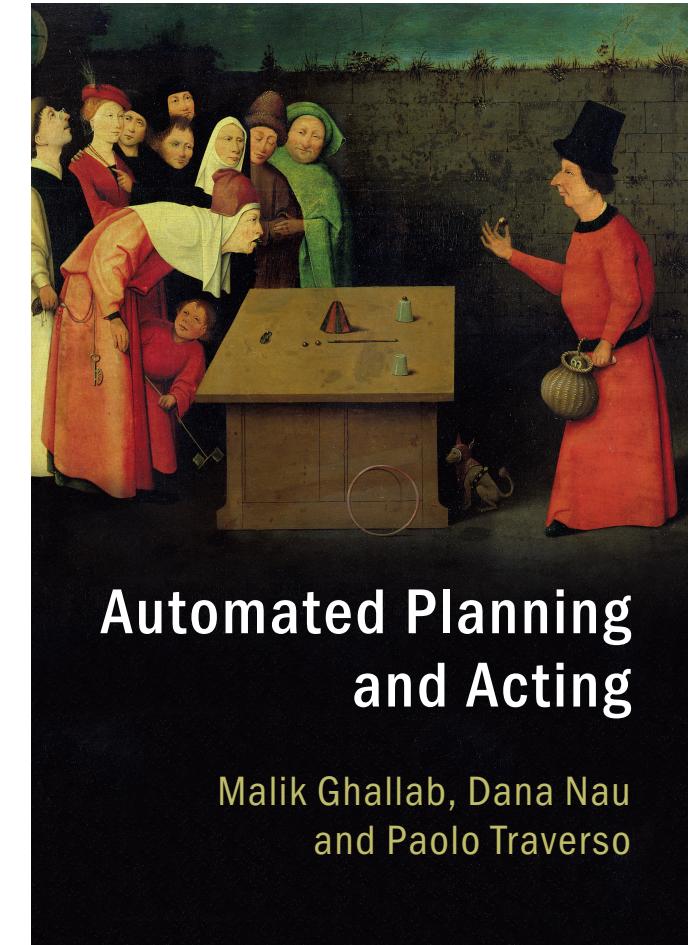


Chapter 2

Deliberation with Deterministic Models

Sections 2.1, 2.2, 2.6

Adapted from Dana Nau
University of Maryland



<http://www.laas.fr/planning>

Motivation

- How to model a complex environment?
 - ▶ Generally need **simplifying** assumptions
- *Classical planning*
 - Finite, static world, just one actor
 - No concurrent actions, no explicit time
 - Determinism, no uncertainty
- Sequence of states and actions $\langle s_0, a_1, s_1, a_2, s_2, \dots \rangle$
- Avoids many complications
- Most real-world environments don't satisfy the assumptions
 - ⇒ Errors in prediction
- OK if they're infrequent and don't have severe consequences

Outline

- Next →*
- Chapter 2, part *a* (chap2a.pdf):
- 2.1 State-variable representation
 - Comparison with PDDL
 - 2.2 Forward state-space search
 - 2.6 Incorporating planning into an actor
-
- Chapter 2, part *b* (chap2b.pdf):
- 2.3 Heuristic functions
 - 2.4 Backward search
 - 2.5 Plan-space search
-
- Additional slides:
- 2.7.7 HTN_planning.pdf
 - 2.7.8 LTL_planning.pdf

Domain Model

State-transition system or *classical planning domain*:

- $\Sigma = (S, A, \gamma, \text{cost})$ or (S, A, γ)

- ▶ S - finite set of *states*
- ▶ A - finite set of *actions*
- ▶ $\gamma: S \times A \rightarrow S$

prediction (or *state-transition*) function

- *partial function*: $\gamma(s, a)$ is not necessarily defined for every (s, a)
 - ▶ a is *applicable* in s iff $\gamma(s, a)$ is defined
 - ▶ $\text{Domain}(a) = \{s \in S \mid a \text{ is applicable in } s\}$
 - ▶ $\text{Range}(a) = \{\gamma(s, a) \mid s \in \text{Domain}(a)\}$
- ▶ **cost**: $S \times A \rightarrow \mathcal{R}^+$ or $\text{cost}: A \rightarrow \mathcal{R}^+$
 - optional; default is $\text{cost}(a) \equiv 1$
 - money, time, something else

- *plan*:

- ▶ a sequence of actions $\pi = \langle a_1, \dots, a_n \rangle$

- π is *applicable* in s_0 if the actions are applicable in the order given

$$\gamma(s_0, a_1) = s_1$$

$$\gamma(s_1, a_2) = s_2$$

...

$$\gamma(s_{n-1}, a_n) = s_n$$

- ▶ In this case define $\gamma(s_0, \pi) = s_n$

- *Classical planning problem*:

- ▶ $P = (\Sigma, s_0, S_g)$

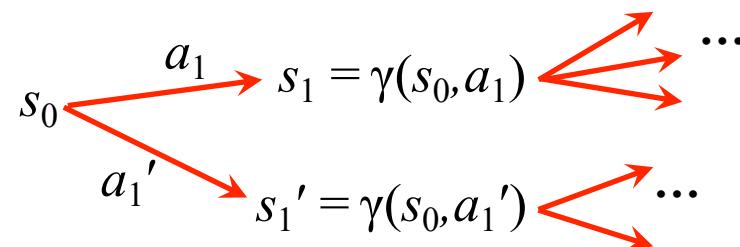
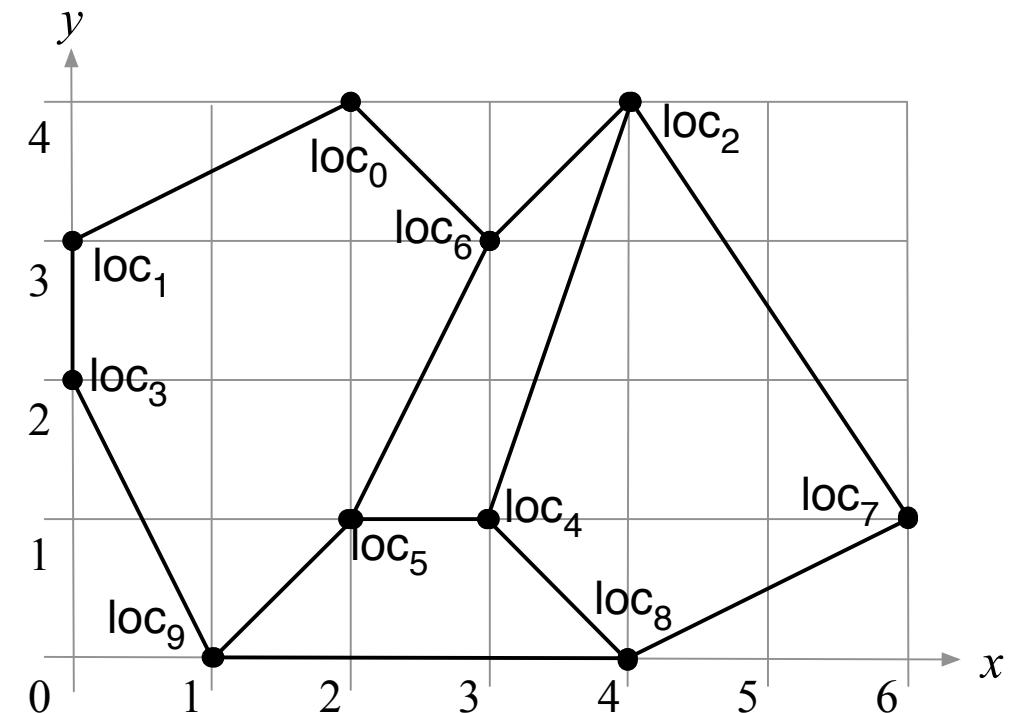
- ▶ planning domain, initial state, *set of goal states*

- *Solution* for P :

- ▶ a plan π such that $\gamma(s_0, \pi) \in S_g$

Representing Σ

- If S and A are small enough
 - ▶ Give each state and action a name
 - ▶ For each s and a , store $\gamma(s,a)$ in a lookup table
- In larger domains, don't represent all states explicitly
 - ▶ Language for describing properties of states
 - ▶ Language for describing how each action changes those properties
 - ▶ Start with initial state, use actions to produce other states

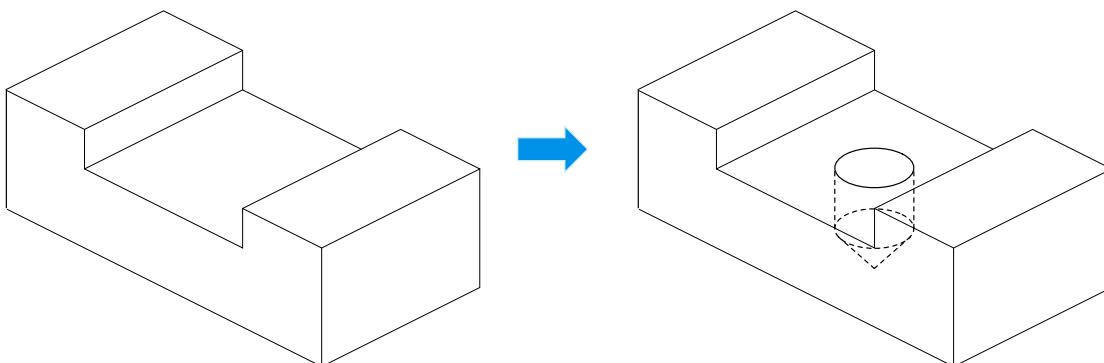


Domain-Specific Representation

- Tailor-made for a specific environment
- **State**: arbitrary data structure
- Action: (head, preconditions, effects, cost)
 - ▶ *head*: name and parameter list
 - Get actions by instantiating the parameters
 - ▶ *preconditions*:
 - Computational tests to predict whether an action can be performed
 - Should be necessary/sufficient for the action to run without error
 - ▶ *effects*:
 - Procedures that modify the current state
 - ▶ *cost*: procedure that returns a number
 - Can be omitted, default is cost = 1

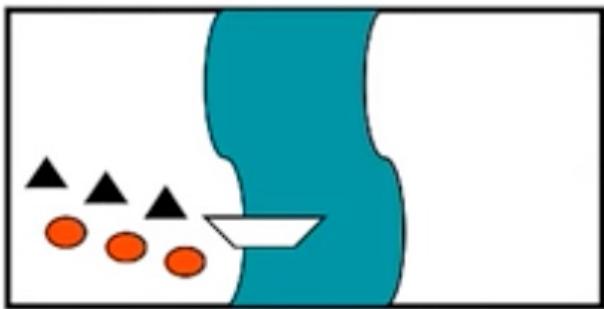
Example

- Drilling holes in a metal workpiece
 - ▶ A state
 - geometric model of the workpiece
 - ▶ *annotated* with dimensions, tolerances, etc.
 - capabilities and status of drilling machine and drill bit
 - ▶ Several actions
 - clamp the workpiece onto the drilling machine
 - load a drill bit into the machine
 - drill a hole
- Name: drill-hole
- Arguments:
 - ▶ ID codes for the machine and drill bit
 - ▶ annotated geometric model of the workpiece
 - ▶ description of the hole to be drilled
- Preconditions
 - ▶ *Capabilities*: can the machine and drill bit produce the desired hole?
 - ▶ *Current state*: Is the drill bit installed? Is the workpiece clamped onto the table? Etc.
- Effects
 - ▶ annotated geometric model of modified workpiece
- Cost
 - ▶ estimate of time or monetary cost



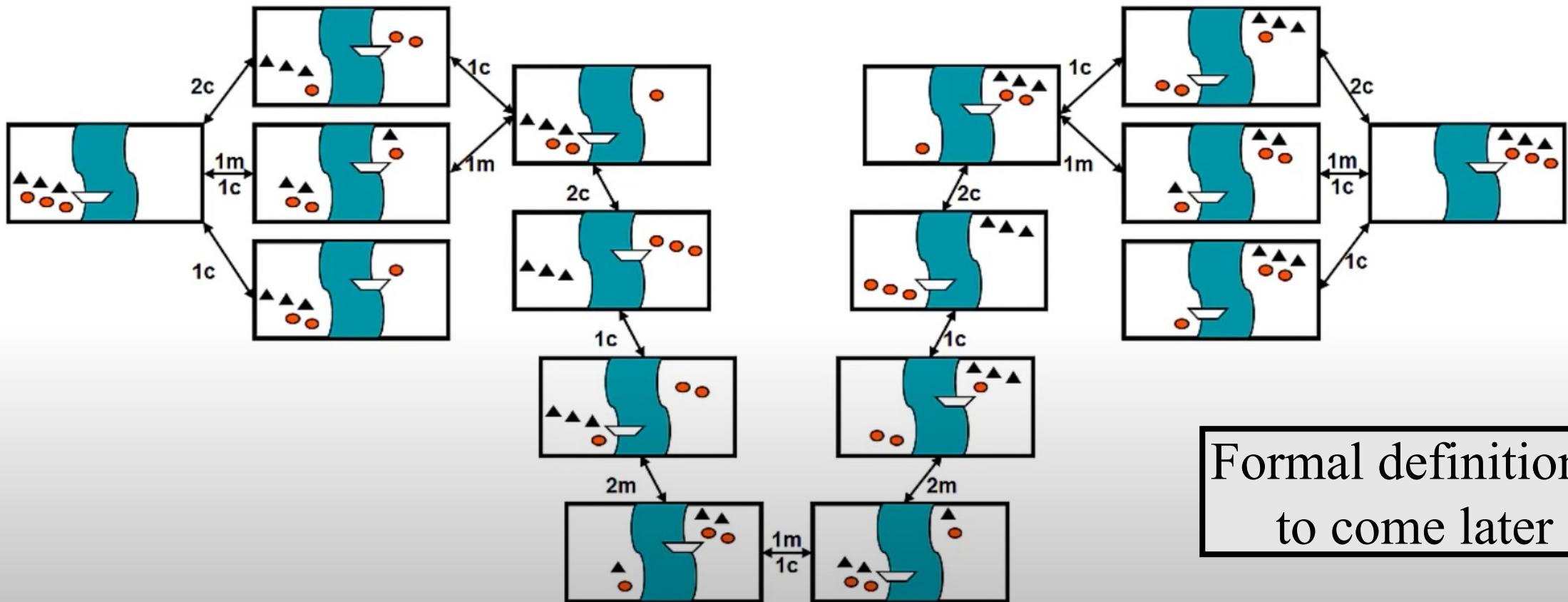
You have
5 minutes!

Toy Problem: Missionaries and Cannibals



On one bank of a river are three missionaries (black triangles) and three cannibals (red circles). There is one boat available that can hold up to two people and that they would like to use to cross the river. If the cannibals ever outnumber the missionaries on either of the river's banks, the missionaries will get eaten. How can the boat be used to safely carry all the missionaries and cannibals across the river?

State-Transition Graph Example: Missionaries and Cannibals



Formal definitions
to come later

Discussion

- Advantage of domain-specific representation?
- Disadvantage?
- Alternative?

You have
3 minutes!

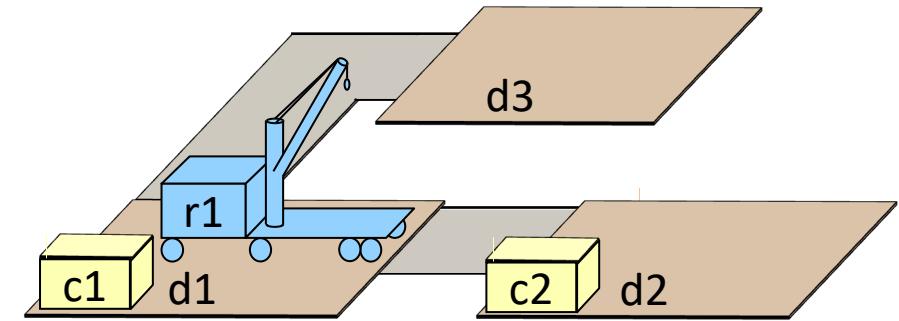
Discussion

- Advantage of domain-specific representation:
 - ▶ use whatever works best for that particular domain
- Disadvantage:
 - ▶ for each new domain, need new representation and deliberation algorithms
- Alternative: *domain-independent* representation
 - ▶ Try to create a “standard format” that can be used for many different planning domains
 - ▶ Deliberation algorithms that work for anything in this format
- *State-variable* representation
 - ▶ Simple formats for describing states and actions
 - ▶ Limited representational capability
 - But easy to compute, easy to reason about
 - ▶ Domain-independent search algorithms and heuristic functions that can be used in all state-variable planning problems

State-Variable Representation

- B : set of symbols called *objects*
 - ▶ names for objects in *the environment*, mathematical constants, ...

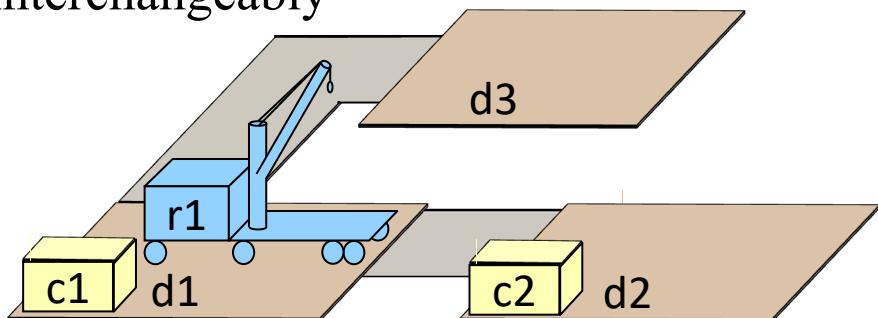
- Example
 - ▶ $B = \text{Robots} \cup \text{Containers} \cup \text{Locs} \cup \{\text{nil}\}$
 - *Robots* = {r1}
 - *Containers* = {c1, c2}
 - *Locs* = {d1, d2, d3}



- B only needs to include **objects that matter** at the current level of abstraction
- Can omit lots of details
 - ▶ physical characteristics of robots, containers, loading docks, roads, ...

