**Derivative**

- Pricing and Valuation of Forward Commitments
- Valuation of Contingent Claims
- Derivative Strategies

## Pricing and Valuation of Forward Commitments

**Summary**

- Forward
  - Long and short
  - Contract price 合同价格
    - Stock, bond, interest, currency
  - Contract value(long) 合同价值
- General
  - V = S – P(Benefit) + PV(Cost) – PV(FP)
  - $V_t = \frac{FP_t - FP}{(1+R_f)^{T-t}}$
- Equity Forward
  - Stock price and dividend (discrete and continuous)
- Bond Forward
  - Bond price and coupon
  - Quotes are **full** prices
- **Bond Futures (clean -> full -> clean)**
  - Clean and full price (full = clean + accrued interest)
  - Quotes are **clean** prices: quoted current price and conversion ratio CF
  - Compound using full price
  - **Clean** -> full ($+\mathrm{AI}_0$) -> future full ($\times \left(1+R_f\right)^T - \mathrm{FVC_T}$) -> future clean ($-\mathrm{AI}_T$)-> future **quoted** $\left(\frac{1}{\mathrm{CF}}\right)$
- **FRA Forward - LIBOR**
  - Forward loan interest rate
  - Contract period
    - Begin: contract **price** (**forward** rate)
    - End: contract **value** (new contract price, difference, discount)
  - Loan period
- **Currency Forward**
  - Currency **exchange** rate
- Interest Rate Swap - LIBOR
  - Swap fixed rate
- Currency Rate Swap - LIBOR
  - Two interest rate swaps, linked by exchange rate, exchange principle
- Equity Swap - LIBOR
  - Fixed and equity: interest rate swap
  - Equity for equity: difference in return

**Long and short**

- Long: Party who buy the financial or physical asset
- Short: party who will sell the financial or physical asset

**Contract Price**

- Price of underlying asset
  - Interest rate, discount, yield to maturity, LIBOR, exchange rate

**Contract Value**

- at the inception (V=0)
  - Forward: no money exchange
  - Future: margin
- After
  - Negative value owe money

**No-Arbitrage Principle**

- No riskless profit to be gained by a combination of forward contract position with positions in other assets
- Assumes
  - Transaction costs are zero
  - No restrictions on the short sales or on the use of short sale proceeds
  - Borrowing and lending can be done in unlimited amounts at the risk-free rate of interest
- Forward price = price that prevents profitable riskless arbitrage in frictionless markets

**Cost-of-Carry Model**

- Zero-coupon bond
  - $\mathrm{FP} = \mathrm{S}_0 \times (1 + R_f)^T$

**Arbitrage**

- **Hints**
  - Starts with **nothing**: no cash and no securities
  - Buy low sell high: Buy under-priced assets and sell overpriced assets
  - Take **opposite** positions in the spot and forward markets
- **Cash and carry - forward contract is overpriced**
  - Forward contract is trade **higher** than expected
  - Borrow money -> buy bond -> short forward (sell bond in the future)
  - Today
    - Borrow money
    - Buy bond
    - Short forward
  - Future
    - Settle forward: deliver bond
    - Replay loan
- Reverse cash and carry – forward contract is under-priced
  - Forward contract is trade **lower** than expected
    - Borrow bond -> short (sell) bond -> lend money -> long forward
  - Today
    - Borrow bond
    - Sell bond
    - Lend money (invest)
    - Long forward
  - Future

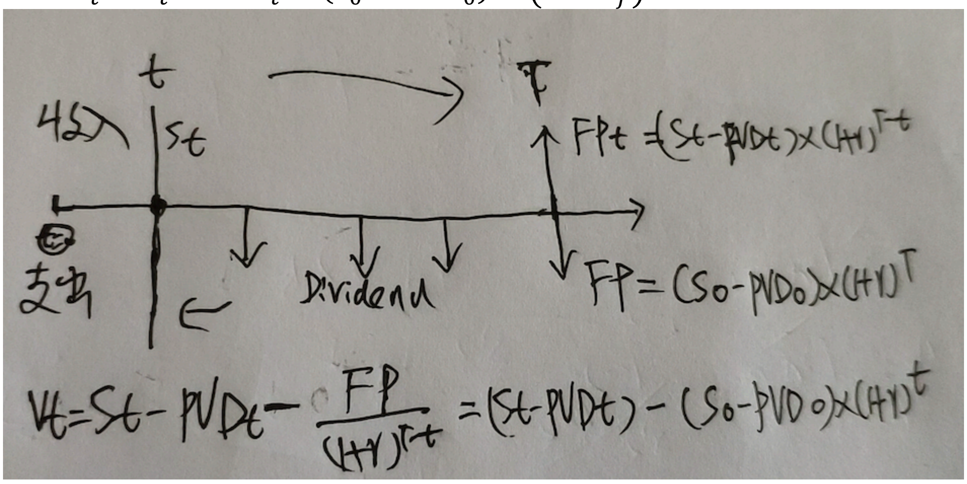
        - Settle forward: buy bond
        - Deliver bond to close short position
        - Receive investment proceeds

**Day Count and Compounding**

- LIBOR-based
    - FRA, swaps, caps, floors
    - 360 day per year and simple interest
    - $1 + \frac{day}{365} \times R_{annnual}$
- Equities, bonds, currencies, and stock options
    - 365 days per year and periodic compounding interest
    - $(1 + R_{annual})^{\frac{day}{365}}$
- Equity **indexes**
    - 365 day and continuous compounding
    - $e^{\frac{day}{365} \times R_{annual}}$

**Equity Forwards – Discrete Dividend**

- Dividend
    - $PVD_0$: present value of dividend at time zero
    - $FVD_T$: future value of dividend at time T
- Contract Price 合同价格
    - $FP = (S_0 - PVD_0) \times (1 + R_f)^T$
    - $FP = S_0 \times (1 + R_f)^T - FVD_T$
- Contact Value at t 合同价值
    - $FP_t = (S_t - PVD_t) \times (1 + R_f)^{T-t}$ 新的平衡价格
        - 在时刻 t，按照开始的方式重新计算合同价格
    - $V_t = \frac{FP_t - FP}{(1+R_f)^{T-t}}$ 新的价格和合同里的价格之差的折现
    - $V_t = S_t - PVD_t - \frac{FP}{(1+R_f)^{T-t}}$
        - 当前价格，未来分红的折现，未来购买价格 FP 的折现
    - $V_t = S_t - PVD_t - (S_0 - PVD_0) \times (1 + R_f)^t$



**Equity Forwards – Continuous Dividend**

- Dividend yield $\delta$
- Compound Risk free rate $R_f$
- Continuous risk-free rate $R_f^c = \ln(1 + R_f)$
- **Contract Price** 合同价格（都需要用连续利率）
  - $FP = S_0 \times e^{(R_f^c - \delta)\times T} = S_0 \times e^{-\delta \times T} \times e^{R_f^c \times T}$
- **Contract Value at t**
  - $FP_t = S_t \times e^{(R_f^c - \delta)\times(T-t)}$
  - $V_t = (FP_t - FP) \times \boldsymbol{e^{-R_f^c \times (T-t)}} =$
  - $V_t = S_t \times e^{-\delta \times (T-t)} - FP \times e^{-R_f^c \times (T-t)}$

