

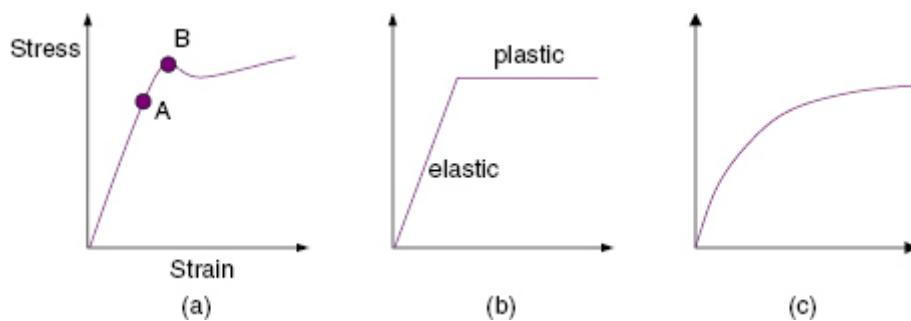
Chapter 3

Total and Effective Stress

3.1 State of Stress in a Soil Mass

3.1.1 Stress–Strain Relationships

Before commencing a study of the material in this chapter it is best to become familiar with the main terms used to describe the stress–strain relationships of a material. It is useful to begin by examining a typical stress–strain plot obtained for a metal ([Fig. 3.1a](#)).



[Fig. 3.1](#) Stress–strain relationships.

Results such as those indicated in the figure would normally be obtained by subjecting a specimen of the metal to a tensile test and plotting the values of tensile strain against the nominal values of tensile stress, as the stress–strain relationship obtained is equally applicable in tension or compression in the case of a metal.

Note: Nominal stress = actual load/original cross-sectional area of specimen, i.e. no allowance is made for reduction in area, due to necking, as the load is increased.

From the plot it is seen that in the early stages of loading, up to point B, the stress is proportional to the strain. Unloading tests can also demonstrate that, up to the point A, the metal is elastic in that it will return to its original dimensions if the load is removed. The limiting stress at which elasticity effects are not quite complete, is known as the elastic limit, represented by point A. The limiting stress at which linearity between stress and strain ceases is known as the limit of proportionality, point B.

In most metals points A and B occur so close together that they are generally assumed to coincide, i.e. elastic limit is assumed equal to the limit of proportionality.

Point C in [Fig. 3.1a](#) represents the yield point, i.e. the stress value at which there is a sudden drop of load, as illustrated, or the stress value at which there is a continuing extension with no further significant increase in load.

[Figure 3.1a](#) can be approximated to [Fig. 3.1b](#) which represents the ideal elastic–plastic material.

In this diagram, point 1 represents the limit of elasticity and proportionality and the point at which plastic behaviour occurs. The form of the compressive stress–strain relationships typical for all types of soil up to their peak values is as shown in [Fig. 3.1c](#).

It is seen that the stress–strain relationship of a soil is never linear and, in order to obtain solutions, the designer is forced either to assume the idealised conditions of [Fig. 3.1b](#) or to solve a particular problem directly from the results of tests that subject samples of the soil to conditions that closely resemble those that are expected to apply *in situ*.

In most soil problems the induced stresses are either low enough to be well below the yield stress of the soil and it can be assumed that the soil will behave elastically (e.g. immediate settlement problems), or they are high enough for the soil to fail by plastic yield (bearing capacity and earth pressure problems), where it can be assumed that the soil will behave as a plastic material.

With soils, even further assumptions must be made if one is to obtain a solution. Generally it is assumed that the soil is both homogeneous and isotropic. As with the assumption of perfect elasticity these theoretical relationships do not apply in practice but can lead to realistic results when sensibly applied.

3.1.2 Stresses within a Soil Mass

A major problem in geotechnical analysis is the estimation of the state of stress at a point at a particular depth in a soil mass.

A load acting on a soil mass, whether internal, due to its self-weight, or external, due to a load applied at the boundary, creates stresses within the soil. If we consider an elemental cube of soil at the point considered, then a solution by elastic theory is possible. Each plane of the cube is subjected to a stress, σ , acting normal to the plane, together with a shear stress, τ , acting parallel to the plane. There are therefore a total of six stress components acting on the cube (see [Fig. 3.2a](#)). Once the values of these components are determined then they can be compounded to give the magnitudes and directions of the principal stresses acting at the point considered.

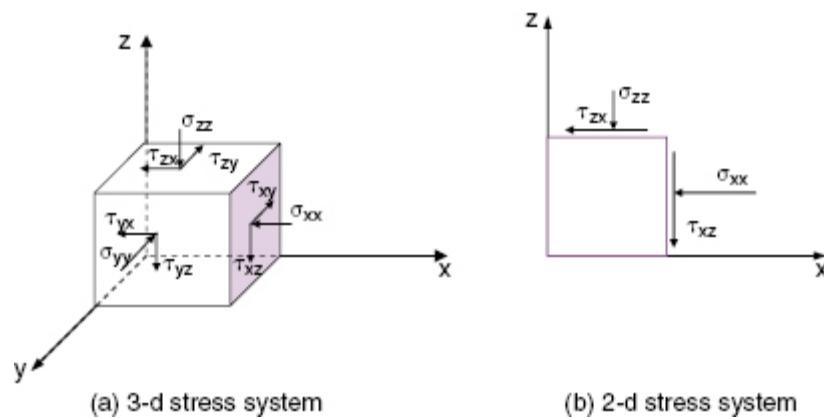


Fig. 3.2 Three- and two-dimensional stress states.

Many geotechnical structures operate in a state of plane strain, i.e. one dimension of the structure is large enough for end effects to be ignored and the problem can be regarded as one of two dimensions. The two-dimensional stress state is illustrated in Fig. 3.2b.

3.2 Total Stress

The total vertical stress acting at a point in the soil (e.g. stress σ_{zz} in Fig. 3.2b) is due to the weight of everything that lies above that point including soil, water and any load applied to the soil surface. Stresses induced by the weight of the soil subject the elemental cube to vertical stress only and they cannot create shear stresses under a level surface.

