Cryptography

Segurança Informática em Redes e Sistemas
2022/23

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Roadmap

- Introduction
- Symmetric Ciphers
- Hash Functions
- Message Authentication Codes
- Asymmetric Ciphers
- Digital Signatures
- Homomorphic Ciphers

Roadmap

- **Introduction**
- Symmetric Ciphers
- Hash Functions
- Message Authentication Codes
- Asymmetric Ciphers
- Digital Signatures
- Homomorphic Ciphers

Cryptography: terminology

- Cryptography
  - Art or science of writing in a concealed form
    - from Greek: kryptós, hidden + graph, r. de graphein, write
  - Used to assure confidentiality of data (until the 1970s)
  - Steganography
    - from Greek: steganós, covered + graph, r. de graphein, write
- Cryptanalysis
  - The art or science of breaking cryptographic systems or ciphered data
- Cryptology
  - Cryptography + Cryptanalysis

Cryptography

- Widespread and dangerous belief:
  - Encrypting everything provides protection against anything
- A simple example to prove the contrary:
  - Money transfer from one bank to the other
    - The bank encrypts the whole message
  - The attacker:
    - Might not be able to understand the message! (or would he?)
    - But he might be able to:
      - Divert the message into his account (maybe not!)
    - Could get rich by:
      - Diverting or stopping debit messages
      - Allow the passage of all credit messages
      - He might be able to distinguish the two merely by looking at their size
    - Crash the bank by:
      - Injecting random messages

Information

- Unlike mass or energy, information (and data) is non-conservative
  - It can be created
  - It can be destroyed
  - It can be duplicated
- This is the root of most security issues
  - e.g., we cannot infer that is has not be stolen merely because we still have it!
- Conclusion: we need more than just encryption

Attacks on information

- We want to protect the information (or data) against:
  - Unauthorized insertion of information
    - Loss of authenticity
  - Unauthorized modification of information in transit
    - Loss of integrity
  - Unauthorized replay of information
        - From an earlier legitimate data transmission
    - Loss of authenticity
  - Unauthorized access to information
    - Loss of confidentiality

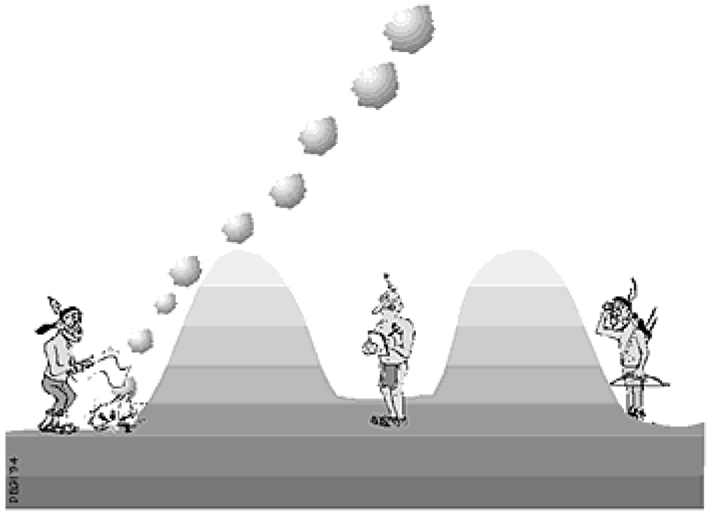
- Which cryptographic services can we use to prevent this?

Cryptographic services

- We need the following cryptographic services
  - Confidentiality
  - Data Integrity
  - Authentication
    - Data origin authentication
    - Entity authentication
      - Identification
      - Authorization
  - Non-Repudiation

Confidentiality

- Confidentiality:
  - Is a service used to keep the content of the information from all, but those entities authorized to have it
    - i.e. making the information unintelligible to all but those who possess some secret.
  - Typically operates by encryption:
    - Encryption: the process of converting plaintext to ciphertext
      - Using:
        - » Cryptographic algorithms
        - » Cryptographic key

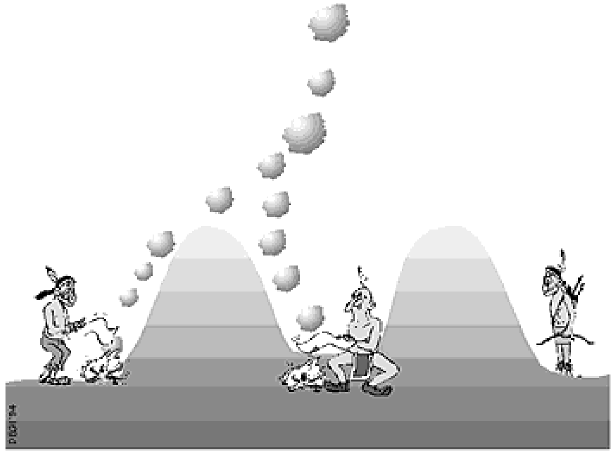


Confidentiality

- Problems:
  - Makes debugging harder
    - Software
    - Systems
    - Protocols
  - Information loss
    - If the key is permanently lost, so is the information
  - Sometimes misused
    - Other, more appropriate, services can be used

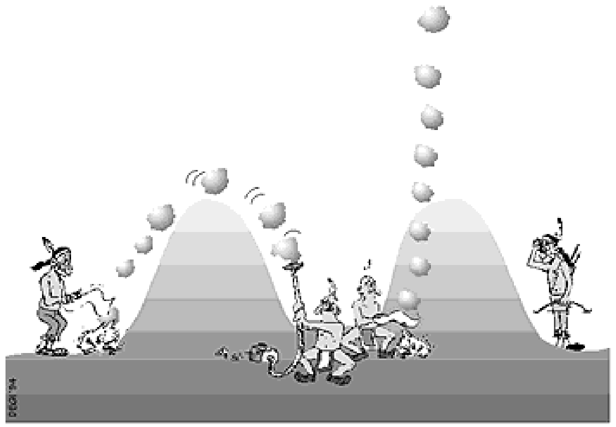
Data Integrity

- Data integrity:
  - Addresses the unauthorized modification of data
    - Detects data manipulation by unauthorized entities
    - An intruder should not be able to substitute a false message for a legitimate one
  - Not the same thing as error detection codes
    - E.g. Cyclic Redundancy Codes (CRC)
      - Are not cryptographically strong
        - » do not protect against intentional alterations of the message



Authentication

- Authentication of messages:
  - Authentication includes Integrity and Freshness
  - It is a service used to ascertain the identity or the origin of a message:
    - Guaranties that entities are who they claim to be
      - Data origin authentication
      - Entity authentication (Identification)
    - An intruder should not be able to masquerade as someone else
  - Authentication ≠ secrecy
    - Until mid 1970's secrecy and authentication intrinsically connected
      - Related with the misconception that:
        - » encrypting everything achieves all goals



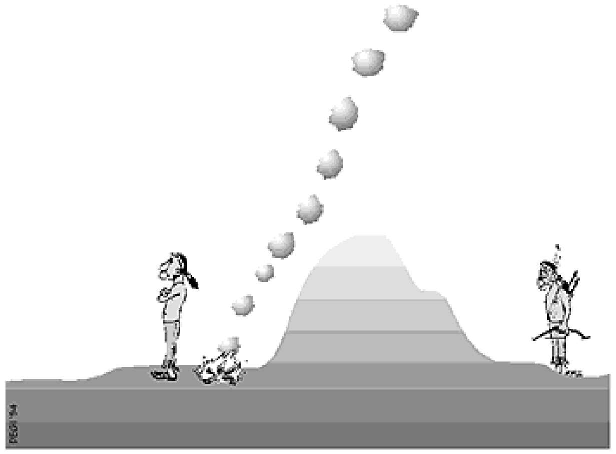
Authentication

- Entity authentication (aka Identification):
  - Applies to real time message exchange
  - If Alice authenticates herself to Bob, then Bob accepts Alice's identity
  - Must be infeasible for a third party to impersonate Alice
  - All these features remain true even after a large number of honest exchanges between Alice and Bob

- Data origin authentication:
  - identify the originator/creator of the message

Non-Repudiation

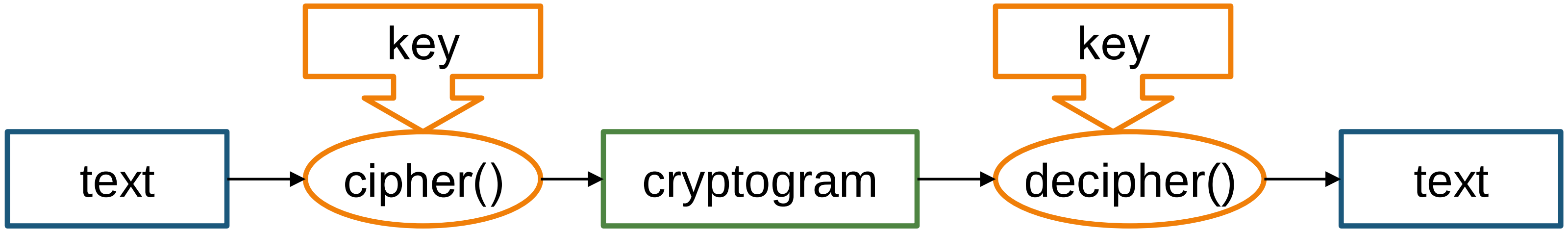
- Non-Repudiation:
  - It is a service which prevents an entity from denying previous commitments or actions
    - Such as:
      - a sent message
      - a signed document
      - …



Ciphers: terminology

- Cipher
  - Specific cryptographic technique
- Cipher Procedure
  - Cipher: plaintext → cryptogram (aka ciphertext)
  - Decipher: cryptogram → plaintext

  - Algorithm: data transformation procedure
  - Key: algorithm parameter



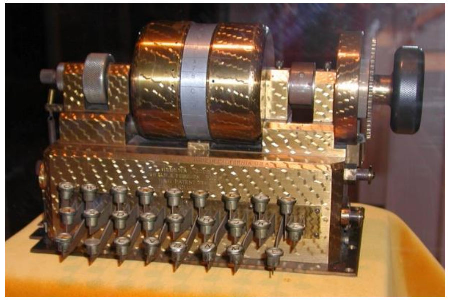
Cryptanalysis
what cryptography has to protect from

- Basic assumption: the algorithm is known
  - If not public, might be obtained (e.g., stolen)
- Attacks:
  - Ciphertext-only: cryptanalyst has access to ciphertexts
    - Without them, no cryptanalysis is possible
  - Known-plaintext: cryptanalyst has a set of ciphertexts to which he knows the corresponding plaintext
    - Often easy to get at least partial plaintext, e.g., message beginning
  - Chosen-plaintext: cryptanalyst can obtain the ciphertexts corresponding to plaintexts of his choice; or:
    - Chosen-ciphertext: cryptanalyst can obtain the plaintexts corresponding to ciphertexts of his choice

less probable and easier

Old (broken) ciphers

- Caesar cipher
  - Shift by 3
- Substitution cipher
  - A -> C
  - …
- Vigenere cipher (1500s)
  - + mod 26
- Rotor machines (1870-1943)
  - Single rotor: Hebern
  - 3-5 rotors: Enigma
- DES (Digital Encryption Standard) - 1974
  - 56 bits key, 64 bits block


Hebern machine

Enigma machine

Modern cipher types

- Regarding the procedure
  - Stream
  - Block
- Regarding the type of key
  - Symmetric (secret key, a shared secret)
  - Asymmetric (public key and private key)

Ciphers	**Block**	**Stream**
Symmetric	yes	yes
Asymmetric	yes	no

Roadmap

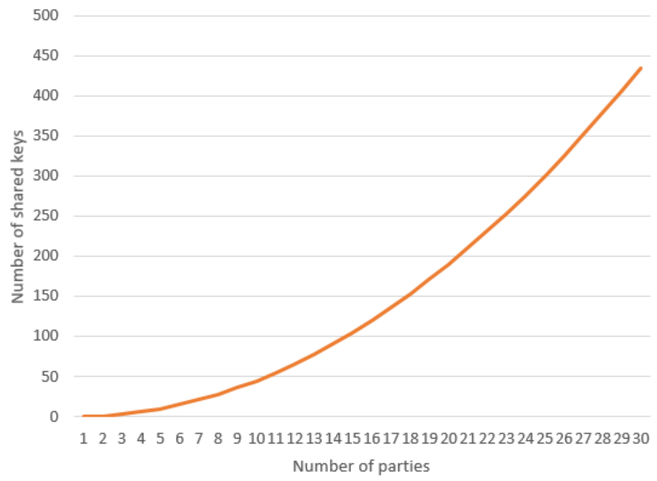
- Introduction
- **Symmetric Ciphers**
- Hash Functions
- Message Authentication Codes
- Asymmetric Ciphers
- Digital Signatures
- Homomorphic Ciphers

Symmetric Ciphers

- Symmetric Ciphers
  - Stream ciphers
  - Block ciphers
    - DES
    - AES
  - Block cipher modes

Symmetric Ciphers

- Secret key
  - Shared by 2 or more communicating parties
- Allow for
  - Confidentiality to all who possess the key
  - Message authentication
- Advantages
  - Performance (typically very efficient)
- Disadvantages
  - N communicating parties, 1 to 1 secretly
    ➔ N x (N-1)/2 keys
- Problems
  - Key distribution

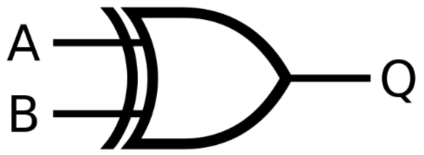


Perfectly Secure Cipher: One-Time Pad

- Mauborgne/Vernam [1917]

- XOR (⊕):
  - 0 ⊕ 0 = 0   1 ⊕ 0 = 1 →   a ⊕ 0 = a
  - 0 ⊕ 1 = 1   1 ⊕ 1 = 0 →   a ⊕ 1 = not a

  a ⊕ b ⊕ b = a



- Encrypt
  - E(P, K) = P ⊕ K = C
    - P = plaintext; K = key
- Decrypt
  - D(C, K) = C ⊕ K = (P ⊕ K) ⊕ K = P

Perfectly Secure Cipher: One-Time Pad

- One-Time Pad (XOR message with key)

- Example:
  - Message: ONETIMEPAD
  - Key: TBFRGFARFM
  - Ciphertext: IPKLPSFHGQ

  - The key TBFRGFARFM decrypts the message to ONETIMEPAD
  - The key POYYAEAAZX decrypts the message to SALMONEGGS
  - The key BXFGBMTMXM decrypts the message to GREENFLUID

One-Time Pad problems

- Security is based on the assumption that K is never reused
  - What if one has two encrypted messages:

$$C_1 = P_1 \oplus K \text{ and } C_2 = P_2 \oplus K$$

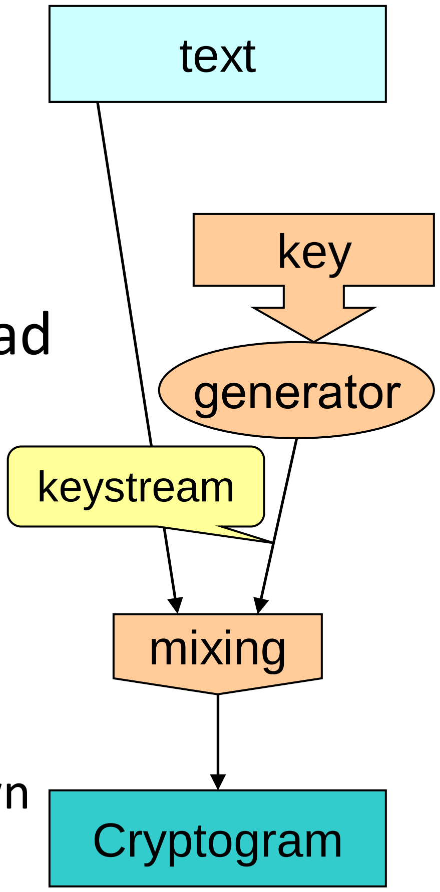
$$\begin{aligned} C_1 \oplus C_2 &= P_1 \oplus K \oplus P_2 \oplus K \\ &= P_1 \oplus P_2 \end{aligned}$$

- Need to generate truly random bit sequence
  - As long as all messages

- Need to securely distribute key bit sequence (!)