**Fixed-income Bond Forwards**

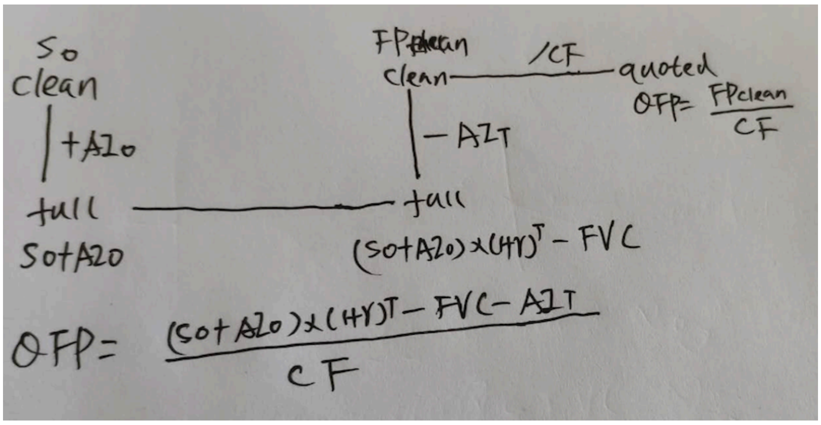
- Coupon-paying bond
- Compared to equity
  - Divided -> coupon
  - Stock price -> **Full** price
- Contract Price 合同价格
  - $FP = (S_0 - PVC_0) \times (1 + R_f)^T$
  - $FP = S_0 \times (1 + R_f)^T - FVC_T$
- Contact Value at t (long position)
  - $FP_t = (S_t - PVC_t) \times (1 + R_f)^{T-t}$ 新的平衡价格
  - $V_t = \frac{FP_t - FP}{(1+R_f)^{T-t}}$ 新的价格和合同里的价格之差的折现
  - $V_t = S_t - PVC_t - \frac{FP}{(1+R_f)^{T-t}}$

**Fixed-income Bond Futures**

- Delivery option
  - Allow the **short** an option to deliver any of several bonds
  - Valuable to short
  - CTD: The underlying deliverable bond in a US Treasury futures contract consists of a basket of bonds from which the short position can deliver the **cheapest** bond
  - MtM: Long and short positions are marked to market each day. Therefore, the contract's market value at the end of each day is zero.
- Conversion factor
  - Each bond is given a conversion factor to adjust the long's payment at delivery so more valuable bonds receive a larger payment
  - **Multipliers** for futures price at settlement
  - Long pays: quotes futures price * conversion factor
- Prices 价格
  - quoted in **clean** prices 报价的永远是 clean 价格
  - calculated using full prices 计算永远是 full 价格
- At **settlement** – buyer pays **full** price

- accrued interes (AI) $= \frac{\text{days since last coupon payment}}{\textit{days between coupon payment}} \times \text{Coupon}$
- $\textbf{full price} = \textbf{clean price} + \textbf{accured interst}$

- Contract Price
  - $FP = (S_0 + \boldsymbol{AI_0}) \times (1 + R_f)^T - FVC_T - \boldsymbol{AI_T}$
    - Begin: $full\ price = S_0 + AI_0$
    - End: $AI_T$ accured interst
  - Quoted **futures** price $QFP = \frac{FP}{CF}$
- Contract Price – Steps (clean -> full -> future full -> future clean -> QFP)
  - $S_0$: **quoted clean** price
  - $S_0^{full} = S_0 + AI_0$ 变成 full price
  - $FP^{full} = S_0^{full} \times (1 + R_f)^T - FVC_T$ 未来的 full price
  - $FP = FP^{clean} = FP^{full} - AI_T$ 未来的 clean price
  - $QFP = \frac{FP}{CF} = \frac{FP^{clean}}{CF}$ 未来的报价
    - Quoted futures price



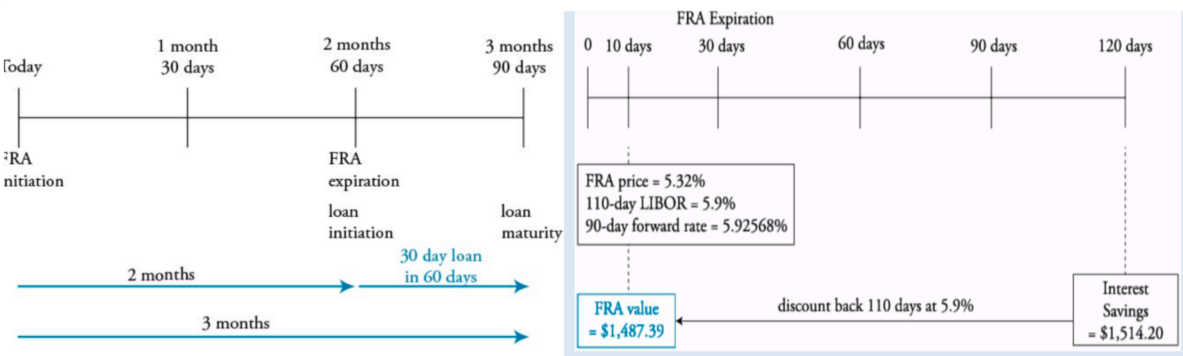

**Forward Rate Agreement (FRA) 远期贷款利率**

- LIBOR 简单利率
  - 30/360, simple interest, add-on rate
- FRA (contract + loan)
  - Long: borrow money in the future
  - Fixed rate payer is long LIBOR
  - Time: $t_1 \times t_2$ FRA
  - **interest** rate $R_1 \times R_2$
  - **Contract** period: $0 \rightarrow t_1$ 合同期间（期初决定利率，期末结算）
    - Time 0: determine the contract price
    - Time $t_1$: determine the **new contract** price, contract value, and settle
  - **Loan** period: $t_1 \rightarrow t_2$ 贷款期间（期初决定利率，期末支付利息）
    - Interest rate is **based** on $t_1$ 利率是基于时刻$t_1$
    - Interest is paid at **maturity** $t_2$ 利息是在时刻$t_2$支付
- Contract price 合同开始：确定合同价格（远期利率）
  - 在时刻 0 决定，从时刻$t_1$到时刻$t_2$的利率

- Although an FRA can be done in conjunction with a LIBOR **deposit**, it is **not** a requirement
- the forward rate $\mathbf{R}$
- $(\mathbf{1}+\mathbf{R_1}\times \mathbf{t_1})\times(\mathbf{1}+\mathbf{R}\times(\mathbf{t_2}-\mathbf{t_1}))=\mathbf{1}+\mathbf{R_2}\times \boldsymbol{t_2}$
- $\mathrm{R}\times(\mathrm{t_2}-\mathrm{t_1})=\frac{1+\mathrm{R_2}\times \mathrm{t_2}}{1+R_1\times t_1}-1$
- $\mathrm{R}=\frac{\frac{1+\mathrm{R_2}\times \mathrm{t_2}}{1+R_1\times t_1}-1}{\mathrm{t_2}-\mathrm{t_1}}$
- Interest at $\mathrm{t_2}$ $\mathrm{P}\times R\times(\mathrm{t_2}-\mathrm{t_1})$ 支付期间

