So since textboxes were not a thing on LearnIT I have put all my answers in the same document, including my thoughts on the multiple choice questions. I hope it is no hazzle to scroll down to the correct task.

# Q1:

Construct a non-deterministic finite-state automaton that recognizes all bitstrings that end with 1 and contain an odd number of 0s.

Let our automaton be :

States:

Input:

Transitions:

Initial state:

Accepting states:

State diagram:

|  |  |  |
| --- | --- | --- |
| State | Input | |
| 0 | 1 |
|  |  |  |
|  |  |  |
|  |  |  |

The above state diagram constructs the below non-deterministic automaton:

Diagram

Description automatically generated

# Q2:

Answer is c by rule of elimination:

Set multiplication produces a set of tuples so a is incorrect.

Set multiplication of produces a set of tuples which cardinality is as follows . Since b has two sets with a cardinality of 3 on both, the answer should give tuples. Since b has only 3 tuples, it is incorrect.

An equivalence relation is symmetric thus for . We can give a counterexample of this: but .

Thus only c is left and correct.

# Q3:

With 29 letters in the alphabet, we have permutations of 6-letter-strings with no repeating letters.

Now consider the sequence ÆØÅ. It can be placed in either of the following ways:

That is, we can either have the sequence starting in the first, second, third or fourth position of the 6-letter-string where \* denotes another letter.

Consider the remaining 26 letters in the alphabet such that we can choose the remaining 3 letters in ways.

Since we had 4 different ways to put the sequence ÆØÅ, we have permutations.

# Q4:

Consider

We can also say and .

Since , we have

Since , the two sequences thus have the same definitions.

Assume and to prove .

Let us calculate .

Thus option d is correct. QED.

# Q5:

Consider the events:

: An individual is infected.

: An individual is not infected.

: An individual tests positive.

: An individual tests negative.

: An individual is infected after they have tested positive.

: An individual tests positive when they are infected.

: An individual tests negative when they are not infected.

: An individual is infected and tests positive.

We have the following probabilities:

We want to know the probability of an individual being infected if they have tested positive. That is, the event .

We know that conditional probability can be determined using . Also for : . Thus we can construct .

We also know that by Bayes’ Theorem, that .

This also works if . Thus we have .

We thus have .

We can calculate :

We now have enough to calculate

(I’m sorry for the many unnecessary calculations, I lost track of what was used and what wasn’t. But my answer will be the approx. 79%).

# Q6:

Rule of elimination again:

A: Counterexample: A = . Then and which is a contradiction to the statement.

B: The union of A and B will be but since then . Thus false.

C: True by rule of elimination. (also consider and thus ).

D: As described above, and since then we have a contradiction:

# Q7:

A is false, just watch this counterexample:

Then

But and and thus .

# Q8:

Prove by induction that .

Basecase: . Then which means the basecase holds.

Assume the statement holds for to prove it holds for .

Then to prove

We thus have that .

Now let us simplify this statement.

We can factorize :

The parenthesis can be simplified to .

Likewise we can show that while Is the same but without the preceding . Thus, in other words, Which we will use to replace on the right side of the equation.

Thus we have shown that it holds for the basecase and that it holds for any subsequent integer .

# Q9:

Option d allows me to construct since:

The only production for S is forcing me to have 10 in the middle.

Now, because A has the productions where it either turns into the emptystring or adds a 0 or 1 before the A (.

Likewise we have because B can only produce the emptystring or 01 followed by B. This allows for any number of including none. Thus option d is correct.

# Q10:

The graph is bipartite because we can put the vertices into two separate sets such that there are no edges between two vertices of the same set as follows:

Also rule of elimination:

“There is a unique simple path between any two vertices”: Counterexample, and thus has two simple paths.

“The graph has no Hamiltonian circuit”: Counterexample, and thus has a Hamiltonian circuit.

“The graph is a rooted tree with root a”: No, because we just showed there is a Hamiltonian circuit which is a circuit and thus the graph is not a tree.

# Q11:

A close up of a piece of paper with writing on it

Description automatically generated with medium confidence

I hope it is read-able.

I know it is not completely correct, but hopefully good enough to submit anyway ;)

# Q12:

We assume that n is odd and thus substitute n with the definition of odd numbers (2k+1 for integer k). This must be equal to the definition of an odd number, i.e.

We write out the exponentiation:

We multiply the parentheses:

We simplify:

We factorize the left side of the parenthesis with 2:

We know that multiplication and addition of integers produce a new integer, so we will write .

We thus have that an even integer is equal to an uneven integer and thus we have a contradiction .

# Q13:

Since the cardinality of then we have that there are more elements in the codomain than in the domain . Thus it is not onto/surjective.

For the question about injectivity, I must confess it is a somewhat qualified guess.

My thoughts are:

1. Since the codomain is (much) larger than the domain, there is plenty of possibilities for two-tuples to each natural number and thus this speaks for a one-to-one function.

2. Since the definition for always differs from another, I believe is no way for two different to give the same tuple of the co-domain .

(Sort of inspired by the same theory about the cardinality of between 0 and 1 that is greater than the cardinality of shown in class).

# Q14:

A pure guess based on the fact that the first/last implication introduction (depending on which way you read it from), i.e. gives the hypothesis . If either Q or R is false, then will be true. Thus for this to be false, both Q and R must be true. That is the option a in Q14 which is why I guessed on that answer.