Hints and Answers Tutorial Sheet 10

1. (i) 2,2

    (ii) 2,1

    (iii) 2,2

    (iv) 2,2

    (v) 2,3

2. (i) $y'' - 2ky' + k^2 y = 0$

    (ii) $y'' + y'^2 + 1 = 0$

    (iii) $y'^3 - y'(x+1) + y = 0$

    (iv) $(1 + y'^2)^3 = a^2 y''^2$

    (v) $y'(x^2 - y^2) = 2xy$

    (vi) $y'(a^2 - b^2) = (xy' - y)(x + yy')$

3. (i) $xy \cos\left(\frac{y}{x}\right) = -\pi$
    Hint: Rearrange the terms first.

    (ii) $3y^2 = 2x^2 (e^{\frac{1}{x^3}} - e)$

    (iii) $xy = \sin x$
    Hint: Find the Integrating factor

4. (i) $\ln x - \cos\left(\frac{y}{x}\right) = c$
    Hint: Put $y = vx$

    (ii) $x(x^2 + y^2) = cx^2$

    (iii) $(5x - 1)(5y - 7) = (5x - 1)^2 - (5y - 7)^2 + c$
    Hint: Put $x = x' + h, y = y' + k$

5. (i) $y = Ae^{\frac{x^3}{3y^3}}$

    (ii) $\frac{1}{4}(x^4 + y^4) + \frac{3}{2}x^2 y^2 = c$

(iii) $xy(x + 1) + \tan y(1 - x) = c$

(iv) $y = ce^{\frac{y}{x}}$

(v) $yxe^{3x} = e^x + c$

6. (i) $y = x^2(1 + e^{\frac{1}{x}})$

(ii) $y(x + \sqrt{a^2 + x^2}) = a^2 \log_e |x + \sqrt{a^2 + x^2}| + c$

(iii) $\sin y = (1 + x)(e^x + c)$
   Hint: Put $\sin y = t$

(iv) $-\frac{1}{y} = e^{-x}(A + x^2)$
   Hint: Rearrange the terms of the equation

(v) $xy = (y + 1) + Ae^{\frac{1}{y}}$
   Hint: Rearrange the terms

(vi) $\frac{1}{x \log y} = \frac{1}{2x^2} + A$
   Hint: Put $\frac{1}{\log y} = t$

(vii) $\frac{\sec x}{y} = \tan x - A$
   Hint: Put $-\frac{1}{y} = t$

(viii) $y^2(4b^2 + 1) = 2a(2b \cos x + \sin x) + ce^{-2bx}$
   Hint: Put $y^2 = t$

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