Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Hints and solutions - 5
Autumn 2020

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1. Value of $\dfrac{df}{dt}$ at $t = 0$:

   (a) $\dfrac{df}{dt} = e$

   (b) $\dfrac{df}{dt} = 3$

   (c) $\dfrac{df}{dt} = 3 + \log 3$

2. (a) Value of $\dfrac{dy}{dx}$:

   i. $\dfrac{dy}{dx} = -\dfrac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}$

   ii. $\dfrac{dy}{dx} = -\dfrac{[1 + (x + y)^2][y^2 + \exp(x)\sin(y^2)] + 1}{[1 + (x + y)^2][2xy + 2y\exp(x)\cos(y^2)] + 1}$,

   iii. $\dfrac{dy}{dx} = -\dfrac{b^2 x}{a^2 y}$

   iv. $\dfrac{dy}{dx} = -\dfrac{2x - y}{2y + x}$

   (b) Values of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$:

   i. $\dfrac{\partial z}{\partial x} = \dfrac{z^2 e^{xz^2} - y^2 z^2}{2xy^2 z + y\cos(yz) - 2xze^{xz^2}}$ and $\dfrac{\partial z}{\partial y} = \dfrac{2xyz^2 + z\cos(yz)}{2xze^{xz^2} - 2xy^2 z - y\cos(yz)}$

   ii. $\dfrac{\partial z}{\partial x} = -\dfrac{\tan^{-1}(\frac{y}{z}) - \frac{yz}{x^2+z^2} + \frac{yz}{x^2+y^2}}{\tan^{-1}(\frac{x}{y}) - \frac{xy}{y^2+z^2} + \frac{xy}{x^2+z^2}}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{\tan^{-1}(\frac{z}{x}) - \frac{xz}{x^2+y^2} + \frac{xz}{y^2+z^2}}{\tan^{-1}(\frac{x}{y}) - \frac{xy}{y^2+z^2} + \frac{xy}{x^2+z^2}}$

   iii. $\dfrac{\partial z}{\partial x} = -\dfrac{y^2 + yz\cos(xyz)}{3z^2 + xy\cos(xyz)}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{2xy + xz\cos(xyz)}{3z^2 + xy\cos(xyz)}$

   iv. $\dfrac{\partial z}{\partial x} = -\dfrac{1 - yz\sin(xyz) - 2xz^2}{-y - xy\sin(xyz) - 2zx^2}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{-z - xz\sin(xyz)}{-y - xy\sin(xyz) - 2zx^2}$

3. Just find the partial derivatives.

4. Apply Euler's theorem and the fact that v is a function of u.

5. Ans:
   (a) Function is homogeneous and of degree 0.
   (b) Function is homogeneous and of degree 0.
   (c) Function is homogeneous and of degree 1.
   (d) Function is homogeneous and of degree 0.
   (e) Function is homogeneous and of degree 2.
   (f) Function is homogeneous and of degree $1/20$.
   (g) Function is not homogeneous.
   (h) Function is homogeneous and of degree -1.
6. Apply Euler's theorem.
7. Apply Euler's theorem; $k = 3/2$.
8. Find higher order partial derivatives of y.
9. Find partial derivatives of u.
10. Apply log to both sides and find partial derivatives.
11. Find higher order partial derivatives of u.
12. Use Euler's theorem on $\tan u$.
13. Use Euler's theorem on $\sin u$ and then find partial derivatives.
14. Use Euler's theorem.
15. Let $U = \dfrac{(ax^3 + by^3)^n}{3n(3n-1)}$ and $V = xf(\dfrac{y}{x})$, then use Euler's theorem on both.
16. Let $\alpha(x, y) = x^m f(\dfrac{y}{x})$ and $\beta(x, y) = y^n g(\dfrac{x}{y})$, take help of Euler's theorem.
17. Use Euler's theorem and take $\alpha = -\dfrac{n}{m}$.
18. Find partial derivatives and substitute the values.
19. Find partial derivatives using chain rules.
20. $k = 0$ or $k = 1 - \dfrac{n}{2}$
21. Find partial derivatives using chain rule.
22. Find second order partial derivatives and substitute the values.