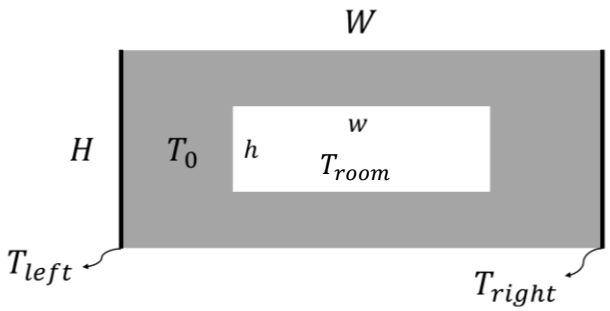
1. INTRODUCTION

In this problem, we will try to visualize what would happen to a metal plate with a hole at the center of it if two temperature sources are connected to its opposing sides. To do so, we will make use of the relaxation method.

2. PROBLEM STATEMENT

A rectangular metal plate with width W and height H has a rectangular hole at its center, which has width w and height h. Initially, the plate is at temperature T_0. Inside the hole, we have air with temperature T_{room}. It is assumed that the room is very large and thus, T_{room} stays constant.

We will connect a source with a constant temperature of T_{left} to its left side, while connecting a source with a constant temperature of T_{right} to the opposing side. Then, we will wait for a long time, so that the plate reaches a steady state.

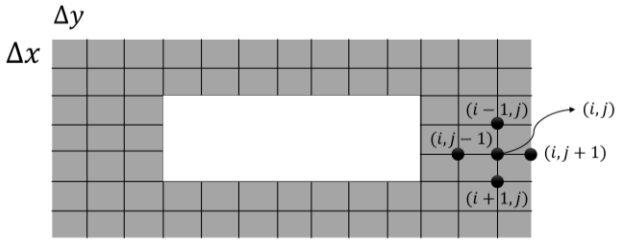


In order to describe how heat evolves with respect to time, we make use of the 2D heat equation:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1}$$

where α is thermal diffusivity.

To treat this equation numerically, we have to discretize the plate and define a grid of nodes in this way:



We will assume that $\Delta x = \Delta y$. After discretizing Eq. 1, we could arrive at:

$$T_{i,j} = \frac{T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1}}{4} \tag{2}$$

To see how Eq. 2 was derived, please refer to this link. Basically, the temperature at each node would be the average of its neighboring nodes. Note that the $\Delta x = \Delta y$ condition has to be satisfied in the simulation. Otherwise, Eq. 2 does not hold.

In this problem, you are asked to use Eq. 2 to plot the temperature of the plate at each node **when the plate reaches its steady state**. A great way to visualize the temperature would be to plot the temperature surface as a function of position (x and y on the plate).

3. CHECKING YOUR RESULTS

In order to check your results, you can compare your results with the provided solution files in the GitHub repository. Note that in the surface plot of the solution, the temperature inside the hole has been set to an arbitrarily low temperature so that it could be visually distinguished from the plate.

4. TABLE OF CONSTANTS

W	$0.1\ m$
H	$0.1\ m$
w	$0.04\ m$
h	$0.04\ m$
T_0	$303\ K$
T_{room}	$303\ K$
T_{left}	$323\ K$
T_{right}	$283\ K$