Secure Computing of Table with FLUTE

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2024년 9월 6일

Lookup Table

LUT (Lookup Table)

δ-to-σ lookup table T is a function $\mathsf{T}\colon \{0,1\}^{\delta} \to \{0,1\}^{\sigma}$.

x_1	x_2	x_3	y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Example 3-to-1 LUT T

Goal Given secret shares of x_1, x_2, and x_3, we want to compute the secret shares of $\mathsf{T}(x_1, x_2, x_3)$.

Question What secret sharing scheme should we use?

Question How can we optimize the online and setup cost? (Up to 2σ bits per LUT caculation in the online phase!)

$[\cdot]$ Secret Sharing

P_0, P_1: parties

$[\cdot]$ Secret Sharing

A bit $v \in \mathbb{Z}_2$ is said to be $[\cdot]$-*shared* between P_0 and P_1 if P_i holds $[v]_i$ such that

$$v = [v]_0 \oplus [v]_1.$$

$[\cdot]$ Secret Sharing

The $[\cdot]$-sharing is *linear* in the sense that:

1. For $[\cdot]$-shared $u, v \in \mathbb{Z}_2$, $[\cdot]$-sharing of $u \oplus v$ can be locally computed by:
$$[u \oplus v]_i = [u]_i \oplus [v]_i \text{ for each } i \in \{0,1\}.$$
2. For $[\cdot]$-shared $v \in \mathbb{Z}_2$ and public $b \in \mathbb{Z}_2$, $[\cdot]$-sharing of bv can be locally computed by:
$$[bv]_i = b[v]_i \text{ for each } i \in \{0,1\}.$$
3. For $[\cdot]$-shared $v \in \mathbb{Z}_2$ and public $b \in \mathbb{Z}_2$, $[\cdot]$-sharing of $v \oplus b$ can be locally computed by:
$$[v \oplus b]_i = [v]_i \oplus ib \text{ for each } i \in \{0,1\}.$$

In particular, $[\cdot]$-sharing of $\overline{v} = v \oplus 1$ can be locally computed.

Multiplication of $[\cdot]$-shared Values

Beaver's Multiplication Triple

Suppose P_0 and P_1 have $[\cdot]$-shares of $u, v \in \mathbb{Z}_2$. Suppose additionally that they have $[\cdot]$-shares of a, b, c where $c = ab$.

1. P_i computes $[u \oplus a]_i$ and $[v \oplus b]_i$, and sends them to P_{1-i}.
2. P_0 and P_1 now know $d := u \oplus a$ and $e := v \oplus b$.
3. P_i computes $[z]_i = i \cdot de \oplus d[b]_i \oplus e[a]_i \oplus [c]_i$.
4. Then, $[z]_0 \oplus [z]_1 = z = uv$.

With the prepared *multiplication triple*, a $[\cdot]$-share of uv is calculated with the cost of four bits in a single round.

$$\begin{aligned} z = uv &= (d \oplus a)(e \oplus b) \\ &= \underbrace{de}_{\text{public}} \oplus db \oplus ea \oplus \underbrace{ab}_{=c} \end{aligned}$$

$\langle\cdot\rangle$ Secret Sharing

$\langle\cdot\rangle$ Secret Sharing

A bit $v \in \mathbb{Z}_2$ is said to be $\langle\cdot\rangle$-*shared* if:

1. A value $\lambda_v \in \mathbb{Z}_2$ is $[\cdot]$-shared between P_0 and P_1.
   - $[\lambda_v]_i$'s are locally set arbitrarily.
2. A value $\mathrm{m}_v \in \mathbb{Z}_2$ is known to both parties.
3. $v = \mathrm{m}_v \oplus \lambda_v$

The $\langle\cdot\rangle$-share of v is denoted $\langle v\rangle_i = (\mathrm{m}_v, [\lambda_v]_i)$.

