The Time Value of Money

Five Elements

- Compound interest (interest on interest) (I/Y)
  - Interest rate (Percentage) per compounding period
- Compounding periods (N)
- Present Value (PV)
- Annuity Payment (PMT)
- Future Value (FV)

Cash Flow

- **Time Line**
  - Start time is 0, cash flow usually occurs at the **end** of a period
  - Singed Amount: Inflow (+), outflow (-)
- **Compounding**: PV -> FV
- **Discounting**: FV -> PV

Interest Rate Interpretation

- Compounding: Required rate of return 复利率
- Discounting: discounted rate 折现率
- **Opportunity** cost 机会成本
  - The value that investor forgone when consumption than saving

Interest Rate Composition/Premium

- **Real risk-free rate 真实无风险利率**
  - Theoretical rate on a single-period that have no inflation in it
- **Nominal risk-free rate 名义无风险利率**
  - Nominal risk-free rate = real risk-free rate + **inflation** rate/premium
  - Inflation premium 通货膨胀溢价
- **Types of Risks**
  - Default risk: 违约风险
  - Liquidity risk: 流动性风险 liquid is better
  - Maturity risk: 到期风险 long term more risk

Interest Compounding

- **Stated/Quoted Annual interest rate**: r 名义、挂牌年利率
- **Compounding Frequency**: m 复利频次
- **Periodic interest rate**: r/m 期间利率
- **Effective Annual Rate** (EAR) 有效年利率
  - $\left(1+\frac{r}{m}\right)^{m}-1$ 一年真实回报率
- **Continuous Compounding** 连续复利
  - Effective annual rate (EAR): $e^{r}-1$
- **Multiple Years 多期**

- Discrete compounding EAR: $\left(1+\frac{r}{m}\right)^{m\times t}-1$多年利率
- Continuously compounding EAR: $e^{r\times t}-1$多年利率

PV and FV

- **Simple Interest 单利**
  - $FV = PV \times (1 + r \times t)$
- **Discrete Compounding 复利**
  - $FV = PV \times \left(1+\frac{r}{m}\right)^{m\times t}$
- **Continuous Compounding**
  - $FV = PV \times e^{r\times t}$
- **Factor**
  - FV = PV * **future** value (interest) factor
  - PV = FV * **present** value (interest) factor
  - PV = FV * **discount** factor

Annuity 年金

- **Annuity**
  - **Equal** cash flow that occurs at **equal** interval over a given **period**.
  - **Finite** set of **constant** sequential cash flow
- **Ordinary Annuity 普通年金**
  - Cash flow occurs at the **end** of each period
- **Annuity Due 期初年金**
  - Cash flow occurs at the **beginning** of each period
  - 计算器里用 BEN 模式
- **Annuity Due vs Ordinary Due**
  - 多一次复利=少一次折现
  - Annuity Due > Ordinary Due 恒大
  - $FV_D = FV_O \times \left(1+\frac{r}{m}\right)$
  - $PV_D = PV_O \times \left(1+\frac{r}{m}\right)$
- **Perpetuity 永续年金**
  - Cash flow last forever: English Bond, Preferred dividend
  - $PV = \frac{PMT}{\frac{r}{m}}$

Unequal Cash Flow 非常规现金流

- use cash flow mode, NPV

TVM Problem 时间价值问题

- **Beginning Cash Flow**
  - Cash flow occurs at the **beginning**
  - MUST use **BEN** mode to compute discount rate or number of compounding period
- **Loan Payment and Amortization 按揭贷款**

- **PMT = interest + principle**
- Interest = beginning balance（剩余本金） * r/m
- Beginning balance at time=0 is PV
- **Ending balance = beginning balance + interest – PMT**
  - = beginning balance * (1 + r/m) – PMT
  - = beginning balance – **principle 本金的降低**

TVM Calculator
- 2^{nd} + FC -> Clear TVM
- 2^{nd} + CPT -> Quit
- 2^{nd} + BEG + 2^{nd} + END -> END
- Keep P/Y = 1 fixed

Cash Flow Calculator
- 2^{nd} + CF -> start cash flow
  - CF_0 initial at time=0
  - C01: payment at time=1
  - F01: repeated time
- 2^{nd} + CF + 2^{nd} + CE|C -> Clear CF
- IRR + CPT -> IRR

Discounting Cash Flow Applications

NPV and IRR - Definition
- **Net Present Value (NPV) 净现值**
  - NPV = $\sum \frac{CF_i}{(1+r)^n}$
  - Costs: outflow, benefits: inflow
- **Internal Rate of Return (IRR) 内部收益率**
  - NPV=0 => IRR
  - Problems: can have multiple IRRs or no IRR.

