MATH 3312 Fall 2024

Programming 3 Due: Oct 16, 2024

Question 1 Consider the linear equation $Ax = b$. Here b is the vector of all ones, A is the tridiagonal matrix,

$$A = \begin{pmatrix} 2.001 & -1 & & & \\ -1 & 2.001 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2.001 & -1 \\ & & & -1 & 2.001 \end{pmatrix}_{1000\times 1000}$$

Hint: You should allocate A as a sparse matrix to save memory and time. Otherwise, this could be very slow.

(a) Use Thomas algorithm to solve it. This will be the exact solution.

In matlab, this can be done by backslash.

(b) Use Jacobi Method to solve the system, with the initial guess $x = 0$, stop when the backward error is less than 10^{-5}. For the ith iteration step, calculate the forward error $e_f(i)$ and the backward error $e_b(i)$. Plot the curve of $e_f(i)$ and the curve of $e_b(i)$. Please use logarithm scale in the y-axis. Does the slope tell you anything about the convergence speed?

(c) Repeat part(b) using Gauss–Seidel Method, and compare with Jacobi Method.

(d) Repeat part(b) using SOR Method. Find the convergence speed for different $\omega = 0.1, 0.2, \cdots, 1.8, 1.9$. Plot the curve showing the convergence speed as a function of ω.

Depending on the performance of the program on your computer, you might use relatively larger or smaller values for the backward error, if needed.