INTRODUCTION TO
COMPUTER SCIENCE
Rutgers University

10. Creating Data Types

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INTRODUCTION TO
COMPUTER SCIENCE
Rutgers University

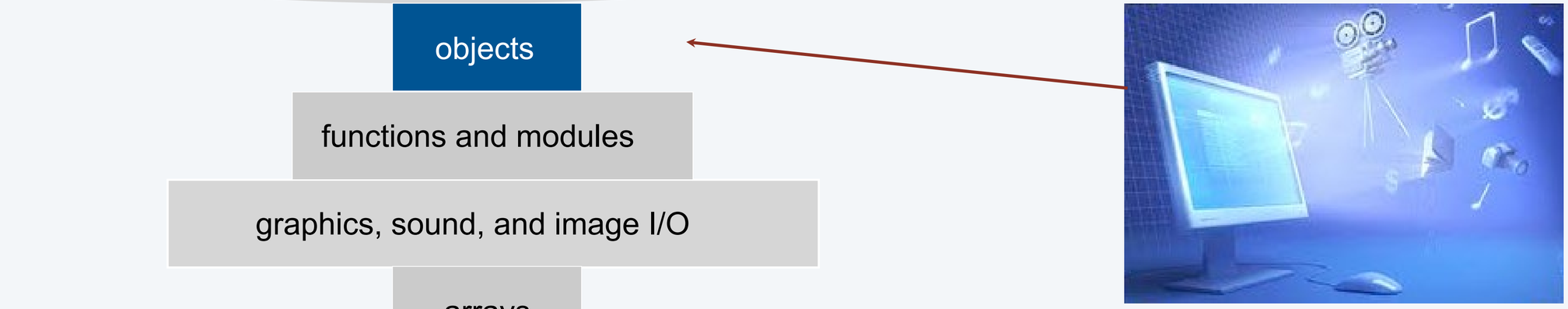
10. Creating Data Types

- Overview
- Point charges (optional)
- Turtle graphics
- Complex numbers (optional)

Adopted and modified from Slides by Sedgewick and Wayne

Basic building blocks for programming

any program you might want to write



Ability to bring life to your own abstractions

Object-oriented programming (OOP)

Object-oriented programming (OOP).

- Create your own data types.
- Use them in your programs (manipulate *objects*).

An object holds a data type value.
Variable names refer to objects.

Examples

data type	*set of values*	*examples of operations*
Color	three 8-bit integers	get red component, brighten
Picture	2D array of colors	get/set color of pixel
String	sequence of characters	length, substring, compare

C	A	T	A	G	C	G	C

An abstract data type is a data type whose representation is *hidden from the client*.

Impact: We can use ADTs without knowing implementation details.

- Previous lecture: how to write client programs for several useful ADTs
- This lecture: how to implement your own ADTs

Implementing a data type

To create a data type, you need to provide code that

- Defines the set of values (instance variables).
- Implements operations on those values (methods).
- Creates and initialize new objects (constructors).

In Java, a data-type implementation is known as a *class*.

Instance variables

- Declarations associate variable names with types.
- Set of type values is "set of values".

Methods

- Like static methods.
- Can refer to instance variables.

Constructors

- Like a method with the same name as the type.
- No return type declaration.
- Invoked by new, returns object of the type.

A Java class

Anatomy of a class


```
public class Point
{
  private double x;
  private double y;

  public Point()
  {
      x =0;
      y = 0;
  }

  public Point(double x, double y)
  {
     this.x =x;
     this.y = y;
  }

public double getX( )
  {
     return x;
  }
public double getY( )
  {
     return y;
  }

// other methods
```

Text file named Point.java

Attributes of a point – instance variables

Constructors

Accessor method – not static

Accessor method – not static

Constructors

- Like a method with the same name as the type.
- No return type declaration.
- Invoked by new, returns object of the type.
- Constructor body is executed when new object is created with new

Designing a class

- Accessor methods provide access to private attributes of a class
- Accessor methods protects the private attributes from unauthorized changes

Example 1: Designing a simple class from given specifications continued..

LO 10.1jk

```
public class Point
{
   private double x;
   private double y;

   public Point()
   {  … }
   public Point(double x, double y)
   { ….}
   public double getX( )
   { … }
  public double getY( )
   {  … }

  // other methods
  public double findSlope(Point other) {
    return ((y - other.getY())/(x - other.getX()));
 }

  public Point findMidPoint (Point other){
        double midX = (x + other.getX())/2;
        double midY = (y + other.getY())/2;
        return new Point(midX, midY);
}
```

object as a parameter

object as a return value

Objects as parameters and return types

- Objects can be passed as parameters to a function.
- Objects can be returned from functions.

Pre and post conditions

LO 10.1o

```
public class Point
{
   ….

  // other methods
  public double findSlope(Point other) {
       //pre-condition. asert( other instanceof Point)
      double slope = ((y - other.getY())/(x - other.getX()));
      // post-condition. none
     return slope;
}

  public Point findMidPoint (Point other){
        //pre-condition. asert( other instanceof Point)
        double midX = (x + other.getX())/2;
        double midY = (y + other.getY())/2;
        Point midPoint = new Point(midX, midY);
        //post condition  aseert(midPoint instanceof Point)
        return
}
```

Pre and post conditions

- Pre-condition. What is known about function state at the beginning-*what information does the function need to complete its task?*
- Post-condition. What is known about function state at end (if all preconditions are met)

Comparing objects

```
public class Point
{
    ….
    public boolean equals(Object obj)  {
       if (obj instanceof Point) {
          Point p = (Point) obj;
          return (this.x = p.x && this.y == p.y);
       }
       else
          return false;
}
```

test to assure obj is indeed a Point

Definition for Point equality

Comparing objects

- Define. What it means for objects of the same type to be equal.
- Warning. Do not compare objects to objects (as they only compare the references)
- *E.g. If (point1 == point2) ….*

only compares references

Displaying objects - toString method

```
public class Point
{
     ….

//       overrides Object toString
public String toString() {
                return (   "("+ this.x + ", " + this.y + ")" );
}
```

System.out.println(p) will print

(3.0, 4.0)

Without a toString method, System.out.println(p) will print

Point@38cccef

getClass().getName() + '@' +
Integer.toHexString(hashCode())

returns a string consisting of the name of the class of which the object is an instance, the at-sign character `@', and the unsigned hexadecimal representation of the hash code of the object.

PointTester.java. *// Client Program*

```
public class PointTester {

 public static void main ( String[] args ) {
      Point p1 = new Point(3,4);
      Point p2 = new Point();
      Point p3 = new Point (3,4);
      System.out.println("p1 = " + p1);
      System.out.println("p2 = " + p2);
      System.out.println("p3 = " + p3);

           Point midpoint = p1.findMidPoint ( p2 ) ;
           System.out.println ( midpoint + " is midpoint of " + p1 + " and " + p2 );

           System.out.println ( p1.findDistanceFromOrigin() + " is the
              distance between the origin and " + p1 );

           System.out.println ( p1.findDistanceBetween(p2) + " is the distance
              between " + p1 + " and " + p2 );

           System.out.println(p2.findSlope (p1) + " is the slope
              between " + p2 + " and " + p1);
           System.out.println ( p1.equals ( p3) );
           System.out.println ( p1 == p3 );
   }
}
```

Example 2. Designing a simple class from given specifications


```
public class Student
{
   private String lastName;
   private String firstName;
   private double gpa;
    private int studentID;

   public Student()
   {
      lastName = “Doe”;
      firstName = “John”;
   }

public Student(String lastName, String firstName)
   {
      this.lastName = lastName;
      this.firstName = firstName;
   }
```

Attributes of a student (as instance variables)

Default constructor????

overloaded constructor

refer to method parameters

refer to instance variables

Designing a class

- Identify class attributes.
- Design class object constructors
- Design behavior of objects (class methods)

Example 3. Designing a simple class from given specifications (FRACTION)

LO 10.1c

```
public class Fraction
{
   private int numerator;
   private int denominator

 public Fraction(int n, int d)
  {
     this.numerator = n;
     this.denominator = d;
  }

public int getNumerator ( )
  {
     return this.numerator;
  }
public int getDenominator( )
  {
     return this.denominator;
  }
```

Attributes of a point

Accessor method

Accessor method

// What other methods?

Designing a class

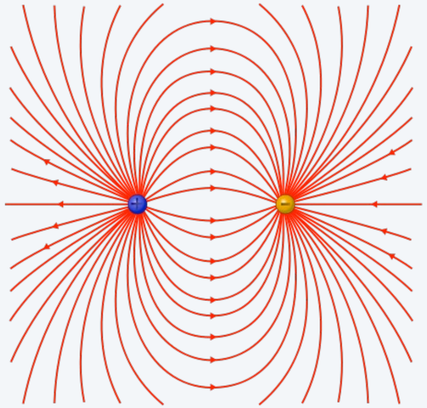
- Accessor methods provide access to private attributes of a class
- Accessor methods protects the private attributes from unauthorized changes

9. Creating Data Types

- Overview
- Point charges (optional)
- Turtle graphics
- Complex numbers (optional)

ADT for point charges

A point charge is an idealized model of a particle that has an electric charge.



An ADT allows us to write Java programs that manipulate point charges.

Values

	examples	
position (x, y)	(.53, .63)	(.13, .94)
electrical charge	20.1	81.9

API (operations)

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	*electric potential at (x, y) due to charge*
String toString()	*string representation of this charge*

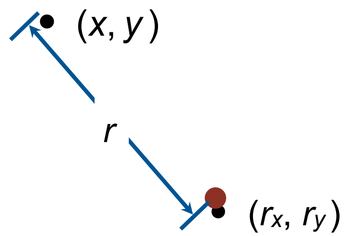
Crash course on electric potential

Electric potential is a measure of the effect of a point charge on its surroundings.

- It increases in proportion to the charge value.
- It decreases in proportion to the *inverse of the distance* from the charge (2D).

Mathematically,

- Suppose a point charge c is located at (r_x, r_y) and has charge q.
- Let r be the distance between (x, y) and (r_x, r_y)
- Let $V_c(x,y)$ be the potential at (x, y) due to c.
- Then $V_c(x, y) = k\frac{q}{r}$ where $k = 8.99\times10^9$ is a normalizing factor.



Q. What happens when multiple charges are present?

A. The potential at a point is the *sum* of the potentials due to the individual charges.

Note: Similar laws hold in many other situations.

← Example. *N*-body (3D) is an inverse *square* law.