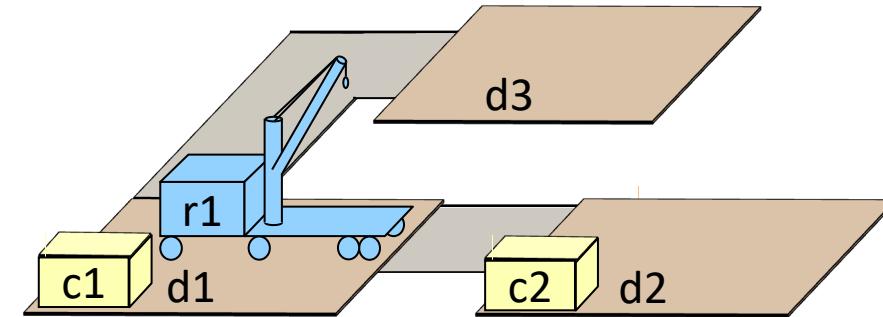
Rigid Properties

- Objects have **two kinds of properties**
 - ▶ *rigid* and *varying*
- *Rigid*: stays the same in every state
 - ▶ Can be described as a **mathematical relation**
 $\text{adjacent} = \{(d1,d2), (d2,d1), (d1,d3), (d3,d1)\}$
 - ▶ Or equivalently, a set of **ground atoms**
 $\text{adjacent}(d1,d2), \text{adjacent}(d2,d1),$
 $\text{adjacent}(d1,d3), \text{adjacent}(d3,d1)$
 - ▶ In practice: the two notations used interchangeably
- Terminology from **first-order logic**:
 - ▶ *atom* \equiv *atomic formula* \equiv *positive literal*
 \equiv predicate symbol with list of arguments
 - e.g., $\text{adjacent}(x,d2)$
 - ▶ an atom is **ground** (or *fully instantiated*) if it contains no variable symbols
 - e.g., $\text{adjacent}(d1,d2)$
 - ▶ **negative literal** \equiv *negated atom* \equiv atom with a negation sign in front of it
 - e.g., $\neg \text{adjacent}(x,d2)$



Varying Properties

- *Varying property* (or *fluent*):
 - a property that may differ in different states
- Represent it using a *state variable*
 - ▶ a term that we can assign a value to
 - e.g., $\text{loc}(r1)$
- Let $X = \{\text{all state variables in the environment}\}$
e.g., $X = \{\text{loc}(r1), \text{loc}(c1), \text{loc}(c2), \text{cargo}(r1)\}$
- *Each state variable $x \in X$ has a range*
 $= \{\text{all values that can be assigned to } x\}$
 - $\text{Range}(\text{loc}(r1)) = \text{Locs}$
 - $\text{Range}(\text{loc}(c1)) = \text{Range}(\text{loc}(c2)) = \text{Robots} \cup \text{Locs}$
 - $\text{Range}(\text{cargo}(r1)) = \text{Containers} \cup \{\text{nil}\}$
- To abbreviate the “range” notation often just say things like
 - ▶ $\text{loc}(r1) \in \text{Locs}$
 - ▶ $\text{loc}(c1), \text{loc}(c2) \in \text{Robots} \cup \text{Locs}$



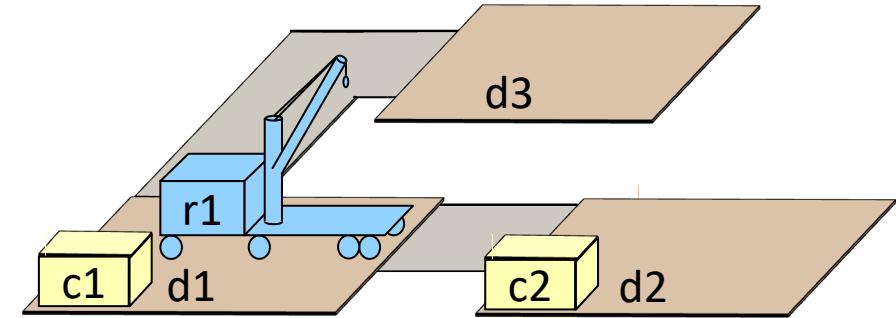
Instead of “domain”,
to avoid confusion
with planning domains

States as Functions

- Represent each state s as a function that assigns values to state variables
 - ▶ For each state variable x , $s(x)$ is one x 's possible values

$$s_1(\text{loc}(r1)) = d1, \quad s_1(\text{cargo}(r1)) = \text{nil},$$

$$s_1(\text{loc}(c1)) = d1, \quad s_1(\text{loc}(c2)) = d2$$



- Mathematically, a function is a set of ordered pairs
$$s_1 = \{(\text{loc}(r1), d1), (\text{cargo}(r1), \text{nil}), (\text{loc}(c1), d1), (\text{loc}(c2), d2)\}$$
- Equivalently, write it as a set of *ground positive literals* (or *ground atoms*):
$$s_1 = \{\text{loc}(r1)=d1, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1, \text{loc}(c2)=d2\}$$
 - ▶ Here, we're using '=' as a predicate symbol

Action Templates

- Action template: a parameterized set of actions

$\alpha = (\text{head}(\alpha), \text{pre}(\alpha), \text{eff}(\alpha), \text{cost}(\alpha))$

- ▶ $\text{head}(\alpha)$: *name, parameters*
- ▶ $\text{pre}(\alpha)$: *precondition literals*
- ▶ $\text{eff}(\alpha)$: *effect literals*
- ▶ $\text{cost}(\alpha)$: *a number* (optional, default is 1)

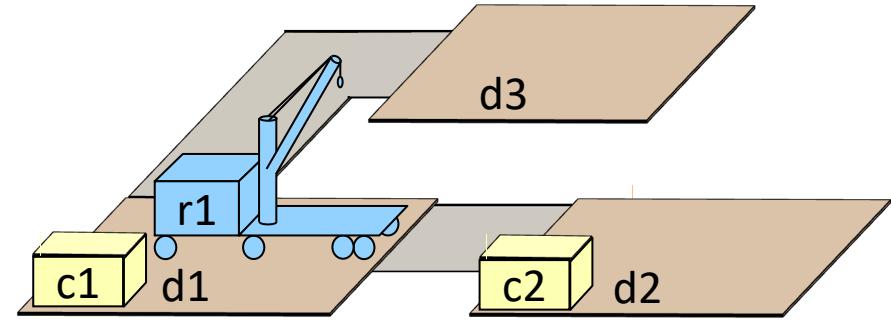
- e.g.,

- ▶ $\text{head}(\alpha) = \text{take}(r,l,c)$
- ▶ $\text{pre}(\alpha) = \{\text{cargo}(r)=\text{nil}, \text{loc}(r)=l, \text{loc}(c)=l\}$
- ▶ $\text{eff}(\alpha) = \{\text{cargo}(r)=c, \text{loc}(c)=r\}$

- Each parameter has a **range of possible values**:

- ▶ $\text{Range}(r) = \text{Robots} = \{r1\}$
- ▶ $\text{Range}(l) = \text{Locs} = \{d1, d2, d3\}$
- ▶ $\text{Range}(l) = \text{Range}(m) = \text{Locs} = \{d1, d2, d3\}$
- ▶ $\text{Range}(c) = \text{Containers} = \{c1, c2\}$

- But we'll usually write it more like **pseudocode**



move(r, l, m)

pre: $\text{loc}(r)=l$, $\text{adjacent}(l, m)$
eff: $\text{loc}(r) \leftarrow m$

take(r, l, c)

pre: $\text{cargo}(r)=\text{nil}$, $\text{loc}(r)=l$, $\text{loc}(c)=l$
eff: $\text{cargo}(r) \leftarrow c$, $\text{loc}(c) \leftarrow r$

put(r, l, c)

pre: $\text{loc}(r)=l$, $\text{loc}(c)=r$
eff: $\text{cargo}(r) \leftarrow \text{nil}$, $\text{loc}(c) \leftarrow l$

$r \in \text{Robots} = \{r1\}$

$l, m \in \text{Locs} = \{d1, d2, d3\}$

$c \in \text{Containers} = \{c1, c2\}$

Actions

- \mathcal{A} = set of action templates

$\text{move}(r,l,m)$

pre: $\text{loc}(r)=l$, $\text{adjacent}(l, m)$
eff: $\text{loc}(r) \leftarrow m$

$\text{take}(r,l,c)$

pre: $\text{cargo}(r)=\text{nil}$, $\text{loc}(r)=l$, $\text{loc}(c)=l$
eff: $\text{cargo}(r) \leftarrow c$, $\text{loc}(c) \leftarrow r$

$\text{put}(r,l,c)$

pre: $\text{loc}(r)=l$, $\text{loc}(c)=r$
eff: $\text{cargo}(r) \leftarrow \text{nil}$, $\text{loc}(c) \leftarrow l$

$r \in \text{Robots} = \{\text{r1}\}$

$l,m \in \text{Locs} = \{\text{d1}, \text{d2}, \text{d3}\}$

$c \in \text{Containers} = \{\text{c1}, \text{c2}\}$

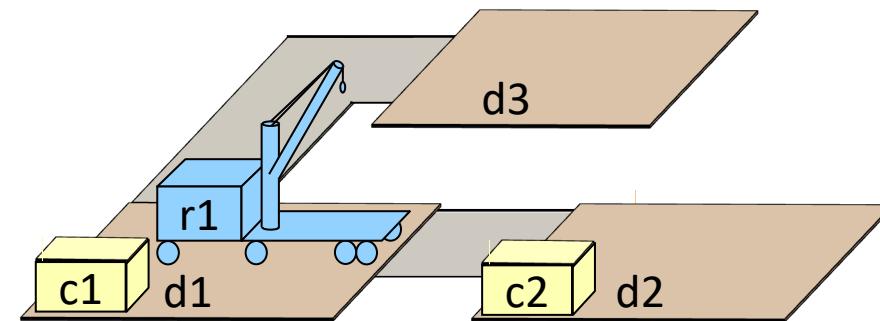
- Action: *ground instance* of an $\alpha \in \mathcal{A}$

► replace each parameter with something in its range

- \mathcal{A} = {all actions we can get from \mathcal{A} }
= {all ground instances of members of \mathcal{A} }

$\text{move}(\text{r1}, \text{d1}, \text{d2})$

pre: $\text{loc}(\text{r1})=\text{d1}$, $\text{adjacent}(\text{d1}, \text{d2})$
eff: $\text{loc}(\text{r1}) \leftarrow \text{d2}$



Actions

- \mathcal{A} = set of action templates

$\text{move}(r,l,m)$

pre: $\text{loc}(r)=l$, $\text{adjacent}(l, m)$
eff: $\text{loc}(r) \leftarrow m$

$\text{take}(r,l,c)$

pre: $\text{cargo}(r)=\text{nil}$, $\text{loc}(r)=l$, $\text{loc}(c)=l$
eff: $\text{cargo}(r) \leftarrow c$, $\text{loc}(c) \leftarrow r$

$\text{put}(r,l,c)$

pre: $\text{loc}(r)=l$, $\text{loc}(c)=r$
eff: $\text{cargo}(r) \leftarrow \text{nil}$, $\text{loc}(c) \leftarrow l$

$r \in \text{Robots} = \{\text{r1}\}$

$l,m \in \text{Locs} = \{\text{d1,d2,d3}\}$

$c \in \text{Containers} = \{\text{c1,c2}\}$

- Action: *ground instance* of an $\alpha \in \mathcal{A}$
 - ▶ replace each parameter with something in its range
- $A = \{\text{all actions we can get from } \mathcal{A}\}$
 $= \{\text{all ground instances of members of } \mathcal{A}\}$

move(r1,d1,d2)

pre: $\text{loc(r1)}=\text{d1}$, adjacent(d1,d2)
eff: $\text{loc(r1)} \leftarrow \text{d2}$

Poll. Let:

$\mathcal{A} = \{\text{the action templates on this page}\}$

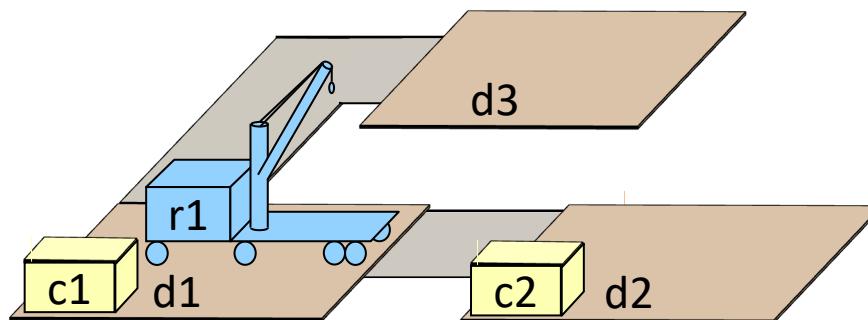
$A = \{\text{all ground instances of members of } \mathcal{A}\}$

How many **move** actions in A ?

Applicability

Poll: How many move actions are applicable in s_1 ?

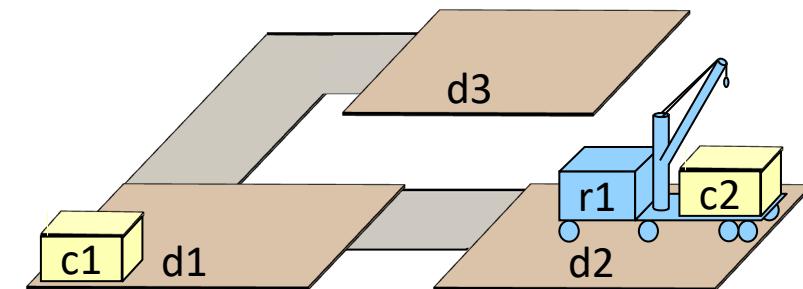
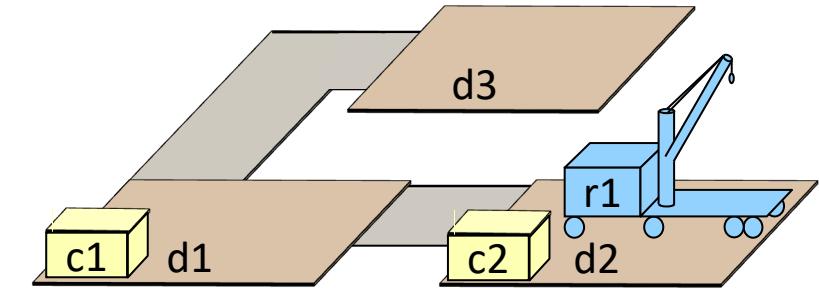
- a is *applicable* in s if
 - ▶ for every positive literal $l \in \text{pre}(a)$,
 $l \in s$ or l is in one of the rigid relations
 - ▶ for every negative literal $\neg l \in \text{pre}(a)$,
 $l \notin s$ and l isn't in any of the rigid relations
- Rigid relation
 $\text{adjacent} = \{(d1,d2), (d2,d1), (d1,d3), (d3,d1)\}$
- *State*
 $s_1 = \{\text{loc}(r1)=d1, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1\}$



- Action template
 $\text{move}(r,l,m)$
pre: $\text{loc}(r)=l$, $\text{adjacent}(l, m)$
eff: $\text{loc}(r) \leftarrow m$
 $\text{Range}(r) = \text{Robots}$
 $\text{Range}(l) = \text{Range}(m) = \text{Locs}$
- Applicable:
 $\text{move}(r1,d1,d2)$
pre: $\text{loc}(r1)=d1$, $\text{adjacent}(d1,d2)$
eff: $\text{loc}(r1) \leftarrow d2$
- Not applicable:
 $\text{move}(r1,d2,d1)$
pre: $\text{loc}(r1)=d2$, $\text{adjacent}(d2,d1)$
eff: $\text{loc}(r1) \leftarrow d1$

State-Transition Function

- If a is applicable in s :
 - ▶ $\gamma(s, a) = \{\text{every literal } x=w \text{ that's in } \text{eff}(a)\}$
U {every literal $x=w$ in s such that x isn't on the left-hand side of a literal in $\text{eff}(a)$ }
- $s_2 = \{\text{loc}(r1)=d2, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1, \text{loc}(c2)=d2\}$
- $\text{take}(r1, d2, c2)$
 - pre: $\text{cargo}(r1)=\text{nil}, \text{loc}(r1)=d2, \text{loc}(c2)=d2$
 - eff: $\text{cargo}(r1) \leftarrow c2, \text{loc}(c2) \leftarrow r1$
- $\gamma(s_2, \text{take}(r1, d2, c2)) =$
{loc(r1)=d2, cargo(r1)=c2, loc(c1)=d1, loc(c2)=r1}



State-Variable Planning Domain

- Let

B = finite set of objects

R = finite set of rigid relations over B

X = finite set of state variables

- for every state variable x , $\text{Range}(x) \subseteq B$

S = state space over X

= {all value-assignment functions that have sensible interpretations}

\mathcal{A} = finite set of action templates

- for every parameter y , $\text{Range}(y) \subseteq B$

A = {all ground instances of action templates in \mathcal{A} }

$\gamma(s,a) = \{(x,w) \mid \text{eff}(a) \text{ contains the effect } x \leftarrow w\}$

$\cup \{(x,w) \in s \mid x \text{ isn't the target of any effect in } \text{eff}(a)\}$

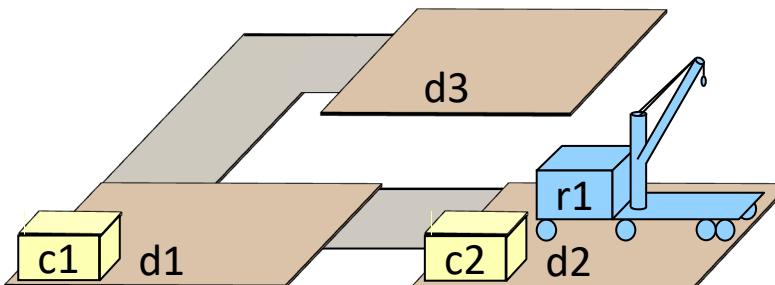


- Then $\Sigma = (S, A, \gamma)$ is a *state-variable planning domain*

Interpretations

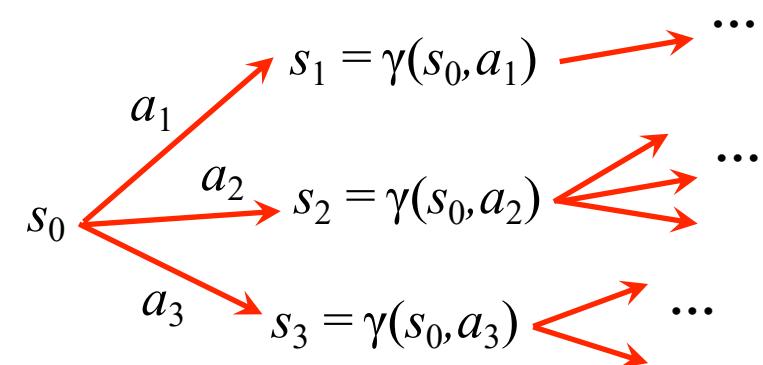
- Let s be a value-assignment function
 - s is a state only if the values make sense in the planning domain we're trying to represent
 - (relation to *model theory*)
- Can $\text{loc}(c1)=r1$ if $\text{cargo}(r1)=\text{nil}$?
 - Not in our intended *interpretation*
 - Mapping of symbols to what they represent

$$s_2 = \{\text{loc}(r1)=d2, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1, \text{loc}(c2)=d2\}$$



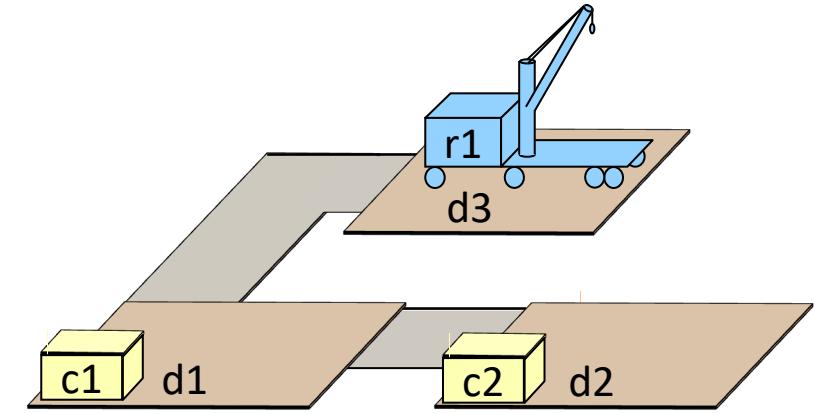
- Can both $\text{loc}(c1)=r1$ and $\text{loc}(c2)=r1$?
 - In our intended interpretation, can a robot carry more than one object at a time?

