Indian Institute of Technology Kharagpur
Department of Mathematics
MA11004 - Linear Algebra, Numeircal and Complex Analysis
Problem Sheet - 7
Spring 2021

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1. (a) Evaluate $\displaystyle\int_{1-i}^{2+i}(2x+iy+1)dz$, along the paths
   (i) $x=t+1, y=2t^2-1$,
   (ii) the straight line joining $1-i$ and $2+i$.

   (b) Evaluate $\displaystyle\int_{0}^{1+i}(x-y+ix^2)dz$ along
   (i) the straight line from $z=0$ and $z=1+i$,
   (ii) real axis from $z=0$ and $z=1$ and then a line parallel to imaginary axis from $z=1$ to $z=1+i$.

   (c) Find the value of the integral $\displaystyle\int_C (z+1)^2 dz$ where C is the boundary of the rectangle with vertices at the points $1+i$, $-1+i$,$-1-i$ and $1-i$.

   (d) Compute $\displaystyle\int_\Gamma |z|dz$ where Γ is the left half of the unit circle $|z|=1$ from $z=-i$ to $z=i$.

   (e) Find the value of $\displaystyle\int_C (z^2-iz)dz$ along the curve C which is $y=x^3-3x^2+4x-1$ joining points $(1,1)$ and $(2,3)$.

2. (a) Verify that the value of the integral $\displaystyle\int_C z^2 dz$ is same for the following cases:
   (i) C is the straight line joining the points $A(0,0)$ and $B(1,2)$.
   (ii) C is the straight line path from $A(0,0)$ to $P(1,0)$ followed by the straight line path from $P(1,0)$ to $B(1,2)$.
   (iii) C is the parabolic path $y=2x^2$ joining the points $A(0,0)$ and $B(1,2)$.

   (b) Integrate the function $f(z)=xz$ along the straight line from $A(1,1)$ to $B(2,4)$ in the complex plane. Is the value same if the path of integration from A to B is along the curve $x=t$, $y=t^2$?

   (c) Evaluate the function f defined by the integral $f(z)=\oint_{|w|=1}\frac{e^{w^2}-1}{w-z}dw$.

3. $F(a)$=$\displaystyle\oint_C \frac{(4z^2+z+5)}{(z-a)}dz$, where C is the ellipse $(\frac{x}{2})^2+(\frac{y}{3})^2=1$ taken in counter clockwise sense. Find $F(3.5)$, $F(i)$, $F'(-1)$ and $F''(-i)$.

4. (a) Evaluate $\displaystyle\oint_C \frac{3z^2+z+1}{(z^2-1)(z+3)}dz$, where C is the circle $|z|=2$.

   (b) Compute $\displaystyle\oint_C \frac{e^{z^2}}{(z-2)}dz$ over the contour C, C: $|z-(2+i)|=3$.

   (c) Compute $\displaystyle\oint_C \frac{1}{(z^2+4)^2}dz$ over the contour C : $|z-i|=\frac{3}{2}$.

(d) Evaluate $\oint_C \frac{1}{(z-a)^n}dz$, where n is any integer and C is any closed curve containing 'a'.

5. (a) Show that $|\oint_C \frac{e^z}{z^2+1}dz| \leq \frac{4}{3}\pi e^2$ where C: $|z| = 2$.

   (b) Estimate an upper bound for $|\oint_{|z|=3} \frac{Log(z)}{z-4i}dz|$.

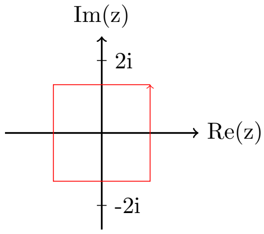
6. (a) Find the value of $\oint_{|z|=1} \frac{4z^2-4z+1}{(z-2)(z^2+4)}dz$.

   (b) Compute $\oint_{|z+1-i|=2} \frac{z+4}{z^2+2z+5}dz$.

   (c) Compute $\oint_{|z|=6} (\frac{e^{2iz}}{z^4} - \frac{z^4}{(z-i)^3})dz$.

   (d) Evaluate the integral $\oint_{|z|=1} \frac{dz}{2-\bar{z}}$.

7. Evaluate $\oint_C \frac{\cos z}{z(z^2+8)}dz$ over the contour shown below:



8. Evaluate $\oint_C \frac{z^2+2}{z^2-2}$ where C is the circle

   (a) $|z| = \frac{3}{2}$.
   (b) $|z-1| = 1$.
   (c) $|z| = \frac{1}{2}$.

9. Verify the Cauchy's theorem for the function $z^3 - iz^2 - 5z + 2i$ if C is

   (a) the circle $|z| = 1$.
   (b) the circle $|z-1| = 2$.
   (c) the ellipse $|z-3i| + |z+3i| = 20$.

10. If $0 < r < R$, evaluate the integral $I = \oint_C \frac{R+z}{z(R-z)}dz$,where C:$|z| = r$. Further using this result deduce that

    (a) $\int_0^{2\pi} \frac{d\theta}{R^2 - 2rR\cos\theta + r^2} = \frac{2\pi}{R^2-r^2}$.
    (b) $\int_0^{2\pi} \frac{\sin\theta d\theta}{R^2 - 2rR\cos\theta + r^2} = 0$.

11. By integrating $f(z) = \frac{1}{(R-z)}$ over C:$|z| = r$, $0 < r < R$ and using Problem 10, show that $\int_0^{2\pi} \frac{R\cos\theta}{R^2 - 2rR\cos\theta + R^2}d\theta = \frac{2\pi r}{R^2-r^2}$.