Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 11
Autumn 2020

---

1 Solve the following homogeneous differential equations:

a. $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 4y = 0$

b. $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 0$

c. $\dfrac{d^2y}{dx^2} + a^2y = 0$

d. $2\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 8y = 0$

e. $\dfrac{d^4y}{dx^4} + 2\dfrac{d^2y}{dx^2} + y = 0$

f. $\left(\dfrac{d^2y}{dx^2} + y\right)^3 \left(\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y\right)^2 = 0$

g. $\dfrac{d^5y}{dx^5} - 3\dfrac{d^4y}{dx^4} + 3\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} = 0$

h. $\dfrac{d^4y}{dx^4} = m^4y$

i. $\dfrac{d^4y}{dx^4} - 5\dfrac{d^3y}{dx^3} + 6\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 8y = 0$

2 Solve the following initial value problems:

a. $\dfrac{d^2x}{dt^2} - 3\dfrac{dx}{dt} + 2x = 0;$ $\quad x(0) = 2, x'(0) = 0$

b. $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 0;$ $\quad y(0) = 1, y'(0) = 0$

c. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 10y = 0;$ $\quad y(0) = 4, y'(0) = 1$

d $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} - 8y = 0;$ $\quad y(0) = \alpha, y'(0) = 2\pi$

e. $\dfrac{d^3y}{dx^3} - 5\dfrac{d^2y}{dx^2} - 22\dfrac{dy}{dx} + 56y = 0;$ $\quad y(0) = 1, y'(0) = -2, y''(0) = -4$

f. $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 0$ $\quad y(0) = 1, y^{'}(0) = 1$

3 Solve the following differential equations:

a. $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = e^{3x}$

b. $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = \cosh x$

c. $\dfrac{d^2y}{dx^2} - 4y = e^x + \sin 3x$

d. $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - y = \cos 2x$

e. $\dfrac{d^2y}{dx^2} - 4y = x \sin hx$

f. $\dfrac{d^2y}{dx^2} - 4y = x^2$

g. $\dfrac{d^2y}{dx^2} + 4y = e^x + \sin 3x + x^2$

h. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$

i. $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = e^{-2x} \sin 2x$

j. $(D^2 + 1)y = \text{cosec}x$

k. $\dfrac{d^3y}{dx^3} + y = \sin 3x - \cos^2 \dfrac{x}{2}$

4 Solve the following problems:

a. Show that the substitution $z = \sinh^{-1} x$ transforms the equation $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 4y$ into $\dfrac{d^2y}{dz^2} = 4y$.

b. Show that all circle of radius r are represented by the differential equation $\left(1 + (\dfrac{dy}{dx})^2\right)^{\frac{3}{2}} = r\dfrac{d^2y}{dx^2}$.

c. Construct a linear homogenous second order Differential equation such that the given functions are solution of differential equation

   (i) $u(x) = x, v(x) = e^x$,

   (ii) $u(x) = \frac{1}{x}, v(x) = e^{-x}$.