Point charge implementation: Test client

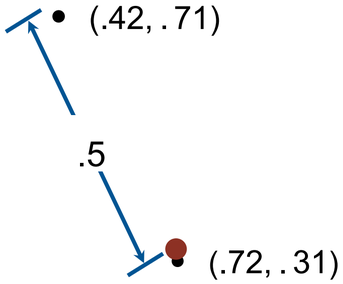
Best practice. Begin by implementing a simple test client.


```
public static void main(String[] args)
{
   Charge c = new Charge(.72, .31, 20.1);
   StdOut.println(c);
   StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
}
```

$$V_c(x, y) = k\frac{q}{r}$$

$$r = \sqrt{(r_x - x)^2 + (r_y - y)^2}$$
$$= \sqrt{.3^2 + .4^2} = .5$$

$$V_c(.42, .71) = 8.99 \times 10^9 \frac{20.1}{.5}$$
$$= 3.6 \times 10^{11}$$




```
% java Charge
20.1 at (0.72, 0.31)
3.61e+11
```

← What we *expect*, once the implementation is done.

Point charge implémentation: Instance variables

Instance variables define data-type values.

Values

	examples	
position (x, y)	(.53, .63)	(.13, .94)
electrical charge	20.1	81.9

```
public class Charge
{
   private final double rx, ry;
   private final double q;
...
}
```

Modifiers control access.

- private denies clients access and therefore makes data type abstract.
- final disallows any change in value and documents that data type is *immutable*.

Key to OOP. Each *object* has instance-variable values.

Point charge implementation: Constructor

Constructors create and initialize new objects.

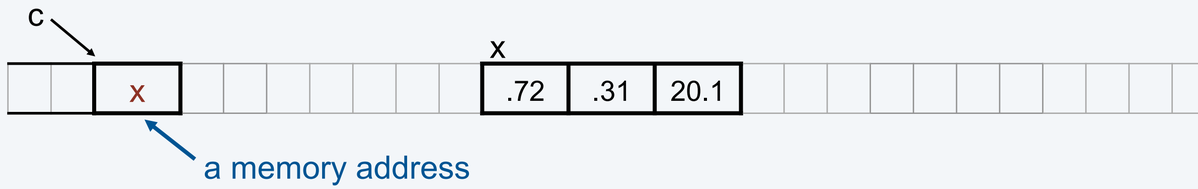
```
public class Charge
{
...
  public Charge(double x0, double y0, double q0)
  {
     rx = x0;
     ry = y0;
     q  = q0;
  }
```

Clients use new to invoke constructors.

- Pass arguments as in a method call.
- Return value is reference to new object.

Possible memory representation of

Charge c = new Charge(.72, .31, 20.1);



Point charge implementation: Methods

public class Charge	
Charge(double x0, double y0, double q0)	
double potentialAt(double x, double y)	*electric potential at (x, y) due to charge*
String toString()	*string representation of this charge*

Methods define data-type operations (implement APIs).


```
public class Charge
{
  ...
   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }
   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }
```

$$V_c(x, y) = k\frac{q}{r}$$

Key to OOP. An instance variable reference in an instance method *refers to the value for the object that was used to invoke the method*.

Point charge implementation

text file named Charge.java

```
public class Charge
{
   private final double rx, ry;   // position
   private final double q;        // charge value

   public Charge(double x0, double y0, double q0)
   {
      rx = x0;
      ry = y0;
      q  = q0;
   }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx;
      double dy = y - ry;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }
   public String toString()
   {  return q + " at " + "(" + rx + ", " + ry + ")";  }

   public static void main(String[] args)
   {
      Charge c = new Charge(.72, .31, 20.1);
      StdOut.println(c);
      StdOut.printf("%6.2e\n", c.potentialAt(.42, .71));
   }
}
```

instance variables

constructor

methods

test client

```
% java Charge
20.1 at (0.72, 0.31)
3.61e+11
```

Point charge client: Potential visualization (helper methods)

Read point charges from StdIn.

- Uses Charge like any other type.
- Returns an array of Charges.

```
public static Charge[] readCharges()
{
   int N = StdIn.readInt();
   Charge[] a = new Charge[N];
   for (int i = 0; i < N; i++)
   {
      double x0 = StdIn.readDouble();
      double y0 = StdIn.readDouble();
      double q0 = StdIn.readDouble();
      a[i] = new Charge(x0, y0, q0);
   }
   return a;
}
```

Convert potential values to a color.

- Convert V to an 8-bit integer.
- Use grayscale.

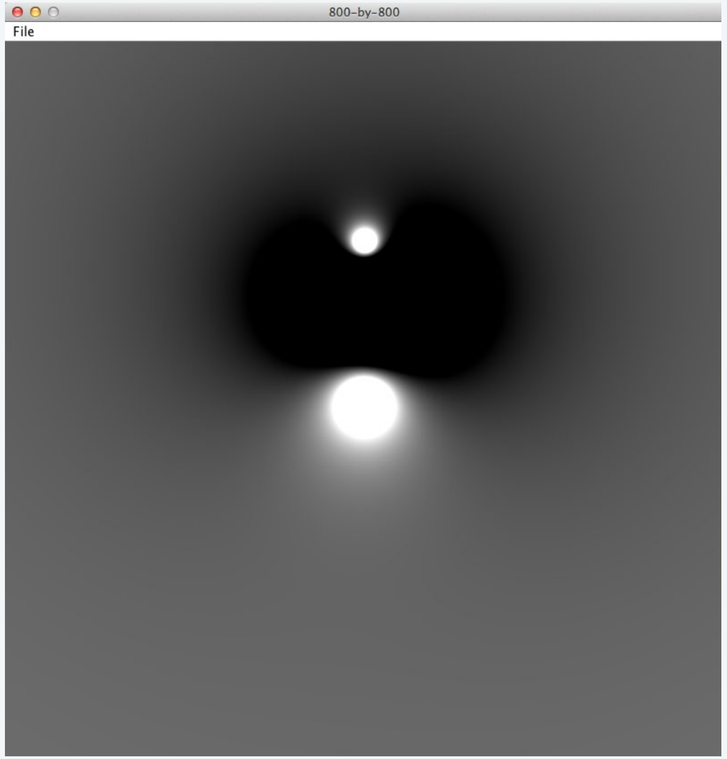
```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   return new Color(t, t, t);
}
```

V											
t	0	1	...	37	38	39	...	128	...	254	255

Point charge client: Potential visualization

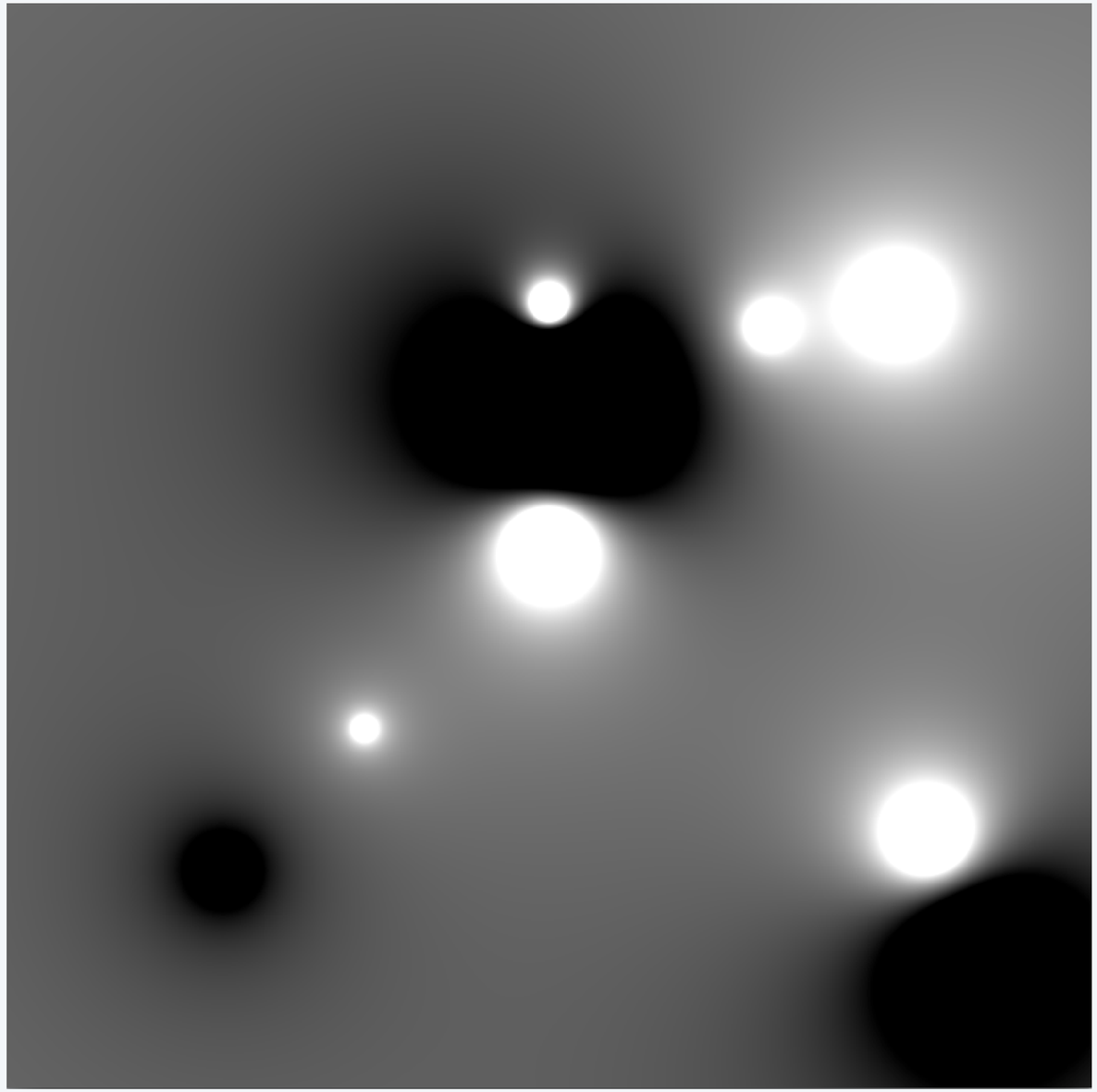
```
import java.awt.Color;

public class Potential

{

   public static Charge[] readCharges()

   { // See previous slide. }

   public static Color toColor()

   { // See previous slide. }

   public static void main(String[] args)

   {

      Charge[] a = readCharges();

      int SIZE = 800;

      Picture pic = new Picture(SIZE, SIZE);
         for (int col = 0; col < SIZE; col++)
            for (int row = 0; row < SIZE; row++)
            {
               double V = 0.0;
      for (int k = 0; k < a.length; k++)
               {
                  double x = 1.0 * col / SIZE;
                  double y = 1.0 * row / SIZE;
                  V += a[k].potentialAt(x, y);
               }
               pic.set(col, SIZE-1-row, toColor(V));
            }
         pic.show();
      }
   }
```

```
% more charges3.txt
3
.51 .63 -100
.50 .50   40
.50 .72   20
% java Potential < charges3.txt
```



