FRM Notebook 4

http://www.ster.kuleuven.be/~pieterd/python/html/core/scipystats.html

Quantifying Volatility in VaR Models

- VaR 在险价值
  - Potential loss over a given time and for a given distribution of returns
  - The loss >=VaR only X percent of the time
  - 1%, 5%, 10% (daily)
- Calculating
  - Assume a standard normal distribution
  - **Z: -2.33 (1%), -1.65(5%), -1.28 (10%)**
  - VaR(x%) = Z_(x%) sigma = z*sigma
    - One-tailed test
- Expected return is given
  - VaR(x%) = E – |z| * sigma (default is 0)
- J-day: VaR(x%) = sqrt(J) * VaR(daily)
  - Weekly: sqrt(5)
  - Monthly: sqrt(20)
  - Semiannual: sqrt(125)
  - Yearly: sart(250)
- Confidence level
  - Var(x%) / z(x%) = sigma
- Methods
  - Linear methods
    - Replace positions with linear exposures on risk factor
    - Options: delta, bond positions: duration
    - Fast and efficient
    - Good for non option-like exposures
  - Full valuation
    - Reprice each scenario over a historical period
    - Historical simulation, monte carlo simulation
    - Accurate and expensive
    - Good for option-like exposures, a wider range of risks, a longer-term
- Delta-Normal valuation method
  - V_0 = V(s_0)
  - dV = delta_0 dS
  - VaR = |delta_0| (z sigma S_0) = |delta_0| VaR_s
  - VaR = modified duration * z * annualized yield volatility * portfolio value
    - Good for short period
  - convexity: delta-gamma method
  - fail? Underestimate
    - Skewed return distributions (options)
    - Leptokurtosis (fat tails)
  - Props
    - Easy to implement, quick
  - Cons

        - Need a normal distribution
        - Fat tails
        - Non-linear
- Historical simulation
    - Historical data
    - Props
        - Easy to use historical data
        - Simple and quick
        - No model risk
    - Cons
        - Not enough data
        - Only one path is used
        - Cannot represent the future
        - Slow to adapt to new volatilities EWMA
        - Small number lead to tails
- monte Carlo simulation
    - more distributions samples
    - more portfolio follow normal distribution -> u and sigma
    - then it is similar to delta-normal method
    - props
        - more powerful, both linear and nonlinear risks
        - can include time variation in risk and correlations
        - flexible
    - cons
        - computational expensive
        - model risk chosen
        - sampling

Quantifying Volatility in VaR Models
- deviations
    - fat-tailed (kurto), skewed, unstable
- fat tail
    - reasons: **volatility** of **unconditional dis**, mean change over time
    - unconditional distribution
    - conditional distribution (change over time)
    - second reason
        - second moment or volatility is time=varying
- market regimes and conditional distributions
    - regime-switching volatility model
    - capture conditional normally and may resolve fat-tailed problem
    - different market exists with high or low volatility
    - constant mean, but either high or low volatility
    - stress testing, scenario analysis
    - extreme value theory (EVT)
- value at risk
    - historical approach or implied-volatility-based
- historical-based approach

    - parametric approach
        - distribution: normal or lognormal
        - variance: a window of K most recent
        - example: delta-normal
        - if random walk, mean=0
    - non-parametric
        - no assumption about distribution
        - use historical simulation method
    - hybrid approach
        - use both to estimate volatility using historical data
- implied-volatility-based
    - use pricing model such as BSM to estimate
    - based on current market data instead of historical data
- Parametric approach for VaR
    - RiskMetrics EWMA (exponentially weighted moving average)
    - GARCH exponentially smoothing
    - Parametric, estimate conditionally volatility, use historical data
    - Historically std assume K returns with equal weight
        - Weight=1/K
    - Exponentially: Decay factor lambda, 0.9 – 1, lambda^k
    - RiskMetrics
        - Sigma^2 = (1-lambda) (lambda^0 r_(t-1,t) + lambda^i r_(t-i-1,t-i))
        - Weight = (1-lambda) lambda ^t, t=0,1,2…
    - Shorter window
        - More volatile, can adapt to change
    - Smaller lambda
        - Higher weight to recent observations and a smaller window
    - GRACH(p,q)
        - Sigma^2 = a + b_1 r^2(t-1,t) + b_2 r^2(t-2,t-1) + .. bp
            - + c_1 sigma_(t-1)^2 + … c_q sigma_(t-q)^2
        - p lagged terms on historical returns squared
        - and q lagged terms on historical volatility
        - sigma_t^2 = a + b r_(t-1,t)^2 + c sigma_(t-1)^2
- Nonparametric vs parametric approach
    - Props
        - No assumption about distribution
        - Fat tails, skewness and other derivations are no longer a concern
        - MDE allows for weights to vary based on relevance to current market data
        - MDE introducing dependency on economic variables
        - Hybrid approach no assumption, use historical simulation and exponential weighting scheme
    - Cons
        - **Data** is used more efficiently in parametric
        - Need a large sample sizes of data
        - Separating full sample data into different market regimes reduce the amount of data