- How to enforce the intended interpretation?
 - Explicitly
 - Mathematical axioms
 - Integrity constraints
- Implicitly
 - Write an initial state s_0 that satisfies the interpretation
 - Write the actions in such a way that whenever s satisfies the interpretation, $\gamma(s,a)$ will too



Plans

- *Plan:* sequence of actions $\pi = \langle a_1, a_2, \dots, a_n \rangle$
 - ▶ $\text{cost}(\pi) = \sum_i \text{cost}(a_i)$
- π is *applicable* in s_0 if the actions can be applied in the order given, i.e.,
 - ▶ a_1 is applicable in s_0 , so let $s_1 = \gamma(s_0, a_1)$,
 - ▶ a_2 is applicable in s_1 , so let $s_2 = \gamma(s_1, a_2)$,
 - ▶ a_3 is applicable in s_2 , so let $s_3 = \gamma(s_2, a_3)$,
 - ▶ ...
 - ▶ a_n is applicable in s_{n-1} ,
 - so let $s_n = \gamma(s_{n-1}, a_n)$
- If so, then $\gamma(s_0, \pi) = s_n$

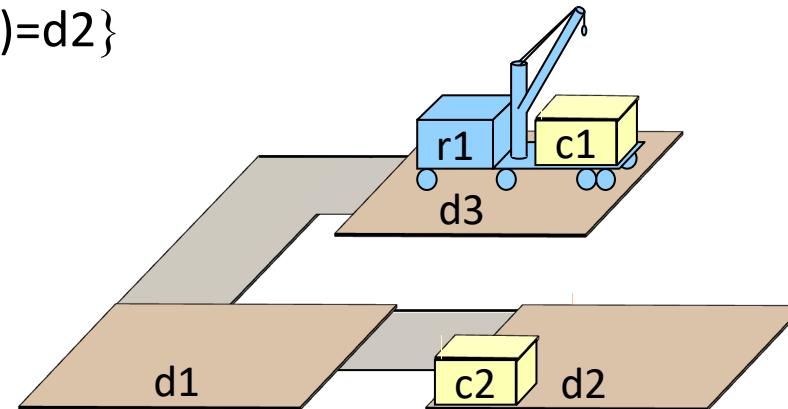


Suppose

- $s_0 = \{\text{loc}(r1)=d3, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1, \text{loc}(c2)=d2\}$
- $\pi = \langle \text{move}(r1, d3, d1), \text{take}(r1, d1, c1), \text{move}(r1, d1, d3) \rangle$

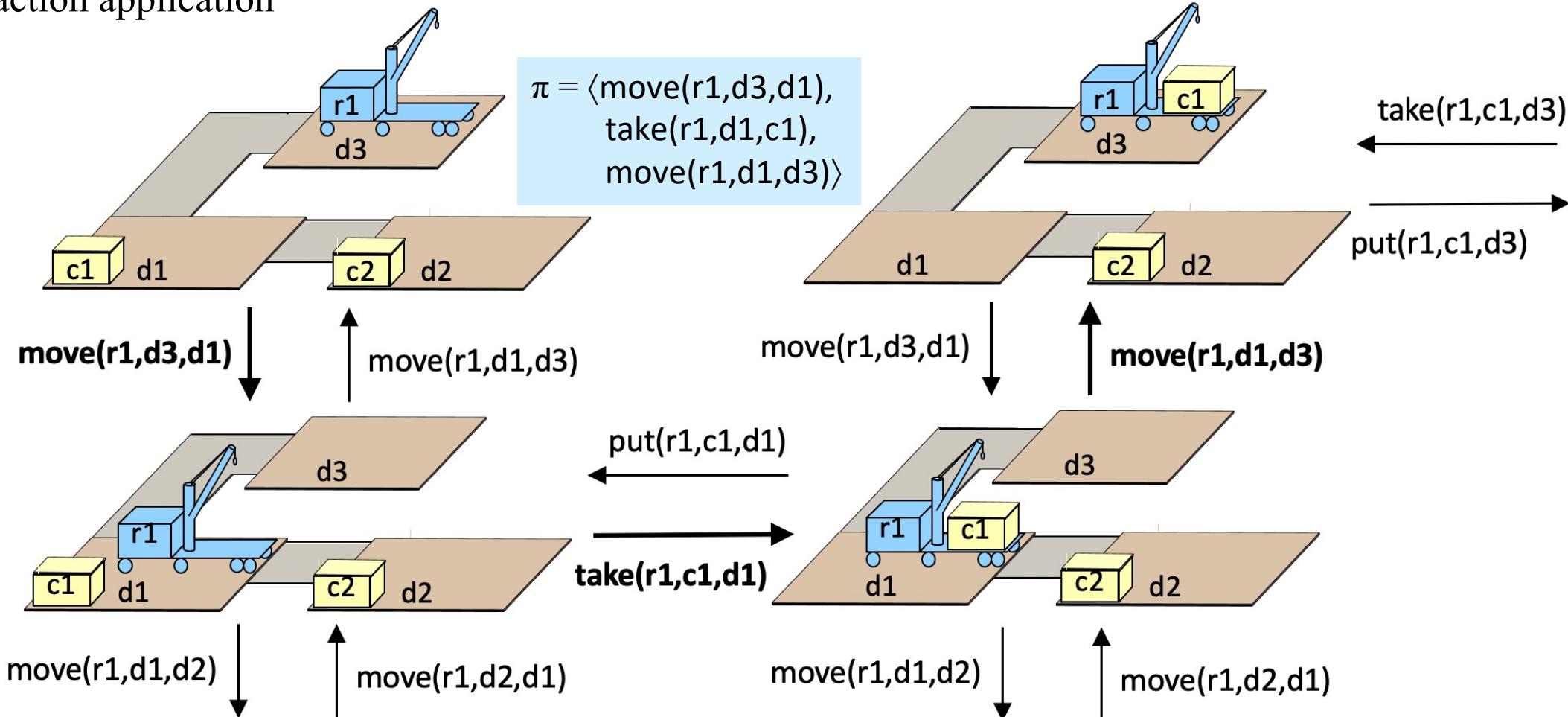
Then

- $\gamma(s_0, \pi) = \{\text{loc}(r1)=d3, \text{cargo}(r1)=c1, \text{loc}(c1)=r1, \text{loc}(c2)=d2\}$



State Space

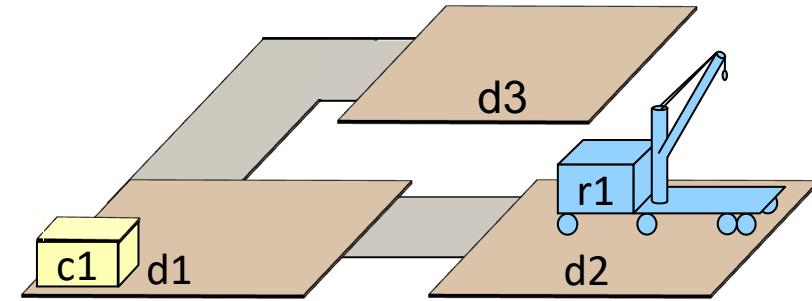
- *State Space*: a directed graph
 - ▶ Nodes = states of the world
 - ▶ Arcs: action application



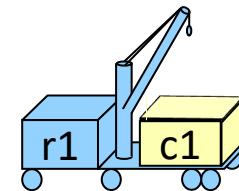
Planning Problems

- *State-variable planning problem* $P = (\Sigma, s_0, g)$
 - state-variable representation of a classical planning problem
 - ▶ $\Sigma = (S, A, \gamma)$ is a state-variable planning domain
 - ▶ $s_0 \in S$ is the *initial state*
 - ▶ g is a set of ground literals called the *goal*
- $S_g = \{\text{all states in } S \text{ that satisfy } g\}$
= $\{s \in S \mid s \cup R \text{ contains every positive literal in } g, \text{ and none of the negative literals in } g\}$
- If $\gamma(s_0, \pi)$ satisfies g then π is a *solution* for P

$\text{adjacent} = \{(d1, d2), (d2, d1), (d1, d3), (d3, d1)\}$



$s_0 = \{\text{loc}(r1)=d2, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1\}$



$g = \{\text{cargo}(r1)=c1\}$

$\pi = \langle \text{move}(r1, d2, d1),$
 $\quad \text{take}(r1, d1, c1) \rangle$

Poll: How many solutions of length 3?

Classical Representation

- Motivation
 - ▶ The field of AI planning started out as **automated theorem proving**
 - ▶ It still uses a lot of that **notation**
- Classical representation is equivalent to state-variable representation
 - ▶ Represents both rigid and varying properties using logical predicates

$\text{adjacent}(l,m)$

- location l is adjacent to m

$\text{loc}(r) = l \rightarrow \text{loc}(r,l)$

- robot r is at location l

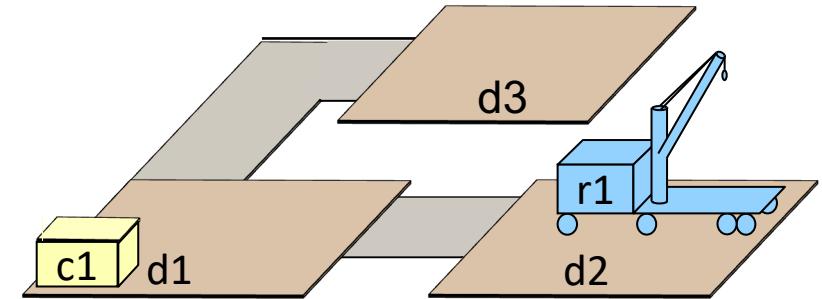
$\text{loc}(c) = r \rightarrow \text{loc}(c,r)$

- container c is on robot r

$\text{cargo}(r) = c \rightarrow \text{loaded}(r)$

- there's a container on r

why not $\text{loaded}(r,c)$?



- State s = a set of ground atoms
 - ▶ Atom a is true in s iff $a \in s$

why?

$s_0 = \{\text{adjacent}(d1,d2), \text{adjacent}(d2,d1), \text{adjacent}(d1,d3), \text{adjacent}(d3,d1), \text{loc}(c1,d1), \text{loc}(r1,d2)\}$

Poll: Should s_0 also contain
 $\neg \text{loaded}(r1)$?

Classical planning operators

- Action templates

$\text{move}(r,l,m)$

pre: $\text{loc}(r)=l$, $\text{adjacent}(l, m)$
eff: $\text{loc}(r) \leftarrow m$

$\text{take}(r,l,c)$

pre: $\text{cargo}(r)=\text{nil}$, $\text{loc}(r)=l$, $\text{loc}(c)=l$
eff: $\text{cargo}(r) \leftarrow c$, $\text{loc}(c) \leftarrow r$

$\text{put}(r,l,c)$

pre: $\text{loc}(r)=l$, $\text{loc}(c)=r$
eff: $\text{cargo}(r) \leftarrow \text{nil}$, $\text{loc}(c) \leftarrow l$

$\text{Range}(r) = \text{Robots} = \{r1\}$

$\text{Range}(l) = \text{Range}(m) = \text{Locs} = \{d1,d2,d3\}$

$\text{Range}(c) = \text{Containers} = \{c1,c2\}$

- Classical planning operators

$\text{move}(r,l,m)$

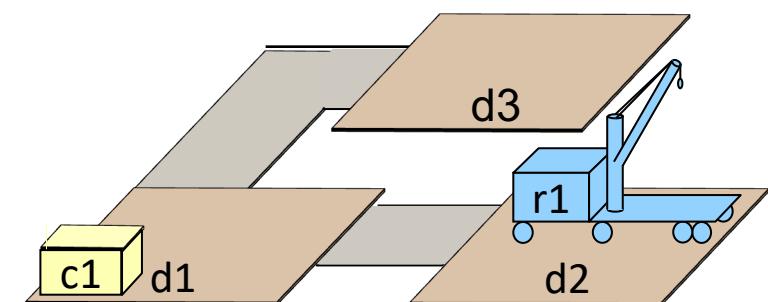
pre: $\text{loc}(r,l)$, $\text{adjacent}(l, m)$
eff: $\neg\text{loc}(r,l)$, $\text{loc}(r;m)$

$\text{take}(r,l,c)$

pre: $\neg\text{loaded}(r)$, $\text{loc}(r,l)$, $\text{loc}(c,l)$
eff: $\text{loaded}(r)$, $\neg\text{loc}(c,l)$, $\text{loc}(c,r)$

$\text{put}(r,l,c)$

pre: $\text{loc}(r,l)$, $\text{loc}(c,r)$
eff: $\neg\text{loaded}(r)$, $\text{loc}(c,l)$



Actions

- Planning operator:

o : move(r,l,m)

pre: loc(r,l), adjacent(l,m)

eff: \neg loc(r,l), loc(r,m)

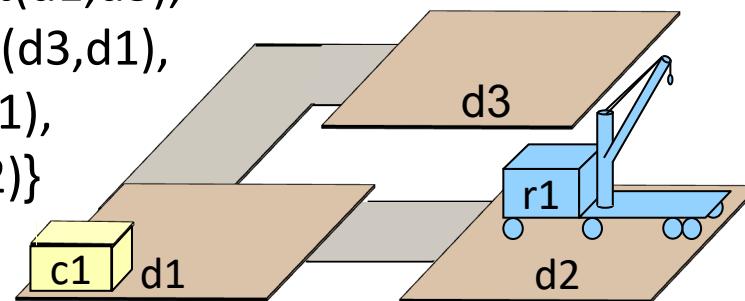
- Action:

a_1 : move(r_1,d_2,d_1)

pre: loc(r_1,d_2), adjacent(d_2,d_1)

eff: \neg loc(r_1,d_2), loc(r_1,d_1)

$s_0 = \{\text{adjacent}(d_1,d_2),$
 $\text{adjacent}(d_2,d_1),$
 $\text{adjacent}(d_1,d_3),$
 $\text{adjacent}(d_3,d_1),$
 $\text{loc}(c_1,d_1),$
 $\text{loc}(r_1,d_2)\}$



- Let

➤ $\text{pre}^-(a) = \{a\text{'s negated preconditions}\}$

➤ $\text{pre}^+(a) = \{a\text{'s non-negated preconditions}\}$

- a is applicable in state s iff

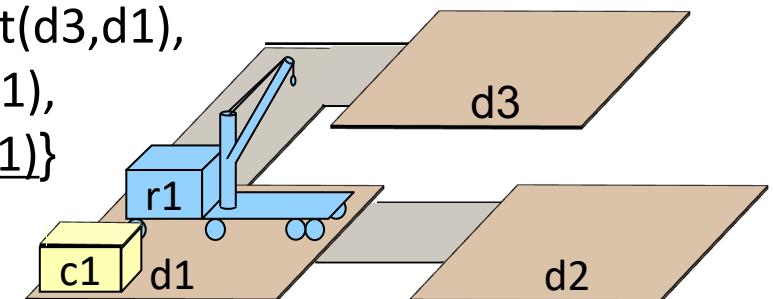
$s \cap \text{pre}^-(a) = \emptyset$ and $\text{pre}^+(a) \subseteq s$

- If a is applicable in s then

➤ $\gamma(s,a) = (s \setminus \text{eff}^-(a)) \cup \text{eff}^+(a)$

meaning?

$\gamma(s_0, a_1) = \{\text{adjacent}(d_1,d_2),$
 $\text{adjacent}(d_2,d_1),$
 $\text{adjacent}(d_1,d_3),$
 $\text{adjacent}(d_3,d_1),$
 $\text{loc}(c_1,d_1),$
 $\underline{\text{loc}(r_1,d_1)}\}$



Discussion

$x(b_1, \dots, b_{n-1}) = b_n$ becomes $P_x(b_1, \dots, b_{n-1}, b_n)$

State-variable
rep.

Classical
rep.

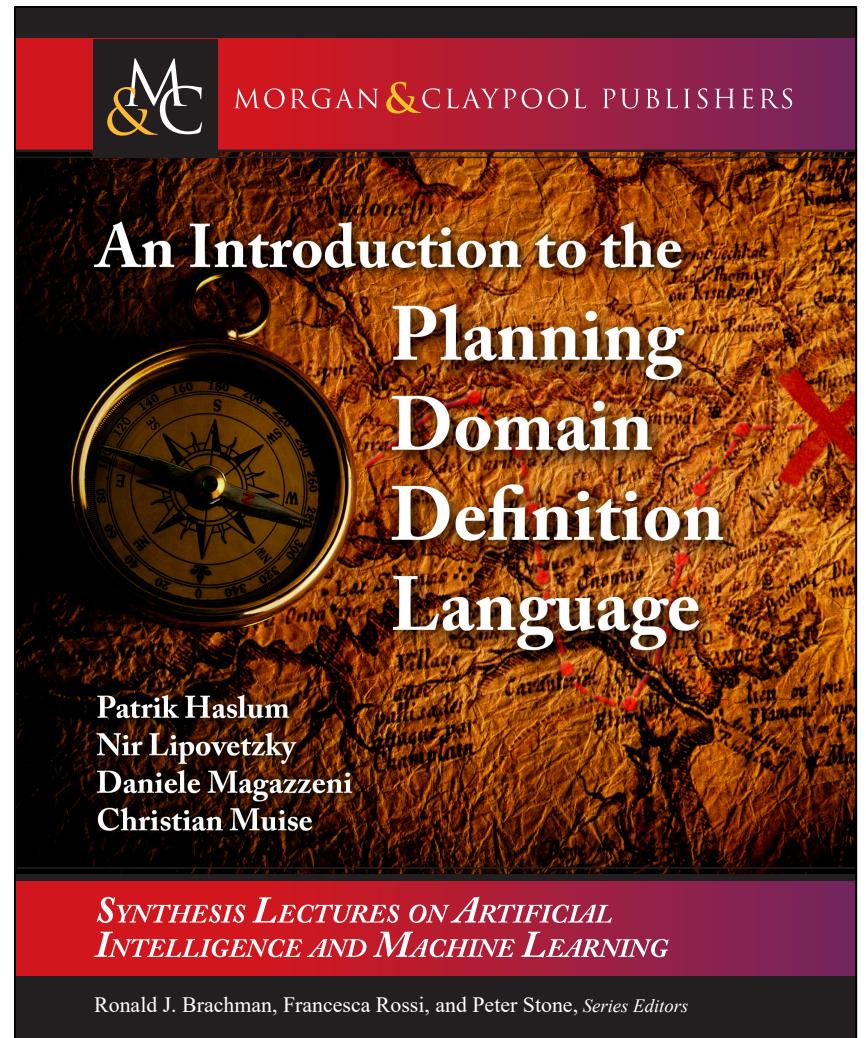
$P(b_1, \dots, b_k)$ becomes $x_P(b_1, \dots, b_k) = 1$

- Equivalent expressive power
 - ▶ Each can be converted to the other in linear time and space
- Classical representation
 - ▶ More natural for logicians
 - ▶ Don't require single-valued functions
- State variables
 - ▶ More natural for engineers and computer programmers
 - ▶ When changing a value, don't have to explicitly delete the old one
- Historically, classical representation has been more widely used
 - ▶ That's starting to change

Poll: Could we instead use
 $x_P(b_1, \dots, b_{k-1}) = b_k$?