- **Contract value** at **Maturity** 合同结束时：确定合同价值（结算）
  - Time is now $\mathrm{t}=\mathrm{t_1}$
  - New forward price is $\mathrm{R_{t_1}}$ ($\mathrm{t_2}-t_1$ month LIBOR forward rate)
  - Annual interest rate gain is $\mathrm{R_{t_1}}-R$
  - interest gain at time $\mathrm{t}=\mathrm{t_2}$ is $\mathrm{P}\times(\mathrm{R_{t_1}}-R)\times(t_2-t_1)$
  - contract value at $\mathrm{t}=\mathrm{t_1}$ is $\mathrm{P}\times\frac{(\mathrm{R_{t_1}}-\mathrm{R})\times(\mathrm{t_2}-\mathrm{t_1})}{1+R_{t_1}\times(t_2-t_1)}$ discount using the LIBOR
- Contract value before Maturity 合同开始到合同结束前 $(0\le \mathrm{t}<\mathrm{t_1})$
  - Time is now $0\le \mathrm{t}<\mathrm{t_1}$
  - Calculate new forward price $\mathrm{R_t}$
    - Given LIBOR $\mathrm{t_1}-t$ and LIBOR $\mathrm{t_2}-t$
    - Use the same method to calculate new contract price $\mathrm{R_t}$
  - Annual interest rate gain is $\mathrm{R_t}-R$
  - interest gain at time $\mathrm{t}=\mathrm{t_2}$ is $\mathrm{P}\times(\mathrm{R_t}-R)\times(t_2-t_1)$
  - contract value at t is $\mathrm{P}\times\frac{(\mathrm{R_t}-\mathrm{R})\times(\mathrm{t_2}-\mathrm{t_1})}{1+LIBOR_{t_2-t}\times(t_2-t)}$ discount using the LIBOR

gure 39.3: Illustration of a 2 × 3 FRA




**Currency Forward**

- price/base currency, home/foreign currency
- Contract Price
  - $F_T=S_0\times\frac{(1+R_p)^T}{(1+R_b)^T}$ price currency per base currency
  - T use 365 day
- Contract Value
  - $V_t=\frac{F_t-F}{(1+R_p)^{T-t}}\times contract\ size$ (price currency)
  - Long side, will buy foreign currency and get price currency

**Futures Contracts**

- Futures
  - Trade on **exchange**
  - Exchange has a clearinghouse
  - Split each trade and act as the counterparty
  - Safeguard: post **margin** and settle daily
- Market to market
  - Adjust marginal balance each day for the change in the value of the contract from the previous trading day, based on the settlement price
- Similar
  - **No** value at initiation
- Difference: MtM
  - Market to market: value after adjustment is **zero**
  - Do not **accumulate** value changes over the term of the contract
  - Future price at any point
    - Makes the value of a new contract equal to zero
  - Value stay away from zero only between the times at which the account is marked to market
    - **Value of futures contract = current futures price – previous mark-to-market**

**Interest Rate Swaps – Swap Rate - LIBOR**

- Parties
  - **Fixed-rate payer (long)**: pay float and receive fixed 支付固定收浮动
  - Float (short): pay fixed and receive float
  - a swap contract through either a portfolio of underlying instruments or a portfolio of forward contracts.
- One IRS = n – 1 FRA 一个等价于多个远期利率协议
- Floating side 面值回归
  - 浮动方支付的现值永远是名义本金，面值回归
  - $\mathrm{FV_{floating}} = NP \times \left(1 + \mathrm{R_{floating}}\right) \rightarrow$
  - $PV = \frac{FV_{floating}}{1+R_{floating}} = NP$
- Floating Rate 每期利率浮动 LIBOR
  - Based on LIBOR rate
  - Discount factor $\mathrm{Z_t} = \frac{1}{1+\mathrm{LIBOR_t}\times \mathrm{t}} (\mathrm{t} = \frac{days}{360})$
- Swap fixed Rate 固定互换利率
  - fixed rate / swap rate / swap fixed rate
  - $PV = \sum_t \frac{C}{1+LIBOR_t \times t} + \frac{1}{1+LIBOR_t \times t} = \sum_t C \times Z_t + Z_t = 1 \rightarrow C = \frac{1-Z_t}{\sum_t Z_t}$
  - $\mathbf{SFR_{periodic}} = \frac{1-Z_t}{\sum_t Z_t}$ 每期的利率
  - $\mathrm{SFR_{annual}} = \mathrm{SFR_{periodic}} \times \#periods$ 年华利率
- Contract Value on settlement days (**fixed-rate payer**) 新和旧的 SFR 之差
  - $\mathrm{V_t} = (\sum_{\mathrm{i>t}} Z_{i-t}) \times \sum_{\mathrm{i>t}} (SFR_{new} - SFR_{old}) \times \frac{days}{360} \times Principal$
    - Use new discount rate

        - For future periods
        - Discount factor for the remaining periods
    - $V_t = (\sum_{i>t} Z_{i-t}) \times \sum_{i>t}(SFR_{periodic} - SFR_{periodic}) \times Principal$
- Contract Value - PV fixed and floating 老方法
    - $PV(\text{fixed}) = \sum_{i>t} Z_{i-t} \times \boldsymbol{SFR_{old}} \times \frac{days}{360} \times Principal$
        - 以前支付的钱，在新的折旧利率下的值
    - $PV(\text{floating}) = Z_{i-1} \times \left(1 + \mathbf{f_1} \times \frac{\mathbf{days}}{\mathbf{360}}\right) \times Principal$
        - 用**以前**浮动利率来算 coupon
        - 用新的利率来折旧

**Currency Swaps**

- 原则
    - 当成两个独立的利率互换，把双方通过**汇率**匹配一起，要交换本金
    - 互相作为交易对手，收本国货币的固定利率
    - Two currency swaps, there are two yield curves and two swap fixed rates, one for each currency
    - Principal amount must be adjusted for the current exchange rate
- **Combination**
    - Pay fixed and receive fixed 最复杂
    - Pay fixed and receive floating
    - Pay floating and receive fixed
    - Pay floating and receive floating 不需要定价
- 步骤
    - 交换本金
    - 支付利息
    - 返回本金
- 2 种货币，先当成 2 个 IRS，然后连接
    - 对于一种货币
        - 知道浮动利率 LIBOR，算出等价的**固定利率**
        - 这就是借款人需要**定期**支付的利息
    - 然后把 2 种货币连接起来，互相交换
    - 每种货币自己是收到固定利率，floater payer - fixed receiver，然后互相作为对手
- **Contract Price** - Base currency 出借浮动利率，**收到固定利率** fixed rate receiver
    - **Lend principal** in base currency
    - Receive **fixed cash** flow in base currency 算出$SFR_b$
    - Receive **principal** in base currency
- **Contract Price** - Price currency 出借浮动利率，**收到固定利率** fixed rate receiver
    - Lend **principal** in price currency
    - Receive **fixed cash** flow in price currency $SFR_p$
    - Receive **principal** in price currency
- **Contract Price - Link**
    - Price currency 支付 base 汇率利息，收到自己的利息

        - Receive $\mathrm{SFR_p}$, pay $\mathrm{SFR_b}$
    - Base currency 支付对方，收到自己
        - Receive $\mathrm{SFR_b}$, pay $\mathrm{SFR_p}$
- Contract Value
    - 会给定新的折现利率和汇率
    - 对于每种货币，先分别算出余下的现金流（利息和本金），用新的利率折旧，计算出价值
        - $\mathrm{V_t} = \sum_{\mathrm{i>t}} Z_{i-t} \times SFR \times \frac{days}{360} \times Principal + Z_{T-i} \times Principal$
        - *折旧Z*的时间点是支付时间和当前时间之差
        - 不用算新的 SFR，假定不变
    - 最后按照新的汇率把一种货币价值转换成另一种货币价值

