1

Rewriting the equations, we get

$$x_1 = \frac{1 - 3x_2 + 5x_3}{12}$$

$$x_2 = \frac{28 - x_1 - x_3}{5}$$

$$x_3 = \frac{76 - 3x_1 - 7x_2}{13}$$

$\therefore$ Gauss - Siedel scheme for the given equations is

$$x_1^{(k+1)} = \frac{1 - 3x_2^{(k)} + 5x_3^{(k)}}{12}$$

$$x_2^{(k+1)} = \frac{28 - x_1^{(k+1)} - x_3^{(k)}}{5}$$

$$x_3^{(k+1)} = \frac{76 - x_1^{(k+1)} - 7x_2^{(k+1)}}{13}$$

where, (x_1^k, x_2^k, x_3^k) is the solution at k th iteration

Assuming an initial guess of

$$\begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \\ x_3^{(0)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(2 — . For Gauss - ...

(2)

Iteration 1

$$x_1^{(1)} = \frac{1 - 3(0) + 5(1)}{12} = 0.50000$$

$$x_2^{(1)} = \frac{28 - (0.50000) - 3(1)}{5} = 4.9000$$

$$x_3^{(1)} = \frac{76 - 3(0.50000) - 7(4.90000)}{13} = 3.0923$$

Iteration 2

$$x_1^{(2)} = \frac{1 - 3(4.9) + 5(3.0923)}{12} = 0.14679$$

$$x_2^{(2)} = \frac{28 - (0.14679) - 3(3.0923)}{5} = 3.7153$$

$$x_3^{(2)} = \frac{76 - 3(0.14679) - 7(3.7153)}{13} = 3.8118$$

Iteration	x_1	x_2	x_3
1	0.50000	4.9000	67.662
2	0.14679	3.7153	18.874
3	0.74275	3.1644	4.0064
4	0.94675	3.0281	0.65772
5	0.99177	3.0034	0.074383
6	0.99919	3.0001	0.00101

∴ The approximate solution at sixth iteration is

$$x^* = (0.99919,\ 3.0001,\ 0.00101)$$

2

For Gauss - Siedel

Rewriting the equations, Gauss - Siedel scheme becomes

$$x_1^{(k+1)} = -\frac{1}{5} + \frac{2}{5}x_2^{(k)} - \frac{3}{3}x_3^{(k)}$$

$$x_2^{(k+1)} = \frac{2}{9} + \frac{3}{9}x_1^{(k+1)} - \frac{1}{9}x_3^{(k)}$$

$$x_3^{(k+1)} = -\frac{3}{7} + \frac{2}{7}x_1^{(k+1)} - \frac{1}{7}x_2^{(k+1)}$$

where (x_1^k, x_2^k, x_3^k) is the solution at kth iteration

Assuming initial guess as $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0,0,0)$

Iteration 1

$$x_1^{(1)} = -\frac{1}{5} + \frac{2}{5}(0) - \frac{3}{5}(0) = -2.00$$

$$x_2^{(1)} = \frac{2}{9} + \frac{3}{9}(-2.00) - \frac{1}{9}(0) \simeq 0.156$$

$$x_3^{(1)} = -\frac{3}{7} + \frac{2}{7}(-2.00) - \frac{1}{7}(0.156) = -0.508$$

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iterations	x_1	x_2	x_3
0	0.000	-0.000	0.000
1	-0.200	0.156	-0.508
2	0.167	0.334	-0.429
3	0.191	0.333	-0.422
4	0.186	0.331	-0.423
5	0.186	0.331	-0.423

$\therefore$ The solution correct upto 3 decimal places is

$$x^* = (0.186, 0.331, -0.423)$$

2

Rewriting the equations, the Jacobi 1 scheme becomes

~~write the system in the form~~

$$x_1^{(k+1)} = -\frac{1}{5} + \frac{2}{5}x_2^{(k)} - \frac{3}{5}x_3^{(k)}$$

$$x_2^{(k+1)} = \frac{2}{9} + \frac{3}{9}x_1^{(k)} - \frac{1}{9}x_3^{(k)}$$

$$x_3^{(k+1)} = -\frac{3}{7} + \frac{2}{7}x_1^{(k)} - \frac{1}{7}x_2^{(k)}$$

Assume initial approximation

as $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0,0,0)$

First approximation

$$x_1^{(1)} = -\frac{1}{5} + \frac{2}{5}(0) - \frac{3}{5}(0) = -0.2$$

$$x_2^{(1)} = \frac{2}{9} + \frac{3}{9}(0) - \frac{1}{9}(0) = 0.222$$

$$x_3^{(1)} = -\frac{3}{7} + \frac{2}{7}(0) - \frac{1}{7}(0) = -0.429$$

Continuing, we obtain the sequence of approximations

Iterations	x_1	x_2	x_3
0	0.000	0.000	0.000
1	-0.200	0.222	-0.429
2	0.146	0.203	-0.517
3	0.192	0.328	-0.416
4	0.181	0.332	-0.421