PDDL

- Language for defining planning domains and problems
- Original version of PDDL ≈ 1996
 - ▶ Just classical planning
- Multiple revisions and extensions
 - ▶ Different subsets accommodate different kinds of planning
- We'll discuss the classical-planning subset
 - ▶ Chapter 2 of the PDDL book



Example domain

- Classical representation:

$\text{move}(r,l,m)$

Precond: $\text{loc}(r,l)$, $\text{adjacent}(l,m)$

Effects: $\neg\text{loc}(r,l)$, $\text{loc}(r,m)$

$\text{take}(r,l,c)$

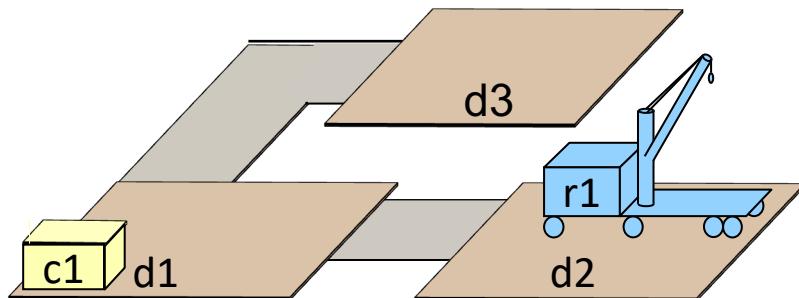
Precond: $\text{loc}(r,l)$, $\text{loc}(c,l)$, $\neg\text{loaded}(r)$

Effects: $\text{loc}(c,r)$, $\neg\text{loc}(c,l)$, $\text{loaded}(r)$

$\text{put}(r,l,c)$

Precond: $\text{loc}(r,l)$, $\text{loc}(c,r)$

Effects: $\text{loc}(c,l)$, $\neg\text{loc}(c,r)$, $\neg\text{loaded}(r)$



```
(define (domain example-domain-1)
  (requirements :negative-preconditions)

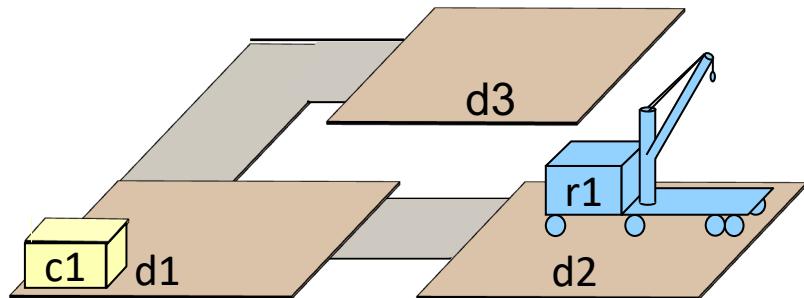
  (:action move
    :parameters (?r ?l ?m)
    :precondition (and (loc ?r ?l)
                        (adjacent ?l ?m))
    :effect (and (not (loc ?r ?l))
                  (loc ?r ?m)))

  (:action take
    :parameters (?r ?l ?c)
    :precondition (and (loc ?r ?l)
                        (loc ?c ?l)
                        (not (loaded ?r)))
    :effect (and (not (loc ?r ?l))
                  (loc ?r ?m)))

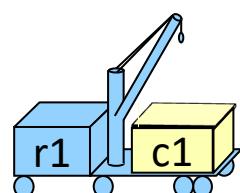
  (:action put
    :parameters (?r ?l ?c)
    :precondition (and (loc ?r ?l)
                        (loc ?c ?r))
    :effect (and (loc ?c ?l)
                  (not (loc ?c ?r))
                  (not (loaded ?r))))))
```

Example problem

- Classical representation:



$s_0 = \{\text{adjacent}(d1, d2), \text{adjacent}(d2, d1), \text{adjacent}(d1, d3), \text{adjacent}(d3, d1), \text{loc}(c1, d1), \text{loc}(r1, d2)\}$



$g = \{\text{loc}(c1, r1)\}$

```
(define (problem example-problem-1)
  (:domain example-domain-1))
```

```
(:init
  (adjacent d1 d2)
  (adjacent d2 d1)
  (adjacent d1 d3)
  (adjacent d3 d1)
  (loc c1 d1)
  (loc r1 d2))
```

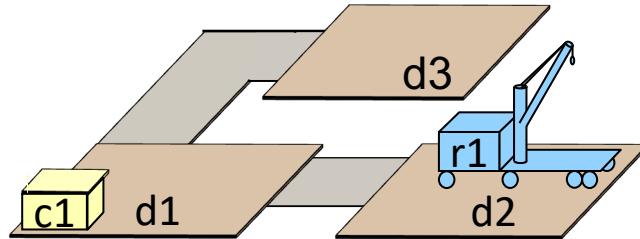
```
(:goal (loc c1 r1)))
```

Example typed domain

State-variable planning:

- Sets of objects
 - ▶ $B = \text{Movable_objects} \cup \text{Locs}$
 - ▶ $\text{Movable_objects} = \text{Robots} \cup \text{Containers}$
 - ▶ $\text{Robots} = \{r_1\}$
 - ▶ $\text{Containers} = \{c_1\}$
 - ▶ $\text{Locs} = \{d_1, d_2, d_3\}$

```
(define (domain example-domain-2)
  (:requirements
    :negative-preconditions
    :typing)
  (:types
    [location movable-obj - object
     robot container - movable-obj] like saying
     Locations, Movable_objects ⊆ B
     Robots, Containers ⊆ Movable_objects
  (:predicates
    (loc ?r - movable-obj
        ?l - location)
    (loaded ?r - robot)
    (adjacent ?l ?m - location))
```



- Parameter ranges
 - ▶ $r \in \text{Robots}$
 - ▶ $l, m \in \text{Locs}$
 - ▶ $c \in \text{Containers}$

```
(:action move
  :parameters (?r - robot
               ?l ?m - location)
  :precondition (and (loc ?r ?l)
                      (adjacent ?l ?m))
  :effect (and (not (loc ?r ?l))
                (loc ?r ?m)))

(:action take
  :parameters (?r - robot
               ?l - location
               ?c - container)
  :precondition (and (loc ?r ?l)
                      (loc ?c ?l)
                      (not (loaded ?r)))
  :effect (and (not (loc ?r ?l))
                (loc ?r ?m)))

(:action put
  :parameters (?r - robot
               ?l - location
               ?c - container)
  :precondition (and (loc ?r ?l)
                      (loc ?c ?r))
  :effect (and (loc ?c ?l)
                (not (loc ?c ?r))
                (not (loaded ?r)))))

like saying  $r \in \text{Robots}$ ,  

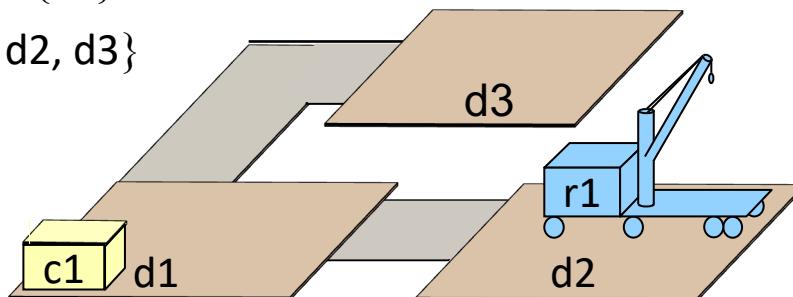
 $l \in \text{Locs}$ ,  

 $c \in \text{Containers}$ 
```

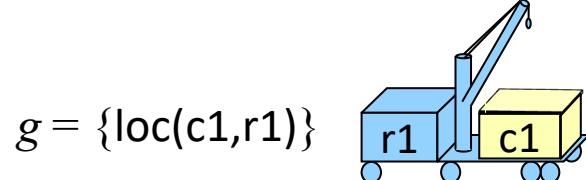
Example typed problem

State-variable planning:

- Sets of objects
 - ▶ $B = \text{Movable_objects} \cup \text{Locs}$
 - ▶ $\text{Movable_objects} = \text{Robots} \cup \text{Containers}$
 - ▶ $\text{Robots} = \{r1\}$
 - ▶ $\text{Containers} = \{c1\}$
 - ▶ $\text{Locs} = \{d1, d2, d3\}$



$s_0 = \{\text{adjacent}(d1, d2), \text{adjacent}(d2, d1), \text{adjacent}(d1, d3), \text{adjacent}(d3, d1), \text{loc}(c1, d1), \text{loc}(r1, d2)\}$



$g = \{\text{loc}(c1, r1)\}$

```
(define (problem example-problem-2)
  (:domain example-domain-2))

  (:objects
    r1 - robot
    c1 - container
    d1 d2 d3 - location)

  (:init
    (adjacent d1 d2)
    (adjacent d2 d1)
    (adjacent d1 d3)
    (adjacent d3 d1)
    (loc c1 d1)
    (loc r1 d2))

  (:goal (loc c1 r1)))
```

Summary

Section 2.1 of Ghallab *et al.* (2016)

- State-Variable Representation
 - ▶ State-transition systems, classical planning assumptions
 - ▶ Classical planning problems, plans, solutions
 - ▶ Objects, rigid properties
 - ▶ Varying properties, state variables, states as functions
 - ▶ Action templates, actions, applicability, γ
 - ▶ State-variable planning domains, plans, problems, solutions
 - ▶ Comparison with classical representation

Chapter 2 of Haslum *et al.* (2019)

- Classical fragment of PDDL
 - ▶ Planning domains, planning problems
 - ▶ untyped, typed

Outline

Chapter 2, part *a* (chap2a.pdf):

- Next →*
- 2.1 State-variable representation
 - Comparison with PDDL
 - 2.2 Forward state-space search
 - 2.6 Incorporating planning into an actor
-

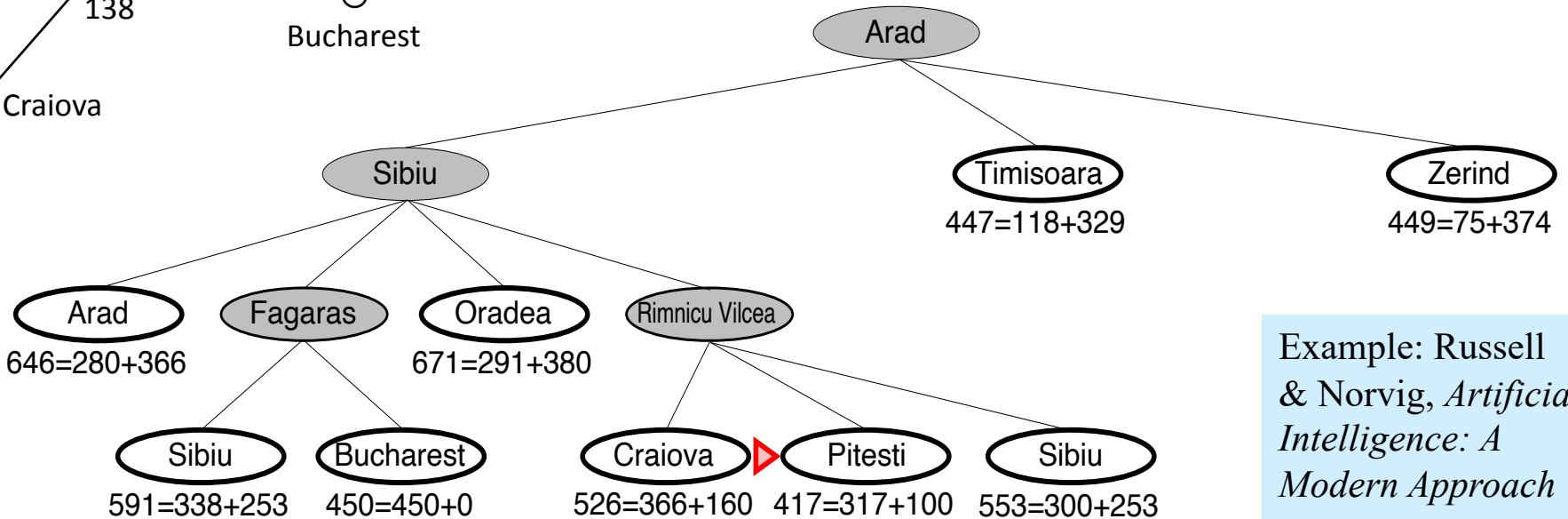
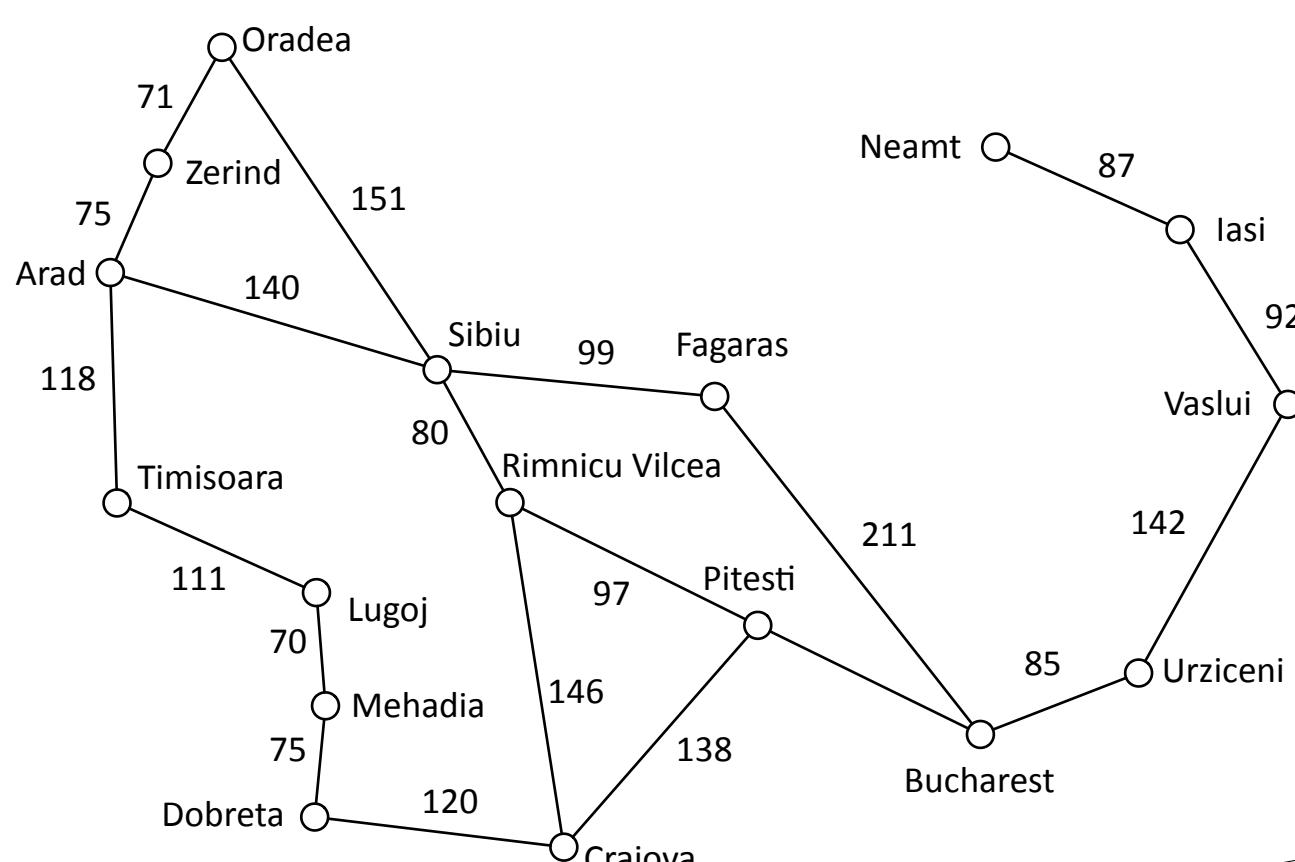
Chapter 2, part *b* (chap2b.pdf):

- 2.3 Heuristic functions
 - 2.4 Backward search
 - 2.5 Plan-space search
-

Additional slides:

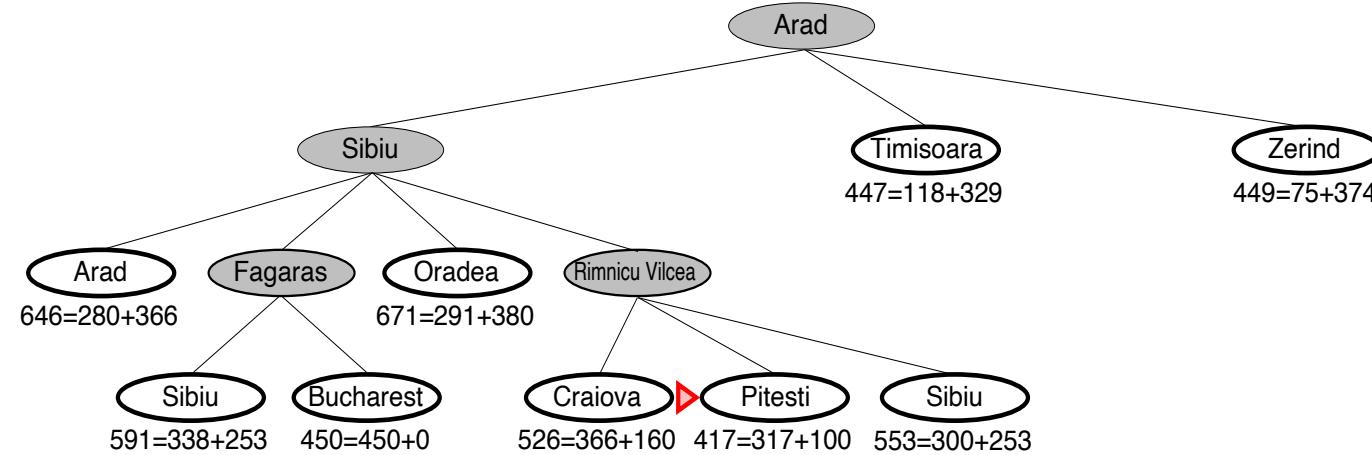
- 2.7.7 HTN_planning.pdf
- 2.7.8 LTL_planning.pdf

Planning as Search



Example: Russell & Norvig, *Artificial Intelligence: A Modern Approach*

Search-Tree Terminology



- **Node** \approx a pair $v = (\pi, s)$, where $s = \gamma(s_0, \pi)$
 - ▶ In practice, v will contain other things too
 - $\text{depth}(v)$, $\text{cost}(\pi)$, pointers to parent and children, ...
 - ▶ π isn't always stored explicitly, can be computed from the parent pointers
- **children** of $v = \{(\pi.a, \gamma(s, a)) \mid a \text{ is applicable in } s\}$
- **successors** or **descendants** of v :
children, children of children, etc.

