1. INTRODUCTION

In this problem, we are going to look at the motion of a charged particle in a uniform magnetic field. To do so, we will have to numerically solve two differential equations with help of our recently-learnt techniques (RK4 or the Euler method(s)).

Note that you do not have to be familiar with the physics of magnetic fields to solve this problem.

2. PROBLEM STATEMENT

Let's assume that we have a charged particle ($q_{particle} = e$) with a velocity of $\vec{v}_0 = v_{x_0}\hat{\imath} + v_{y_0}\hat{\jmath} + v_{z_0}\hat{k}$ at $t = 0$. We also have a uniform magnetic field described by $\vec{B} = B_0\hat{k}$, where B_0 is a constant value. Using Newton's second law of motion, we can arrive at these three equations:

$$eB_0 v_y = m\dot{v}_x \quad (1)$$
$$-eB_0 v_x = m\dot{v}_y \quad (2)$$
$$0 = m\dot{v}_z \quad (3)$$

The third equation can be easily solved by integration and will not be discussed in this problem. Instead, we will focus on the first two equations.

In this problem, you are asked to numerically solve the first two differential equations and plot each of v_x and v_y with respect to t. To do this, you can use any of the standard plotting libraries in Python, such as Matplotlib.

3. CHECKING YOUR RESULTS

In order to check your results, you can do either 1) compare your solutions with the provided solution files in the GitHub repository, or 2) compare your results with analytic solutions of the first two differential equations. The second approach is recommended.

Using techniques of solving differential equations (see "Separation of Variables"), one could find the solutions of Eqs. 1-2 to be:

$$v_x = \sqrt{2}v_{x_0}\sin(\omega t + \frac{\pi}{4}) \quad (4)$$
$$v_y = \sqrt{2}v_{y_0}\cos\left(\omega t + \frac{\pi}{4}\right) \quad (5)$$

where $\omega = \frac{eB_0}{m}$.

You can overlay these solutions on your numerical results and see whether they match each other.

4. TABLE OF CONSTANTS

e	$1.60 \times 10^{-19}\, C$
m	$1.67 \times 10^{-23}\, kg$
v_{x_0}	$5.65\, m.s^{-1}$

v_{y_0}	$5.65\ m.s^{-1}$
B_0	$1.50 \times 10^{-4}\ T$

5. APPENDIX

One might be curious about what the motion of the particle would look like in the magnetic field. To know this, you can (numerically) integrate your results to get x and y as functions of t. You can do the same to the result of Eq. 3 and get z as a function of t as well. You can look up “Numerical Integration” to find suitable methods for numerically integrating your results.

If you do this and draw a 3D plot of the results, you will probably see something like this:

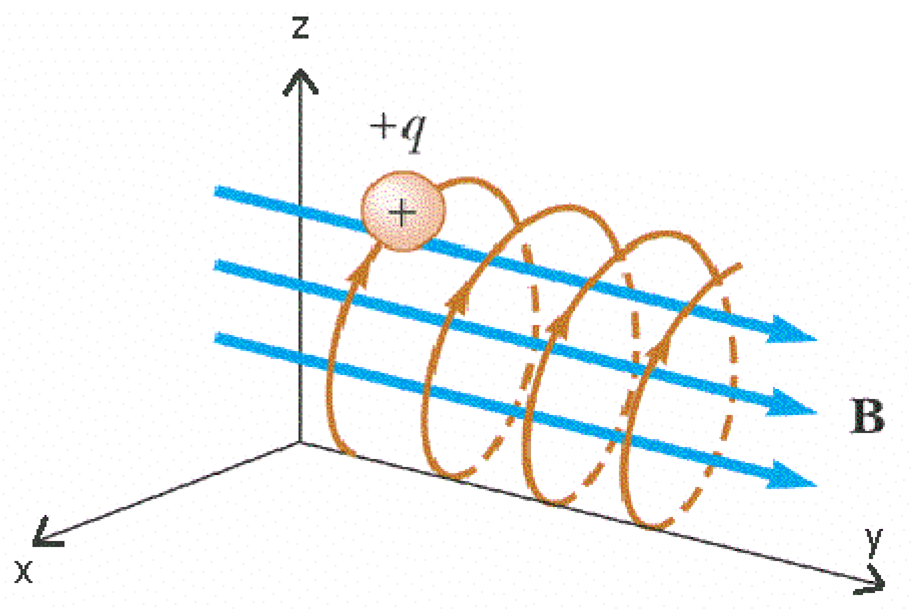

Image taken from: https://www.researchgate.net/figure/Helix-motion-is-produced-if-constant-electric-field-and-constant-magnetic-field-are_fig4_275055314

Please note that the direction of the magnetic field in this figure does not match that of our problem.