Portfolio

- Portfolio Management Process and Investment Policy Statement
- An introduction to multifactor models
- Measuring and managing market risk
- Economics and Investment markets
- Analysis of active portfolio management
- Algorithmic trading and high-frequency trading

Portfolio Management Process and Investment Policy Statement

Elements of Portfolio Management

- Evaluating investor and market characteristics
    - Objective and constraints of the investor
        - Objective: risk and return
    - Economic environment
        - Macro: growth, inflation, unemployment
        - Micro: sector, industry, security-specific
- Developing an investment policy statement (IPS)
- Determining an asset **allocation** strategy
- Measuring and evaluating **performance**
- **Monitoring** dynamic investor objectives and capital market conditions

Importance

- Risk-return trade-off
- Only care about systematic risk

Management Steps

- Planning
    - Analyse objective and constraints
    - Develop IPS
    - Investment strategy
    - **Strategic** Asset allocation strategy
- Execution
    - **Tactical** asset allocation
    - Security selection
- Feedback
    - Monitoring and rebalance
    - Performance evaluation

IPS

- goal
    - long-term discipline for portfolio decisions
    - protect against short-term shifts
- includes
    - objectives and constraints
    - duties and responsibilities
    - calendar schedule for both portfolio performance and IPS review
    - guideline for adjustments and rebalancing

Strategic Asset Allocation

- long-term target weight
- **passive** investment strategies
    - **indexing and buy-and-hold**
- **active** investment strategies

- generating alpha
- **semi-active**, risk-controlled active, or enhanced index strategies
  - index **tilting**
    - match benchmark portfolio but **deviate** from weight
- time horizon
  - short -> less risky
- risk-return profile

Investment Objective
- risk and return

Investment Objective - Risk Objective
- **risk tolerance**
  - **willingness** and **ability** to take risk
- measures
  - absolute risk objective
    - standard deviation
  - relative risk objective
    - benchmark or index, tracking

	Ability to Take Risk	
Willingness to Take Risk	Below Average	Above Average
Below Average	Below-average risk tolerance	Education/resolution required
Above Average	Below-average risk tolerance	Above-average risk tolerance

Investment Objective - Return objective
- desired return > required return
- real and nominal return
- pre-tax and after-tax return
- total return
  - income and capital gains

Investment Constraints (RRTTLLU)
- time horizon
- tax
- liquidity
- legal and regulatory
- unique circumstances

Investor	Return Requirement	Risk Tolerance
Individual investor	Depends on life-cycle stage and financial position	Depends on life-cycle stage and financial position
Defined benefit pension plan	Sufficient to fund pension liability while accounting for inflation	Depends on plan features, age of workforce, and funding status of plan
Defined contribution pension plan	Depends on life-cycle stage of beneficiaries	Depends on risk tolerance of beneficiaries
Endowments and foundations	Sufficient to cover spending needs, expenses, and inflation	Generally average or above average
Life insurance companies	Function of policyholder reserve rates	Below average because of significant regulatory constraints
Non-life insurance companies	Function of policy pricing and financial strength	Below average because of significant regulatory constraints
Bank	Function of cost of funds	Depends on business model and financial strength

An introduction to multifactor models

Summaries

- APT
    - cross-sectional data
    - Intercept: risk free rate
    - error term: no
- Multifactor Models - regression
    - Macroeconomic factor
        - Time series data
        - Intercept: **expected return**
        - Factor sensitivity: estimated **slope**
        - Factor: **surprise** (real – expected)
    - Fundamental factor
        - cross-sectional data
        - Intercept: no meaning
        - Standardized sensitivity: Z-score
        - Factor: estimated **slope**
    - Stats factor
- Return Attribution
- Risk Attribution
- Carhart Model

Arbitrage Pricing Theory (APT)

- linear model with multiple systematic risk factors provided by the market
- unlike CAPM, APT does not identify the specific risk factors
- Assumptions
    - **Unsystematic** risk can be diversified away
        - well-diversified portfolios that eliminate asset-specific risk, not factor risk.
    - Returns are generated using a **factor** model
    - No arbitrage opportunities exist
    - absence of an arbitrage opportunity indicates a condition of financial market equilibrium.
    - A well-diversified portfolio will not contain any arbitrage opportunities