- **ancestors** of v
= {nodes that have v as a successor}
- **initial** or **starting** or **root** node $v_0 = (\langle \rangle, s_0)$
 - ▶ root of the search tree
- **path** in the search space: sequence of nodes $\langle v_0, v_1, \dots, v_n \rangle$ such that each v_i is a child of v_{i-1}
- **height** of search space
= length of longest acyclic path from v_0
- **depth** of v
= $\text{length}(\pi) = \text{length of path from } v_0 \text{ to } v$
- **branching factor** of v
= number of children of v
- **branching factor** of a search tree
= max branching factor of the nodes
- **expand** v : generate all children

Forward Search

Forward-search (Σ, s_0, g)

$s \leftarrow s_0; \pi \leftarrow \langle \rangle$

loop

 if s satisfies g then return π

$A' \leftarrow \{a \in A \mid a \text{ is applicable in } s\}$

 if $A' = \emptyset$ then return failure

 nondeterministically choose $a \in A'$

$s \leftarrow \gamma(s, a); \pi \leftarrow \pi.a$

- Nondeterministic algorithm

- ▶ *Sound*: if an execution trace returns a plan π , it's a solution
- ▶ *Complete*: if the planning problem is solvable, at least one of the possible execution traces will return a solution

- Represents a class of deterministic search algorithms

- ▶ They'll all be sound
- ▶ Whether they're complete depends on how you implement the nondeterministic choice
 - Which leaf node to expand next
 - Which nodes to prune from the search space

Forward Search

Forward-search (Σ, s_0, g)

```
 $s \leftarrow s_0; \pi \leftarrow \langle \rangle$ 
loop
  if  $s$  satisfies  $g$  then return  $\pi$ 
   $A' \leftarrow \{a \in A \mid a \text{ is applicable in } s\}$ 
  if  $A' = \emptyset$  then return failure
  nondeterministically choose  $a \in A'$ 
   $s \leftarrow \gamma(s, a); \pi \leftarrow \pi.a$ 
```

- Nondeterministic algorithm
 - ▶ *Sound*: if an execution trace returns a plan π , it's a solution
 - ▶ *Complete*: if the planning problem is solvable, at least one of the possible execution traces will return a solution
- Represents a class of deterministic search algorithms
 - ▶ Deterministic versions of the nondeterministic choice
 - ▶ Which leaf node to expand next
 - ▶ Which nodes to prune from the search space
 - ▶ They'll all be sound, but not necessarily complete

Many of the algorithms in this class:

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{\langle \rangle, s_0\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

$\{(\pi.a, \gamma(s, a)) \mid s \text{ satisfies pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$ (ii)

$Frontier \leftarrow Frontier \cup Children$

return failure

Deterministic Version

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

$\{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$ (ii)

$Frontier \leftarrow Frontier \cup Children$

return failure

- Special cases:
 - ▶ depth-first, breath-first, A*, many others
- Classify by
 - ▶ how they *select* nodes (i)
 - ▶ how they *prune* nodes (ii)
- Pruning often includes *cycle-checking*:
 - ▶ Remove from $Children$ every node (π,s) that has an ancestor (π',s') such that $s' = s$
- In classical planning problems, S is finite
 - ▶ Cycle-checking will guarantee termination

Breadth-First Search (BFS)

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

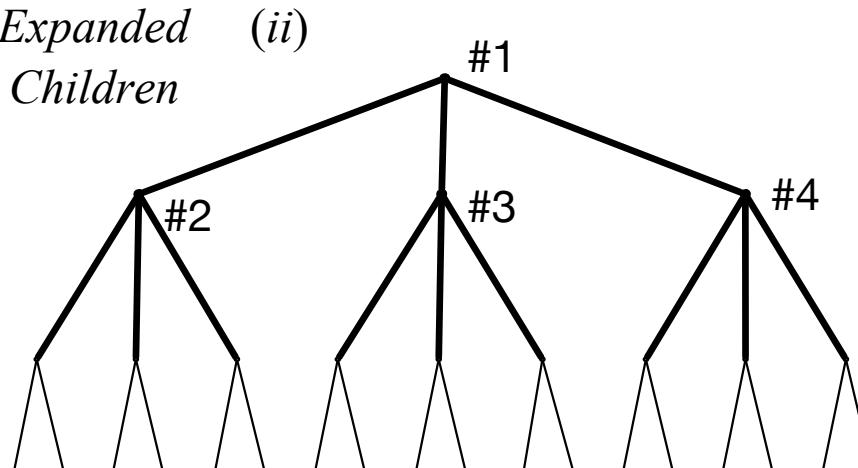
$\{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$

$Frontier \leftarrow Frontier \cup Children$

return failure



- (i): Select $(\pi, s) \in Frontier$ that has the **smallest length(π)**, i.e., smallest number of edges
 - ▶ Tie-breaking rule: select oldest

- (ii): Remove every $(\pi, s) \in Children \cup Frontier$ such that $s \in Expanded$
 - ▶ Thus expand states at most once

- Properties

- ▶ Terminates
- ▶ Returns solution if one exists
 - shortest, but not least-cost
- ▶ Worst-case complexity:
 - memory $O(|S|)$
 - running time $O(b|S|)$
- ▶ where
 - b = max branching factor
 - $|S|$ = number of states in S

Depth-First Search (DFS)

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

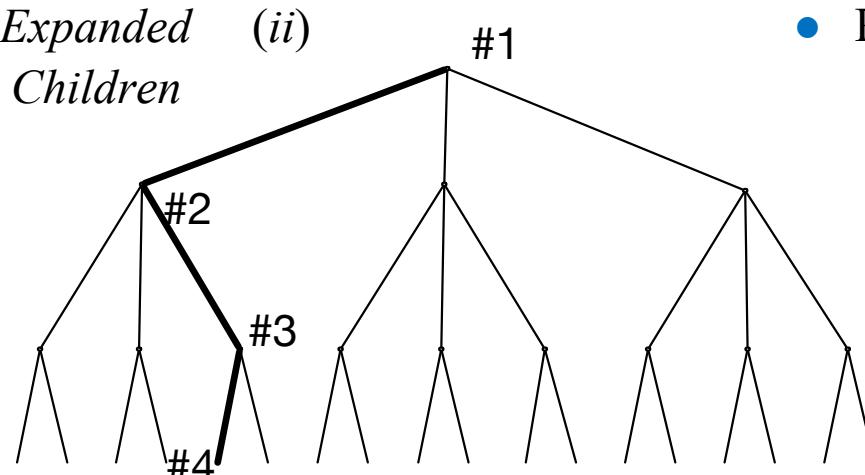
$\{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$

$Frontier \leftarrow Frontier \cup Children$

return failure



(i): Select $(\pi, s) \in Frontier$ that has **largest length(π)**, i.e., largest number of edges

- ▶ Possible tie-breaking rules: left-to-right, **smallest $h(s)$**

- heuristic function, will discuss later

(ii): **Do cycle-checking**, then prune all nodes that recursive depth-first search would discard

- ▶ Repeatedly remove from $Expanded$ any node that has no children in $Children \cup Frontier \cup Expanded$

- Properties

- ▶ Terminates
- ▶ Returns solution if there is one
 - No guarantees on quality
- ▶ Worst-case running time $O(b^l)$
- ▶ Worst-case memory $O(bl)$
 - b = max branching factor
 - l = max depth of any node

Uniform-Cost Search

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

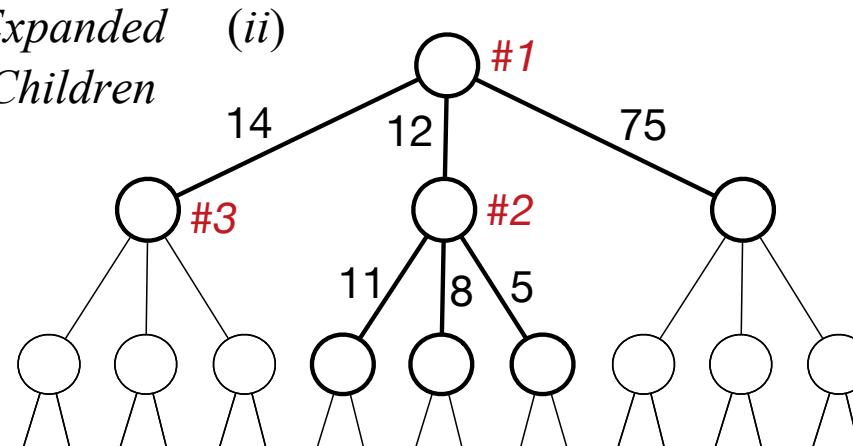
$\{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$

$Frontier \leftarrow Frontier \cup Children$

return failure



(i): Select $(\pi, s) \in Frontier$ that has **smallest cost(π)**

(ii): Prune every $(\pi, s) \in Children \cup Frontier$
such that $Expanded$ already contains a node (π', s)

- Properties

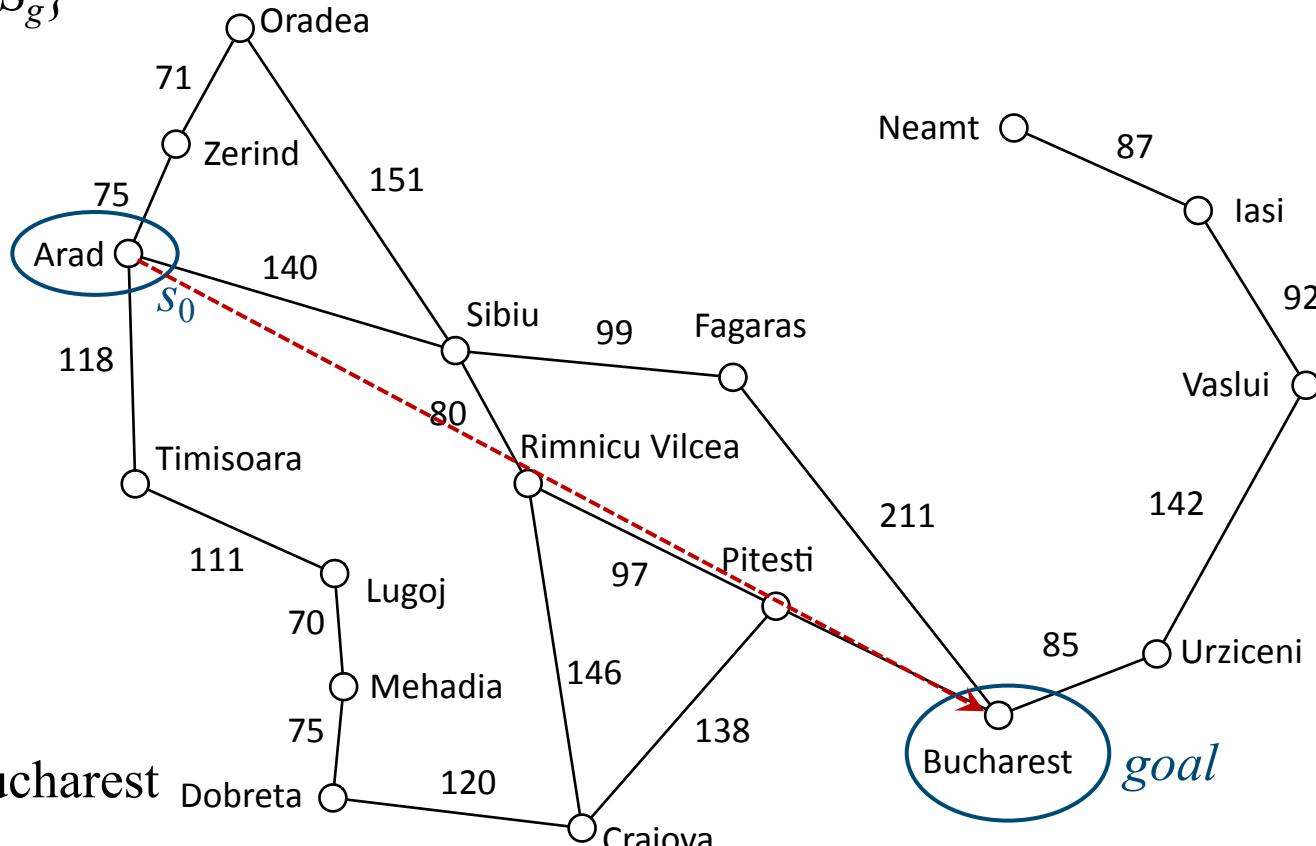
- Terminates
- Finds **optimal** (i.e., least-cost) solution if one exists
- Worst-case time $O(b|S|)$
- Worst-case memory $O(|S|)$

Poll: If node v is expanded before node v' ,
then how are $\text{cost}(v)$ and $\text{cost}(v')$ related?

- $\text{cost}(v) < \text{cost}(v')$
- $\text{cost}(v) \leq \text{cost}(v')$
- $\text{cost}(v) > \text{cost}(v')$
- $\text{cost}(v) \geq \text{cost}(v')$
- none of the above

Heuristic Functions

- Idea: estimate the cost of getting from a state s to a goal
- Let $h^*(s) = \min\{\text{cost}(\pi) \mid \gamma(s, \pi) \in S_g\}$
 - Note that $h^*(s) \geq 0$ for all s
- heuristic function $h(s)$:*
 - Returns estimate of $h^*(s)$
 - Require $h(s) \geq 0$ for all s
- Example:
 - s = the city you're in
 - Action: follow road from s to a neighboring city
 - $h^*(s)$ = smallest distance to Bucharest using roads
 - $h(s)$ = straight-line distance from s to Bucharest



from Russell & Norvig, *Artificial Intelligence: A Modern Approach*

Greedy Best-First Search (GBFS)

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

$\{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$ (ii)

$Frontier \leftarrow Frontier \cup Children$

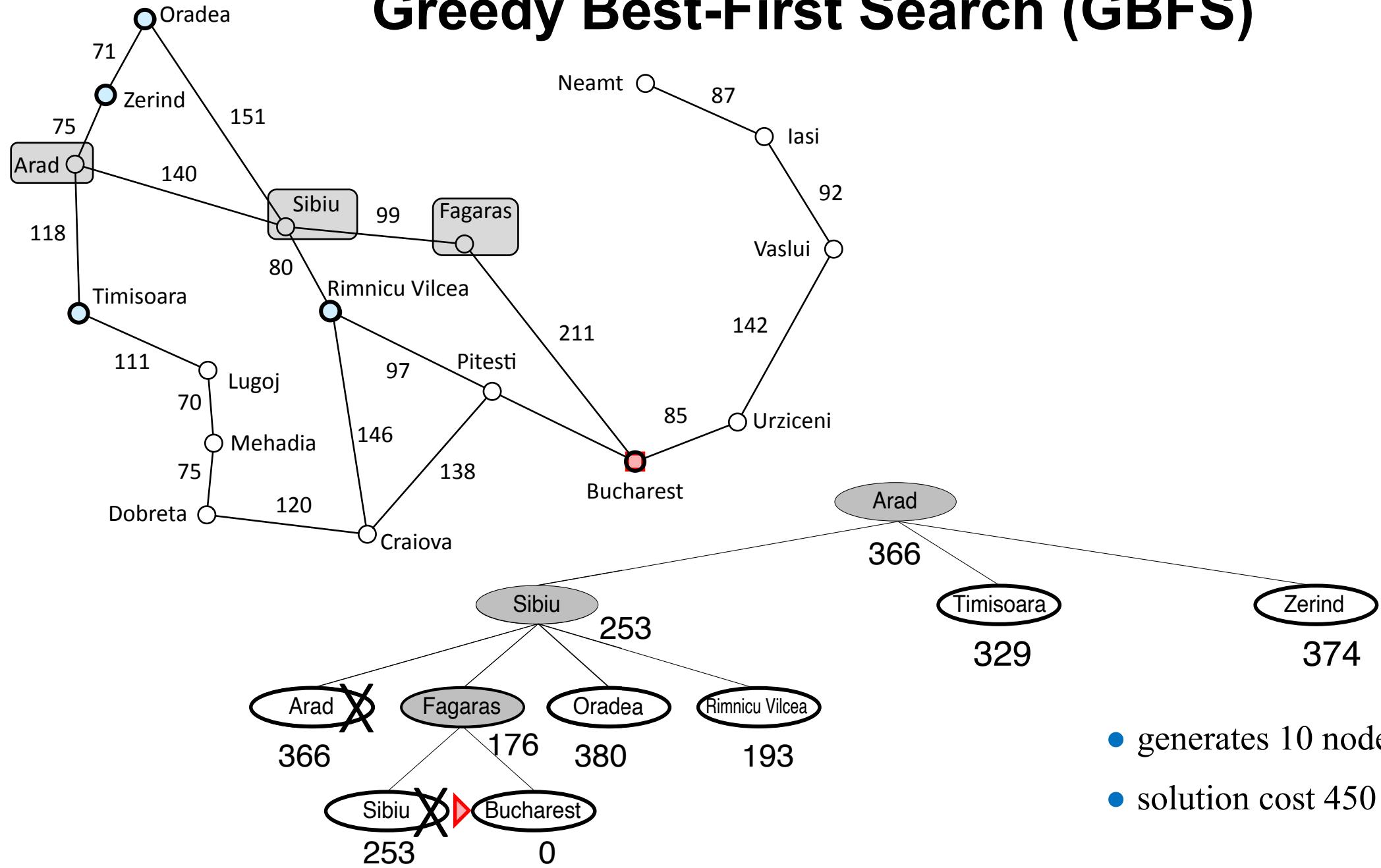
return failure

- Idea: choose a node that's **likely to be close to a goal**
- Node selection
 - ▶ Select a node $v = (\pi, s) \in Frontier$ for which $h(s)$ is **smallest**
- Pruning: for every node $v = (\pi, s)$ in $Children$:
 - ▶ If $Children \cup Frontier \cup Expanded$ contains another node with state s , then we've found **multiple paths** from s_0 to s
 - ▶ **Keep only the one with the lowest cost**
 - ▶ If more than one such node, keep the oldest
- Properties
 - ▶ Terminates; returns a solution if one exists
 - ▶ **Solution is usually found quickly, often near-optimal**

Poll: Have you seen GBFS before?

