Problem set- II Hints and Answers

1. a. Ans: $y = (C_1 + C_2 x)e^{2x} + C_3 e^{-x}$
   b. Ans: $y(x) = C_1 e^x + C_2 e^{2x}$
   c. Ans: $y(x) = C_1 \cos ax + C_2 \sin ax$
   d. Ans: $y(x) = e^x(C_1 \cos \sqrt{3}x + C_2 \sin \sqrt{3}x)$
   e. Ans: $(A + Bx)\cos x + (C + Dx)\sin x$
   f. Ans: $y(x) = (C_1 + C_2 x + C_3 x^2)\cos x + (C_4 + C_5\, x + C_6\, x^2)\sin x + \{(C_7 + C_8\, x)\cos \frac{\sqrt{3}x}{2} + (C_9 + C_{10}\, x)\sin \frac{\sqrt{3}}{2}x\}e^{-\frac{1}{2}x}$
   g. Ans: $y = C_1 + xC_2 + e^x(C_3 + xC_4 + C_5 x^2)$
   h. Ans :$y = Ae^{mx} + Be^{-mx} + C\cos mx + D\sin mx$
   i. Ans :$y = (A + Bx + Cx^2)e^{2x} + De^{-x}$

2. a. Ans: $x(t) = 4e^t - 2e^{2t}$
   b) Ans:$y(x) = e^{-2x}(\cos x + 2\sin x)$
   c) Ans: $y(x) = e^x(4\cos 3x - \sin 3x)$
   d) Ans: The general solution is $y(t) = C_1 e^{-2t} + c_2\, e^{4t}$ and the solution is depend on the co-efficient of C_2 only since $e^{-2t} \to \infty$ as $t \to \infty$
   e) Ans :$y(x) = \frac{14}{33}e^{-4x} + \frac{13}{15}e^{2x} - \frac{16}{55}e^{7x}$
   f) Ans :$y(x) = e^{-\frac{x}{2}}(\cos \frac{\sqrt{3}}{2}x + \sqrt{3}\sin \frac{\sqrt{3}}{2}x)$

3. a. Ans: $y = Ae^{3x} + Be^{2x} + xe^{3x}$
   b. Ans: $y = Ae^x + Be^{-2x} - \frac{x}{2}e^x + \frac{1}{12}e^{-x}$
   c. Ans :$Ae^{2x} + Be^{-2x} + \frac{e^x}{-3} + \frac{1}{13}\sin 3x$
   d. Ans:$y = Ae^x + (B + Cx)e^{-x} - \frac{1}{25}(2\sin 2x + \cos 2x)$
   e. Ans:$y = Ae^{2x} + Be^{-2x} - \frac{x}{3}\sin hx - \frac{2}{9}\cos hx$
   f. Ans:$y = Ae^{2x} + Be^{-2x} - \frac{1}{4}x^2 - \frac{1}{8}$
   g. Ans:$y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{5}e^x - \frac{1}{5}\sin 3x + \frac{1}{4}x^2 - \frac{1}{8}$
   h. Ans:$y = (A + Bx)e^x - e^x(x\sin x + 2\cos x)$
   i. Ans :$y = Ae^{-2x} + Be^{-3x} + \frac{e^{-2x}}{-8}[2\cos 2x + 4\sin 2x]$
   j. Ans:$A\cos x + B\sin x + \log(\sin x)\sin x - x\cos x$
   k. Ans $Ae^{-x} + e^{\frac{x}{2}}(B\cos \frac{\sqrt{3}}{2}x + C\sin \frac{\sqrt{3}}{2}x) + \frac{1}{730}(\sin 3x + 27\cos 3x) - \frac{1}{2} - \frac{1}{4}(\cos x - \sin x)$

4. a. Ans: Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in term of $\frac{dy}{dz}$ and $\frac{d^2y}{dz^2}$ and put in the given differential equation.
   b. Ans: Differentiate $(x - a)^2 + (y - b)^2 = r^2$ twice and try to remove the constant a and b. [$(a, b)$is center of the circle]

c. Ans: If u and v be two function that possess two continuous derivative on the interval I and such that $W(u, v, x) \neq 0$ for all $x \in I$. Then the equation $\det H = 0$ is a linear homogenous second order differential equation for which u, v are solution, where H consider as the below matrix.

$$H = \begin{bmatrix} y & y' & y'' \\ u & u' & u'' \\ v & v' & v'' \end{bmatrix}$$