3

$f(x) = 8x^3 - 12x^2 - 2x + 3 = 0$

x	-2	-1	0	1	2	3
$f(x)$	-105	-15	3	-3	15	105

from table

$\therefore f(-1) < 0$ & $f(0) > 0$

$\Rightarrow$ $f(x) = 0$ has root in $(-1, 0)$

$f(0) > 0$ & $f(1) < 0$

$\Rightarrow$ $f(x) = 0$ has root in $(0, 1)$

$f(1) < 0$ & $f(2) > 0$

$\Rightarrow$ $f(x) = 0$ has root in $(1, 2)$

4

$f(x) = x^3 - 5x + 1 = 0$

Since $f(0) > 0$ and $f(1) < 0$

the smallest +ve root lies in the interval $(0,1)$.

Taking $a_0 = 0$, $b_0 = 1$, we get

$$m_1 = \frac{1}{2}(a_0 + b_0)$$

$$= \frac{1}{2}(0+1) = 0.5$$

$$f(m_1) = -1.375$$

and $f(a_0) f(m_1) < 0$

Thus, the root lies in $(0, 0.5)$

Taking $a_1 = 0$, $b_1 = 0.5$, we get

$$m_2 = \frac{1}{2}(a_1 + b_1)$$

$$= \frac{1}{2}(0 + 0.5) = 0.25$$

$$f(m_2) = f(0.25) = -0.234375$$

and $f(a_1) f(m_2) < 0$

Thus the root lies in the interval $(0, 0.25)$

Taking $a_2 = 0$, $b_2 = 0.25$

$$m_3 = \frac{1}{2}(0 + 0.25) = 0.125$$

$$f(m_3) = f(0.125) = 0.37695$$

$f(a_2)\, f(m_3) > 0$

$\therefore$ root lies in $(0.125, 0.25)$

and approximate root

$$x^* = \frac{1}{2}(0.125 + 0.25)$$

$$= 0.1875$$

5

$f(x) = \cos x - xe^{x} = 0$

Since $f(0) = 1 > 0$, and $f(1) = -2.1780 < 0$,

the root lies in the interval $(0,1)$.

Taking the initial approximations as $a_0 = 0$, $b_0 = 1$ we get

$$m_1 = \frac{1}{2}(a_0 + b_0) = \frac{1}{2}(0+1) = 0.5$$

$$f(m_1) = f(0.5) = 0.0532$$

and $f(a_0)\, f(m_1) > 0$

Therefore, the root lies in the interval $(0.5, 1.0)$

$a_1 = 0.5$, $b_1 = 1.0$, we get

$$m_2 = \frac{1}{2}(a_1 + b_1) = \frac{1}{2}(0.5 + 1.0) = 0.75$$

$$f(m_2) = -0.8561$$

and $f(a_1)\, f(m_2) < 0$

$\therefore$ The root lies in the interval $(0.5, 0.75)$

$a_2 = 0.5$, $b_2 = 0.75$

$$m_3 = \frac{1}{2}(a_2 + b_2)$$

$$= \frac{1}{2}(0.5 + 0.75)$$

$$= 0.625$$

$$f(m_3) = f(0.625) = -0.1677$$

and $f(a_2)\, f(m_3) < 0$

$\therefore$ root lies $(0.5, 0.625)$

and the root

$$x^* = \frac{0.5 + 0.625}{2} = 0.5625$$

6

we have,

$$f(x) = x^3 - 7x^2 + 8x - 3$$

Newton's Method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Now $\quad f'(x) = 3x^2 - 14x + 8$

1st Iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{f(5)}{f'(5)} = 5 - \frac{-13}{13} = 6$$

2nd Iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{f(6)}{f'(6)}$$

$$= 6 - 9/32 = 5.71875$$

$$\boxed{x_2 = 5.71875}$$

7. We have

$$f(x) = x^5 - x^3 + 2x^2 - 1$$

As, we have to find the root near 1, take

$x_0 = 1$ as our initial point

Now, $f'(x) = 5x^4 - 3x^2 + 4x$

Newton's formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^5 - x_n^3 + 2x_n^2 - 1}{5x_n^4 - 3x_n^2 + 4x_n}$$