1. yes
2. no
3. yes, but I don't remember it very well

Greedy Best-First Search (GBFS)



straight-line dist. from s to Bucharest
Arad 366
Bucharest 0
Craiova 160
Dobreta 242
Fagaras 176
Iasi 226
Lugoj 244
Mehadia 241
Neamt 234
Oradea 380
Pitesti 100
Rimnicu Vilcea 193
Sibiu 253
Timisoara 329
Urziceni 80
Vaslui 199
Zerind 374

- generates 10 nodes
- solution cost 450

A*

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow$

$\{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$ (ii)

$Frontier \leftarrow Frontier \cup Children$

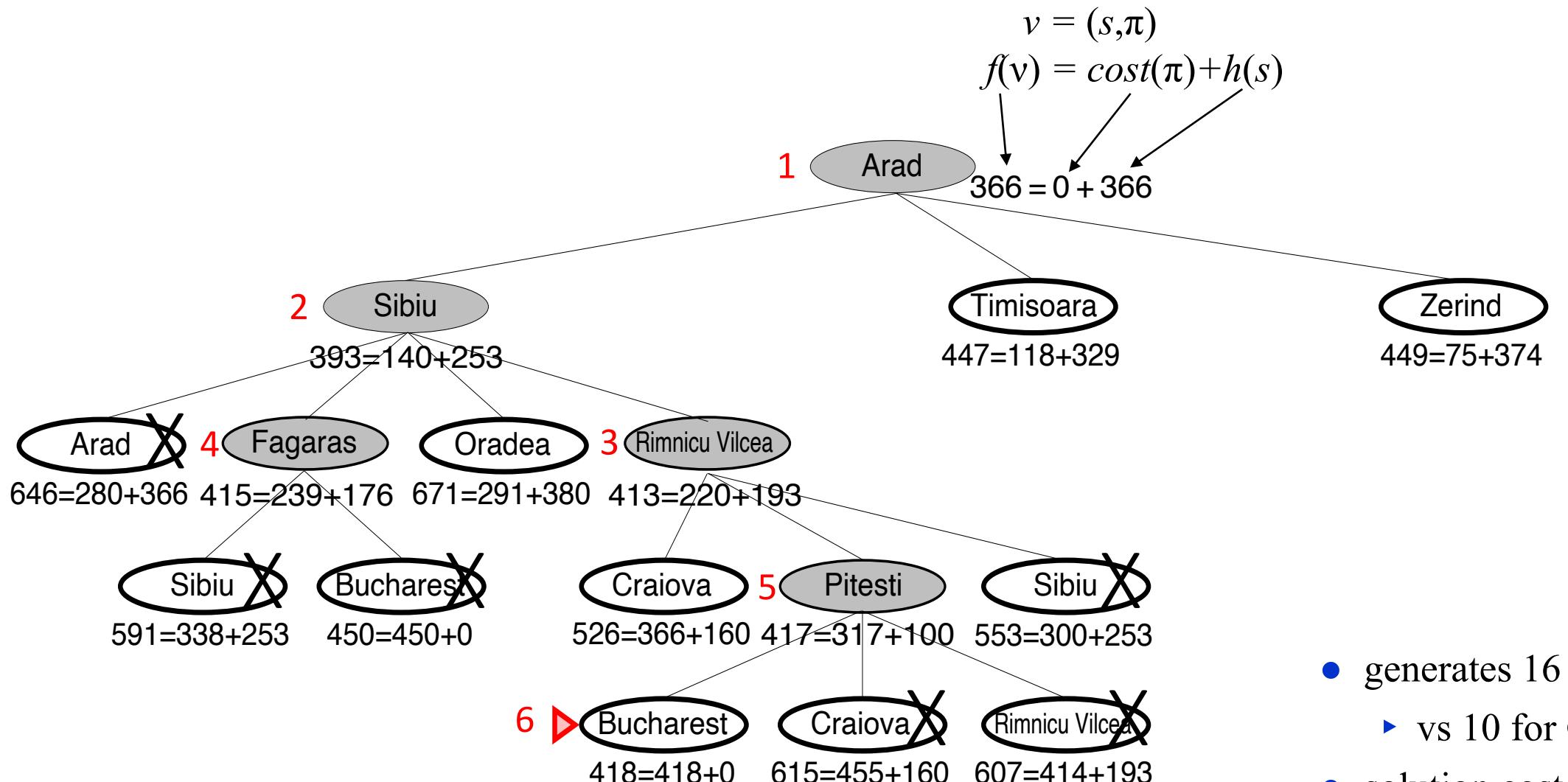
return failure

Poll: Have you seen A* before?

1. yes
2. no
3. yes, but I don't remember it very well

- Idea: try to choose a node on an **optimal path** from s_0 to goal
- Node selection
 - ▶ Select a node $v = (\pi,s)$ in $Frontier$ that has smallest value of $f(v) = \text{cost}(\pi) + h(s)$
 - Tie-breaking rule: choose oldest
- Pruning: same as in GBFS
 - ▶ for every node $v = (\pi,s)$ in $Children$:
 - If $Children \cup Frontier \cup Expanded$ contains another node with the same state s , then we've found multiple paths to s
 - Keep only the one with the lowest cost
 - If more than one such node, keep the oldest
- Properties (in classical planning problems):
 - ▶ *Termination*: Always terminates
 - ▶ *Completeness*: returns a solution if one exists
 - ▶ *Optimality*: under certain conditions (I'll discuss later), can guarantee optimality

	straight-line dist. from s to Bucharest
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Fagaras	176
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



- generates 16 nodes
 - ▶ vs 10 for GBFS
- solution cost 418
 - ▶ vs 450 for GBFS

Admissibility

- Notation:

- $v = (\pi, s)$, where π is the plan for going from s_0 to s
- $h^*(s) = \min \{ \text{cost}(\pi') \mid \gamma(s, \pi') \text{ satisfies } g\}$
- $f^*(v) = \text{cost}(\pi) + h^*(s)$
- $f(v) = \text{cost}(\pi) + h(s)$

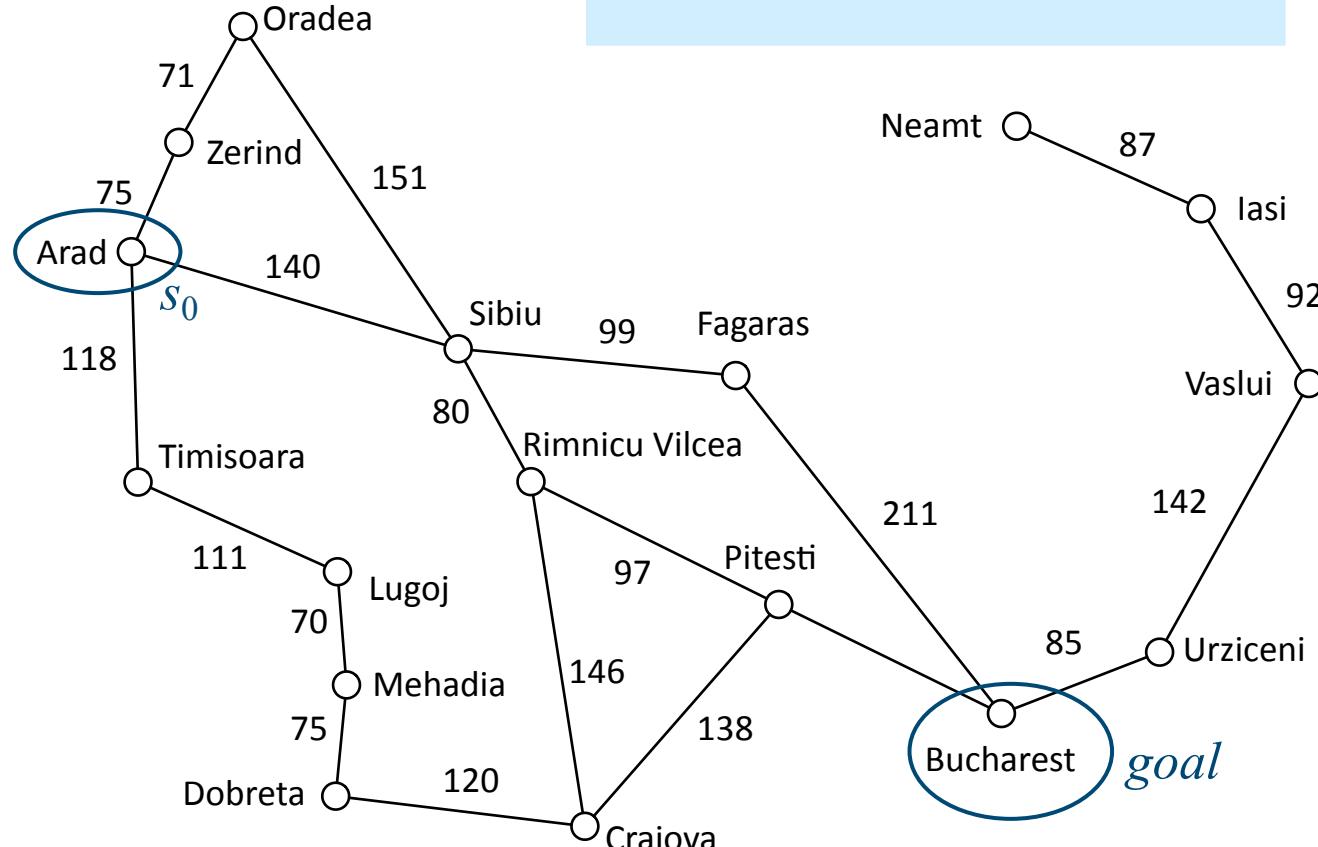
- Definition: h is *admissible* if for every s , $h(s) \leq h^*(s)$

- Optimality:

- in classical planning problems, if h is admissible then any solution returned by A* will be optimal (least cost)

Poll: If $h(s) = \text{straight-line distance from } s \text{ to Bucharest}$, is h admissible?

1. Yes.
2. No.
3. Not sure.



Admissibility

- Notation:

- $v = (\pi, s)$, where π is the plan for going from s_0 to s
- $h^*(s) = \min \{ \text{cost}(\pi') \mid \gamma(s, \pi') \text{ satisfies } g\}$
- $f^*(v) = \text{cost}(\pi) + h^*(s)$
- $f(v) = \text{cost}(\pi) + h(s)$

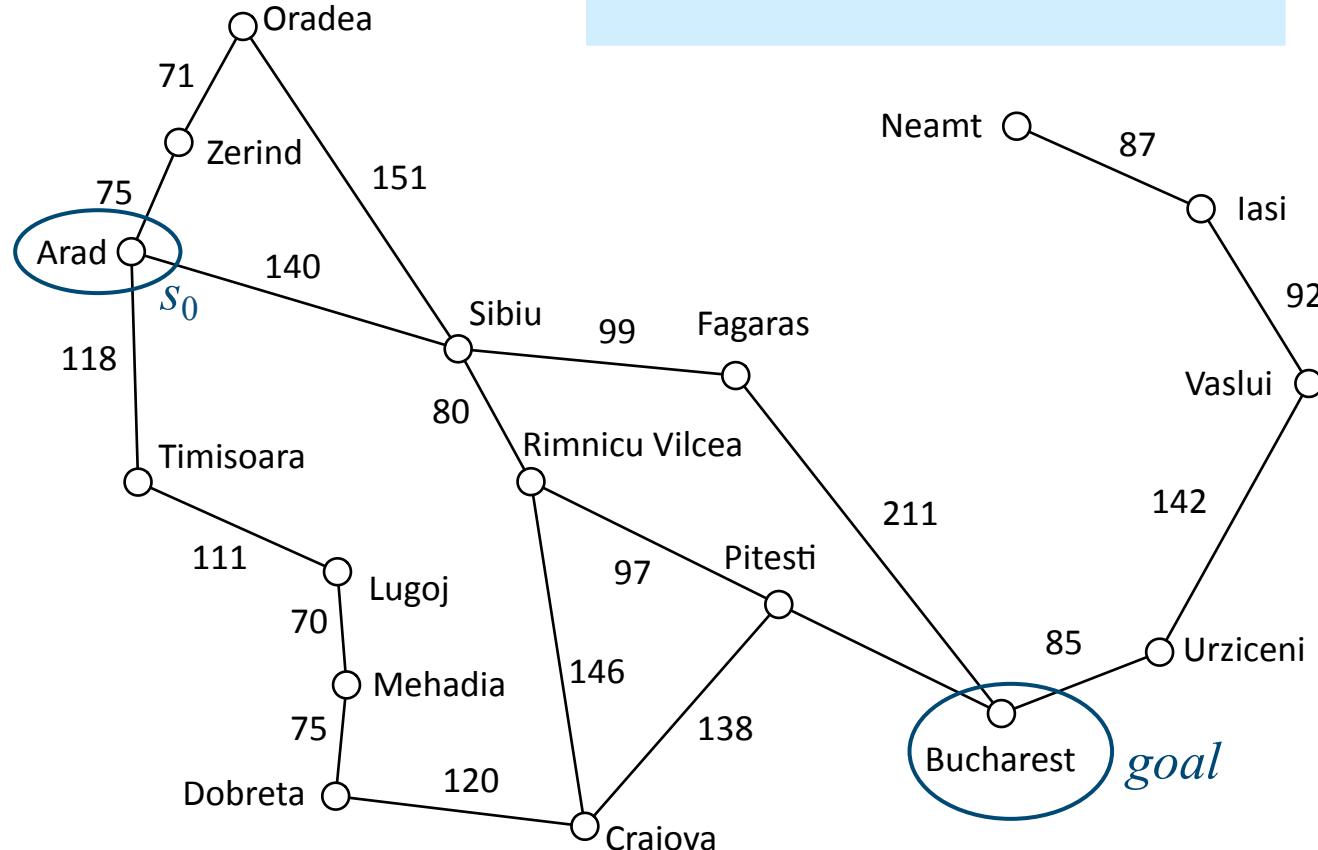
- Definition: h is *admissible* if for every s , $h(s) \leq h^*(s)$

- Optimality:*

- in classical planning problems, if h is admissible then any solution returned by A* will be optimal (least cost)

Poll: If h is admissible, does it follow that for every node v , $f(v) \leq f^*(v)$?

1. Yes. 2. No. 3. Not sure.



straight-line dist. from s to Bucharest	
Arad	366
Bucharest	0
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Fagaras	176
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Dominance

- Definition:

- Let h_1, h_2 be admissible heuristic functions
- h_2 dominates h_1 if $\forall s, h_1(s) \leq h_2(s) \leq h^*(s)$

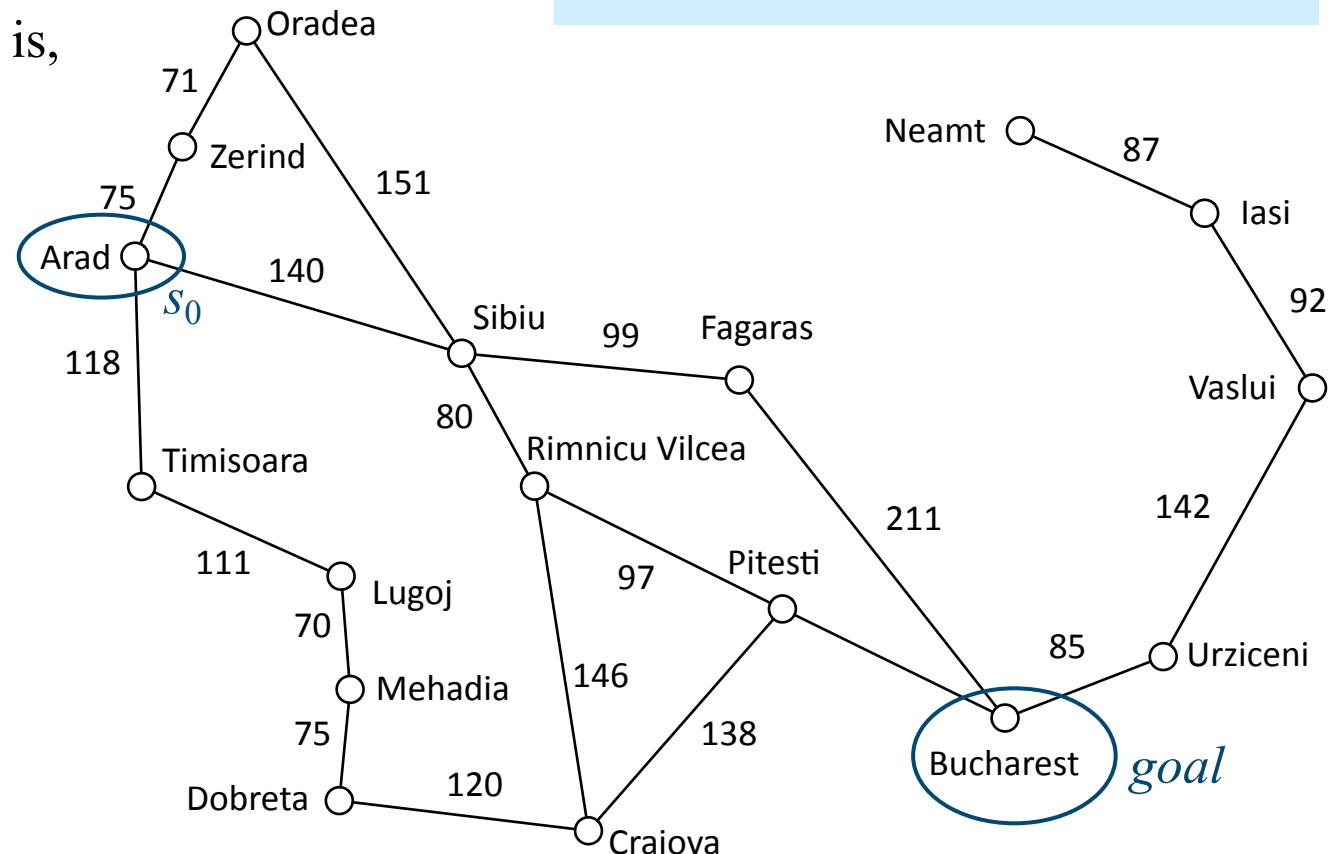
- Assume that regardless of what h is, A* always resolves ties in favor of the same node

- If h_2 dominates h_1 then

- A* with h_2 will never expand more nodes than A* with h_1
- In most cases, A* with h_2 will expand fewer nodes than A* with h_1

Poll: Let $h_1(s) = 0$ and $h_2(s) =$ straight-line distance from s to Bucharest. Does h_2 dominate h_1 ?