        - MDE lead to snooping or over-fitting
        - MDE a large number of data w.r.t. #conditioning variables used
- Nonparametric approach
    - Historical simulation
        - Six lowest returns for window of 100 days
    - Hybrid approach
        - Assign weight 1, r, r^2, r^(k-1)
            - sum = 1+ r + … + r^(k-1) = (1-r^k)/(1-r)
        - order the returns
        - accumulating weights until x percentages is reached
    - Multivariate density estimation (MDE)
        - Sigma^2 = \sum_{i=1^K} w(x_t-1) r_(t-1)^2
- Return aggregation
    - Portfolio.
    - Parametric: Delta-normal
        - Estimate N variances and n(n-1)/2 covariance for N positions
    - Historical simulation
        - Estimate return
        - Weight return by relative size of each position. Current
    - Third
        - Estimate volatility of vector aggregated returns
        - Normality based on strong law of large number
- Implied volatility
    - Estimate future volatility using historical data
        - Time to adjust to changes in the market
    - BSM model,
    - Props
        - Predictive nature, react immediately to changing
    - Cons
        - Assume return a continuous time lognormal diffusion process
        - Volatility is constant
        - Implied volatility is > realized volatility
        - Available data is limited
- Mean reversion
    - X_i = a + b X_i-1 (auto regressive model)
    - Long-run mean  = a / (1-b)
    - B=1
        - Infinite
        - Two periods: 2s^2
    - B<1
        - Mean reversion
        - Two periods: S^2, (1+b^2)s^2 < 2s^2
- Backtesting VaR
    - Unbiased
        - Random events, exception 1 or 0
    - Adaptable
        - Return increase its sizes, the VaR should be adjusted

- Robust
  - Deviation | real – expected| small

Putting VaR to Work

- Linear and non-linear
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Measures of Financial Risk

- Mean-variance framework
  - Efficient frontier
  - Risk-free security, market portfolio
  - Limitation: elliptical (i.e., normal) distribution
- VaR
  - Maximum loss at a confidence level over a time period
  - U – z sigma
  - Elements
    - Return distribution:
    - Confidence level:  increase
    - Holding period:
      - u=0, sqrt(n)
      - u>0, lower rate
  - Model risk, implementation risk
  - Major limitation
    - No amount or magnitude of the actual loss
  - Work well with elliptical distribution
  - Cannot satisfy subadditivity when distribution is non elliptical
- Coherent Risk measures
  - monotonicity: higher return less risk
  - **subadditivity**: r(R1+R2) <= r(R1) + z(R2)
  - positive homogeneity: r(bR) = b r(R)
  - translation invariance r(c+R) = r(R) -c
- Expected shortfall/expected tail loss (ETL)/conditional VaR
  - Expected loss when return lie below the worst case quantile
  - Convex: adjust holding period and confidence level
  - Better
    - Satisfy all properties of risk measurements, VaR only when normal
    - Portfolio risk surface is convex
    - Magnitude of a loss
    - less assumptions about the risk/return decision rules
- spectral risk
  - weighted average of return quantiles from loss distribution
  - ES: 1/(1-confidence) for tail loss, rest=0. Risk neutral
  - VaR: 1 for p=confidence, rest=0. Risk seekers
- Scenario analysis
  - Assign probability to loss outcomes, compute ES for each
  - Highest from m Is still coherent

Binomial Trees

- One-step binomial model
    - Step
        - Bankruptcy-free portfolio: stock-pus-borrowing
            - Borrow S_D/(1+r)^t, buy stock S
                - 到期还 S_D。
            - **Net cash outlay = S – S_D/(1+r)^t 最少付出**
            - Net stock value: S _U – S or 0 盈利表
        - Replicating the future return option
            - Bankruptcy-free portfolio = x * options
        - 等价 Align the dollar cost of the option and the portfolio
            - **Net cash outlay / x**
        - Value the option
            - **Net cash outlay / x**
- Perfect hedge
    - Hedge ratio/delta – delta hedging
        - #asset units eliminate price volatility of one **call** option
    - delta: # stocks to hold per call option to be shorted
        - delta=(C_U – C_D) / (S_U – S_D)
    - Synthetic call replication
        - Call price = delta * (stock price – PV(borrowing))
        - Borrowing = S_D
- Risk-neutral valuation
    - Stock price
        - U = size of up-move factor  = exp(s * sqrt(t) )
            - S: annualized volatility
        - D = size of down-move factor = 1/U
        - S_U = S*U, S_D = S*D
    - Option value
        - C_U, C_D
    - Probability
        - P_U = probability of an up move = (exp(rt) - D) / (U-D)
        - P-D = 1-P_U
    - Expected value of the option
        - C_U * P_U + C_D * P_D
    - PV
- Two-step binomial model
    - P^2, S U^2
    - 2P(1-P), S U D
    - (1-P)^2, S D^2
- Modifying the binomial model
    - Dividend yield q
        - P_U = (exp( (r-q)t) – D / (U_D)
    - Currency
        - P_U = (exp( (r_D – R_F)t) – D / (U_D)

    - Future
        - P_U = ( 1 – D / (U_D)
- American Options
    - Whether early  exercise if optimal at the end of the first step
    - 正向推导 stock tree
    - 反向推导 option tree
        - 第一次结束后，和第二次的分别对比，是否需要提前执行。
        - 最后结束后，和根节点对比，是否需要提前执行。
- increasing the number of time periods
    - time small, -> continuous -> converge BSM