APT Equation

- $\mathrm{E}(\mathrm{R_p}) = R_f + \beta_1 \times \lambda_1 + \cdots + \beta_k \lambda_k$
- Intercept: risk free rate
- λ_k factor **sensitivity**
- λ_k expected **risk premium** for risk factor
- pure factor portfolio λ_k
    - $\beta_k = 1$ and the rest beta are zeros
- Portfolio beta
    - $\beta = \sum_i w_i \beta_i$

- Expected return per unit sensitivity
  - $\frac{R_k}{\lambda_k}$
  - **Short smallest one 卖掉单位收益低的**
- **套利**
  - **一般给 3 个，算出**$\frac{R}{\lambda_k}$，卖掉最低的那个，比如是第三个。
  - **余下的 2 个一个是 w，另一个是 1-w。让 sensitivity 相同**
    - $\boldsymbol{w \times \lambda_1 + (1 - w) \times \lambda_2 = \lambda_3}$

Multifactor Models

- Macroeconomic factor
  - **Surprise** (shocks, real - expected)
  - GDP, interest, inflation
- Fundamental factor
  - Firm-specific factors (P/E, leverage, market cap, earnings growth rate)
- Statistical factor
  - Factor analysis: **covariance** in asset returns
  - Principal component: **variance** in asset returns
  - Weakness: no economic interpretation

Macroeconomic factor

- Time series
- $\mathrm{R} = E(\mathrm{R}) + b_1 \times F_{gdp} + b_2 \times F_{qs} + \epsilon$ 期望 + 非期望(偏差, surprise)
- $E(\mathrm{R})$ **expected** return
- $F_{gdp} = real\ GDP - expected\ GDP$ **真实 – 预期**
- $F_{qs} = real\ credit\ quality\ spread - expected\ credit\ quality\ spread$
  - $quality\ spread = \text{BB rated bond yield} - \text{treasury bond yield}$
- ϵ firm-specific surprise, unsystematic risk
- Priced risk factors
  - Systematic risk factors
  - Unsystematic risk – no price

Fundamental Factors Models

- Cross-sectional data
- $\mathrm{R} = a + b_1 \times F_{p/e} + b_2 \times F_{size} + \epsilon$ 期望 + 非期望(偏差, surprise)
- a: intercept
- $F_{p/e}$: **return** associated with P/E factor
- b_1: standardized sensitivity of stock to P/E factor
- F_{size}: return associated with size factor
- b_2: **standardized sensitivity** of stock to Size factor
- ϵ return not explained by the factor model
- **standardized sensitivity (similar to z-statistics)**
  - $b = \frac{P/E - \overline{P/E}}{\sigma_{p/e}} = \frac{\mathrm{x-u}}{\mathrm{\sigma}}$
  - 标准化
- Factor returns

- $F_{p/e}$ rate of return associated with each factor
  - Difference in rate of return between low and high P/E stocks
- They are **slopes** estimated using multiple **regression**

- Intercept term
  - No economic meaning, not expected return
  - Just to make unsystematic risk of asset equal to zero
- 这是推导模型
  - 系数是计算出，标准化的，不是拟合出来的
  - 因子回报反而是回归出来的

	APT	Multifactor Model
Characteristics	**Cross-sectional** equilibrium that explains variations across asset expected returns 横截面	**Time-series** regression that explains **variation** overtime 时间序列
Assumptions	**Equilibrium** pricing model no arbitrage	**Regression** model
Intercept 截距	**Risk-free return 无风险利率**	**Macroeconomic: Expected return 期望收益**
Error term 参差		Regression error term

Difference

- fundamental factor model 公司财报
  - return on equity, capital investment, and equity returns.
- Macroeconomic factors 经济学宏观
  - interest rates, inflation risk, business cycle risk, or credit spreads
- statistical factor
  - historical covariances

Absolute Return and Risk

- $\mathrm{R_P}$ and σ_P
- Sharpe ratio $\frac{\mathrm{R_P}-R_f}{\sigma_\mathrm{P}}$
  - Information ratio of risk-free as benchmark

Active/relative Return and Risk

- Benchmark $\mathrm{R_b}$ and σ_b
- Define a new variable $\mathrm{X} = \mathrm{R}_p - \mathrm{R_b}$
- Active return $\mathrm{R_P} - \mathrm{R_b} = \mathrm{R_X}$
- Active risk, tracking error, $\sigma_{r_p - r_b} = \sigma_X$
- Information ratio
  - $\mathrm{IR} = \frac{\mathrm{u_x}}{\sigma_x} = \frac{R_p - R_b}{\sigma_{r_p - r_b}}$
- Passive management
  - Use absolute value of alpha
- Active management
  - Use information ratio