**Equity Swaps**

- Contract Price
    - SFR
- Float pay for equity
    - No need
- Fixed pay for Equity
    - Fixed rate payer: value is discounted cash flow
    - Equity index value $\mathrm{V_1} = V_0 \times (1 + R) \rightarrow R = \frac{V_1}{V_0}$
- Equity for equity
    - A for B
    - Returns are $R_A$ and $\mathrm{R_B}$
    - Value for A is $\mathrm{V_0} \times (\mathrm{R_B} - \mathrm{R_A})$

# Valuation of Contingent Claims

**Summary**

- **Binomial Model - Valuation**
  - Two approaches
    - **expectation** approach and **arbitrage approach**
  - Stock Option
    - One-period and two periods
    - European and American Options
  - Call-Put Parities
    - C+PV(X)=P+S, synthetic and arbitrage
  - Interest Rate Option
    - Up and down probability are 0.5
  - Arbitrage Approach
    - Hedge ratio (number of stocks per option)
- **BSM**
  - Stock
  - Stock with dividend
  - Currencies
- **Black Model**
  - Futures
  - Interest Rate Option
  - Swaps: swaptions – interest rate swap
- Option Greeks and Implied Volatility
- Dynamic Hedging – Delta Neutral (1/delta)
- Gamm Risk

**Binomial Model Valuation**

- Returns
  - Up return $\mathrm{R_u}$
  - Down return $R_d$
  - Risk free return $R_f$
- Move Factor
  - Up factor $\mathrm{U} = 1 + \mathrm{R_u}$
  - Down factor $\mathrm{D} = 1 + R_d$
  - Risk free factor $1 + R_f$
  - If only give up or down, then
    - $\mathbf{U \times D = 1}$
- Probability
  - **Move up** $\mathbf{p_u} = \frac{R_f - R_d}{R_u - R_d} = \frac{1 + R_f - D}{U - D}$
  - Move up $\mathrm{p_d} = 1 - p_u$
- Stock Price
  - Initial stock price $\mathrm{S_0}$
  - Up price $\mathrm{S^+} = \mathrm{S_0} \times \mathrm{U}$
  - Down price $\mathrm{S^-} = \mathrm{S_0} \times \mathrm{D}$
- Option Value – Call option
  - Up price $C^+ = \max(0, \mathrm{S^+} - K)$

- Down price $C^{-} = \max(0, S^{-} - K)$
- Option Value – Put option
  - Up price $P^{+} = \max(0, K - S^{+})$
  - Down price $P^{-} = \max(0, K - S^{-})$
- Weighted future Value
  - $V = p_u \times V^{+} + p_d \times V^{-}$
- Value
  - $V_0 = \frac{V}{(1+R_f)^t}$

**Call-Put Parity**

- $C - P = S - PV(K) = S - \frac{K}{(1+R_f)^T}$
  - T = actual days / 365
- $PV(K) + C = S + P$
- Fiduciary call $PV(K) + C$
  - Hold a bond and call option -> can gain more in up
- Protective put $S + P$
  - Hold a stock and a call option -> can loss less in down
- Synthetic – Replication
- Arbitrage
  - Buy low sell high
  - Market vs replication

**European and American Options**

- European Options
  - Compute option value at the **last** node and then discount
- American Options
  - Compute option value at **every** node, and decide whether to exercise
- Early Exercise: dividend-paying call and deep-in-the-money put
  - Call option
    - No dividend -> not valuable
    - Dividend -> possible (right before the dividend pay-out)
  - Put
    - Deep in the money (close to zero) -> valuable
- Early Exercise: capture intrinsic value
  - Capture intrinsic value and ignore time value
  - Intrinsic value can be invested at risk free rate, but interest earned is usually **less** than time value
  - Deep-in-the-money put, upside is limited, and intrinsic interest **can exceed** time value

**Binomial Model – Arbitrage Approach - Hedge Ratio**

- Portfolio Delta = 0 w.r.t stock price
  - $\Delta\text{Call} > 0, \Delta\text{put} < 0, \Delta\text{stock} = 1$
  - Call: $\Delta\text{portfolio} = \Delta\text{call} - h\Delta\text{stock} = 0 \rightarrow h = \frac{\Delta\text{call}}{\Delta stock} = \Delta$

- Put: $\Delta\text{portfolio} = \Delta\text{put} + \text{h}\Delta\text{stock} = 0 \rightarrow \text{h} = \frac{-\Delta\text{put}}{\Delta stock} = -\Delta$
  - Call is better than put (earn premium than pay premium)
- Call option (buy option sell stock or short option buy stock)
  - $\text{h} = \frac{\text{C}^{+} - C^{-}}{\text{S}^{+} - S^{-}}$
- Arbitrage
  - ending portfolio value is the same regardless of up or down move
- if option is overpriced
  - begin
    - borrow money $V_0$ 借钱
    - buy stock and sell option 买股票，卖期权
    - portfolio value is $V_0 > 0$
  - ending
    - portfolio value $V^{+} > 0$
    - pay $V_0 \times (1 + \text{R}_\text{f})^\text{T}$
    - profit is $V^{+} - V_0 \times (1 + \text{R}_\text{f})^\text{T}$
  - Present value
    - Profit $\frac{V^{+} - V_0 \times (1+\text{R}_\text{f})^\text{T}}{(1+\text{R}_\text{f})^{T}} = \frac{V^{+}}{(1+\text{R}_\text{f})^{T}} - V_0 = \frac{h \times S^{+} - C^{+}}{(1+\text{R}_\text{f})^{T}} - (h \times S_0 - C_0)$
- **Arbitrage Pricing**
  - $\frac{h \times S^{+} - C^{+}}{(1+\text{R}_\text{f})^{T}} - (h \times S_0 - C_0) = 0 \rightarrow \text{C}_0 = h \times S_0 + \frac{h \times S^{+} - C^{+}}{(1+\text{R}_\text{f})^{T}}$

| **Status** | **Stock** | **#stocks** | **Option Value** | **Portfolio Value** (long stock short option) | **Present value** |
|---|---|---|---|---|---|
| Initial | $\text{S}_0$ | h | $\text{C}_0$ | $h \times S_0 - \text{C}_0$ | $\text{V}_0 = h \times S_0 - \text{C}_0$ |
| up | $\text{S}^{+}$ | h | $\text{C}^{+}$ | $h \times S^{+} - \text{C}^{+}$ | $PV^{+} = \frac{h \times S^{+} - \text{C}^{+}}{(1 + R_f)^T}$ |
| down | $\text{S}^{-}$ | h | $\text{C}^{-}$ | $h \times S^{-} - \text{C}^{-}$ | $PV^{-} = \frac{h \times S^{-} - \text{C}^{-}}{(1 + R_f)^T}$ |

**Binominal Interest Rate Trees**

- risk neutral
  - probability of move up and move down are 0.5
- interest rate at each node: one-period **forward** rate

