Rocket Propulsion Laboratory: Ariel Thrust Chamber Assembly Calculations
Draft

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Abstract

This document covers the design of the combustion chamber and nozzle of RPL's $500lb_f$ methalox engine, Ariel, with an 80% bell nozzle approximation. **Note: Gas Pressures are reported in atmospheres for easy comparison to curves sourced from braeunig.com. Future iterations of this document will be translated to English units as is common in for aerospace literature.**

Nomenclature

A_c	=	Chamber Area (in^2)
A_e	=	Nozzle Exit Area (in^2)
A_t	=	Nozzle Throat Area (in^2)
D_c	=	Chamber Diameter (in)
D_e	=	Nozzle Exit Diameter (in)
D_t	=	Nozzle Throat Diameter (in)
c^*	=	Characteristic Velocity ($\frac{ft}{s}$)
C_f	=	Thrust Coefficient (unitless)
L^*	=	Characteristic Length (in) = Length necessary for complete combustion.
L_c	=	Chamber Length (in)
F	=	Thurst (lb_f)
F_{oS}	=	Factor of Safety (Unitless)
g_e	=	Gravity of Earth ($\frac{ft}{s^2}$)
I_{sp}	=	Specific Impulse (s)
P_a	=	Ambient Pressure (atm)
P_c	=	Chamber Pressure, (atm)
P_e	=	Exit Pressure (atm)
P_t	=	Nozzle Throat Pressure, (atm)
R	=	Universal Gas Constant, ($\frac{lb*lb_f}{^\circ R*lb-mol}$)
S_y	=	Yield Stress of Chamber Lining (psi)
T_c	=	Chamber Temperature ($^\circ R$)
T_t	=	Nozzle Throat Temperature ($^\circ R$)
t_w	=	Chamber Wall Thickness (in)
V_c	=	Chamber Volume (in^3)
$\dot{m}$	=	Propellant Mass Flow Rate ($\frac{lb}{s}$)
$\dot{m}_F$	=	Fuel Mass Flow Rate ($\frac{lb}{s}$)
$\dot{m}_O$	=	Oxidizer Mass Flow Rate($\frac{lb}{s}$)
M_r	=	Mixture Ratio (unitless)
M_w	=	Molecular Weight of the Exhaust (amu)
ϵ_c	=	Contraction Area Ratio (unitless)
ϵ_e	=	Expansion Area Ratio (unitless)
γ	=	Adiabatic Index-Ratio of Specific Heats (unitless)

Constants

$F_{oS} = 1.4$
$g_e = 32.1740 \frac{ft}{s^2}$
$P_a = 1\ atm$
$R = 1545.348963 \frac{lb*lb_f}{^\circ R*lb-mol}$
$S_y = 80000\ psi$

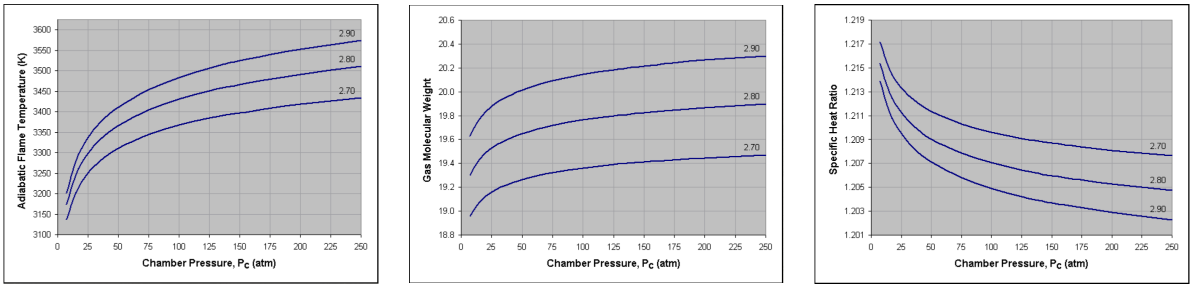
1 Introduction

Compared to other traditional rocket propellants, LOX and liquid CH_4 is advantageous for deep space missions for several reasons. While it has a lower specific vacuum impulse than LOX/LH_2, it presents other favorable properties such as a higher boiling point. This requires a less severe temperature differential to be maintained, reducing the risk of boil off. Methalox's density is also significantly higher than LOX/LH_2, allowing for smaller-less massive-propellant tanks. Further more, the propellant required for an additional trip(s) from an carbon dioxide rich atmosphere can be produced on-site with a small amount of seed hydrogen, allowing for considerable weight-propellant savings important for mission design. Here the Rocket Propulsion Laboratory goes through the initial design of a 2000 lb_f methalox engine, dubbed Ariel.

