One-Dimensional Kinematics

$$\vec{v}_{average} = \frac{\Delta\vec{x}}{\Delta t}$$

Constant velocity

$$\vec{a} = 0$$

$$x = x_0 + vt$$

Constant acceleration

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \tfrac{1}{2}(v + v_0)t$$

Free Fall

$$g = 9.8\frac{m}{s^2}$$

$$y = y_0 + v_0 t + \tfrac{1}{2}a_g t^2$$

$$v = v_0 + a_g t$$

$$v^2 = v_0^2 + 2a_g(y - y_0)$$

$$y - y_0 = \tfrac{1}{2}(v + v_0)t$$

$$a_g = -g \text{ if the y axis points upward}$$

Projectile Motion

$$x = x_0 + v_x t$$

$$y = y_0 + v_{0y} t + \tfrac{1}{2}a_g t^2$$

$$v_y = v_{0y} + a_g t$$

$$v_y^2 = v_{0y}^2 + 2a_g(y - y_0)$$

$$y - y_0 = \tfrac{1}{2}(v_y + v_{0y})t$$

$$a_g = -g \text{ if the y axis points upward}$$

Relative Velocity

$$\vec{V}_{AB} = \vec{V}_{AC} + \vec{V}_{CB}$$

Dynamics of Newton's Laws

$$\sum \vec{F} = m\vec{a}$$

$$\text{Third Law} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$$

Gravitational Force

$$\vec{W} = m\vec{g}$$

$$g = G\frac{M}{r^2}$$

$$F_g = G\frac{Mm}{r^2}$$

$$G = 6.67 \times 10^{-11}\,\frac{N.m^2}{kg^2}$$

Static Friction

$$f_s^{max} = \mu_s F_N$$

$$f_s \le \mu_s F_N$$

Kinetic Friction

$$f_k = \mu_k F_N$$

Circular Motion

$$s = r\theta$$

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{1}{f}$$

$$v = \frac{2\pi r}{T} = 2\pi r f = \omega r$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Work and Energy

$W = Fd\cos\theta$

$PE = mgh$

$KE = \frac{1}{2}mv^2$

$W_{Net} = \Delta KE = KE_f - KE_i$

$W_{gravity} = -\Delta PE = -mg(h_f - h_i)$

$W_{nc} = \Delta KE + \Delta PE$

$W_{nc} = (KE_f - KE_i) + (PE_f - PE_i)$

$if\ W_{nc} = 0 \implies KE_i + PE_i = KE_f + PE_f$

$P = \dfrac{W}{t}$

Linear Momentum

$\vec{p} = m\vec{v}$

$Impulse \Rightarrow \vec{I} = \vec{F}\Delta t$

$\vec{F}_{Net} = \dfrac{\Delta\vec{p}}{\Delta t} = \dfrac{\vec{p}_f - \vec{p}_i}{\Delta t}$

$x_{center\ of\ mass} = \dfrac{m_1x_1 + m_1x_1 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n}$

Rotational Kinematics

$s = r\theta$

$v = r\omega$

$a_T = r\alpha$

<u>Constant angular velocity</u>

$\alpha = 0$

$\theta = \theta_0 + \omega t$

<u>Constant angular acceleration</u>

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t$

Rotational Dynamics

$\tau = lF = rF\sin\theta$

$\sum\tau = I\alpha$

$I = \sum_i m_i r_i^2 = m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2$

$W_{Rot} = \tau\theta$

$KE_{Rot} = \frac{1}{2}I\omega^2$

$L = I\omega$

$if\ \vec{\tau}_{Net} = 0 \Rightarrow I_i\omega_i = I_f\omega_f$

Elasticity and Simple Harmonic Motion

$Hooke's\ Law \Rightarrow F = kx$

$PE_{elastic} = \frac{1}{2}kx^2$

$x = A\cos\omega t$

$v = -A\omega\sin\omega t$

$v_{max} = A\omega$

$a = -A\omega^2\cos\omega t$

$a_{max} = A\omega^2$

$\omega = 2\pi f = \dfrac{2\pi}{T}$

<u>Simple Harmonic Oscillator (SHO)</u>

$\omega = \sqrt{\dfrac{k}{m}}$

$T = 2\pi\sqrt{\dfrac{m}{k}}$

<u>Simple pendulum</u>

$\omega = \sqrt{\dfrac{g}{l}}$

$T = 2\pi\sqrt{\dfrac{l}{g}}$

Fluids

$P_{atm} = 1.013 \times 10^5\ Pa$

$\rho = \dfrac{m}{V}$

$P = \dfrac{F}{A}$

$P = P_{atm} + \rho g h$

$F_B = W_{Fluid} = \rho_{Fluid}\, V_{Fluid\ Disp.}\, g$

$Volume\ Flow\ Rate \implies A_1 v_1 = A_2 v_2$

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$\rho_{water} = 1000\,\frac{kg}{m^3}$

