on LearnIT I have put all my answers in the same document, including my thoughts on the multiple choice questions. I hope it is no hazzle to scroll down to the correct task.

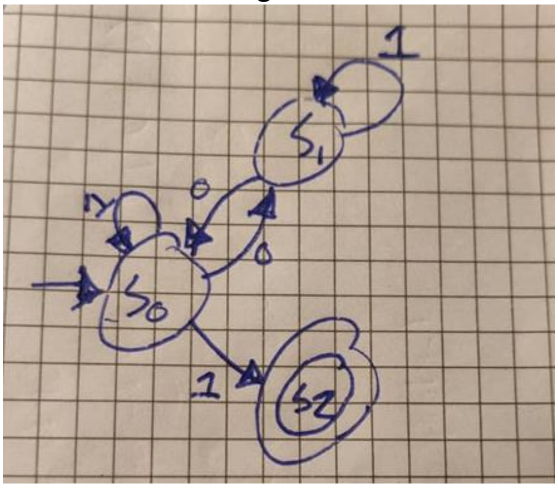
Q1:

Construct a non-deterministic finite-state automaton that recognizes all bitstrings that end with 1 and contain an odd number of 0s.
Let our automaton be $M = (S, I, f, s_0, F)$:
States: $S = \{s_0, s_1, s_2\}$
Input: $I = \{0,1\}$
Transitions: $See\ table\ below$
Initial state: s_0
Accepting states: $F = \{s_2\}$

State diagram:

State	Input	
	0	1
s_0	s_1	s_0, s_2
s_1	s_0	s_1
s_2		

The above state diagram constructs the below non-deterministic automaton:



Q2:

Answer is c by rule of elimination:
Set multiplication produces a set of tuples so a is incorrect.
Set multiplication of $A \times B$ produces a set of tuples which cardinality is as follows $|A \times B| = |A| * |B|$. Since b has two sets A, B with a cardinality of 3 on both, the answer should give $3 * 3 = 9$ tuples. Since b has only 3 tuples, it is incorrect.
An equivalence relation is symmetric thus for $\forall(a, b) \in R, (b, a) \in R$. We can give a counterexample of this: $(1,4) \in R$ but $(4,1) \notin R$.
Thus only c is left and correct.

Q3:

With 29 letters in the alphabet, we have $\frac{29!}{(29-6)!} = 29 * 28 * 27 * 26 * 25 * 24 = 342.014.400$ permutations of 6-letter-strings with no repeating letters.

Now consider the sequence ÆØÅ. It can be placed in either of the following ways:

$$ÆØÅ ***$$
$$* ÆØÅ **$$
$$** ÆØÅ *$$
$$*** ÆØÅ$$

That is, we can either have the sequence starting in the first, second, third or fourth position of the 6-letter-string where * denotes another letter.

Consider the remaining 26 letters in the alphabet such that we can choose the remaining 3 letters in $\frac{26!}{(26-3)!} = 26 * 25 * 24 = 15.600$ ways.

Since we had 4 different ways to put the sequence ÆØÅ, we have $4 * \frac{26!}{(26-3)!} = 4 * 15.600 = 62.400$ permutations.

Q4:

Consider $a_0 = 3\ and\ a_1 = 3$

We can also say $a_0 = 3 * 1$ and $a_1 = 3 * 1$.

Since $f_0 = 1\ and\ f_1 = 1$, we have $a_0 = f_0 * 3\ and\ a_1 = f_1 * 3$

Since $a_n = a_{n-1} + a_{n-2}\ and\ f_n = f_{n-1} + f_{n-2}$, the two sequences thus have the same definitions.

Assume $f_{n-1} * 3 = a_{n-1}$ and $f_{n-2} * 3 = a_{n-2}$ to prove $f_n * 3 = a_n$.

Let us calculate $a_n = f_{n-1} * 3 + f_{n-2} * 3 = 3(f_{n-1} + f_{n-2}) = 3 * f_n$.

Thus option d is correct. QED.

Q5:

Consider the events:

A: An individual is infected.