2 Approach

To ensure accuracy and reliability in the calculations, several reference texts were used to baseline the design. In particular, *Modern Engineering for Design of Liquid-Propellant Engines* by Dieter K. Huzel and David H. Huang and *Rocket Propulsion Elements,* 8^{th} *Ed.* by George P. Sutton were heavily referenced to bridge the knowledge gap and guide the designs of the combustion chamber and the nozzle.

Both texts show that the overall design of a given thrust chamber assembly is characterized by the desired propellant, chamber pressure, exit pressure, thrust, and nozzle contour for a given engine. From a given propellant and chamber pressure, key parameters of the engine's exhaust can be computed. In particular they are: the adiabatic index, adiabatic flame temperature, molecular weight of the exit gas, and the optimum mixture ratio of the propellant. However neither text goes into detail about characterizing the chemical properties of propellants. For this, software like Stanford's StanJan and NASA's CEA application are used. braeunig.us hosts the relevant curves for methalox, calculated with StanJan . This data was cross checked with CEAweb to verify accuracy.



(a) Adiabatic Flame Temp. (b) Molecular Weight of Gas (c) Specific Heat Ratio

Figure 1: Three Data-sets in Reference

Determining the optimum mixture ratio requires that Specific Impulse vs Mixture Ratio curves be calculated, holding Chamber Pressure constant. The global maxima for these curves provide the optimal mixture ratio for a given pressure. This is shown in another graph from braeunig.us , where they have graphed Optimal Mixture Ratio vs Pressure, holding Exit Pressure constant:

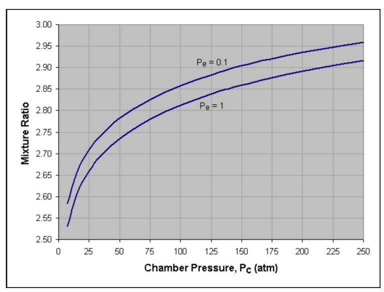


Figure 2: M_R versus P_c

2.1 Thrust

The desired thrust acts as an indirect scaling factor for the engine, influencing the diameter of the chamber and diameter of the nozzle's exit relative to the diameter of the nozzle's throat. With this in mind, Ariel was designed to produce only $500lbf$ as this would reduce the cost in both materials and time to produce as well as mitigate risks that would scale with the engine size.

2.2 Nozzle Contour

The nozzle follows suit in that an an 80% bell nozzle approximation is used as it provides a reasonable trade off between performance and weight. Furthermore, considering how thoroughly bell nozzles have been studied and documented, they are a safe choice for RPL's first attempt at constructing a liquid rocket engine.

2.3 Exit Pressure

As [citation needed, Huzel+Huang] explains, having an exit pressure different from the ambient pressure results in over or under expansion of the exhaust which are both unfavorable. Ariel was designed to be fired at sea level, so the desired Exit Pressure was defined as one atm.

2.4 Chamber Pressure

Looking at Figure 1 and Figure 2 and Equations ?? and ??, it's apparent that a higher chamber pressure and corresponding mixture ratio would result in higher specific impulse. However, given that Ariel will be a small motor and a first for RPL, high pressures were considered an unnecessary safety risk. At the other end of the spectrum, while lowering chamber pressure mitigates pressure related hazards, minor fluctuations in the mixture ratio would result in increasing large perturbations of chamber pressure. With this in mind, looking at Figure 2 it can be seen that a reasonable range for chamber pressure is 50 to 90 atmospheres.

Ariel was designed to have a chamber pressure of 68 atmospheres. 68 is a conservative average, it falls in the middle of the above range but leans slightly towards the lower end.

2.5 Exhaust Properties

With the characteristic engine parameters defined, exhaust properties can discerned from the data generated from StanJan or CEAweb.

3 Calculations

3.1 Characteristic Engine Parameters

Having defined the independent characteristic parameters for Ariel's combustion chamber the optimal mixture ratio of methalox to feed the engine was approximated by examining Figure 2. Using these values for thrust, chamber pressure, exit pressure, and the mixture ratio the exhaust parameters where calculated with CEAweb ??.