Stream ciphers

- Practical approximation to the One-Time Pad
- Keystreams generated in a deterministic way
  - From a fixed size key
  - Approximation to real random sequence generators
- Encryption and decryption as with a one-time pad
  - i.e., by doing XOR with the keystream (mixing)
- Practical aspects of stream ciphers security:
  - If the plain text is known, the keystream is exposed
  - The repetition of cycles (reuse of the keystream) facilitates cryptanalysis
    - if the cycle period or part of the plain text is known
  - Integrity control must exist
    - Easy to modify the cryptogram in a deterministic way

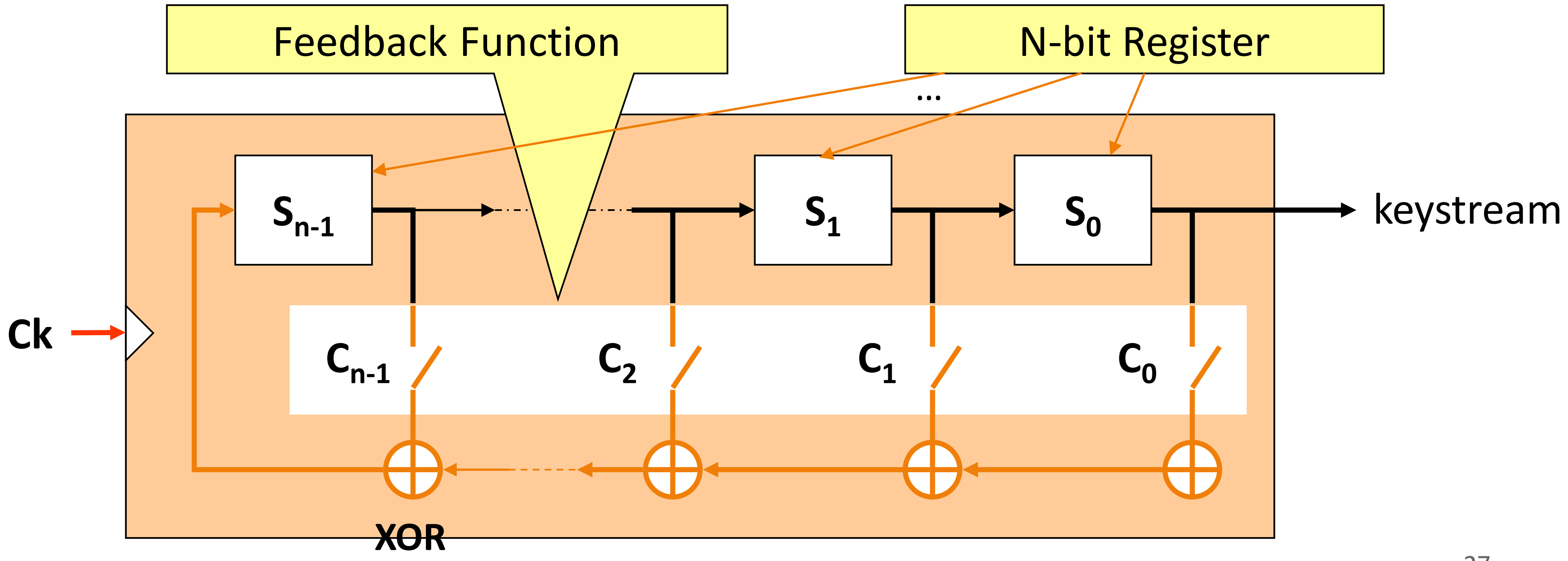


Symmetric stream ciphers

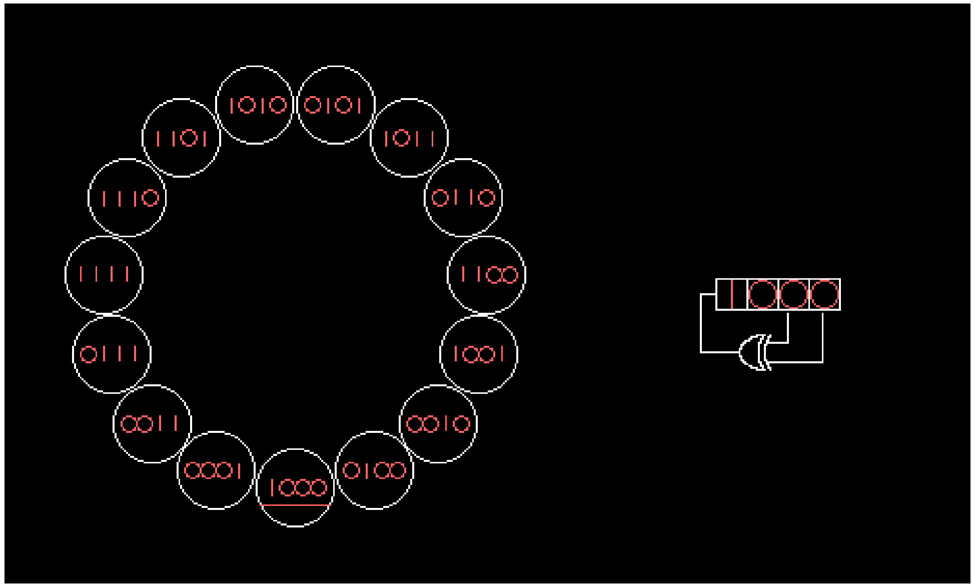
- Used approximations
  - Secure pseudo-random generators
    - Based on LFSRs (Linear Feedback Shift Registers) → next
    - Based on block ciphers → later
    - Other approximations (nonlinear functions, etc.)
  - Usually without self-synchronization
    - Receiver must know when encrypted data begins
  - Typically without the possibility of fast random access
- Most common algorithms
  - A5 (GSM)
  - RC4
  - SEAL (with fast random access)
  - Salsa20
  - Trivium

Linear Feedback Shift Register (LFSR)

- State machine that produces a cyclic sequence of bits
  - The sequence depends on the **key** = initial state of the register
  - $S_0, \ldots, S_{n-1}$ = **register**'s bits; $C_0, \ldots, C_{n-1}$ = **coefficients** of the function
  - Max. period of the cycle is 2^n-1



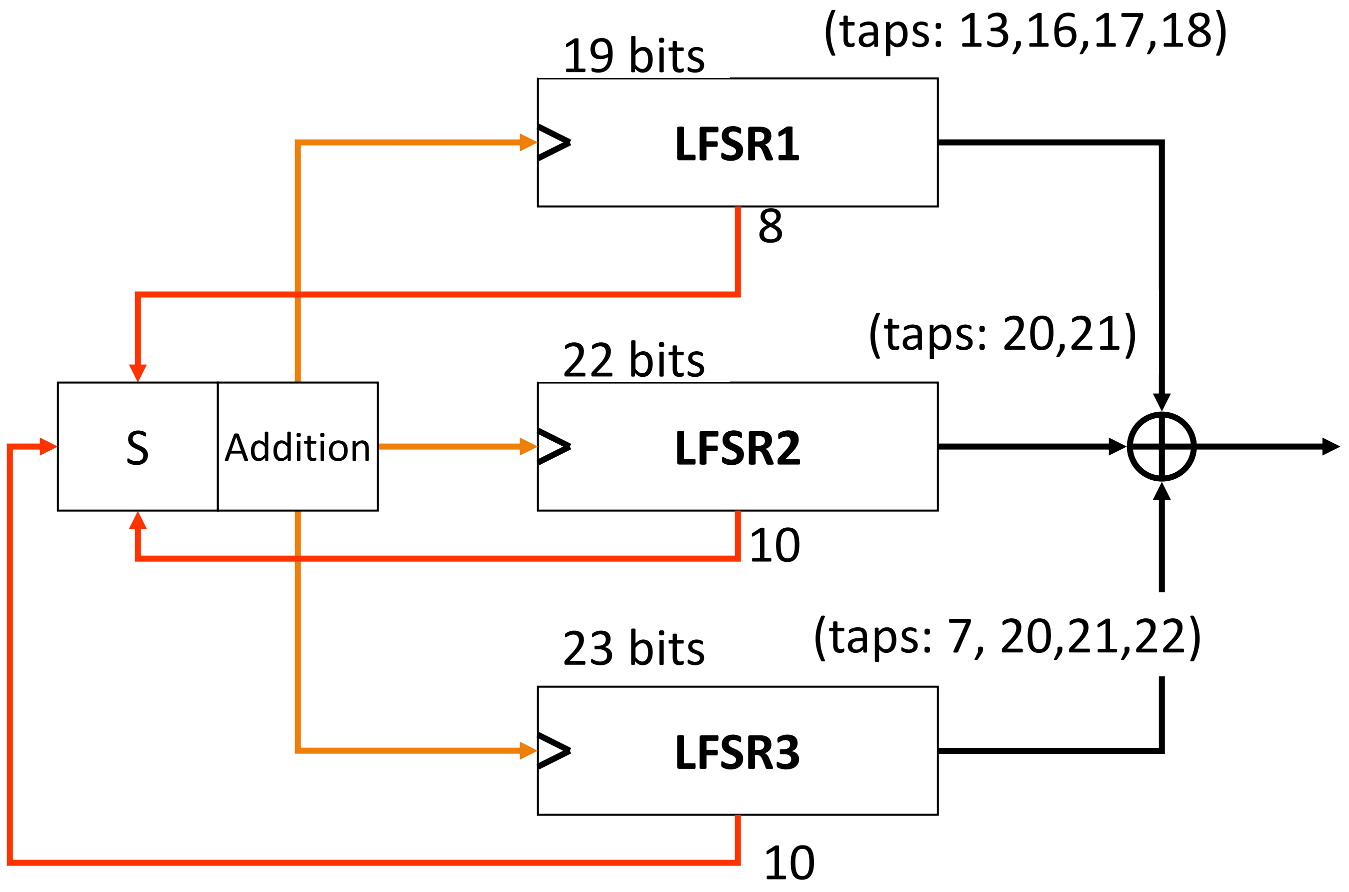
4 bit LFSR example



Bits 2 and 3 are **tapped**

Taps are the bits that affect the next state

LFSR structure: A5/1 (GSM)



Symmetric block ciphers

- Also based on approximations, using Shannon's notions of confusion and diffusion:
  - Confusion: repeated application of a complex function to a large block (e.g. 64 bits)
  - Diffusion: basic operations:
    - Permutation: exchange bits without losing or adding bits
    - Substitution: change bits for others using a substitution table
    - Expansion: introducing new bits
    - Compression: deleting some bits
- Some relevant symmetric encryption algorithms:
  - DES – Data=64; Key=56 – insecure, never use
  - AES – D=128; K=128, 192, 256 – current standard
  - Others (IDEA, Blowfish, CAST, RC5, etc.)

DES Avalanche

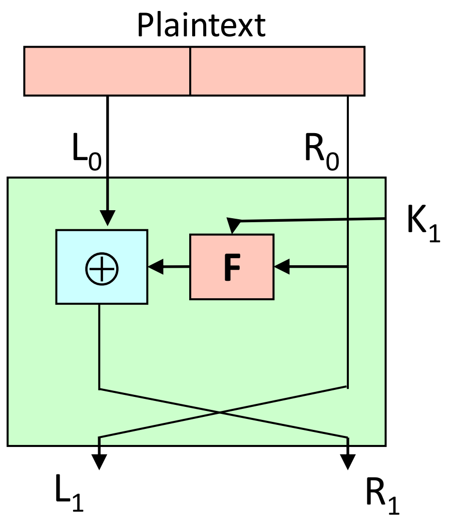
```
Input:     ...............................................................*      1
Permuted:  .......................................*........................      1
Round 1:   .......*........................................................      1
Round 2:   .*..*...*.....*........................*........................      5
Round 3:   .*..*.*.**..*.*.*.*....**.....**.*..*...*.....*.................     18
Round 4:   ..*.*****.*.*****.*.*......*.....*..*.*.**..*.*.*.*....**.....**     28
Round 5:   *...**..*.*...*.*.*.*...*.***..*..*.*****.*.*****.*.*......*....     29
Round 6:   ...*..**.....*.*..**.*.**...*..**...**..*.*...*.*.*.*...*.***..*     26
Round 7:   *****...***....**...*..*.*..*......*..**.....*.*..**.*.**...*..*
Round 8:   *.*.*.*.**.....*.*.*...**.*...*******...***....**...*..*.*..*...
Round 9:   ***.*.***...**.*.****.....**.*..*.*.*.*.**.....*.*.*...**.*...**
Round 10:  *.*..*.*.**.*..*.**.***.**.*...****.*.***...**.*.****.....**.*..
Round 11:  ..******......*..******....*....*.*..*.*.**.*..*.**.***.**.*...*
Round 12:  *..***....*...*.*.*.***...****....******......*..******....*....
Round 13:  **..*....*..******...*........*.*..***....*...*.*.*.***...****..
Round 14:  *.**.*....*.*....**.*...*..**.****..*....*..******...*........*.
Round 15:  **.*....*.*.*...*.**.*..*.*.**.**.**.*....*.*....**.*...*..**.**
Round 16:  .*..*.*..*..*.**....**..*..*..****.*....*.*.*...*.**.*..*.*.**.*
Output:    ..*..**.*.*...*....***..***.**.*...*..*..*.*.*.**.*....*.*.*.**.   ~50%
```

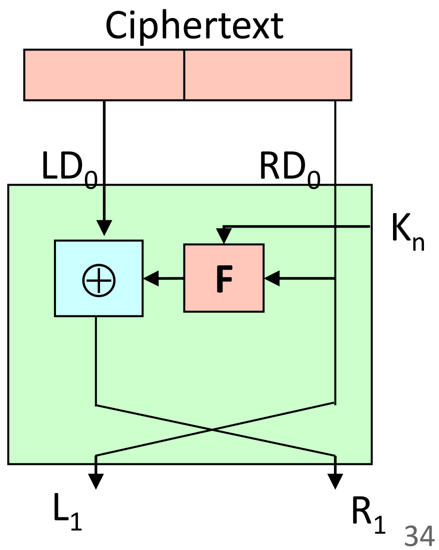
* is a changed bit that propagates changes

Feistel Network

- Complex function most commonly used in block cipher algorithms

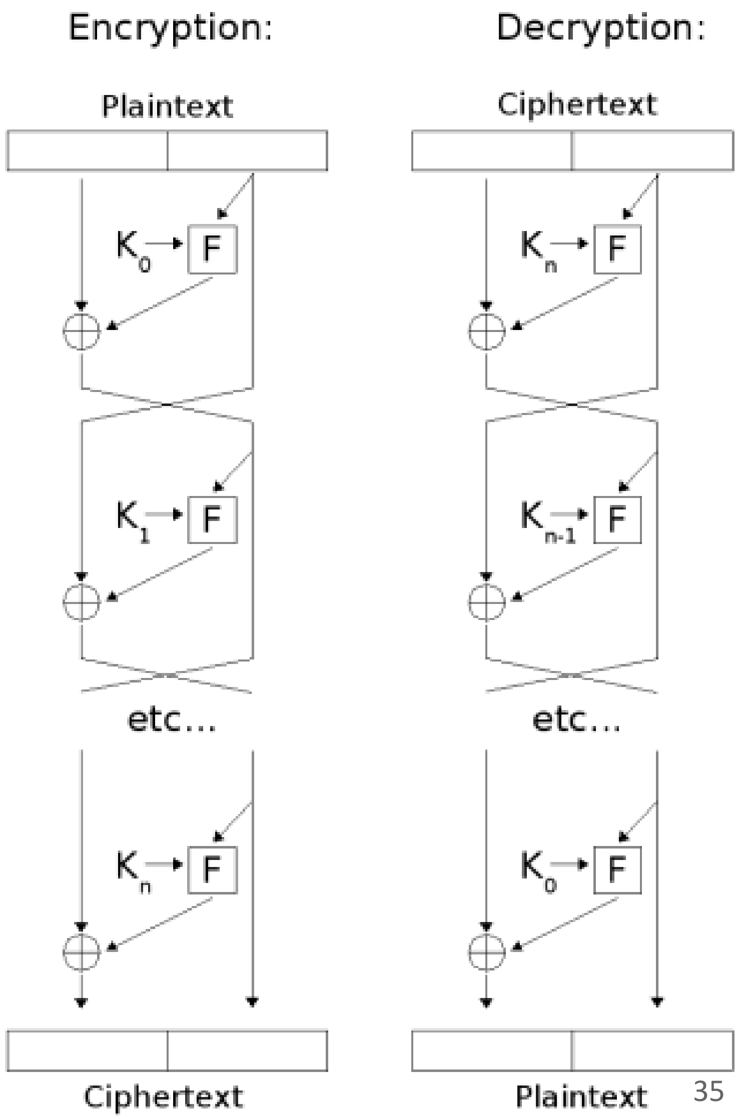
- Applies a round function (F) over multiple rounds
  - F can be a pseudo-random generator
  - Each round uses a different round key (K_i)
  - Round keys are obtained from the key (K)
  - Text is split in left (L) and right (R) parts





Feistel Network

- Cipher and decipher processes are the same
  - Keys are used in the inverse order



DES Algorithm

- DES:
  - 64-bit blocks
  - Key is 56 bits
- Possible key combinations
  - 2^{56} = 7.2 x 1016 = 72,000,000,000,000,000
  - Try 1 per second = 9,000 Million years to search the entire key space
- Distributed attacks on DES:
  - RSA's DES challenges:
    - 1997: 96 days (using 70,000 machines)
    - 1998: 41 days (distributed.net)
    - 2008: less than 1 day (array of 128 Spartan FPGAs ~ €5k)
    - 2016: about 15 days on a standard PC with a GPU (~€800)

Block ciphers: reinforcement

- Multiple ciphers
  - Double cipher
    - Breakable by brute force in max. 2^{n+1} attempts instead of expected 2^{2n}
  - Triple cipher, typ. EDE = Encrypt-Decrypt-Encrypt – 3DES-EDE
    - $C_i = E_{K1}(D_{K2}(E_{K3}(P_i)))$
    - $P_i = D_{K3}(E_{K2}(D_{K1}(C_i))$
    - Typically, $K_1=K_3$ is used
    - Effective key size = 56 + 56 bits = 112 bits
- Key whitening (DESX)
    - DES-X$(P_i) = K_2 \oplus DES_K(K_1 \oplus P_i)$
      - $C_i = E_K(K_1 \oplus P_i) \oplus K_2$
      - $P_i = K_1 \oplus D_K(K_2 \oplus C_i)$

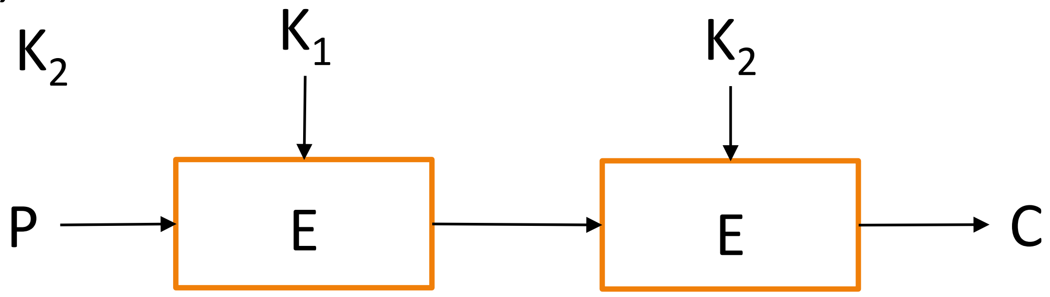
Meet-in-the-middle attack

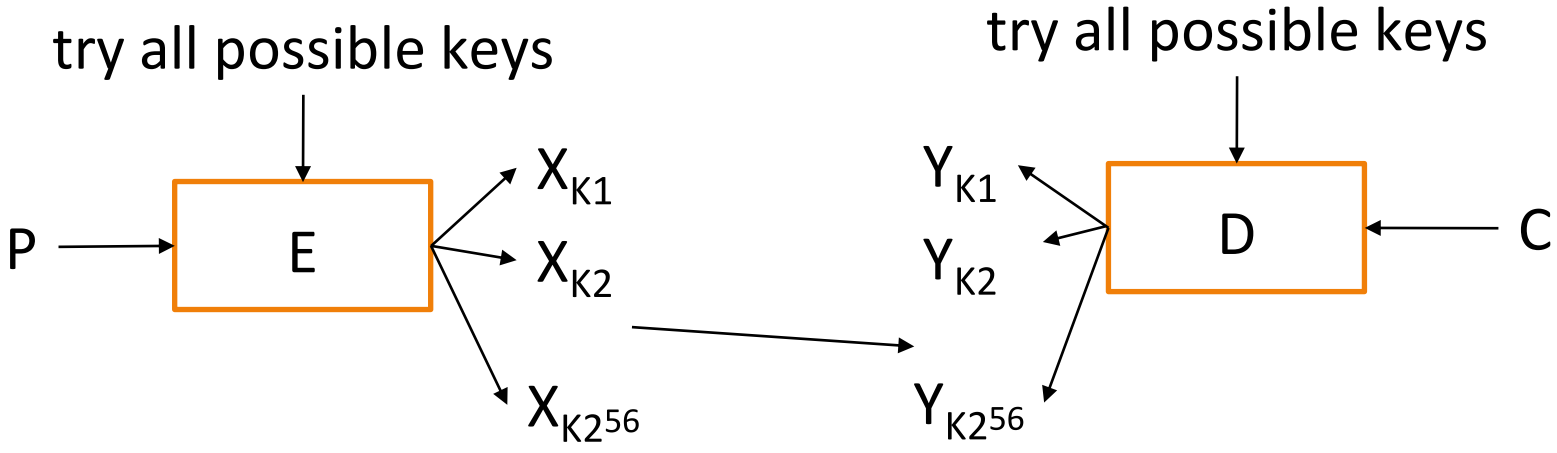
- Double DES
  - $C = E_{K2} (E_{K1} (P))$
  - Effective key size of Double DES?

