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MTH 3312

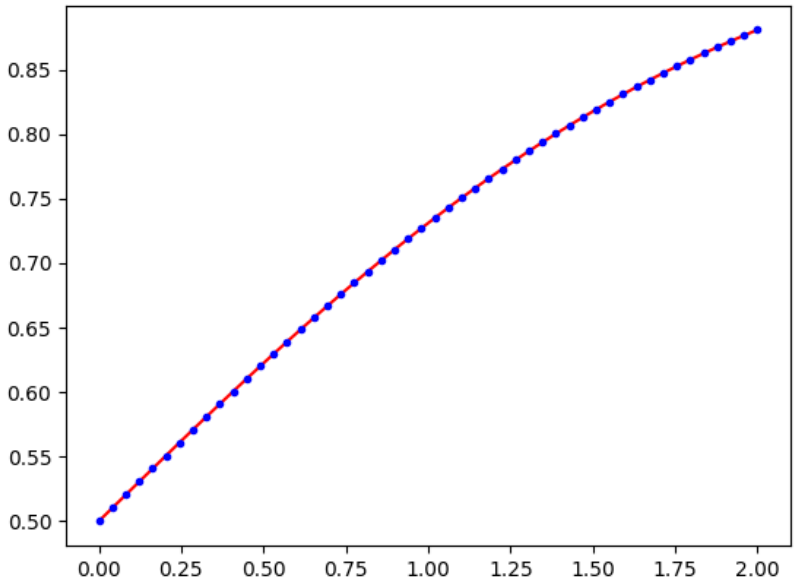
Due: 12/2/2024

All code documented in the python files as well.

Q1)

A)

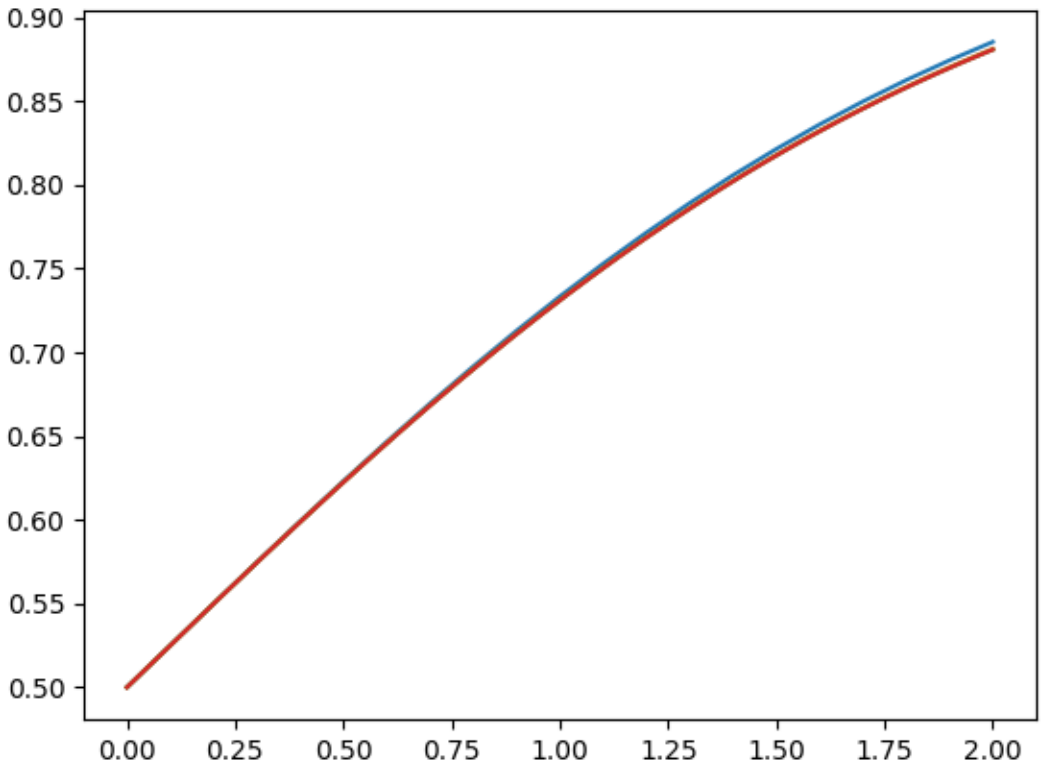
In the code, the function *partA()*, calculates the exact solution of the IVP using the *solve_ivp()* function from the scipy package. The graph below shows the *solve_ip()* function in the blue dots and the exact solution in the red solid line. Based on the graph, the IVP very closely matches the exact solution.



