Assignment 2
Linear Algebra, Numerical & Complex analysis (MA11004)
Department of Mathematics, Indian Institute of Technology Kharagpur

Q1. (a) Consider the following system of linear equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Solve the system (carry your calculation keeping results rounded off to 6 decimal places) using Jacobi's iteration method starting from initial guess $(x, y, z) = (0, 0, 0)$. After how many iterations the result comes close to (error tolerance $< 10^{-3}$) the exact solution?

Q1. (b) Use Bisection method to find a root of the equation $x^3 + 4x^2 - 10 = 0$ in the interval $[1, 2]$ correct upto three decimal places.

Q2. (a) Consider the following system of linear equations

$$3x + 7y + 13z = 76$$
$$x + 5y + 3z = 28$$
$$12x + 3y - 5z = 1$$

Perform 3 iterations (correct upto 4 decimal places) using Gauss-Seidel iteration method starting from initial guess $(x, y, z) = (1, 0, 1)$. Do the values tend to converge? Make the system diagonally dominant and carry the iteration keeping result rounded off to 5 decimal places. After how many iterations the result comes close (error tolerance $< 10^{-3}$) to exact solution?

Q2. (b) The equation $x^3 - 2x - 5 = 0$ has a root near $x = 2$. Use Newton-Raphson method to compute the root correct upto three decimal places.

Q3. Let $f(x) = x^4 - x - 10$.

(a) Show that the fixed point iterates $x_{n+1} = g(x_n)$, of $f(x)$ converges to a root α of $f(x)$, with $x_0 = 4$, where $g(x) = (x + 10)^{\frac{1}{4}}$. Also find the root α correct up to four decimal places by using the iterates $x_{n+1} = g(x_n)$.

(b) Let $g(x) = x + \beta f(x)$. Find the possible values (or range) of β such that the fixed point iterates $x_{n+1} = g(x_n)$ converges, where $x_0 = 4$.

Q4. Consider the following data:

x	0	2	4	6	8	10
$f(x)$	-1	3	-2	5	4	-7

(i) Write the Newton forward difference table. Find the Newton forward difference interpolating polynomial and estimate the value of $f(5)$.

(ii) Find the Newton backward difference interpolation polynomial. Do the Newton forward and backward difference interpolating polynomials coincide?

Q5. (a) Using Lagrange's interpolation formula, determine the curve passing through the points $(0,0)$, $(1,1)$ and $(2,20)$.

(b) Using Lagrange's interpolation formula, find $y(9.5)$ from the following table

x:	7	8	9	10
y:	3	1	1	9

Q6. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$. Hence, obtain an approximate value of π.

(b) By dividing the range into ten equal parts, evaluate $\int_0^{\pi} \sin x \, dx$ by Simpson's $\frac{1}{3}$ rd rule.

Q7. a) Examine whether the function $f(z) = (2x^2 + y) + i\,(y^2 - x)$ is analytic at any point.

b) Find out whether the function $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane. If so, find the harmonic conjugate function of u.

Q8. a) Evaluate $\int_C |z|\bar{z}\, dz$, where C consists of the line segment $-1 \le x \le 1$ and C^+, the upper half of the circle $|z| = 1$, positively oriented.

b) Evaluate the integral $\int_0^{1+i} (x - y + ix^2)\, dz$,

i) along the straight line from $z = 0$ to $z = 1 + i$

ii) along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

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