$\langle\cdot\rangle$ Secret Sharing

The $\langle\cdot\rangle$-sharing is *linear* in the sense that:

1. For $\langle\cdot\rangle$-shared $u, v \in \mathbb{Z}_2$, $\langle\cdot\rangle$-sharing of $u \oplus v$ can be locally computed by:
$$\mathsf{m}_{u\oplus v} = \mathsf{m}_u \oplus \mathsf{m}_v \text{ and } [\lambda_{u\oplus v}]_i = [\lambda_u]_i \oplus [\lambda_v]_i$$
2. For $\langle\cdot\rangle$-shared $v \in \mathbb{Z}_2$ and public $b \in \mathbb{Z}_2$, $\langle\cdot\rangle$-sharing of bv can be locally computed by:
$$\mathsf{m}_{bv} = b\mathsf{m}_v \text{ and } [\lambda_{bv}]_i = b[\lambda_v]_i.$$
3. For $\langle\cdot\rangle$-shared $v \in \mathbb{Z}_2$ and public $b \in \mathbb{Z}_2$, $\langle\cdot\rangle$-sharing of $v \oplus b$ can be locally computed by:
$$\mathsf{m}_{v\oplus b} = \mathsf{m}_v \oplus b.$$
In particular, $\langle\cdot\rangle$-sharing of $\overline{v} = v \oplus 1$ can be locally computed.

LUT Revisited

x_1	x_2	x_3	y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Example 3-to-1 LUT T

$$(\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3}) \\ \vee(\overline{x_1} \wedge x_2 \wedge x_3) \\ \vee(x_1 \wedge \overline{x_2} \wedge x_3)$$

DNF representation of T

$$(\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3}) \\ \oplus(\overline{x_1} \wedge x_2 \wedge x_3) \\ \oplus(x_1 \wedge \overline{x_2} \wedge x_3)$$

An equivalent circuit.

$$y = \begin{pmatrix} \overline{x_1} \\ \overline{x_1} \\ x_1 \end{pmatrix} \odot \begin{pmatrix} \overline{x_2} \\ x_2 \\ \overline{x_2} \end{pmatrix} \odot \begin{pmatrix} \overline{x_3} \\ x_3 \\ x_3 \end{pmatrix}$$

“Multi-Fan-In Inner Product”

Multiplication of $\langle\cdot\rangle$-shared Values

$$z = uv = (\mathsf{m}_u \oplus \lambda_u)(\mathsf{m}_v \oplus \lambda_v)$$
$$= \mathsf{m}_u\mathsf{m}_v \oplus \mathsf{m}_u\lambda_v \oplus \mathsf{m}_v\lambda_u \oplus \lambda_u\lambda_v$$

Multiplication of $\langle\cdot\rangle$-shared Values

Input $\langle\cdot\rangle$-shares of $u, v \in \mathbb{Z}_2$, $[\cdot]$-shares of $\lambda_u\lambda_v$, and $[\cdot]$-shares of λ_z.

Output $\langle\cdot\rangle$-shares of $z := uv$.

1. P_i computes $[z]_i = i \cdot \mathsf{m}_u\mathsf{m}_v \oplus \mathsf{m}_u[\lambda_v]_i \oplus \mathsf{m}_v[\lambda_u]_i \oplus [\lambda_u\lambda_v]_i$.
2. P_i computes $[\mathsf{m}_z]_i = [z]_i \oplus [\lambda_z]_i$ and sends it to P_{i-1}.
3. Then, P_0 and P_1 have $\langle\cdot\rangle$-shares of $z = uv$.

With the prepared multiplication triple, a $\langle\cdot\rangle$-share of uv is calculated with the cost of <u>two bits in a single round</u>.

If z is an operand of multiplication in following steps through the circuit, λ_z must be settled in the setup phase to optimize the online communication cost.

Inner Product of $\langle\cdot\rangle$-shared Values

Inner Product of $\langle\cdot\rangle$-shared Values

Input $\langle\cdot\rangle$-shares of $u^1, \cdots, u^N, v^1 \cdots, v^N \in \mathbb{Z}_2$, $[\cdot]$-shares of $\lambda_{u^1}\lambda_{v^1}, \cdots, \lambda_{u^N}\lambda_{v^N}$, and $[\cdot]$-shares of λ_z.

Output $\langle\cdot\rangle$-shares of $z := \bigoplus_{k=1}^{N} u^k v^k$.

1. P_i computes $[u^k v^k]_i$ for $k \in [N]$ as before.
2. P_i computes $[\mathsf{m}_z]_i = [\lambda_z]_i \oplus \bigoplus_{k=1}^{N} [u^k v^k]_i$ and sends it to P_{1-i}.
3. Then, P_0 and P_1 have $\langle\cdot\rangle$-shares of $z = \bigoplus_{k=1}^{N} u^k v^k$.