**Interest Rate Options 利率期权**

- LIBOR Arrears
  - interest rate is determined at the beginning 利率开始决定
  - interest is paid at the end, but interest can be changed using option 利率计算利息，期权可以改变利息
  - interest is discounted using the beginning interest rate 利率用来折旧
- call option - cap
  - $\text{payoff} = \text{notional principal } \times \max(0, \text{reference rate} - \text{ecercise rate})$
  - Borrower has an interest rate **cap 借款利息不会高于行权值**
- put option

  - $\text{payoff} = \text{notional principal} \times \max(0, \text{ecercise rate} - \text{reference rate})$
  - lender has an interest rate **floor 投资利息不会低于行权值**
- just change stock price -> interest (interest rate * principal)
  - 把股票价格换成利息即可

**BSM Assumptions**

- Continuous time
- Asset price follow geometric Brownian motion process
- Asset return follows a **lognormal** distribution
- Logarithmic continuously compounded return is normally distributed
- **Volatility** of asset return is constant and known
- Markets are **frictionless**
  - No tax, no transaction cost, no restrictions on short sales
  - Continuous trading, no arbitrate
- The asset **yield** is constant
- Options are **European** options

**Stock Options**

- $C - P = S - X \times e^{-r\times T}$
- $\mathbf{C_0 = S \times N(d_1) - X \times e^{-r\times T} \times \boldsymbol{N(d_2)}}$ **用钱买股票**
  - Buy $\mathrm{N(d_1)}$ stocks using $\mathrm{X \times e^{-r\times T}} \times N(d_2)$ of borrowed funds
    - A short position in $N(d_2)$ bonds
    - 借钱买股票或者发行债券买股票
  - $\mathrm{N(d_1)}$ 到期前股票大于行权价格的概率
  - $\mathrm{N(d_2)}$ 到期日股票大于行权价格的概率
  - $\mathrm{N(x)}$是累计概率分布
- $\mathrm{P_0 = C_0 - S - X \times e^{-r\times T} = X \times e^{-r\times T}} \times N(-d_2) - \mathrm{S \times N(-d_1)}$
  - Short $\mathrm{N(-d_1)}$ stocks, and long $\mathrm{N(-d_2)}$ bonds
  - 卖股票买债券，出借钱
- $\mathrm{d_1} = \frac{\ln\left(\frac{S}{X\times e^{-r\times T}}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}} = \frac{\ln\frac{S}{X}+\left(r+\frac{\sigma^2}{2}\right)\times T}{\sigma\sqrt{T}}$
- $\mathrm{d_2} = d_1 - \sigma\sqrt{T}$
- $N(-d_1) = 1 - N(d_1)$

**Stock Options with Dividend**

- $C - P = S \times \mathbf{e^{-\delta\times T}} - X \times e^{-r\times T}$
- $\mathbf{C_0 = S \times e^{-\delta\times T} \times N(d_1) - X \times e^{-r\times T} \times \boldsymbol{N(d_2)}}$
- $\mathrm{P_0 = X \times e^{-r\times T}} \times N(-d_2) - \mathrm{S} \times \mathbf{e^{-\delta\times T}} \times \mathrm{N(-d_1)}$
- $\mathrm{d_1} = \frac{\ln\left(\frac{S\times e^{-\delta\times T}}{X\times e^{-r\times T}}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}} = \frac{\ln\frac{S}{X}+\left(r-\delta+\frac{\sigma^2}{2}\right)\times T}{\sigma\sqrt{T}}$
- **因此**
  - 把$S$替换成$S \times e^{-\delta T}$
  - 在计算$\mathrm{d_1}$时，$r$替换成$r - \delta$

**Currencies Options – pricing currency and interest on base currency**

- Exchange rates
  - Spot rate is $S_0$
  - Future rate is $X$
- Derive Process
  - In the future, use $X$ to exchange 1 base currency
  - X price currency worth $\mathrm{X} \times \mathrm{e}^{-r_p \times T}$ now
  - 1 base currency worth $\mathrm{e}^{-r_b \times T}$ in base currency, and it worth $S_0 \times \mathrm{e}^{-r_b \times T}$ in price currency now
- $\mathbf{C_0} = \mathbf{S} \times \boldsymbol{e^{-r_b \times T}} \times \mathbf{N(d_1)} - \mathbf{X} \times \mathbf{e^{-r_p \times T}} \times \boldsymbol{N(d_2)}$
- $\mathrm{P_0} = \mathrm{X} \times \mathrm{e}^{-\mathrm{r_p} \times \mathrm{T}} \times N(-d_2) - \mathrm{S} \times \boldsymbol{e^{-r_b \times T}} \times \mathrm{N}(-\mathrm{d}_1)$
- 技巧
  - 可以把$r_p$当成分红，但其实是 interest earned on the foreign currency

| Status | Price/home | Base/foreign | Notes |
|---|---|---|---|
| Now | $S_0 \times \mathrm{e}^{-r_b \times T}$<br>$\mathrm{X} \times \mathrm{e}^{-r_p \times T}$ | $\mathrm{e}^{-r_b \times T}$ | 推导 |
| future | X | 1 | 给定 |

**Future Options – Black Model**

- The price is the **future** price $\mathbf{S} = \mathbf{F_T} \times \boldsymbol{e^{-r \times T}}$
- $\mathbf{C_0} = \mathbf{S} \times \mathbf{N(d_1)} - \mathbf{X} \times \mathbf{e^{-r \times T}} \times \boldsymbol{N(d_2)} = \mathbf{e^{-r \times T}}(\boldsymbol{F_T} \times \boldsymbol{N(d_1)} - \boldsymbol{X} \times \boldsymbol{N(d_2)})$
- $\mathrm{d}_1 = \frac{\ln\left(\frac{F \times e^{-r \times T}}{X \times e^{-r \times T}}\right) + \frac{\sigma^2}{2} T}{\sigma\sqrt{T}} = \frac{\ln\frac{F_T}{X} + \frac{\sigma^2}{2} \times T}{\sigma\sqrt{T}}$
- 技巧：可以把分红利率考虑成 risk free rate $\boldsymbol{r}$

**Interest Rate Options – actual/365 convention**

- Options on forward rates (FRA)
- FRA uses 30/360, but options on FRA uses actual/365 convention
- $\mathrm{m} \times \mathrm{n}$ forward
- $\mathrm{C_0} = \mathrm{e}^{-r \times T}\left(S \times N(d_1) - X \times N(d_2)\right) \times \frac{actual}{365} \times \textit{notional principal}$
  - $\textit{actual period} = \frac{actual}{365} = \frac{(n-m) \times 30}{365}$ the loan periods
  - $T = \frac{n \times 30}{360}$ to the beginning of the time

**IRO - Combinations**

- Long interest rate call and a short interest rate put -> long FRA
  - A forward contract: fixed rate payer
- A series of interest rate call options with different maturities and the same exercise price -> interest rate **cap**
- Interest rate floor: a series of interest rate put
- Cap and floor with the same exercise rate -> payer swap

**Swaptions – option on interest rate swap**

- On option that give the holder the right to enter an interest rate swap
- Payer swaption

