ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2021-2022**
DURATION: 3 HOURS **FULL MARKS: 200**

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) i. What do you mean by a differential equation? Classify the types of differential equations. 4 (CO1) (PO1)

   ii. Verify that $y = e^{3x} \cos 2x$ is a solution to the linear equation $y'' - 6y' + 13y = 0$. 4 (CO2) (PO1)

   iii. Find an explicit solution to the following initial value problem: 4 (CO2) (PO1)
   $$\frac{dx}{dt} = 4(x^2 + 1);\; x\left(\frac{\pi}{4}\right) = 1.$$

   b) i. Define the degree and order of a differential equation. 4 (CO1) (PO1)

   ii. Solve the differential equation $(x+y+1)dx-(2x+2y+1) = 0$ by separation of variables method. 8 (CO2) (PO1)

   c) Solve the differential equation $(6x - 5y + 4)dy - (2x - y + 1) = 0$ by a suitable method. 10 (CO2) (PO1)

2. a) i. When is a differential equation said to be an exact differential equation? Write down its mathematical formulation with an example. 4 (CO1) (PO1)

   ii. Solve the differential equation $(3x^2y - 6x)dx + (x^3 + 2y)dy = 0$. 8 (CO2) (PO1)

   b) i. Define an integrating factor. When do we need them in solving a differential equation? 4 (CO1) (PO1)

   ii. Solve the inexact differential equation $xydx + (2x^2 + 3y^2 - 20)dy = 0$ using a suitable technique. 8 (CO2) (PO1)

   c) Solve the initial value problem $(e^x + y)dx + (2 + x + ye^y)dy = 0$, $y(0) = 1$ using a suitable technique. 10 (CO2) (PO1)

3. a) i. Define Cauchy-Euler's form of linear differential equation. 3 (CO1) (PO1)

11 ii. Solve the following Cauchy-Euler's differential equation: 8 (CO2) (PO1)

$$x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3$$

6 b) Solve the following differential equation using the method of variation of parameters: 10 (CO2) (PO1)

$$(D^2 - 3D + 2)y = \sin(e^{-x}).$$

27 c) i. Define ordinary point of a second order linear differential equation. 2 (CO1) (PO1)

12 ii. Find the power series solution of the differential equation $\frac{d^2y}{dx^2} + xy = 0$ about the ordinary point $x = 0$. 10 (CO2) (PO1)

4. 4 a) Define regular and irregular singular points of a linear differential equation. 4 (CO1) (PO1)

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12 b) Use the method of Frobenius to obtain two linearly independent power series solution of the differential equation $2x^2y'' - xy' + (x-5)y = 0$ about the singular point $x = 0$. 15 (CO2) (PO1)

23 7 c) i. Write down the Rodrigue's formula for Legendre polynomial. Evaluate $P_3(x)$ using this formula. 7 (CO2) (PO1)

5 ii. Prove that $P_{2n}(0) = (-1)^n\frac{(2n)!}{2^{2n}(n!)^2}$, where $P_n(x)$ is a Legendre polynomial of degree n. 7 (CO2) (PO1)

5. a) What do you mean by Bessel's differential equation? Define Bessel's function of first kind and second kind. 6 (CO1) (PO1)

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8 b) For Bessel's polynomial $J_n(x)$, prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. 9 (CO2) (PO1)

31 6 c) i. Define a Partial Differential Equation (PDE). Write down the general form of a second order PDE. Classify them with proper naming. 6 (CO1) (PO1)

12 ii. Find the general solution to the PDE $p\tan x + q\tan y = \tan z$ using Lagrange's method. 12 (CO2) (PO1)

$6+7=13$

6. a) Define the complete and particular integral of a PDE. 6 (CO1) (PO1)

   b) Find the integral surface of the PDE $(x-y)p+(y-x-z)q=z$ through the curves $z=1$, $x^2+y^2=1$. 9 (CO2) (PO1)

$\frac{8+6}{15}$

   c) i. Show that the two functions, $f(x,y,z,p,q)=xp-yq=0$ and $g(x,y,z,p,q)=z(xp+yq)-2xy=0$, are compatible and find the solution. 9 (CO2) (PO1)

      ii. Find a complete integral of $p^2x+q^2y=z$ using Charpit's method. 9 (CO2) (PO1)