* Find the number of intervals n such that the error for the composite Simpson's $\frac{1}{3}$rd rule is less than 5×10^{-7} when evaluating $\int_2^5 \log x \, dx$. Find also the corresponding step size.

Ans: $f(x) = \log x$, $f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$, $f'''(x) = \frac{2}{x^3}$, $f''''(x) = -\frac{6}{x^4}$

$$|\varepsilon| = \left| \frac{(b-a) f^{(iv)}(\xi) h^4}{180} \right| \leq \frac{(5-2) h^4}{180} \cdot \frac{3}{8} \quad [\because f^{iv}(\xi) \leq f^{iv}(2)]$$

$$= \frac{h^4}{160}$$

$$h = \frac{b-a}{n} = \frac{3}{n}$$

$$\frac{h^4}{160} = \frac{81}{n^4 \times 160} \leq 5 \times 10^{-7}$$

$$\therefore n^4 \geq \frac{81}{160} \cdot \frac{1}{5 \times 10^{-7}}$$

$$\therefore n \geq 31.72114$$

i.e. we need to choose $n = 32$ and the corresponding step size is $h = \frac{3}{n} = \frac{3}{32} = 0.09375$.

Error in trapezoidal rule $\varepsilon = -\frac{b-a}{12} h^2 f''(\xi)$, $x_0 < \xi < x_n$

" " Simpson's rule $\varepsilon = -\frac{b-a}{180} h^4 f''''(\xi)$, $x_0 < \xi < x_n$