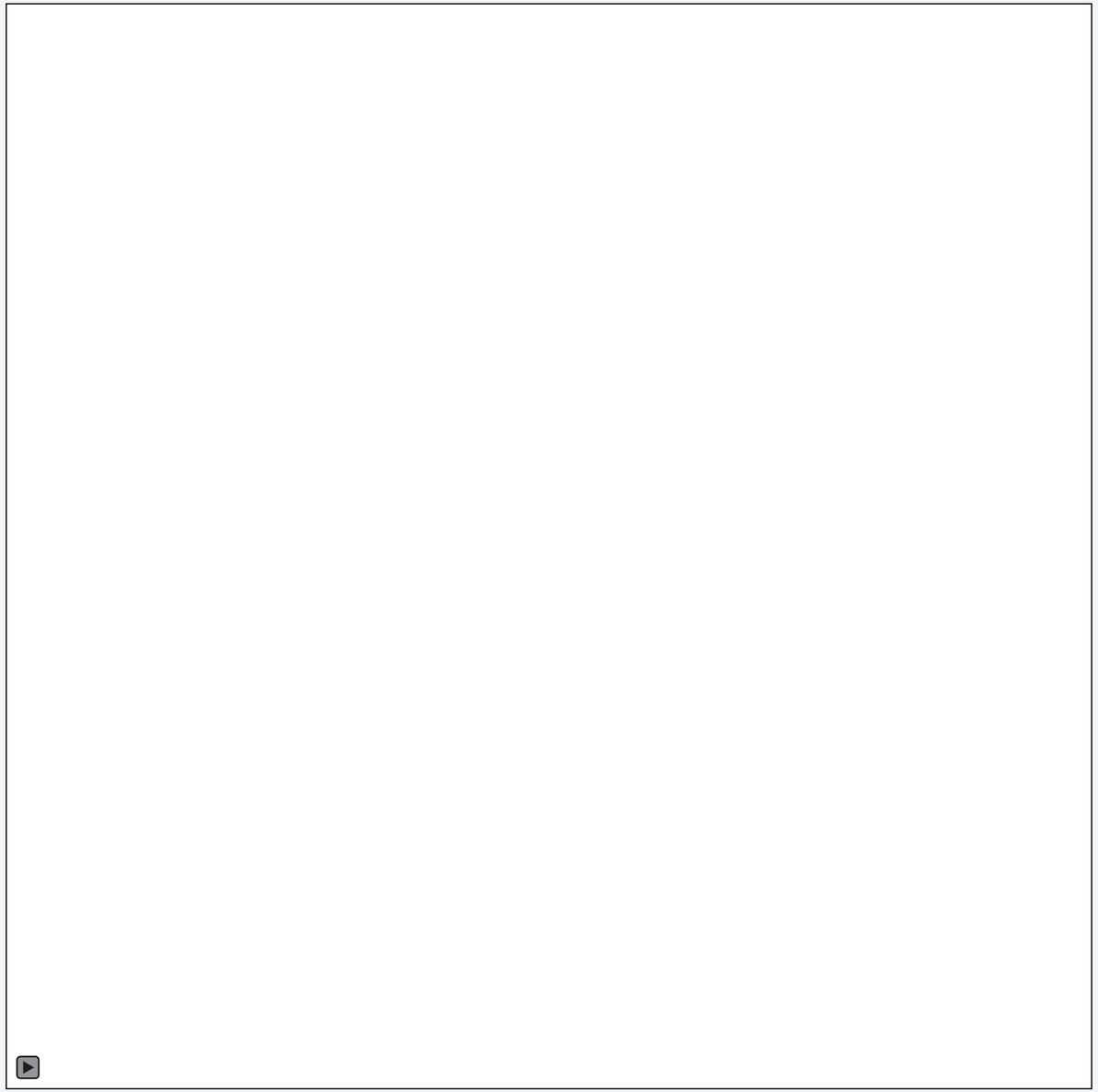
Potential visualization I

```
% more charges9.txt
9
.51 .63 -100
.50 .50   40
.50 .72   20
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50

% java Potential < charges9.txt
```



Potential visualization II: A moving charge

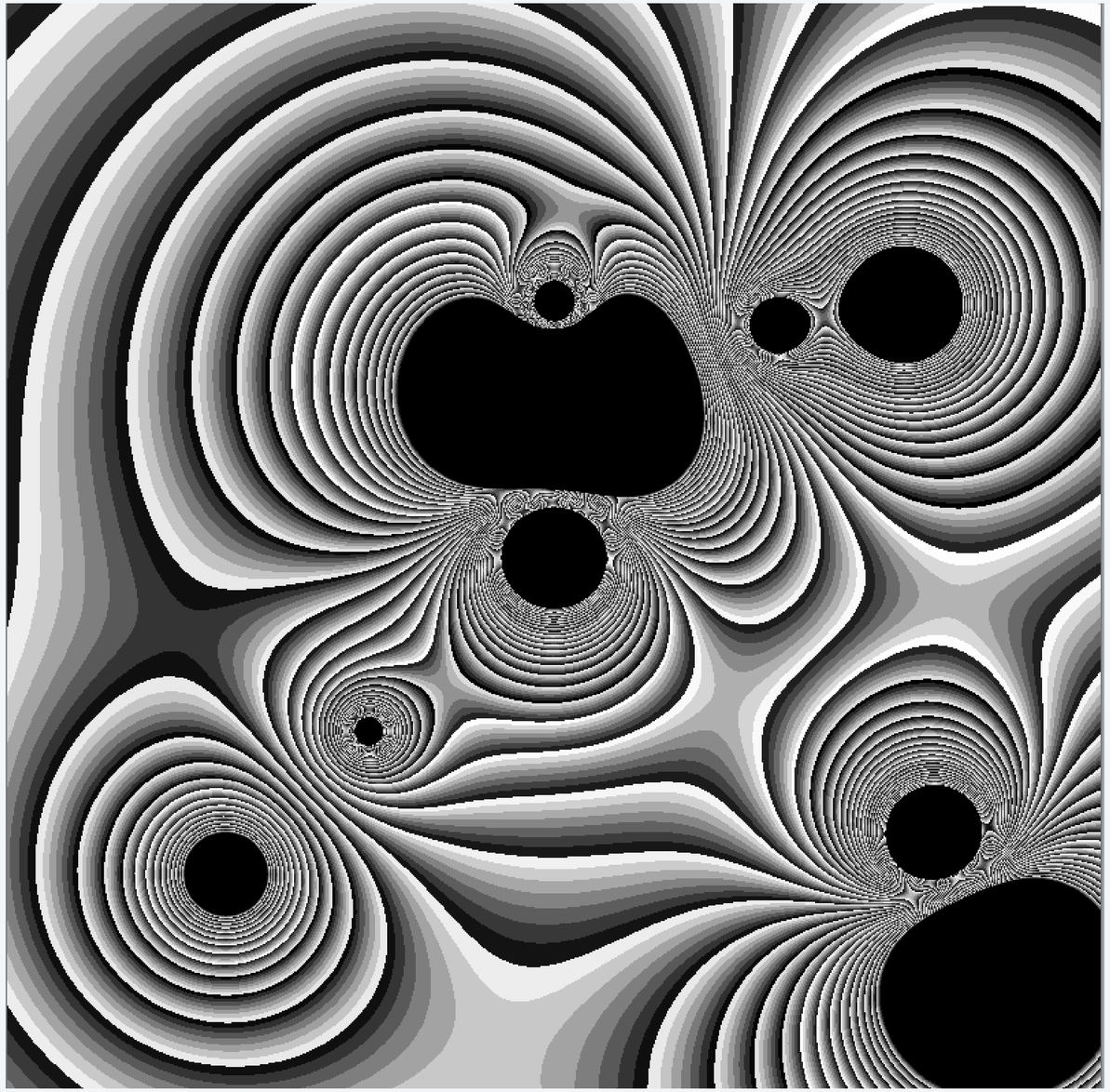
```
% more charges9.txt
9
.51 .63 -100
.50 .50   40
.50 .72   20
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50

% java PotentialWithMovingCharge < charges9.txt
```



Potential visualization III: Discontinuous color map

```
public static Color toColor(double V)
{
   V = 128 + V / 2.0e10;
   int t = 0;
   if (V > 255) t = 255;
   else if (V >= 0) t = (int) V;
   t = t*37 % 255
   return new Color(t, t, t);
}
```

V											
t	0	1	2	3	4	5	6	7	8	9	...



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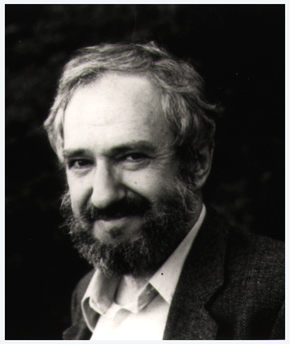
- Overview
- Point charges (optional)
- Turtle graphics
- Complex numbers (optional)

ADT for turtle graphics

A turtle is an idealized model of a plotting device.

An ADT allows us to write Java programs that manipulate turtles.



Seymour Papert
1928–

Values

position (*x*, *y*)	(.5, .5)	(.25, .75)	(.22, .12)
orientation	90°	135°	10°
			

API (operations)

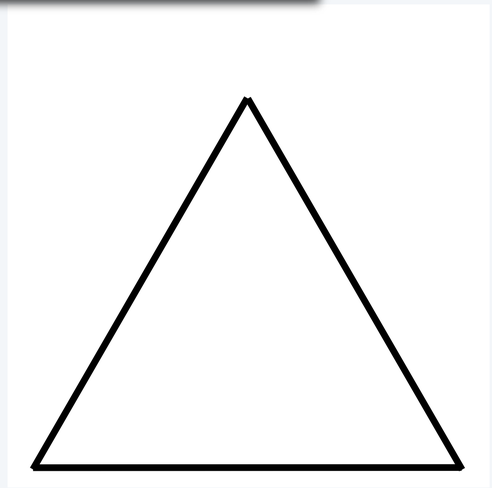
public class Turtle	
Turtle(double x0, double y0, double q0)	
void turnLeft(double delta)	*rotate* delta *degrees counterclockwise*
void goForward(double step)	*move distance* step*, drawing a line*

Turtle graphics implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
   Turtle turtle = new Turtle(0.0, 0.0, 0.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
   turtle.goForward(1.0);
   turtle.turnLeft(120.0);
}
```


```
% java Turtle
```



Note: Client drew triangle without computing √3

What we *expect*, once the implementation is done.

Turtle implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.


