MATH 3312 Fall 2024

Programming 6 Due: Dec 9th, 2024

Question 1 Consider a game where each round results in either a gain of \$1 or a loss of \$1. The probability of winning each round is 49%, and you start with \$100. Your goal is to reach \$150, at which point you leave the table happily. However, if your balance reaches \$0, you can no longer continue playing.

This could be represented as a first passage time problem of a biased random walk,

$$\begin{cases} W_0 = 0, \\ W_t = W_{t-1} + s_t. \end{cases} \qquad s_t = \begin{cases} 1, & p = 0.49, \\ -1, & p = 0.51, \end{cases}$$

on the interval $W \in [-100, 50]$.

- Simulate the probability of achieving your goal, by generating $N = 10000$ realizations of W_t.
  Compare your result with the exact value given by the formula,
  $[(q/p)^b - 1]/[(q/p)^{(a+b)} - 1]$.
  In this case, $p = 0.49$, $q = 1 - p = 0.51$, $a = 50$, $b = 100$.
- Suppose each round takes 1 minute, estimate how long it takes for you to leave the table, by finding the average time cost of $N = 10000$ realizations of W_t.
  Compare your result with the exact value given by the formula,
  $[b - (a+b)(1 - (q/p)^b)/(1 - (q/p)^{(a+b)})]/(q - p)$.

Question 2 Numerically solve the Langevin equation on the interval $t \in [0, 1]$,

$$dy = -y dt + 2 dB_t, \quad y(0) = 1.$$

- Use Euler–Maruyama's method with step sizes $\Delta t = 0.1$, 0.01, and 0.001. For each step size, run 5000 realizations.
  Find the mean and standard deviation of $y(1)$.
- Repeat using Milstein method.