Return Attribution

- Active return $\mathrm{R_P} - \mathrm{R_b}$
- Factor return (factor tilts) $\sum_{\mathrm{i}}(\beta_p - \beta_p) \times \lambda_i$
    - Factor exposure different from those of the benchmark
- **Security** selection return $\sum_{\mathrm{i}}(\lambda_p - \lambda_b) \times \beta_p$
    - Choosing a different weight compared to weight of those securities in the benchmark
- The active return consists of two parts: (1) the sum of each factor return multiplied by the difference between the portfolio's sensitivity to that factor and the benchmark's sensitivity and (2) the active return resulting from security selection.

Risk Attribution

- Active risk squared σ_X^2
- Active **factor** risk
    - Active factor tilts attributed to deviations of portfolio's factor **sensitivities** from the benchmark sensitivities to the same set of factors
- Active **specific** risk
    - Deviations of portfolio's individual asset weightings versus the benchmark's individual asset weightings, after controlling for differences in factor sensitivities
    - $active\ specific\ risk = \sum_{\mathrm{i}}(\beta_p^i - \beta_b^i)^2 \times \sigma_{i^2}$ within a factor, the individual stock weight
- Active risk **squared** = active factor risk + active specific risk

Use Multifactor Models

- Passive management
    - **Tracking** portfolio: have the same set of factor exposures to match a benchmark
- Active management
    - Make specific bets on desired factors while hedging for other factors
    - **Factor** portfolio: for hedging purpose
- **Rule-based** or algorithmic active management (alterative indices)

CarHart Model

- Extends Fama and French three-factor model to include momentum
    - $\mathrm{E(R)} = R_f + \beta_1 RMRF + \beta_2 SMB + \beta_3 HML + \beta_4 WML$
    - $\mathrm{E(R)} = R_f + \beta_1(R_M - R_f) + \beta_2(R_s - R_b) + \beta_3(R_h - R_{low}) + \beta_4(R_w - R_{losser})$
    - HML: high book-to-market (value-to-growth)

Portfolios

- Market
- Factor
- tracking

Multifactor Benefits

- Select efficient portfolios
- Adjust risk tolerance
    - Zero on risks that has a comparative advantage
    - Avoid risks that is incapable of absorbing

Measuring and managing market risk

Summaries

- VaR

Value at Risk (VaR)

- Measures the downside risk of a portfolio
- Components
    - Loss size: the minimum amount of loss
    - The probability of a loss greater than or equal to the specified loss size
    - Time frame
- Loss size
    - Absolute loss
    - Percentage loss
- Probability or confidence level
    - Confidence level = 1 – probability
- Distribution
    - 5% left-hand tail is bounded by 5% VaR
    - VaR is the upper limit of the specified left tail
- VaR is an expression of a minimum loss. It is incorrect for Patterson to state that the policy will limit fund losses to xxx.

VaR Estimation

- **Risk factors**
    - Market risk, interest rate risk, currency risk
- Parametric or variance-covariance
- Historical Simulation
- Monte Carlo Simulation

Parametric or variance-covariance

- Normal distribution
    - Mean, variance and covariances

- Estimation
    - Lookback period: the return periods
- $VaR = u - z \times \sigma < 0$ 先算出分布的均值和标准差
- Two securities
    - $u_p = w_1 u_1 + w_2 u_2$
    - $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2cov_{12}$
    - $VaR = u_p - z \times \sigma_p < 0$
    - One-tail
        - 5%: 1.65
- Daily and N-days
    - u_d and σ_d
    - $u_n = N \times u_d$ and $\sigma_n = \sqrt{N} \times \sigma_d$
    - N =250 trading days in a year
    - $VaR_d = u_d - z \times \sigma_d < 0$
    - $VaR_n = u_n - z \times \sigma_n < 0$

Historical Simulation

- Based on actual changes in risk factors over a lookback period
- VaR(5%) Steps
    - Compute daily changes
    - Order them from smallest (negative means loss) to largest
    - Find the lowest 5% of losses
- Advantages
    - No assumptions about distribution
    - No adjustment for difference between lookback period and longer prior period
    - Can be used to estimate the VaR for portfolios with options
- VaR depend on **lookback** period