```
public class Turtle
{
   private double x, y;
   private double angle;
   public Turtle(double x0, double y0, double a0)
  {
     x = x0;
     y = y0;
     angle = a0;
  }
...
}
```

instance variables are *not* final

Values

position (*x*, *y*)	(.5, .5)	(.75, .75)	(.22, .12)
orientation	90°	135°	10°

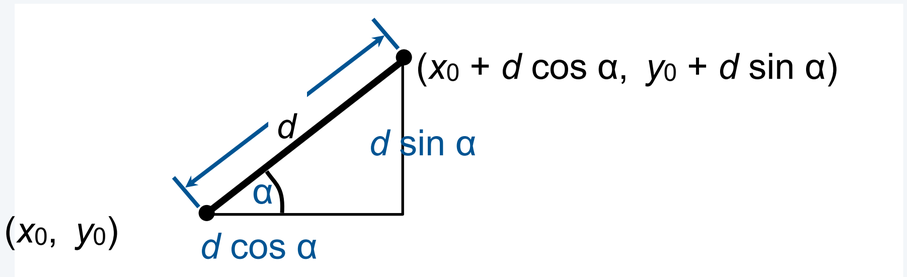
Turtle implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Turtle
{
 ...
   public void turnLeft(double delta)
   { angle += delta; }
   public void goForward(double d)
   {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
 ...
```

API

public class Turtle	
Turtle(double x0, double y0, double q0)	
void turnLeft(double delta)	
void goForward(double step)	



Turtle implementation

Write a client program that will create and use objects of a class that you wrote.
.. Write client programs that test all methods of your newly created data type.

LO 10.1mn

text file named Turtle.java

```
public class Turtle
{
  private double x, y;
  private double angle;

  public Turtle(double x0, double y0, double a0)
  {
    x = x0;
    y = y0;
    angle = a0;
  }

  public void turnLeft(double delta)
  { angle += delta; }
  public void goForward(double d)
  {
    double oldx = x;
    double oldy = y;
    x += d * Math.cos(Math.toRadians(angle));
    y += d * Math.sin(Math.toRadians(angle));
    StdDraw.line(oldx, oldy, x, y);

  }
  public static void main(String[] args)
  {
   Turtle turtle = new Turtle(0.0, 0.0, 0.0);
   turtle.goForward(1.0); turtle.turnLeft(120.0);
   turtle.goForward(1.0); turtle.turnLeft(120.0);
   turtle.goForward(1.0); turtle.turnLeft(120.0);
  }
```

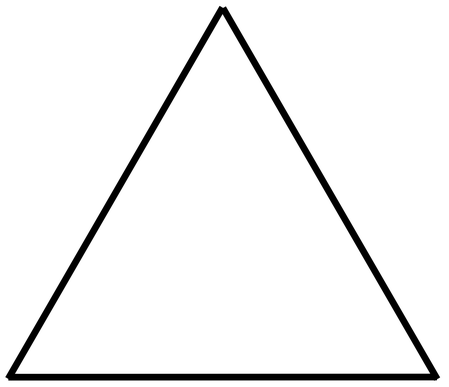
instance variables

constructor

methods

test client

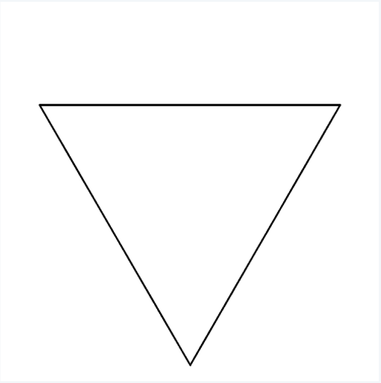
```
% java Turtle
```



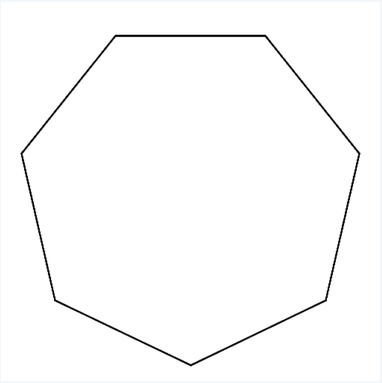
Turtle client: N-gon

```
public class Ngon
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++)
      {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```

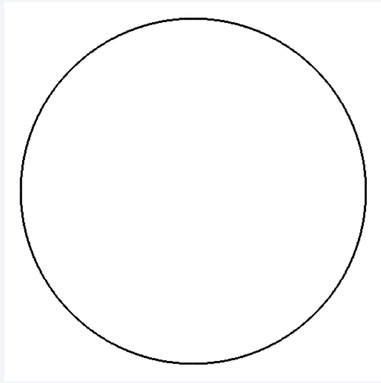
% java Ngon 3



% java Ngon 7



% java Ngon 1440

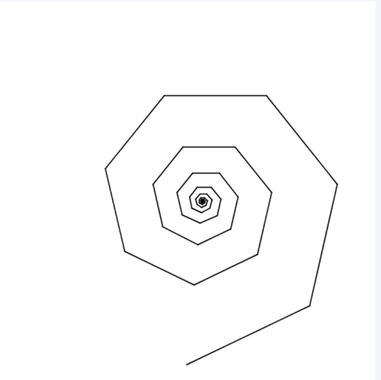


Turtle client: Spira Mirabilis

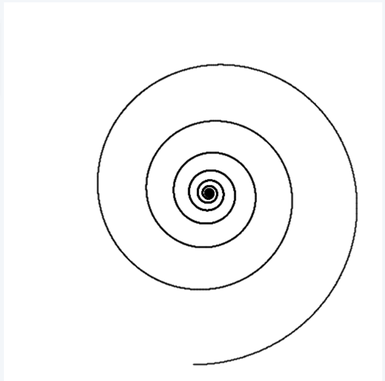
```
public class Spiral
{
   public static void main(String[] args)
   {
      int N        = Integer.parseInt(args[0]);
      double decay = Double.parseDouble(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++)
      {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
```

% java Spiral 3 1.2

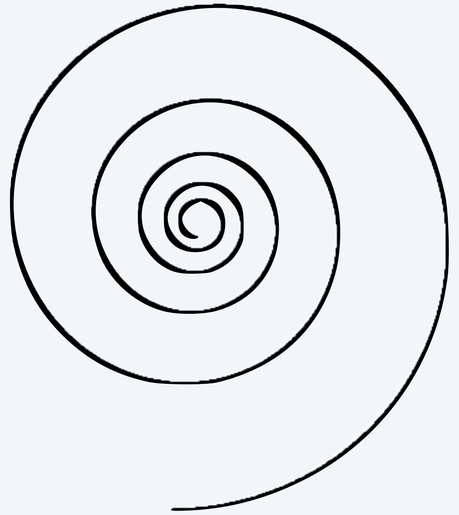
% java Spiral 7 1.2

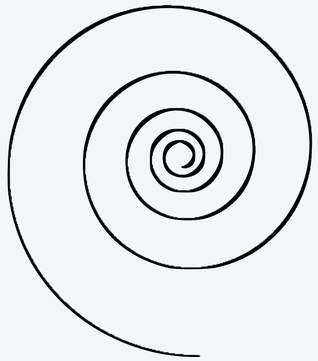


% java Spiral 1440 1.0004



Spira Mirabilis in the wild





Pop quiz 1 on OOP

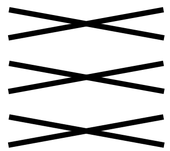
Q. Fix the serious bug in this code:

```
public class Turtle
{
   private double x, y;
   private double angle;
   public Turtle(double x0, double y0, double a0)
  {
     double x = x0;
     double y = y0;
     double angle = a0;
  }
...
}
```

Pop quiz 1 on OOP

Q. Fix the serious bug in this code:

```
public class Turtle
{
   private double x, y;
   private double angle;
   public Turtle(double x0, double y0, double a0)
  {
     double x = x0;
     double y = y0;
     double angle = a0;
  }
...
}
```



A. Remove type declarations.

They create local variables, which are *different* from the instance variables!

Object-oriented programmers pledge. "I *will not* shadow instance variables"

Every programmer makes this mistake, and it is a difficult one to detect.

9. Creating Data Types

- Overview
- Point charges (optional)
- Turtle graphics
- Complex numbers (optional)

Crash course in complex numbers

A complex number is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

Complex numbers are a *quintessential mathematical abstraction* that have been used for centuries to give insight into real-world problems not easily addressed otherwise.

Leonhard Euler
1707−1783

A. L. Cauchy
1789−1857

To perform *algebraic operations* on complex numbers, use real algebra, replace i^2 by −1 and collect terms.

- Addition example: $(3 + 4i) + (-2 + 3i) = 1 + 7i$.
- Multiplication example: $(3 + 4i) \times (-2 + 3i) = -18 + i$.

Example: $|3 + 4i| = 5$

The *magnitude* or *absolute value* of a complex number $a + bi$ is $|a + bi| = \sqrt{a^2 + b^2}$.

Applications: Signal processing, control theory, quantum mechanics, analysis of algorithms...

ADT for complex numbers

A complex number is a number of the form $a + bi$ where a and b are real and $i \equiv \sqrt{-1}$.

An ADT allows us to write Java programs that manipulate complex numbers.

Values

complex number	3 + 4*i*	−2 + 2*i*
real part	3.0	−2.0
imaginary part	4.0	2.0

API (operations)

public class Complex	
Complex(double real, double imag)	
Complex plus(Complex b)	*sum of this number and* b
Complex times(Complex b)	*product of this number and* b
double abs()	*magnitude*
String toString()	*string representation*

Complex number data type implementation: Test client

Best practice. Begin by implementing a simple test client.

```
public static void main(String[] args)
{
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("a * b = " + a.times(b));
}
```


```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

What we *expect*, once the implementation is done.

Complex number data type implementation: Instance variables and constructor

Instance variables define data-type values.

Constructors create and initialize new objects.


```
public class Complex
{
   private final double re;
   private final double im;
   public Complex(double real, double imag)
  {
     re = real;
     im = imag;
  }
...
}
```

instance variables *are* final

Values

complex number	3 + 4*i*	−2 + 2*i*
real part	3.0	−2.0
imaginary part	4.0	2.0

Complex number data type implementation: Methods

Methods define data-type operations (implement APIs).

```
public class Complex
{
 ...
   public Complex plus(Complex b){
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }
   public Complex times(Complex b){
      double real = re * b.re - im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }
   public double abs()  {  return Math.sqrt(re*re + im*im);  }
   public String toString()
   {  return re + " + " + im + "i";  }
 ...
}
```

might also write "this.re"
or use Complex a = this

Java keyword "this" is a reference to "this object" and is implicit when an instance variable is directly referenced

$$a = v + wi$$
$$b = x + yi$$
$$a \times b = vx + vyi + wxi + wyi^2$$
$$= vx - wy + (vy + wx)i$$

API

public class Complex	
Complex(double real, double imag)	
Complex plus(Complex b)	*sum of this number and* b
Complex times(Complex b)	*product of this number and* b
double abs()	*magnitude*
String toString()	*string representation*

Complex number data type implementation

text file named Complex.java

```
public class Complex{
   private final double re;
   private final double im;

   public Complex(double real, double imag) {
      re = real; im = imag;
   }
   public Complex plus(Complex b){
      double real = re + b.re;
      double imag = im + b.im;
      return new Complex(real, imag);
   }
   public Complex times(Complex b){
      double real = re * b.re - im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }
   public double abs()
   {  return Math.sqrt(re*re + im*im);  }

   public String toString()
  {  return re + " + " + im + "i";  }
   public static void main(String[] args){
      Complex a = new Complex( 3.0, 4.0);
      Complex b = new Complex(-2.0, 3.0);
      StdOut.println("a = " + a);
      StdOut.println("b = " + b);
      StdOut.println("a * b = " + a.times(b));
   }
}
```

instance variables

constructor

methods

test client

```
% java Complex
a = 3.0 + 4.0i
b = -2.0 + 3.0i
a * b = -18.0 + 1.0i
```

The Mandelbrot set

The *Mandelbrot set* is a set of complex numbers.