With the prepared N multiplication triples, a $\langle\cdot\rangle$-share of $\bigoplus_{k=1}^{N} u^k v^k$ is calculated with the cost of two bits in a single round.

Multi-Fan-In Product of $\langle\cdot\rangle$-shared Values

Multi-Fan-In Product of $\langle\cdot\rangle$-shared Values

Input $\langle\cdot\rangle$-shares of $u^1, \cdots, u^M \in \mathbb{Z}_2$, $\boxed{???}$, and $[\cdot]$-shares of λ_z.

Output $\langle\cdot\rangle$-shares of $z := \bigwedge_{j=1}^{M} u^j$.

Let $\mathcal{Q}$ be a set of $\langle\cdot\rangle$-shared values (wires). We use the following notation:

$$\mathrm{m}_{\mathcal{Q}} \triangleq \bigwedge\nolimits_{u\in\mathcal{Q}} \mathrm{m}_u \quad \text{and} \quad \lambda_{\mathcal{Q}} \triangleq \bigwedge\nolimits_{u\in\mathcal{Q}} \lambda_u.$$

Observation

$$\bigwedge_{j=1}^{M} u^j = \bigwedge_{j=1}^{M} (\mathrm{m}_{u^j} \oplus \lambda_{u^j}) = \bigoplus_{\mathcal{Q}\subseteq\mathcal{I}} (\mathrm{m}_{\mathcal{Q}} \cdot \lambda_{\mathcal{I}\setminus\mathcal{Q}})$$

where $\mathcal{I} := \{u^1, u^2, \cdots, u^M\}$.

Multi-Fan-In Product of $\langle\cdot\rangle$-shared Values

Multi-Fan-In Product of $\langle\cdot\rangle$-shared Values

Input $\langle\cdot\rangle$-shares of $u^1, \cdots, u^M \in \mathbb{Z}_2$, $[\cdot]$-shares of $\lambda_{\mathcal{Q}}$ for each $\mathcal{Q} \subseteq \mathcal{I}$ with $|\mathcal{Q}| > 1$ where $\mathcal{I} = \{u^1, \cdots, u^M\}$, and $[\cdot]$-shares of λ_z.

Output $\langle\cdot\rangle$-shares of $\bigwedge_{j=1}^{M} u^j$.

1. P_i computes
$$[z]_i = i \cdot \mathsf{m}_{\mathcal{I}} \oplus \bigoplus_{\mathcal{Q} \subsetneq \mathcal{I}} (\mathsf{m}_{\mathcal{Q}} \cdot [\lambda_{\mathcal{I} \setminus \mathcal{Q}}]_i).$$
2. P_i computes $[\mathsf{m}_z]_i = [z]_i \oplus [\lambda_z]_i$ and sends it to P_{1-i}.
3. Then, P_0 and P_1 have $\langle\cdot\rangle$-shares of $\bigwedge_{j=1}^{M} u^j$.

With the prepared $2^M - M - 1$ multiplication triples, a $\langle\cdot\rangle$-share of $\bigwedge_{j=1}^{M} u^j$ is calculated with the cost of two bits in a single round.

Multi-Fan-In Inner Product of $\langle\cdot\rangle$-shared Values

Multi-Fan-In Inner Product of $\langle\cdot\rangle$-shared Values

Input $\langle\cdot\rangle$-shares of $\mathbf{u}^1, \cdots, \mathbf{u}^M \in (\mathbb{Z}_2)^N$, $[\cdot]$-shares of $\lambda_{\mathcal{Q}}$ for each $\mathcal{Q} \subseteq \mathcal{I}_k$ with $|\mathcal{Q}| > 1$ where $\mathcal{I}_k = \{\mathbf{u}_k^1, \cdots, \mathbf{u}_k^M\}$ for $k \in [N]$, and $[\cdot]$-sharing of λ_z.

Output $\langle\cdot\rangle$-shares of $z := \mathbf{u}^1 \odot \cdots \odot \mathbf{u}^M = \bigoplus_{k=1}^N \bigwedge_{j=1}^M \mathbf{u}_k^j$.

1. P_i computes
$$[z]_i = i \cdot \bigoplus_{k=1}^{N} \mathsf{m}_{\mathcal{I}_k} \oplus \bigoplus_{k=1}^{N} \bigoplus_{\mathcal{Q} \subsetneq \mathcal{I}_k} (\mathsf{m}_{\mathcal{Q}} \cdot [\lambda_{\mathcal{I} \setminus \mathcal{Q}}]_i).$$
2. P_i computes $[\mathsf{m}_z]_i = [z]_i \oplus [\lambda_z]_i$ and sends it to P_{1-i}.
3. Then, P_0 and P_1 have $\langle\cdot\rangle$-shares of $\mathbf{u}^1 \odot \cdots \odot \mathbf{u}^N$.

