Contents

Contextual Natural Deduction – $\mathbf{ND^c}$ (2013)

$$\frac{}{\Gamma, a : A \vdash a : A}$$

$$\frac{\Gamma, a : A \vdash b : \mathscr{C}_\pi[B]}{\Gamma \vdash \lambda_\pi a^A.b : \mathscr{C}_\pi[A \to B]} \to_I (\pi)$$

$$\frac{\Gamma \vdash f : \mathscr{C}^1_{\pi_1}[A \to B] \quad \Gamma \vdash x : \mathscr{C}^2_{\pi_2}[A]}{\Gamma \vdash (f\ x)^{\rightarrow}_{(\pi_1;\pi_2)} : \mathscr{C}^1_{\pi_1}[\mathscr{C}^2_{\pi_2}[B]]} \to^{\rightarrow}_E (\pi_1;\pi_2)$$

$$\frac{\Gamma \vdash f : \mathscr{C}^1_{\pi_1}[A \to B] \quad \Gamma \vdash x : \mathscr{C}^2_{\pi_2}[A]}{\Gamma \vdash (f\ x)^{\leftarrow}_{(\pi_1;\pi_2)} : \mathscr{C}^2_{\pi_1}[\mathscr{C}^1_{\pi_2}[B]]} \to^{\leftarrow}_E (\pi_1;\pi_2)$$

π, π_1 and π_2 must be positive positions. a is allowed to occur in b only if π is strongly positive.

Clarifications: $\mathscr{C}_\pi[F]$ denotes a formula with F occurring in the hole of a *context* $\mathscr{C}_\pi[]$. π is the position of the hole. It is: *positive* iff it is in the left side of an even number of implications; *strongly positive* iff this number is zero.

History: Contextual Natural Deduction [1] combines the idea of deep inference {2} with Gentzen's natural deduction {3}.

Technicalities: Soundness and completeness w.r.t. minimal logic are proven [1] by providing translations between $\mathbf{ND^c}$ and Gentzen's natural deduction. Proofs in $\mathbf{ND^c}$ can be quadratically shorter than proofs in Gentzen's natural deduction.

References

[1] Bruno Woltzenlogel Paleo. "Contextual Natural Deduction". In: *Logical Foundations of Computer Science, International Symposium, LFCS 2013, San Diego, CA, USA, January 6-8, 2013. Proceedings*. Ed. by Sergei N. Artëmov and Anil Nerode. Vol. 7734. Lecture Notes in Computer Science. Springer, 2013, pp. 372–386. ISBN: 978-3-642-35721-3. DOI: 10.1007/978-3-642-35722-0_27. URL: http://dx.doi.org/10.1007/978-3-642-35722-0_27.

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Clarifications:

History:

Technicalities:

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