Monte Carlo Simulation

- Parametric Assumption
    - Based on assumed probability **distribution** for each risk factor
    - Can make assumptions about **correlations** between risk factors
- Generate
    - Generate random values for each risk factor, and pricing model
    - Repeat thousands of times
- Historical Simulations
    - just as **historical simulations,** we can order the outcomes and identify the fifth percentile to estimate 5% VaR.
- Parametric
    - Compute the mean and std, then apply parametric method

VaR Advantages

- simple and easy to explain
- risk of different portfolios, asset classes, or trading operations to be compared to gain a sense of relative riskiness

- performance evaluation
    - consider income to VaR
    - risk-adjusted performance
- risk budgeting
    - maximum allocation of VaR for a business line
- accepted by global banking regulators
- reliability of VaR can be verified by backtesting

VaR Disadvantages

- require many choices
    - loss percentage, lookback period, distribution assumptions, parameter estimates
- normality leads to **underestimates** of downside risk
- liquidity risk not included
- correlation increases during financial risks
    - increasing correlation, VaR estimated based on normal levels of correlation will overestimate diversification benefits
- many aspects are not quantified or included
- focus on downside and extreme negative outcomes

VaR Extensions

- Conditional VaR (expected tail loss/expected shortfall)
- Incremental VaR
    - Change in portfolio allocation to a security
- Marginal VaR
    - Slope of a VaR versus security weight
    - VaR change for 1% change in weight
- Ex ante tracking error / relative VaR
    - VaR of tracking portfolio
    - Long portfolio and short benchmark

Sensitivity Analysis

- Change in one factor
- Relative exposure

Scenario Analysis

- A set of changes of significant magnitude in multiple risk factors
- Consider correlation and simultaneous change in risk factors
- Historical scenario
    - Actual occurred in the past
- Hypothetical scenario
- Hard to compare, different assets have different factors
- Consider liquidity
- **does** not need to rely on history

Stress Test

- Effect on value of a scenario of extreme risk factor changes

Reverse Stress Testing
- Identify largest risk exposures

Risk Factors
- Equity
  - Beta in CAPM
- Fixed-income
  - Duration (to yield)
  - Convexity (second order)
  - $\Delta_{\mathrm{p}} = -D \times \Delta y + \frac{1}{2} \times Conexity \times \Delta y^2$
- Option
  - Delta (to stock price)
  - Gamma (second order)
  - Vega (volatility)
  - $\Delta_{\mathrm{p}} = Delta \times \Delta S + \frac{1}{2} \times Gamma \times \Delta S^2 + Vega \times \Delta\sigma$

Risk Measures Application
- Bank
  - Liquidity gap
  - VaR: trading securities, economic capital
- Asset managers
  - Relative risk measure
  - Position limits, sensitivities
  - **active share** (difference in weight)
  - Ex-post tracking error (backward looking)
    - Performance evaluation
  - Ex-ante tracking error (forward looking)
    - Risk estimation
- Hedge funds
  - Non-normal distribution: **maximum drawdown**
  - Gross exposure, leverage, VaR
- Defined benefit pension funds
  - **Surplus-at-risk**, VaR for assets minus liabilities
  - **Glide path**
- Insurance
  - P&C
    - Market risk, VaR, capital at risk
  - Life insurers
    - Market risk, mortality risk, **asset and liability** matching

Constraints
- Risk budgeting
  - Maximum loss for a firm, and then allocate to different divisions/strategies/asset classes

- Position limits - to diversify
    - Individual securities within an asset class
    - Can be expressed as currency or percentage amounts
    - Can be based on liquidity measure
- Scenarios limits
    - Expected loss for a given scenario
- Stop-loss limits
    - A risk exposure is reduced if losses exceed a specified amount over a certain period of time
    - Portfolio insurance: index fall -> hedge

Capital Allocation

- Limit the overall risk of all activities
- Adjust expected returns for risk

Economics and Investment markets

Summaries

- **Taylor Rule**
- Real risk-free rate: GDP growth
- Inflation (BEI)
  - Expected **inflation**
  - Uncertainty about inflation – **maturity**
- **Credit** risk – bond
- **Equity risk premium – equity**
- **Liquidity** risk – real estate