The Black-Scholes-Merton Model

- stock price: lognormal distributed
- ln S_T = N(u,s)
    - u = ln S_0 + (u-s^2/2)T
    - s' = s * sqrt(T)
- stock price: lognormal, stock return: normal
- continuous return distribution
    - S_T = S_0 exp(uT) => u = (lnS_T-ln S_0)/T = N(u1,s1)
    - u – s^2/2
    - s/sqrt(T)
- Expected Value
    - E(S_T) = S_0 exp(u T), u is the expected rate of return
- Expected return u (arithmetic), mean return u – s^2/2 (geometric)
- Realized return: chain-like return (geometric mean)
    - ((1+r1)(1+r2)…(1+r_n))^(1/n) - 1
- estimating historical volatility
    - Price S_1, S_2,
    - Return lg S_2/ S_1, …
- BSM
    - No-arbitrage, portfolio yield risk-free rate
    - Price lognormal
    - Risk-free rate is constant and known
    - Volatility is constant and known
    - Market are frictionless (no tax, no txn cost, )
    - Asset has no cash flow
    - European options
- BSM
    - c = S* N(d_1) – X exp(-rT) * N(d_2) 累计概率分布
    - p = X exp(-rt) (1-N(d_2)) – S(1-N(d_1))
        - p = X exp(-rt) N(-d_2) – S N(-d_1)
    - d1 = (ln(S/X) + (r +s^2/2)T) / (s sqrt(T))
    - d2 = d1 – (s * sqrt(T))
- BSM with dividend
    - S_0 -> S_0 exp(-qT) 连续利率
    - S_0 -> S_0 – PV(dividend) 折旧法

        - Decrease call value, increase put value
- American Options
    - No dividend: no difference between EUR and USD call options
        - S – X exp(-rT) > S – X
    - Assume the last dividend date before expiration t, exercised value
        - S(t) – X
    - Unexercised
        - S(t) – D(t) – X exp(-r (T-t))
    - Exercise when, 以时刻 t 来看，折现到 t 时刻
        - S(t) – X > S(t) – D(t) – X exp(-r (T-t))
        - => D > X (1 – exp(-r (T-t)))
        - Black's approximation, use the larger of the two
            - Price on t, price on T
- Valuation of warrants
    - Warrants, bond issue, holder can purchase a security at a stated price
    - Buy warrant from firm, increase outstanding
    - 原则
        - n \detal_p = m c' 价格下降的市值 = 新增的 warrant 市值
        - \dela_p = \delta_c 价格下降的幅度= warrant 下降的幅度
        - n (c-c')=mc' => c'= n/(n+m)c
    - Value of warrant
        - C'= N / (N+M)* C (value of regular call option)，m 的单调递减函数
        - N: # shares outstanding, M: #new warrants
    - 价格的下降 = 新发的 warrant
        - n delta_p = m * c' = m n /(m+n)c
        - => delta_p = m / (m+n) c = c – c'
- volatility estimation
    - historical volatility
        - n price to returns
            - R_i = (P_i – P_(i-1))/P_(i-1),I = 1 to NH
        - Return to continuously compound return
            - r_i = ln(1+R_i) = ln (P_i / P_(i-1))
        - s^2 = \sum (r_i – avg(r)) / (n-1)
    - implied volatility
        -

The Greek Letters

- naked position
    - sell a call option without owning the asset
- covered position
    - sell a call option owning the asset
- stop-loss strategy (cover a naked position)
    - purchase stock when price > strike price
    - sold as soon as it goes below the strike price
    - hold a naked position when option is out-of-money
    - hold a covered position when the option is in-the-money

    - draw back
        - the transaction cost and price uncertainty
- Delta hedging
    - Delta, the price change of a call option c to the change of asset s
        - Delta = d c / d s
    - Call, delta > 0, increase as s increase
    - Put, delta < 0, decrease as s increase
    - Options Delta
        - Hedge a long stock or short call, buy delta * # options sold
        - Delta-neutral, delta=N(d1), put delta=N(-d1) =1 – N(d1)
        - Pay dividend q, call delta = exp(-q T) N(d1), put delta=exp(-qT)N(-d1)
    - Forward Delta
        - Delta=1
    - Futures Delta F=S exp(rT)
        - Delta = exp(rT) or delta=exp ((r-q) T) with q dividend
- Dynamic hedge: cost high
    - Stock, sell call #option =#stock / delta
- Static hedge: hedge-and-forget
- Other portfolio hedging
    - Buy put option 1/(#call delta - 1)
    - Sell 1/#call delta call options
    - C – P = S \delta => delta call – delta put =1
- Delta of a portfolio
    - Delta = \sum w_i delta_i
- Theta 时间
    - Theta  = dc / dt, time decay
    - Function of time and price
    - Stock price: - 正态分布
    - Time: - 指数分布
- Gamma 股票价格 2 次
    - Gamma = d^2 c / d S^2
        - = f(d1) / (S_0 s sqrt(T)),
        - N(x) = 1/(2 sqrt(pi)) exp(-x^2/2)
    - Largest at the money
    - 像正态分布
    - gamma-neurtal，用非线性的 option，不能用线性的 asset, forward
    - – Gamma_p / Gamma_T
        - gamma_p of a existing portfolio,
        - gamma_T of a traded option that can be added
        - 买了 trade option 后，改变了 delta，因此需要卖以前的资产
- relation
    - r P = theta + r S delta + 0.5 s^2 S^2 gamma
    - dollar risk-free return: rP
- Vega 波动
    - Vega = dc / ds, price change in the volatility
        - Call European, vega = S_0 f(d1) sqrt(T)