With the prepared $N(2^M - M - 1)$ multiplication triples, a $\langle\cdot\rangle$-share of $\bigodot_{j=1}^N \mathbf{u}^j$ is calculated with the cost of two bits in a single round.

Input $\langle\cdot\rangle$-shares of $\mathbf{u}^1, \cdots, \mathbf{u}^M \in (\mathbb{Z}_2)^N$, $[\cdot]$-shares of $\lambda_{\mathcal{Q}}$ for each $\mathcal{Q} \subseteq \mathcal{I}_k$ with $|\mathcal{Q}| > 1$ where $\mathcal{I}_k = \{ \mathbf{u}_k^1, \cdots, \mathbf{u}_k^M \}$ for $k \in [N]$, and $[\cdot]$-sharing of λ_z.

Output $\langle\cdot\rangle$-shares of $z := \mathbf{u}^1 \odot \cdots \odot \mathbf{u}^M = \bigoplus_{k=1}^{N} \bigwedge_{j=1}^{M} \mathbf{u}_k^j$.

With the prepared $\underline{N(2^M - M - 1) \text{ multiplication triples}}$, a $\langle\cdot\rangle$-share of $\bigodot_{j=1}^{N} \mathbf{u}^j$ is calculated with the cost of two bits in a single round.

$$y = \begin{pmatrix} \overline{x_1} \\ \overline{x_1} \\ x_1 \end{pmatrix} \odot \begin{pmatrix} \overline{x_2} \\ x_2 \\ \overline{x_2} \end{pmatrix} \odot \begin{pmatrix} \overline{x_3} \\ x_3 \\ x_3 \end{pmatrix}$$

For δ-to-1 LUT,
$N \leq 2^{\delta-1}$ and $M = \delta$??
$2^{\delta-1}(2^{\delta} - \delta - 1)$ triples??

We now exploit the fact that elements of $\mathbf{u}^j$ are closely related.

$\boxed{\lambda_v \text{ is unchanged when calculating shares of } \overline{v}!}$

Some Definitions

Fix a δ-to-σ LUT T.

x^1	x^2	x^3	y^1	y^2
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	0
1	1	1	0	1
$\mathbf{e}^1$	$\mathbf{e}^2$	$\mathbf{e}^3$	$\mathbf{y}^1$	$\mathbf{y}^2$

For instance, in this example,

$$\mathbf{e}^2 = \begin{pmatrix} 0\\0\\1\\1\\0\\0\\1\\1 \end{pmatrix} \quad \text{and} \quad \mathbf{y}^1 = \begin{pmatrix} 1\\0\\0\\1\\1\\0\\0\\0 \end{pmatrix}.$$

Expressing LUT with Multi-Fan-In Inner Product

Now, in the previous example, note that:

x^1	x^2	x^3	y^1
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$$\mathbf{z}^1 = \begin{pmatrix} \overline{x^1} \\ \overline{x^1} \\ x^1 \end{pmatrix} \odot \begin{pmatrix} \overline{x^2} \\ x^2 \\ \overline{x^2} \end{pmatrix} \odot \begin{pmatrix} \overline{x^3} \\ x^3 \\ x^3 \end{pmatrix}$$

$$= \begin{pmatrix} \overline{x^1} \\ \overline{x^1} \\ \overline{x^1} \\ \overline{x^1} \\ \vdots \end{pmatrix} \odot \begin{pmatrix} \overline{x^2} \\ \overline{x^2} \\ x^2 \\ x^2 \\ \vdots \end{pmatrix} \odot \begin{pmatrix} \overline{x^3} \\ x^3 \\ \overline{x^3} \\ x^3 \\ \vdots \end{pmatrix} \odot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}$$

$$= (\overline{x^1} \oplus \mathbf{e}^1) \odot \cdots \odot (\overline{x^3} \oplus \mathbf{e}^3) \odot \mathbf{y}^1$$

$\overline{x^j} \oplus \mathbf{e}^j$ is an element-wise addition.

