Set - 7

1. (a) (i) $4 + \frac{25}{3}i$
   (ii) $4 + 8i$

   (b) (i) $\frac{1}{3}(i-1)$
   (ii) $-\frac{1}{2} + \frac{5}{6}i$

   (c) compute integration over all four edges and then add.
   Ans: 0

   (d) $2i$

   (e) $\frac{29i-58}{6}$

2. (a) Function is analytic,hence value is same. Ans:$-\frac{11+2i}{3}$

   (b) Ans: $-\frac{29}{3} + 11i$.
   No(function is not analytic). Ans:$-\frac{151}{15} + \frac{45i}{4}$

   (c) consider cases where $|z| > 1$ and $|z| < 1$

$$f(Z) = \begin{cases} 2\pi i(e^{z^2} - 1) & |z| < 1 \\ 0 & |z| > 1 \end{cases}$$

3. Use Cauchy's integral formula.
   $F(3.5) = 0$, $F(i) = 2\pi(i+1)$, $F'(-1) = -14\pi i$ and $F''(-i) = 16\pi i$

4. (a) Use Cauchy's integral formula,ans: $-\frac{\pi i}{4}$

   (b) Use Cauchy's integral formula,ans: $2\pi i e^4$

   (c) Use Cauchy's integral formula for derivative, ans : $\frac{\pi}{16}$

   (d)

$$\oint_C \frac{1}{(z-a)^n} dz = \begin{cases} 2\pi i & n = 1 \\ 0 & n \neq 1, n \in Z \end{cases}$$

   , where n is any integer and C is any closed curve containing 'a'.

5. (a) Use ML-inequality.

(b) Usue ML-inequality. $|\oint_{|z|=3} \frac{Log(z)}{z-4i}dz| \leq 6\pi(ln3+\pi)$

6. (a) Use Cauchy's integral formula. $\oint_{|z|=1} \frac{4z^2-4z+1}{(z-2)(z^2+4)}dz$ =0

   (b) Use Cauchy's integral formula. $\oint_{|z+1-i|=2} \frac{z+4}{z^2+2z+5}dz$ =$\frac{\pi}{2}(3+2i)$

   (c) Use Cauchy's integral formula . $\oint_{|z|=6} (\frac{e^{2iz}}{z^4} - \frac{z^4}{(z-i)^3})dz$=$\frac{8\pi}{3} + 12\pi i$

   (d) Use Cauchy's integral formula $\oint_{|z|=1} \frac{dz}{2-\bar{z}}$=$\frac{\pi i}{2}$

7. $\oint_C \frac{cosz}{z(z^2+8)}dz$=$\frac{\pi i}{4}$

8. Use Cauchy's integral formula

   (a) 0

   (b) $2\pi i$

   (c) 0

9. show that for all three cases it is equal to 0

10. Use Cauchy's integral formula. ans: $2\pi i$

    (a) substitute $z = re^{i\theta}$ and compare the real parts.

    (b) compare the imaginary part.

11. substitute $z = re^{i\theta}$ and compare the imaginary parts.Also use previous result 10(a)