- 和 gamma 很像
- RHO
  - Rho = d c / dr
    - EUR call, rho = X Texp(-rT) N(d2)
  - 影响最小。但是对于 fixed-income 很大
- hedging in practice
  - delta-neutral position, and monitor exposure to other Greeks
- portfolio insurance
  - an underlying instrument and either cash/derivative that generates value.
  - Buy put options for portfolio
    - A synthetic put position: sell index future contracts
    - Lower trading cost, high liquidity

**Prices, Discount Factors, and Arbitrage**

- Steps
  - Estimate the cash flow
  - Determine the discount rate
    - Treasury: A single rate or a series of rate
    - Non-treasury: risk premium
  - Compute the PV
- Single yield
  - PMT: Annuity (principal * coupon)
  - Future value: principal
- Price-yield curve
  - Option-free bond, convex, half
- Bond price quotations
  - Percentage relative to par value (100)
  - Treasury notes and bonds: 32nds
    - 97-6, 97:06, 97.6 –> 97+6/32=
  - corporate and municipal bonds: 8ths
    - 102-1 -> 102+1/8
  - + = half
    - 101-12+ => 101+12.5/32
- **discount factors 折现因子，零息 zero-coupon**
  - bootstrap，从到期快的推算, d(t1), d(t2)
  - 和 spot rate 类似 discount factor = exp(-s t)
- law of one price
- Strip
  - Zero-coupon bond = Strips (separate trading of registered interest and principal securities)
  - Coupon = P-Strip + c-strip
  - C-strip can be put with any bond
  - P-strip are identified with specific bonds
  - Props
    - Create any type of cash flow and match cash flows with liability cash flows. Mitigates reinvestment risk

        - Sensitive to interest change
    - Cons
        - Illiquid
        - Shorter c-strip trade rich, longer c-strip trade cheap
        - P-strip trade at fair value
        - Large institutions can profit from strip mispricing
- Replicating portfolio
    - 现金流线性组合，按照 expiry date，倒推法
- compute price between coupon dates
    - 前面假设 bond valuation 发生在 coupon date
    - accrued interest, fractional period compounding, day-count convention
- accrued interest (AI)
    - AI = c (#days since last coupon to settlement date) / # days in coupon period
- Day-count convention
    - Government bond
        - Actual / Actual
    - Corporate and municipal bond
        - 30 / 360
    - government agencies annually, semi-annual, quarterly
        - 30 / 360
- Dirty price/full price/invoice price
    - w, 1+w, 2+w, …, n-1+w
        - w: #days until the next payment/ # days in coupon period
    - Dirty price P = C/(1+y)^w + C/(1+y)^(1+w) + … + C/(1+y)^(n-1+w), M(1+y)^(n-1+w)
    - 这种思路好理解
        - 折价到上次付息日期 payment date，再折算到开始日期
        - P2 = C/(1+y) + … + C/(1+y)^n
        - P = P2 * (1+y)^(1-w)
- Clean price/flat price/quoted price
    - = dirty price – accrued interest C*(1-w)
    - =P2*(1+y)^(1-w) – c(1-w)
        - w=1, P2=P，上次付息之后，就是 clean price
        - w=0, P2(1+y)-c 刚好付一次息

Spot, Forward, and Par Rates

- Compounding
    - FV = PV(1+r/m)^(mn)
- Holding period return
    - r = m((FV/PV)^(1/mn)—1)
- Interest rate swap
    - No exchange of principal
    - If hypothetically exchange the notional amount, it is similar to bond
    - 交换本金的话，等价于 bond 的 return
- spot rate and discount rate
    - PV = FV d(t)

- PV (1+z(t)/m)^(mt) = FV
    - d(t) (1+z(t)/m)^(mt) = 1
    - z(t) = m(1(/d(t))^(1/mt) – 1)
- PV exp(z(t) * t) = FV
    - d(t) exp(z(t) * t) = 1

- forward rate
    - bootstrapping: spot rate, derive forward rate
    - compounding
        - (1+z2/m)^T = (1+z1/m)^t * (1+f/m)^(T-t)
- par rate
    - semi-annual
        - par rate/2 (d(0.5) + d(1) + d(1.5)+d(2)) + 100 d(2) = 100
    - par rate = spot rate, when bond is issued at its par value
    - C_T / 2 \sum_{i=1^T} d(t/2) + d(T) = 1
    - A_t =\sum_{i=1^T} d(t/2) annuity factor
- Discount factor, Spot, forward, par rates
    - A bond can be split into a series of strips
    - Coupon rate -> zero-coupon
    - FV = PV (1+z(t)/m)^(mn)
    - PV = FV d(t)
    - (1+z2/m)^T = (1+z1/m)^t * (1 + f(t,T-t) /m)^(T-t)
    - par rate
        - par rate/2 (d(0.5) + d(1) + d(1.5)+d(2)) + 100 d(2) = 100
        - coupon rate /2 (d(0.5) + d(1) + d(1.5)+d(2)) + 100 d(2) = bond price
        - bond price = 100 + (coupon - par)/2 (d(0.5) + d(1) + d(1.5)+d(2))
        - bond price = 100 + (coupon - par)/2 annuity factor
    - upward-sloping curve, spot rate > par rate
    - downward-sloping spot, par < spot rate
- effect of maturity on bond price and return
    - coupon rate vs. forward rate
    - coupon rate > forward rate
        - price increase
    - short-term rate > forward rate
        - short-term return > long-term return
- yield curve shapes
    - normal, flat, inverted
    - parallel shift
    - nonparallel shifts
        - twists
            - flatten: spread narrowed
                - upward flatten
            - steepening:  spread widen
                - downward steepening
            - if upward steepen, long-term rate increase, bond price drop
                - sell long-term and buy-short term
        - butterfly
            - positive butterfly: yield curve less curved