Expressing LUT with Multi-Fan-In Inner Product

Fixing a δ-to-σ LUT T, and given $\langle\cdot\rangle$-shares of $x^1, \cdots, x^\delta$, let

$$\mathbf{u}^j := \overline{x^j} \oplus \mathbf{e}^j \in (\mathbb{Z}_2)^{2^\delta}$$

for $j \in [\delta]$ so that

$$\langle \mathbf{u}_k^j \rangle_i = \left(\overline{\mathsf{m}_{x^j}} \oplus \mathbf{e}_k^j, [\lambda_{x^j}]_i \right)$$

gives a $\langle\cdot\rangle$-sharing of $\mathbf{u}_k^j$ for $j \in [\delta]$, $k \in [2^\delta]$.

With this, we have

$$\lambda_{\mathbf{u}_{k_1}^{j_1} \cdots \mathbf{u}_{k_n}^{j_n}} = \lambda_{x^{j_1} \cdots x^{j_n}}!$$

Expressing LUT with Multi-Fan-In Inner Product

As previously shown, the w-th output $\mathbf{z}^w$ can be calculated by

$$\begin{aligned}
\mathbf{z}^w &= (\overline{x^1} \oplus \mathbf{e}^1) \odot \cdots \odot (\overline{x^\delta} \oplus \mathbf{e}^\delta) \odot \mathbf{y}^w \\
&= \mathbf{u}^1 \odot \cdots \odot \mathbf{u}^\delta \odot \mathbf{y}^w \\
&= \bigoplus_{k=1}^{2^\delta} \left[\left(\bigwedge_{j=1}^{\delta} \mathbf{u}_k^j \right) \wedge \mathbf{y}_k^w \right] \\
&= \bigoplus_{k=1}^{2^\delta} \left[\left(\bigwedge_{j=1}^{\delta} \left(\mathsf{m}_{\mathbf{u}_k^j} \oplus \lambda_{\mathbf{u}_k^j} \right) \right) \wedge \mathbf{y}_k^w \right] \\
&= \bigoplus_{k=1}^{2^\delta} \left[\mathbf{y}_k^w \cdot \bigoplus_{\mathcal{Q}_k \subseteq \mathcal{I}_k} \left(\mathsf{m}_{\mathcal{Q}_k} \cdot \lambda_{\mathcal{I}_k \setminus \mathcal{Q}_k} \right) \right]
\end{aligned}$$

where $\mathcal{I}_k := \{ \mathbf{u}_k^1, \cdots, \mathbf{u}_k^\delta \}$ for each $k \in [2^\delta]$.

Calculating LUT

Calculating δ-to-σ LUT

Input A δ-to-σ LUT T, $\langle\cdot\rangle$-shares of $x^1, \cdots, x^\delta \in \mathbb{Z}_2$, $[\cdot]$-shares of $\lambda_{\mathcal{Q}}$ for each $\mathcal{Q} \subseteq \mathcal{I}$ with $|\mathcal{Q}| > 1$ where $\mathcal{I} = \{x^1, \cdots, x^\delta\}$, and $[\cdot]$-shares of λ_z.

Output $\langle\cdot\rangle$-shares of $z := \mathrm{T}(x^1, \cdots, x^\delta)$.

1. P_i computes its share of $\mathbf{u}_k^j = \overline{x^j} \oplus \mathbf{e}_k^j$ for $j \in [\delta]$ and $k \in [2^\delta]$.
2. P_i computes, for each $w \in [\sigma]$,
$$[\mathbf{z}_w]_i = i \cdot \bigoplus_{k=1}^{2^\delta} (\mathbf{y}_k^w \cdot \mathrm{m}_{\mathcal{I}_k}) \oplus \bigoplus_{k=1}^{2^\delta} \left[\mathbf{y}_k^w \cdot \bigoplus_{\mathcal{Q}_k \subsetneq \mathcal{I}_k} \left(\mathrm{m}_{\mathcal{Q}_k} \cdot [\lambda_{\mathcal{I}_k \setminus \mathcal{Q}_k}]_i \right) \right].$$
3. P_i computes $[\mathrm{m}_{\mathbf{z}_w}]_i = [\mathbf{z}_w]_i \oplus [\lambda_{\mathbf{z}_w}]_i$ for $w \in [\sigma]$ and sends them to P_{1-i}.
4. Then, P_0 and P_1 have $\langle\cdot\rangle$-shares of $\mathbf{z} = \mathrm{T}(x^1, \cdots, x^\delta)$.