Discount rate

- Real risk-free discount rate
- Expected inflation
- Risk premium

Risk-free rate

- Default-free bond: compensate an investor for **forgoing** their **current** consumption
- Inter-temporal rate of substitution $\mathrm{m_t}$ 内部替代率
- Marginal utility of real consumption now $\mathrm{u_0}$
- Marginal utility of real consumption in the future $\mathrm{u_t}$
  - $\mathbf{m_t = \frac{u_t}{u_0}}$
- Investors always prefer current consumption over future consumption
  - $\boldsymbol{u_0 > u_t \rightarrow \mathrm{m_t} < 1}$
- Current price P_0 of a zero-coupon, inflation-indexed, risk-free bond that will pay $1 at time t can be expressed as
  - $P_0 \times u_0 = 1 \times u_1 \rightarrow P_0 = \frac{u_1}{u_0}$
  - $\boldsymbol{P_0 = E(m_t) < 1}$
  - $\boldsymbol{R = \frac{1-P_0}{P_0} = \frac{1}{E(m_t)} - 1 = E\left(\frac{u_0}{u_t}\right) - 1 > 0}$
- Higher relative current consumption
  - High real rate
- Higher income in the future
  - Margin future utility decrease -> consume more, save less -> high real rate
- Increase savings when
  - Expected returns are high
  - Uncertainty about future income increases

Risky Cash flows and Risk Premiums

- Risk-aversion
- $\boldsymbol{P_0 = \frac{E(P_1)}{1+R} + \mathrm{cov}(\mathrm{P_1}, \mathrm{m_1}) = \frac{\mathrm{E}(\mathrm{P_1})}{1+R+premium}}$
- Risk aversion: $\mathrm{cov}(\mathrm{P_1}, \mathrm{m_1}) < 0$

GDP Growth Rate – positive correlation

- GDP growth rate is high

- Future consumption utility is low, m_t fall
- Save less, consume more
- Higher interest rate

- Interest rate (**TIPS**)
  - Positive correlated with GDP growth
  - Positive correlated to volatility due to high risk premium

Inflation

- Expected inflation π
- Uncertainty about actual inflation θ – related to maturity
- Short-term risk free (T-bills)
  - $R_{short} = R + \pi$
- Short-term risk free (T-bills)
  - $R_{long} = R + \pi + \theta$

Break-even Inflation Rate (BEI)

- Use risk-free bond such as Treasury bond
- EBI = yield on non-inflation-indexed bond – yield on inflation-indexed bond
- $BEI = \pi + \theta$
  - Expected inflation
  - Risk premium for uncertainty about actual inflation

Taylor Rule

- Setting policy rate
  - Price stability
  - Maximum sustainable level of employment
- Central bank policy rate
  - $\boldsymbol{r = R_n + \pi + 0.5 \times (\pi - \pi^*) + 0.5 \times (y - y^*)}$
  - $\boldsymbol{r = R_n + \pi - 0.5 \times (\pi^* - \pi) - 0.5 \times (y^* - y)}$
  - $\boldsymbol{R_n}$ **neutral real policy interest rate**
  - π current inflation rate
  - π^* target inflation rate
  - $y = \log GDP$ log of current level of output
  - $y^* = \log GDP^*$ log of current target/sustainable output

Term Spread - Maturity

- Expected future GDP growth -> positive
- Difference between the yield on a long-term bond yield and the yield on a shot-term bond
- Positive term spread
  - Increasing inflation for long periods

Risky Bond Yield (risk free + inflation + maturity + credit)

- $R_{risky} = R + \pi + \theta + \gamma$
  - Risk free rate R GDP growth
  - Expected inflation $\pi = R_{nomimal} - R_{real}$

- Term spread $\theta = R_{long} - R_{short}$
- Credit spread $\gamma = R_{risky} - R_{risk\ free}$

- credit spread widens
    - high rated bonds outperform

Business Cycle

- credit quality
    - downturn, consumer cyclical sector spread increase more
- corporate earning
    - cyclical (durable goods and consumer discretionary) more sensitive to defensive or non-cyclical (consumer staples, consumer non-discretionary)
    - downturn, can short cyclical firms
- equity
    - $R_{equity} = R + \pi + \theta + \gamma + k$
        - k additional risk premium relative to risky debt
        - $\gamma + k$ risk premium
    - assets provide a higher payoff during economic downturns are highly valued, because of **consumption hedging property**
    - Equity prices are generally cyclical, high value during expansion
    - Equity risk premium is positive
    - Equity is bad to hedge consumption hedging property
- Multiples (P/E and P/B)
    - Positively related to growth
    - Negatively related to required return
- Style strategy
    - Growth stocks
        - High P/E, high growth, and low dividend
        - Perform well during expansion
    - Value stock
        - Low P/E, low growth, and high dividend
        - Perform well during recession
    - Small-cap stocks tend to have higher volatility and higher risk premium
- Sector rotation
    - Market timing
    - Aftermath of a recession -> bad
    - Economic is better
        - Value -> growth
        - Small -> big
        - Noncyclical -> cyclical
- Real-estate
    - Bond-like: credit quality of tenants
    - Equity-like: **uncertainty** about terminal value
    - **Illiquidity**
    - $R_{real\ estate} = R + \pi + \theta + \gamma + k + \phi$
        - k risk premium for uncertainty about terminal value of property
        - ϕ risk premium for illiquidity
    - rental income is stable