- negative butterfly: more curvature

Returns, Spreads, and Yields

- gross realized return
    - $r = (P2 + C - P1)/P1$, P1 bond initial price, C is coupon
- net realized return (-finance cost)
    - r – borrow rate /t
- reinvestment risk
    - if interest rate go down across the board, the reinvestment rate will be lower
    - $P2 + C1 + C2*(1 + r_{invest})$
- bond spread
    - market price – calculated price
    - $P = C / (1 + f(1)) + (C+P) / (1+f(1)) (1+f(2))$
    - $market = C / (1 + f(1) + s) + (C+P) / (1+f(1) + s) (1+f(2) + s)$
    - spread
- yield to maturity 复利回报率
    - yield to maturity, = internal rate of return (for fixed-income security)
    - $P = C1/(1+y) + C2 / (1+y)^2 + … + Cn / (1+y)^n$
    - $P(1+y)^n = C1 (1+y)^{(n-1)} + … \_+ Cn$ 等价于中途的现金流又被重新投资了
    - 等于 realized return， 中途金流重新被投资
- bond equivalent yield (BEY) – semiannual YTM or semiannual-pay YTM
- make annual payment: annual-pay yield to maturity
- traditional yield measures
    - reinvestment risk: coupon, repayment of principal
    - long-term bond with a larger coupons
- annuity
    - N, PMT, YTM => PV
- Perpetuity
    - PV = PMT / YTM (y)
- Spot rates and YTM
    - $PV = C/(1+z1) + C/(1+z2)^2 + … + (C+P) / (1+zn)^n$ (discount)
    - $PV = C/(1+y) + C/(1+y)^2 + … + (C+P) / (1+y)^n$ (discount)
- Ytm, coupon rate, price
    - Issue at par: market rate = par rate
    - Market rate drop coupon rate > ytm
        - Coupon higher, price increase, premium bond
    - Market rate increase coupon rate < ytm
        - Coupon lower, price drop, discount bond
    - Pull to par
- Coupon effect
    - Smaller coupon, more sensitive to interest rate change
    - Lower coupon rate, greater interest-rate risk
- Return decomposition
    - Overall
        - $P(R_t, S_t) – P(R_{(t-1)},S_{(t-1)})$
    - Carry-and-roll (original to expected)

        - P(R_t’, S_(t-1)) – P(R_(t-1),S_(t-1))
    - Rate change
        - P(R_t, S_(t-1)) – P(R_t’,S_(t-1))
    - Spread change (trade cheap or high)
        - P(R_t, S_t) – P(R_t,S_(t-1))
- Call-roll-down scenarios
    - Realized forward rate vs. implied forward rate
    - Unchanged term structure
    - Unchanged yield

One-Factor Risk Metrics and Hedges

- 所有的不同时期的利率都一起变化
- DV01, duration, convexity
- Dollar value of a basis point DV01 (1 basis point, 0.0001)
    - PVBP (PV of one basis point)
    - DV01 = - delta P / delta y /10000 相对值
- Hedge ratio
    - HR = DV01 (position) / DV01 (hedging instrument)
    - Face value * DV01(bond) = position * DV01 (option)
        - Face value = position * HR
- Duration 百分比 – investing 反向定义
    - Volatility: coupon, maturity, initial yield -> duration
    - Macaulay duration (option-free bond)
        - In years
    - Modified duration
        - Modified = macaulay / (1+market yield)
        - Modified = delta P / delta y / P
    - Effective
        - Effective  = (P(-delta y) – P(+delta y)) / 2 delta y / P_0
- DV01 vs duration
    - DV01: dollar value of every 0.01% change in rates
    - Duration: **percentage** change of 1 unit in rates
    - - DV01 / 0.0001 = bond value * duration
    - DV01 = - duration * 0.0001 * bond value
- Convexity
    - Convexity = 1/P D^2 P / Dy^2
    - Convexity = 1/P * (P(-dy) + P(dy) – 2)/ dy^2
- Percentage change
    - Percentage change = duration + convexity
    - Percentage change = (- duration * dy + 0.5 convexity * dy^2)*100%
- Portfolio duration
    - Duration portfolio = \sum_i w_i D_i
        - By market value
    - Assumption: they are perfectly correlated
- Negative convexity
    - Callable bond, (left side)
- Barbell portfolio

- Babell portfolio: use short and long bonds - volatily
- Bullet strategy: use intermediate maturity range