- Represent each complex number $x + yi$ by a point (x, y) in the plane.
- If a point is *in* the set, we color it BLACK.
- If a point is *not* in the set, we color it WHITE.

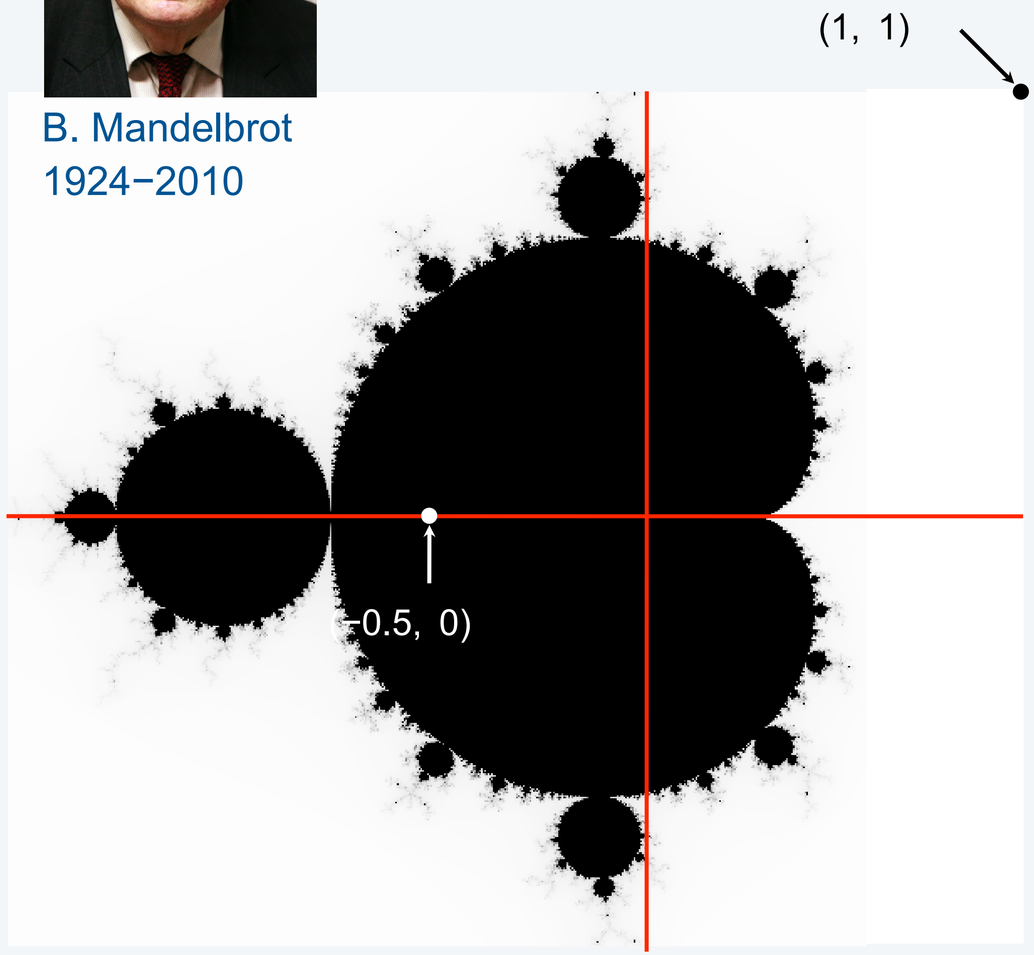
Examples

- *In* the set: $-0.5 + 0i$.
- *Not* in the set: $1 + i$.

Challenge

- No simple formula exists for testing whether a number is in the set.
- Instead, the set is defined by an *algorithm*.

B. Mandelbrot
1924−2010



Determining whether a point is in the Mandelbrot set

Is a complex number z_0 in the set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ *diverges to infinity*, z_0 is *not in* the set.
- If not, z_0 is *in* the set.

t	z_t
0	$-1/2 + 0i$
1	$-1/4 + 0i$
2	$-7/16 + 0i$
3	$-79/256 + 0i$
4	$-26527/65536 + 0i$

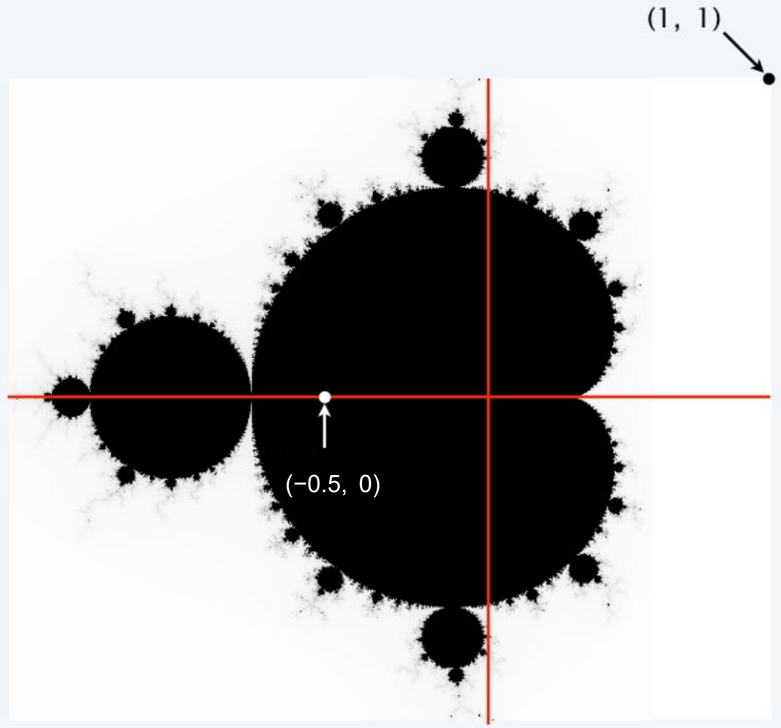
always between −1/2 and 0

$z = -1/2 + 0i$ is *in* the set

t	z_t
0	$1 + i$
1	$1 + 3i$
2	$-7 + 7i$
3	$1 - 97i$
4	$-9407 - 193i$

diverges to infinity

$z = 1 + i$ is *not* in the set



$(1+i)^2 + (1+i) = 1 + 2i + i^2 + 1 + i = 1+3i$

$(1+3i)^2 + (1+i) = 1 + 6i + 9i^2 + 1 + i = -7+7i$

Plotting the Mandelbrot set

Practical issues

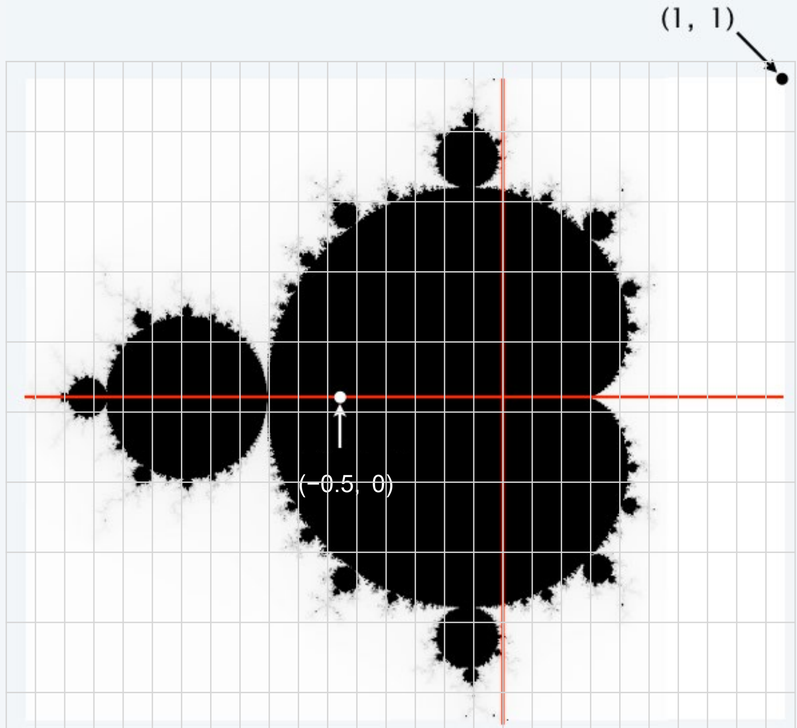
- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.

Approximate solution for first issue

- Sample from an *N*-by-*N* grid of points in the plane.
- Zoom in to see more detail (stay tuned!).

Approximate solution for second issue

- Fact: if $| z_t | > 2$ for any t, then z *is not* in the set.
- Pseudo-fact: if $| z_{255} | \leq 2$ then z is "likely" in the set.



Important note: Solutions imply significant computation.

Complex number client: Mandelbrot set visualization (helper method)

Mandelbrot function of a complex number.

- Returns WHITE if the number is not in the set.
- Returns BLACK if the number is (probably) in the set.

```
public static Color mand(Complex z0)
{
   Complex z = z0;
   for (int t = 0; t < 255; t++)
   {
      if (z.abs() > 2.0) return Color.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return Color.BLACK;
}
```

For a more dramatic picture,
return new Color(255-t, 255-t, 255-t)
or colors picked from a color table.

