MATH 3312 Fall 2024

Programming 4 Due: Nov 6th, 2024

Question 1 Consider the interpolation on $[-2, 2]$.

- For evenly spaced nodes, $-2 = x_1 < \cdots < x_n = 2$, plot
  $g_n(x) = (x - x_1) \cdots (x - x_n)$.
  For $n = 10, 20, 30, \cdots, 100$, find the maximum value, $M(n) = \max_{-1 \le x \le 1} |g_n(x)|$. Plot $M(n)$, use log scale in y-axis. You might find the maximum by evaluating $g_n(x)$ at evenly spaced $N = 1000$ points.
- For Chebyshev nodes, $-2 = x_1 < \cdots < x_n = 2$, plot $(x - x_1) \cdots (x - x_n)$.
- Compare interpolation using evenly spaced nodes with interpolation using Chebyshev nodes, what happens when n becomes large.

Question 2 Build a program to find the cubic spline that interpolates, $y = \ln x$, at $N = 21$ evenly spaced points in $[1, 3]$, including the two ends. You might use the built-in spline function in matlab.

(a) Use natural boundary condition. Divide the interval $[1, 3]$ into $M = 10^6$ equal pieces, Let $\{x_i\}_{i=1}^{M+1}$ be the end-points of the sub-intervals. Let $\{e_i\}_{i=1}^{M+1}$ be the error of the spline. Plot the curve of $(x_i, e_i)_{i=1}^{M+1}$.

(b) Repeat Part (a) with clamped boundary condition. You might use the exact value of the derivatives.