Contents

Contextual Natural Deduction – $\mathbf{ND^c}$ (2013)

$$\overline{\Gamma, a : A \vdash a : A}$$

$$\frac{\Gamma, a : A \vdash b : \mathscr{C}_\pi[B]}{\Gamma \vdash \lambda_\pi a^A . b : \mathscr{C}_\pi[A \to B]} \to_I (\pi)$$

$$\frac{\Gamma \vdash f : \mathscr{C}^1_{\pi_1}[A \to B] \quad \Gamma \vdash x : \mathscr{C}^2_{\pi_2}[A]}{\Gamma \vdash (f\ x)^{\rightharpoonup}_{(\pi_1;\pi_2)} : \mathscr{C}^1_{\pi_1}[\mathscr{C}^2_{\pi_2}[B]]} \to^{\rightharpoonup}_E (\pi_1;\pi_2)$$

$$\frac{\Gamma \vdash f : \mathscr{C}^1_{\pi_1}[A \to B] \quad \Gamma \vdash x : \mathscr{C}^2_{\pi_2}[A]}{\Gamma \vdash (f\ x)^{\leftharpoonup}_{(\pi_1;\pi_2)} : \mathscr{C}^2_{\pi_1}[\mathscr{C}^1_{\pi_2}[B]]} \to^{\leftharpoonup}_E (\pi_1;\pi_2)$$

π, π_1 and π_2 must be positive positions. a is allowed to occur in b only if π is strongly positive.

Clarifications: $\mathscr{C}_\pi[F]$ denotes a formula with F occurring in the hole of a *context* $\mathscr{C}_\pi[]$. π is the position of the hole. It is: *positive* iff it is in the left side of an even number of implications; *strongly positive* iff this number is zero.

History: Contextual Natural Deduction [1] combines the idea of deep inference $\{2\}$ with Gentzen's natural deduction $\{3\}$.

Technicalities: Soundness and completeness w.r.t. minimal logic are proven [1] by providing translations between $\mathbf{ND^c}$ and Gentzen's natural deduction. Proofs in $\mathbf{ND^c}$ can be quadratically shorter than proofs in Gentzen's natural deduction.

References

[1] Bruno Woltzenlogel Paleo. "Contextual Natural Deduction". In: *Logical Foundations of Computer Science, International Symposium, LFCS 2013, San Diego, CA, USA, January 6-8, 2013. Proceedings*. Ed. by Sergei N. Artëmov and Anil Nerode. Vol. 7734. Lecture Notes in Computer Science. Springer, 2013, pp. 372–386. ISBN: 978-3-642-35721-3. DOI: 10.1007/978-3-642-35722-0_27. URL: http://dx.doi.org/10.1007/978-3-642-35722-0_27.

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ToDo – (ToDo)

ToDo

Entry by: ToDo

Natural Deduction – ToDo(1934)

ToDo

Clarifications:

History:

Technicalities:

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Sequent Calculus LJ – (1935)

$$\frac{}{A \vdash A}\ init \qquad \frac{\Gamma_1 \vdash P \quad \Gamma_2, P \vdash C}{\Gamma_1, \Gamma_2 \vdash C}\ cut$$

$$\frac{\Gamma \vdash P}{\Gamma, \neg P \vdash}\ \neg_l \qquad \frac{\Gamma, P \vdash}{\Gamma \vdash \neg P}\ \neg_r$$

$$\frac{P_i, \Gamma \vdash C}{P_1 \wedge P_2, \Gamma \vdash C}\ \wedge_{li} \qquad \frac{\Gamma \vdash P \quad \Gamma \vdash Q}{\Gamma \vdash P \wedge Q}\ \wedge_r$$

$$\frac{P, \Gamma \vdash C \quad Q, \Gamma \vdash C}{P \vee Q, \Gamma \vdash C}\ \vee_l \qquad \frac{\Gamma \vdash P_i}{\Gamma \vdash P_1 \vee P_2}\ \vee_{ri}$$

$$\frac{\Gamma_1 \vdash P \quad Q, \Gamma_2 \vdash C}{P \rightarrow Q, \Gamma_1, \Gamma_2 \vdash C}\ \rightarrow_l \qquad \frac{\Gamma, P \vdash Q}{\Gamma \vdash P \rightarrow Q}\ \rightarrow_r$$

$$\frac{P\{x \leftarrow \alpha\}, \Gamma \vdash C}{\exists x.P, \Gamma \vdash C}\ \exists_l \qquad \frac{\Gamma \vdash P\{x \leftarrow t\}}{\Gamma \vdash \exists x.P}\ \exists_r$$

$$\frac{P\{x \leftarrow t\}, \Gamma \vdash C}{\forall x.P, \Gamma \vdash C}\ \forall_l \qquad \frac{\Gamma \vdash P\{x \leftarrow \alpha\}}{\Gamma \vdash \forall x.P}\ \forall_r$$

$$\frac{P, P, \Gamma \vdash C}{P, \Gamma \vdash C}\ c_l \qquad \frac{\Gamma \vdash C}{P, \Gamma \vdash C}\ w_l \qquad \frac{\Gamma \vdash}{\Gamma \vdash P}\ w_r$$

Clarifications: Assuming that α is a variable not contained in P, Γ or C, t does not contain variables bound in P and C stands for one formula or the empty set.

History: Proposed by Gentzen in [1] by restricting the succedent of sequents in to have at most one formula. In the original paper, he notes that this restriction is equivalent to removing the principle of excluded middle from the natural deduction system $\{3\}$ in order to obtain . The cut is admissible in LJ and this result is known as *Hauptsatz*.

Technicalities: Soundness and completeness of LJ can be proved using a translation of LJ derivations into . Decidability of the propositional fragment and consistency of intuitionistic logic follows from the cut admissibility in this calculus.

References

[1] Gerhard Gentzen. “Untersuchungen über das logische Schließen I”. In: *Mathematische Zeitschrift* 39.1 (Dec. 1935), pp. 176–210.

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