- commercial value -> cyclical
- **bad consumption hedging property**

Analysis of active portfolio management

Summaries

- Active Return Decomposition
- Information Ratio and Sharpe Ratio
- Unconstrained Optimal Active Portfolio
    - $\sigma_A^* = \frac{IR}{SR_B} \times \sigma_B$
    - $SB_p^* = \sqrt{SR_B^2 + IR^2}$
    - $R_A^* = IR \times \sigma_A^*$ (IR unchanged)
    - $\sigma_p^* = \sqrt{\sigma_A^{*2} + \sigma_B^2}$
    - $w^* = \frac{\sigma_A^*}{\sigma_A}$ (risky portfolio weight, optimal and previous active risk)
- Fundamental Law
    - $\mathrm{IR} = \mathrm{TC} \times \mathrm{IC} \times \sqrt{BR}$
    - $\mathrm{TC} = 2 \times \mathrm{correct}\% - 1$
    - $\mathrm{IC} = 1$ for unconstrained portfolio
    - $\mathrm{BR} = \frac{\mathrm{N}}{1+(N-1)\times\rho}$

Active Return

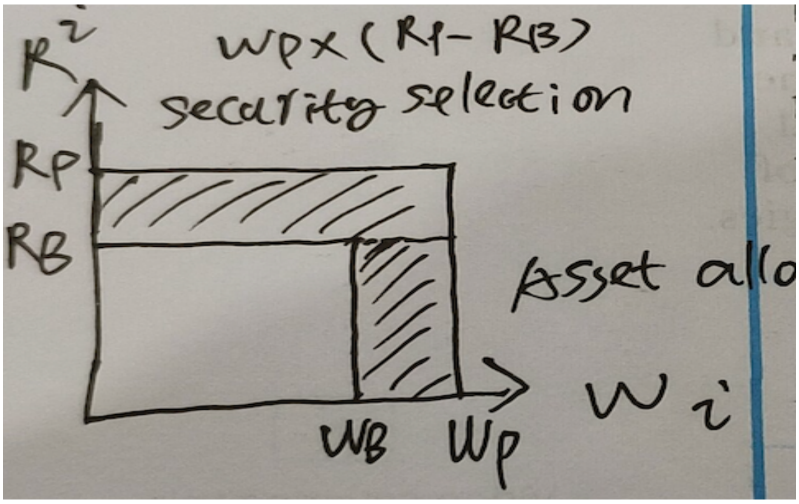
- $\mathrm{E(R_A)} = \mathrm{E}\left(\mathrm{R_p}\right) - \mathrm{E(R_b)}$

Active Weights

- $\Delta w = \mathrm{w_a} = \mathrm{w_p} - \mathrm{w_b}$
- $\mathrm{E(R_A)} = \sum \Delta \mathrm{w_i} \times E(R_i)$
- $\sum \Delta \mathrm{w_i} = \sum w_i^p - \sum w_i^b = 0$

Active Return Components

- $\mathrm{E(R_A)} = \sum w_i^p E(R_i^p) - \sum w_i^b E(R_i^b)$
    - $\mathrm{E}\left(\mathrm{R_p}\right) = \sum w_i^p E(R_i^p)$
    - $\mathrm{E(R_b)} = \sum w_i^b E(R_i^b)$
- Asset allocation return
    - $\sum \Delta w_i \times E(R_i^b)$ active weight * benchmark return
- Security selection return
    - $\sum \mathrm{w_i^p} \times \Delta E(R_i^a)$ active return * portfolio weight
    - $E(R_i^a) = E\left(R_i^p\right) - E\left(R_i^b\right)$



