Spring 2021

Answers and Hints for Problem Set - 6

MATHEMATICS-I (MA10001)

1. Find the following limits (if exists)
   (a) Limit does not exist.
   (b) Limit does not exist.
   (c) $5 - 3i$
   (d) Limit does not exist.
2. Test the continuity of the following functions at $z = 0$
   (a) Not continuous at $z = 0$.
   (b) Not continuous at $z = 0$.
   (c) Not continuous at $z = 0$.
3. Test the differentiability of the following functions at $z = 0$.
   (a) Not differentiable at $z = 0$.
   (b) Not differentiable at $z = 0$.
   (c) differentiable at $z = 0$.
4. (a) $\lim_{x\to 0} f(x) = f(0)$.
   (b) The complex derivative $f'(0)$ does not exist.
5. To show $f'(0)$ does not exist, put $y = mx$.
6. (a) Show that $u_{xx} + u_{yy} = 0$. To find its harmonic conjugate use $dv = v_x dx + v_y dy = -u_y + u_x dy$.
   (b) Show that $u_{xx} + u_{yy} = 0$. To find its harmonic conjugate use $dv = v_x dx + v_y dy = -u_y + u_x dy$.
   (c) Show that $u_{xx} + u_{yy} = 0$. To find its harmonic conjugate use $dv = v_x dx + v_y dy = -u_y + u_x dy$.
   (d) Show that $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$. To find its harmonic conjugate use $dv = v_r dr + v_\theta d\theta = -\frac{1}{r}u_\theta dr + r u_r d\theta$.
7. Apply Cauchy-Riemann equations.
8. $\lim_{x\to 0} f(x) = f(0)$ and C-R equations is not satisfied.
9. (a) Use $f'(z) = u_x + iv_x = 0 = v_y - iu_y$.
   (b) Apply Cauchy-Riemann equations on the expression of $f'(z)$.
   (c) Apply Cauchy-Riemann equations on the expression of $f'(z)$.
   (d) Differentiate(partially) $|f(z)| = c$ with respect to x and y and then eliminate u_y.
10. Use harmonic property of u and v.
11. Show that $\operatorname{Re} f(z)$ is not harmonic.
12. (a) Limit is not unique.
    (b) By definition of continuity.
    (c) Apply Cauchy-Riemann equations.
    (d) Apply Cauchy-Riemann equations.
    (e) Use Cauchy-Riemann equations and $\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\big[(u_x - v_y) + i(u_y + v_x)\big]$.