Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 6
Autumn 2020

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1. Expand $f(x, y) = e^{(2x+xy+y^2)}$ in powers of x and y upto second order term.

2. Expand $f(x, y) = sin(xy)$ in powers of $(x - 1)$ and $(y - \pi/2)$ up to second degree term, and then find the remainder term.

3. Expand $f(x, y) = e^y \sin x$ in Taylor's series upto second order term about $(\frac{\pi}{2}, 1)$. Also estimate the value of $f(x, y) = e^y \sin x$ when $x = \frac{51}{100}\pi,\ y = 0.99$.

4. Expand $f(x, y, z) = e^z \sin(x+y)$ in Taylor's series upto second order term about the point $(0, 0, 0)$.

5. Expand $f(x, y) = x^2y+\sin y+e^x$ in powers of $(x-1)$ and $(y-\pi)$ upto second order terms using taylor's theorem and find the remainder term.

6. Show that
$\sin x \, \sin y = xy - \dfrac{1}{6}[(x^3 + 3xy^2)\cos(\theta x)\sin(\theta y) + (y^3 + 3x^2y)\sin(\theta x)\cos(\theta y)]$, where $0 < \theta < 1$.

7. Classify the local extremum of the following functions:

   (a) $f(x, y) = x^2y - 2xy^2 + 3xy + 4$.

   (b) $f(x, y) = 2(x - y)^2 - x^4 - y^4$.

   (c) $f(x, y) = x^3 - 12x + y^3 + 3y^2 - 9y$.

   (d) $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

   (e) $f(x, y) = x^2y - xy^2 + 4xy - 4x^2 - 4y^2$.

8. Verify that $x^3y^2(1 - x - y)$ has a maximum at $(\frac{1}{2}, \frac{1}{3})$.

9. Find the absolute maximum and minimum values of $f(x, y) = 4x^2+9y^2-8x-12y+4$ over thr rectangle in the first quadrant bounded by the lines $x = 2, y = 3$ and the co-ordinate axes.

10. Find the global extremum of $f(x, y) = x^2 + xy + y^2$ over the circular region $R = \{(x, y)/\ x^2 + y^2 \leq 1\}$.

11. Find the absolute maximum and minimum value of the function $f(x,y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \leq 1$.

12. Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

13. Prove that the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2}$ is $\dfrac{8abc}{3\sqrt{3}}$.

14. Find the extreme value of $f(x,y,z) = 2x + 3y + z$ such that $x^2 + y^2 = 5$ and $x + z = 1$.

15. Find the extremum value of $a^3x^2 + b^3y^2 + c^3z^2$ s.t. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ where $a > 0, b > 0, c > 0$.

16. Of all triangles, with the same perimeter, determine the triangle with greatest area.

17. Find the smallest and the largest distance between the points P and Q such that P lies on the plane $x + y + z = 2a$ and Q lies on the sphere $x^2 + y^2 + z^2 = a^2$ where a is any constant.

18. A rectangular box open at the top is to have a volume of 32 c.c. Find the dimensions of the box that requires the least material for its construction.

19. Find the shortest and the longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$.

20. Using Lagrange's method of multiplier, show that the maximum and minimum value of $ax + by$ (where the constants $a, b > 0$) subject to the constraint $x^2 + y^2 = 1$ are $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$ respectively.

21. For the extremum values of $x^2 + y^2 + z^2$ subject to the constraints $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$, show that the stationary points satisfy the relation $\frac{l^2}{1-\lambda a} + \frac{m^2}{1-\lambda b} + \frac{n^2}{1-\lambda c} = 0$.

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