$= 2^{56} * 2^{56} = 2^{112}$ WRONG!

Meet-in-the-middle attack
a known-plaintext attack

Attacker knows P, C
doesn't know K_1, K_2





One $X_{Ki} = Y_{Kj}$ means $K_1 = K_i$ and $K_2 = K_j$
so maximum effort is $2 * 2^{56}$ tries

Meet-in-the-middle attack

- Double DES
  - $C = E_{K2}(E_{K1}(P))$
  - Effective key size of Double DES?

    $= 2^{56} * 2^{56} = 2^{112}$ WRONG!

- Meet-in-the-Middle Attack
  - $C = E_{K2}(E_{K1}(P))$
  - $X = E_{K1}(P) = D_{K2}(C)$
  - Brute force attack (given one P/C pair):
    1. calculate EK1 (P) for all keys (2^{56} work)
    2. calculate DK2 (C) for all keys (2^{56} work)
    3. the match gives the keys
  - Total work = $2 * 2^{56} = 2^{57}$

AES – current encryption standard

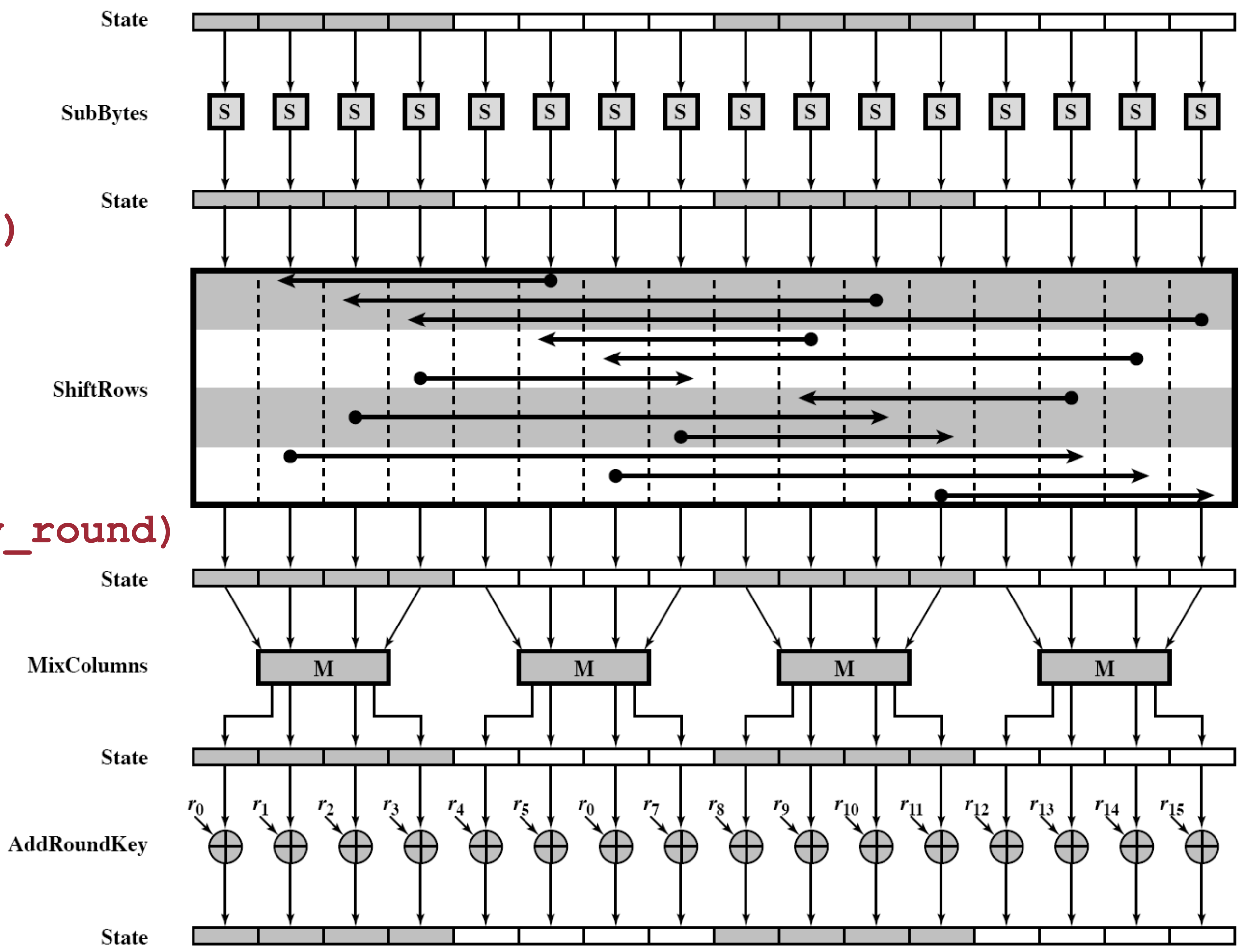
- AES (Advanced Encryption Standard) parameters:
  - Key size: 128, 192, 256 bits
  - Input block length: 128 bits

- Runs 10/12/14 rounds in which it:
  - substitutes bytes (one S-box used on every byte)
  - shifts rows (permute bytes between columns)
  - mixes columns (substitute using column multiplication)
  - adds round key (XOR *state* with key material)
    - round key size: 128

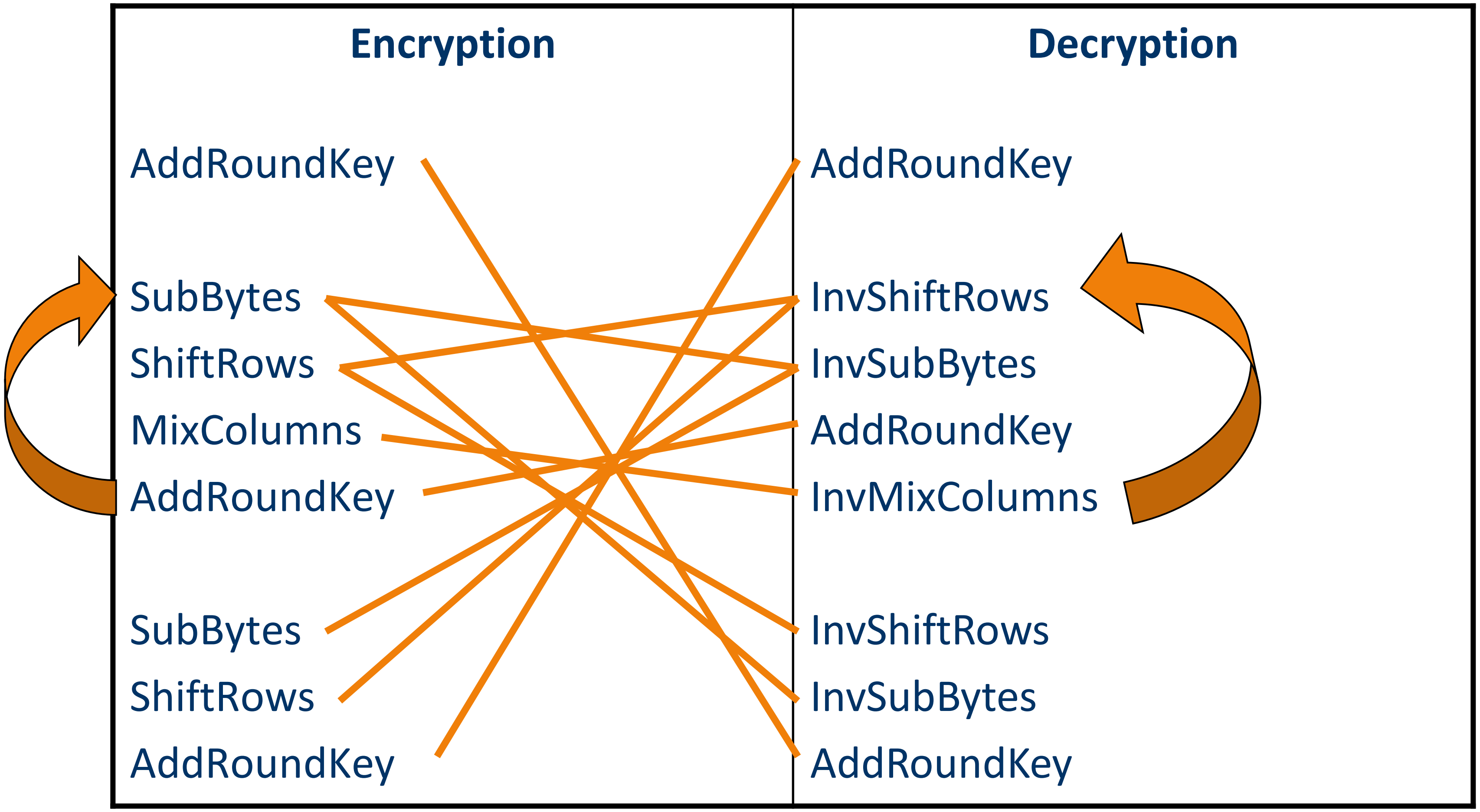
  - With fast XOR & table lookup implementations

AES - Main Encryption Round

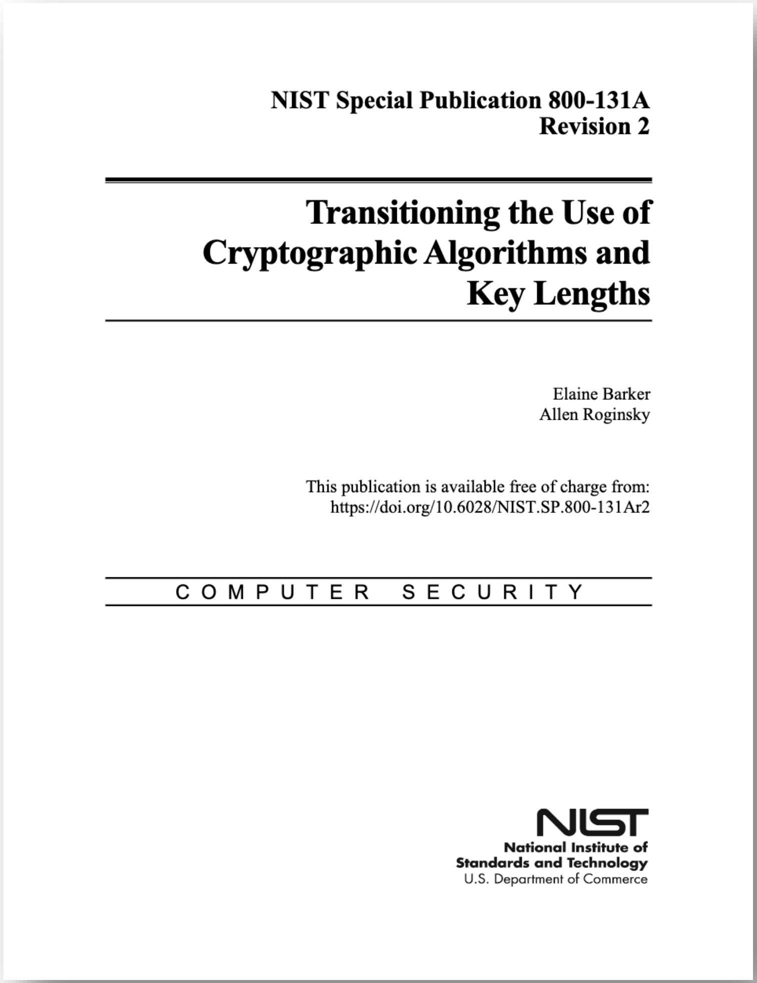
```
state = input

AddRoundKey(Key0)

for #rounds{
 SubBytes()
 ShiftRows()
 MixColumns()
 AddRoundKey(Key_round)
}

out = state
```


State
SubBytes
S
State
ShiftRows
State
MixColumns
M
State
AddRoundKey
r_0 r_1 r_2 r_3 r_4 r_5 r_0 r_7 r_8 r_9 r_{10} r_{11} r_{12} r_{13} r_{14} r_{15}
State

AES – Encrypt *vs* Decrypt



Which algorithm / key size to use?


NIST Special Publication 800-131A
Revision 2

Transitioning the Use of Cryptographic Algorithms and Key Lengths

Elaine Barker
Allen Roginsky

This publication is available free of charge from:
https://doi.org/10.6028/NIST.SP.800-131Ar2

COMPUTER SECURITY

NIST
National Institute of Standards and Technology
U.S. Department of Commerce

March 2019

Table 1: Approval Status of Symmetric Algorithms Used for Encryption and Decryption

Algorithm	Status
Two-key TDEA Encryption	Disallowed
Two-key TDEA Decryption	Legacy use
Three-key TDEA Encryption	Deprecated through 2023 Disallowed after 2023
Three-key TDEA Decryption	Legacy use
SKIPJACK Encryption	Disallowed
SKIPJACK Decryption	Legacy use
AES-128 Encryption and Decryption	Acceptable
AES-192 Encryption and Decryption	Acceptable
AES-256 Encryption and Decryption	Acceptable

Cipher modes

- Problem
  - How to use a block cipher with a fixed block size with plaintext of a different size?
    - Plaintext may be much larger than the block size
    - Size of the plaintext may not be a multiple of the size of the block
- The problem is solved by cipher modes

Block cipher modes

- Initially proposed for DES
  - ECB (Electronic Code Book)
  - CBC (Cipher Block Chaining)
  - OFB (Output FeedBack mode)
  - CTR (CounTeR mode)
  - GCM (Galois Counter Mode)

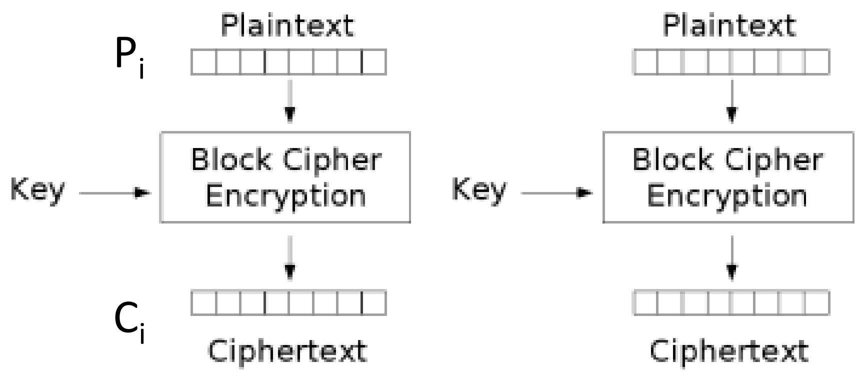
- Final sub-block processing
  - Alignment with padding
    - Augments the cryptogram by adding padding to the plaintext

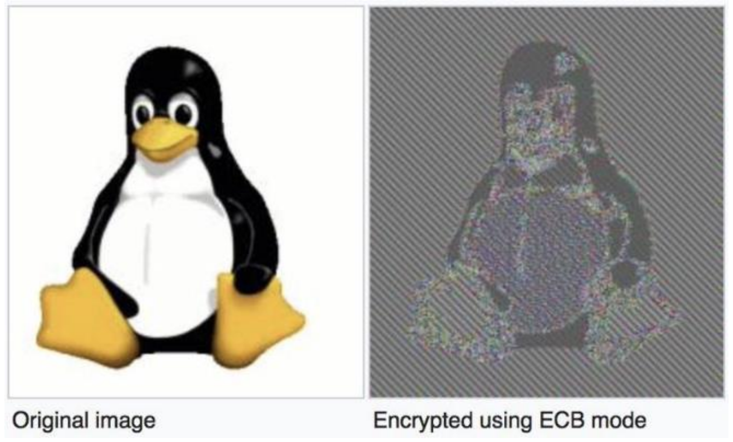
Block cipher modes: ECB

ECB (Electronic Code Book)

$C_i = E_k(P_i)$

$P_i = D_k(C_i)$





Original by Larry Ewing's

- Strength:
  - Simple
- Weakness:
  - Repetitive information in the plaintext may show in the ciphertext, if aligned with blocks
  - If same message is encrypted with the same key and resent, their cipher texts are the same
- Typical use: sending short pieces of data, e.g., encryption key