    - Option to be a fixed-rate payer 支付固定收浮动
    - A **call** option on **floating** swap 浮动利率上涨好
    - A put option on a coupon bond 债券看跌
- Receiver swaption
    - Option to be a fixed-rate receiver 支付浮动收到固定
    - A **call** option on a **coupon** bond
    - A put option on floating swap 浮动利率下降好
- Swaption
    - An option on a series of cash flows (annuity)
    - One for each settlement date of the swap, equal to the difference between exercise rate on the swaption and the market swap fixed rate
- Payer swaption 未来现金流之差，再折现
- $\text{pay} = \mathbf{PVA} \times \left(\boldsymbol{SFR} \times N(d_1) - X \times N(d_2)\right) \times AP \times \ notional\ principal$
    - $PVA = \sum_\mathrm{i} Discount\ Factor_i$ present value of such an annuity，折现之和
    - $AP = actual\ period = \frac{1}{\#settlement\ period\ per\ year}$
    - $SFR$ current market swap **annual** fixed rate
    - Discount using risk-free rate
    - Swaption time to expire is m for a m*n forward rate
- $X$ exercise rate specified in the payer swaption
- $\text{receiver} = \mathbf{PVA} \times \left(X \times N(-d_2) - \boldsymbol{SFR} \times N(-d_1)\right) \times AP \times \ notional\ principal$

**Equivalencies**

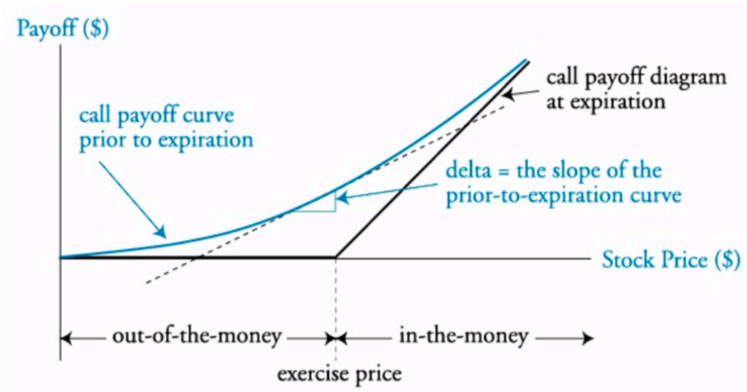
- **Convert Rule**
    - Swaption: payer -> call
    - Swaption: receiver -> put
    - Swap: payer -> S
    - Swap: receiver -> -S
- payer swap (S)
    - a long payer swaption (c)
    - a short receiver swaption (-p)
    - with the same exercise rates
- receiver swap (-s)
    - long receiver swaption (p)
    - a short payer swaption (-c)
    - with the same exercise rates
- long callable bond
    - a long option-free bond
    - a short receiver swaption

**Option Greeks - Delta – Slope**

- $\Delta = \frac{\Delta \text{option price}}{\Delta \text{stock price}}$
- Call: positive, put: negative
- Call-put parity
    - $\Delta_\text{call} - \Delta put = 1$
- $\Delta_\text{call} = e^{-\delta \times T} \times N(d_1), \Delta_{put} = -e^{-\delta \times T} \times N(-d_1)$

- $\Delta_{\text{call}} - \Delta put = e^{-\delta \times T}$

- Call
  - Deep out of money: $\Delta = 0$
  - Deep in the money: $\Delta = 1$
  - At the money: $\Delta \approx 0.5$
- put
  - Deep out of money: $\Delta = 0$
  - Deep in the money: $\Delta = -1$
- Call option with no dividend
  - Stock price increase, out -> in, then delta 0->1
- Put option with no dividend
  - Stock price increase, in -> out, then delta -1 -> 0




**Option Greeks – Gamma – Curvature – Convex - Positive**

- Rate of change in delta
- Convex: always positive
- Moneyless
  - At the money -> largest
  - Deep in- or out- money -> 0
- $\Delta \mathrm{P} \approx \Delta \times \Delta \mathrm{S} + \frac{1}{2} \times Gamma \times \Delta S^2$

**Option Greeks – Vega - Positive**

- Change in volatility
- 高波动、高收益

**Option Greeks – Rho**

- Change in risk free rate
- Call: positive
- Put: negative

**Option Greeks – Theta - Negative**

- **Passage of time (0 -> current time t) 开始到当前时刻**
- time-to-maturity (current time t -> exercise date T) 当前到行权
- Time decay
  - Speculative value decline

- It is negative, value decrease
- Deep in-the-money put option may actually **increase** in value
- Notes
    - Option value and time-to-maturity: positive
        - Long maturity has higher value
    - Option value and passage of time: negative
        - As time passes and option approaches maturity, value decay

| Sensitivity Factor (Greek) | Input | Calls | Puts |
|---|---|---|---|
| Delta | Asset price (S) | Positively related Delta > 0 | Negatively related Delta < 0 |
| Gamma | Delta | Positive Gamma > 0 | Positive Gamma > 0 |
| Vega | Volatility (σ) | Positively related Vega > 0 | Positively related Vega > 0 |
| Rho | Risk-free rate (r) | Positively related Rho > 0 | Negatively related Rho < 0 |
| Theta | Time to expiration (T) | Time value → \$0 as call → maturity Theta < 0 | Time value →\$0 as put → maturity Theta < 0* |
| | Exercise price (X) | Negatively related | Positively related |

**Dynamic Hedging - Delta-neutral**

- Hege stock price risk
    - portfolio value does not change when stock price change
    - one option contract = 100 options
- Call option
    - Combine a stock with a short position in call
    - 1 stock should **sell** $\frac{1}{\Delta\text{call}} = \frac{1}{N(d_1)}$ option
    - Positive Gamma Risk
- Put options
    - Combine a stock with a long position in put
    - 1 stock should **buy** $-\frac{1}{\Delta\text{put}} = -\frac{1}{\Delta\text{call}-1} = \frac{-1}{N(d_1)-1}$option
        - because put delta is negative
    - Negative Gamma Risk
- Drawback
    - Risk free only for **small** change in stock price
    - Must be **continually** rebalanced to maintain the hedge
    - Significant **transaction** costs

**Gamma Risk**

- BSM assumptions hold -> no abrupt change -> no gamma risk
- Gamma risk: risk of **abruptly** jump in price
    - Leaving a delta-hedged portfolio unhedged
- Long stock and short call
    - Stock price drop abruptly
    - Stock: Delta 1, gamma: 0

    - Portfolio: negative gamma

**Implied Volatility**
- Derived from market price
- If future implied volatility increase, option value will increase
- Gauge market perceptions

# Derivative Strategies

**Summaries**

- Application
  - Hedge - Modify risk and return
    - Swap: exchange 交换
    - Future: **change 改变**
  - Synthetic Asset
    - C – P = S - PV(X)
- Strategy
  - Covered call
  - Protective put
  - Bull spread
  - Bear spread
  - Straddle
  - Collar
  - Calendar spread

**Hedge - Risk and Return Profile**

- Interest rate swaps
  - Modify duration
  - $payer\ swap\ value = floating\ rate\ value - fixed\ rate\ value$
  - $Duration_{fixed} > Duratiion_{floating}$
  - Payer swap has negative duration
  - If future interest will increase?
    - Interest increase, price decrease, decrease duration, use payer swap
- interest rate future 改变（long 增加 Duration, short 减少 Duration）
  - modify **duration** of a portfolio
  - future interest rate decline -> buy future -> increase duration
  - sell future -> decrease duration
- Currency swaps
  - Use the **relative advantage** in borrowing from own market than foreign market
- Currency Futures
  - Hedge an asset or liability in a foreign currency
  - US company has a euro liability, worry about euro appreciating
    - Purchase euro futures
- Equity Swaps 互换风险
  - Exchange equity return for another asset return
  - Reduce equity exposure
    - Temporarily without liquidating holdings
  - Total return swap (TRS)
- Stock Index Futures
  - Change the exposure of equities
  - Rotate out equities
    - Short future contracts

