

## 1. INTRODUCTION

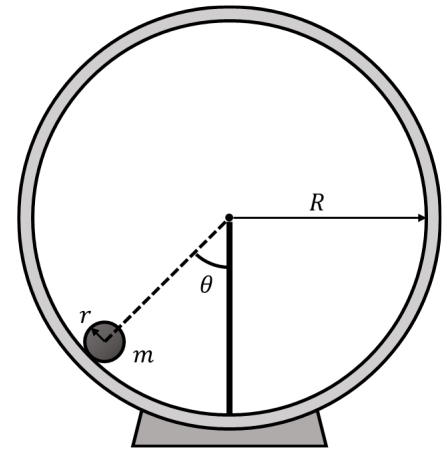
In this problem, we are going to visualize the motion of a spherical bead rolling back and forth in a stationary circular hoop. To do so, we will animate the motion in Matplotlib with the help of our recently-learned techniques.

## 2. PROBLEM STATEMENT

A solid bead with radius  $r$  and mass  $m$  is released at one side of a circular hoop with radius  $R$ . The bead rolls without slipping down and back up the ramp, making an angle  $\theta$  with the vertical as shown in the figure on the right.

In the assumption that  $\theta$  is small and  $R \gg r$ , the equation of motion for the hoop is as written below:

$$\theta(t) = \theta_0 \cos\left(\sqrt{\left(\frac{5}{7}\right)\frac{mg}{R}} t\right)$$



Using Matplotlib, animate the motion of the bead on the hoop and create a plot of  $\theta$  with respect to time.

## 3. CHECKING YOUR RESULTS

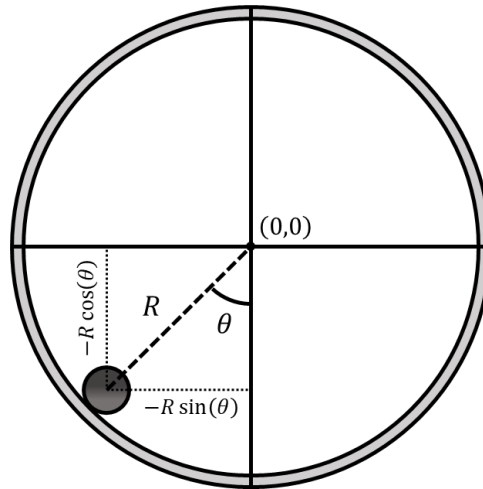
To check your results, compare your solution with the provided solution files in the GitHub repository. Not all solutions will be identical – the provided solution is meant to be used as a guide if you get stuck.

## 4. TABLE OF CONSTANTS

$\theta_0$	$\frac{\pi}{4} \text{ rad}$
$m$	$3.15 \times 10^{-2} \text{ kg}$
$g$	$9.81 \text{ m} \cdot \text{s}^{-2}$
$R$	$1.25 \times 10^{-1} \text{ m}$
$r$	$1.50 \times 10^{-2} \text{ m}$

## 5. APPENDIX

This problem involves converting from polar to rectangular coordinates. The angle of the bead on the hoop with respect to the vertical must be expressed in terms of  $x$  and  $y$  coordinates to be plotted. As shown below, assuming  $R \gg r$  and placing the center of the hoop at the origin, the  $x$  and  $y$  positions of the bead are  $-R \sin(\theta)$  and  $-R \cos(\theta)$  respectively.



When animating the motion, we must offset the bead by a distance  $r$  from the hoop so that the edge of the bead, not the center, is in line with the hoop. In rectangular coordinates, this involves adding  $r \sin(\theta)$  and  $r \cos(\theta)$  to the  $x$  and  $y$  positions respectively as depicted below.