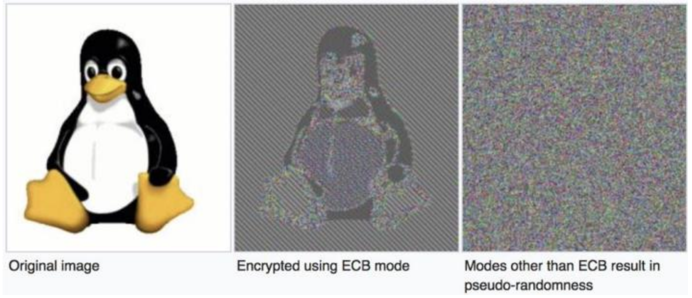
Block cipher modes: CBC

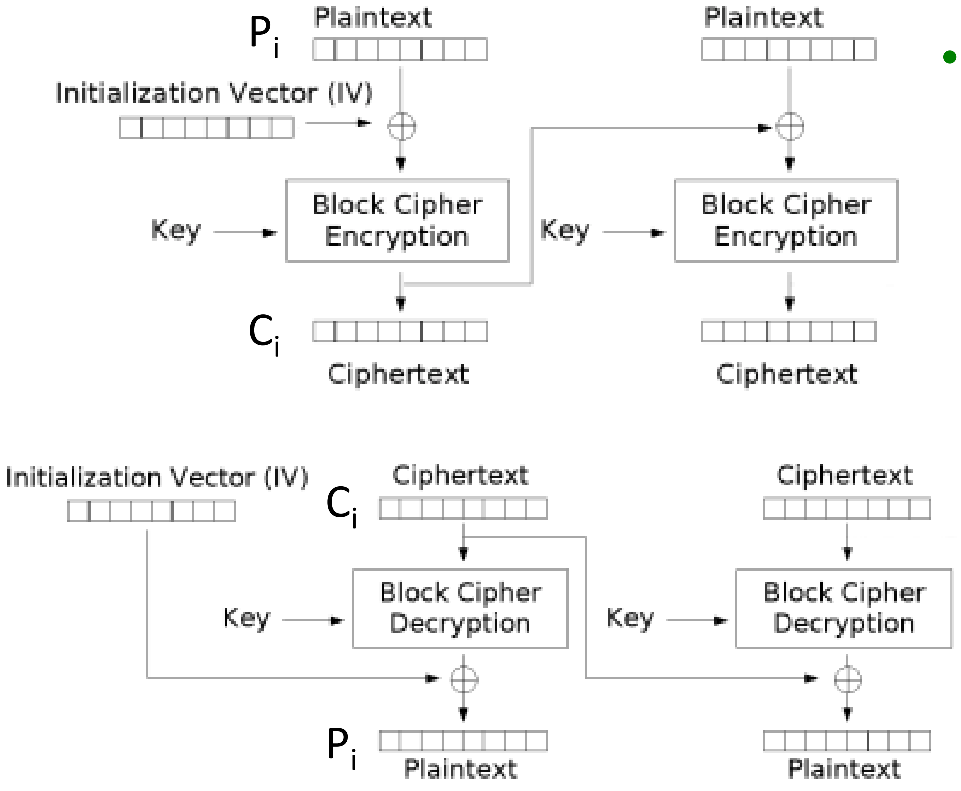
CBC (Cipher-Block Chaining)

$C_i = E_k(P_i \oplus C_{i-1})$

$P_i = C_{i-1} \oplus D_k(C_i)$

$C_0 = IV$





- Strength:
  - Repeated plaintext blocks result in different ciphered blocks
  - A ciphertext block depends on **all blocks** before it

Weakness:
  - More complex
  - A corrupted bit in the ciphertext will affect all bits in its block and **one bit** in the next block

Block cipher modes: OFB / CTR

OFB (Output FeedBack)

$C_i = P_i \oplus O_i$

$P_i = P_i \oplus O_i$

Transforms block cipher into stream cipher

$O_i = E_k(O_{i-1})$

$O_0 = IV$

CTR (CounTer) mode *or*

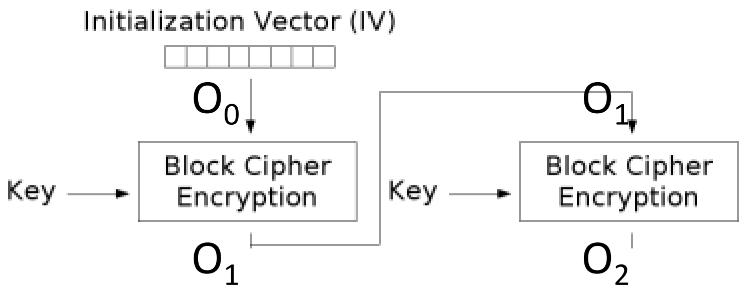
ICM (Integer Counter Mode)

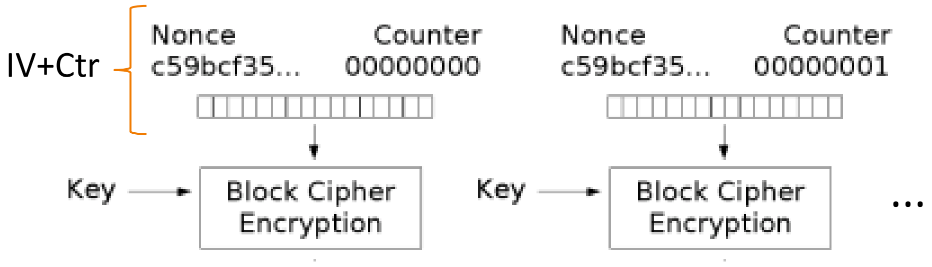
$C_i = P_i \oplus E_k(IV+Ctr_i)$

$P_i = C_i \oplus E_k(IV+Ctr_i)$

Transforms block cipher into stream cipher

$Ctr_i = Ctr_i + 1$





Block cipher modes: OFB / CTR

OFB (Output FeedBack)

$$\left.\begin{array}{l} C_i = P_i \oplus O_i \\ P_i = P_i \oplus O_i \end{array}\right\} \text{Transforms block cipher into stream cipher}$$

$O_i = E_k(O_{i-1})$

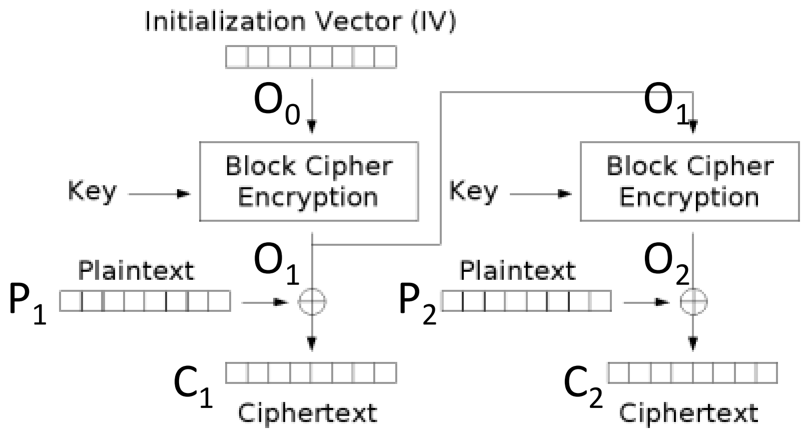
$O_0 = IV$

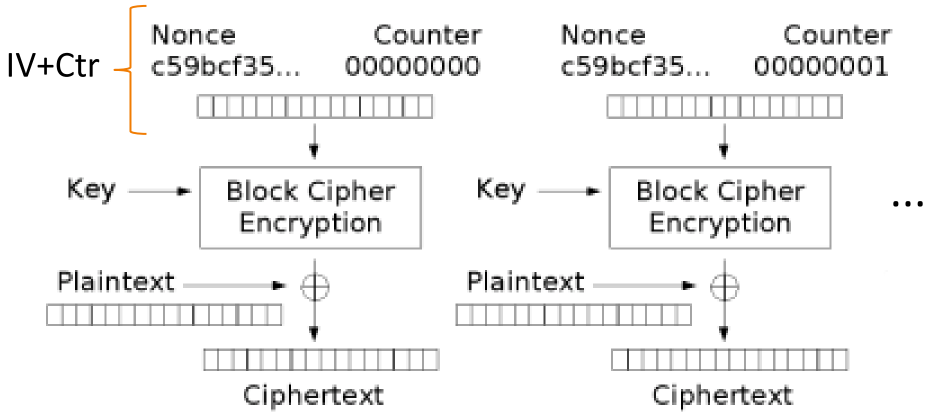
CTR (CounTer) mode *or*

ICM (Integer Counter Mode)

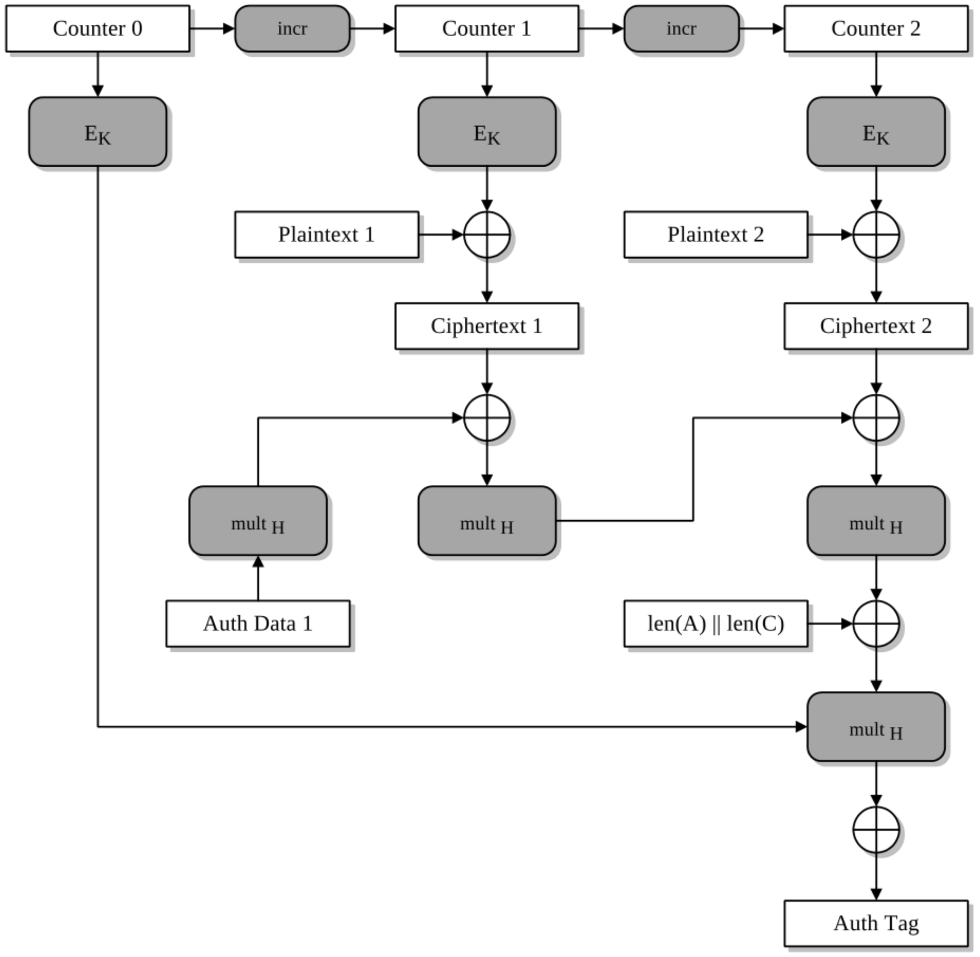
$$\left.\begin{array}{l} C_i = P_i \oplus E_k(IV+Ctr_i) \\ P_i = C_i \oplus E_k(IV+Ctr_i) \end{array}\right\} \text{Transforms block cipher into stream cipher}$$

$Ctr_i = Ctr_i + 1$



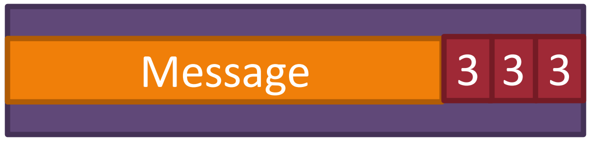


Block cipher modes: GCM
Galois Counter Mode



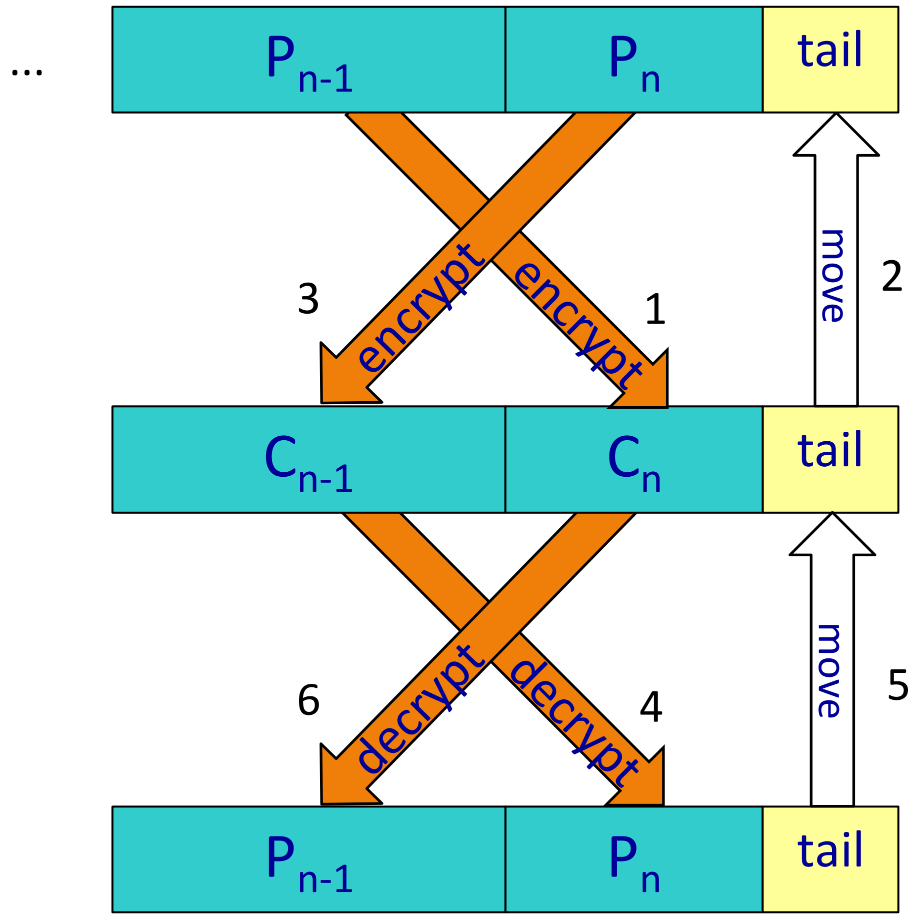
Padding Schemes

- $M \bmod B \neq 0$ => **padding** of last block (M=message size; B=block size)
- Options:
  - Pad with zero (null) bytes, spaces (0x20), all bytes of the same value
  - Pad with random bits
  - Pad with 0x80 (1000 0000) followed by zero (null) characters
  - **PKCS#5 scheme**
    - Sequence of bytes, each of which equal to the number of padding bytes
    - Example: if 24 bits of padding need to be added, the padding string is "03 03 03" (3 bytes times 8 bits equals 24 bits)

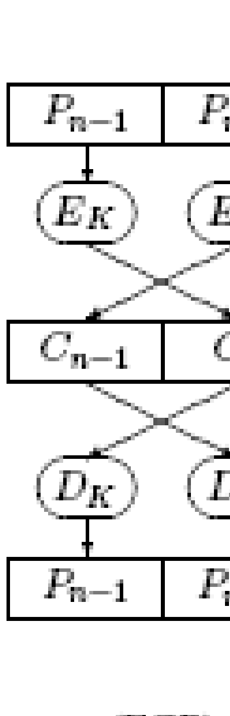


Cipher Text Stealing: ECB mode

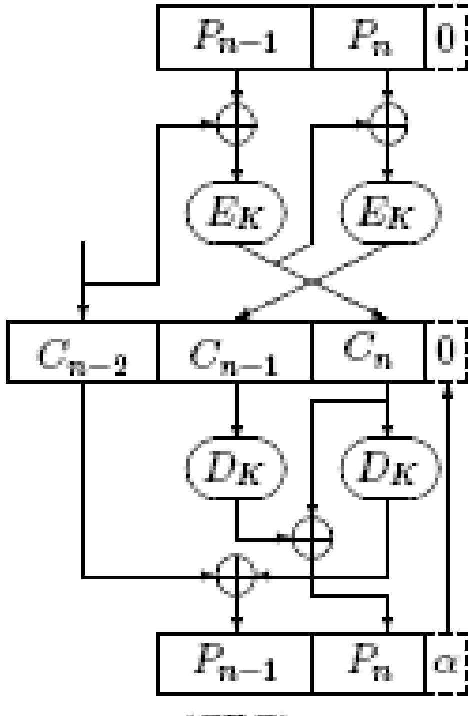
- Avoids the need of padding
- Plays with the last 2 blocks
- Benefit: no need to send the padding bits!



