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### **Contextual Natural Deduction – ND<sup>c</sup> (2013)**

$$\overline{\Gamma,a:A\vdash a:A}$$

$$\frac{\Gamma,a:A\vdash b:\mathscr{C}_{\pi}[B]}{\Gamma\vdash\lambda_{\pi}a^{A}.b:\mathscr{C}_{\pi}[A\rightarrow B]}\rightarrow_{I}(\pi)$$

$$\frac{\Gamma \vdash f : \mathscr{C}^1_{\pi_1}[A \to B] \quad \Gamma \vdash x : \mathscr{C}^2_{\pi_2}[A]}{\Gamma \vdash (f \ x)^{\rightharpoonup}_{(\pi_1;\pi_2)} : \mathscr{C}^1_{\pi_1}[\mathscr{C}^2_{\pi_2}[B]]} \ \to^{\rightharpoonup}_E (\pi_1;\pi_2)$$

$$\frac{\Gamma \vdash f : \mathscr{C}^1_{\pi_1}[A \to B] \quad \Gamma \vdash x : \mathscr{C}^2_{\pi_2}[A]}{\Gamma \vdash (f \: x)^{\leftarrow}_{(\pi_1;\pi_2)} : \mathscr{C}^2_{\pi_1}[\mathscr{C}^1_{\pi_2}[B]]} \, \to_E^{\leftarrow} (\pi_1;\pi_2)$$

 $\pi$ ,  $\pi_1$  and  $\pi_2$  must be positive positions. a is allowed to occur in b only if  $\pi$  is strongly positive.

**Clarifications:**  $\mathscr{C}_{\pi}[F]$  denotes a formula with F occurring in the hole of a *context*  $\mathscr{C}_{\pi}[]$ .  $\pi$  is the position of the hole. It is: *positive* iff it is in the left side of an even number of implications; *strongly positive* iff this number is zero.

**History:** Contextual Natural Deduction [1] combines the idea of deep inference  $\{2\}$  with Gentzen's natural deduction  $\{3\}$ .

**Technicalities:** Soundness and completeness w.r.t. minimal logic are proven [1] by providing translations between  $\mathbf{ND^c}$  and Gentzen's natural deduction. Proofs in  $\mathbf{ND^c}$  can be quadratically shorter than proofs in Gentzen's natural deduction.

#### References

[1] Bruno Woltzenlogel Paleo. "Contextual Natural Deduction". In: Logical Foundations of Computer Science, International Symposium, LFCS 2013, San Diego, CA, USA, January 6-8, 2013. Proceedings. Ed. by Sergei N. Artëmov and Anil Nerode. Vol. 7734. Lecture Notes in Computer Science. Springer, 2013, pp. 372–386. ISBN: 978-3-642-35721-3. DOI: 10.1007/978-3-642-35722-0\_27. URL: http://dx.doi.org/10.1007/978-3-642-35722-0\_27.

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## ToDo - (ToDo)

ToDo

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# Natural Deduction - ToDo(1934)

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Clarifications:		
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