$\sim A$: An individual is not infected.

A: An individual tests positive.

$\sim B$: An individual tests negative.

$A|B$: An individual is infected after they have tested positive.

$B|A$: An individual tests positive when they are infected.

$\sim B|\sim A$: An individual tests negative when they are not infected.

$A \cap B$: An individual is infected and tests positive.

We have the following probabilities:

$P(A) = 0{,}04$

$P(\sim A) = 1 - P(A) = 0{,}96$

$P(B|A) = 0{,}9$

$P(\sim B|A) = 1 - P(B|A) = 0{,}1$

$P(\sim B|\sim A) = 0{,}99$

$P(B|\sim A) = 1 - P(\sim B|\sim A) = 0{,}01$

We want to know the probability of an individual being infected if they have tested positive. That is, the event $A|B$.

We know that conditional probability can be determined using $P(A|B) = \frac{P(A \cap B)}{P(B)}$. Also for $B|A$: $P(B|A) = \frac{P(A \cap B)}{P(A)}$. Thus we can construct $P(A \cap B) = P(B|A) * P(A) = 0.9 * 0.04 = 0{,}036$.

We also know that by Bayes' Theorem, that $P(A|B) = \frac{P(B|A)*P(A)}{P(B)}$.

This also works if $A = {\sim}A$. Thus we have $P({\sim}A|B) = \frac{P(B|{\sim}A)*P({\sim}A)}{P(B)}$.

We thus have $P({\sim}A|B) = \frac{0{,}01*0{,}96}{P(B)}$.

We can calculate $P(B)$:

$$P(B) = P(B|A) * P(A) + P(B|{\sim}A) * P({\sim}A) = 0{,}9 * 0{,}04 + 0{,}01 * 0{,}96 = 0{,}0456$$

We now have enough to calculate $P(A|B) = \frac{P(B|A)*P(A)}{P(B)} = \frac{0{,}9*0{,}04}{0{,}0456} \approx 0{,}79$

(I'm sorry for the many unnecessary calculations, I lost track of what was used and what wasn't. But my answer will be the approx. 79%).

Q6:

Rule of elimination again:

A: Counterexample: A = $\emptyset, B = \{1,2\}$. Then $A \subseteq B$ and $A \cap B = \emptyset$ which is a contradiction to the statement.

B: The union of A and B will be $\emptyset$ but since $\emptyset = \{\,\}$ then $\emptyset \notin \{\,\}$. Thus false.

C: True by rule of elimination. (also consider$\mathcal{P}(\emptyset) = \{\emptyset\}$ and thus $\emptyset \in \mathcal{P}(\emptyset)$).

D: As described above, $|A \times B| = |A| * |B|$ and since $|A \times B| = 3, |A| = 3, |B| = 3$ then we have a contradiction: $3 * 3 \neq 3$

Q7:

A is false, just watch this counterexample:

$a = 5, b = 7, m = 2$

Then $a \ mod \ m = 1, b \ mod \ m = 1 \ and \ thus \ a \equiv b \ (mod \ m)$

But $a + b = 12$ and $12 \ mod \ m = 0$ and thus $(a + b) \ mod \ m \neq b \ mod \ m$.

Q8:

Prove by induction that $\sum_{i=0}^{a}(i * i!) = (a + 1)! - 1$.

Basecase: $a = 0$. Then $\sum_{i=0}^{a}(i * i!) = 0 * 0! = 0 \ and \ (a + 1)! - 1 = 1! - 1 = 0$ which means the basecase holds.

Assume the statement holds for n to prove it holds for $n + 1$.

Then $\sum_{i=0}^{a}(i * i!) = (a + 1)! - 1$ to prove $\sum_{i=0}^{a}(i * i!) + (a + 1) * (a + 1)! = (a + 2)! - 1$

We thus have that $\sum_{i=0}^{a}(i * i!) + (a + 1) * (a + 1)! = (a + 1)! - 1 + (a + 1) * (a + 1)!$.