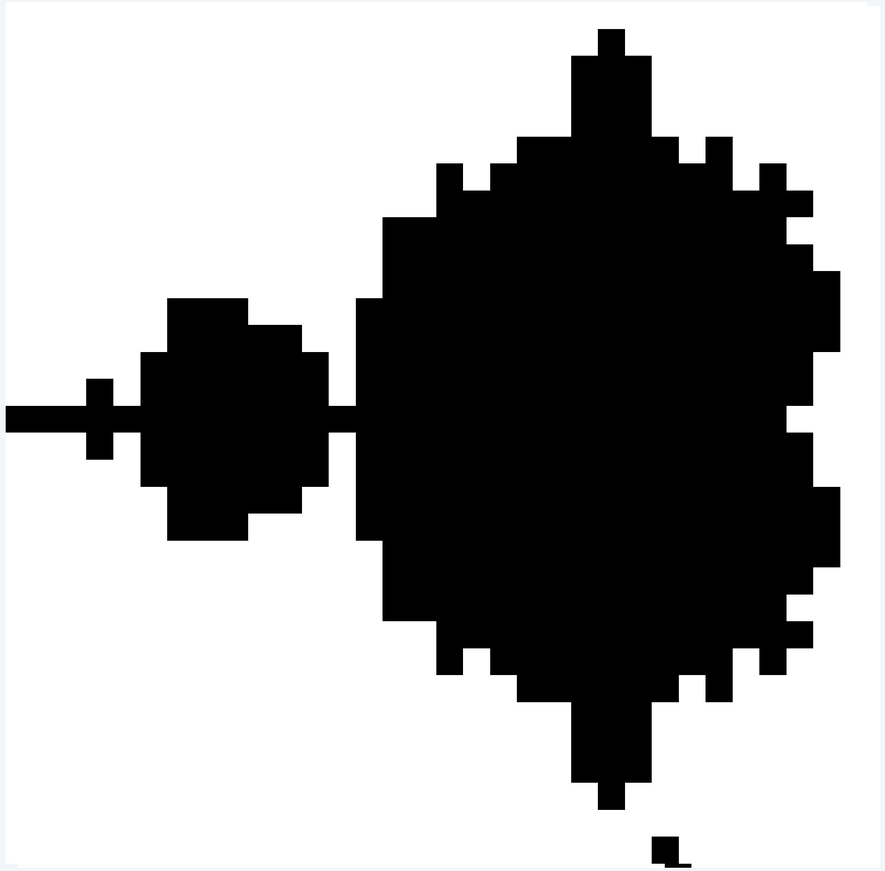
Complex number client: Mandelbrot set visualization

```
import java.awt.Color;

public class Mandelbrot{
   public static Color mand(Complex z0)
   { // See previous slide. }
   public static void main(String[] args) {
      double xc   = Double.parseDouble(args[0]);
      double yc   = Double.parseDouble(args[1]);
      double size = Double.parseDouble(args[2]);
      int N   = Integer.parseInt(args[3]);
      Picture pic = new Picture(N, N);
      for (int col = 0; col < N; col++)
         for (int row = 0; row < N; row++) {
            double x0 = xc - size/2 + size*col/N;
            double y0 = yc - size/2 + size*row/N;
            Complex z0 = new Complex(x0, y0);
            Color color = mand(z0);
            pic.set(col, N-1-row, color);
         }
      pic.show();
   }
}
```

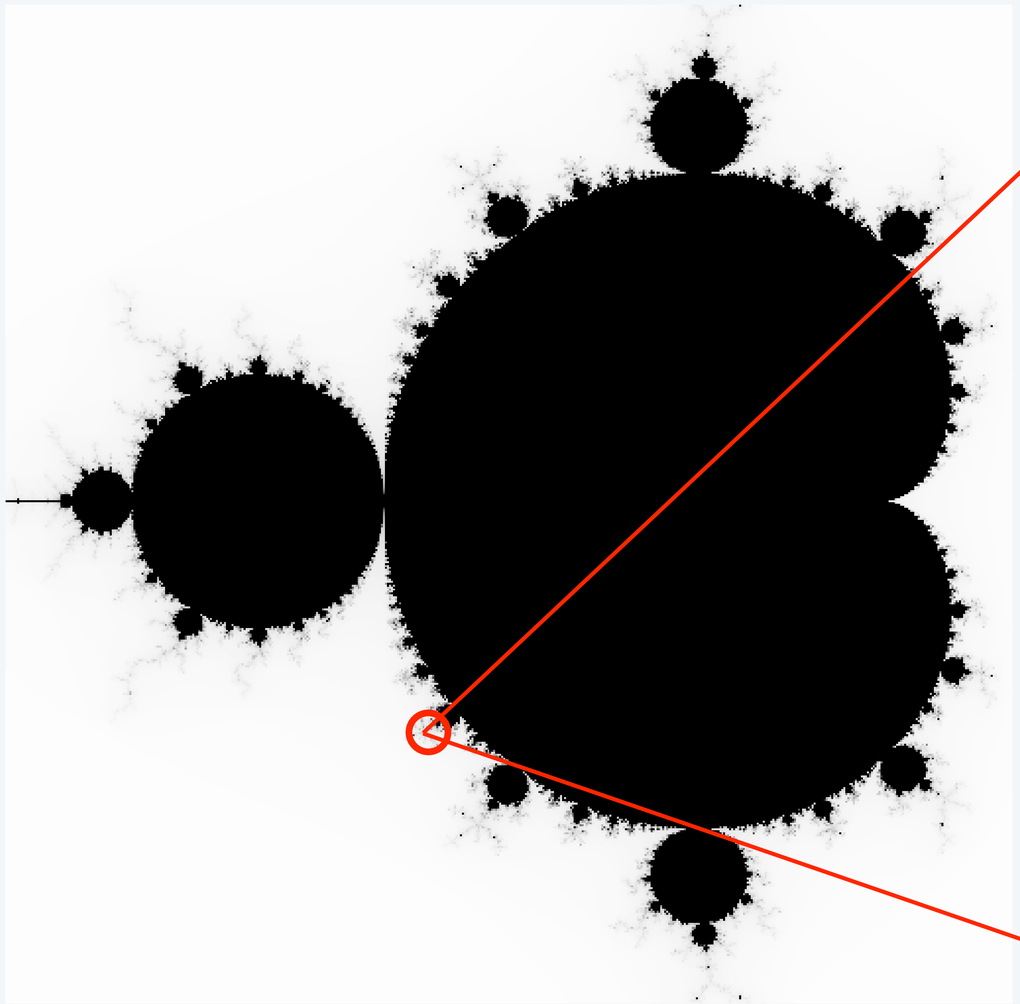
```
% java Mandelbrot –.5 0 2 32
```

```
% java Mandelbrot –.5 0 2 64
```

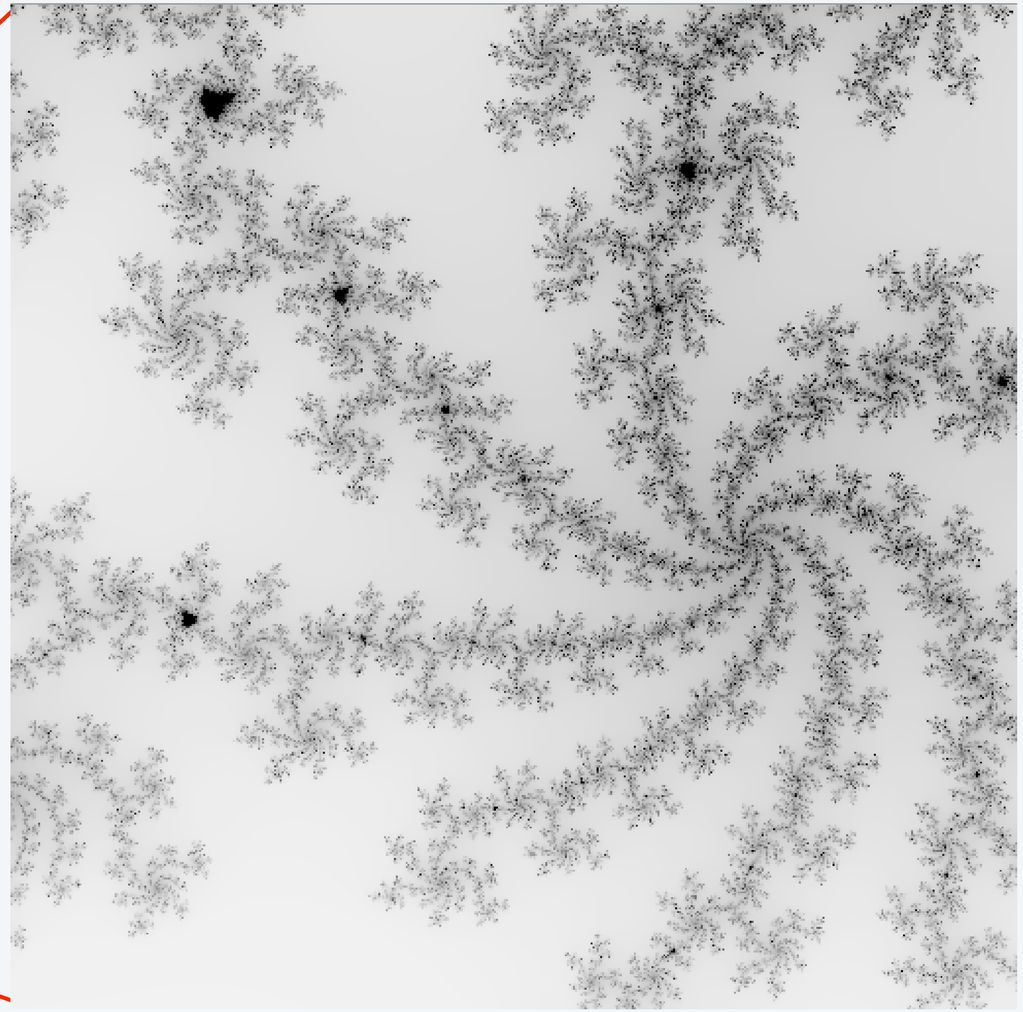


Mandelbrot Set

% java GrayscaleMandelbrot –.5 0 2



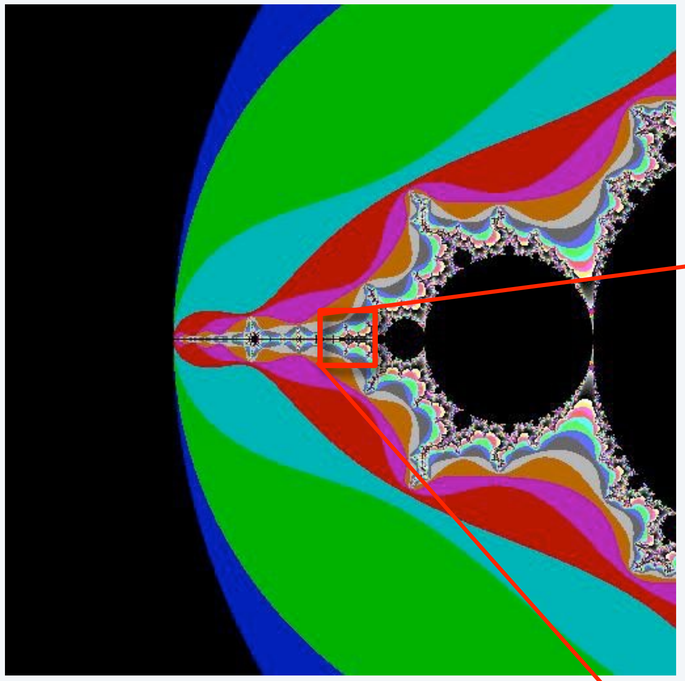
% java GrayscaleMandelbrot .1045 -.637 .01



