

Question 1

Consider the IVP

$$\begin{cases} y' = y(1 - y) \\ y(0) = 1/2; \\ t \in [0, 2]. \end{cases}$$

- (a) Verify the exact solution is $y(t) = \frac{e^t}{1 + e^t}$.
- (b) Apply Euler's method, $w_{i+1} = w_i + hf(t_i, w_i)$, to solve the IVP numerically.
Find the global truncation error at $t = 2$ for step size $h = 0.1, 0.01, 0.001, 0.0001$.
Is the calculation consistent with the order of the method?
- (c) Repeat part (b) using Implicit Euler Method.

$$w_{i+1} = w_i + hf(t_{i+1}, w_{i+1}).$$

Please explain how you solve the implicit equation.

- (d) Repeat part (b) using Explicit Trapezoid Method.

$$w_{i+1} = w_i + \frac{h}{2}[f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i))].$$

- (e) Repeat part (b) using Adam-Bashforth Two-step Method.

$$w_{i+1} = w_i + h \left[\frac{3}{2}f(t_i, w_i) - \frac{1}{2}f(t_{i-1}, w_{i-1}) \right].$$