Total stress increases with depth and with unit weight and the total vertical stress at depth z in the soil due to the weight of the soil acting above, as depicted in Fig 3.3, is defined

$$\sigma_z = \gamma z$$

where γ = unit weight of the soil.

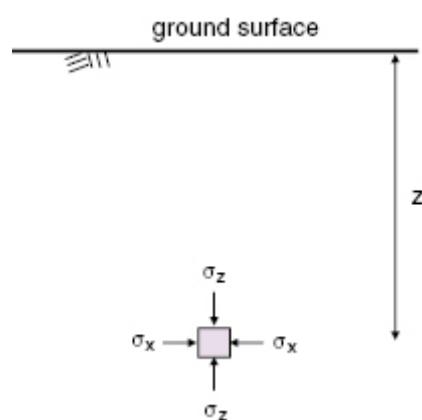


Fig. 3.3 Total stress in a homogeneous soil mass.

If the soil is multi-layered, the total vertical stress is determined by summing the stresses induced by each layer of soil (see [Examples 3.1](#) and [3.2](#)).

3.3 Pore Pressure

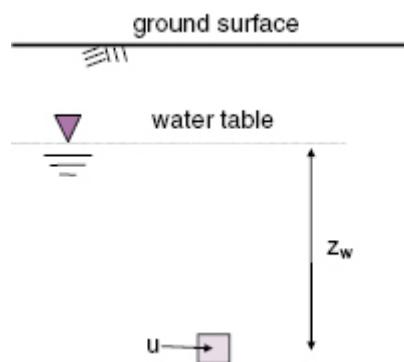
The term pore water was introduced in Section 2.2.1. Pore water experiences pressure known as the pore pressure or pore water pressure, u . The magnitude of the pore pressure at a point in the soil depends on the depth below the water table and the flow conditions. In the case of a horizontal ground water table, we may be able to assume that no flow is taking place and the pore pressure at a point beneath the ground water table can be established from the hydrostatic pressure acting. The magnitude of the pore pressure at the water table is zero.

In [Fig. 3.4](#) the pore pressure is given by the hydrostatic pressure.

$$u = \gamma_w z_w$$

where

z_w = the depth below the water table.



[Fig. 3.4](#) Hydrostatic pore water pressure.

In situations where seepage is taking place, the pore pressures can be established from a flow net.

It is clear that pore pressures are positive below the water table. Above the water table however, the soil is saturated with capillary water in a state of suction, and here the pore pressures will be negative.

$$u = -\gamma_w h_c$$

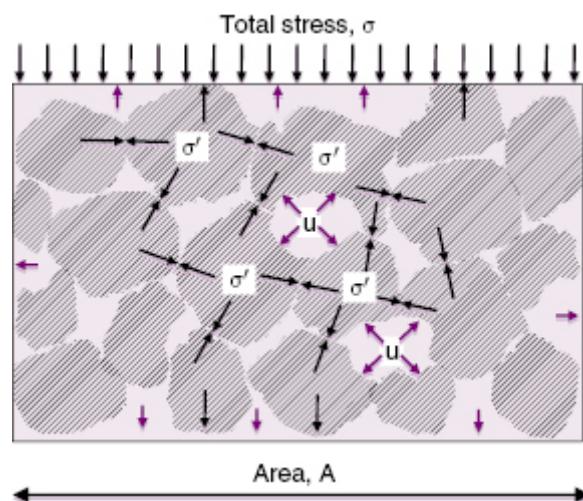
where

h_c = height of capillary rise.

3.4 Effective Stress

The stress that controls changes in the volume and strength of a soil is known as the *effective stress*. In [Chapter 1](#) it was seen that a soil mass consists of a collection of mineral particles with voids between them. These voids are filled with water, air and water, or air only (see Fig. 1.10).

For the moment let us consider saturated soils only. When a load is applied to such a soil, it will be carried by the water in the soil voids (causing an increase in the pore water pressure) or by the soil skeleton (in the form of grain to grain contact stresses), or else it will be shared between the water and the soil skeleton as illustrated in [Fig. 3.5](#). The portion of the total stress carried by the soil particles is known as the effective stress, σ' . The load carried by the water gives rise to an increase in the pore water pressure, u , which, depending on the permeability, leads to water flowing under pressure out of the soil mass. This is called drainage and leads to soils possessing different strength characteristics before, during and at the end of the drainage period. This in turn necessitates the need for us to understand the behavior of the soil both immediately at the point of loading (i.e. when the soil is in an *undrained state*) and at a point in time long after the load has been applied (i.e. when the soil is in a *drained state*). The effects of undrained and drained conditions on soil strength are covered in [Chapter 4](#).



[Fig. 3.5](#) Load carried by soil particles and pore water.

Terzaghi first presented the concept of effective stress in 1925 and again, in 1936, at the First International Conference on Soil Mechanics and Foundation Engineering, at Harvard University. He showed, from the results of many soil tests, that when an undrained saturated soil is subjected to an increase in applied normal stress, $\Delta\sigma$, the pore water pressure within the soil increases by Δu , and the value of Δu is equal to the value of $\Delta\sigma$. This increase in u caused no measurable

changes in either the volumes or the strengths of the soils tested, and Terzaghi therefore used the term *neutral stress* to describe u , instead of the now more popular term pore water pressure.

Terzaghi concluded that only part of an applied stress system controls measurable changes in soil behaviour and this is the balance between the applied stresses and the neutral stress. He called these balancing stresses the effective stresses. He further explained that if a saturated soil fails by shear, the normal stress on the plane of failure, σ , also consists of the neutral stress, u , and an effective stress, σ' which led to the equation known to all soils engineers:

$$\text{effective stress} = \text{total stress} - \text{pore pressure}$$

$$\sigma' = \sigma - u$$

where the prime represents ‘effective stress’.

This equation is applicable to all saturated soils.

