

Question 1

Set $f(x) = 54x^6 + 45x^5 - 102x^4 - 69x^3 + 35x^2 + 16x - 4$.

- (a) Plot the function on the interval $[-2, 2]$.
- (b) Use Newton's Method to find all five roots in the interval (correct to eight decimal places).
- (c) For each root, use the value obtained in part (b) to approximate its exact value r . Let $e_i = |x_i - r|$ denote the (absolute) error at step i in Part (b). Plot the curve (e_i, e_{i+1}) . Please use logarithm scale for both the x -axis and y -axis. What can we say about the convergence based on the slope of these curves.
- (d) For the one with linear convergence, redo part (c) using secant method.

Question 2

Let H denote the $n \times n$ Hilbert matrix, whose (i, j) entry is $1/(i + j - 1)$. Use Gauss Elimination to solve $Hx = b$, where b is the vector of all ones, for $n = 1000, 2000, \dots, 10000$. Record the time cost as $t(n)$. Plot $t(n)$, using proper scale of the axes to verify $O(n^3)$.

Hint: If your computer is too slow to solve $n = 10000$, you might use smaller n .