ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **SUMMER SEMESTER, 2021-2022**

DURATION: 3 HOURS **FULL MARKS: 150**

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 6 (six)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) Let p, q, and r be the propositions — 2×4 (CO1) (PO1)

   p : You have the flu.

   q : You miss the final examination.

   r : You pass the course.

   Express each of these following propositions as English sentences:

   i. $p \rightarrow q$

   ii. $\neg p \leftrightarrow q$

   b) Express the following statements using mathematical and logical operators, predicates, and quantifiers, where the domain consists of all integers: — 2×4 (CO1) (PO1)

   i. The sum of two negative integers is negative.

   ii. The sum of the squares of two integers is greater than or equal to the square of their sum.

   c) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. — 9 (CO1) (PO1)

2. a) List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$. — 8 (CO2) (PO1)

   Determine whether R is:

   i. Reflexive relation.

   ii. Symmetric relation.

   iii. Transitive relation.

   b) Determine and analyze Big-O representation for the following function which represents the complexity of an algorithm: — 8 (CO2) (PO2)

   $$f(n) = (n! + 2^n)(n^2 + log(n^2 + 1))$$

   c) Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n. — 9 (CO2) (PO1)

3. a) Show that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod m$, where a and b are integers, then $a \equiv b \pmod n$. — 8 (CO3) (PO1)

   b) What are the solutions to the linear congruence $3x \equiv 4 \pmod 7$? — 8 (CO3) (PO1)

   c) Use the extended Euclidean algorithm to express $gcd(144, 89)$ as a linear combination of 144 and 89. — 9 (CO3) (PO1)

4 a) A graph has 21 edges, 3 vertices with a degree of 4, 2 vertices with a degree of 3, and all other vertices with a degree of 2. Determine the number of total vertices. 8 (CO4) (PO1)

b) Can a simple graph exist with 11 vertices, each with degree 3? Provide proper arguments in favor of your answer. 8 (CO4) (PO1)

c) Define a bipartite graph. Determine whether the graph in Figure 1 is a bipartite graph. 9 (CO4) (PO1)

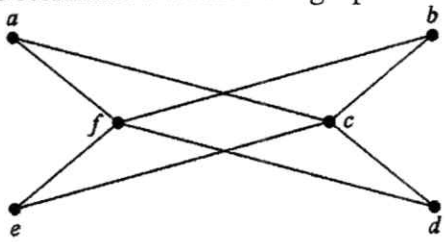


Figure 1: Graph for Question 4.c)

5 a) Represent the graph in Figure 2 using an adjacency matrix. 7 (CO4) (PO1)

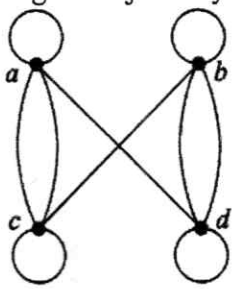


Figure 2: Graph for Question 5.a)

b) Determine whether the graph in Figure 3 is a Euler graph. If it is a Euler graph, then construct the Euler circuit. If it is a not a Euler graph, then determine whether the graph has a Euler path and construct such a path if one exists. 9 (CO4) (PO1)

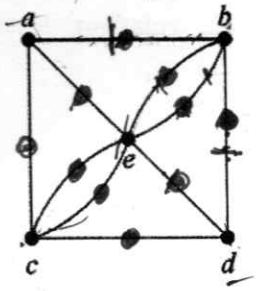
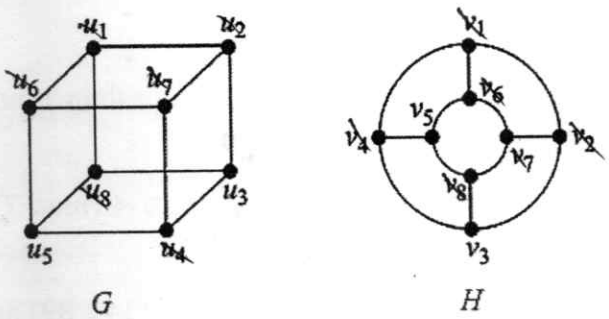


Figure 3: Graph for Question 5.b)

c) Analyze the graphs G and H illustrated in Figure 4 and determine whether they are isomorphic. Provide a rigorous argument in favor of your answer. 9 (CO4) (PO2)



G *H*

Figure 4: Graph for Question 5.c)

$m = 4 \quad L = 100$

$n \quad I$

6. a) Suppose that Alex starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letter. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter? — 8 (CO4) (PO1)

b) What is the value of the prefix expression $+ - * \ 235/\uparrow \ 2\ 3\ 4$? — 7 (CO4) (PO1)

c) Answer following questions about the rooted tree illustrated in Figure 5. — 2 × 5 (CO4) (PO1)

i. Which vertices are internal?
ii. Which vertices are leaves?
iii. Which vertices are children of j?
iv. Is the tree in Figure 5 a full m-ary tree for some positive integer m?
v. What is the height of the tree?

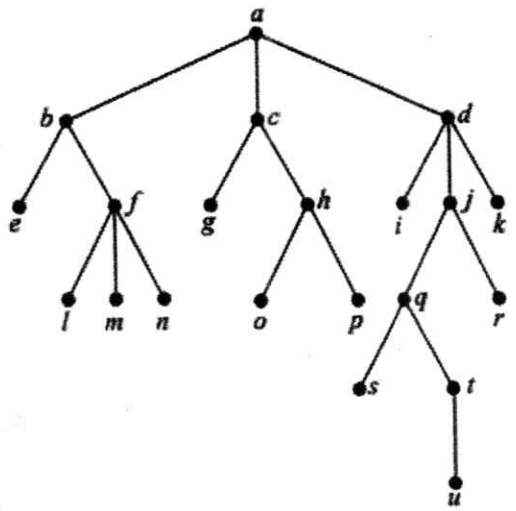


Figure 5: Rooted tree for Question 6.c)