Indian Institute of Technology Kharagpur
Department of Mathematics
MA11004 - Linear Algebra, Numerical and Complex Analysis
Problem Sheet - 1 - Hints
Spring 2021

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1. Determine which of the following sets form vector spaces under the given operations:

   (a) Check the properties of vector space from the definition.
   (b) Check associative property.
   (c) Check the properties of vector space from the definition.
   (d) Check closure property.
   (e) Check the usual properties of vector space for the two functions f_1, f_2 with period p_1, p_2 and check the linearity property for $f_1 + f_2$ taking period $p^{'} = lcm(p_1, p_2)$.

2. Determine which of the following subsets are the subspaces of the given vector spaces:

   (a) $\{(a, a + c, c) : a, c \in \mathbb{R}\}$ forms subspace in $\mathbb{R}^3$.
   (b) $(A + B)^T = A^T + B^T$ and $(\alpha A)^T = \alpha A^T$.
   (c) $S = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ forms a subspace in $M_{2\times 2}$.
   (d) Give a counter example to show that $\det(A_1 + A_2) \neq 0$.
   (e) $(a_1, 0, c_1), (0, b_2, c_2) \in S$.
   (f) $(a_1, a_1, c_1), (a_2, a_2, c_2) \in W$.

3. Every subspace contains null vector and for the converse use the linearity of integration.

4. Take $g(x) = \alpha f_1(x) + \beta f_2(x)$ and show that $g^{'}(-1) = 3g(2)$.

5. (a) for $E = \alpha A + \beta B + \gamma C$, find α, β and γ.
   (b) Take $p = \alpha p_1 + \beta p_2 + \gamma p_3$ and find α, β, γ.
   (c) Take each vectors as $\alpha u + \beta v$ and solve.

6. Solve the expression $u_3 = \alpha u_1 + \beta u_2$ for some $\alpha, \beta \in \mathbb{R}$.

7. (a) Find the scalars $\alpha, \beta, \gamma, \delta$ such that $\alpha(v_1 - v_2) + \beta(v_2 - v_3) + \gamma(v_3 - v_4) + \delta v_4 = av_1 + bv_2 + cv_3 + dv_4$ holds.
   (b) Find the scalars α, β, γ such that $\alpha u_1 + \beta(u_1 + u_2) + \gamma(u_1 + u_2 + u_3) = au_1 + bu_2 + cu_3$ holds.

8. (a) Take $c_1(4, -4, 8, 0) + c_2(2, 2, 4, 0) + c_3(6, 0, 0, 2) + c_4(6, 3, -3, 0) = (0, 0, 0, 0)$ and $c_1 = c_2 = c_3 = c_4 = 0$ is the only solution.
   (b) Follow the same step as (a) and then differentiate it with respect to x two times and find the values of scalars from these three equations.
   (c) Follow the same step as (a).
   (d) In each of the intervals $[-1, 0], [0, 1], f_1 = kf_2$ but in the interval $[-1, 1]$ we can not say that.
   (e) Show that $c_1 = c_2 = 0$ is the only solution of $c_1(1+i)+c_2(i-i) = 0$ when $c_1, c_2 \in \mathbb{R}$.Then give an example of $c_1, c_2 \neq 0$ is a solution of the above equation when $c_1, c_2 \in \mathbb{C}$.
   (f) For $x = 2$ we can have non-zero scalars.