Indian Institute of Technology Kharagpur
Department of Mathematics
MA11004 - Linear Algebra, Numerical and Complex Analysis
Problem Sheet - 2 - Hints and Answers
Spring 2021

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1. (b) and (c) form a basis.

2. (a) Basis :
$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$
dim $V = 3$.
   (b) Basis : $\{(1, 0, 1, -2), (0, 1, 2, -1)\}$, dim $U = 2$.
   (c) Basis : $\{x, x^2 - \frac{1}{3}, x^3, x^4 - \frac{1}{5}\}$, dim $U = 4$.

3. dim $U = 2$, dim $W = 2$, dim $U + W = 3$ and dim $(U \cap W) = 1$.

4. (a) No.
   (b) Yes.

5. $\phi(z) = Im(z)$ where $z \in \mathbb{C}$.

6. (a) $N(T) = L\{0\}$, dim $N(T) = 0$.
   $R(T) = L\{3x, 2 + \frac{3}{2}x^2, 4x + x^3\}$,dim $R(T) = 3$.
   (b) $N(T) = L\{(1, 1, 0)\}$,dim $N(T) = 1$.
   $R(T) = L\{(\frac{1}{2}, 0), (-\frac{1}{2}, \frac{1}{2})\}$, dim $R(T) = 2$.
   (c) $N(T)$ =set of all 2×2 symmetric matrices. Dim $N(T) = 3$.
   $R(T)$ =set of all 2×2 skew symmetric matrix. Dim $R(T) = 1$.

7. (a) $T(x, y) = (2x - y, x - y, 2x)$.
   (b) $T(x, y, z) = (x + 2y + 3z, x + 3y + 2z)$.
      (i) $T(1, 1, 0) = (3, 4), T(6, 0, -1) = (3, 4)$,
      (ii) $KerT = L\{(-5, 1, 1)\}, Im(T) = L\{(1, 1), (2, 3), (3, 2)\}$.

8. (a) $\begin{bmatrix} 0 & 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 & 72 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.
   (b) $\begin{bmatrix} 0 & 0 & 4 & 0 \\ 1 & 3 & 9 & 27 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 12 \end{bmatrix}$.

9. Rank-Nullity theorem.

10. (a) $(-\frac{1}{5}, 4, -\frac{4}{5})$.

(b) $(-2c + 10, c, -5, 2)$.

11. (a) Rank of $A = 2$.
    (b) Rank of $A = 2$.
12. (a) 2.
    (b) 3.
13. $-\frac{1}{2}, 1, 1$.
14. $k = -4$.
15. $(1 - 3k, -k, 5k)$, k is an integer.
16. Not possible
17. (a) $a \neq 8$.
    (b) $a = 8, b \neq -1, 3$.
    (c) $a = 8, b = 3$ or $a = 8, b = -1$.

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