Simplified Version

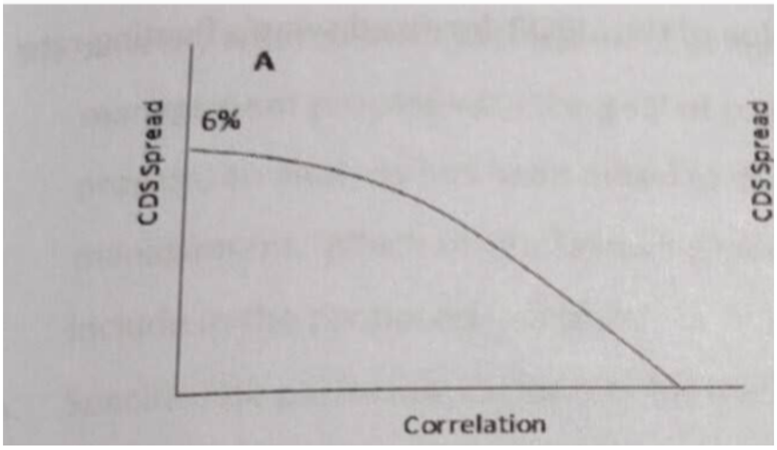
- Buyer:
- Asset A: binomial distribution P(A) default probability of A
- CDS seller: B binomial distribution P(B) default probability of B
- Buyer pay
    - Spread
- CDS seller will pay
    - $\mathrm{PD} \times \mathrm{RR}$
- Equal
    - $\mathrm{spread} = \mathrm{PD} \times \mathrm{RR}$
    - $\rightarrow \frac{\mathrm{spread}}{\mathrm{RR}} = \mathrm{PD}$ （RR是常数的）
- PD (asset default but CDS seller does not default)
    - $\mathrm{PD} = \mathrm{P(A\ default\ but\ B\ not\ default)} = \mathrm{P(A)} - \mathrm{P}(AB)$
    - $\rightarrow \mathrm{P(A)} - (P(A)P(B) + \rho\sigma_A\sigma_B)$
    - $\rightarrow \mathrm{P(A)} - \mathrm{P(A)P(B)} - \rho\sigma_A\sigma_B$ (inverse function of correlation)
- Case study (assume $\mathrm{RR} = 1$ for simplicity)
    - $\rho = 0 \rightarrow \mathrm{spread} = \mathrm{P(A)} - \mathrm{P(A)P(B)} > 0$
    - $\rho = 1 \rightarrow \mathrm{spread} = \mathrm{P(A)} - \mathrm{P(A)P(B)} - \sigma_A\sigma_B$
        - let $\mathrm{P(A)} = P(B) = p$
    - $= p - p^2 - \sqrt{p \times (1-p)} \times \sqrt{p \times (1-p)} = p - p^2 - p(1-p) = 0$
- **Another way 另一种方法**
    - $\mathrm{spread} = \mathrm{PD} \times \mathrm{LGD} = \mathrm{PD} \times (1 - \mathrm{RR}) \rightarrow \frac{spread}{LGD} = PD$
    - 核心是 PD 如何计算，用相关性公式，结论不变化的
- $1 + \mathrm{r} = \mathrm{P(A)} \times \mathrm{RR} + (1 - \mathrm{P(A)}) \times (1 + \mathrm{r} + \mathrm{s})$
    - $\mathrm{s} = \mathrm{YTM} - R_f = \frac{R_f \times \mathrm{PD} + \mathrm{PD} \times \mathrm{LGD}}{1 - PD} \sim \frac{k}{\frac{1}{PD} - 1}$??

Figure 1: Default Probabilities for Two Firms

Event	x_1	x_2	$(x_1\ x_2)$	*Default Probability*
Firm 1 Defaults	1	0	0	$\pi_1 - \pi_{12}$
Firm 2 Defaults	0	1	0	$\pi_2 - \pi_{12}$
Both Default	1	1	1	π_{12}
No Default	0	0	0	$1 - \pi_1 - \pi_2 + \pi_{12}$

$$\rho_{12} = \frac{\pi_{12} - \pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

A hedge fund holds a subordinate unsecured note issued by an oil refinery and is negotiating with a bank to buy protection on the note using a CDS. Based on internal analysis at the fund, a fair value for the spread would be 6% assuming no correlation between a default on the note and a default of the bank. Which of the following graphs shows the correct relationship between the default correlation and the CDS spread?



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