Multi-Factor Risk Metrics and Hedges

- Yield-curve risk
- Key rate exposure
  - Key rate exposure: most liquid
  - Partial’01
    - **Swap** portfolio: bond, swaps
  - Forward-bucket’01
    - **Swaps and predefined regions**
    - Change in the shape of the yield curve
- Key rate shift
  - 2,5,10,30
  - if one of these rates shifts by one basis point, -> key rate shift
  - 1,7,20
  - 1 year -> [0,7]
  - 7 year -> [1,20]
  - 20 year –> [7,]
  - a combination of rates that’s close to them
- hedging
  - rates of a given term is affected by key rates surround it.
    - Rates are not perfectly linear
- Partial and forward-bucket ‘01s
  - Swaps, use partial and forward-bucket
  - Fit a par rate curve daily or more frequently.
  - Measure the change in One basis point decrease, and refitting the curve
- Forward-bucket
  - Divide into bucket, shift one region at a time, each by 0.01
- Estimating portfolio volatility
  - Can include volatility impacts of each key rate
  - But also the correlation between each key rate
  - Bucket: estimate forward rate

Country Risk: Determinants, Measures and Implications

- Country risk 政治、法律、经济
  - Financial market: RETF, ADR, GDR
  - Company are global
- Risk
  - Economic, political (structure, efficiency), legal, disproportionate reliance of commodity or service
- Economic growth life cycle 经济周期
  - Early growth and emerging market
- Political risk
  - Continuous vs discontinuous 政治结构
    - Democracy: risk continuous but low
    - Autocratic: risk discontinuous but sever

    - Corruption 腐败
    - Physical violence 暴力
        - Economic cost (insurance, security cost)
        - Physical cost (physical harm)
    - Nationalization and expropriation risk
- Legal 法律结构和效率
    - Property rights (structure) and the speed with which disputes are settled
- Economic Structure 经济结构
    - Reliance on a single commodity or service
- Evaluating country risk
    - Political Risk Services (PRS)
    - Euromoney
        - Survey 4000 economists
    - The economist: currency, sovereign debt risk, banking risk
    - The world bank
- Sovereign defaults
    - Government-issued debt
    - Foreign currency default
        - Countries more likely to default on funds borrowed from banks than on sovereign bond issues
        - Latin America accounts
    - Local currency
        - Can print money. But why?
        - Gold standard, 1971, follow gold standard.
            - Gold reserve to back currency
        - Shared currency
            - Euro,
        - Currency debasement
    - Consequences
        - Military actions
        - GDP growth falls between 0.5% to 2%, the decline is short-lived
        - Sovereign Rating and borrowing cost
            - Ratings defaulted one or two grades lower
            - Borrowing cost 0.5% to 1% higher
        - Trade retaliation, up to 15 years
        - Fragile banking system
        - Political change
- FACTORS
    - Level of indebtedness
    - Pension funds and social services
        - Higher, old population-> more defaults
    - Tax receipts
        - High tax -> low default
    - Stability of tax receipts
        - Diversified -> stable tax
    - Political risk
    - Backing from other countries/entities

- Rating Agencies and default risk
    - Moody and S&P
    - Local currency >= foreign currency
- How rating agencies measure risk
    - Faced by banks and private bondholders
    - S&P
        - Probability of default
    - Moody's
        - Probability of default and the severity of the default
    - Default
        - Outright default：
        - Rescheduling or restructuring the debt
    - Process
        - Factors that contribute to default
            - Political risk, economic structure, economic growth
            - Fiscal flexibility, debt burden
            - Offshore and contingent liabilities
        - Ratings recommendation
            - 5-10 people, debates each
        - foreign currency vs local currency ratings
            - notch-up approach
                - foreign currency is the key
            - notch-down approach
                - local currency is the key,  foreign is down
    - review
        - periodic basis
        - news can trigger a ratings review
- How Ratings measure risk？
    - Ratings are biased upward – too optimistic
    - Herd behaviour
    - Not timely enough
    - Overreaction leads to a vicious cycle – lower too much
    - Rating failures
        - Bad information
        - Overburdened analyst
        - Conflicts of interests
- Sovereign Default spread
    - Same currency and maturity
    - Compared to us
- spread vs rating
    - props
        - change occur in real time
        - granularity
        - adjust quickly to new information
    - cons
        - need a risk-free security
        - cannot compare local currency bonds

        - greater volatility
            - unrelated to default risk of sovereign
- Latin America is responsible for the greatest number of foreign currency defaults over the last five decades with more than 60% of defaults in each decade with the exception of the 1990s.

External and Internal Ratings

- External rating
    - Specific instrument: Issue-specific credit rating
    - Entity that issued the instrument: issuer credit rating
- Rating scale – Moody's rating
    - Aaa, Aa, A, Baa, Ba, B, Caa, Ca, C
    - Each move increase in expected loss caused by a default
    - bonds
        - Investment grade: >=Baa
        - Non-investment grade (junk bond): <Ba
- Rating scale - S&P
    - AAA, AA, A, BBB, BB, B, CCC, CC, C, D
    - Investment grade: >=BBB
    - D is the default rating
- Rating Process
    - Qualitative analysis
    - Quantitative analysis (financial ratio analysis)
    - Meeting with firm's management
    - Meeting of the committee in the rating agency assigned to rating the firm
    - Notifying the rated firm of the assigned rating
    - Opportunity for the firm to appeal or offer new information
    - Disseminating the rating to the public via the news media
- Rating Review
    - After the initial rating, ratings agency monitors the firm and adjusts the ratin as needed
- Rating transition matrix
    - Row: Rating from,
    - Column: rating to (The last column: **default**)
    - Value: percentage
- Impact of time horizon, economic cycle, industry, geography
    - Increase with horizon
    - Economic and industrial cycles
        - Average cycle
        - Over-or underestimating when deviate too far
        - default rate of lower-grade bonds is correlated with cycle
        - high-grade bond sis fairly stable
    - vary from industry
    - geographic location does not seem to cause a similar variation of default for
- impact of rating change on bond and stock prices
    - bond
        - rating downgrade -> bond price decrease (stronger evidence)

