CDS Spread

核心思想

- 无套利原则
  - 等式：支出和流出**现金流**等价：可以是折现 PV 或者复利 FV。
  - **概率、时间、数量、折现**。
- 注意
  - 折现或者复利法结果不一定一样，主要是 RR 的定义。
  - 要判断 spread 是如何定义的。
  - 要注意 RR 是如何定义的。

一年折现法，教材

- 假定**未来**收到 1 块钱
- 考虑违约和回收
  - $(1-\mathrm{PD})\times 1+\mathrm{PD}\times \mathrm{RR}=(1-\mathrm{PD})\times 1+\mathrm{PD}\times \mathrm{RR}\times 1$
  - $=1-\mathrm{PD}+\mathrm{PD}\times \mathrm{RR}=1-\mathrm{PD}\times(1-\mathrm{RR})=1-\mathrm{PD}\times \mathrm{LGD}=1-\mathrm{EL}$
  - 注意：回收的 recover rate 是基于未来值(可能是本金加利息)的
  - 折现到现在是 $\mathrm{PV}=\frac{1-\mathrm{EL}}{1+R_f}$
- 补偿风险，加上风险溢价，未来收到 1 块，现值是
  - $PV=\frac{1}{1+YTM}$
- 等式
  - $\frac{1}{1+YTM}=\frac{1-\mathrm{EL}}{1+R_f}\rightarrow 1+\mathrm{YTM}=\frac{1+\mathrm{R_f}}{1-EL}\rightarrow YTM=\frac{\mathrm{R_f}+EL}{1-EL}$
- Spread
  - $z=YTM-R_f=\frac{EL}{1+YTM}$
  - $z=YTM-R_f=\frac{\mathrm{R_f}+EL}{1-EL}=\frac{R_f\times EL+EL}{1-EL}$

一年复利法，考题

- 假定**现值**是 1 块
- 无风险时
  - 按照自然利率复利 $\mathrm{FV}=1+\mathrm{R_f}$
- 有风险时
  - 理论收入是 $1+\mathrm{YTM}$，考虑到违约后是
    - $\mathrm{FV}=(1-\mathrm{PD})\times(1+\mathrm{YTM})+\mathrm{PD}\times \mathrm{RR}$
      $=(1-\mathrm{PD})\times(1+\mathrm{YTM})+\mathrm{PD}\times \mathrm{RR}\times 1$
  - 注意：回收的 recover rate 是基于**现值（本金）**的
- 等式
  - $1+R_f=(1-\mathrm{PD})\times(1+\mathrm{YTM})+\mathrm{PD}\times \mathrm{RR}$
  - $\rightarrow 1+R_f=1-\mathrm{PD}+\mathrm{YTM}\times(1-\mathrm{PD})+\mathrm{PD}\times \mathrm{RR}$
  - $\rightarrow R_f=\mathrm{YTM}\times(1-\mathrm{PD})-\mathrm{PD}\times \mathrm{LGD}=\mathrm{YTM}\times(1-\mathrm{PD})-\mathrm{EL}$
  - $\rightarrow \mathbf{YTM}=\frac{R_f+\mathrm{EL}}{1-PD}$
- spread
  - $\mathrm{z}=\mathrm{YTM}-R_f=\frac{R_f\times \mathrm{PD}+\mathrm{EL}}{1-PD}$

CDS spread

- 2 种
- 第一是支付的 spread，钱
    - PD 增加，CDS 价值增加
    - 相关性增加，价值降低。
    - Higher PD, higher CDS, but more WWR.
    - The higher the correlation risk, the lower the CDS spread s.
    - 如果 r=1，价值为 0。
    - 相关性增加，价值不是单增的：有时增加、有时减少
    - 相关性从-1 到-0.4，价值增加 slightly
        -
- 第二是 bond 的 spread premium YTM-rf

一年复利法，考虑 CDS seller 的违约

- 假定**现值**是 1 块
- 假定
    - 按照自然利率复利 $\mathrm{FV} = 1 + \mathrm{R_f}$
    - $\mathrm{P_b}$ is the default probability of bond
    - $\mathrm{P_s}$ is the default probability of CDS seller
    - $\mathrm{P_{bs}}$ is the joint default probability of bond and CDS
    - ρ is the correlation between Bond and CDS seller
        - $\rho = \frac{\mathrm{P_{bs}} - P_b \times P_s}{\sigma_b \times \sigma_s} \rightarrow \mathrm{P_{bs}} = P_b \times P_s + \rho \times \sigma_b \times \sigma_s$
- 不考虑 CDS seller 违约，理论收入是 $1 + \mathrm{R_f} + z$
    - $\mathrm{FV} = (1 - P_b) \times (1 + \mathrm{R_f} + z) + \mathrm{P_b} \times \mathrm{RR}$
- 考虑 CDS seller 违约，理论收入是 $1 + \mathrm{R_f} + z'$
    - $\mathrm{FV} = (1 - P_b) \times (1 + \mathrm{R_f} + z') + (\mathrm{P_b} - \mathrm{P_{bs}}) \times \mathrm{RR} + \mathrm{P_{bs}} \times 0$
        - $\mathrm{P_{bs}}$ both bond and CDS seller default, get **zero**
        - $\mathrm{P_b} - \mathrm{P_{bs}}$is the probability that bond default but CDS seller does not default -> recovery RR
- 让上面 2 个公式相等
    - $(1 - P_b) \times (1 + \mathrm{R_f} + z) + \mathrm{P_b} \times \mathrm{RR} = (1 - P_b) \times (1 + \mathrm{R_f} + z') + (\mathrm{P_b} - \mathrm{P_{bs}}) \times \mathrm{RR}$
    - $\rightarrow (1 - P_b) \times (1 + \mathrm{R_f} + z) + \mathrm{P_{bs}} \times \mathrm{RR} = (1 - P_b) \times (1 + \mathrm{R_f} + z')$
    - $\rightarrow z' = z + \frac{\mathrm{P_{bs}} \times \mathrm{RR}}{1 - \mathrm{P_b}} = z + \frac{\mathrm{P_b} \times \mathrm{P_s} \times \mathrm{RR}}{1 - P_b} + \rho \times \frac{\sigma_b \times \sigma_s \times \mathrm{RR}}{1 - P_b}$