B) When solving for the global truncation error at *t=2*, I used the exact solution at *t=2* and passed it to the function which applied Euler's Method. When doing so for each h, my program gives the output:

```
Euler's Method:
Global error for 0.1 is: 0.0046255545565762779
Global error for 0.01 is: 0.00045147368816533806
Global error for 0.001 is: 4.084789696068292e-05
Global error for 0.0001 is: 1.4914469759208515e-07
```

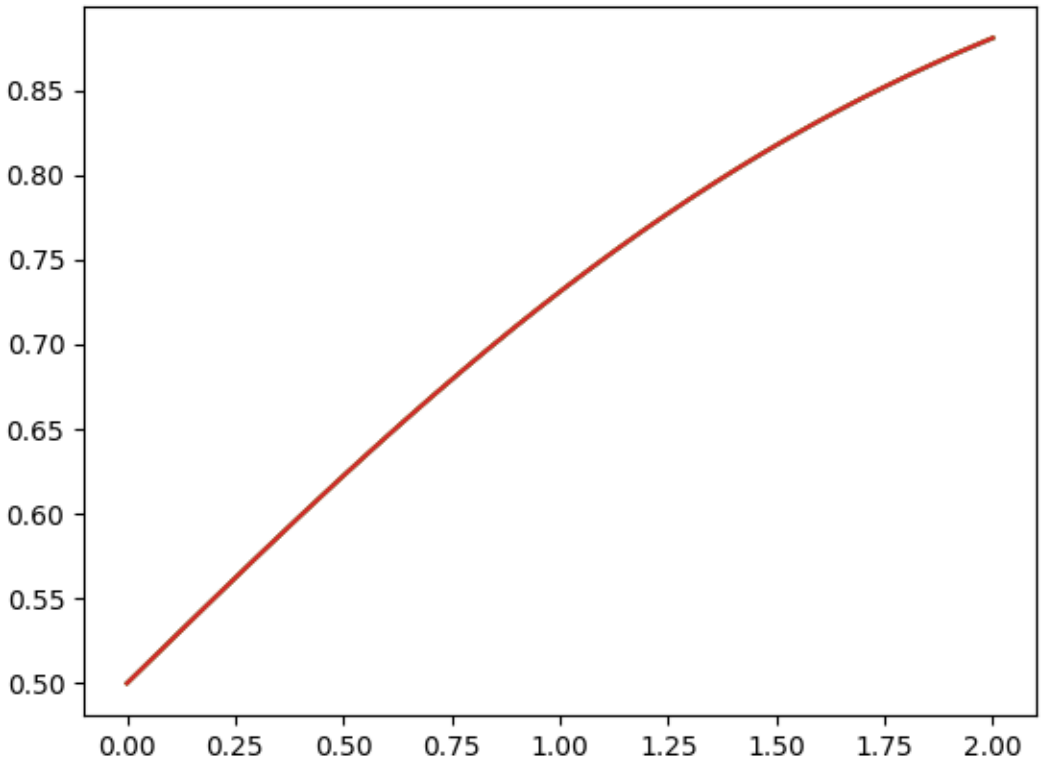
The formula I used to get the global error was from slide 14 on the ODE slides where $g_i = |w_i - y_i|$. The graph of the IVP's at each step is shown below. Euler's Method is first order and the global error after calculating the method is not necessarily consistent but it is relatively close. There are four separate lines, but they very closely overlap each other because the error is very small.



C) For Implicit Euler Method, it is first order, and the calculation from the program is:

```
Implicit Euler's method:
Global error for 0.1 is: 0.00016256972169814699
Global error for 0.01 is: 6.21196309102956e-06
Global error for 0.001 is: 4.7186532725618235e-06
Global error for 0.0001 is: 4.703788533566566e-06
```

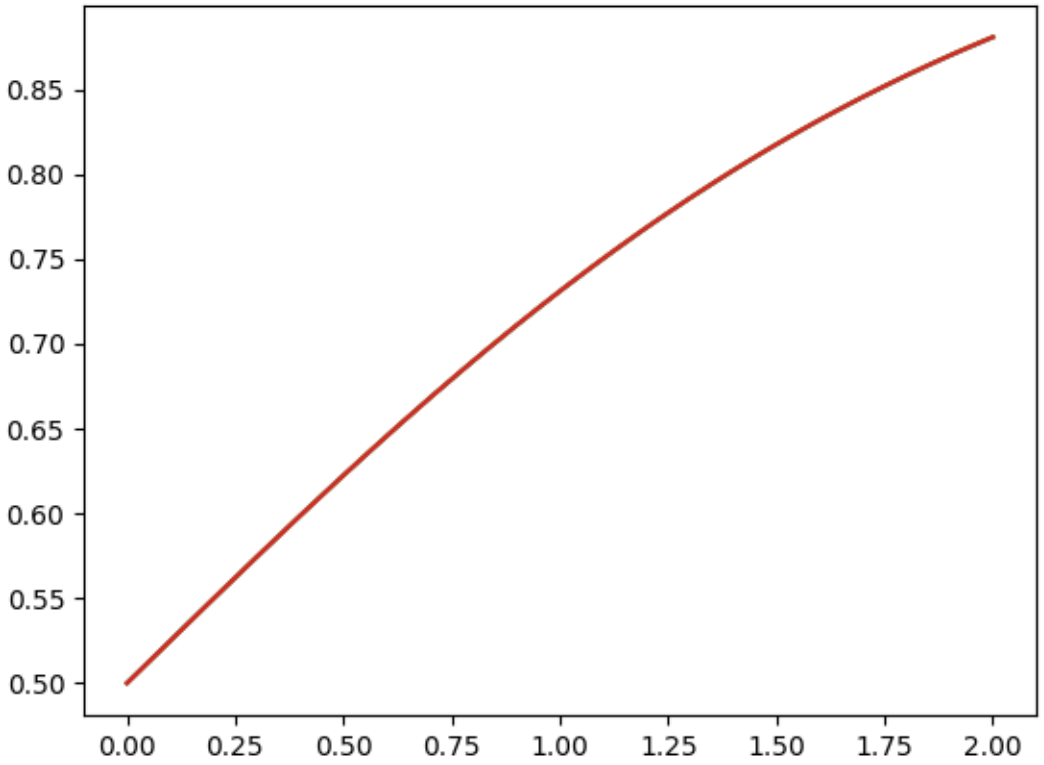
The calculations are not consistent with the order of error in these calculations. To solve the implicit equation I used the formula from slide 17. Below is the graph of all the lines from implicit Euler's method. All of the lines are extremely similar to each other.