Characteristic Engine Parameters

$$
\begin{aligned}
F &= 2000\ lb_f \\
P_c &= 68\ atm\ (999.321\ psi) \\
P_e &= 1\ atm\ (14.6959\ psi) \\
T_c &= 6051.60\ ^\circ R \\
M_r &= 2.77 \\
M_w &= 20.158\ amu \\
\gamma &= 1.2161
\end{aligned}
$$

3.2 Area Contraction and Expansion Ratios, ϵ_c and ϵ_e

$$\epsilon_c = \frac{A_c}{A_t} = 1.317303 + \frac{32346880 - 1.317303}{1 + (\frac{D_t}{1.041567*10^{-10}})^{0.6774809}} = 6.0510 \tag{1}$$

This equation for the contraction area ratio comes from a rough fitting of Figure 4-9 from Huzel and Huang .

$$\epsilon_e = \frac{A_e}{A_t} = \frac{(\frac{2}{\gamma+1})^{\frac{1}{\gamma-1}}(\frac{P_c}{P_e})^{\frac{1}{\gamma}}}{[(\frac{\gamma+1}{\gamma-1})[1-(\frac{P_e}{P_c})^{\frac{\gamma-1}{\gamma}}]]^{\frac{1}{2}}} = 8.5917 \tag{2}$$

3.3 Thrust-Chamber Performance Parameters

3.3.1 Characteristic Velocity, c*

$$c^* = \frac{\sqrt{g_e \gamma R T_c}}{\gamma\sqrt{\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{\gamma-1}}}} = 5929\,\frac{ft}{s} \tag{3}$$

Characteristic Velocity is a measure of the combustion performance, similar to Specific Impulse.

3.3.2 Thrust Coefficient, C$_f$

$$C_f = \lambda * \sqrt{\frac{2\gamma^2}{\gamma-1}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \epsilon_e\left[\frac{P_e - P_a}{P_c}\right] = 1.5640 \tag{4}$$

Where $\lambda = .985$ is the correction factor associated with a 80% bell nozzle for an engine of this size . It applies only to the momentum term of C_f as it accounts for divergence from uniform axial flow (the contour of the nozzle at it's exit isn't parallel to the central axis of the thrust chamber).

3.3.3 Specific Impulse, I$_{sp}$

$$I_{sp} = \frac{c^* C_f}{g_e} = 288s \tag{5}$$

Specific Impulse is the amount of thrust generated per unit of propellant, or, in essence, the performance of our engine.

3.4 Mass Flow, $\dot{m}$

The Propellant Mass Flow Rate

$$\dot{m} = \frac{F}{I_{sp}} = 6.9394\frac{lb}{s} \tag{6}$$

the Fuel Mass Flow Rate,

$$\dot{m}_F = \frac{\dot{m}}{r+1} = 1.841\frac{lb}{s}$$

and the Oxygen Mass Flow Rate.

$$\dot{m}_O = \frac{r * \dot{w}}{r+1} = 5.099\frac{lb}{s}$$

3.5 Nozzle Throat Parameters

Nozzle Throat Temperature,

$$T_t = T_c \left[\frac{2}{1+\gamma}\right] = 5461.5°R \tag{7}$$

Nozzle Throat Pressure,

$$P_t = P_c \left[\frac{\gamma+1}{2}\right]^{\frac{-\gamma}{\gamma-1}} = 38.173atm \tag{8}$$

Nozzle Throat Area,

$$A_t = \frac{\dot{m}I_{sp}}{P_c C_f} = 1.2796in^2 \tag{9}$$

and Nozzle Throat Diameter.

$$D_t = \left(\frac{4A_t}{\pi}\right)^{\frac{1}{2}} = 1.2764in \tag{10}$$

Nozzle Exit Area,

$$A_e = \epsilon_e A_t = 10.9942in^2 \tag{11}$$

and Nozzle Exit Diameter.

$$D_e = \left(\frac{4A_e}{\pi}\right)^{\frac{1}{2}} = 3.7414in \tag{12}$$

3.6 Chamber Dimensions

a) Calculate the Chamber Volume.