Temperature and Heat

$T_F = \left(\frac{9}{5}\right) T_C + 32$

$T_K = T_C + 273$

$\Delta L = \alpha L_0 \Delta T$

$\Delta V = \beta V_0 \Delta T$

$Q = mc\Delta T$

$Q = mL_f$

$Q = mL_v$

$Q_{gained} = Q_{lost}$

$Energy = Pt$

Constants for water and ice

$c_i = 2000\,\frac{J}{kg.℃}$

$c_w = 4186\,\frac{J}{kg.℃}$

$L_f = 33.5 \times 10^4\,\frac{J}{kg.℃}$

$L_v = 22.6 \times 10^5\frac{J}{kg.℃}$

Ideal Gas and Kinetic Theory of Gases

$PV = nRT$

$PV = Nk_BT$

$\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

Monoatomic ideal gases

$PV = \dfrac{2}{3}N\left(\frac{1}{2}mv_{rms}^2\right)$

$\overline{KE} = \frac{3}{2}k_BT = \frac{1}{2}mv_{rms}^2$

$U = \frac{3}{2}nRT = \frac{3}{2}Nk_BT$

$R = 8.31\,\frac{J}{mol.K} = 0.0821\,\frac{atm.l}{mol.K}$

$n = \frac{N}{N_A} = \frac{m_{gas}}{MM} \Rightarrow MM \equiv Molar\ Mass$

$N_A = 6.022 \times 10^{23}\ mol^{-1}$

$k_B = 1.38 \times 10^{-23}\,\frac{J}{K}$

Thermodynamics

$\Delta U = Q - W$

$Q = nC_P\Delta T$

$Q = nC_V\Delta T$

$Monoatomic\ ideal\ gas \Rightarrow \begin{cases} C_P = \frac{5}{2}R \\ C_V = \frac{3}{2}R \end{cases}$

$isobaric\ process \Rightarrow W = P\Delta V$

$isothermal\ process \Rightarrow W = nRTln\left(\frac{V_f}{V_i}\right)$

$adiabatic\ process \Rightarrow W = -\frac{3}{2}nR\left(T_f - T_i\right)$

$adiabatic\ process \Rightarrow P_iV_i^\gamma = P_fV_f^\gamma$

$\gamma = \dfrac{C_P}{C_V}$

Heat Engines

$Q_H = W + Q_C$

$Heat\ Engine\ Efficiency \Rightarrow e = \frac{W}{Q_H}$

$Refrigerators \Rightarrow \eta = \frac{Q_C}{W}$

$Heat\ Pumps \Rightarrow \eta = \frac{Q_H}{W}$

Carnot Engine

$\frac{Q_H}{T_H} = \frac{Q_C}{T_C}$

$e_C = 1 - \frac{T_C}{T_H}$

$Entropy \Rightarrow \Delta S = \left(\frac{Q}{T}\right)_{reversible}$

Waves and Sound

$\lambda = \frac{v}{f} = vT$

$v = \sqrt{\frac{F}{m/L}}$

$y = Asin\left(2\pi ft \mp \frac{2\pi x}{\lambda}\right)$

$Sound \Rightarrow v = 331 + 0.6T(°C)$

$Intensity \Rightarrow I = \frac{P}{A}$

$Sound\ generated\ uniformly\ in\ all\ directions$

$\Rightarrow I = \frac{P_T}{4\pi r^2}$

$\beta = 10log\left(\frac{I}{I_0}\right)$

$I_0 = 1.0 \times 10^{-12}\ \frac{W}{m^2}$

Doppler Effect

$Listener\ moving\ toward\ (+)\ and\ away\ (-)$

$from\ a\ stationary\ source$

$f' = f\left(1 \pm \frac{v_o}{v}\right)$

$Source\ moving\ toward\ (-)\ and\ away\ (+)$

$from\ a\ stationary\ Listener$

$f' = f\left(\frac{1}{1 \mp \frac{v_s}{v}}\right)$

$Both\ Source\ and\ Listener\ are\ moving$

$f' = f\left(\frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}}\right)$

The Principle of Linear Superposition

In phase speakers interference

$Constructive\ interference \Rightarrow n = 0,1,2,3, \ldots$

$|L_2 - L_1| = n\lambda$

$Destructive\ interference \Rightarrow n = 1,3,5, \ldots$

$|L_2 - L_1| = n\frac{\lambda}{2}$

Out of phase speakers interference

$Constructive\ interference \Rightarrow n = 1,3,5, \ldots$

$|L_2 - L_1| = n\frac{\lambda}{2}$

$Destructive\ interference \Rightarrow n = 0,1,2,3, \ldots$

$|L_2 - L_1| = n\lambda$

Diffraction

$Rectangular\ opening \Rightarrow sin\theta = \frac{\lambda}{W}$

$Circular\ opening \Rightarrow sin\theta = 1.22\frac{\lambda}{D}$

Standing Waves

$String\ fixed\ at\ both\ ends \Rightarrow n = 1,2,3, \ldots$

$$L = n\frac{\lambda}{2}$$

$$f_n = n\left(\frac{v}{2L}\right)$$

$String\ fixed\ at\ one\ end\ only \Rightarrow n = 1,3,5, ..$

$$L = n\frac{\lambda}{4}$$

$$f_n = n\left(\frac{v}{4L}\right)$$

$Tube\ open/closed\ at\ both\ ends$

$\Rightarrow n = 1,2,3, \ldots$

$$L = n\frac{\lambda}{2}$$

$$f_n = n\left(\frac{v}{2L}\right)$$

$Tube\ open\ at\ one\ end\ only \Rightarrow n = 1,3,5, \ldots$

$$L = n\frac{\lambda}{4}$$

$$f_n = n\left(\frac{v}{4L}\right)$$

Beats

$Beat\ frequency \Rightarrow f_b = |f_1 - f_2|$

Additional Formulas

$Area\ of\ a\ circle \Rightarrow A = \pi r^2$

$Area\ of\ a\ sphere \Rightarrow A = 4\pi r^2$

$Volume\ of\ a\ sphere \Rightarrow V = \frac{4}{3}\pi r^3$

$Volume\ of\ a\ cylinder \Rightarrow V = \pi r^2 h$