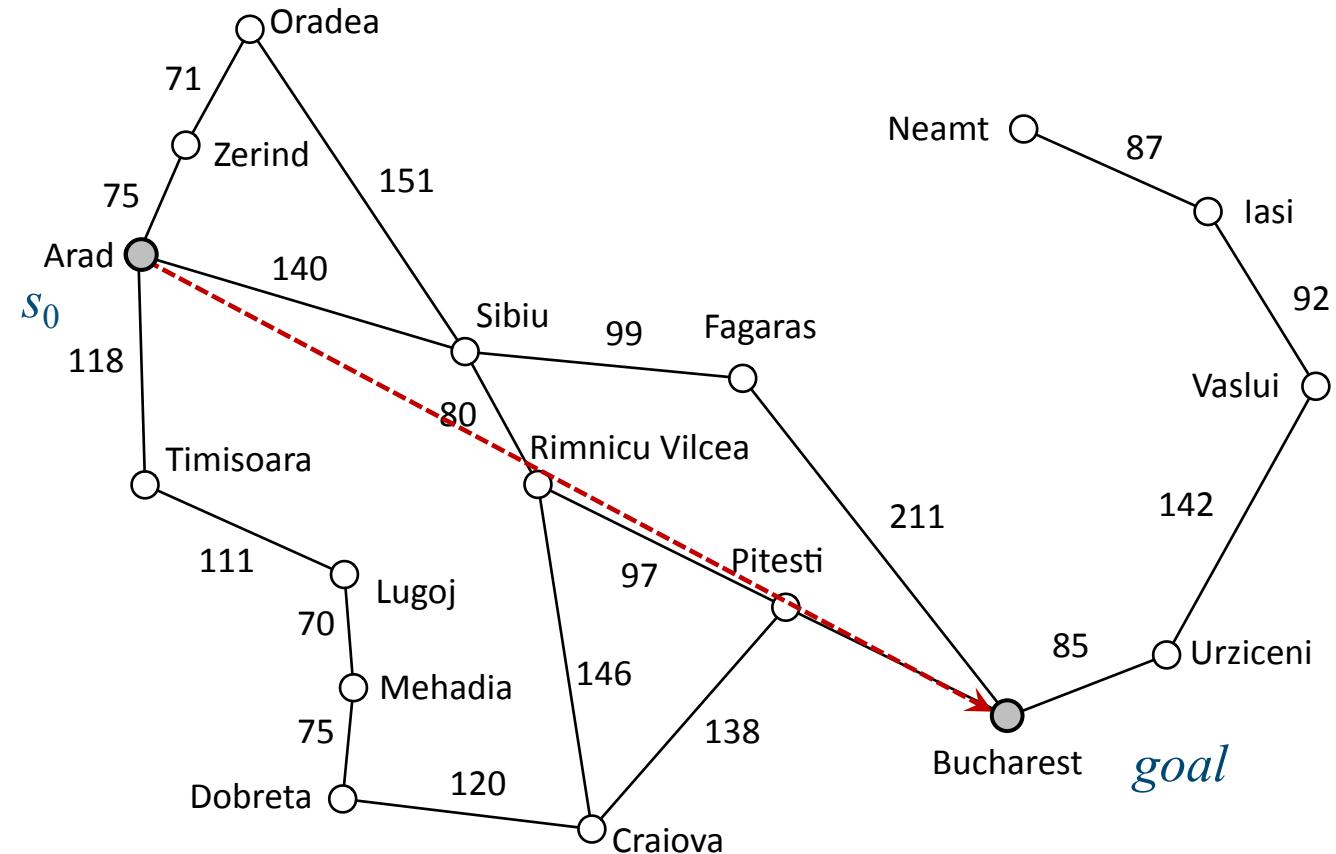
1. Yes.
2. No.
3. Not sure.



straight-line dist. from s to Bucharest
Arad 366
Bucharest 0
Craiova 160
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Timisoara 329
Urziceni 80
Vaslui 199
Zerind 374

Digression

- Straight-line distance to Bucharest is a ***domain-specific heuristic function***
 - ▶ OK for planning a path to Bucharest
 - ▶ Not for other planning problems
- ***Domain-independent*** heuristic function:
 - ▶ A heuristic function that can be used in any classical planning domain
 - ▶ Many such heuristics (see Section 2.3)



Properties of A*

In classical planning problems:

- *Termination*: A* will always terminate
- *Completeness*: if the problem is solvable, A* will return a solution
- *Optimality*: if h is admissible then the solution will be optimal (least cost)
- *Dominance*: If h_2 dominates h_1 then (assuming A* always resolves ties in favor of the same node)
 - ▶ A* with h_2 will never expand more nodes than A* with h_1
 - ▶ In most cases, A* with h_2 will expand fewer nodes than A* with h_1

- A* needs to store every node it visits
 - ▶ Running time $O(b|S|)$ and memory $O(|S|)$ in worst case
 - ▶ With good heuristic function, usually much smaller
- The book discusses additional properties

Comparison

- If h is admissible, A* will return optimal solutions
 - ▶ But running time and memory requirement grow exponentially in b and d
- GBFS returns the first solution it finds
 - ▶ There are cases where GBFS takes more time and memory than A*
 - But with a good heuristic function, such cases are rare
 - ▶ On classical planning problems with a good heuristic function
 - GBFS usually near-optimal solutions
 - GBFS does very little backtracking
 - Running time and memory requirement usually much less than A*
 - ▶ GBFS is used by most classical planners nowadays

Depth-First Branch and Bound (DFBB)

```
Deterministic-Search( $\Sigma, s_0, g$ )
   $Frontier \leftarrow \{(\langle \rangle, s_0)\}$ 
   $Expanded \leftarrow \emptyset$ 
   $c^* \leftarrow \infty; \pi^* \leftarrow \text{failure}$ 
  while  $Frontier \neq \emptyset$  do
    select a node  $v = (\pi, s) \in Frontier$  (i)
    remove  $v$  from  $Frontier$  and add it to  $Expanded$ 
    if  $s$  satisfies  $g$  then return  $\pi$ 
    if  $s$  satisfies  $g$  and  $\text{cost}(\pi) < c^*$  then
       $c^* \leftarrow \text{cost}(\pi); \pi^* \leftarrow \pi$ 
    else if  $f(v) < c^*$  then
       $Children \leftarrow \{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$ 
      prune 0 or more nodes from  $Children, Frontier, Expanded$  (ii)
       $Frontier \leftarrow Frontier \cup Children$ 
  return  $\text{failure } \pi^*$ 
```

Poll: Have you seen DFBB before?

1. yes
2. no
3. yes, but I don't remember it very well

Basic ideas:

- depth-first search, guided by h
- π^* = best solution so far
- $c^* = \text{cost}(\pi^*)$
- prune v if $\text{cost}(v) \geq c^*$
- when frontier is empty, return π^*

- Node (step i) selection like DFS:
 - ▶ Select $v = (\pi, s) \in Children$ that has largest $\text{length}(\pi)$
 - ▶ Tie-breaking: smallest $h(s)$
- Pruning (step ii)
 - ▶ Like DFS, do cycle-checking and prune what recursive depth-first search would discard
- Additional pruning during node expansion:
 - ▶ If $f(v) \geq c^*$ then discard v
- Properties
 - ▶ Termination, completeness, optimality same as A*
 - ▶ Comparison to A*: usually less memory, more time
 - ▶ Worst-case is like DFS: $O(bl)$ memory, $O(b^l)$ time

Comparisons

- If h is admissible, both A* and DFBB will return optimal solutions
 - ▶ Usually DFBB generates more nodes, but A* takes more memory
 - ▶ DFBB does badly in highly connected graphs (many paths to each state)
 - Can have exponentially worse running time than A* (generates nodes exponentially many times)
 - ▶ DFBB best in problems where S is a tree of uniform height, all solutions at the bottom (e.g., constraint satisfaction)
 - DFBB and A* have similar running time
 - A* can take exponentially more memory than DFBB
- DFS returns the first solution it finds
 - ▶ can take much less time than DFBB
 - ▶ but solution can be very far from optimal

Iterative Deepening (IDS)

$\text{IDS}(\Sigma, s_0, g)$

for $k = 1$ to ∞ do

do a depth-first search, backtracking at every node of depth k

if the search found a solution then return it

if the search generated no nodes of depth k then return failure

Poll: Have you seen Iterative Deepening before?

1. yes
2. no
3. yes, but I don't remember it very well

- Nodes generated:

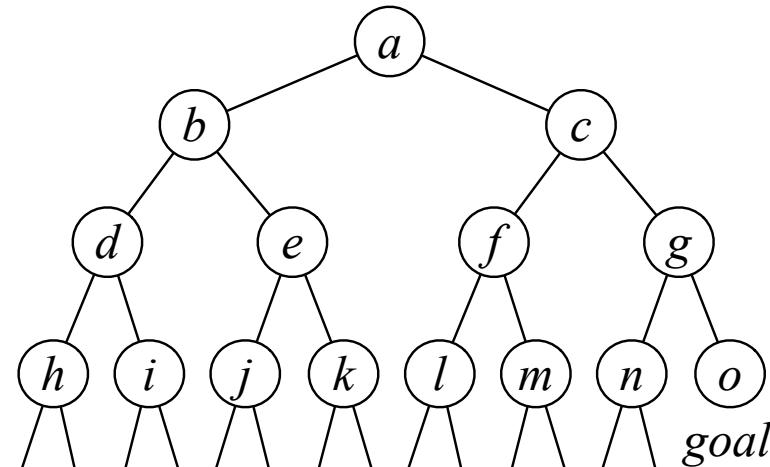
a
 a,b,c
 a,b,c,d,e,f,g
 $a,b,c,d,e,f,g,h,i,j,k,l,m,n,o$

- Solution path $\langle a,c,g,o \rangle$
- Total number of nodes generated:

$$1+3+7+15 = 26$$

- If goal is at depth d and branching factor is 2:

► $\sum_{i=1}^d (2^i - 1) = \sum_{i=1}^d 2^i - \sum_{i=1}^d 1 = (2^{d+1} - 2) - d = O(2^d)$



Iterative Deepening (IDS)

$\text{IDS}(\Sigma, s_0, g)$

for $k = 1$ to ∞ do

do a depth-first search, backtracking at every node of depth k

if the search found a solution then return it

if the search generated no nodes of depth k then return failure

- Nodes generated:

a

a, b, c

a, b, c, d, e, f, g

$a, b, c, d, e, f, g, h, i, j, k, l, m, n, o$

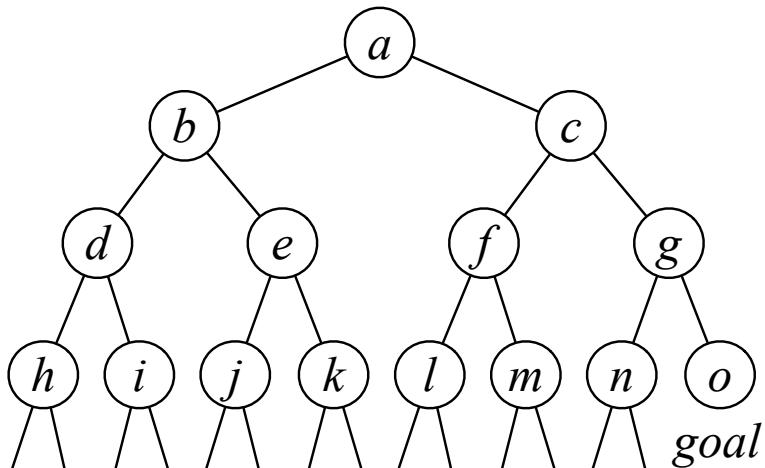
- Solution path $\langle a, c, g, o \rangle$

- Total number of nodes generated:

$$1 + 3 + 7 + 15 = 26$$

- If goal is at depth d and branching factor is 2:

$$\triangleright \sum_{i=1}^d (2^i - 1) = \sum_{i=1}^d 2^i - \sum_{i=1}^d 1 = (2^{d+1} - 2) - d = O(2^d)$$



Properties:

- Termination, completeness, optimality
 - same as BFS
- Memory (worst case): $O(bd)$
 - vs. $O(b^d)$ for BFS
- If the number of nodes grows exponentially with d :
 - worst-case running time $O(b^d)$, vs. $O(b^l)$ for DFS
 - b = max branching factor
 - l = max depth of any node
 - d = min solution depth if there is one, otherwise l

Summary

- 2.2 Forward State-Space Search
 - ▶ Forward-search, Deterministic-Search
 - ▶ cycle-checking
 - ▶ Breadth-first, depth-first, uniform-cost search
 - ▶ A*, GBFS
 - ▶ DFBB, IDS

Outline

- Chapter 2, part *a* (chap2a.pdf):
- 2.1 State-variable representation
 - Comparison with PDDL
 - 2.2 Forward state-space search
 - Next →* 2.6 Incorporating planning into an actor
-

Chapter 2, part *b* (chap2b.pdf):

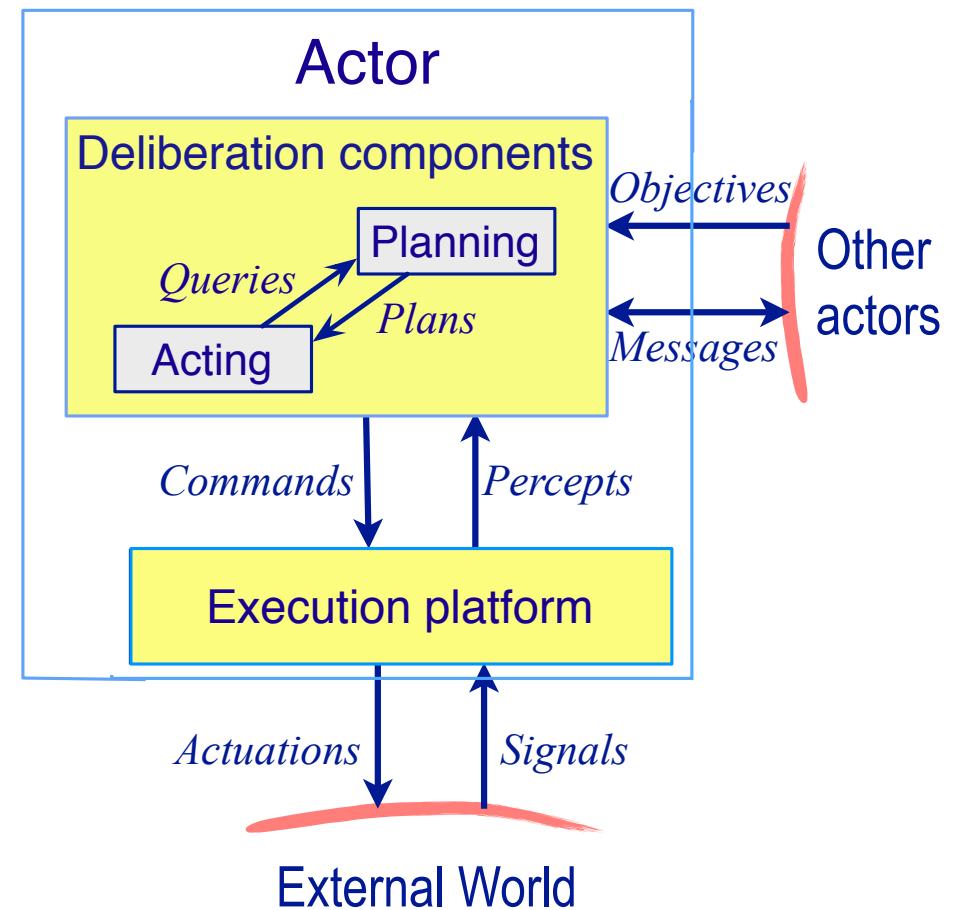
- 2.3 Heuristic functions
 - 2.4 Backward search
 - 2.5 Plan-space search
-

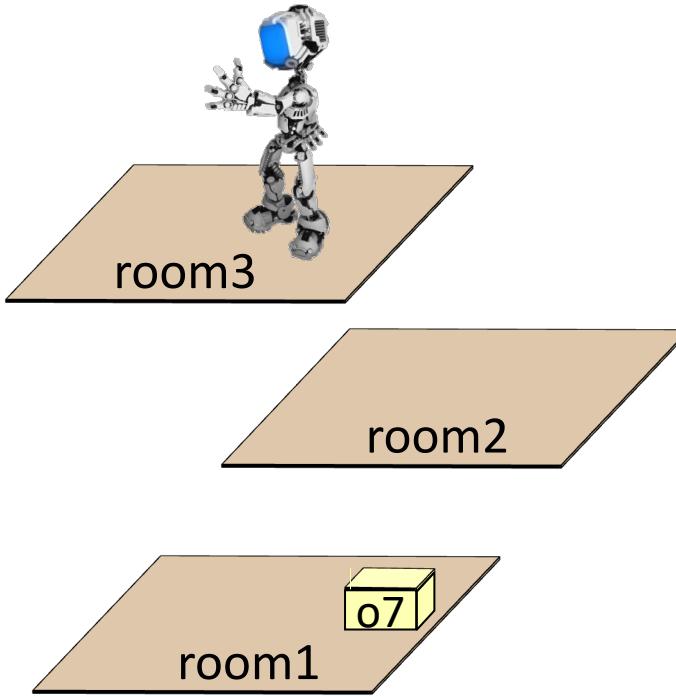
Additional slides:

- 2.7.7 HTN_planning.pdf
- 2.7.8 LTL_planning.pdf

2.6 Incorporating Planning into an Actor

- For classical planning we assumed
 - Finite, static world, just one actor
 - No concurrent actions, no explicit time
 - Determinism, no uncertainty
 - ▶ Sequence of states and actions $\langle s_0, a_1, s_1, a_2, s_2, \dots \rangle$
- Most real-world environments don't satisfy the assumptions
⇒ Errors in prediction
- OK if
 - ▶ errors occur infrequently, and
 - ▶ they don't have severe consequences
- What to do if an error *does* occur?