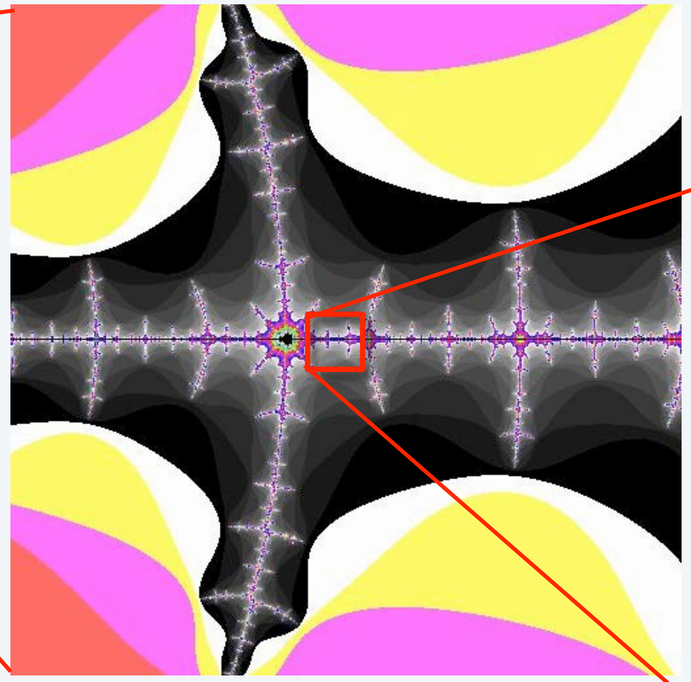
Mandelbrot Set

```
% java ColorMandelbrot –.5 0 2 < mandel.txt
```