        - rating upgrade -> increase (weaker evidence)
    - stock price
        - rating downgrade -> price decrease (moderate)
        - upgrade -> increase (mixed)
- internal credit rating
    - at-the-point
        - short time
        - quantitative
        - procyclical - amplify the business cycle
            - economic down –> downgrade -> decrease in loan
    - through-the-cycle
        - long time
        - qualitative
    - lag economic cycle
- process
    - weights to financial ratios and risk factors
    - purpose
        - determine the credit risk of a loan
        - manage loan portfolio
    - should reflect information from cumulative default probability tables
- backtest – link between rating and default rates
    - 11-18 years data, validate these ratings
- biases
    - time horizon bias: mix rating from different approaches
    - homogeneity bias: inability to maintain consistent rating
    - principallagent bias: moral hazard
    - information: insufficient information
    - criteria: unstable criteria to set rating
    - scale: rating unstable over time
    - backtesting: incorrectly link
    - distribution: incorrect distribution to model PD

Capital Structure in Banks
- probability of default (PD)
    - EDF
- Exposure amount (EA)/ Exposure At default (EAD)
    - Loan balance outstanding/dollar amount
    - Can be percentage of the nominal amount/maximum amount
- Loss rate(LR)/loss given default (LGD)
    - Percentage loss if borrow defaults
    - LGD = 10 recover rate (RR)
- Expected loss
    - EL = PD * LGD * LR
    - Fail to capture the variation of the asset value
        - Incidence of default
        - Credit migration
            - deterioration

- Unexpected loss
    - Variation in expected loss
    - UL = sqrt(Var)
    - UL = EAD * sqrt(PD*sigma_LR^2 + LR^2  sig_PD^2)
        - Sig_PD^2 = PD *(1-PD) binomial
- Portfolio
    - EL_p = sum EL_i
    - UL_p^2 = \sum \sum_j r(I,j) UL_i UL_j
- Risk contribution RC/ULC
    - RC_i = UL_i d UL_p/ d UL_i
        - = UL_i/UL_p \sum UL_j p(I,j)
- effect of correlation
    - concentration risk
- Economic capital
    - 经济资本是描述在一定的置信度水平上（如 99%），一定时间内（如一年），为了弥补银行的非预计损失（unexpected losses）所需要的资本
    - Hold capital reserve to buffer unexpected loss
    - Economic capital = excess capital
    - Capital multiplier (CM)
    - = UL * CM
- modelling credit risk
    - beta distribution
        - mean: EL, variance: UL
        - shape, a, and b （symmetric or skewed）
- challenge
    - credits are presumed to be illiquid assets
        - with bottom-up, risk contribution, not influenced by the correlation
    - one-year horizon. Should use more years
    - other risks are separated

Operational Risk

- define
    - not credit or market
    - direct and indirect loss resulting from inadequate or failed internal processes, people, and system or from external events.
    - Includes legal risk, not reputational risk or strategic ris
    - 不包含 credit and market risks
- 7 种风险，8 个业务线
- Risk capital
    - Basic indicator approach 小银行 gross income
    - Standardized approach 8 个业务线
    - Advanced measurement 大银行，减少 capital
- Basic indicator
    - Capital = 15% * annual gross income over a 3-year period
- Standard
    - Eight business lines with different beta factors

- Have
    - Operational risk mgt
    - Document losses for each line
    - Report loss on a regular basis
    - Have a system
    - Conduct independent audits with both internal and external auditors
- Advanced
  - Approximate unexpected losses
  - Use operational VaR
  - 99.9% percentile in 1-year time
- Operational Risk Categories
  - Clients, products, business
    - Failure to perform obligations. Confidential information, money laundering
  - Internal fraud: insider trading,
  - External fraud: robbery, computer hacking
  - Damage to physical assets: natural disasters, fires, earthquakes
  - Execution, delivery, and process management
  - Business disruption and system failures
    - Computer failures, hardware/software
  - Employment practices and workplace safety
    - Not follow laws related to employment or health and safety
- Loss frequency and loss severity
  - Loss frequency: #losses over a period/one year
    - **Poisson distribution**
      - Short period, #=lambda t
      - P(n) = exp(-lamba T) (lambda T)^n / n!
  - Loss severity: value of loss suffered / size of loss
    - **Lognormal distribution**
      - Asymmetric, fat-tailed
  - Covolution
    - Monte Carlo simulation
- Data limitation
  - Insufficient
  - Frequency: internal data
  - Severity: internal and external
    - Sharing agreements with other banks
    - Public data
      - Large losses from weak controls
  - Should adjust for inflation
    - Loss Y = loss Z (revenue Y/ revenue Z)^0.23
- Scenario analysis
  - Incorporate events that have not yet occurred
  - Drawback: time
- Forward-looking approaches
  - Risk Control Self-Assessment (RCSA)
  - Key Risk Indicators (KRI)