Information Ratio and Sharpe Ratio

- Sharpe Ratio
  - $\mathrm{SR} = \frac{\mathrm{R_p} - R_f}{\sigma_p}$
  - Unaffected by **addition** of cash or leverage
- Information Ratio -> for active portfolio
  - $\mathrm{IR} = \frac{\mathrm{R_p} - R_b}{\sigma_{r_a - r_b}} = \frac{R_A}{\sigma_A} = \frac{active\ return}{active\ risk}$
  - Unaffected by **aggressive** of active weight
- **Closet index fund**
  - **Sharpe ratio** similar to that of the benchmark index
  - Very low active risk and low information risk
  - After fee, IR is often negative

Optimal Portfolio

- Unconstrained active portfolio, optimal amount of **active** risk is the level of active risk that maximizes the portfolio's Sharpe ratio
  - Original fund (track benchmark)
    - absolute return R_1 and absolute risk σ_1
    - active return $R_a = R_1 - R_b$, active risk σ_a
    - information ratio $IR = \frac{R_a}{\sigma_a}$
  - Benchmark (track risk-free)
    - Absolute return R_b, absolute risk σ_b
    - Hypothesis Information Ratio $IR_b = \frac{R_b - R_f}{\sigma_{R_b - R_f}} = \frac{R_b - R_f}{\sigma_b}$
    - Sharpe ratio $SR_b = \frac{R_b - R_f}{\sigma_b}$
  - New optimal active risk (min squared error -> mean/variance)
    - $IR^* = \frac{R_{a^*}}{\sigma_a^*} = IR$ (IR is unaffected by change in portfolio)
    - $\frac{R_{a^*}}{{\sigma_a^*}^2} = \frac{R_b - R_f}{\sigma_b^2} \rightarrow \frac{\mathrm{IR}}{\sigma^*} = \frac{SR_b}{\sigma_b}$
- Optimal unconstrained

- $\frac{SR_P}{\sigma_P}(absolute) = \frac{IR}{\sigma_A^*}(relative) = \frac{SR_B}{\sigma_B}(\text{benchmark})$
  - $\neq \frac{IR}{\sigma_A}(original\ fund)$ 除法
  - $\sigma_P = \sqrt{\sigma_A^{*2} + \sigma_B^2}$
  - $SR_P = \sqrt{IR^2 + SR_B^2}$
  - $\sigma_A^* = \frac{IR}{SR_B} \times \sigma_B$
  - $R_P - R_F = R_P - R_B + R_B - R_F \rightarrow SR_p \times \sigma_p = IR \times \sigma_A^* + SR_B \times \sigma_B$ 乘法
- active risk $\sigma^* = \frac{IR}{SR_b} \times \sigma_b$
- active return $R_a^* = R_p - R_b = IR \times \sigma^* = \frac{IR^2}{SR_b} \times \sigma_b$
- risk free return $R_p - R_f = IR \times \sigma_p^* + SR_b \times \sigma_b$
  - $R_p - R_f = (R_p - R_b) + (R_b - R_f) = IR \times \sigma_p^* + SR_b \times \sigma_b$
  - $= \frac{IR^2 + SB_b^2}{SB_b} \times \sigma_b$
- Sharpe ratio $SR_p = \sqrt{SR_b^2 + IR^2}$
- Total portfolio risk: $\sigma_p^2 = \sigma_b^2 + \sigma_a^{*2}$
- Allocation weight to original fund $w = \frac{\sigma_a^*}{\sigma_a} = \frac{\frac{IR}{\sigma_a}}{\frac{SR_b}{\sigma_b}}$
- allocation to benchmark is $1 - w$

Fundamental Law - Information Ratio (IR)

- Information coefficient (IC) – $\mathrm{Corr}(R_a, E(R_a))$ active return 预测能力
  - Manager's skill
  - Ex-ante (expected), risk-weighted **correlation between active returns** and forecasted active return
  - Ex-post measures **actual correlation** between active return and expected active return
- Transfer coefficient (TC) - $\mathrm{Corr}(w_a, w_a^*)$ active weight 执行能力
  - portfolio construction
  - Correlation between actual active weight and optimal active weights
- Breadth (BR)
  - Number of independent **active bets** per **year**
- Active risk - aggressiveness
- Grinold rule – expected active return
  - $u_i = IC \times \sigma_i \times S_i$
    - S_i score of security I (standardized with an assumed variance 1)
- $IR = TC \times IC \times \sqrt{BR}$
- Return (four factors)
  - $E(R_A) = IR \times \sigma_A$
  - skill (information coefficient), portfolio construction (transfer coefficient), breadth (number of independent decisions per year), and aggressiveness (benchmark tracking risk