NPV and IRR - Reinvestment
- NPV
  - reinvestment at the cost of **capital**, reflect market-based **opportunities** of capital, **more** realistic
- IRR
  - reinvestment at the required rate of return by **investor**, not realistic

NPV and IRR - Decision Rule
- **NPV**
  - NPV > 0 => accept
  - NPV < 0 => reject
  - Mutually exclusive project => **higher** positive NPV
- **IRR**
  - IRR > required rate => accept
  - IRR < required rate => reject

- Mutually exclusive project => higher IRR
- **NPV and IRR conflicts**
  - Select the one with highest **NPV**

Holding Period Return

- **Holding period return (HPR) 持有期收益率**
  - $\mathrm{HRP} = \frac{P_2 - P_1 + D}{P_1}$
- **Return**
  - Price Appreciation 价格升值: $\mathrm{P}_2 - P_1$
  - Cash Received 现金收益: D
- **Total Return 总回报**: $\mathrm{P}_2 - P_1 + D$

Multiple Period Return

- **Methods**
  - Money-Weighted return: IRR
  - Time-weighted Return
    - **geometric** mean (If more than one year)
      - $\mathrm{HPR} = \sqrt[n]{(1 + \mathrm{HPR}_1) \times (1 + \mathrm{HPR}_2) \cdots (1 + HRP_n)} - 1$
    - Simple compounding (less than one year)
      - $\mathrm{HPR} = (1 + \mathrm{HPR}_1) \times (1 + \mathrm{HPR}_2) \cdots (1 + HRP_n) - 1$
- **Comparison**
  - Fund contributed before a poor performance: IRR < HPR
  - Fund contributed at a favourable time: IRR > HPR
- **Practice**
  - Most investment fund use **HPR** since it is not affected by **timing** of cash flow
  - If fund manager has control over the cash flow **timing**, use **IRR** may be better

Other Measures

- **Assumption**
  - Beginning price: P_1 (present value)
  - Ending price: P_2 (face value)
  - Holding time: t
- **Holding Period Return**
  - $\mathrm{HPR} = \frac{P_2 - P_1}{P_1}$
- **Bank Discount Yield (BDY)**
  - US T-bill quoted based on a discount on the face value
    - $\mathrm{BDY} = \frac{P_2 - P_1}{P_2} \times \frac{360}{t}$
  - **Discussion**
    - Not a real yield
    - Use simple interest
    - Use 360 days
- **Money Market Yield (MMY)**
  - Money market quoted based on a return on the value
    - $\mathrm{MMY} = \frac{P_2 - P_1}{P_1} \times \frac{360}{t} = HPR \times \frac{360}{t}$

- **Discussion**
  - Use simple interest
  - Use 360 days

- **DBY and MMY**
  - Return $\mathrm{P2} - \mathrm{P1} = \mathrm{BDY} \times \mathrm{P_2} = MMY \times P_1$

- **Effective Annual Yield (EAY)**
  - **Assume daily compounding**
    - $\mathrm{EAY} = \frac{P_2}{P_1}^{\frac{365}{t}} - 1 = (1 + \mathrm{HPR})^{\frac{365}{t}} - 1$
  - **Formula**
    - Given annualized rate: EAY
    - **Effectively Daily Rate EDY and EAR**
      - **每天的回报 365 天复利是年回报**
      - $(1 + EDY)^{365} = 1 + EAR$ **(geometric compounding)**
      - $\mathrm{EDY} = (1 + EAY)^{\frac{1}{365}} - 1$
    - **Holding Period Return**: $1 + HPR = (1 + EDY)^{\mathrm{t}} = (1 + EAY)^{\frac{t}{365}}$
    - $\mathrm{EAY} = (1 + HPR)^{\frac{365}{t}} - 1$
  - **Discussion**
    - Use compounding
    - Use 365 days

- **Bond Equivalent Yield (BEY)**
  - BEY: twice the semi-annual bond, US bond usually quote using this
    - $\mathrm{BEY} = 2 \times \left(\frac{P_2}{P_1}^{\frac{180}{t}} - 1 \right)$
  - 利用 EAY 计算出半年的 Effective Semi-annually yield (ESY)
  - **EAY 是 ESY 的 2 次复利**
    - $\mathbf{1} + \boldsymbol{EAY} = (\mathbf{1} + \boldsymbol{ESY})^{\mathbf{2}}$
    - $\boldsymbol{BEY} = \mathbf{2}\ \boldsymbol{ESY} = \mathbf{2}\sqrt[2]{\mathbf{1} + \boldsymbol{EAY}} - \mathbf{1}$
  - **Detailed Formula**
    - $(1 + EDY)^{180} = 1 + ESY$ **(geometric compounding)**
    - $\mathrm{EDY} = (1 + EAY)^{\frac{1}{180}} - 1$
    - $1 + HPR = (1 + EDY)^{\mathrm{t}} = (1 + ESY)^{\frac{t}{180}}$
    - $\mathrm{ESY} = (1 + HPR)^{\frac{180}{t}} - 1$