    - Prepare for future losses
        - Learn from mistakes of other companies
    - Causal relationships
        - Correlation between firm actions and operational risk losses
        - RCSA and KRI
- Scorecard data
    - Allocate risk capital
    - Scorecard approach
        - Ask managers questions, answered, scored
    - Questions
        - Ratio of supervisors to staff
        - Employee turnover rate
        - #open positions
        - presence of confidential information
- Power law
    - Extreme value theory (EVT)
    - P(V > X) = K X^(-alpha)
- Insurance
    - Use AMA can use insurance to reduce capital charge
    - Moral hazard
        - Deductible, policy limits, coinsurance
        - Do not reveal insurance to traders
    - Adverse selection
        - Bad banks with poor internal controls more likely to desire
        -

Governance Over Stress Testing

- Key elements of effective governance and controls
    - Governance structure
    - Policy and procedures
    - Documentation
    - Validation and independent review
    - Internal audit
- Responsibilities of BOD and SM
    - Separation of duties between BOD and SM
- BOD
    - Oversight for key strategies and decisions
    - Critical of stress tests by actively challenging assumptions
    - Sufficiently knowledgeable about stress testing
    - Stress testing results
        - Risk appetite and risk profile
        - Capital and liquidity adequacy and capital funding plans
        - Early warning sign
        - Forward-looking assessments
- SM
    - Implementation of stress testing
    - Not a single stress test, but a series of stress tests

    - Results aggregated, remedial actions, documented
    - Activities, assumptions, limitations
- Policies, procedures and documentation
    - Clear and comprehensive
    - Policies and procedures, clear, concise, reviewed and approved annually
    - Documentation
        - Desc, results, assumptions, limitations, constraints
        - Track, and analyse results
- Validation and independent review
    - Nonstress periods in models to test predictive power
    - Model validation
        - Expert-based judgment, sensitivity analysis, simulation tech
- Role of internal audits
    - Integrity and reliability
- Key aspects of ST
    - Coverage
        - Portfolios, liabilities, exposures
        - Detect risk concentrations
        - Short and long term
    - Types and approaches
        - Scenario analysis
        - Firm-specific and system-wide
    - Capital and liquidity ST
        - Earnings, losses, cash flows, capital and liquidity

Stress Testing and Other Risk Management Tools

- ST vs VAR/EC
    - ST define losses from an **accounting** perspective
    - EC take a **market** view of losses
    - ST look **longer** time horizon, VAR point-in-time losses only
    - ST not probability, but **ordinal** rankings, VAR cardinal probabilities and MC
    - ST **conditional** scenarios, VAR unconditional
- Using VAR in ST
    - EL = PD * LGD * EAD
        - **Merton** model
        - A factor model
- Stressed inputs and VAR
    - Stressed inputs: **market** risk
        - 10-day, 99%, one-tailed confidence
    - stressed **parameters** -> counterparty credit risk
    - CVA （capital valuation adjustments）
- Pros
    - **Conservative**
- Cons
    - Respond to current market **conditions**

Principles for Sound Stress Testing Practices and Supervision

- Internal Models Approach (IMA) -> market risk
- Infernal ratings-based (IRB) -> credit risk
- **ST and risk governance**
- Weakness
  - Lack of involvement of BOD and SM
  - No overall organizational view
    - Separated in function or line
  - No fully developed ST
    - Market > credit > operational
    - Correlated exposures and risk concentrations
  - No adequate response to crisis
- Recommendations
  - BOD and SM actively and fully engaged in ST
  - Comprehensive ST program
    - Risk identification & mgt
    - Alternative risk perspective
    - Liquidity and capital mgt
    - Communication
  - Multiple perspective and tech
    - Sensitivity analysis
    - Scenario analysis
    - Risk simulation
  - Written policies and documentation
  - Sound infra
  - Regular assessment
- ST methodologies
  - Weakness
    - Inadequate infra
    - Inadequate risk assessment
    - Inadequate recognition of interaction effects
    - Inadequate firm-wide perspective
  - Recom
    - Comprehensive ST
      - Business areas and risk exposures
    - Risk concentration
      - Name, industry, region, correlated factors, off blance sheet
    - Multiple measures
- ST scenarios
  - Weakness
    - Lack of depth and breadth
    - Lack of tech
    - Lack of forward-looking
  - Recommendations
    - A variety of events
    - Futuristic outlook
      - New products/emerging risks
    - Synergy effect

        - Time horizon
        - Reverse ST
            - Outcome, events, hedging
- ST handling
    - Structured products: MBS, ASB, CDO
    - Market condition: prepayment risk
    - Contractual obligation
    - Basis risk
        - Future contract to hedge
        - Short hedge, sell future contract
        - Basis risk = S – F
    - Counterparty credit risk
        - Swaps, options, forward, insurance
        - Monoline issuer: default protection insurance
        - Wrong-way risk: PD increase as of market conditions
    - Pipeline risk
        - Securitization
        - Pipeline/warehouse risk
    - Contingent risk
        - Off-balance sheet exposure
        - Off-balance sheet commitments: liquidity, credit, market
    - Funding liquidity risk
        - Nature, size, duration, intensity
        - Interrelationship between liquidity and market risk
        - Correlation
    - Recommendation
        - assess stress testing methods
        - take corrective action
        - challenge firm-wide scenario
        - evaluate capital and liquidity needs
        - apply additional stress scenarios
        - consult additional resources

market values (par value x price as percent of par)

- par value 2m, mkt price: 93, market value: 93%*2

one basis point change