Now let us simplify this statement.

$$\sum_{i=0}^{a+1}(i * i!) = (a + 1)! + (a + 1) * (a + 1)! - 1$$

We can factorize $(a+1)!$:

$$\sum_{i=0}^{a+1}(i*i!) = (a+1)!*\big(1+(a+1)\big)-1$$

The parenthesis $\big(1+(a+1)\big)$ can be simplified to $(a+2)$.
Likewise we can show that $(a+2)! = (a+2)*(a+1)*(a)*\ldots*1$ while $(a+1)!$ Is the same but without the preceding $(a+2)*$. Thus, in other words, $(a+2)! = (a+2)*(a+1)!$ Which we will use to replace on the right side of the equation.

$$\sum_{i=0}^{a+1}(i*i!) = (a+2)!-1$$

Thus we have shown that it holds for the basecase $a = 0$ and that it holds for any subsequent integer w^5.

Q9:

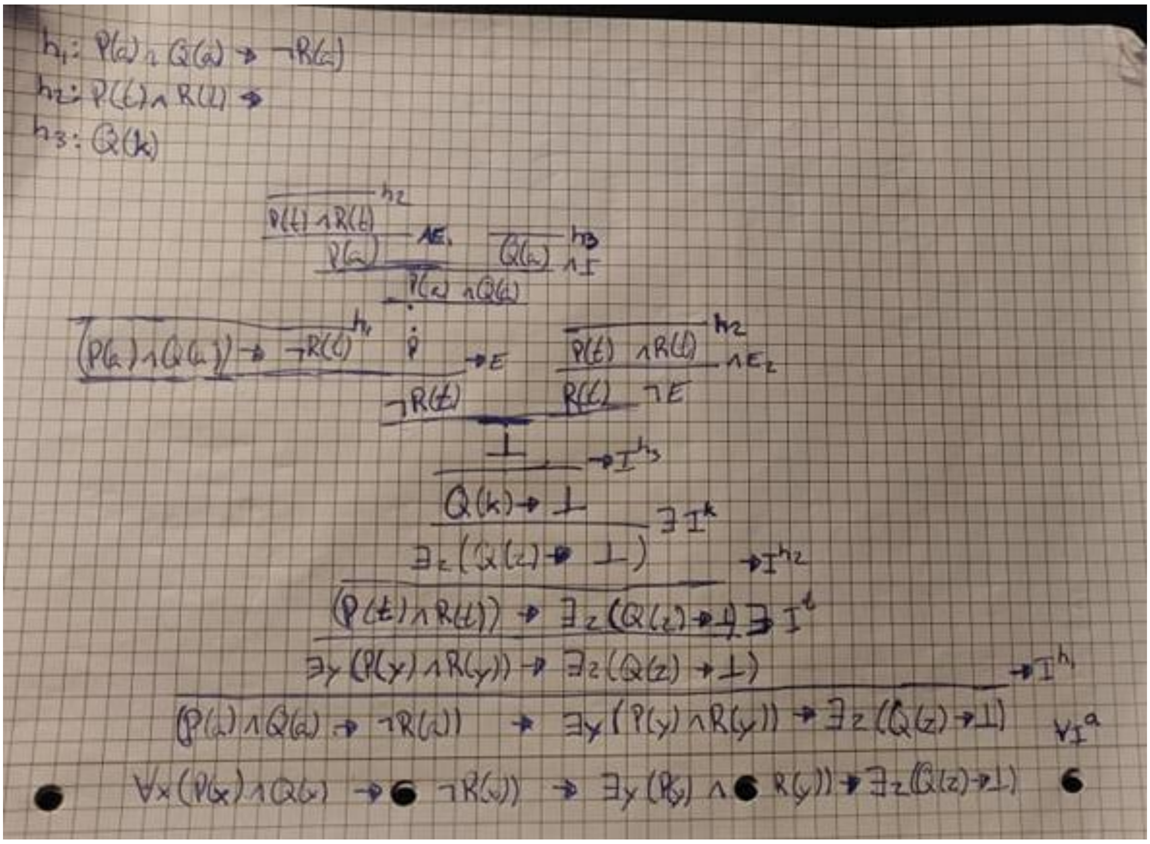
Option d allows me to construct $(0 \cup 1)^*10(01)^*$ since:
The only production for S is $S \to A10B$ forcing me to have 10 in the middle.
Now, $A = (0 \cup 1)^*$ because A has the productions where it either turns into the emptystring or adds a 0 or 1 before the A ($A \to 0A, A \to 1A$).
Likewise we have $B = (01)^*$ because B can only produce the emptystring or 01 followed by B. This allows for any number of (01) including none. Thus option d is correct.