$$x_1 = x_0 - \frac{x_0^5 - x_0^3 + 2x_0^2 - 1}{5x_0^4 - 3x_0^2 + 4x_0}$$

$$= 1 - \frac{(1 - 1 + 2 \times (1) - 1)}{5(1) - 3(1) + 4(1)}$$

$$\approx 0.83333333$$

Again

$x_2 \approx 0.77541271$

$x_3 \approx 0.77005822$

$x_4 \approx 0.77001784$

$x_5 \approx 0.77001784$

$\therefore$ root near 1 upto eight decimal places is $\boxed{x^* = 0.77001784}$

"7", correct upto 3 decimal ,

8

we have

$$\sin x = 10(x-1)$$

let

$$f(x) = \sin x - 10(x-1)$$

Now,

$$f(1) = 0.0175 > 0 \quad \& \quad f(1.5) = -4.97 < 0$$

we write the equation as

$$x = \frac{1}{10}(\sin x + 10) = \phi(x)$$

Thus

$$|\phi'(x)| = \left|\frac{\cos x}{10}\right| \leq \frac{1}{10} < 1$$

Thus $x = \dfrac{\sin x + 10}{10} = \phi(x)$ gives us a convergent sequence of iteration

Iteration formula

$$x_{n+1} = \phi(x_n)$$

We take
$x_0 = 1$

n	x_n	$\phi(x_n)$
0	1	1.0017
1	1.0017	1.0017
2	1.0017	1.0017

Thus, 1.0017 is the root of the given eqn, correct upto 3 decimal places

9

Let $x = N^{1/2}$

$x^2 = N$

$\therefore \quad x^2 - N = 0$

$f(x) = 0$

$f(x) = x^2 - N$

$f'(x) = 2x$

Now,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - N}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

$$= \frac{x_n^2 + N}{2x_n}$$

$$= \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$$

Now $x = \sqrt{2}$

$x^2 = 2$

By above

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$$

$$x_1 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3/2}\right)$$

$$= \frac{17}{12} = 1.416666$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{2}{x_1}\right)$$

$$= \frac{1}{2}\left(\frac{17}{12} + \frac{24}{17}\right)$$

$$= \frac{1}{2} \times \frac{17^2 + 12 \times 24}{12 \times 17}$$

$$= 1.414215$$

10

$x = \sqrt[3]{7}$

$\Rightarrow x^3 - 7 = 0$

$f(x) = x^3 - 7$

$f'(x) = 3x^2$

Now

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^3 - 7}{3x_n^2}$$

$$= \frac{2x_n^3 + 7}{3x_n^2}$$

$$= \frac{2}{3}x_n + \frac{7}{3x_n^2}$$

$$= \frac{1}{3}\left[2x_n + \frac{7}{x_n^2}\right]$$

$x_0 = 2$

$$x_1 = \frac{1}{3}\left[4 + \frac{7}{4}\right]$$

$$= \frac{23}{3} = 7.6666$$

$$x_2 = \frac{1}{3}\left[2 \times \frac{23}{3} + 7 \times \frac{9}{(23)^2}\right]$$

$$= 5.1508$$

$$\phi(x) = \frac{x_n^3 + 1}{3}$$

$$|\phi'(x)| = |x_n^2| \, 1/3 = 1/9 < 1$$

$\therefore$ it will converge to the root

(12)

(i)

$f(x) = x^4 - x - 10 = 0$

$\Rightarrow \quad x = (x+10)^{1/4}$

$g(x) = (x+10)^{1/4}$

$x_{n+1} = (x_n+10)^{1/4} \qquad n=0,1,2,\ldots$

initial guess $x_0 = 1$

$x_1 = (1+10)^{1/4} = 1.82116$

$x_2 = (1.82116+10)^{1/4} = 1.85424$

$x_3 = (1.85424+10)^{1/4} = 1.85553$

$x_4 = (1.85553+10)^{1/4} = 1.85558$

$x_5 = (1.85558+10)^{1/4} = 1.85558$

root is 1.85558

12

we write the given equation as

$$x = x + \alpha(3x^3 + 4x^2 + 4x + 1) = \phi(x)$$

where α is an arbitrary constant to be determined such that

$$|\phi'(x)| = |1 + \alpha(9x^2 + 8x + 4)| < 1 \quad \forall x \in (-1, 0)$$

since, $9x^2 + 8x + 4 > 0 \quad \forall$ ~~x ∈ (-1,0)~~ $x \in (-1, 0)$

$\therefore \alpha < 0$

The condition $|\phi'(x)| < 1$. must also be satisfied at the initial approximation $x_0 = -0.5$. using this condition, we get

$$|\phi'(-0.5)| = \left|1 + \frac{9\alpha}{4}\right| < 1$$

$$\Rightarrow \quad -1 < 1 + \frac{9\alpha}{4} < 1$$

$$\Rightarrow \quad -2 < \frac{9\alpha}{4} < 0$$

$$\Rightarrow \quad \frac{-8}{9} < \alpha < 0$$

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