Ciphertext Stealing: ECB and CBC



(ECB)



(CBC)

Roadmap

- Introduction
- Symmetric Ciphers
- **Hash Functions**
- Message Authentication Codes
- Asymmetric Ciphers
- Digital Signatures
- Homomorphic Ciphers

Hash functions / message digests

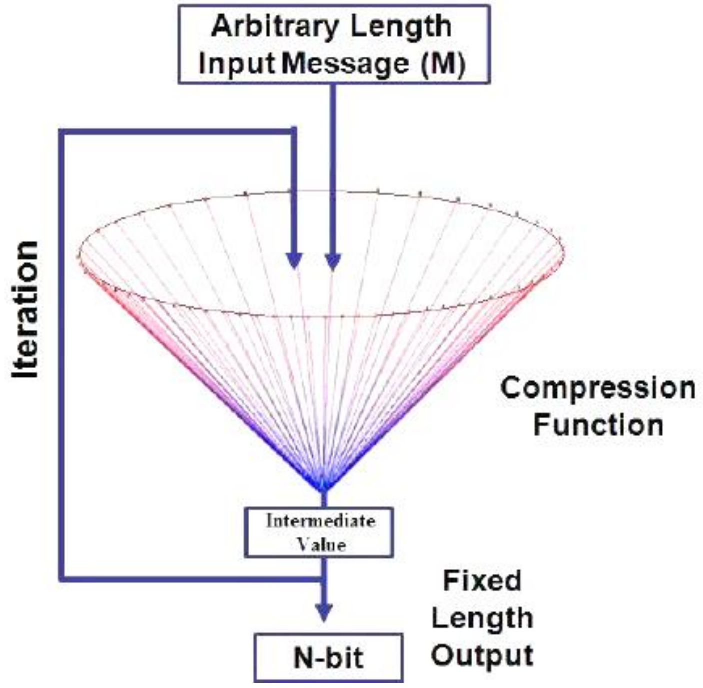
- Aka cryptographic hash functions or collision-resistant hash functions
  - don’t confuse with hash functions used in hash tables (that you learned in IAED)
- They are cryptographic, but not ciphers
  - not used for encryption

Hash functions properties

- Generate very different output values – **hash** value – for similar inputs
- One-way (non-invertible):
  - Collision resistance
    - Computationally infeasible to find two inputs that give the same hash
  - Preimage resistance (strong collision resistance)
    - Given a hash, it’s computationally infeasible to find an input that produces that hash
  - 2nd preimage resistance (weak collision resistance)
    - Given a hash value and the corresponding input, it’s computationally infeasible to find a second input that generates that same hash

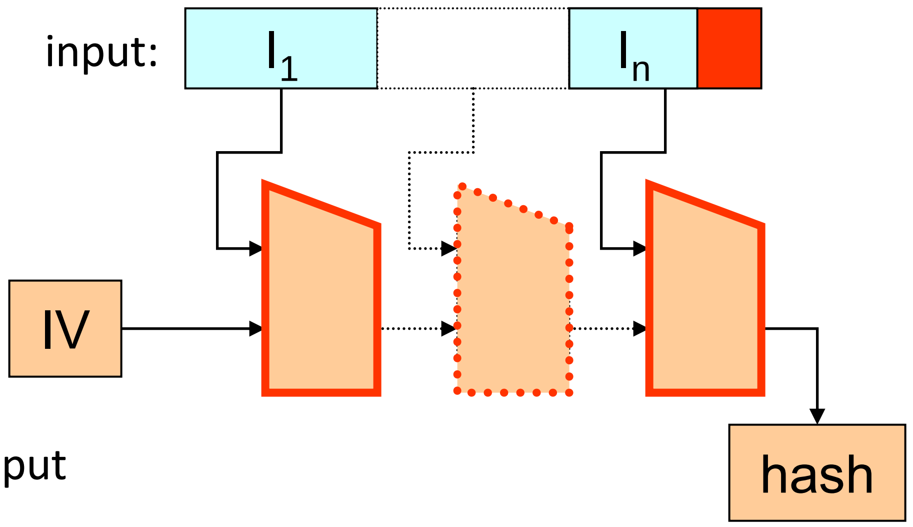
Hash functions

- Generate a fixed size value based on arbitrary size input
  - Iterative usage of a compression function with a fixed parameter input
  - The input text is aligned to the input blocks



Hash functions

- Some mechanisms used:
  - Shannon's diffusion & confusion
  - Iterative compression
  - MD Strengthening
    - Padding with 10000….000
    - Plus the number of bits of the input

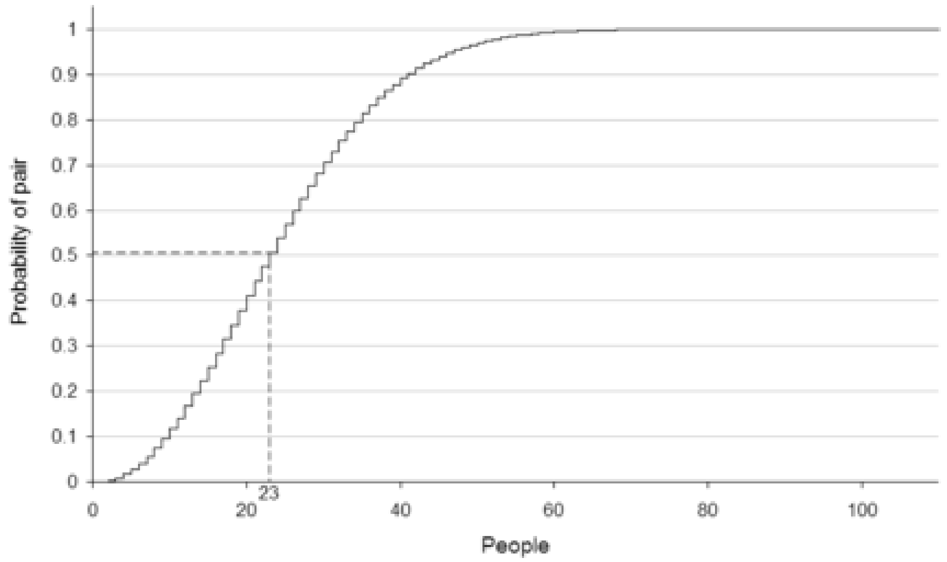


Hash functions

- Most used algorithms
  - Brute force attacks find collisions in $2^{m/2}$ tries, where m is the n. of bits
  - **MD5** (128 bits) *Very weak*
    - Collisions found in 2^{39} tries << 2^{64}
  - **SHA-1** (Secure Hash Algorithm, 160 bits) *Weak*
    - Collisions found in 2^{63} tries << 2^{80}
  - **SHA-2** (256 to 512 bits) *Ok*
    - Collisions found in 2^{128} or 2^{256} tries (secure, for now)
  - **SHA-3** (256 to 512 bits) *Ok*

Birthday paradox

- For there to be a 50% chance that someone in a room shares your birthday, you need 253 people
- However, for 50% chance that any two people in the room have the same birthday, you only need 23 people
  - It only takes 23 people to form 253 pairs when cross-matched



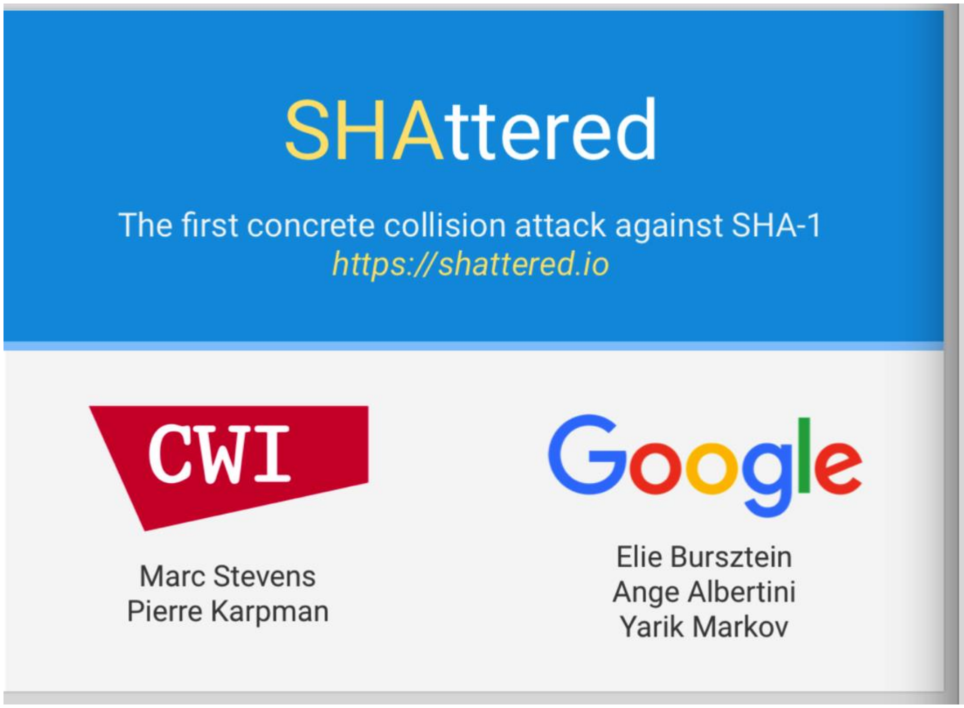
Birthday paradox

- Paradox
  - *P(me): probability n students having the same birthday as me (“collision”)*
    - Probability A’s birthday date is the same as mine = 1/365 ~= 0.003
    - Probability A’s birthday date different from mine = 1 - 1/365 ~= 0.997
    - Probability A and B’s dates different from mine = $(1 - 1/365)^2$ ~= 0.994
    - P(me) = $1-(1-\frac{1}{365})^n$
    - P(me) > 0,5 $\Leftrightarrow$ n $\geq$ 253
  - *P(pair): probability of pair with the same birthday*
    - P(pair) = $1-(1-\frac{1}{365})(1-\frac{2}{365})\ldots(1-\frac{n-1}{365})$
    - P(pair) > 0,5 $\Leftrightarrow$ n $\geq$ 23 !!!

Hash functions attacks

- Objective is to find a **collision**
- **Brute forcing**
  - <u>Attack</u>: pick H(M) and see if it's equal to H(M'), H(M''), H(M'''),…
  - P(collision) > 0.5 = $2^m/2$ = $\mathbf{2^{m-1}}$
  - where **m** is the number of bits of the hash function
- **Birthday attack**
  - Allows finding collision much faster
  - <u>Attack</u>: pick M, M', M'', M'''… and obtain hashes until any 2 are identical
  - Finding 2 messages with H(M) = H(M'):
  - P(collision) > 0.5 ~= $\mathbf{2^{m/2}}$ tries (only)
- Cryptanalysis leads to even faster attacks

SHA-1 is not secure



SHAttered

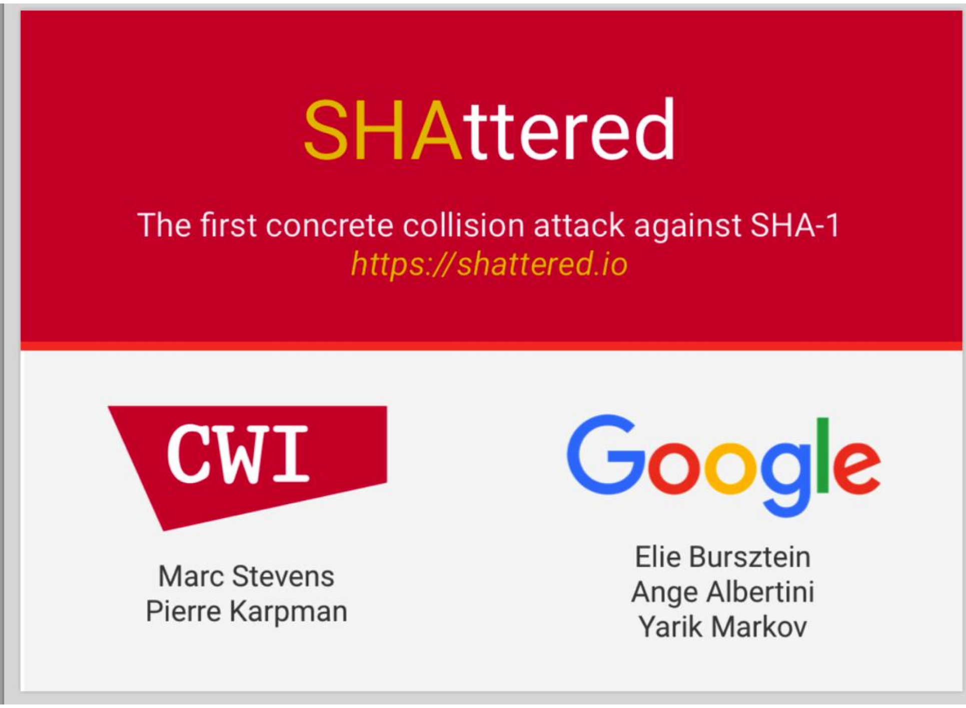
The first concrete collision attack against SHA-1
https://shattered.io

CWI

Marc Stevens
Pierre Karpman

Google

Elie Bursztein
Ange Albertini
Yarik Markov



SHAttered

The first concrete collision attack against SHA-1
https://shattered.io

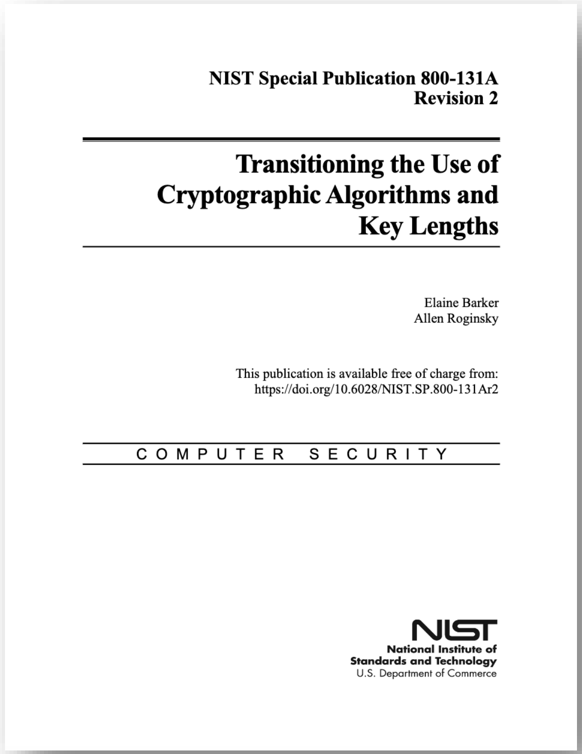
CWI

Marc Stevens
Pierre Karpman

Google

Elie Bursztein
Ange Albertini
Yarik Markov

Different PDF files, same SHA-1 hash



NIST Special Publication 800-131A
Revision 2

Transitioning the Use of Cryptographic Algorithms and Key Lengths

Elaine Barker
Allen Roginsky

This publication is available free of charge from:
https://doi.org/10.6028/NIST.SP.800-131Ar2

C O M P U T E R   S E C U R I T Y

National Institute of Standards and Technology
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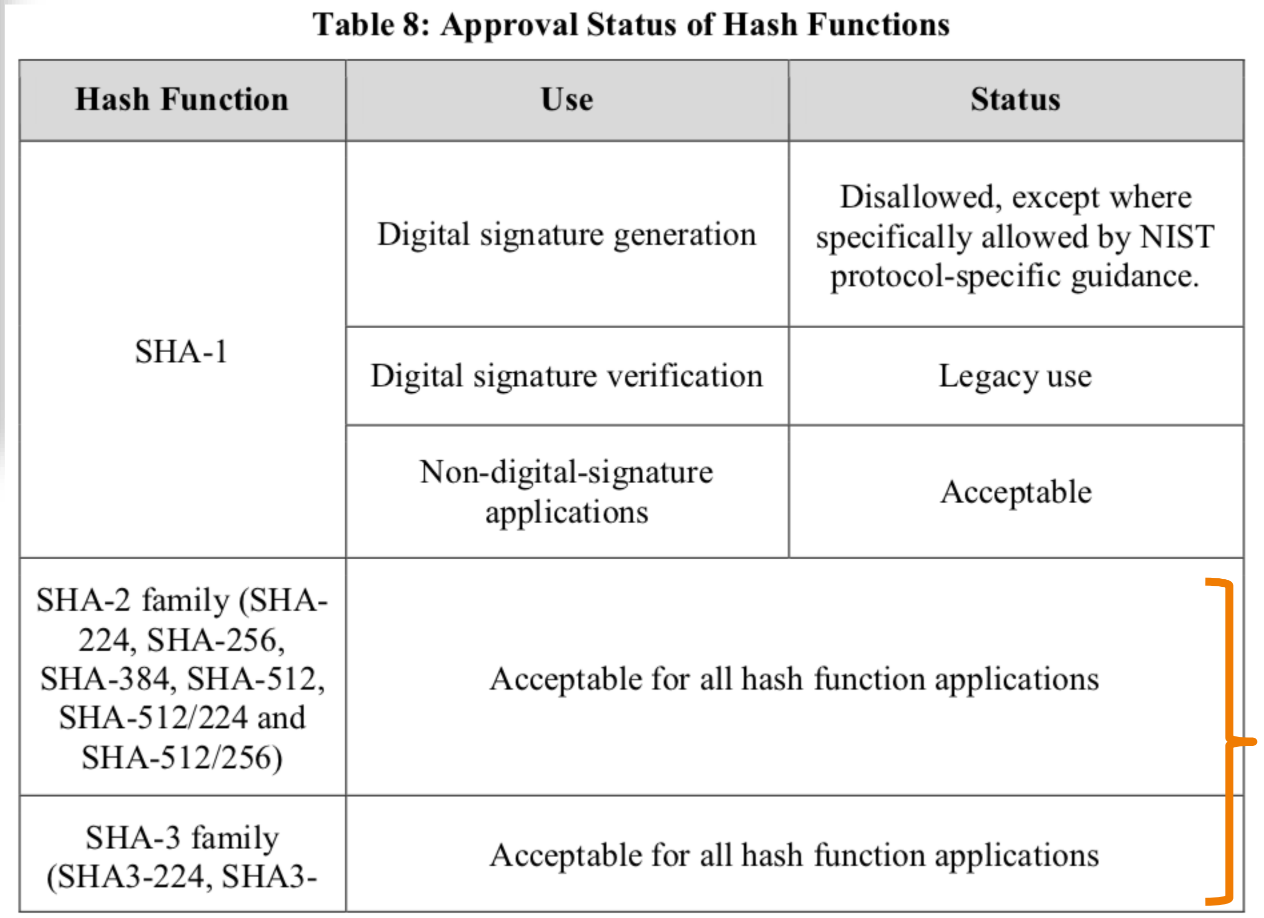


Table 8: Approval Status of Hash Functions

Hash Function	Use	Status
SHA-1	Digital signature generation	Disallowed, except where specifically allowed by NIST protocol-specific guidance.
	Digital signature verification	Legacy use
	Non-digital-signature applications	Acceptable
SHA-2 family (SHA-224, SHA-256, SHA-384, SHA-512, SHA-512/224 and SHA-512/256)	Acceptable for all hash function applications	
SHA-3 family (SHA3-224, SHA3-	Acceptable for all hash function applications	

Roadmap

- Introduction
- Symmetric Ciphers
- Hash Functions
- **Message Authentication Codes**
- Asymmetric Ciphers
- Digital Signatures
- Homomorphic Ciphers

Message Authentication Code (MAC)

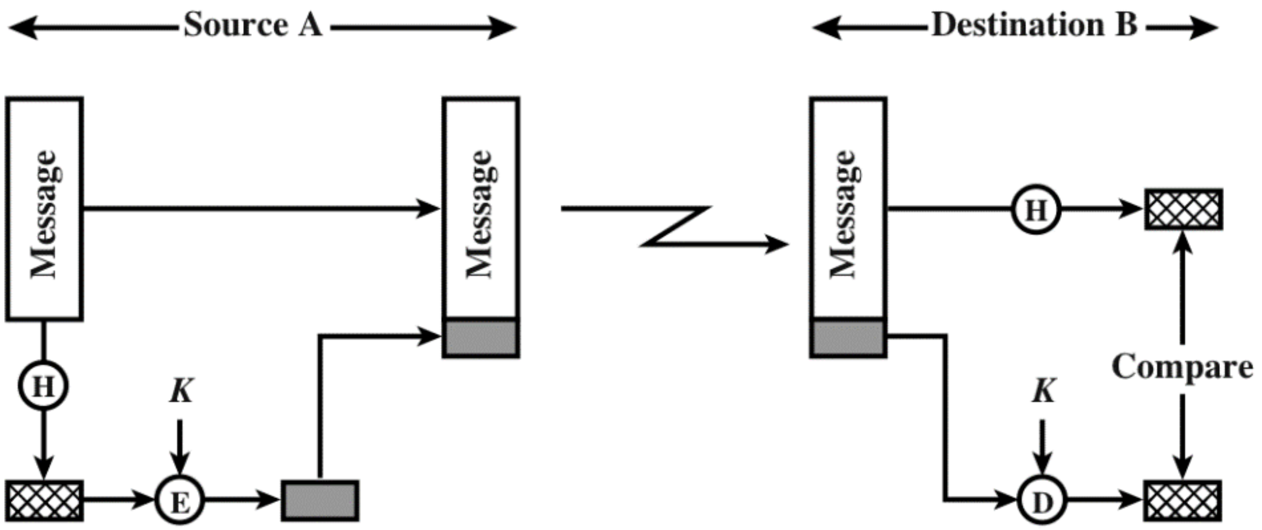
- Objective: **authenticating** a message or equivalent
  - Allows checking its authenticity and integrity
- Assumption: sender and recipient have a shared secret key K
- Idea:
  - Send the message + MAC
  - If the message (or the MAC) is modified by an attacker, the recipient will be able to detect it
  - Attacker can't create a valid MAC because he doesn't have K
- What about CRCs and checksums?
  - Attacker can modify the message and CRC/checksum accordingly, as he knows the algorithm and there is no secret involved

