Introduction to Derivatives in Calculus

1 What is a Derivative?

A derivative represents the instantaneous rate of change of a function with respect to its variable. In simpler terms, it tells us how quickly a function is changing at any given point.

1.1 Mathematical Definition

The derivative of a function $f(x)$ is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

This limit represents the slope of the tangent line to the curve at point x.

1.2 Example: Power Rule

For a simple function like $f(x) = x^2$:

- The derivative is $f'(x) = 2x$
- This means at any point x, the slope of the curve is $2x$
- At $x = 3$, the slope is 6
- At $x = 0$, the slope is 0 (horizontal tangent)

1.3 Real-World Applications

1. **Velocity**: If position is $s(t)$, then velocity is $s'(t)$
2. **Acceleration**: If velocity is $v(t)$, then acceleration is $v'(t)$
3. **Optimization**: Finding maximum and minimum values
4. **Economics**: Marginal cost and revenue analysis

1.4 Practice Problems

1. Find the derivative of $f(x) = 3x^3 + 2x^2 - 5x + 1$
2. What is the slope of $y = x^2$ at the point $(2, 4)$?
3. If the position of an object is $s(t) = 16t^2$, find its velocity at $t = 3$ seconds.

This foundational concept opens the door to understanding rates of change in all areas of science and mathematics.