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# Random notes

**Variables** are placeholders in mathematics to generalize.

**Propositions / Statements** are sentences that are either *True* or *False.*

**Judgement** are conclusions derived by logical reasoning using propositions.

**Propositional logic:**

Atomic Statement (“A is true”)

Conjunction: (“A and B”)

Disjunction: (“A or B”)

Implication: (“if A then B”)

Negation: (“Not A”)

# Chapter 6 – Set theory

## 6.1 Definitions and element method of proof

Read as: A is the set of all *x* such that *x* has the property P.

A set can be written as such: A={1,3,5,7} where |A| denotes the size which in this case is 4.

Here A is the set of evens up to and including 12 where the use of … replaces an obvious repetition. A set is unordered.

**Subsets** are when all elements of one set is also in another set.

Imagine set being all positive odd numbers and being all positive integers, then *A* is a subset of *B*.

A **proper subset** is a subset if the subset is not equal to the set which it is a subset to. So {1, 3, 5} is not a proper subset of {1, 3, 5}, but it is a proper subset of {1, 3, 5, 7}.

The **union** of two sets is another set containing all elements of the two sets. For example, the union of all positive even and odd numbers is the set of natural numbers:

The **intersection** of two sets is a set of all elements that are both in the two sets.

The **difference** (or relative complement) is as follows:

In other words, the difference is the set of all elements of *A* that are not in *B*.

The **complement** of a set *A* is all elements that are not in *A* written as follows:

The complement can also be written

Symbolically, these are written:

Using **Venn Diagrams**, the before 4 symbols are depicted:

Graphical user interface, application

Description automatically generated

**Intervals** in sets can be written using regular parenthesis and square brackets as follows:

In other words, a regular parenthesis is used to show a limit up until but excluding the number, where as a square bracket shows that the number is included in the subset.

infinity is shown using a regular parenthesis in an interval.

An empty set is called the **null set**:

Other sets are also *null sets* (ie. You can use other definitions than the number squared must be negative one).

A **power set** is a set containing all subsets of a set *A*. Let

The powerset of (a set that contains the emptyset) is as follows:

The size of a powerset can be calculated as follows:

Two **disjoint sets**are two set where the intersection is empty:

A **partition** of a set is a set *A* separated into subsets that are mutually disjoint such that *A­1, A2 & A3* all add up to the set of *A* but being mutually disjoint. Note that there can be a finite or infinite number of subsets.

## 6.2 Properties of sets

*Inclusion of Intersection*

In other words, the intersection of A and B will always be a subset of both A and B.

*Inclusion in Union*

Both subsets A and B will be a subset of the union of the same sets.

*Transitive Property of Subsets*

If a set A is a subset of B, and B is a subset of C, then A is also a subset of C.

**For all following laws, the sets A, B, C etc. are all subsets of the universal set *U*.**

*Commutative Laws*

*Associative Laws*

*Distributive Laws*

*Identity Laws*

*Complement Laws*

*Idempotent Laws*

*Universal Bound Laws*

*De Morgan’s Laws*

*Absorption Laws*

*Complements of U and Ø*

*Set Difference Law*

# Chapter 7 – Properties of functions

## 7.1 Functions defined on general sets

A function *f* from a set *X* to a set *Y* is denoted

A function satisfies two conditions:

1. Every element in *X* is related to some element in *Y*.
2. No element in *X* is related to more than one element in *Y*.

A function maps an element *x* in *X* to an element *y* in *Y* as such:

The **domain** of this function would be the set *X*.

The **co-domain** of this function is the set *Y* (including non-used elements of *Y* that aren’t mapped to from the set *X*).

The **range** of the function is the set that are mapped to from the elements of *X* through *f*. This means that the difference of the *range* and the *co-domain* is that the range does not include the elements of *Y* that can’t be produced with the set *X* through the function *f*.

**Function equality** is when

**Logarithms** and logarithmic functions are defined as follows:

That is valid for all real numbers *b* where

To find an approximate value for *y* in where *b* is not *e* or 10, it can be hard to use a computer. Instead the following formula can be used:

**Boolean Functions** are functions whose co-domain consists of two values; {0, 1} or

A **not well defined function** is a function that fails to satisfy at least one of the conditions for being a function as stated above.

