Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 9
Autumn 2020

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1. Evaluate the integrals over the region D.
   (i) $\int\int_D xydA$, where D is region bounded by x-axis, ordinate $x = 2a$ and curve $x^2 = 4ay$,
   (ii) $\int\int_D e^{\frac{x}{y}}dA$, where $D = \{(x, y)|1 \le y \le 2, y \le x \le y^3\}$.
   (iii) $\int\int_D (4xy - y^3)dA$, where D is region bounded by $y = \sqrt{x}$ and $y = x^3$,
   (iv) $\int\int_D (6x^2 - 40y)dA$, where D is the triangle with vertices $(0, 3), (1, 1)$ and $(5, 3)$,
   (v) $\int\int_D (x^2 + 2xy^2 + 2)dA$, where D is region bounded by $y = x - x^2$, $y = 0$, $x = 0$ and $x = 2$.

2. Evaluate the following integrals by changing the order of integration
   (i) $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{a}x} dydx$,
   (ii) $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}dydx$,
   (iii) $\int_0^3\int_{x^2}^9 x^3e^{y^3}dydx$,
   (iv) $\int_0^8\int_{3\sqrt{y}}^2 \sqrt{x^4+1}dxdy$.

3. Evaluate $\int_0^1\int_0^{1-x} e^{\frac{y}{x+y}}dydx$, using the transformation $x + y = u$ and $y = uv$.

4. Consider the transform T from the xy-plane to the uv-plane given by
$$T : x = \frac{1}{2}(u + v), y = \frac{1}{2}(u - v).$$
   (i) Calculate the Jacobian of the transform T.
   (ii) Evaluate $\int\int_D (x - y)\cos^2(x + y)dA$ using transformation T, where D is the square in xy-plane with vertices $(0, 0), (\pi, \pi), (0, 2\pi)$ and $(-\pi, \pi)$.

5. Evaluate the integral by making an appropriate change of variables
   (i) $\int\int_D x^2dxdy$, D is elliptic region $\{(x, y) : \frac{x^2}{4} + \frac{y^2}{9} \le 1$.
   (ii) $\int\int_D y^2dxdy$, D is region bounded by curves $xy = 1, xy = 2, xy^2 = 1$ and $xy^2 = 2$.
   (iii) $\int\int_D (x + y)^2dxdy$, D is parallelogram bounded by the lines $x + y = 0, x + y =$

$1, 2x - y = 0$ and $2x - y = 3$.

6. Find the area lying between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

7. Find the volume of the region bounded by the cylinder $x^2+y^2 = 4$ and the planes $y+z = 4$ and $z = 0$, using double integral.

8. Find the area of the paraboloid $2z = \frac{x^2}{a} + \frac{y^2}{b}$ inside the cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

9. Calculate the area of the region bounded by the upper half of the circle $x^2 + y^2 = 25$, the x-axis and the ordinates $x = -3$ and $x = 4$.