Example 3.1: Total and Effective Stress

A 3 m layer of sand, of saturated unit weight 18 kN/m^3 , overlies a 4 m layer of clay, of saturated unit weight 20 kN/m^3 . If the groundwater level occurs within the sand at 2 m below the ground surface, determine the total and effective vertical stresses acting at the centre of the clay layer. The sand above groundwater level may be assumed to be saturated.

Solution:

For this sort of problem it is usually best to draw a diagram to represent the soil conditions (see Fig. 3.6).

$$\text{Total vertical stress at centre of clay} = \text{Total weight of soil above}$$

$$\sigma_v = 2 \text{ m saturated clay} + 3 \text{ m saturated sand}$$

$$= 2 \times 20 + 3 \times 18 = 94 \text{ kPa}$$

$$\text{Effective stress} = \text{Total stress} - \text{Water pressure}$$

$$\sigma'_v = 94 - 9.81(2+1) = 64.6 \text{ kPa}$$

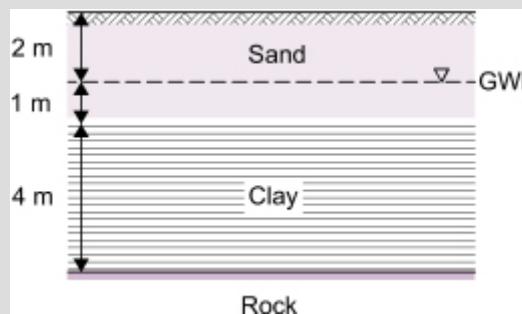


Fig. 3.6 Example 3.1.

Example 3.2: Distributions of Total Stress, Pwp and Effective Stress

A 5 m deep deposit of silty sand lies above a 4 m deep deposit of gravel. The gravel is underlain by a deep layer of stiff clay. The ground water table is found 2 m below the ground surface. The soil properties are:

$$\rho_b \text{ sand above GWT} = 1.7 \text{ Mg/m}^3$$

$$\rho_{\text{sat}} \text{ sand below GWT} = 1.95 \text{ Mg/m}^3$$

$$\rho_{\text{sat}} \text{ gravel} = 2.05 \text{ Mg/m}^3$$

Draw the distributions of vertical total stress, pore water pressure, and vertical effective stress with depth down to the clay layer.

Solution:

The values of total stress, pore pressure and effective stress are calculated at the salient points through the soil profile. These points are where changes in conditions occur, such as the horizon between two soils.

<u>Depth = 0 m</u>	$\sigma_z = 0; u = 0; \sigma' = 0$
<u>Depth = 2 m</u>	$\sigma_z = 1.70 \times 9.81 \times 2 = 33.4 \text{ kPa}$
	$u = 0$
<u>Depth = 5 m</u>	$\sigma_z = 33.4 + (1.95 \times 9.81 \times 3) = 90.8 \text{ kPa}$
	$u = 9.81 \times 3 = 29.4 \text{ kPa}$
<u>Depth = 9 m</u>	$\sigma_z = 90.8 + (2.05 \times 9.81 \times 4) = 171.2 \text{ kPa}$
	$u = 9.81 \times 7 = 68.7 \text{ kPa}$

The distributions are then plotted (see [Fig. 3.7](#)). Note that the values of total stress, pore pressure and effective stress all increase linearly between successive depths in the profile.

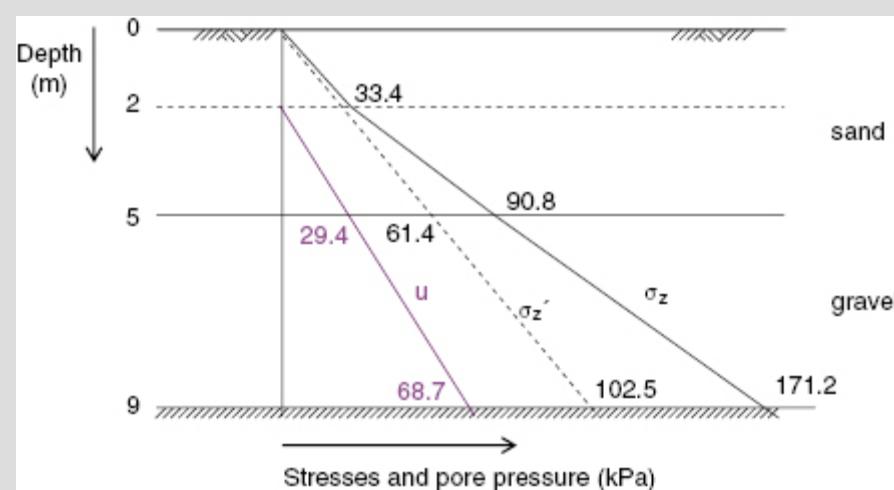


Fig. 3.7 Example 3.2.

3.5 Stresses Induced by Applied Loads

3.5.1 Stresses Induced by Uniform Surface Surcharge

In the case of a uniform surcharge spread over a large area it can be assumed that the increase in vertical stress resulting from the surcharge is constant throughout the soil. Here, the vertical total stress at depth z , is given by

$$\sigma_z = \gamma z + q$$

where q is the magnitude of the surcharge (kPa).

Example 3.3: Effective Stress with Surface Loading

Details of the subsoil conditions at a site are shown in Fig. 3.8 together with details of the soil properties. The ground surface is subjected to a uniform loading of 60 kPa and the groundwater level is 1.2 m below the upper surface of the silt. It can be assumed that the gravel has a degree of saturation of 50% and that the silt layer is fully saturated. NB $\gamma_w = 9.81 \text{ kN/m}^3$.

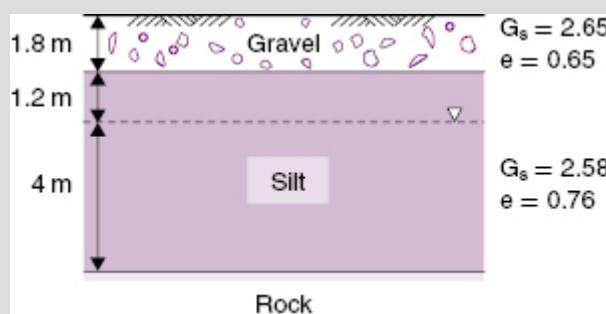


Fig. 3.8 Example 3.3.