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The value of the CDS, i.e., the fixed CDS spread S, is mainly determined by the default probability of the

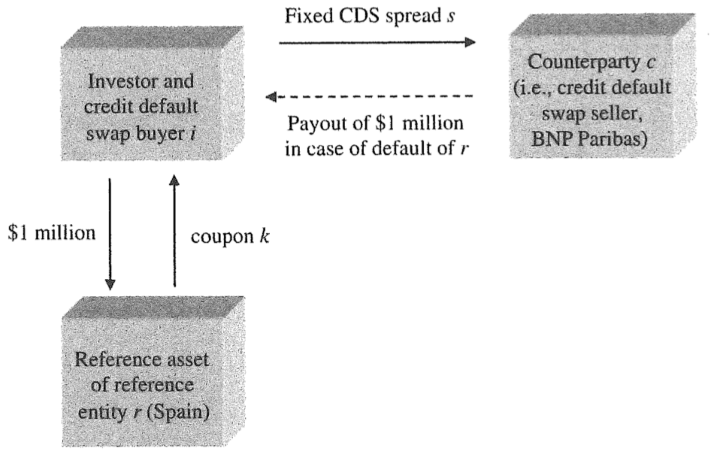

FIGURE 6-1 An investor hedging his Spanish bond exposure with a CDS.

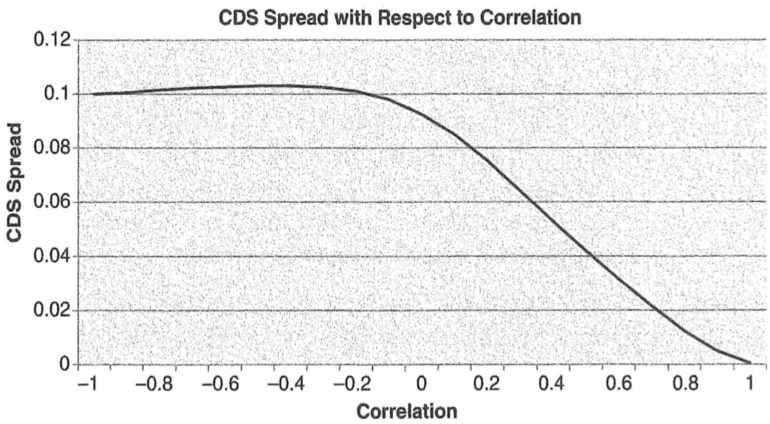

FIGURE 6-2 CDS spread s of a hedged bond purchase (as displayed in Figure 6-1) with respect to the default correlation between the reference entity r and the counterparty c.

	折现法	复利法
Recover Rate	基于本金加利息	基于本金
YTM	$\frac{R_f + EL}{1 - EL}$	$\frac{R_f + EL}{1 - PD}$
spread	$\frac{R_f \times EL + EL}{1 - EL}$	$\frac{R_f \times PD + EL}{1 - PD}$
复利法 FV 等式	$(1 - PD) \times (1 + YTM) + PD \times RR = 1 + R_f$	
折现法 PV 等式	$\frac{1}{1 + YTM} = \frac{(1 - PD) \times 1 + PD \times RR}{1 + R_f} = \frac{1 - EL}{1 + R_f}$	

