ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination
Course Number: Math 4241
Course Title: Integral Calculus and Differential Equations

Summer Semester: 2022-2023
Full Marks: 200
Time: 3 Hours

Answer all the **6 (Six)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) Sketch the region enclosed by the curves $y = x - 1$ and $y^2 = 2x + 6$. Then find the area of the region by integrating with respect to x. **(11)** (CO3) (PO2)

b) Use cylindrical shell method to find the volume of the solid that is generated when the region enclosed by the curves $y = 2x - 1$, $y = -2x + 3$ and $x = 2$ is revolved about the y-axis. **(12)** (CO3) (PO2)

c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis. **(10)** (CO3) (PO2)

2. a) Find the area of the region that is inside the cardioid $r = 2 + 2\cos\theta$ and outside the circle $r = 3$. **(10)** (CO3) (PO2)

b) Find the nature of singularity of the differential equation **(4)** (CO2) (PO2)

$$2x^2 y'' + xy' - (2x + 1)y = 0$$

c) Solve the following differential equation by Fröbenius method: **(20)** (CO3) (PO2)

$$2xy'' + (x + 1)y' + y = 0$$

3. a) A small metal bar, whose initial temperature was $20^0 C$, is dropped into a large container of boiling water How long will it take the bar to reach $90^0 C$ if it is known that its temperature increases 2^0 in 1 second? How long will it take the bar to reach $98^0 C$? **(11)** (CO3) (PO2)

b) Find the charge $q(t)$ on the capacitor in an LRC-series circuit when $L = 0.25$ henry $(h), R = 10$ ohms $(\Omega), C = 0.001$ farad (f), $E(t) = 0, q(0) = q_0$ coulombs, $and\ i(0) = 0$. **(11)** (CO3) (PO2)

c) Eliminate arbitrary function ϕ from the equation **(11)** (CO1) (PO1)
$$\phi(\tan x + \sin^{-1} y - \log z, e^x - \sec y + z^3) = 0.$$

4. a) Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(11)** (CO2) (PO2)

b) Prove that $\int_{-1}^{1} x^2 P_{n-1}(x) P_{n+1}(x)\,dx = \dfrac{2n(n+1)}{(4n^2-1)(2n+3)}$.   (12)   (CO2) (PO2)

c) Prove that $J_2{'}(x) = \left(1 - \dfrac{4}{x^2}\right)J_1(x) + \dfrac{2}{x}J_0(x)$.   (11)   (CO2) (PO2)

5. a) Solve $p\cos(x+y) + q\sin(x+y) = z$, using Lagrange's method   (11)   (CO1) (PO1)

b) Find the general integral of $p^2 x^2 + q^2 y^2 = z^2$.   (11)   (CO1) (PO1)

c) Apply Charpit's method to find the complete integral of
$$z = px + qy + p^2 + q^2.$$
     (11)   (CO2) (PO1)

6. a) Solve $\left(D_x^2 + 4D_x D_y + 4D_y^2\right)z = e^{2x+y}$.   (13)   (CO1) (PO1)

b) Find the solution to the heat conduction problem
$$T_{xx} = 4T_t;\ \ 0 < x < 2, t > 0$$
$$T(0,t) = 0, T(2,t) = 0, t > 0$$
$$T(x,0) = 2\sin\tfrac{\pi x}{2} - \sin\pi x + 4\sin 2\pi x,\ \ 0 \le x \le 2.$$
     (20)   (CO3) (PO2)