Determine the vertical effective stress acting at a point 1 m above the silt/rock interface.

Solution:

$$\text{Bulk unit weight of gravel} = \gamma_w \frac{G_s + e S_r}{1+e} = 9.81 \frac{2.65 + 0.65 \times 0.5}{1+0.65} \\ = 17.7 \text{ kN/m}^3$$

$$\text{Saturated weight of silt} = \gamma_w \frac{G_s + e}{1+e} = 9.81 \frac{2.58 + 0.76}{1+0.76} \\ = 18.6 \text{ kN/m}^3$$

Effective vertical stress at 1 m above silt/rock interface

$$\begin{aligned} &= \left[\begin{array}{l} \text{Uniform pressure} \\ \text{applied at ground} \\ \text{surface} \end{array} \right] + \left[\begin{array}{l} \text{Total pressure} \\ \text{due to weight} \\ \text{of soils} \end{array} \right] - \left[\begin{array}{l} \text{Water pressure} \end{array} \right] \\ &= 60 + (1.8 \times 17.7 + 4.2 \times 18.6) - 3 \times 9.81 \\ &= 140.6 \text{ kPa} \end{aligned}$$

3.5.2 Stresses Induced by Point Load

The simplest case of applied loading has been illustrated in Example 3.3. However, most loads are applied to soil through foundations of finite area so that the stresses induced within the soil

directly below a particular foundation are different from those induced within the soil at the same depth but at some radial distance away from the centre of the foundation.

The determination of the stress distributions created by various applied loads has occupied researchers for many years. The basic assumption used in all their analyses is that the soil mass acts as a continuous, homogeneous and elastic medium. The assumption of elasticity obviously introduces errors but it leads to stress values that are of the right order and are suitable for most routine design work.

In most foundation problems, however, it is only necessary to be acquainted with the *increase* in vertical stresses (for settlement analysis) and the *increase* in shear stresses (for shear strength analysis).

Boussinesq (1885) evolved equations that can be used to determine the six stress components that act at a point in a semi-infinite elastic medium due to the action of a vertical point load applied on the horizontal surface of the medium.

His expression for the increase in vertical stress is:

$$\Delta\sigma_z = \frac{3Pz^3}{2\pi(r^2 + z^2)^{\frac{5}{2}}}$$

where

P = concentrated load

$r = \sqrt{x^2 + y^2}$ (see [Fig. 3.9](#), inset).

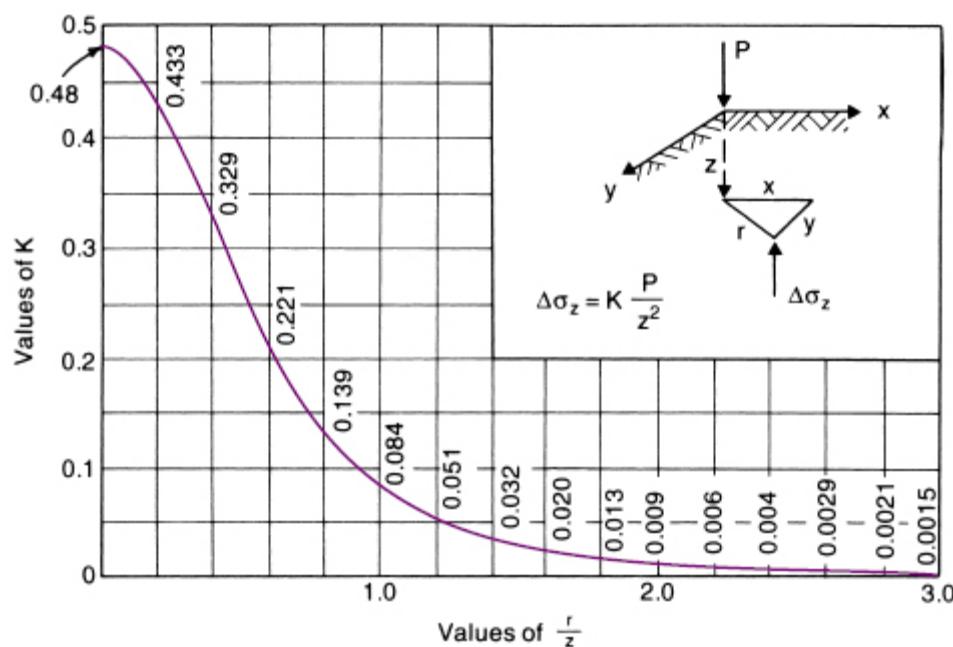


Fig. 3.9 Influence coefficients for vertical stress from a concentrated load (after Boussinesq, 1885).

The expression has been simplified to:

$$\Delta\sigma_z = K \frac{P}{z^2}$$

where K is an influence factor.

Values of K against values of r/z are shown in [Fig. 3.9](#).

Example 3.4: Vertical Stress Increments Beneath a Point Load

A concentrated load of 400 kN acts on the surface of a soil.

Determine the vertical stress increments at points directly beneath the load to a depth of 10 m.

Solution:

For points below the load $r = 0$ and at all depths $r/z = 0$, whilst from [Fig. 3.9](#) it is seen that $K = 0.48$.

z (m)	z^2	$\frac{P}{z^2}$	$\Delta\sigma_z = K \frac{P}{z^2}$ (kPa)
0.5	0.25	1600.0	768.0
1.0	1.00	400.0	192.0
2.5	6.25	64.0	30.7
5.0	25.00	16.0	7.7
7.5	56.25	7.1	3.4
10.0	100.00	4.0	1.9

This method is only applicable to a point load, which is a rare occurrence in soil mechanics, but the method can be extended by the principle of superposition to cover the case of a foundation exerting a uniform pressure on the soil. A plan of the foundation is prepared and this is then split into a convenient number of geometrical sections. The force due to the uniform pressure acting on a particular section is assumed to be concentrated at the centroid of the section, and the vertical stress increments at the point to be analysed due to all the sections are now obtained. The total vertical stress increment at the point is the summation of these increments.