  - Rotate money out of bonds and into equities
    - Long future contracts
- Foreign Currency Options
  - Hedge existing asset or liability denominated in a foreign currency
  - US company has a euro liability, worry about euro appreciating
    - Long call options on euros
  - **Options** are better than futures in managing downside risk
    - The only risk the premium paid

**Synthetic with Options**

- C – p = S - PV(k)
- **Synthetic Stock**
  - Long call + short put = long stock
  - Long put + short call = Short stock
  - At the money: Strike price = current market price
- **Synthetic Puts and calls**
  - Long stock + long put = Long call
  - Long call + short stock = Long put

**Synthetic with Forwards/Futures**

- Synthetic Stock using
  - **risk-free asset + long futures = long stock**
  - 无风险资产 + 购买远期 = 股票
- Synthetic Cash/Risk-free
  - Long stock + short futures = risk-free asset
  - 持有股票 + 卖掉远期 = 无风险资产

**Hedging – Discussion**

- Fixed-income duration
  - Use interest rate swap or futures
- Reduce equity exposure
  - Both a short futures and a synthetic short (with options)

**Option Strategy – Covered call (等价于-put)**

- Covered call = Long stock + short call
- Properties
  - Stock price increase -> sell stock -> earn **income**
  - Stock price decrease -> Earn **premium**
- When to use
  - Price slower increase 看涨
  - Limit gain 止盈
  - Continuous write 连续发行，增加 total return
- Objective
  - Income generation – **out-of-money** call -> premium
    - Write Out of money call

    - If the price of the underlying will remain **flat** (will not increase above the call exercise price). The option premium is considered to be income.
    - However, the investor **gives up all gains** above the exercise price.
  - Improve the market – in-the-money call - Premium > (S-K)
    - Stock seller can sell at a better price with **in-the-money** call
    - Premium > (S-K)
    - Stock price at 50 with exercise price 45, but trading for 8
      - premim:8 = Intrinsic value: 5, **time value**: 3
    - earn time value
  - Target price realization – out-of-money
    - Stock price at 50, strike at 55, premium 2
    - Continue revise price
- Profit & Loss
  - Initial investment $\mathrm{S_0 - C_0}$
  - Value at expiration $\mathrm{S_T - \max(0, S_T - X)} = \min\,(X, S_T)$ 最小值
  - Profit at expiration $\mathrm{\min(X, S_T) - (S_0 - C_0)}$
  - Maximum gain $\mathrm{X - (S_0 - C_0) = C_0 + X - S_0}$ 被行权
    - $\mathrm{S_T = X}$
  - Maximum loss $\mathrm{S_0 - C_0}$ 也是初始成本，全部亏光
    - $\mathrm{S_T = 0}$
  - Breakeven point $\mathrm{S_0 - C_0}$ 就是初始成本，0 利润
    - $\mathrm{S_T} = S_0 - C_0$

**Option Strategy – Protective Put (等价于 call)**

- Protective put = Long stock + long put
- Long put -> **Insurance** policy
  - Deductible: $\mathrm{S_0} - X$ (**exercise** the put) 可亏损金额
  - Premium: $\mathrm{P_0}$
  - Reduce premium by **increasing** deductible
    - Deductible 越大，发生的概率就越小，因此便宜
  - Reduce premium by buy OTM put
- Profit & Loss
  - Initial investment $\mathrm{S_0} + P_0$
  - Value at expiration $\mathrm{S_T + \max(0, X - S_T)} = \max\,(X, S_T)$ 最大值
  - Profit at expiration $\mathrm{\max(X, S_T) - (S_0 + P_0)}$
  - Maximum gain $\mathrm{S_T - (S_0 + P_0)} = +\infty$ 不行权
  - Maximum loss $\mathrm{S_0} + P_0 - X$行权，初始成本全部亏了，不过有 X 收入
    - $\mathrm{S_T = 0}$
  - Breakeven price $\mathrm{S_0} + P_0$ 就是初始成本，0 利润
    - $\mathrm{S_T} = S_0 + P_0$
- Risk
  - Premium reduce total return
  - Consistently insuring will reduce return
- 

**Delta**

- $\Delta\text{stock} = \Delta\text{forward} = 1$
- $\Delta\text{covered call} = 1 - \Delta call = -\Delta\text{put}$
- $\Delta\text{protective put} = 1 + \Delta put = \Delta call$

**Cash-Secured Puts 看涨**

- Hold cash + short put
- Receive premium, but takes the downside
- Same with covered call
- S-C = P+PV(X)

**Spread**

- Long and short of the same type of option
- 对一种期权的买和卖
- They differ in exercise price or maturity
- Bull spreads
  - Higher price in the future
  - Calls or puts where 买入低行权价格，卖出高行权价格
    - long option exercise price < short option exercise price
- bear spreads
  - Lower price in the future
  - Calls or puts where 买入高行权价格，卖出低行权价格
    - long option exercise price > short option exercise price

**Bull Call Spread – pay premium and earn price**

- **buy** a **lower** exercise price call and write a higher price call
- limited upside and limited downside
- gain when price increase
- exercise price: $\mathrm{X_L} < \mathrm{X_H}$ 买低卖高
- premium: $C_L > \mathrm{C_H}$ 行权价格越高越便宜
- Initial investment $\mathrm{C_L} - \mathrm{C_H}$
- Value at expiration $\max(0, \mathrm{S_T} - \mathrm{X_L}) - \max(0, \mathrm{S_T} - X_H)$
- Profit at expiration
- Maximum gain $\mathrm{X_H} - \mathrm{X_L} - (\mathrm{C_L} - C_H)$ 都行权
- Maximum loss $\mathrm{C_L} - \mathrm{C_H}$初始成本全部亏了，都不行权
- Breakeven price $\mathrm{X_L} + \mathrm{C_L} - \mathrm{C_H}$一个行权，一个不行权

**Bear Call Spread – earn premium**

- buy a **higher** exercise price call and write a **lower** price call
- Limited upside and downside
- Gain when price decrease
- exercise price: $\mathrm{X_L} < \mathrm{X_H}$ 卖低买高
- premium: $C_L > \mathrm{C_H}$ 行权价格越高越便宜
- the **counterparty** to bull call spread