Q10:

The graph is bipartite because we can put the vertices into two separate sets such that there are no edges between two vertices of the same set as follows:
$S_1 = \{a, c, e\}$
$S_2 = \{b, d, f\}$

Also rule of elimination:
"There is a unique simple path between any two vertices": Counterexample, $d \to e$ and $d \to c \to f \to d$ thus has two simple paths.
"The graph has no Hamiltonian circuit": Counterexample, $d \to e \to b \to a \to f \to c \to d$ and thus has a Hamiltonian circuit.
"The graph is a rooted tree with root a": No, because we just showed there is a Hamiltonian circuit which is a circuit and thus the graph is not a tree.

Q11:



I hope it is read-able.
I know it is not completely correct, but hopefully good enough to submit anyway ;)

Q12:

$n \in \mathbb{N}$ *and* $n^3 + 13$ *is odd to prove* n *is even by contradiction*

We assume that n is odd and thus substitute n with the definition of odd numbers (2k+1 for integer k). This must be equal to the definition of an odd number, i.e. $2k + 1$

$$(2k_1 + 1)^3 + 13 = 2k_2 + 1$$

We write out the exponentiation:

$$(2k_1 + 1)(2k_1 + 1)(2k_1 + 1) + 13 = 2k_2 + 1$$

We multiply the parentheses:

$$(2k_1 + 1) * (4k_1^2 + 1^2 + 4k_1) + 13 = 2k_2 + 1$$

$$4k_1^2 + 1^2 + 4k_1 + 8k_1^2 + 2k_1 + 8k_1^3 + 13 = 2k_2 + 1$$

We simplify:

$$8k_1^3 + 14k_1^2 + 6k_1 + 4 = 2k_2 + 1$$

We factorize the left side of the parenthesis with 2:

$$2 * (4k_1^3 + 7k_1^2 + 3k_1 + 2) = 2k_2 + 1$$

We know that multiplication and addition of integers produce a new integer, so we will write $4k_1^3 + 7k_1^2 + 3k_1 + 2 = k_3, k_3 \in \mathbb{Z}$.

$$2k_3 = 2k_2 + 1$$

We thus have that an even integer is equal to an uneven integer and thus we have a contradiction w^5.

Q13:

Since the cardinality of $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}| * |\mathbb{N}|$ then we have that there are more elements in the codomain $(\mathbb{N} \times \mathbb{N})$ than in the domain $\mathbb{N}$. Thus it is not onto/surjective.
For the question about injectivity, I must confess it is a somewhat qualified guess.
My thoughts are:
1. Since the codomain is (much) larger than the domain, there is plenty of possibilities for two-tuples to each natural number and thus this speaks for a one-to-one function.
2. Since the definition for $f(n)$ always differs from another, I believe is no way for two different n to give the same tuple of the co-domain $f(n)$.
(Sort of inspired by the same theory about the cardinality of $\mathbb{R}$ between 0 and 1 that is greater than the cardinality of $\mathbb{N}$ shown in class).

Q14:

A pure guess based on the fact that the first/last implication introduction (depending on which way you read it from), i.e. $(\sim Q \vee \sim R) \rightarrow S$ gives the hypothesis $(\sim Q \vee \sim R)$. If either Q or R is false, then $(\sim Q \vee \sim R)$ will be true. Thus for this to be false, both Q and R must be true. That is the option a in Q14 which is why I guessed on that answer.