Indian Institute of Technology Kharagpur
Department of Mathematics
MA11004 - Linear Algebra, Numerical and Complex Analysis
Problem Sheet - 4
Spring 2021

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1. Find the solution of the following linear system of equations using Gauss-Seidel method:

$$\begin{aligned} 12x_1 + 3x_2 - 5x_3 &= 1 \\ x_1 + 5x_2 + 3x_3 &= 28 \\ 3x_1 + 7x_2 + 13x_3 &= 76 \end{aligned}$$

   Take initial guess $(x_1, x_2, x_3) = (0, 1, 0)$, and perform six iterations.

2. Use Jacobi method and Gauss-Seidel method to approximate the solution of the following linear system of equations correct upto 3 decimal places:

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3. \end{aligned}$$

   Take initial guess $(x_1, x_2, x_3) = (0, 0, 0)$.

3. The equation

$$8x^3 - 12x^2 - 2x + 3 = 0$$

   has three real roots. Find the intervals each of unit length containing each one of these roots.

4. Perform three iterations of bisection method to obtain the smallest positive root of the equation

$$f(x) = x^3 - 5x + 1 = 0.$$

5. Perform three iterations of bisection method to obtain a root of the equation

$$f(x) = \cos(x) - xe^x = 0.$$

6. Use Newton-Raphson method to determine the second approximation x_2 for a root of the equation

$$f(x) = x^3 - 7x^2 + 8x - 3 = 0; \quad \text{if} \quad x_0 = 5.$$

7. Consider the function

$$f(x) = x^5 - x^3 + 2x^2 - 1 = 0.$$

   Approximate the root near 1, correct upto 8 decimal places using Newton-Raphson method.

8. Find the root of the equation $\sin(x) = 10(x-1)$ by using fixed point iteration method correct up to three decimal places.

9. Approximate the positive square root of a number N by Newton-Raphson Method. Hence find first and second approximate value of $\sqrt{2}$ using initial approximation equals to 1.5.

10. Using Newton-Raphson method, find first and second approximate values of $\sqrt[3]{7}$ by taking initial approximation equals to 2.

11. Find a root of the equation

$$f(x) = x^4 - x - 10 = 0$$

using fixed point iteration method with initial guess equals to 1.

12. The equation

$$f(x) = 3x^3 + 4x^2 + 4x + 1 = 0$$

has a root in the interval $(-1, 0)$. Determine an iteration function $\phi(x)$, such that the sequence of iterations obtained from

$$x_{k+1} = \phi(x_k) \quad , \quad x_0 = 0.5, \quad k = 0, 1, \ldots$$

converges to the root.

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