Message Authentication Code (MAC)

- MAC is a hash generated using a secret key
  - 1- Hash the message and encrypt the digest
    - For example, with a symmetric block cipher
  - 2- Using a keyed-function
    - ANSI X9.9 (a.k.a. DES-MAC) with DES-CBC (64 bits) – next
  - 3- Hash the message along with a shared key
    - Keyed-MD5 (128 bits)
    - HMAC (size of the used hash function)

Message Authentication Code (MAC)

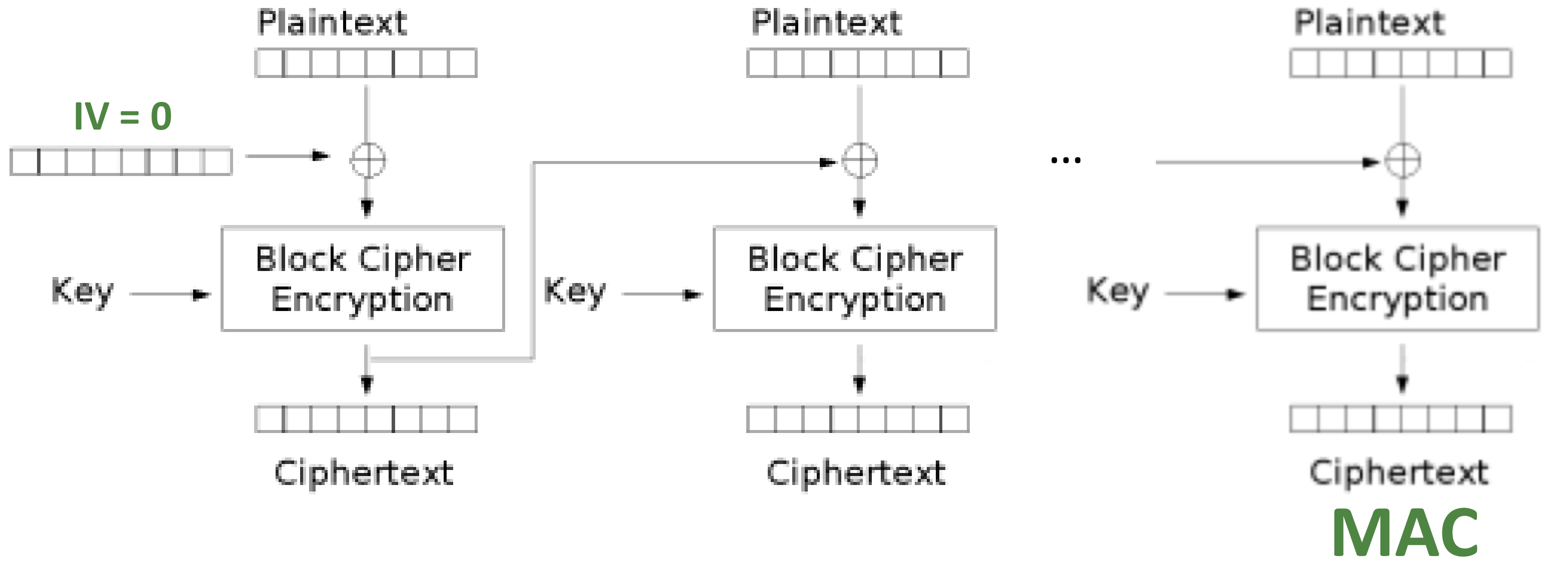
- 1- Hash the message and encrypt the digest



If attacker modifies the message (or the hash), the hash calculated at the destination won't match the hash received

Message Authentication Code (MAC)

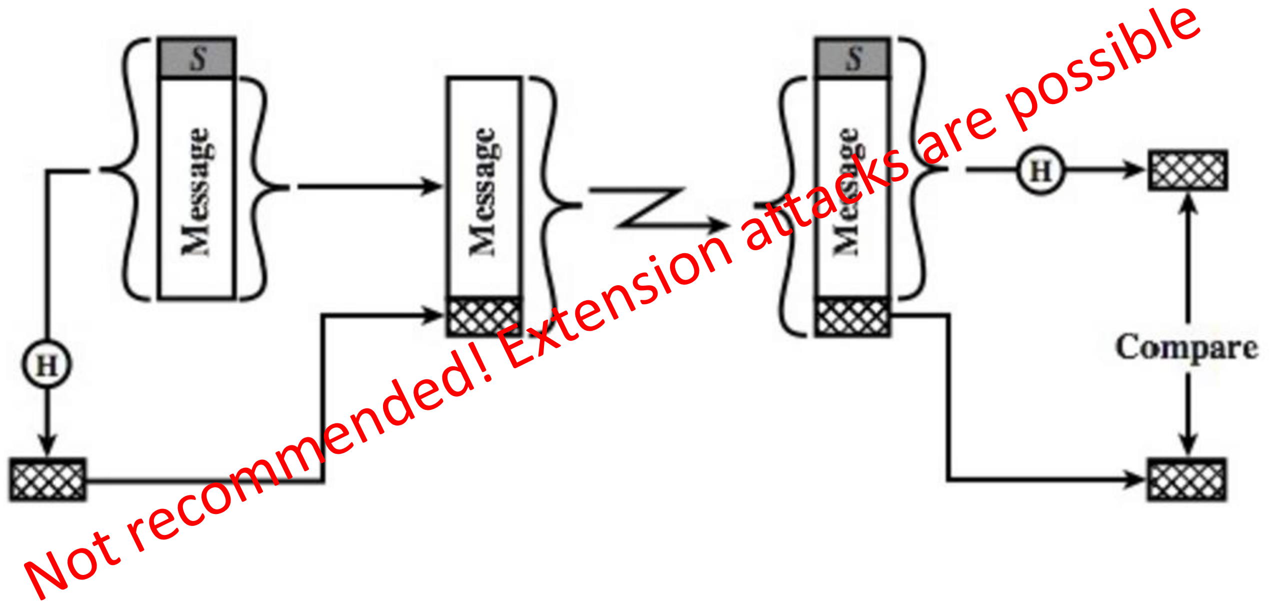
- 2- CMAC (Cipher-based MAC)
  - Use a block cipher in CBC mode
  - MAC is the last block
  - Must be CBC for MAC to depend on the whole message



At the destination do the same and check if the MAC obtained is the same

Message Authentication Code (MAC)

- 3- Hash the message along with a shared key
  - Ad hoc algorithm showing the basic idea:



Message Authentication Code (MAC)

- 3- HMAC algorithm
  - FIPS Standard / RFC 2104: keyed hash MAC
  - HMAC(m,k)= hash(k⊕opad || hash(k⊕ipad || m))
    - ipad – inner padding – 0x36363636
    - opad – outer padding – 0x5c5c5c5c
    - HMAC in practice: SSL/TLS, WTLS, IPSec

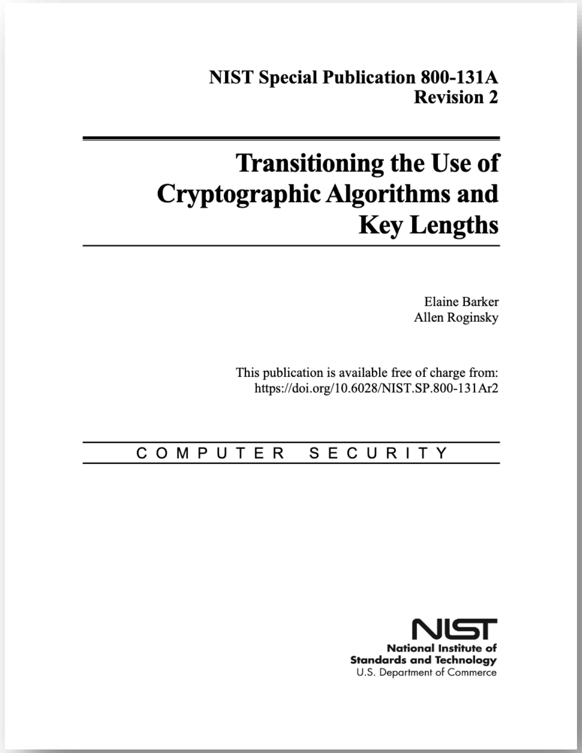


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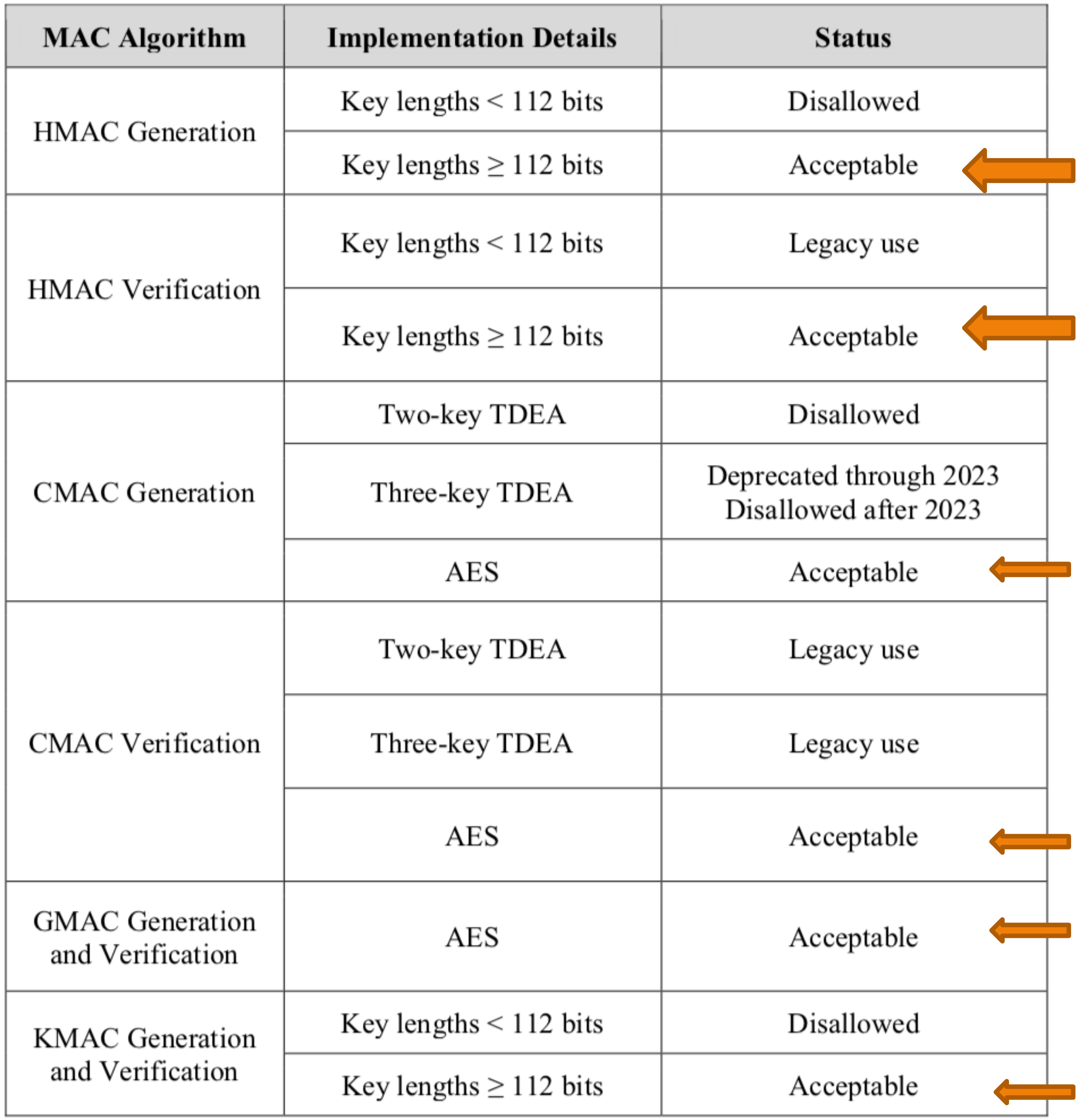
Table 9: Approval Status of MAC Algorithms



MAC Algorithm	Implementation Details	Status
HMAC Generation	Key lengths < 112 bits	Disallowed
	Key lengths ≥ 112 bits	Acceptable
HMAC Verification	Key lengths < 112 bits	Legacy use
	Key lengths ≥ 112 bits	Acceptable
CMAC Generation	Two-key TDEA	Disallowed
	Three-key TDEA	Deprecated through 2023 Disallowed after 2023
	AES	Acceptable
CMAC Verification	Two-key TDEA	Legacy use
	Three-key TDEA	Legacy use
	AES	Acceptable
GMAC Generation and Verification	AES	Acceptable
KMAC Generation and Verification	Key lengths < 112 bits	Disallowed
	Key lengths ≥ 112 bits	Acceptable

CMAC, GMAC, KMAC
são standards do NIST

Roadmap

- Introduction
- Symmetric Ciphers
- Hash Functions
- Message Authentication Codes
- **Asymmetric Ciphers**
- Digital Signatures
- Homomorphic Ciphers

Asymmetric ciphers: overview

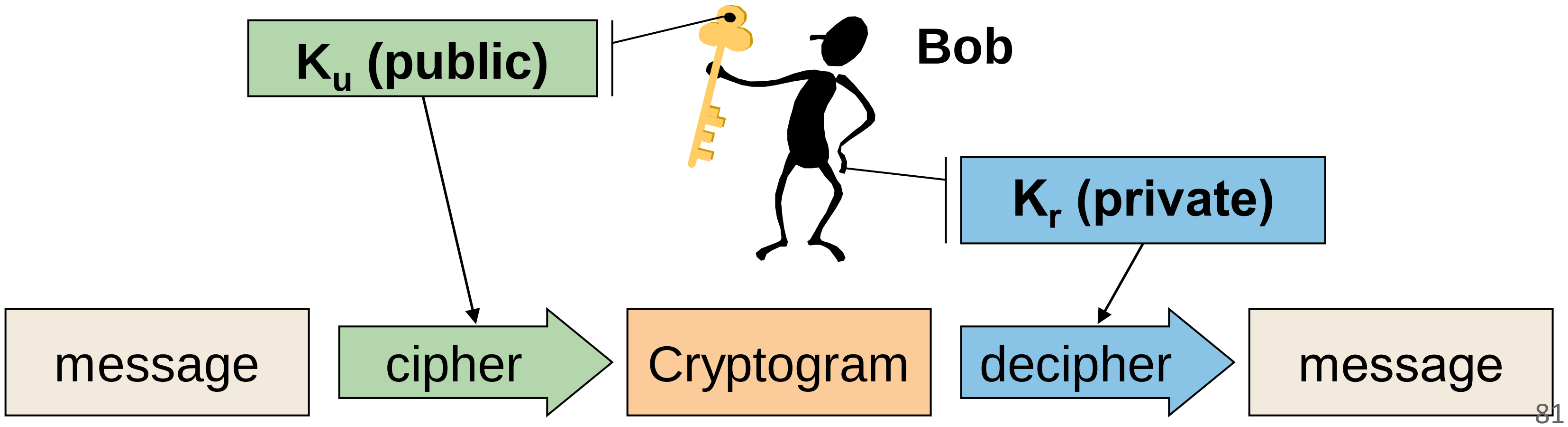
- Aka **public-key cryptography**
- Appeared with the objective of solving a hard problem: **key distribution**
- They use a **pair of keys**
  - One is **private**, personal, non-transmissible
  - One is **public**, can/should be widely known
- Allow for
  - **Confidentiality** with the exchange of secret keys
  - **Authentication** with digital signatures
  - **Integrity** with digital signatures

Asymmetric ciphers: assessment

- **Advantages**
  - N communicating parties ⇒ N key pairs
  - instead of N(N-1)/2 as symmetric ciphers
- **Disadvantages**
  - Performance: typically inefficient in comparison to symmetric ciphers (but signatures are not too slow today)
- **Problems**
  - Distribution of public keys, although much simpler than distributing secret keys
  - Life-time of the key pairs (revocation)

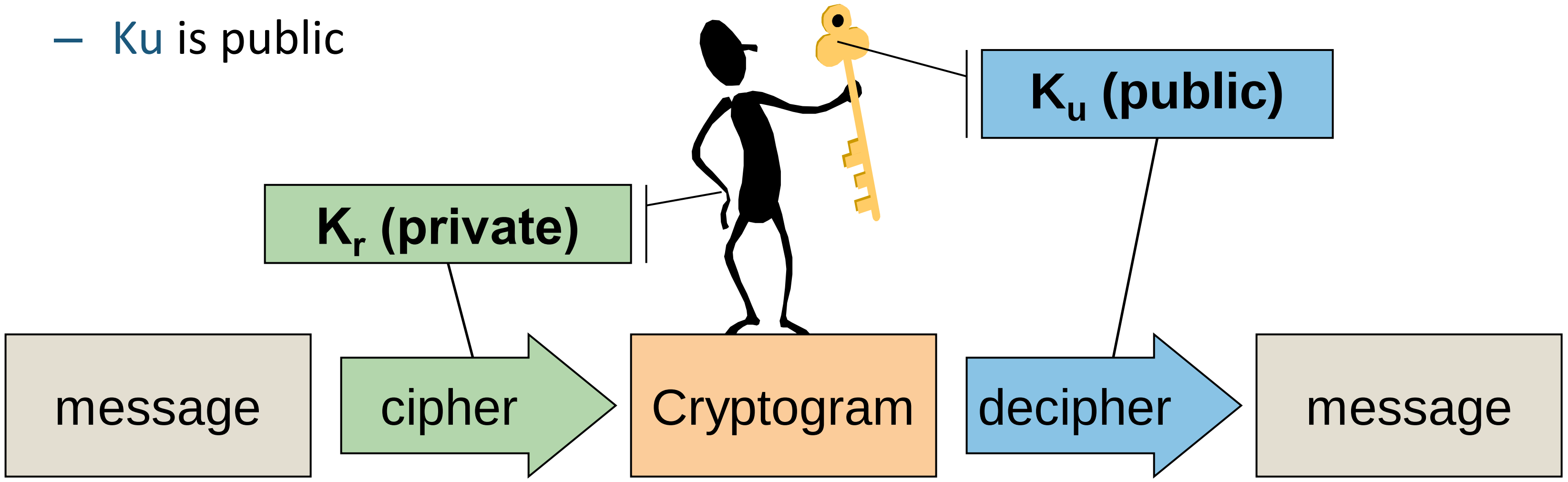
Asymmetric ciphers: Confidentiality

- Cipher / decipher with a pair of keys
  - C = E(Ku, P)   P = D(Kr, C)
  - To communicate with confidentiality with X we only need to know Ku
  - Very inefficient for large P, so not used like this for encrypting messages/files
- No authentication
  - Bob does not know who generated the cryptogram