$a_1 = \text{go(r1, room3, hall)}$
 $a_2 = \text{navigate(r1, hall, room1)}$
 $a_3 = \text{take(r1, room1, o7)}$
 $a_4 = \text{navigate(r1, room1, room2)}$
 $a_5 = \text{put(r1, room2, o7)}$

go(r, l, m)
 pre: adjacent(l, m), loc(r)= l
 eff: loc(r) $\leftarrow m$

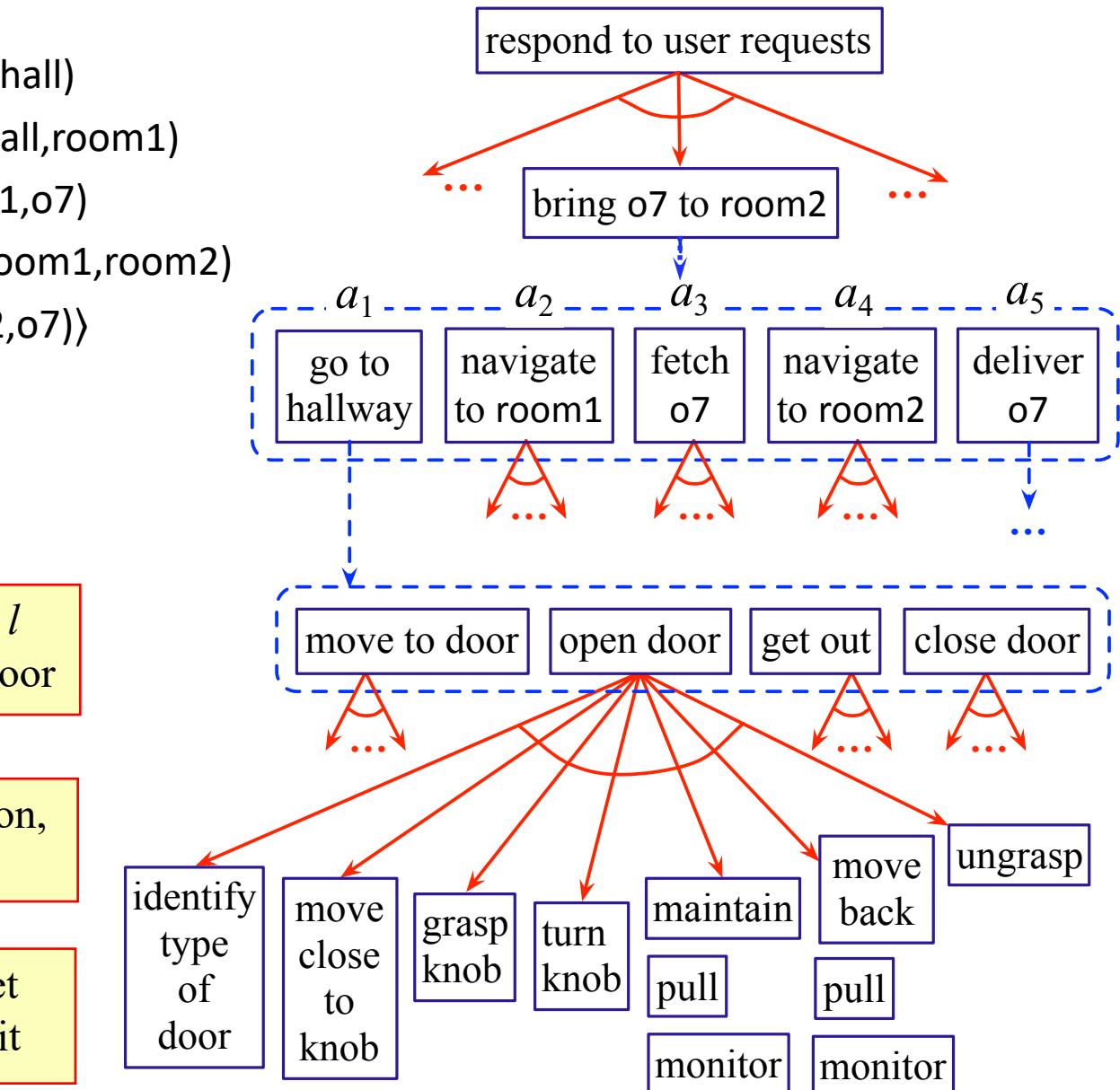
navigate(r, l, m)
 pre: \neg adjacent(l, m), loc(r)= l
 eff: loc(r) $\leftarrow m$

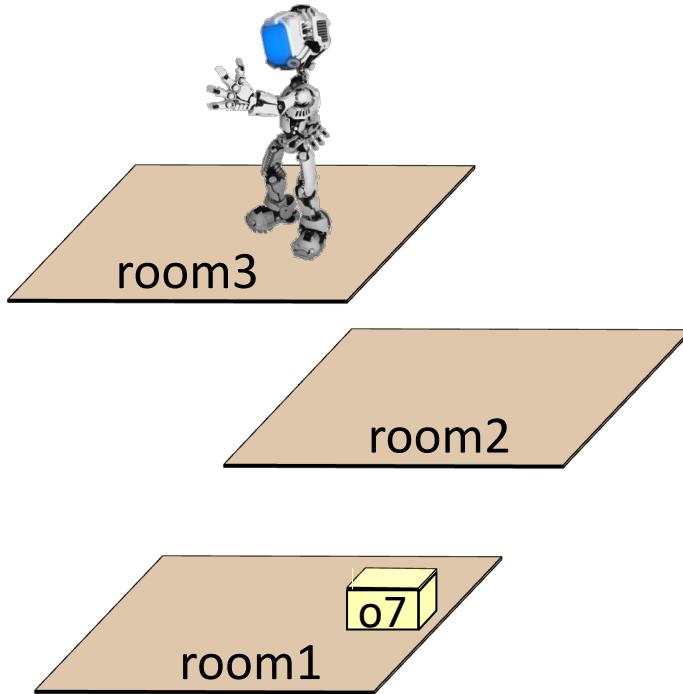
take(r, l, o)
 pre: loc(r)= l , loc(o)= l , cargo(r)=nil
 eff: loc(o) $\leftarrow r$, cargo(r) $\leftarrow o$

ignores how to get from l to m , e.g., opening the door

ignores how do navigation, localization

ignores how to find o , get access to it, grasp it, lift it





$go(r,l,m)$

pre: $\text{adjacent}(l,m)$, $\text{loc}(r)=l$
eff: $\text{loc}(r) \leftarrow m$

$navigate(r;l,m)$

pre: $\neg\text{adjacent}(l,m)$, $\text{loc}(r)=l$
eff: $\text{loc}(r) \leftarrow m$

$take(r;l,o)$

pre: $\text{loc}(r)=l$, $\text{loc}(o)=l$,
 $\text{cargo}(r)=\text{nil}$
eff: $\text{loc}(o) \leftarrow r$, $\text{cargo}(r) \leftarrow o$

$a_1 = go(r1, room3, hall)$
 $a_2 = navigate(r1, hall, room1)$
 $a_3 = take(r1, room1, o7)$
 $a_4 = navigate(r1, room1, room2)$
 $a_5 = put(r1, room2, o7)$

ignores how to get from l to m , e.g., opening the door

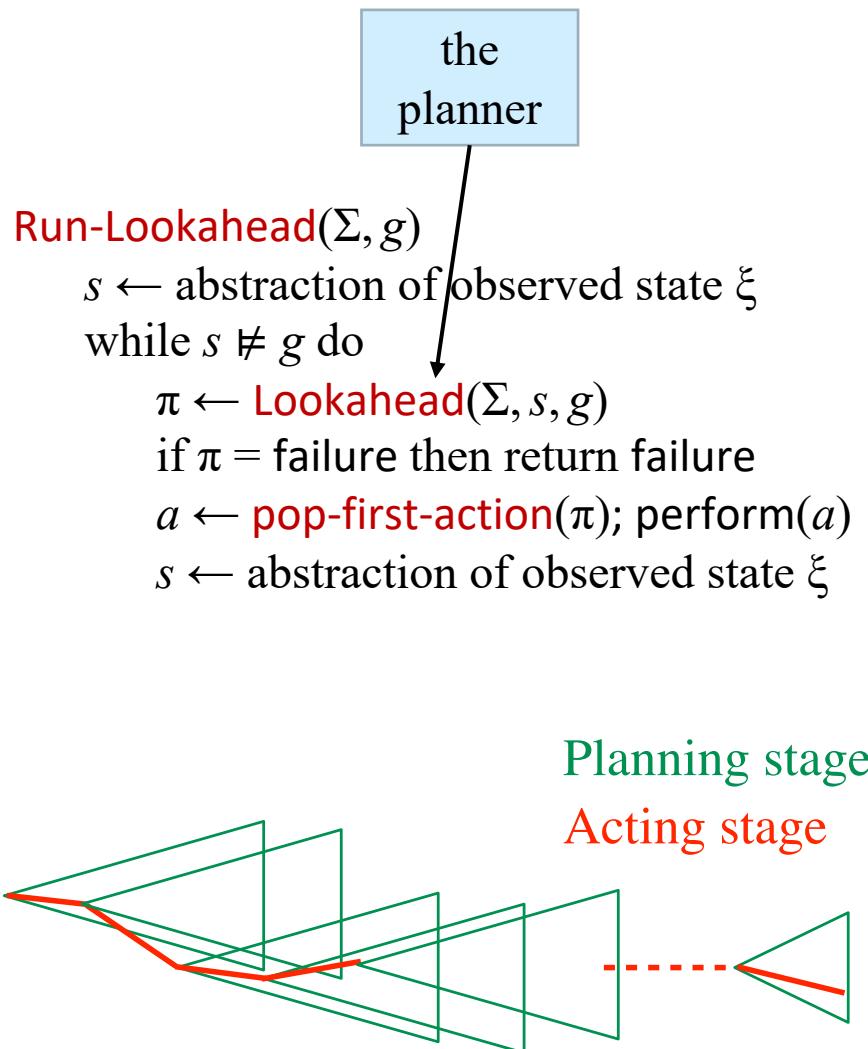
ignores how do navigation, localization

ignores how to find o , get access to it, grasp it, lift it

Service Robot

- Some things that can go wrong:
 - *Execution failures*
 - locked door
 - robot battery goes dead
 - *Unexpected events*
 - class ends, hallway gets crowded
 - someone puts an object onto $r1$
 - *Incorrect information*
 - navigation error, go to wrong place
 - *Missing information*
 - where is $o7$?
- How to detect and recover from errors?

Using Planning in Acting



- Call Lookahead, obtain π , perform 1st action, call Lookahead again ...
- Useful when unpredictable things are likely to happen
 - ▶ Replans immediately
- Also useful with *receding horizon* search (e.g., as in chess programs):
 - ▶ Lookahead looks a limited distance ahead
- Potential problem:
 - ▶ Lookahead needs to return quickly
 - ▶ Otherwise, may pause repeatedly while waiting for Lookahead to return
 - ▶ What if ξ changes during the wait?

Using Planning in Acting

Run-Lazy-Lookahead(Σ, g)

$s \leftarrow$ abstraction of observed state ξ

until s satisfies g do

$\pi \leftarrow$ Lookahead(Σ, s, g)

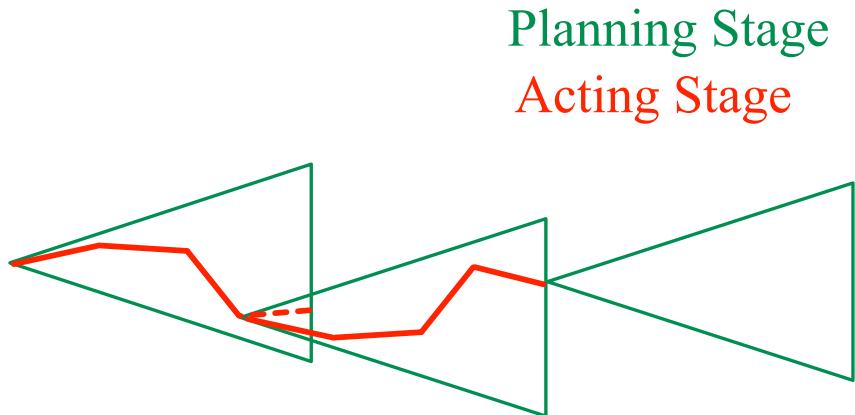
if $\pi =$ failure then return failure

until $\pi = \langle \rangle$ or $s \models g$ or Simulate(Σ, s, g, π) = failure do

$a \leftarrow$ pop-first-action(π); perform(a)

$s \leftarrow$ abstraction of observed state ξ

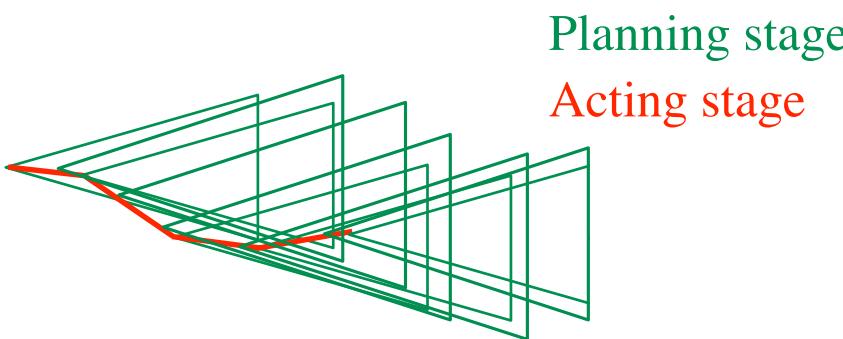
- Call Lookahead, execute the plan as far as possible, don't call Lookahead again unless necessary
- Simulate tests whether the plan will execute correctly
 - ▶ Lower-level refinement, physics-based simulation
- What if you don't have a simulation program?
 - ▶ Could write Simulate(...) to test whether $\gamma(s, \pi) \models g$
 - or test whether $s = \gamma(s', a)$, where s' is the previous state
- Potential problems
 - ▶ Simulate needs to return quickly
 - otherwise, may pause repeatedly, ξ may change
 - ▶ May miss opportunities to replace π with a better plan



Using Planning in Acting

Run-Concurrent-Lookahead (basic idea)

- ▶ global s, π
- ▶ thread 1:
 - loop:
 - ▶ $s \leftarrow$ observed state
 - ▶ $\pi \leftarrow \text{Lookahead}(\Sigma, s, g)$
- ▶ thread 2:
 - loop:
 - ▶ $a \leftarrow \text{pop-first-element}(\pi)$
 - ▶ perform a
 - ▶ return if observed state $\models g$



- Motivation: plan and act in a dynamically changing environment
 - ▶ Want a recent plan, rather than the old one that Run-Lazy-Lookahead would use
 - ▶ Want to get it quickly, rather than waiting like Run-Lookahead
- But there are several problems with the pseudocode
 - ▶ It ignores some implementation details
 - how to do locking
 - whether each thread has correct values for π and s
 - ▶ If thread 2 performs any actions while Lookahead is running, we probably should restart Lookahead
 - Otherwise Lookahead will return a plan that's out-of-date
 - ▶ Another possibility:
 - If thread 2 is going to perform action a , have thread 1 run $\text{Lookahead}(\Sigma, \gamma(s, a), g)$

How to do Lookahead

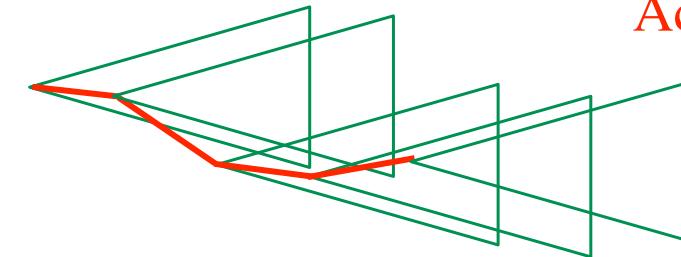
Some possibilities (can also combine these)

- **Receding horizon**

- ▶ **Modify Lookahead to search just part of the way to g (see next page)**
- ▶ E.g., cut off search when one of the following exceeds a **maximum threshold**:
 - plan length, plan cost, computation time

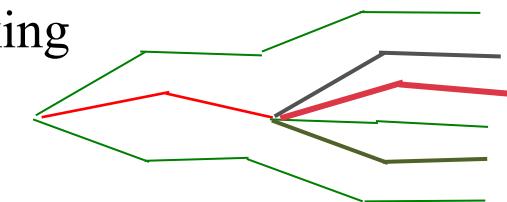
Planning stage

Acting stage



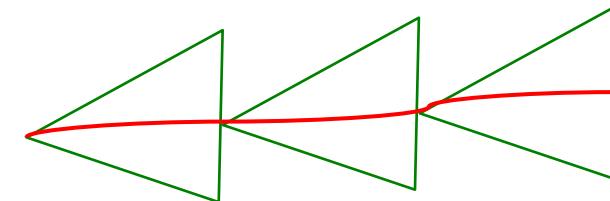
- **Sampling**

- ▶ **Modify Lookahead to do a *Monte Carlo rollout***
 - Depth-first search with random node selection and no backtracking
- ▶ Call Lookahead several times, choose the plan that looks best
- ▶ Best-known example of this: the UCT algorithm (see Chapter 6)



- **Subgoaling**

- ▶ **Tell Lookahead to plan for some subgoal g' , rather than g itself**
- ▶ Once the actor has achieved g' , tell Lookahead to plan for the next subgoal g''
- ▶ And so forth until the actor reaches g



Receding-Horizon Search

Deterministic-Search(Σ, s_0, g)

$Frontier \leftarrow \{(\langle \rangle, s_0)\}$

$Expanded \leftarrow \emptyset$

while $Frontier \neq \emptyset$ do

 select a node $v = (\pi, s) \in Frontier$ (i)

 remove v from $Frontier$

 add v to $Expanded$

 if s satisfies g then return π

$Children \leftarrow \{(\pi.a, \gamma(s,a)) \mid s \text{ satisfies } \text{pre}(a)\}$

 prune 0 or more nodes from

$Children, Frontier, Expanded$ (ii)

$Frontier \leftarrow Frontier \cup Children$

return failure

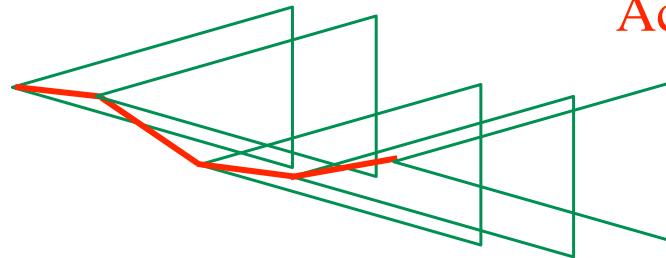
- Lookahead = modified version of Deterministic-Search

► Before line (i), put something like one of these:

- *cost-based cutoff*: if $\text{cost}(\pi) + h(s) > c_{\max}$ then return π
- *length-based cutoff*: if $|\pi| > l_{\max}$ then return π
- *time-based cutoff*: if $\text{time-left}() = 0$ then return π

Planning stage

Acting stage



Subgoaling Example

- Killzone 2
 - ▶ “First-person shooter” game, ≈ 2009
- Special-purpose AI planner
 - ▶ Plans enemy actions at the squad level
 - Subproblems; solution plans are maybe 4–6 actions long
 - ▶ Different planning algorithm from what we’ve discussed so far
 - ▶ HTN planning (see Section 2.7.7)
 - Quickly generates a plan for a subgoal
 - Replans several times per second as the world changes
- Why it worked:
 - ▶ Don’t *want* to get the best possible plan
 - ▶ Need actions that appear **believable** and **consistent** to human users
 - ▶ Need them **very quickly**



Summary

- 2.6 Incorporating Planning into an actor
 - ▶ Things that can go wrong while acting
 - ▶ Algorithms
 - Run-Lookahead,
 - Run-Lazy-Lookahead,
 - Run-Concurrent-Lookahead
 - ▶ Lookahead
 - receding-horizon search
 - sampling
 - subgoaling

Outline

Chapter 2, part *a* (chap2a.pdf):

- 2.1 State-variable representation
 - Comparison with PDDL
 - 2.2 Forward state-space search
 - 2.6 Incorporating planning into an actor
-

Chapter 2, part *b* (chap2b.pdf):

- 2.3 Heuristic functions
 - 2.4 Backward search
 - 2.5 Plan-space search
-

Additional slides:

Next → 2.7.7 HTN_planning.pdf
2.7.8 LTL_planning.pdf