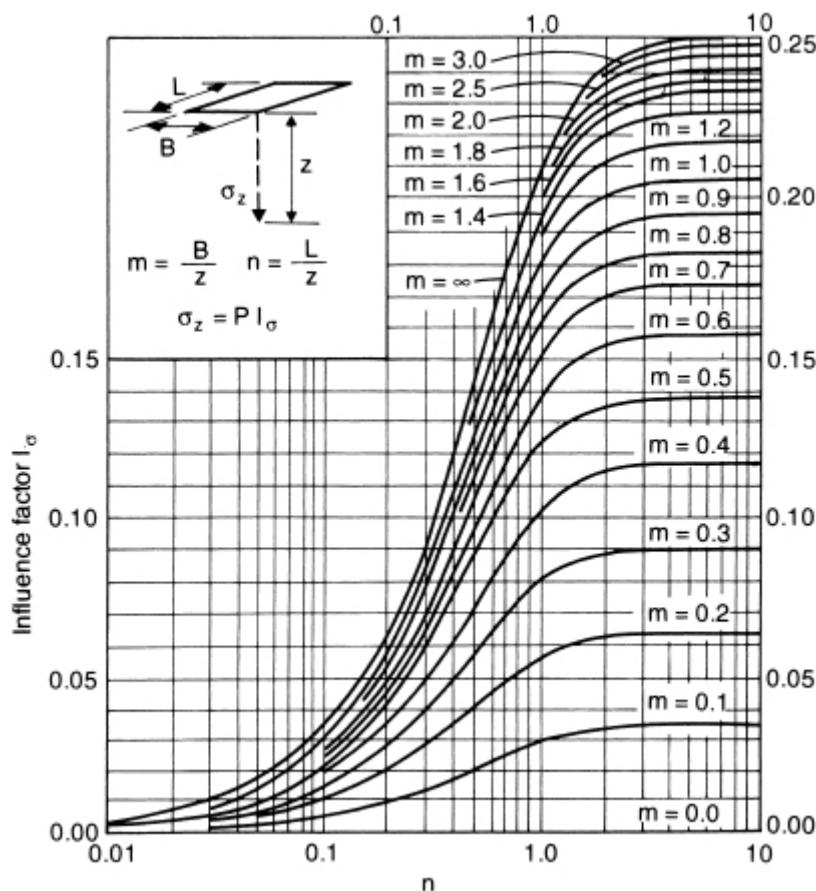
3.5.3 Stresses Induced by Uniform Rectangular Load

These can be established following Steinbrenner's method (1934). If a foundation of length L and width B exerts a uniform pressure, p , on the soil then the vertical stress increment due to the foundation at a depth z below one of the corners is given by the expression:

$$\sigma_z = p I_\sigma$$

where I_σ is an influence factor depending upon the relative dimensions of L , B and z .

I_σ can be evaluated by the Boussinesq theory and values of this factor (which depend upon the two coefficients $m = B/z$ and $n = L/z$) were prepared by Fadum in 1948 ([Fig. 3.10](#)).



[Fig. 3.10](#) Influence factors for the vertical stress beneath the corner of a rectangular foundation (after Fadum, 1948).

With the use of this influence factor the determination of the vertical stress increment at a point under a foundation is very much simplified, provided that the foundation can be split into a set of rectangles or squares with corners that meet over the point considered.

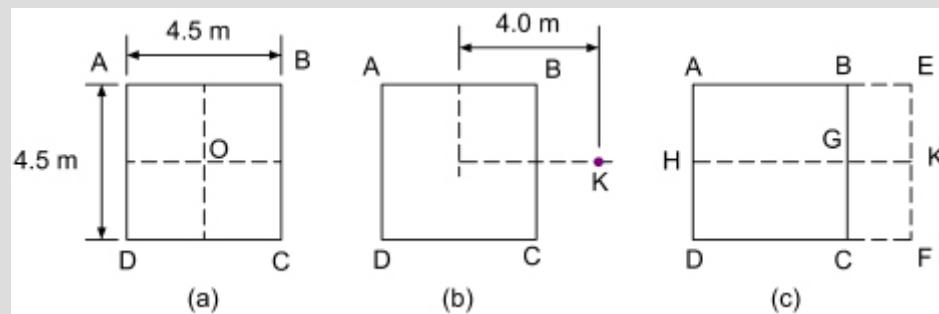
Example 3.5: Vertical Stress Increments Beneath a Foundation

A 4.5 m square foundation exerts a uniform pressure of 200 kPa on a soil. Determine (i) the vertical stress increments due to the foundation load to a depth of 10 m below its centre and (ii) the vertical stress increment at a point 3 m below the foundation and 4 m from its centre along one of the axes of symmetry.

Solution:

- i. The square foundation can be divided into four squares whose corners meet at the centre O ([Fig. 3.11a](#)).

$z \text{ (m)}$	$m = \frac{B}{z}$	$n = \frac{L}{z}$	I_σ	$4I_\sigma$	$\sigma_z \text{ (kPa)}$
2.5	0.9	0.9	0.163	0.652	130
5.0	0.45	0.45	0.074	0.296	59
7.5	0.3	0.3	0.04	0.16	32
10.0	0.23	0.23	0.025	0.1	20



[Fig. 3.11](#) Example 3.5.

- ii. This example illustrates how the method can be used for points outside the foundation area ([Fig. 3.11b](#)). The foundation is assumed to extend to the point K ([Fig. 3.11c](#)) and is now split into two rectangles, AEKH and HKFD.

For both rectangles:

$$m = \frac{B}{z} = \frac{2.25}{3} = 0.75; \quad n = \frac{L}{z} = \frac{6.25}{3} = 2.08$$

From [Fig. 3.10](#), $I_\sigma = 0.176$, therefore $\sigma_z = 0.176 \times 2 \times 200 = 70.4 \text{ kPa}$.

