MATH 4311 Spring 2025

Program 2 Due: Feb 27, 2024

This project considers Interpolation and Least Square Fitting.

Consider M data points $\{(t_i, f(t_i))_{i=1}^{M}\}$. Here t_i are evenly distributed on $[-1, 1]$. For each of the following functions,

$$\text{(a)},\ \ f(t) = \frac{1}{1+12t^2}; \qquad \text{(b)},\ \ f(t) = e^t.$$

(I) Find a polynomial of degree N fits the data points, namely, solve a system of inconsistent equations.

$$a_0 + a_1 t_i + a_2 t_i^2 + \cdots + a_N t_i^N = e^{-t_i^2}, \text{ for } i = 1, 2, \cdots, M.$$

Do NOT use built-in polyfit. You might use built-in polyval. Plot the error between the fitting polynomial and the function $f(x)$.

- Try $M = 51$, $N = 10, 20, 30, 40, 50$. Does it become better with increasing degree? Explain.
- Try $M = 101$, $N = 10, 20, 30, \cdots, 100$. Does it become better after increasing number of data points?

(II) Find the trigonometric interpolation function $P(t)$ using M = 8, 16, 32, 64, 128, 256 data points. Plot the error curves of the interpolation functions, by evaluating $|P(t) - f(t)|$ at $p = 2048$ evenly spaced points. Does the interpolation become better with increasing degree? Explain.

You should use FFT to evaluate the trigonometric interpolation function, instead of adding them one by one.

(III) For $M = 256$, find the best least square approximation using the first $m = 8, 16, 32, 64, 128$ basis functions. Plot the curves of the approximation function and compare with the function $f(t)$, by evaluating $P(t)$ at $p = 2048$ evenly spaced points, using FFT. Does the fitting become better with increasing degree? Explain.