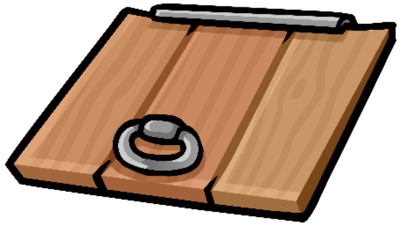


Asymmetric ciphers: Authentication

- The cryptogram cannot be modified
  - C = E(Kr, P)   P = D(Ku, C)
  - Only X knows the key Kr used to generate the cryptogram
  - Better idea to encrypt H(P): C = E(Kr, H(P))
- No confidentiality
  - Whoever knows the key Ku is able to decipher the cryptogram
  - Ku is public



Trapdoor functions



- Functions with the properties:
  - Computing in one direction is easy A $\rightarrow$ B
  - Computing the inverse is very hard (unless you know a secret) A $\nleftarrow$ B

- Trapdoor functions are the core concept of Asymmetric cryptography
  - If you find mathematical functions with these properties, you can build a cryptographic scheme around it

Asymmetric Ciphers

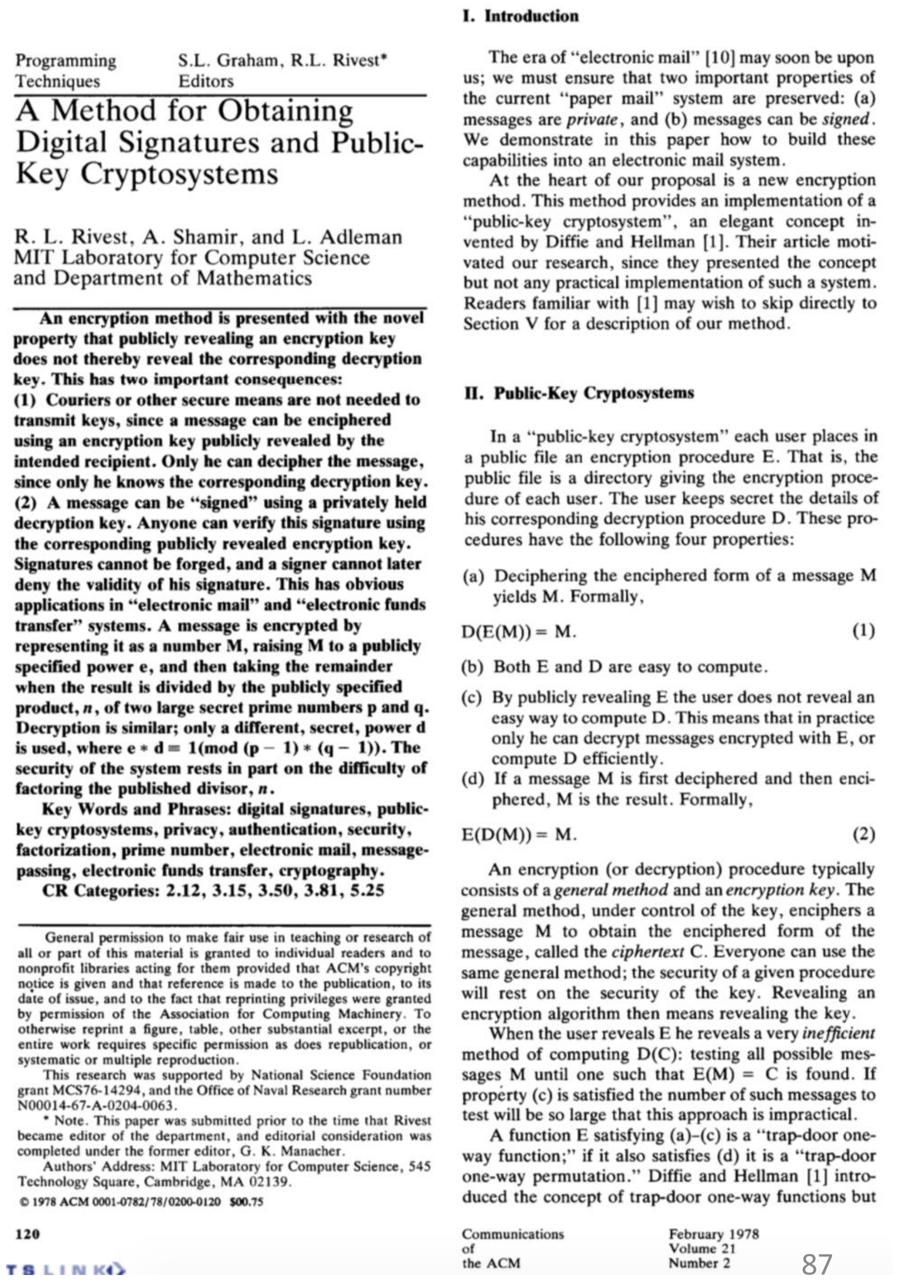
- Used mathematical approximations
  - Factorization of large numbers
  - Discrete logarithm problem
  - Knapsack problem
- Most common encryption algorithms
  - RSA (Rivest-Shamir-Adleman) – first and most used asymmetric encryption algorithm
  - ECC (Elliptic Curve Cryptography) – much used today, especially for signatures
- Other public-key techniques (not encryption)
  - Diffie-Hellman (DH), also known as Diffie-Hellman-Merkle – originated the area of asymmetric crypto

Modular arithmetic

- Idea: do arithmetic normally (+ - × ÷) but result is the mod of what you obtained dividing by a modulus
  - i.e., the result is the remainder of the integer division of the result by the modulus
  - Performs arithmetic operations not on a line but on a circle
  - mod can be applied "as often as you want"
  - Examples:
    - 1537 x 4248 (mod 10) = 6529176 (mod 10) = 6
    - 1537 (mod 10) x 4248 (mod 10) = 7 x 8 (mod 10) = 56 (mod 10) = 6

RSA

- RSA (Rivest-Shamir-Adleman)
- First **asymmetric encryption** algorithm
  - But not the first asymmetric cryptography algorithm (that was Diffie-Hellman-Merkle)
- A major step for cryptography



Programming Techniques — S.L. Graham, R.L. Rivest* Editors

A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

R. L. Rivest, A. Shamir, and L. Adleman
MIT Laboratory for Computer Science and Department of Mathematics

**An encryption method is presented with the novel property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences:
(1) Couriers or other secure means are not needed to transmit keys, since a message can be enciphered using an encryption key publicly revealed by the intended recipient. Only he can decipher the message, since only he knows the corresponding decryption key.
(2) A message can be "signed" using a privately held decryption key. Anyone can verify this signature using the corresponding publicly revealed encryption key. Signatures cannot be forged, and a signer cannot later deny the validity of his signature. This has obvious applications in "electronic mail" and "electronic funds transfer" systems. A message is encrypted by representing it as a number M, raising M to a publicly specified power e, and then taking the remainder when the result is divided by the publicly specified product, *n*, of two large secret prime numbers p and q. Decryption is similar; only a different, secret, power d is used, where $e * d \equiv 1 (\mathrm{mod}\ (p - 1) * (q - 1))$. The security of the system rests in part on the difficulty of factoring the published divisor, *n*.**

**Key Words and Phrases: digital signatures, public-key cryptosystems, privacy, authentication, security, factorization, prime number, electronic mail, message-passing, electronic funds transfer, cryptography.
CR Categories: 2.12, 3.15, 3.50, 3.81, 5.25**

This research was supported by National Science Foundation grant MCS76-14294, and the Office of Naval Research grant number N00014-67-A-0204-0063.

* Note. This paper was submitted prior to the time that Rivest became editor of the department, and editorial consideration was completed under the former editor, G. K. Manacher.

Authors' Address: MIT Laboratory for Computer Science, 545 Technology Square, Cambridge, MA 02139.

I. Introduction

The era of "electronic mail" [10] may soon be upon us; we must ensure that two important properties of the current "paper mail" system are preserved: (a) messages are *private*, and (b) messages can be *signed*. We demonstrate in this paper how to build these capabilities into an electronic mail system.

At the heart of our proposal is a new encryption method. This method provides an implementation of a "public-key cryptosystem", an elegant concept invented by Diffie and Hellman [1]. Their article motivated our research, since they presented the concept but not any practical implementation of such a system. Readers familiar with [1] may wish to skip directly to Section V for a description of our method.

II. Public-Key Cryptosystems

In a "public-key cryptosystem" each user places in a public file an encryption procedure E. That is, the public file is a directory giving the encryption procedure of each user. The user keeps secret the details of his corresponding decryption procedure D. These procedures have the following four properties:

(a) Deciphering the enciphered form of a message M yields M. Formally,

$$D(E(M)) = M. \qquad (1)$$

(b) Both E and D are easy to compute.

(c) By publicly revealing E the user does not reveal an easy way to compute D. This means that in practice only he can decrypt messages encrypted with E, or compute D efficiently.

(d) If a message M is first deciphered and then enciphered, M is the result. Formally,

$$E(D(M)) = M. \qquad (2)$$

An encryption (or decryption) procedure typically consists of a *general method* and an *encryption key*. The general method, under control of the key, enciphers a message M to obtain the enciphered form of the message, called the *ciphertext* C. Everyone can use the same general method; the security of a given procedure will rest on the security of the key. Revealing an encryption algorithm then means revealing the key.

When the user reveals E he reveals a very *inefficient* method of computing D(C): testing all possible messages M until one such that E(M) = C is found. If property (c) is satisfied the number of such messages to test will be so large that this approach is impractical.

A function E satisfying (a)–(c) is a "trap-door one-way function;" if it also satisfies (d) it is a "trap-door one-way permutation." Diffie and Hellman [1] introduced the concept of trap-door one-way functions but

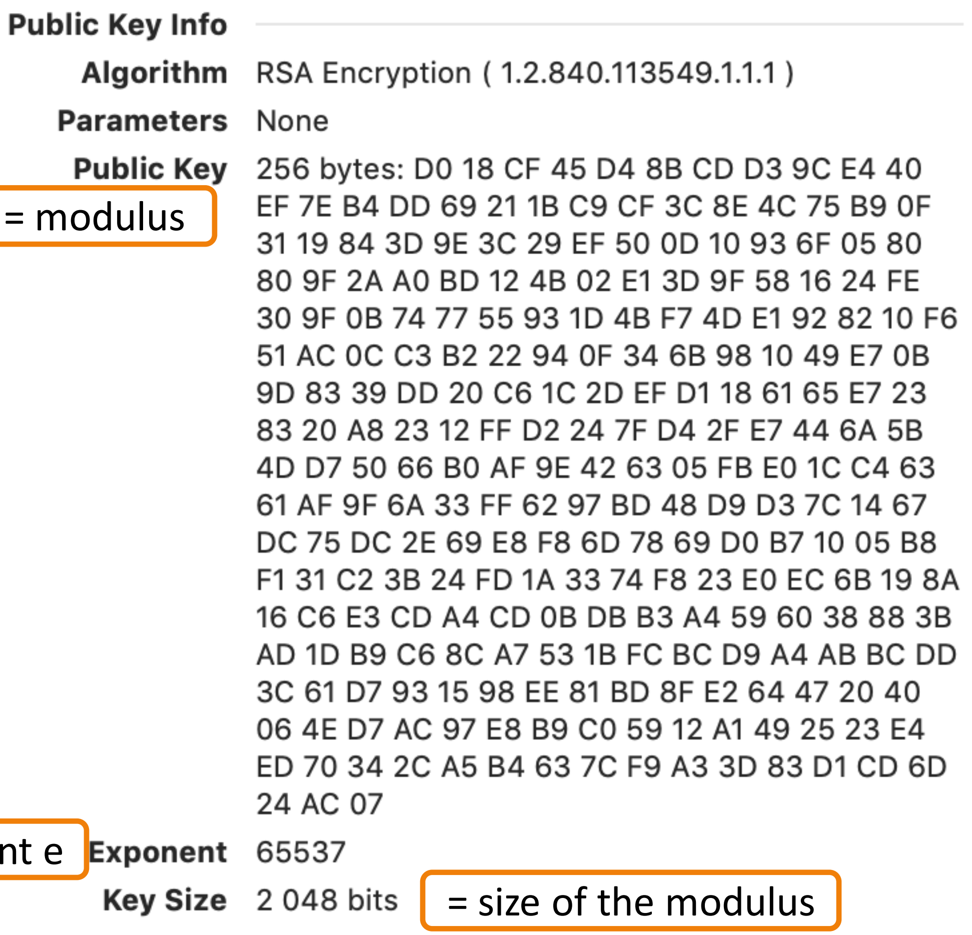
RSA overview

- **Base values and modulus**
  - **p, q** – secret; large prime numbers
  - **n = p × q** – public; large (1K, 2K, 3K bits); used as modulus; key length
- **Public key** – exponent **e** and **n** (e < n)
- **Private key** – exponent **d** (d < n) and primes **p** and **q**
- **Key generation**: choose **p, q** large primes
  - **e** choosen such that it is coprime with **z=** $\boldsymbol{\phi(n) = (p-1)(q-1)}$
    - *coprime* means that the only factor that divides both is 1
    - **ϕ(n)** is the Euler's totient function; counts factors up to n that are coprime with n
  - **d** chosen such that $\boldsymbol{e.d \equiv 1 \pmod{z}}$ $\Leftrightarrow$ $e.d \bmod z = 1$
- **Encryption** with public key: $C = P^e \bmod n$
- **Decryption** with private key: $P = C^d \bmod n$

$= (P^e \bmod n)^d \bmod n = P^{ed} \bmod n = P^{ed \bmod z} \bmod n = P^1 \bmod n = P$

Public key example

- Taken from a browser



```
Public Key Info
      Algorithm  RSA Encryption ( 1.2.840.113549.1.1.1 )
     Parameters  None
     Public Key  256 bytes: D0 18 CF 45 D4 8B CD D3 9C E4 40
                 EF 7E B4 DD 69 21 1B C9 CF 3C 8E 4C 75 B9 0F
                 31 19 84 3D 9E 3C 29 EF 50 0D 10 93 6F 05 80
                 80 9F 2A A0 BD 12 4B 02 E1 3D 9F 58 16 24 FE
                 30 9F 0B 74 77 55 93 1D 4B F7 4D E1 92 82 10 F6
                 51 AC 0C C3 B2 22 94 0F 34 6B 98 10 49 E7 0B
                 9D 83 39 DD 20 C6 1C 2D EF D1 18 61 65 E7 23
                 83 20 A8 23 12 FF D2 24 7F D4 2F E7 44 6A 5B
                 4D D7 50 66 B0 AF 9E 42 63 05 FB E0 1C C4 63
                 61 AF 9F 6A 33 FF 62 97 BD 48 D9 D3 7C 14 67
                 DC 75 DC 2E 69 E8 F8 6D 78 69 D0 B7 10 05 B8
                 F1 31 C2 3B 24 FD 1A 33 74 F8 23 E0 EC 6B 19 8A
                 16 C6 E3 CD A4 CD 0B DB B3 A4 59 60 38 88 3B
                 AD 1D B9 C6 8C A7 53 1B FC BC D9 A4 AB BC DD
                 3C 61 D7 93 15 98 EE 81 BD 8F E2 64 47 20 40
                 06 4E D7 AC 97 E8 B9 C0 59 12 A1 49 25 23 E4
                 ED 70 34 2C A5 B4 63 7C F9 A3 3D 83 D1 CD 6D
                 24 AC 07
       Exponent  65537
       Key Size  2 048 bits
```

= modulus

exponent e

= size of the modulus

RSA example with small numbers

- Key generation
  - Pick primes **p** and **q**: 3 and 11
  - Calculate modulus **n** = $p \times q = 3 \times 11 = 33$
  - Calculate **z** = $\phi(n) = (p-1) \times (q-1) = 2 \times 10 = 20$
  - Choose **e** such that **e** is co-prime with **z** and $e < n$: $e = 7$
  - Public key: **e** = 7, **n** = 33
  - Calculate **d**
    - $e.d = 1 \pmod z$ and $d < n \Leftrightarrow$
    - $7.d \bmod 20 = 1$ and $d < 33 \Rightarrow$
    - **d** = 3
- Encrypt w/ public key P = 14
  - $C = P^e \bmod n$
  - $C = 14^7 \bmod 33$
  - $C = 20$
- Decrypt w/ private key
  - $P = C^d \bmod n$
  - $P = 20^3 \bmod 33$
  - $P = 14$

Why is RSA difficult to break?