The effect of rectangles BEGK and KGCF must now be subtracted.

For both rectangles:

$$m = \frac{2.25}{3} = 0.75; \quad n = \frac{1.75}{3} = 0.58$$

From [Fig. 3.10](#), $I_\sigma = 0.122$ (strictly speaking m is 0.58 and n is 0.75, but m and n are interchangeable in [Fig. 3.10](#)). Hence:

$$\sigma_z = 0.122 \times 2 \times 200 = 48.8 \text{ kPa}$$

Therefore the vertical stress increment due to the foundation

$$= 70.4 - 48.8 = 21.6 \text{ kPa}$$

Circular foundations can also be solved by this method. The stress effects from such a foundation may be found approximately by assuming that they are the same as for a square foundation of the same area.

Example 3.6: Vertical Stress Increments Beneath Circular Foundation

A circular foundation of diameter 100 m exerts a uniform pressure on the soil of 450 kPa.

Determine the vertical stress increments for depths up to 200 m below its centre.

Solution:

$$\text{Area of foundation} = \frac{\pi \times 100^2}{4} = 7850 \text{ m}^2$$

Length of side of square foundation of same area = $\sqrt{7850} = 88.6$ m. This imaginary square can be divided into four squares as in [Example 3.5\(i\)](#). Length of sides of squares = 44.3 m.

z (m)	$n = m = \frac{B}{z}$	I_σ	$4I_\sigma$	σ_z (kPa)
10	4.43	0.248	0.992	446
25	1.77	0.221	0.884	398
50	0.89	0.16	0.64	288
100	0.44	0.071	0.284	128
150	0.3	0.04	0.16	72
200	0.22	0.024	0.096	43

3.5.4 Irregularly Shaped Foundations

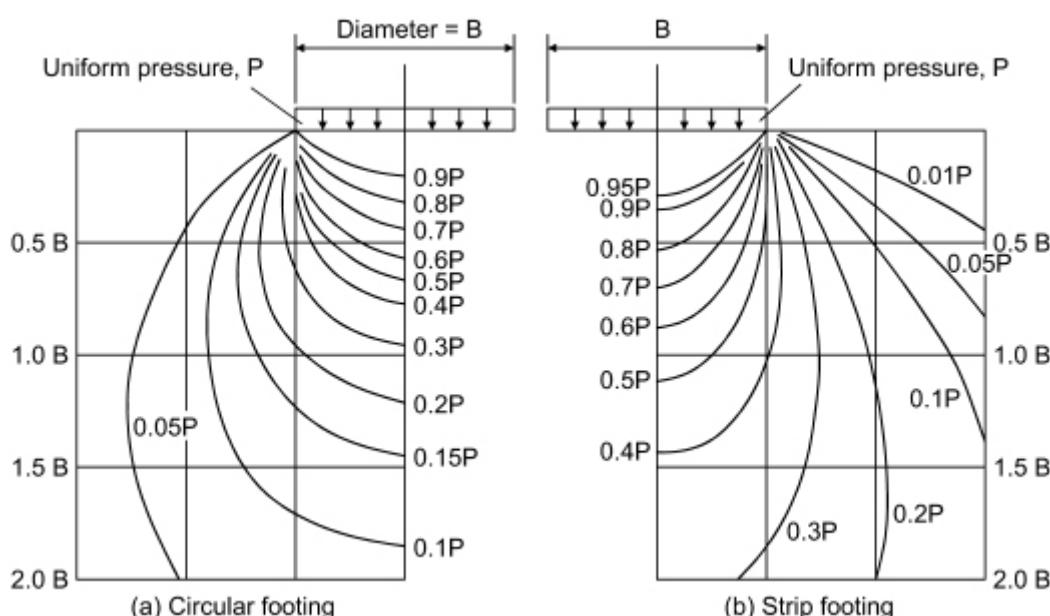
It may not be possible to employ Fadum's method for irregularly shaped foundations, and a numerical solution is then only possible by the use of Boussinesq's coefficients, K, and the principle of superposition.

Computer software for calculating the vertical stress increments beneath irregularly shaped foundations is widely available and nowadays such software is routinely used to determine the

stress values. Historically, however, the Newmark chart (Newmark, 1942) was used to determine the values and any reader interested in the use of the Newmark chart is guided to the seventh, or earlier, editions of this book for a full and detailed explanation.

3.5.5 Bulbs of Pressure

If points of equal vertical pressure are plotted on a cross-section through the foundation, diagrams of the form shown in [Figs 3.12a](#) and [3.12b](#) are obtained.



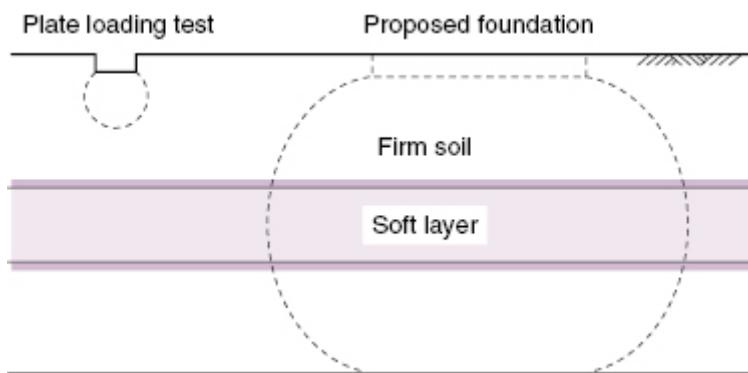
[Fig. 3.12](#) Bulbs of pressure for vertical stress.

These diagrams are known as bulbs of pressure and constitute another method of determining vertical stresses at points below a foundation that is of regular shape, the bulb of pressure for a square footing being obtainable approximately by assuming that it has the same effect on the soil as a circular footing of the same area.