D) For Explicit Trapezoid Method, the calculation yielded the errors below:

```
Explicit Trapezoid method:
Global error for 0.1 is: 0.00016256972169814699
Global error for 0.01 is: 6.21196309102956e-06
Global error for 0.001 is: 4.7186532725618235e-06
Global error for 0.0001 is: 4.703788533566566e-06
```

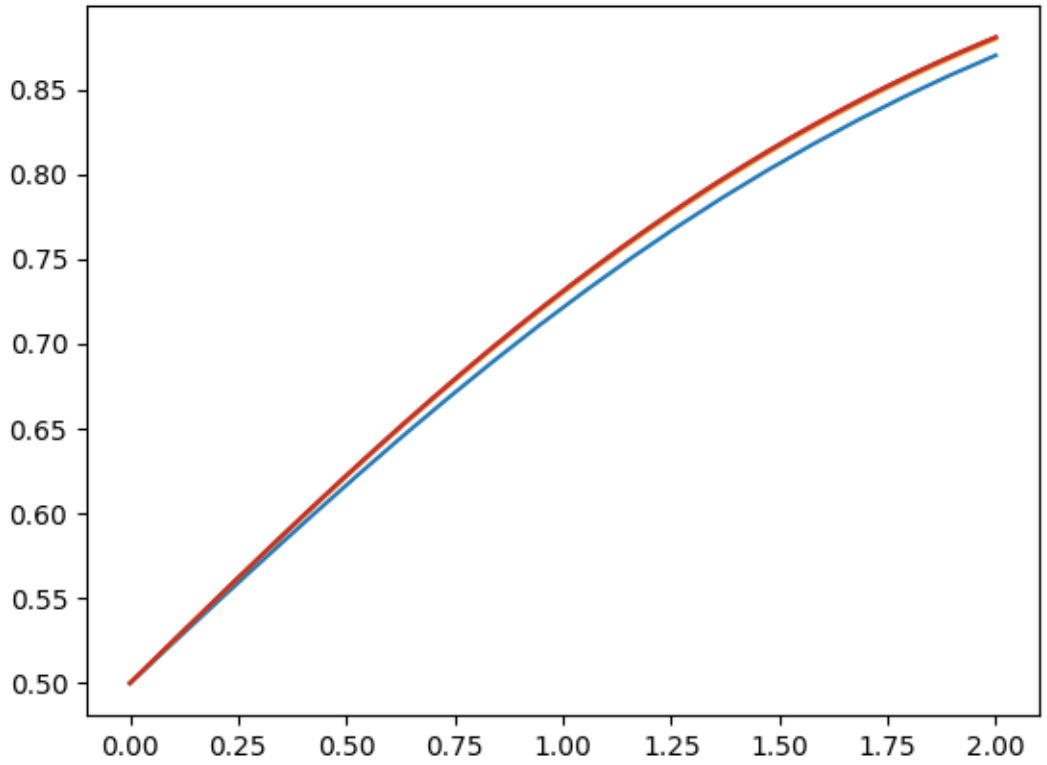
Trapezoid method is second order, and the calculations are relatively consistent with the order for most of the given h values. Below is the graph, which is very similar to the previous one where the errors are very similar to each other for each h.



E) The calculations from the Adam-Bashforth method are:

```
Adam-Bashforth Two-step method:
Global error for 0.1 is: 0.00012038677615444637
Global error for 0.01 is: 3.4568142401880664e-06
Global error for 0.001 is: 4.691178957960673e-06
Global error for 0.0001 is: 4.703513863391073e-06
```

The method is second order, and the calculations are relatively consistent with the order. On the next page is the graph for the method and with h = 0.1, the error is noticeable.


0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00