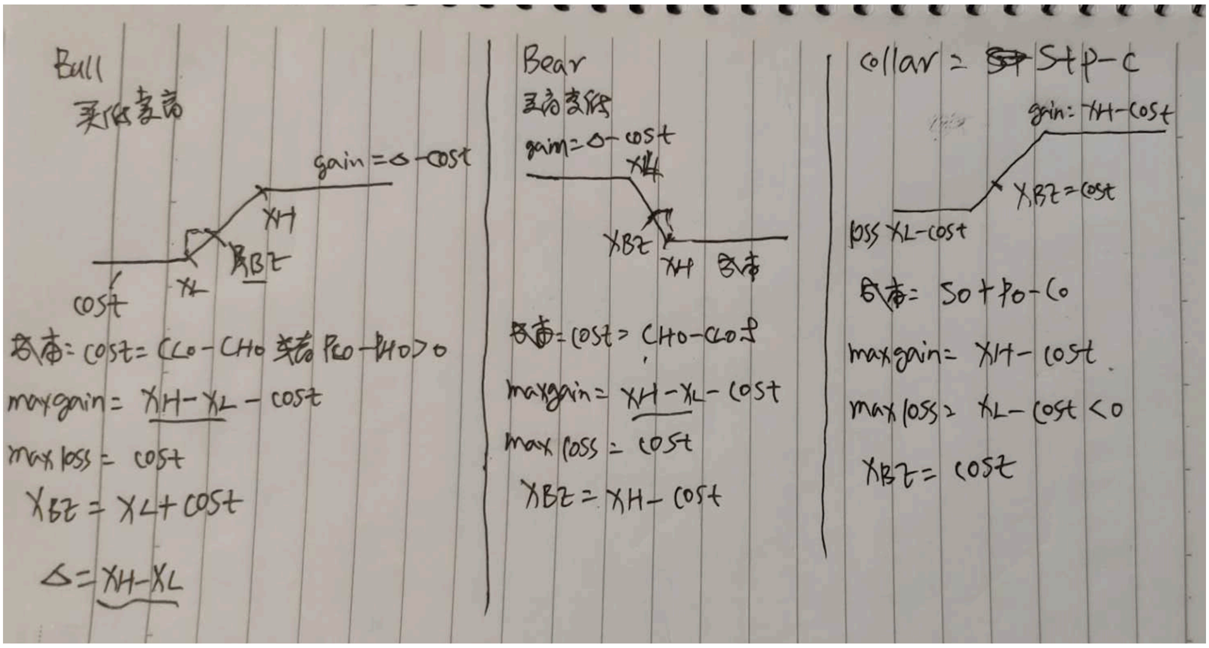
**Bear Put Spread**

- buy a **higher** exercise price put and write a **lower** price put
- limited upside
- exercise price: $X_L < X_H$ 买高卖低
- premium: $P_L < P_H$ 行权价格越高越贵
- Initial cost $P_H - P_L$ 期权费
- Value at expiration $\max(0, X_H - S_T) - \max(0, X_L - S_T)$
- Profit at expiration
- Maximum gain $X_H - X_L - (P_H - P_L)$ 都行权
- Maximum loss $P_H - P_L$初始成本全部亏了，都不行权
- Breakeven price $X_H - (P_H - P_L)$个行权，一个不行权

**Spreads 技巧（会画图）**

- Bull 买低卖高
  - inverted Z style
  - long low, short high
- bear 买高卖低
  - Z style
  - Long high, short low
- earn stock return, reduce cost 顺势赚回报
  - bull call, bear put
- earn premium, reduce risk 逆势赚期权费
  - bull put, bear call
- bull call – earn stock return
  - long low, short high
  - earn stock return, reduce cost
- bull put
  - long low, short high
  - earn premium, reduce risk
- bear put – earn stock return
  - long high, short low
  - earn stock return, reduce cost
- bear call
  - long high, short low
  - earn premium, reduce risk

| | Call | Put | Notes |
|---|---|---|---|
| **Bull** | Long low & Earn return | Long low & Earn premium | Long low short high, inverted Z-payoff curve |
| **Bear** | Long high & Earn premium | Long high & Earn return | Long high short low, Z-payoff curve |
| | Earn return & reduce cost | Earn premium & reduce risk | |




**Risks of spreads**

- upside and downside are limited
- chopping off the tails

**Collar (stock + p - c) 和 bull call 图形一样**

- combines protective put and **covered** call -> similar to a **spread**
- decrease the **volatility** of investment returns
- own stock, buy a protective put, and sell a call to offset the put premium
- if the two premiums are equal -> **zero-cost** collar
- usually put strike is less than put strike $X_L < X_H$
- put a band around the possible returns of a long stock returns
- stock price
  - $> X_H \rightarrow loss$
  - $< X_L \rightarrow gain\ (protective\ put)$
  - Others
- $X_L = X_H = X \rightarrow locked\ in\ profit\ or\ loss\ of\ X - S_0$
- Initial cost $\mathrm{S_0} + P - C$ 期权费
- Value at expiration $\mathrm{S_T} + \max(0, \mathrm{X_L} - \mathrm{S_T}) - \max(0, \mathrm{S_T} - \mathrm{X_H})$
- Profit at expiration
- Maximum gain $\mathrm{X_H} - \mathrm{S_0} - (\mathrm{P} - \mathrm{C})$ call 行权
  - Stock return
- Maximum loss $\mathrm{S_0} - X_L + P - C$初始成本全部亏了，put 行权
- Breakeven price $\mathrm{S_0} + P - C$ 都不行权，覆盖成本

**Straddle**

- **Expect a large price move but unsure of the direction**
- Neutral on market direction, but expect large volatility
- Long straddle
  - Long call and long put on the same stock with the same strike price

- Loss if price does not change much
- Short straddle
  - Short call and short put on the same stock with the same strike price
  - Gain if price does not change much
- Long straddle
  - Initial cost $P + C$ 期权费
  - Value at expiration $\max(0, \mathrm{X} - \mathrm{S_T}) + \max(0, \mathrm{S_T} - \mathrm{X})$
  - Profit at expiration
  - Maximum gain unlimited
  - Maximum loss $P + C$初始成本全部亏了
  - Breakeven prices $\mathrm{S_0} + P + C$ 或者 $\mathrm{S_0} - (P + C)$

**Calendar Spread**

- Two call option on the same stock with the same exercise price but **different maturities**
- Long calendar spread
  - Short near-dated and long longer-dated call
  - Longer-dated premium > short-dated premium -> initial outflow
  - Stock price will be flat in near term but is poised to break out in the longer term
  - the expectation is that a price move is not **imminent**. That is, the expectation is for an upward price move but after a **lag**.
  - The trader attempts to capture the decay in time value by selling the near-dated call option and buying the long-dated call option with the same strike price. If the price does not move up immediately as the trader expects, the near-dated call option will expire worthless and the trader will capture the time value.

**Investment Objective**

- Market direction
  - Strong bullish (bearish) -> long calls (puts)
  - Average bullish (bearish) -> long calls and short puts (write calls and buy puts)
  - Weak bullish (bearish) -> write puts (calls)
- Future volatility
  - Increased volatility -> long straddle

| | | Direction | | |
|---|---|---|---|---|
| | | **bullish** | **Neutral** | **bullish** |
| Volatility | **High** | Buy calls | Buy straddle | Buy puts |
| | **Average** | Buy calls & write puts | Spreads | Buy puts & write calls |
| | **low** | Write puts | Write straddle | Write calls |

**Breakeven Price Analytics**

- The annual volatility needed to **break even** over the number of trading days

- $\sigma_{\text{annual}} = \frac{\%\Delta P}{\sqrt{t}} = \%\Delta P \times \sqrt{\frac{252}{trading\ days\ until\ maturity}}$
    - $\%\Delta P = \frac{|break\ even\ price - current\ price|}{current\ price}$
    - $t = \frac{trading\ days\ until\ maturity}{252}$
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