

Indian Institute of Technology Kharagpur
Department of Mathematics
MA11003 - Advanced Calculus
Problem Sheet - 12
Autumn 2020

1. Solve the following differential equations:

(a) $(x^2 D^2 - 3xD + 4)y = 2x^2$,

(b) $(x^2 D^2 + 7xD + 13)y = \log x$

PI

(c) $(x^2 D^2 - 4xD + 6)y = x^4$

(d) $(x^2 D^2 + xD - 1)y = x^m$, $[m \neq \pm 1]$

(e) $(x^2 D^2 - 3xD + 5)y = \sin(\log x)$

PI

(f) $(x^2 D^2 - xD + 1)y = 2 \log x$

PI

(g) $(x^2 D^2 - (2m - 1)x D + (m^2 + n^2))y = n^2 x^m \log x$

(h) $(x^2 D^2 - xD + 2)y = x \log x$

(i) $(x^2 D^2 - 3xD + 5)y = x^2 \sin(\log x)$

(j) $(x^4 D^4 + 6x^3 D^3 + 9x^2 D^2 + 3xD + 1)y = (1 + \log x)^2$

2. Solve the following differential equations:

(a) $(1 + x)^2 y'' - 4(1 + x)y' + 6y = 6(1 + x)$

(b) $(x + 1)^2 y'' + (x + 1)y' = (2x + 3)(2x + 4)$,

(c) $(1 + 2x)^2 y'' - 6(1 + 2x)y' + 16y = 8(1 + 2x)^2$

3. Apply the method of variation of parameters to solve the following differential equations:

(a) $y'' - 2y' = e^x \sin x$

(b) $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$

(c) $y'' - 2y' + y = e^x \log x$

(d) $y''' + y' = \tan x$

(e) $y''' - 2y'' - 21y' - 18y = 3 + 4e^{-t}$

(f) $y'' - 2y' + 2y = e^x \tan x$

4. Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$

with $y(0) = 0$ and $(\frac{dy}{dx})_{x=0} = 0$