In the case of a rectangular footing the bulb pressure will vary at cross-sections taken along the length of the foundation, but the vertical stress at points below the centre of such a foundation can still be obtained from the charts in [Fig. 3.12](#) by either (i) assuming that the foundation is a strip footing or (ii) determining σ_z values for both the strip footing case and the square footing case and combining them by proportioning the length of the two foundations.

From a bulb of pressure one has some idea of the depth of soil affected by a foundation. Significant stress values go down roughly to 2.0 times the width of the foundation, and [Fig. 3.13](#)

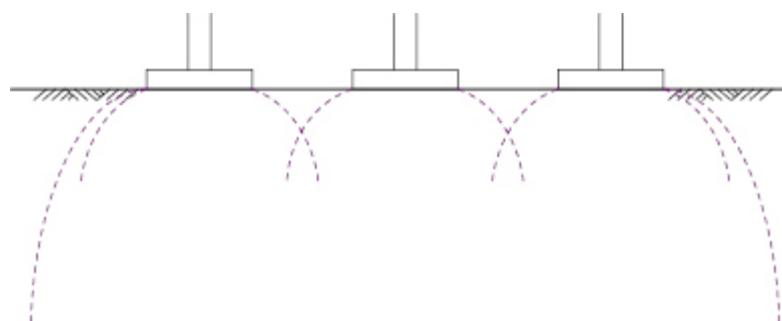
illustrates how the results from a plate loading test (see [Chapter 6](#)) may give quite misleading results if the proposed foundation is much larger: the soft layer of soil in the diagram is unaffected by the plate loading test but would be considerably stressed by the foundation.



[Fig. 3.13](#) Plate loading test may give misleading results.

Boreholes in a site investigation should therefore be taken down to a depth at least 1.5 times the width of the proposed foundation or until rock is encountered, whichever is the lesser.

Small foundations will act together as one large foundation ([Fig. 3.14](#)) unless the foundations are at a greater distance apart (c/c) than five times their width, which is not usual. Boreholes for a building site investigation should therefore be taken down to a depth of approximately 1.5 times the width of the proposed building.



[Fig. 3.14](#) Overlapping of pressure bulbs.

3.5.6 Shear Stresses

In normal foundation design procedure it is essential to check that the shear strength of the soil (see [Chapter 4](#)) will not be exceeded. The shear stress developed by loads from foundations of various shapes can be calculated. Jürgenson obtained solutions for the case of a circular footing and for the case of a strip footing ([Fig. 3.15](#)). It may be noted that, in the case of a strip footing, the maximum stress induced in the soil is p/π , this value occurring at points lying on a semicircle of diameter equal to the foundation width B . Hence the maximum shear stress under the centre of a continuous foundation occurs at a depth of $B/2$ beneath the centre.

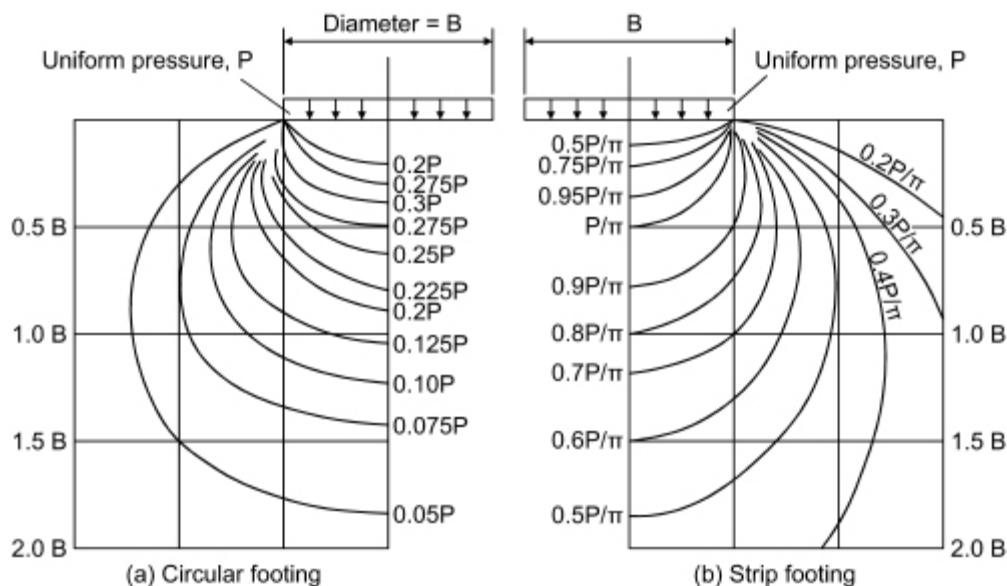


Fig. 3.15 Pressure bulbs of shear stress.

Shear Stresses Under a Rectangular Foundation

It is sometimes necessary to evaluate the shear stresses beneath a foundation in order to determine a picture of the likely overstressing in the soil.

Unfortunately a large number of foundations are neither circular nor square but rectangular, but [Figs 3.15a](#) and [3.15b](#) can be used to give a rough estimate of shear stress under the centre of a rectangular footing.

Example 3.7: Shear Stress Induced by Foundation

Loading

A rectangular foundation has the dimensions $15 \text{ m} \times 5 \text{ m}$ and exerts a uniform pressure on the soil of 600 kPa. Determine the shear stress induced by the foundation beneath the centre at a depth of 5 m.

Solution:

Strip footing

$$\frac{z}{B} = \frac{5}{5} = 1.0$$

From [Fig. 3.15b](#):

$$\tau = \frac{0.81 \times 600}{\pi} = 155 \text{ kPa}$$

For a square footing:

$$\text{Area} = 5 \times 5 = 25 \text{ m}^2$$

Diameter of circle of same area:

$$\sqrt{\frac{25 \times 4}{\pi}} = 5.64 \text{ m}$$

Hence the shear stress under a 5 m square foundation can be obtained from the bulb of pressure of shear stress for a circular foundation of diameter 5.64 m.

$$\frac{z}{B} = \frac{5}{5.64} = 0.89$$

From [Fig. 3.15a](#):

$$\tau = 0.2 \times 600 = 120 \text{ kPa}$$

These values can be combined if we proportion them to the respective areas (or lengths):

$$\tau = 120 + (155 - 120) \frac{15}{15 + 5} = 146 \text{ kPa}$$

The method is approximate but it does give an indication of the shear stress values.