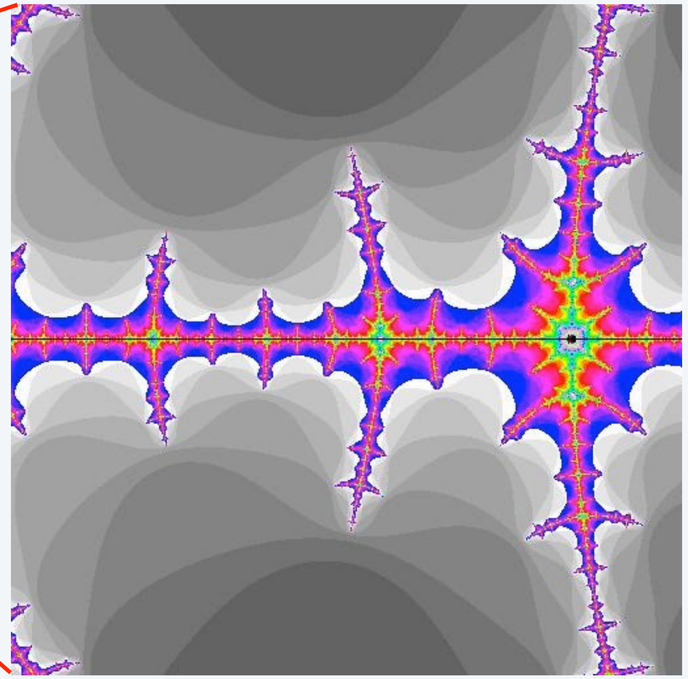
color map

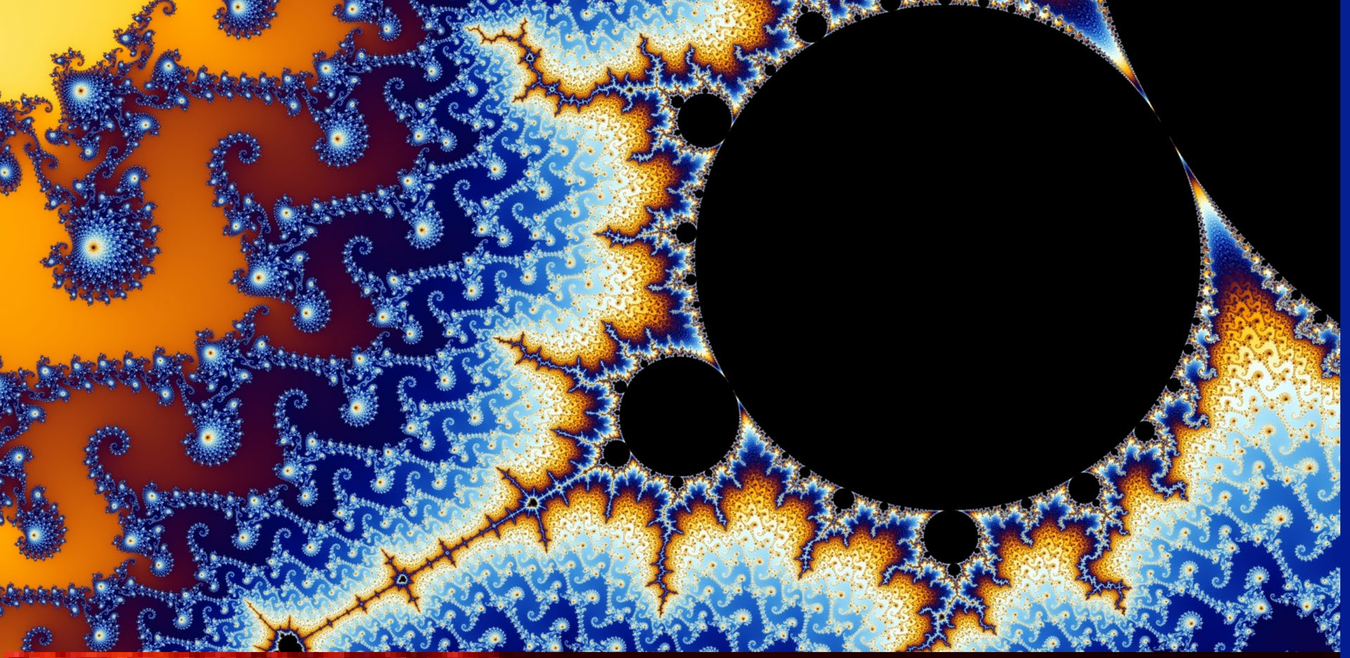


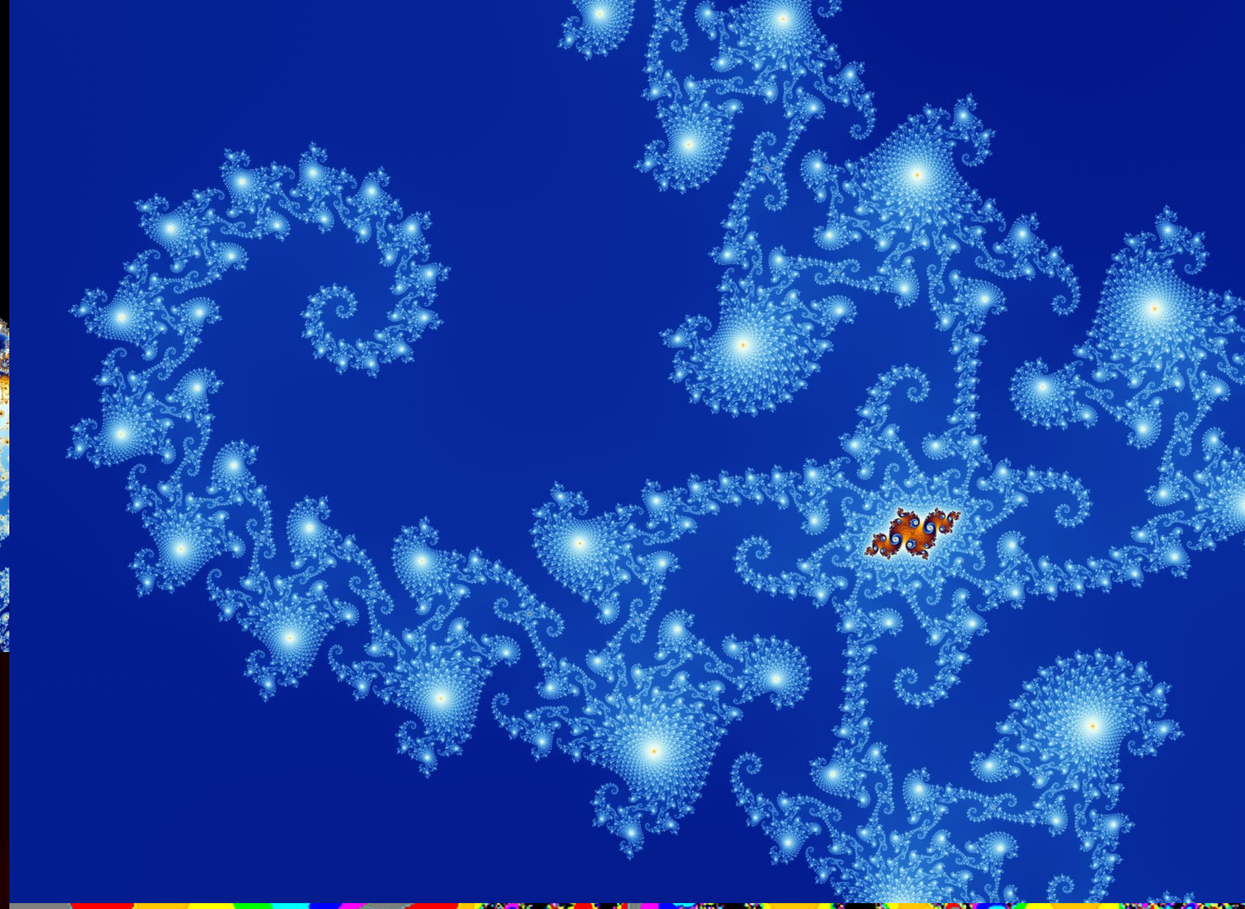
–1.5 0 2

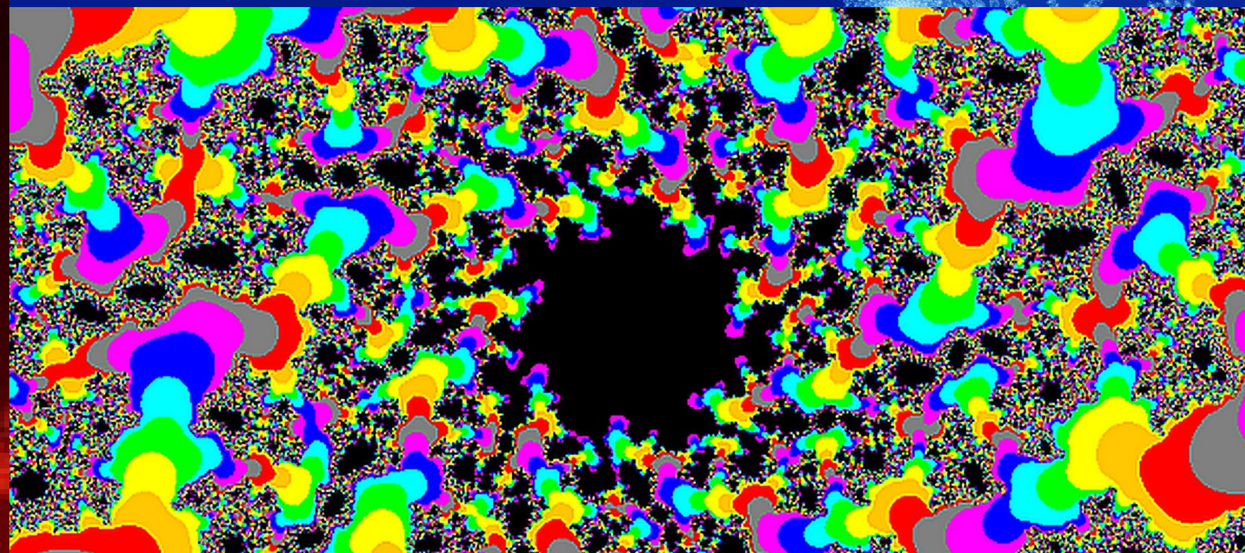


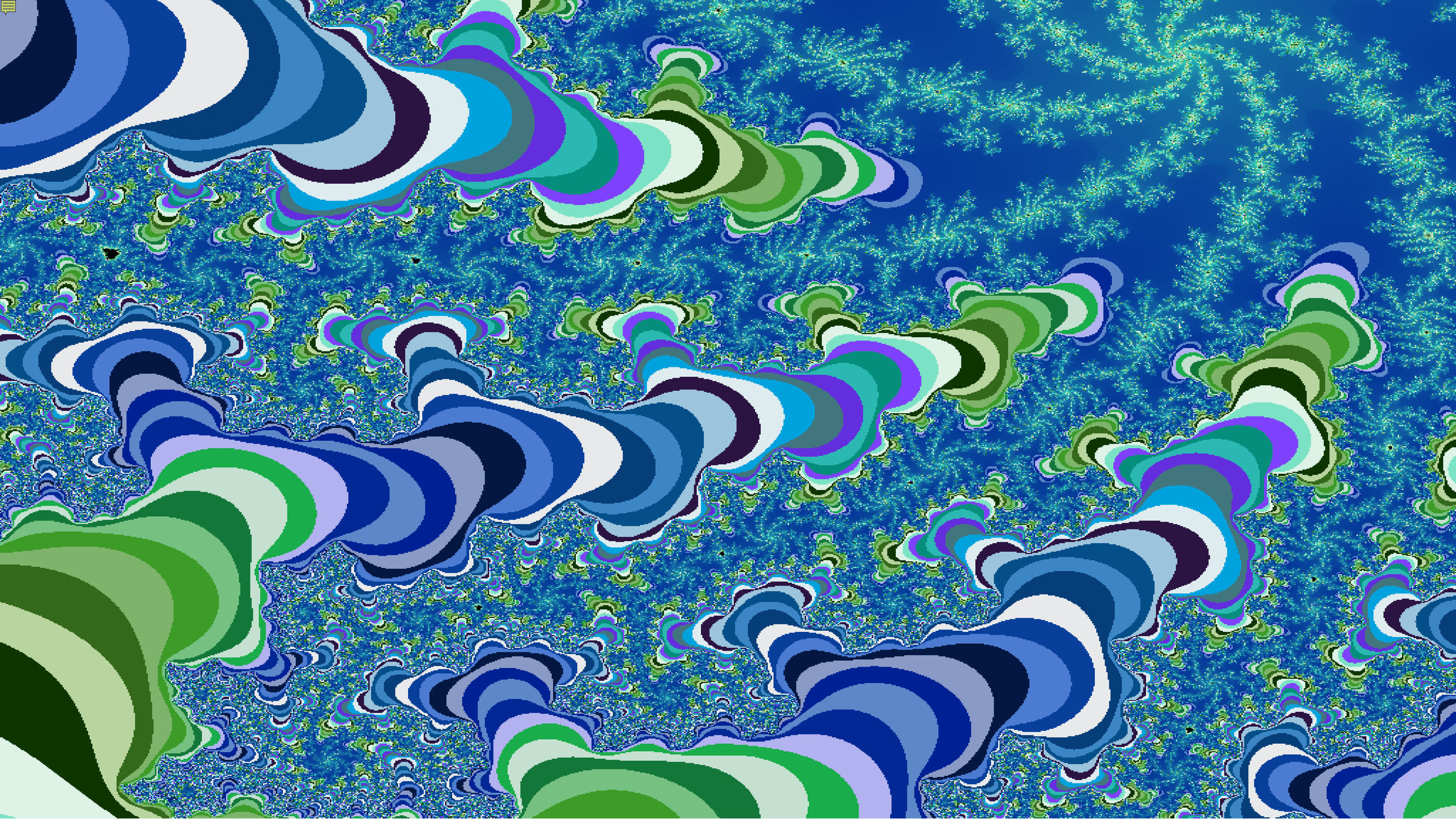
–1.5 0 .002







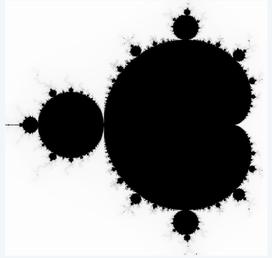




OOP summary

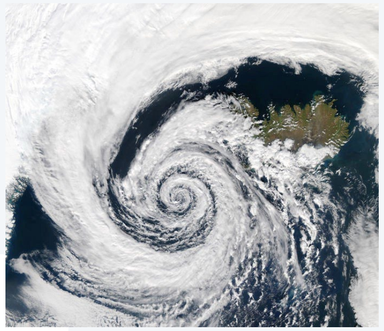
Object-oriented programming (OOP)

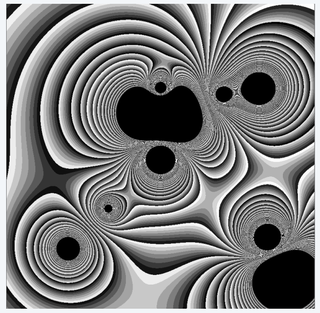
- Create your own data types (sets of values and ops on them).
- Use them in your programs (manipulate *objects*).



OOP helps us simulate the physical world

- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Examples: charged particle, color, sound, genome….





OOP helps us extend the Java language

- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Examples: complex, vector, polynomial, matrix, picture....

T	A	G	A	T	G	**T**	**G**	**C**	T	A	G	C

You have come a long way

any program you might want to write

objects

functions and modules

graphics, sound, and image I/O

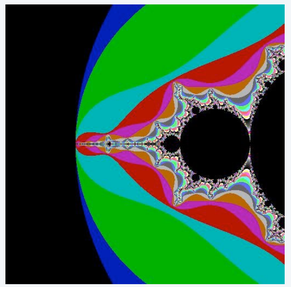
arrays

conditionals and loops

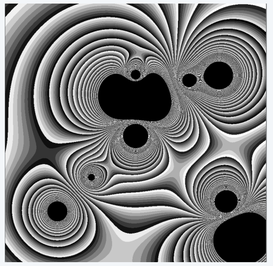
Math | text I/O

primitive data types | assignment statements

```
public class HelloWorld
{
  public static void main(String[] args)
  {
    System.out.println("Hello, World");
  }
}
```



T	A	G	A	T	G	T	G	C	T	A	G	C



Course goal. Open a *whole new world* of opportunity for you (programming).

Image sources

http://en.wikipedia.org/wiki/Leonhard_Euler#/media/File:Leonhard_Euler.jpg

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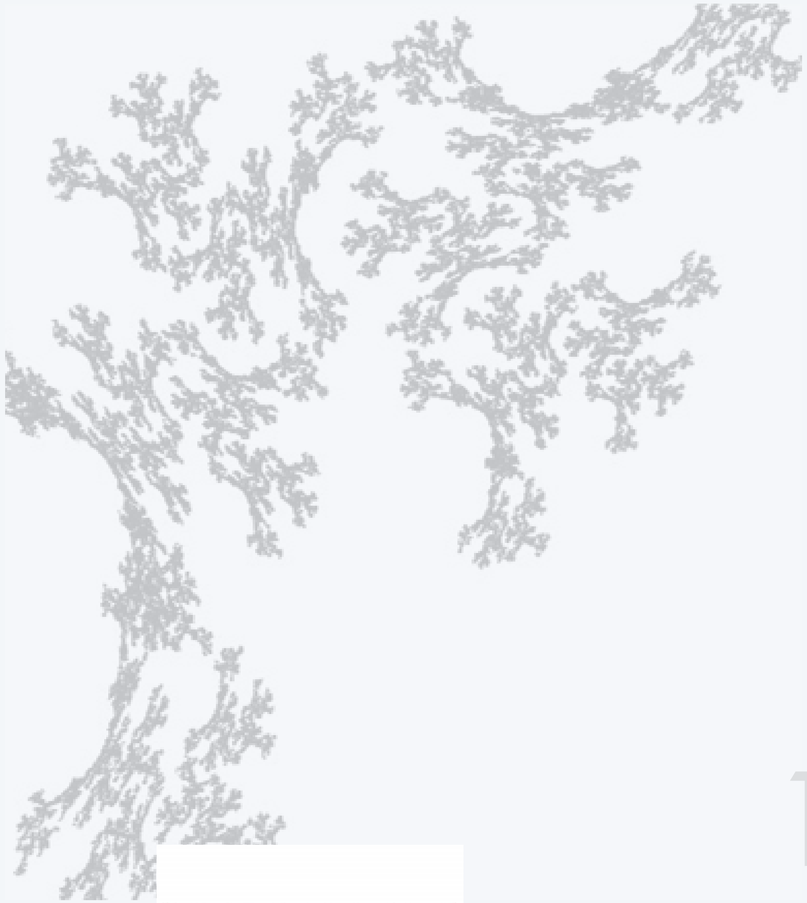
INTRODUCTION TO
COMPUTER SCIENCE

Rutgers University

10. Creating Data Types

- Overview
- Point charges
- Turtle graphics
- Complex numbers



INTRODUCTION TO
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10. Creating Data Types

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http://introcs.cs.princeton.edu