Indian Institute of Technology Kharagpur
Department of Mathematics
MA11004 - Linear Algebra, Numeircal and Complex Analysis
Problem Sheet - 5
Spring 2021

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1. Prove that
$$\Delta \log f(x) = \log[1 + \frac{\Delta f(x)}{f(x)}],$$
where Δ is the forward difference operator.

2. For linear interpolation, in case of equispaced tabular data, show that the error does not exceed $1/8^{\text{th}}$ of the second difference.

3. Consider the following tabular values:

x	0	1	2	3
$f(x)$	1	0	7	28

Find the interpolating polynomial using

(i) Newton's Forward Interpolation Formula.

(ii) Newton's Backward Interpolation Formula.

4. From the following table find the number of students who obtained less than 45 marks :

Marks	No. of students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

5. The population of a town in West Bengal was as given below :

Year:	1891	1901	1911	1921	1931
Population (in thousands):	46	66	81	93	101

Estimate the population of the year 1925.

6. Given that $f(1) = 2$, $f(2) = 4$, $f(3) = 8$, $f(4) = 16$, $f(7) = 128$. Find the value of $f(5)$ using the Lagrange's interpolation formula.

7. Using the Lagrange's interpolation formula, express
$$\frac{3x^2 + x + 1}{x^3 - 6x^2 + 11x - 6}$$
as the sum of partial fraction.

8. Using the Lagrange's formula, prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})].$$

Note: $y_i = y(x_i) = f(x_i)$.

9. Compute the value of following integral

$$\int_{0.2}^{1.4} (\sin x - \ln x + \mathrm{e}^x)\mathrm{d}x$$

using trapezoidal rule with 6 equal subintervals.

10. A car laps a racetrack in 84 seconds. The speed of the car at each 6 seconds interval is determined using a radar gun and is given from the beginning of the lap, in feet/second, in the entries in the following table:

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?
Note: use trapezoidal formula

11. How big should the spacing h be so that the computation of

$$\int_0^1 \mathrm{e}^x \,\mathrm{d}x$$

by trapezoidal rule will be correct to five decimal places.

12. Let

$$y = ax^2 + bx + c$$

be the equation of the parabola passing through the points $(-h, y_0), (0, y_1), (h, y_2)$. Find the area underlying the parabola bounded by the x-axis and the two ordinates $x = -h, x = h$ using Simpson's $1/3^{\text{rd}}$ rule. What conclusion do you draw from the result.

13. Calculate the value of the integral

$$\int_4^{5.2} \ln x \,\mathrm{d}x$$

using Simpson's $1/3^{\text{rd}}$ rule with 6 equal subdivisions of the interval.

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