Constrained Optimal Portfolio

- active risk $\sigma^* = TC \times \frac{IR_{unconstrained}}{SR_b} \times \sigma_b$
- Sharpe ratio $SR_p = \sqrt{SR_b^2 + TC^2 \times IR_{uncontrained}^2}$
- Sharpe ratio of actively managed portfolio > Sharpe ratio of benchmark
- Portfolio with highest I
- R = portfolio with highest SR

Ex-post Performance Measure

- Ex-post information coefficient IC_R
- Expected value added $E(R_A|IC_R) = TC \times IC_R \times \sqrt{BR} \times \sigma_A$
- Actual return $R_A = E(R_A|IC_R) + noise$
- Variance attribution
  - forecast attribution TC^2
  - noise attribution $1 - TC^2$

Information coefficient (IC) 预测能力

- $IC = 2 \times correct\% - 1$

Transfer coefficient (TC) 转换能力

- Unconstrained portfolio $\mathbf{TC = 1}$
- Constrained portfolio $\mathrm{TC} < 1$

Breadth (BR) 投资次数

- For N independent decisions
  - $BR = N$
- For N dependent decisions with correlation r
  - $BR = \frac{N}{1+(N-1)\times r}$
- Comprised by
  - Cross-sectional dependency
  - Time-series dependency

Active Risk (AR) - Sector Rotation 主动风险

- Two sectors X $(\mathrm{u_x}, \sigma_x)$ and Y $(\mathrm{u_y}, \sigma_y)$
- long one and short the other X-Y
- active risk $\sigma_c^2 = \sigma_x^2 - 2\sigma_x\sigma_y\rho + \sigma_y^2$
- Annualized active risk $\sigma_A = \sigma_C \times \sqrt{BR}$
- Annualized active return $\mathrm{R_A} = \sigma_A \times IR = IC \times \sqrt{BR} \times \sigma_A$

Fundamental Law – Applications

- Selecting the highest information ratio manager will produce the highest Sharpe ratio for the portfolio
- Applications
  - Security selection, market timing, sector rotation

- Limitation
    - Ex-ante measurement of skill
        - Tend to **overestimate** skill
    - Independence
        - Value strategy applied to different stocks-> not truly independent
        - Comprised by **time-series** correlation
        - Monthly rebalance -> not necessary independence
        - Cannot be applied to fixed-income
            - Duration interest rate risk to all of them

Algorithmic trading and high-frequency trading

Execution Algorithms – how to trade 算法交易

- minimize market impact
- Algorithms
    - volume-weighted average price algorithms
    - implementation shortfall algorithms
    - market participation algorithms

High-frequency trading algorithm – when and what to trade 高频交易

High-frequency trading – stat arbitrage algorithms

- pair trading
- index trading
- basket trading
- spread trading
    - intra-market
    - inter-market
    - inter-exchange spread
    - multilegged inter-exchange
- mean reversion
- delta neutral

High-frequency trading other

- liquidity aggregation and smart order routing
- real-time pricing of securities
- trading on news
- genetic tuning

Market Fragmentation

- liquidity aggregation
    - Super book
- Smart order routing
    - Direct orders to market with the best combination of liquidity and price

Risk Management

- Real-time pre-trade risk firewall
- Back testing and market simulation

Regulatory Oversight

- Insider trading
- Front running
- Painting the tap
    - Buy to drive up price and then sell
- Ficitions orders

    - Quote stuffing
        - Order and cancel them
    - Layering
        - Place genuine and fake orders
    - Spoofing
        - Place between bid and ask and cancel them before they are executed
- Wash trading
    - Buy and sell to increase volume
- Trader collusion
    - Multiple traders conspire

Positive Impacts

- Facilitate large trades
- Increased liquidity
- Lower cost
- Tighter bid-ask spreads
- Improved pricing efficiency
- Promotes open and competitive markets
- Increased competition between trading venues

Negative Impacts

- Unfair speed advantages
- Magnification of market movements
- Market manipulation
- Risk of trading errors
- Out-of-control algorithms
- Denial-of-service
- Slowed markets due to excessive orders
- Increased difficulty of policing the market
- Unequal access to information