

**ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)**  
**ORGANISATION OF ISLAMIC COOPERATION (OIC)**  
**Department of Computer Science and Engineering (CSE)**

**MID SEMESTER EXAMINATION**  
**DURATION: 1 HOUR 30 MINUTES**

**WINTER SEMESTER, 2022-2023**  
**FULL MARKS: 75**

**Math 4341: Linear Algebra**

**Programmable calculators are not allowed. Do not write anything on the question paper.**

Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

1. Consider the following system of equations for  $Ax = b$  and answer the subsequent questions:

$$x_1 + 2x_2 + 3x_3 = 9$$

$$x_1 + 3x_2 + 4x_3 = 11$$

$$-6x_2 - 10x_3 = -24$$

- 22 a) How many operations do we need to carry out during forward elimination on a matrix  $[A \ b]$ ? 5  
(CO1)  
(PO1)
- b) Solve the linear system of equations using Gaussian elimination. 10  
(CO2)  
(PO1)
- c) Find  $A^{-1}$ . Check if your answer is correct by showing  $AA^{-1} = I$ . 10  
(CO2)  
(PO1)
2. a) Describe the column spaces (lines or planes) of the following matrices: 2 × 3  
(CO1)  
(PO1)
- $$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}$$
- b) Find the conditions on  $b_1, b_2$ , and  $b_3$  for which the following systems are solvable: 5 × 2  
(CO2)  
(PO1)
- i.  $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$
- ii.  $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$
- c) Construct 3 × 3 matrix considering the following requirements: 4.5 × 2  
(CO2)  
(PO1)
- i. Column space contains (1, 1, 0) and (1, 0, 1), but not (1, 1, 1).
- ii. Column space is only a line.

3. a) Determine the values of the variables  $a$  to  $h$  in the matrices  $A$  and  $B$  so that they have rank 1. 2.5 × 2  
(CO1)  
(PO1)

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & a & b \\ 4 & c & d \end{bmatrix}; \quad B = \begin{bmatrix} e & 9 & f \\ 1 & g & h \\ 2 & 6 & -3 \end{bmatrix}$$

- b) Consider the following system of equations for  $Ax = b$  and answer the subsequent questions: 4 × 5  
(CO2)  
(PO1)

$$x_1 + 2x_2 + 3x_3 + 5x_4 = b_1$$

$$2x_1 + 4x_2 + 8x_3 + 12x_4 = b_2$$

$$3x_1 + 6x_2 + 7x_3 + 13x_4 = b_3$$

- i. Reduce  $[A \ b]$  to  $[U \ c]$ , so that  $Ax = b$  becomes a triangular system  $Ux = c$ .
- ii. Find the condition on  $b_1$ ,  $b_2$ , and  $b_3$  for  $Ax = b$  to have a solution.
- iii. Describe the column space of  $A$ . What is the picture of the column space in  $\mathbb{R}^3$ ?
- iv. Describe the null space of  $A$ . Find the special solutions in  $\mathbb{R}^4$ .
- v. Find the complete solution when  $Ax = (0, 6, -6)$ .