Calculating LUT

Calculating δ-to-σ LUT

Input A δ-to-σ LUT T, $\langle\cdot\rangle$-shares of $x^1, \cdots, x^\delta \in \mathbb{Z}_2$, $[\cdot]$-shares of $\lambda_{\mathcal{Q}}$ for each $\mathcal{Q} \subseteq \mathcal{I}$ with $|\mathcal{Q}| > 1$ where $\mathcal{I} = \{x^1, \cdots, x^\delta\}$, and $[\cdot]$-shares of λ_z.

Output $\langle\cdot\rangle$-shares of $z := \mathsf{T}(x^1, \cdots, x^\delta)$.

With the prepared $2^\delta - \delta - 1$ multiplication triples, a $\langle\cdot\rangle$-share of $\mathsf{T}(x^1, \cdots, x^\delta)$ is calculated with the cost of <u>2σ bits in a single round</u>.

Pros and Cons

Pros

- Any complex circuit can be represented by an *interconnection of small LUTs*.
- Evaluation does not depend on the internal logic.
- Setup does not depend on the number of outputs.

Cons

- Exponential setup time/communicaion.
- Exponential internal calculation.

Analytic Comparison with Other Protocols

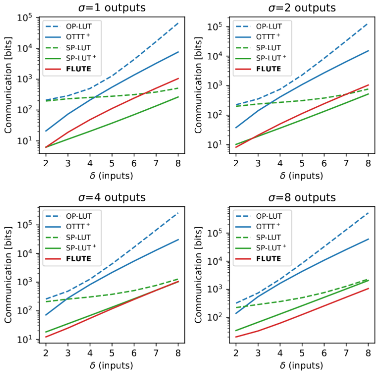


Figure 7: Total communication for different LUT sizes with $2 \leq \delta \leq 8$ inputs and $\sigma \in \{1, 2, 4, 8\}$ outputs.

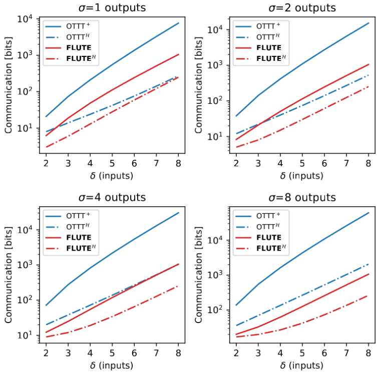
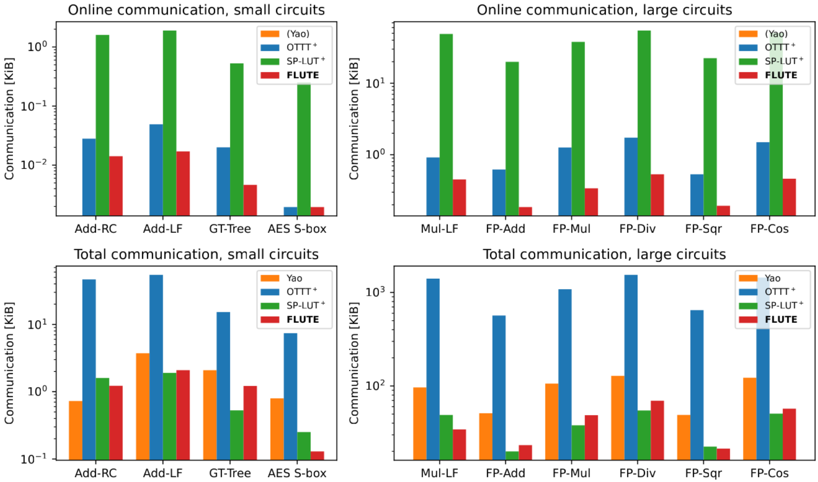


Figure 8: Total communication for different LUT sizes with $2 \leq \delta \leq 8$ inputs and $\sigma \in \{1, 2, 4, 8\}$ outputs using a helper server $S_{\mathcal{H}}$ (cf. §C).

Online/Total Communication Compared with Other Protocols



References

- Brüggemann, A., Hundt, R., Schneider, T., Suresh, A., & Yalame, H. (2023). Flute: Fast and secure lookup table evaluations. *2023 IEEE Symposium on Security and Privacy (SP)*. `https://doi.org/10.1109/sp46215.2023.10179345`