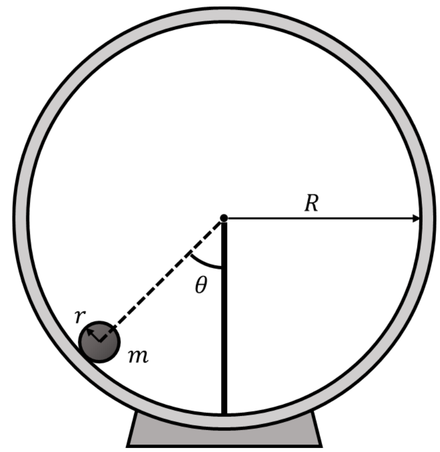
1. INTRODUCTION

In this problem, we are going to visualize the motion of a spherical bead rolling back and forth in a stationary circular hoop. To do so, we will animate the motion in Matplotlib with the help of our recently-learned techniques.

2. PROBLEM STATEMENT

A solid bead with radius r and mass m is released at one side of a circular hoop with radius R. The bead rolls without slipping down and back up the ramp, making an angle θ with the vertical as shown in the figure on the right.



In the assumption that θ is small and $R \gg r$, the equation of motion for the hoop is as written below:

$$\theta(t) = \theta_0 \cos\left(\sqrt{\left(\frac{5}{7}\right)\frac{mg}{R}}\,t\right)$$

Using Matplotlib, animate the motion of the bead on the hoop and create a plot of θ with respect to time.

3. CHECKING YOUR RESULTS

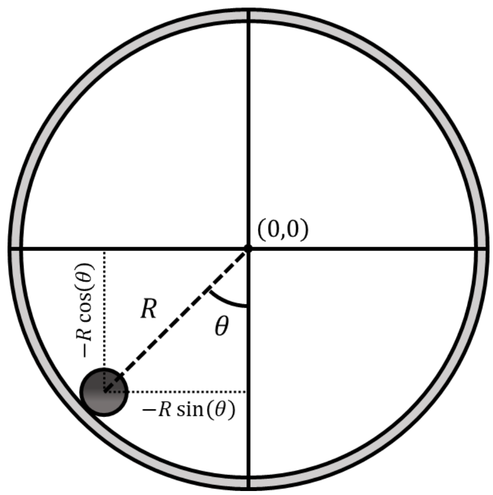
To check your results, compare your solution with the provided solution files in the GitHub repository. Not all solutions will be identical – the provided solution is meant to be used as a guide if you get stuck.

4. TABLE OF CONSTANTS

θ_0	$\frac{\pi}{4}$ rad
m	$3.15\ \times 10^{-2}$ kg
g	$9.81\ \mathrm{m \cdot s^{-2}}$
R	$1.25\ \times 10^{-1}$ m
r	1.50×10^{-2} m

5. APPENDIX

This problem involves converting from polar to rectangular coordinates. The angle of the particle on the hoop with respect to the vertical must be expressed in terms of x and y coordinates to be plotted. As shown below, assuming $R \gg r$ and placing the center of the hoop at the origin, the x and y positions of the bead are $-R\sin(\theta)$ and $-R\cos(\theta)$ respectively.



When animating the motion, we must offset the bead by a distance r from the hoop so that the edge of the bead, not the center, is in line with the hoop. In rectangular coordinates, this involves adding $r\sin(\theta)$ and $r\cos(\theta)$ to the x and y positions respectively as depicted below.