$$V_c = A_t L^* = 51.1848 in^3 \tag{13}$$
$$L^* = 40 in$$

Characteristic Lengths and Volumes are a more intuitive way to express the requirements for efficient combustion of the propellant, which is usally expressed as propellant stay time (τ). Characteristic Length is proportional to the complexity molecules that make up the propellant; examining table from Huzel and Huang, it can be reasoned Methalox will have a characteristic length between that of RP-1 and LOX+LH_2. This is estimated to be 40 in.

b) Calculate the Chamber Length by utilizing the contraction ratio (ϵ_c) to solve for A_c, and thus D_c.

$$V_c = (1.1)A_c * L_c = 51.1848 in^3 \tag{14}$$

Chamber Area

$$A_c = \epsilon_c A_t = 7.7429 in^2 \tag{15}$$

Chamber Diameter

$$D_c = (\frac{4A_c}{\pi})^{\frac{1}{2}} = 3.1398 in \tag{16}$$

Chamber Length

$$L_c = \frac{V_c}{1.1A_c} = 6.0096 in \tag{17}$$

c) Chamber Wall Thickness

From Barlow's law relating the maximum working pressure to a pipe's geometry:

$$F_{oS} * P_c = \frac{2 * S_y * t_w}{D_{outside}} \tag{18}$$

Where $D_{outside} = D_c + 2t_w$

We derived the equation that gives the desired thickness of the chamber lining

$$t_w = \frac{F_{oS} * P_c * D_c}{2 * S_y - 2 * F_{oS} * P_c} = 0.0279 in \tag{19}$$

S_y equals the yield stress of material used for the chamber lining; for Ariel that material is Inconel 718, with a yield stress of 80000 psi

3.7 Nozzle Contour

Most references suggest using a parabolic approximation for bell nozzles . However, the examples provided in Huzel and Huang couldn't be reconciled with the data they provided. A lecture from Georgia Tech suggested to use the general conic equation to approximate the contour of a bell nozzle. This follows the form:

$$y = P * x + Q + (S * x + T)^{\frac{1}{2}} \tag{20}$$

4 Results

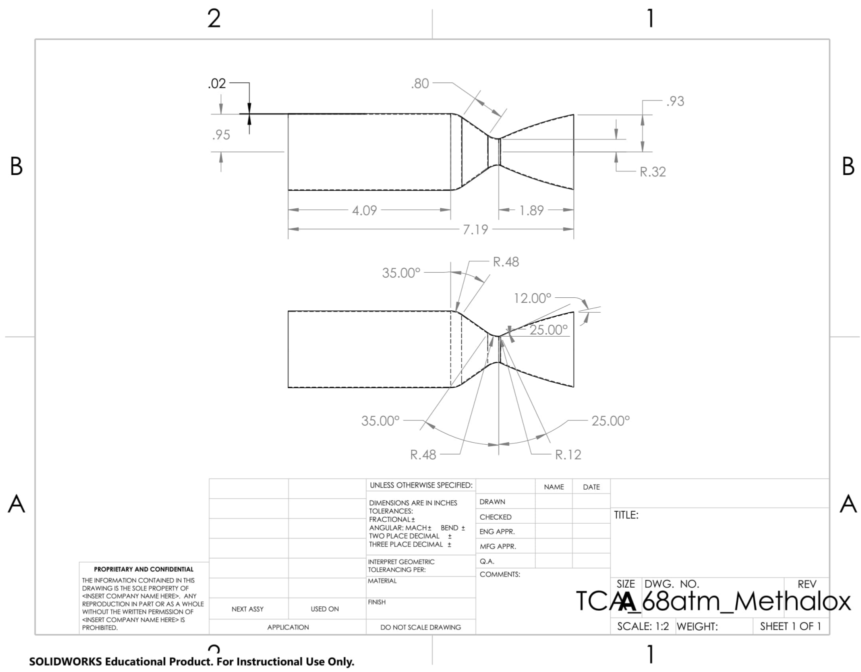


Figure 3: Current Model

5 Conclusions

In order to bridge the knowledge gap between professional industry knowledge and collegiate student knowledge, we have heavily utilized the information from *Modern Engineering for Design of Liquid-Propellant Engines* by Dieter K. Huzel and David H. Huang as well as *Rocket Propulsion Elements, 8th Ed.* by George P. Sutton . This literature provided us with the equations necessary to design and model the first iteration of Ariel. Further iteration will be guided by results from analysis, as an example, CFD can be used to test the approximation made for ϵ_c.