- No one knows how to compute the inverse RSA ("decryption") without knowing the prime factors of the modulus **n**
- No one knows how to efficiently do integer factorization, i.e., recover the prime factors from **n** alone
  - The private key is knowledge of the prime factors
- Knowing the public key does not allow decryption

- Future crisis: Shor’s quantum algorithm for integer factorization
  - Current quantum technology can only factorize small numbers
  - ≥ 2048-bit numbers are safe (for now…)
  - Post-quantum crisis; need for quantum resistance

RSA signatures

- RSA is often used to produce **signatures**
  - Public key – **e, n**
  - Private key – **d, p, q**
- **Signing** the message M:
  - Obtain S such that: **$S = (\text{hash}(M))^d \bmod n$**
  - Only the private key owner can generate the signature
- **Verifying** the signature:
  - Check if: **$\text{hash}(M) == S^e \bmod n$**
  - Anyone with the public key can verify the signature

Elliptic Curve Cryptography (ECC)

- RSA and DH use integer and polynomial arithmetic with very large numbers/polynomials
  - Impose a significant load in storing and processing keys and messages
- **Elliptic curves** are an alternative
  - Offer same security with smaller bit sizes
    - 224-bit ECC ~ 2048-bit RSA
    - 500-bit ECC ~ 15k-bit RSA
  - Newer, not as well analyzed, but commonly used
    - Many curves exist, each with their tradeoffs
    - Not necessarily faster than RSA for the same equivalent security
  - Normally used for **key agreement or signing**, not ciphering

Elliptic Curve Cryptography (ECC)

- Analogy:
  - ECC addition is analogous to modulo multiplication
  - ECC repeated addition is analogous to modulo exponentiation
- The "hard" problem is the **elliptic curve logarithm problem**
  - Q=k·P, where Q,P belong to an elliptic curve; eg *$y^2= x^3-3x+b$ modulo p*
  - Easy to compute Q given k, P
  - Hard to find k given Q, P
- Public Key operations Q(x,y)=kP(x,y):
  - Q = public key
  - k = private key
  - P = base point (a parameter of the curve)

ECC Encryption/Decryption

- There are several alternatives but we consider the simplest: to encode the message **M** as a point $\mathbf{Q_M}$ on the elliptic curve
- Key generation
  - Select suitable curve and base point P
  - Bob chooses a private key: $\mathbf{k_B} < n$
  - Bob computes the public key: $\mathbf{Q_B} = k_B \cdot P$
- Alice encrypts $\mathbf{Q_M}$ using Bob's public key $\mathbf{Q_B}$:

  $\mathbf{C_M} = \{r\cdot P, \mathbf{Q_M} + r\cdot \mathbf{Q_B}\}$ – **r** is a random number; C_M is a point in the curve
- Bob decrypts $\mathbf{C_M}$ using its private key $\mathbf{k_B}$:

$$\underbrace{\underbrace{\mathbf{Q_M} + r\cdot\mathbf{Q_B}} - \underbrace{\mathbf{k_B} r\cdot P}}_{\mathbf{C_M}} = \mathbf{Q_M} + r(\mathbf{k_B}\cdot P) - \mathbf{k_B}(r\cdot P) = \mathbf{Q_M}$$

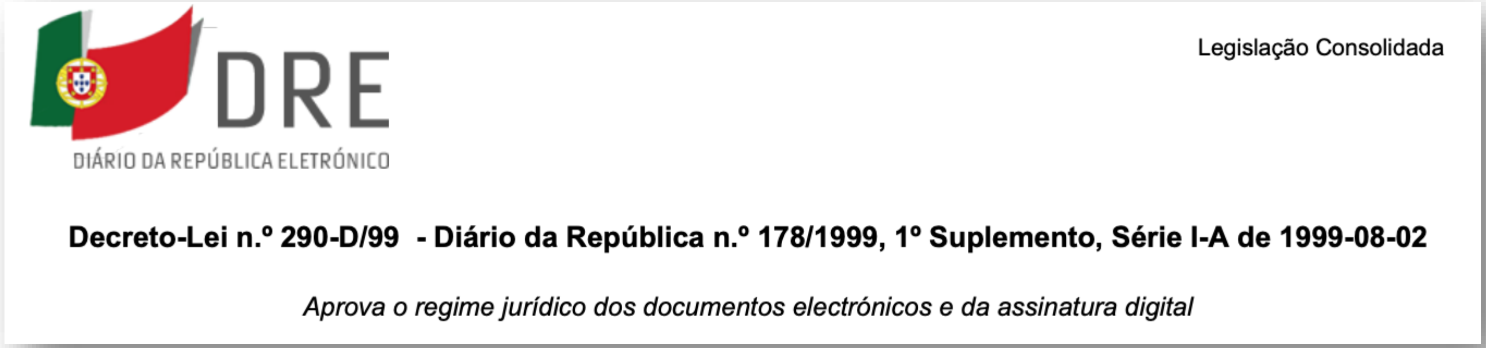
Porque $\mathbf{Q_B} = k_B P$

Roadmap

- Introduction
- Symmetric Ciphers
- Hash Functions
- Message Authentication Codes
- Asymmetric Ciphers
- **Digital Signatures**
- Homomorphic Ciphers

Digital signatures – objectives

- Authenticate the content of a document
- Authenticate its signer
- Being able to assure authentication towards a third party
- Signer cannot repudiate the signature

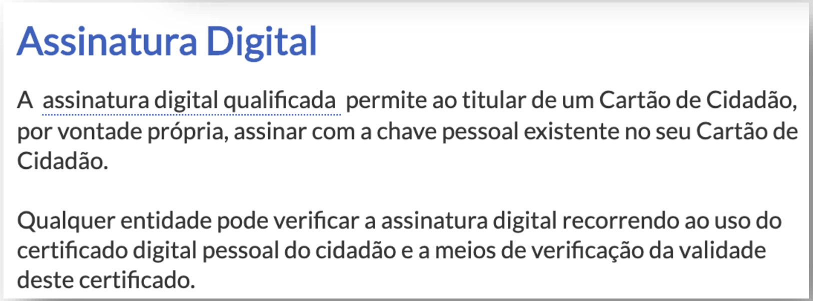


DRE
DIÁRIO DA REPÚBLICA ELETRÓNICO

Legislação Consolidada

Decreto-Lei n.º 290-D/99 - Diário da República n.º 178/1999, 1º Suplemento, Série I-A de 1999-08-02

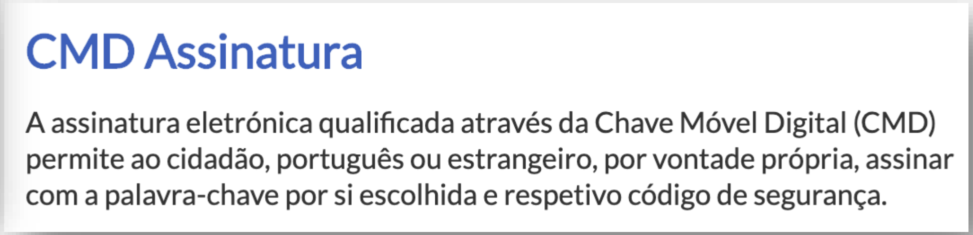
Aprova o regime jurídico dos documentos electrónicos e da assinatura digital



Assinatura Digital

A assinatura digital qualificada permite ao titular de um Cartão de Cidadão, por vontade própria, assinar com a chave pessoal existente no seu Cartão de Cidadão.

Qualquer entidade pode verificar a assinatura digital recorrendo ao uso do certificado digital pessoal do cidadão e a meios de verificação da validade deste certificado.



CMD Assinatura

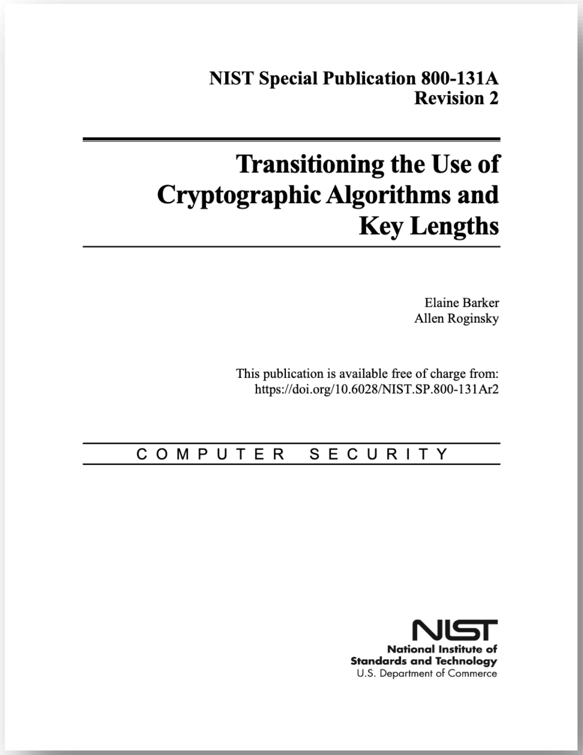
A assinatura eletrónica qualificada através da Chave Móvel Digital (CMD) permite ao cidadão, português ou estrangeiro, por vontade própria, assinar com a palavra-chave por si escolhida e respetivo código de segurança.

Digital signatures

- Used approximations: asymmetric cipher + hash function
- **Basic algorithm** (K_x^r - private key K_x^u - public key)
  - Signature: $S_x(doc) = E(K_x^r, hash(doc))$
  - Validation: check if $D(K_x^u, S_x(doc)) = hash(doc)$
- Why cannot an adversary provide a fake signature?

- Another option:
  - Signature: $S_x(doc) = info + E(K_x^r, hash(doc + info))$
  - Validation: $D(K_x^u, S_x(doc)) = hash(doc + info)$
  - Example info: timestamp, used algorithm, …

- Today often **ECDSA** is adopted (with elliptic curve encryption)


NIST Special Publication 800-131A
Revision 2

Transitioning the Use of Cryptographic Algorithms and Key Lengths

Elaine Barker
Allen Roginsky

This publication is available free of charge from:
https://doi.org/10.6028/NIST.SP.800-131Ar2

C O M P U T E R   S E C U R I T Y

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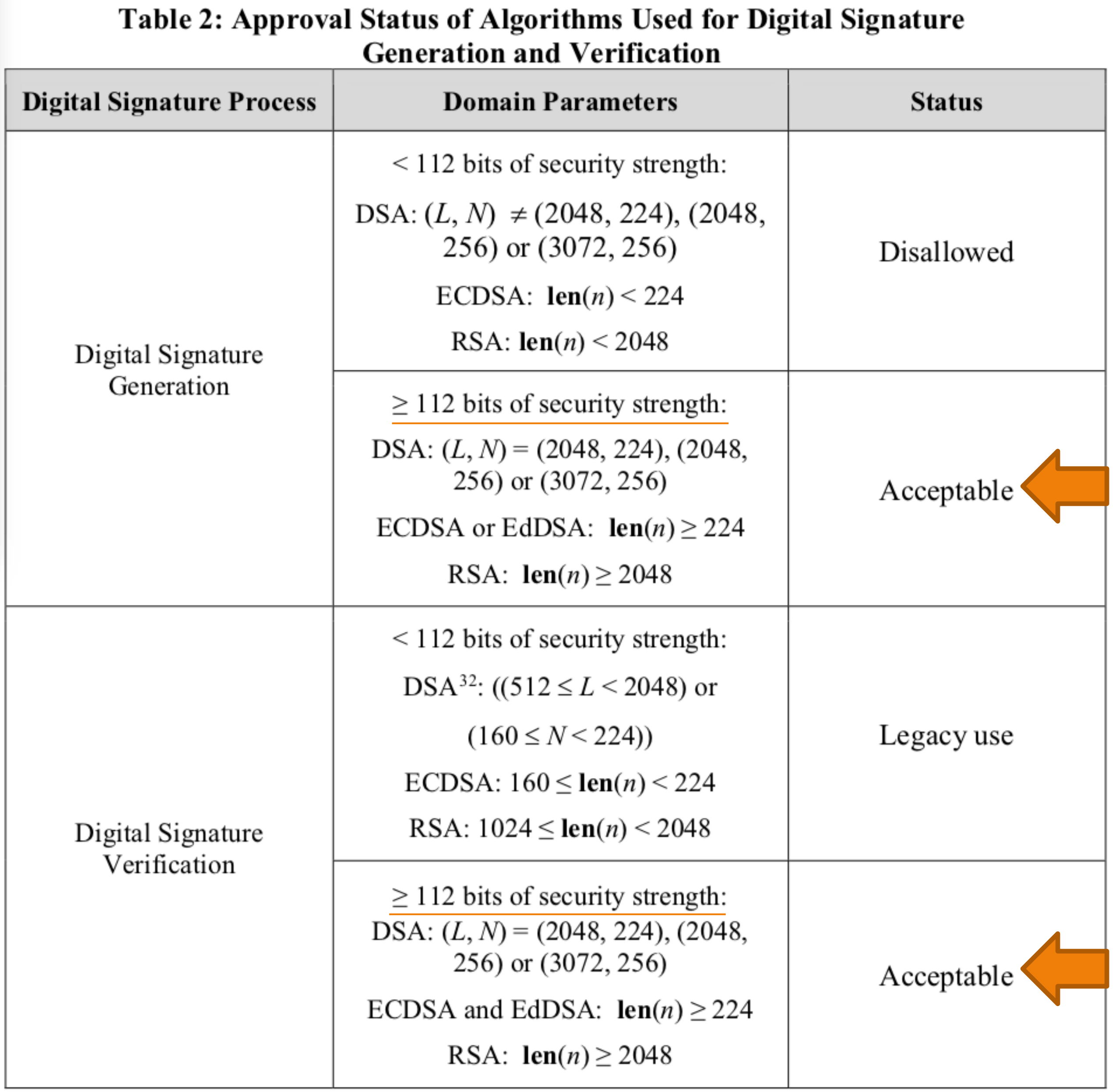
Table 2: Approval Status of Algorithms Used for Digital Signature Generation and Verification

Digital Signature Process	Domain Parameters	Status
Digital Signature Generation	$<$ 112 bits of security strength: DSA: $(L, N) \neq (2048, 224), (2048, 256)$ or $(3072, 256)$ ECDSA: $\mathbf{len}(n) < 224$ RSA: $\mathbf{len}(n) < 2048$	Disallowed
	$\geq$ 112 bits of security strength: DSA: $(L, N) = (2048, 224), (2048, 256)$ or $(3072, 256)$ ECDSA or EdDSA: $\mathbf{len}(n) \geq 224$ RSA: $\mathbf{len}(n) \geq 2048$	Acceptable
Digital Signature Verification	$<$ 112 bits of security strength: DSA[32]: $((512 \leq L < 2048)$ or $(160 \leq N < 224))$ ECDSA: $160 \leq \mathbf{len}(n) < 224$ RSA: $1024 \leq \mathbf{len}(n) < 2048$	Legacy use
	$\geq$ 112 bits of security strength: DSA: $(L, N) = (2048, 224), (2048, 256)$ or $(3072, 256)$ ECDSA and EdDSA: $\mathbf{len}(n) \geq 224$ RSA: $\mathbf{len}(n) \geq 2048$	Acceptable

Roadmap

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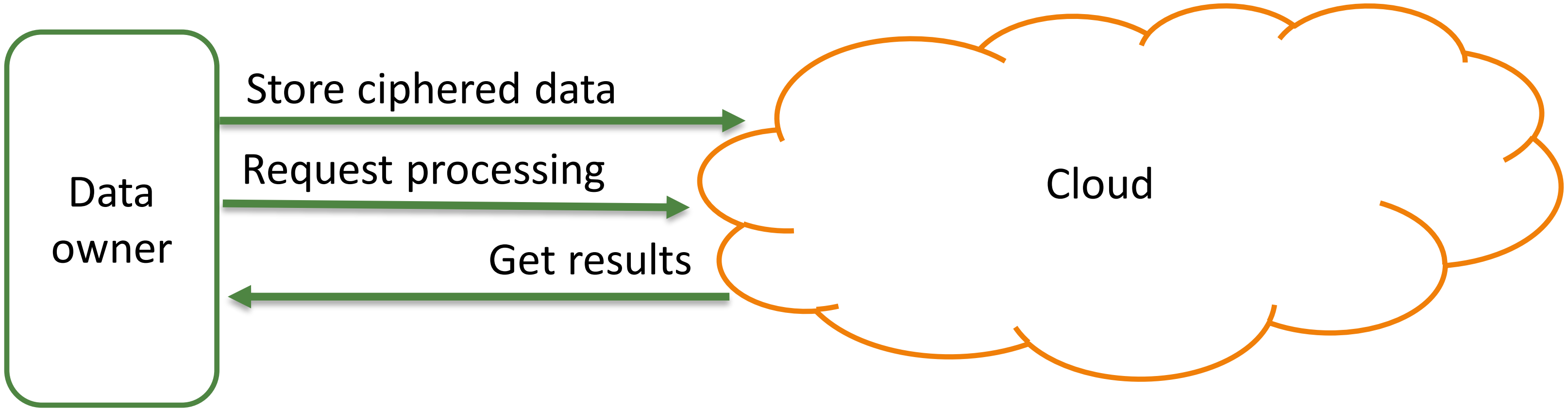
- Digital Signatures
- **Homomorphic Ciphers**

Homomorphic Encryption

- How to **outsource the processing of ciphered data**?
  - Basic assumption: data processed is encrypted with the same key
  - Data is processed in the cloud, not downloaded before processing



Homomorphic Encryption

- **Homomorphic encryption**
  - Generate a ciphered result that, when deciphered, matches the result of the operations as if they had been performed on the plaintext
- **Fully homomorphic encryption (FHE)**
  - A cryptosystem that allows arbitrary computation on ciphertexts, i.e., that allow creating programs with any functionality
  - First implemented by Gentry in 2009
  - Currently very inefficient
- In practice, aim lower, e.g., queries over encrypted databases, e.g., **CryptDB** – https://css.csail.mit.edu/cryptdb/

Homomorphic Encryption

Example practical schemes (1/2)

- **Deterministic encryption** – allows equality comparison
  - Encrypt with some deterministic symmetric cipher E, e.g., AES in CBC mode with fixed IV
  - If E(A, K) = E(B, K) then A = B
  - Less secure than randomized scheme (random IV), reveals equality

- **Order Preserving Encryption** – allows inequality comparison
  - Less secure than deterministic as it reveals order

Homomorphic Encryption
Example practical schemes (2/2)

- **Multiplication** using RSA
  - Both keys are kept secret: **e, d, p, q** (even **e** that is usually public)
  - Recall encryption with RSA: $C = P^e \bmod n$
  - Consider two pieces of data encrypted with **e**: C_a and C_b
  - Multiplication: $C_a \times C_b = P_a^e \times P_b^e \bmod n = (P_a \times P_b)^e \bmod n$
  - Decrypting with **d** we get $P_a \times P_b$
  - i.e., we multiply two encrypted values and, when we decrypt them, we get the multiplication of the two plaintext numbers

Summary

- Introduction
- Symmetric Ciphers
- Hash Functions
- Message Authentication Codes
- Asymmetric Ciphers
- Digital Signatures
- Homomorphic Ciphers