For instance is not a well defined function because for we have no real number that satisfies . Likewise, the function is not well-defined either, because the rational number but using the function, these would give and hence the same element in the domain maps to two different elements in the co-domain.

The **image of** a set is defined for a function such that the image of *A* is a subset of the co-domain such that all values in the image of *A* can be mapped to from at least one element of *A*.

The **inverse image** of a set is the subset in *X* consisting of the elements that map to any element of the inverse image. In the above example, the inverse image of *C* would be a subset of *X* such that all elements *x* in *X* would map to an element of *C*. All elements that map to an element of *C* must be included but if no elements of *X* maps to any element of *C*, then the inverse image would be an empty set .

## 7.2 One-to-one, onto and inverse functions

A **One-to-one** or **injective** function has a domain where all elements of the domain maps to distinct elements of the co-domain:

An **onto** or **surjective** function has a co-domain such that all elements *y* in the co-domain *Y* will be mapped to from an element in the domain *X*. In other words, the co-domain is equal to the range of the function.

A **one-to-one correspondence** or **bijective** function is the name of a function that is both one-to-one and onto (injective and surjective). This means that all elements of the domain maps to a distinct element in the co-domain, and likewise all elements in the co-domain is mapped to from exactly one element of the domain.

A bijective function has an **inverse function** that maps the elements of the co-domain to the domain. In other words, if the bijective function is , then the inverse function is . Or equivalently:

*Diagram

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An inverse function for a mathematical equation can be found by isolating the *x* in the formula y=ax+b (or alike formular).

Let

## 7.3 Composition of functions

**The composition of functions *f* and *g*** is written for the functions , such that the image of the function *f* will be used as the domain of the function *g*.

is read “*g* circle *f*” and the function is called the **composition of *f* and *g*.** Note that when written mathematically it is read with the outer function first, while the name is called the composition of the first function and the latter function.

is not equal to necessarily, only if all elements of the domain will give the same image no matter which function is used first.

Consider .

Let

Hence

An **identity function** is a function such that the element is sent back to itself.

The composition of a function with an identity function is thus:

And likewise, if a function is composed with its inverse function, we get the following

# Chapter 8 – Properties of Relations

## 8.1 Relations on sets

A less-than **relation** *L* from is written as follows:

Said relation is pronounced “Let L be a relation from A to B”

The **inverse of a relation** is defined as below:

For instance, a divide relation defined as where and would give the following ordered pairs:

The reverse relation would then be

Such a relation and inverse relation can be shown as **directed graphs.**

Diagram

Description automatically generatedDiagram

Description automatically generated

A **relation on a set** *A* is a relation from *A* to *A*.

## 8.2 Reflexivity, symmetry and transitivity

A relation on a set is **reflexive** if all elements of the set relates to itself. Imagine a set , then the reflexive relation would at least hold the ordered pairs .

A relation on a set is **symmetric** when all relations from x to y also has a relation from y to x. This doesn’t mean all elements of a set must be related to every other element, but only if x relates to y then y relates to x.

A relation on a set is **transitive** whenever there is a relation from x to y and y to z, then there are also a relation from x to z such that there are no incomplete directed triangles in a directed graph.

Text

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The **transitive closure** of a non-transitive relation is the relation which includes all tuples such that *R* is transitive, including itself. Look at the two relations below, where is on the left and is on the right. Note how (0,2) and (1,3) had to be added, but this created the two relations (0, 2) and (2,3) such that another tuple (0,3) had to be added also.

Box and whisker chart

Description automatically generated with low confidencePolygon

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## 8.3 Equivalence relations

Consider the partition of a set such that and .

The **relation induced by the partition** will be a relation like follows:

Let with the following partitions: .

The relation induced by the partition is as follows:

This means that

A relation induced by a partition of a set satisfies the properties of being reflexive, symmetric and transitive.

A relation is an **equivalence relation** when all three properties are met.

An **equivalence class** is denoted such that for an equivalence relation . See the following example:

A picture containing accessory, necklet

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Here, while . Note that and .

The **distinct equivalence classes** are hence where either or is omitted.

A **representative of a class**  is any element such that .

A **congruence** is written and means that is congruent to where .