

Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 7
Autumn 2020

1. Discuss the convergence of improper integrals using definition:

(i) $\int_1^{\infty} \frac{1}{x^4} dx$	(v) $\int_1^2 \frac{4x}{(x^2 - 4)^{\frac{1}{3}}} dx$	(viii) $\int_{-2}^{\infty} \sin(x) dx$
(ii) $\int_{10}^{\infty} \frac{1}{x \ln x} dx$	(vi) $\int_1^{\infty} \frac{x+1}{x^{\frac{3}{2}}} dx$	(ix) $\int_1^{\infty} \frac{1}{(1+x)\sqrt{x}} dx$
(iii) $\int_0^1 \frac{\ln x}{x^2} dx$		(x) $\int_1^2 \frac{1}{x \ln^2 x} dx$
(iv) $\int_{-\infty}^{\infty} \frac{x}{(x^2 + 1)} dx$	(vii) $\int_{-\infty}^0 \frac{1}{(3-x)^{\frac{1}{2}}} dx$	

2. Discuss the convergence of the following integrals :

(i) $\int_0^1 \frac{1}{x^2 + \sqrt{x}} dx$	(iv) $\int_0^{\infty} \frac{1 - \cos(x)}{x^2} dx$	(viii) $\int_0^{\infty} \frac{\cos x}{e^x} dx$
(ii) $\int_0^{\infty} \frac{1}{x + e^x} dx$	(v) $\int_1^{\infty} \frac{x}{(1+x)^3} dx$	(ix) $\int_1^{\infty} e^{x+x^{-1}} dx$
(iii) $\int_0^{\infty} \frac{1}{x^2 + xe^x} dx$	(vi) $\int_1^{\infty} \frac{x}{3x^4 + 5x^2 + 1} dx$	(x) $\int_0^1 \frac{e^x}{x^2} dx$
	(vii) $\int_{-\infty}^{\infty} e^{- x } dx$	

3. Examine the convergence of the following integrals :

(i) $\int_0^1 \frac{1}{(x+2)\sqrt{x(1-x)}} dx$	(iii) $\int_1^{\infty} \frac{1}{x^{\frac{1}{2}}(1+x)^{\frac{1}{4}}} dx$	(v) $\int_0^1 \frac{x^{p-1}}{1-x} dx$
(ii) $\int_0^{\infty} x^{-\frac{1}{2}} e^{-x} dx$	(iv) $\int_0^{\infty} \frac{\cos(x)}{\sqrt{x^3 + x}} dx$	

4. Prove that $\int_0^{\frac{\pi}{2}} \frac{x^m}{\sin^n x} dx$ is convergent iff $n < m + 1$

5. Show that the improper integral $\int_0^1 \frac{\sin(\frac{1}{x})}{\sqrt{x}} dx$ is convergent.
 6. Prove that the integral $\int_0^\infty \left(\frac{1}{x+1} - \frac{1}{e^x} \right) \frac{1}{x} dx$ is convergent
 7. Test the convergent of $\int_0^\infty e^{-x^2} dx$
 8. Explain the convergence of $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$
 9. Show that $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ is convergent iff m, n are both positive.
 10. Show that $\int_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}(bx)}{x} dx = \frac{\pi}{2} \log\left(\frac{a}{b}\right) \quad 0 < b < a$
 11. Prove that $\int_0^\infty \frac{\sin(x)(1 - \cos(x))}{x^2} dx = \log 2$
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