3.5.7 Contact Pressure

Contact pressure is the actual pressure transmitted from the foundation to the soil. Throughout Section 3.5 it has been assumed that this contact pressure value, p , is uniform over the whole base of the foundation, but a uniformly loaded foundation will not necessarily transmit a uniform contact pressure to the soil. This is only possible if the foundation is perfectly flexible. The contact pressure distribution of a rigid foundation depends upon the type of soil beneath it.

[Figures 3.16a](#) and [3.16b](#) show the form of contact pressure distribution induced in a cohesive soil (a) and in a cohesionless soil (b) by a rigid, uniformly loaded, foundation.

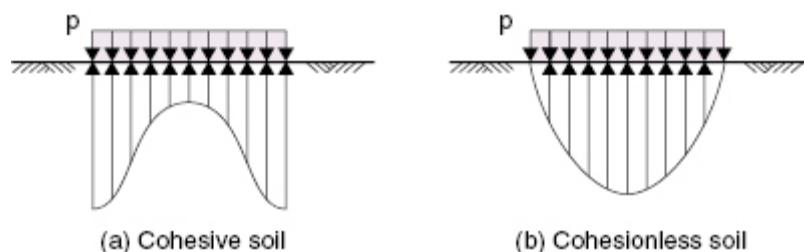


Fig. 3.16 Contact pressure distribution under a rigid foundation loaded with a uniform pressure, p .

On the assumption that the vertical settlement of the foundation is uniform, it is found from the elastic theory that the stress intensity at the edges of a foundation on cohesive soils is infinite. Obviously local yielding of the soil will occur until the resultant distribution approximates to [Fig. 3.16a](#).

For a rigid surface footing sitting on sand the stress at the edges is zero as there is no overburden to give the sand shear strength, whilst the pressure distribution is roughly parabolic ([Fig. 3.16b](#)). The more the foundation is below the surface of the sand the more shear strength there is developed at the edges of the foundation, with the result that the pressure distribution tends to be more uniform.

In the case of cohesive soil, which is at failure when the whole of the soil is at its yield stress, the distribution of the contact pressure again tends to uniformity.

A reinforced concrete foundation is neither perfectly flexible nor perfectly rigid, the contact pressure distribution depending upon the degree of rigidity. This pressure distribution should be considered when designing for the moments and shears in the foundation, but in order to evaluate shear and vertical stresses below the foundation the assumption of a uniform load inducing a uniform pressure is sufficiently accurate.

Exercises

Exercise 3.1

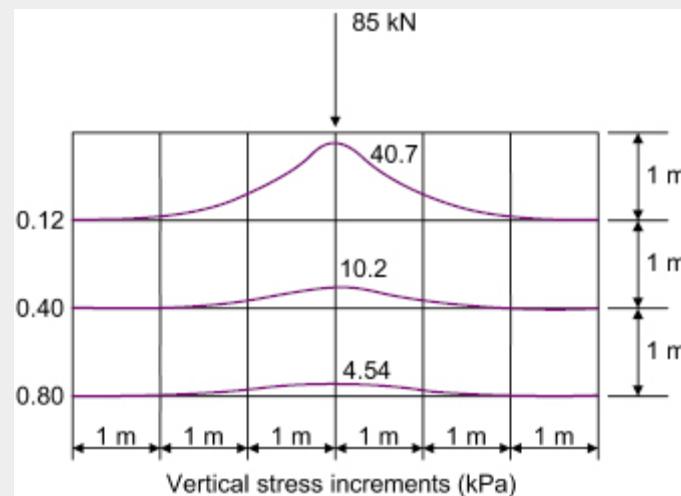
A raft foundation subjects its supporting soil to a uniform pressure of 300 kPa. The dimensions of the raft are 6.1 m by 15.25 m. Determine the vertical stress increments due to the raft at a depth of 4.58 m below it (i) at the centre of the raft and (ii) at the central points of the long edges.

Answer (i) 192 kPa, (ii) 132 kPa

Exercise 3.2

A concentrated load of 85 kN acts on the horizontal surface of a soil. Plot the variation of vertical stress increments due to the load on horizontal planes at depths of 1 m, 2 m and 3 m directly beneath it.

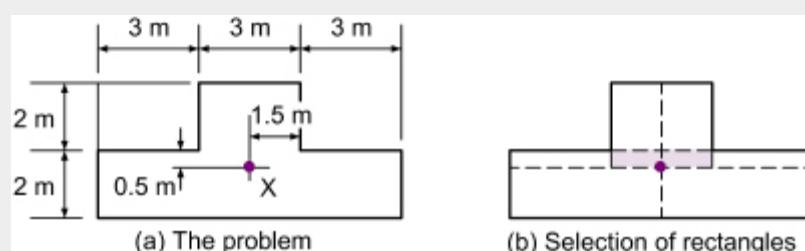
Answer [Fig. 3.17.](#)



[Fig. 3.17](#) Exercise 3.2.

Exercise 3.3

The plan of a foundation is given in [Fig. 3.18a](#). The uniform pressure on the soil is 40 kPa. Determine the vertical stress increment due to the foundation at a depth of 5 m below the point X, using [Fig. 3.10](#).



[Fig. 3.18](#) Exercise 3.3.

Note: In order to obtain a set of rectangles whose corners meet at a point, a section of the foundation area is sometimes included twice and a correction made. For this particular problem the foundation area must be divided into six rectangles ([Fig. 3.18b](#)); the effect of the shaded portion will be included twice and must therefore be subtracted once.

Answer 11.1 kPa

Exercise 3.4

A load of 500 kN is uniformly distributed through a pad foundation of dimensions 1.0 m × 1.5 m. Determine the increase in vertical stress at a depth of 2.0 m below one corner of the foundation.

Answer 36 kPa