Case 1: Yield Spread 收益 spread

- Define
  - Theoretical face value \$1
  - Number of years n
  - Risk free rate R_f
  - Yield-to-maturity YTM
  - **Spread: $\mathbf{spread = YTM - R_f}$**
  - Default only occurs at the end of a year
  - Loss given default is LGD, recover rate is RR
- Risk-premium Approach
  - $FV = 1$
  - $PV = \frac{1}{(1+YTM)^n}$
- Risk-free Approach
  - $\text{Payoff} = \begin{cases} 1 - LGD, & if\ default\ with\ probability\ PD \\ 1, & not\ efualt\ 1 - PD \end{cases}$
  - $FV = PD \times (1 - LGD) + 1 - PD = 1 - PD \times LGD$
  - $PV = \frac{1 - PD \times LGD}{(1+R_f)^n}$
- PV equal
  - $\frac{1}{(1+YTM)^n} = \frac{1 - PD \times LGD}{(1+R_f)^n} \rightarrow \frac{(1+R_f)^n}{(1+YTM)^n} = 1 - PD \times LGD$
- PV of one year
  - $\frac{1+R_f}{1+YTM} = 1 - PD \times lGD \rightarrow PD \times LGD = \frac{YTM - R_f}{1+YTM}$
  - $YTM = \frac{R_f + PD \times LGD}{1 - PD \times LGD}$
- **Spread of one year**
  - $PD \times LGD = \frac{spread}{1+YTM}$
- PV of one year when YTM is small
  - $spread = PD \times LGD$ （这是 notes 里提到的）
  - spread 就是用来补偿风险的

Case 2: Face Value Spread 基于 Face Value 的 Spread

- Assumption 假设
  - CDS lasts for one year 一年的 CDS
  - Default only once at time $0 < t \leq 1$ 只违约一次
  - Spread 定义
    - Spread s is the percentage of face value. Spread 是 FV 占比
  - Buyer pay spread 何时支付？支付多少？
    - Pay the full spread s at the end of a year 在年底支付全部 spread s
    - or pay accrued spread $\mathbf{s \times t}$ when default occurs 或者在违约时支付应计 spread $\mathbf{s \times t}$
  - Seller will pay RR when default happens
- Define
  - Risk free rate R_f
  - Loss given default is LGD, recover rate is RR
- Discount Factor 很多种方法，差别应该不大
  - Continuous 连续复利 $d_t = e^{-R_f \times t}$ （Practice 里用的是这个）

- Annual compounding $d_t = \frac{1}{(1+R_f)^t}$
- Simple compounding 单利 $d_t = \frac{1}{1+R_f \times t}$

- Cash Flow 现金流分析

Time (year)	**Probability 发生概率**	**Discount Factor**	**Spread Accrued %**	**Buyer FV 买方支付**	**Seller FV 卖方支付**
1	$1-p$	d_1	1	s	0
$0 < t \leq 1$	p	d_t	t	$s \times t$	RR
平均概率折现	$d_{buyer} = (1-p) \times d_1 + p \times t \times d_t$				$d_{seller} = p \times d_t$

- **PV Equal**
  - $(1-p) \times s \times d_1 + p \times s \times t \times d_t = p \times \mathrm{RR} \times d_t$
  - $s \times [(1-p) \times d_1 + p \times t \times d_t] = \mathrm{RR} \times p \times d_t$
  - $s = \mathrm{RR} \times d_{seller}/d_{ps}$
- 假设在年底才可能会违约$t = 1$
  - $s \times [(1-p) \times d_1 + p \times 1 \times d_1] = \mathrm{RR} \times p \times d_1$
  - $=> s = \mathrm{RR} \times p$

问题

A risk analyst is valuing a 1-year credit default swap (CDS) contract that will pay the buyer 80% of the face value of a bond issued by a corporation immediately after a default by the corporation. To purchase this CDS, the buyer will pay the CDS spread, which is a percentage of the face value, once at the end of the year. The analyst estimates that the risk-neutral default probability for the corporation is 7% per year. The risk-free rate is 2.5% per year. Assuming defaults can only occur halfway through the year and that the accrued premium is paid immediately after a default, what is the estimate for the CDS spread?

a. 560 basis points
b. 570 basis points
c. 580 basis points
d. 590 basis points

Correct answer: d

Explanation: The key to CDS valuation is to equate the present value (PV) of payments to the PV of expected payoff in the event of default. Let:

Explanation: The key to CDS valuation is to equate the present value (PV) of payments to the PV of expected payoff in the event of default. Let:

r = risk-free rate = 2.5%

s = CDS spread.

π = probability of default during year 1 = 7%

C = contingent payment in case of default = 80%

$d_{0.5}$ =discount factor for half-year = $e^{-0.5*r} = e^{-0.5*0.025} = 0.987578$

$d_{1.0}$ =discount factor for 1-year = $e^{-1.0*r} = e^{-0.025} = 0.975310$

Therefore, to solve for the CDS spread (s):
The PV of payments (premium leg, which includes the spread payment and accrual) is:

$s*[0.5*d_{0.5}*\pi + d_{1.0}*(1-\pi)] = s*[0.034565 + 0.907038] = s*0.941603$

The payoff leg (in the event of default) = $C * d_{0.5} * \pi = 0.8*0.987578*0.07 = 0.055304$
Equating the two PVs and solving for the spread: $s*0.941603 = 0.055304$
Thus, $s = 0.058734$ or a spread of approximately 587 basis points.

建议列出现金流表，不容易出错

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Explanation: This can be calculated by using the formula which equates the future value of a risky bon with
yield (y) and default probability (p) to a risk free asset with yield (r). That is,

$1 + r = (1 – \pi) * (1 + y) + \pi R$

where π = Probability of default and R = Recovery rate