- Portfolio Risk and Return: Part I
- Portfolio Risk and Return: Part II
- Basic of portfolio planning and construction

Portfolio Risk and Return: Part I

Return

- Holding period return
  - $\text{HPR} = \frac{P_1 - P_0 + D}{P_0}$
- Average returns
  - Simple average of a series periodic returns
  - **Unbiased estimator** of true mean
- Geometric mean
  - Compound annual return
- Money-weighted rate of return
  - Internal rate of return based on cash inflow and outflows
  - **Inflow**: **beginning** value, deposit cash
  - **Outflow**: withdraw of cash, interest, dividends, **ending** value
  - Use the shortest period between significant cash flows
  - If monthly -> **compound** to get effective annual rate
- Gross return
  - Total return before deducting fees (management & administration)
- Net return
  - Gross return – fees
- Commissions and other costs
  - **Deducted** in both gross and net returns
- Pre-tax nominal return
  - Return prior to paying taxes
- After-tax nominal return
  - Return after tax liability is deduced
- Real return – nominal return adjusted for inflation
  - Nominal return 7%, inflation is 2% -> approximated real return is 7%-2%=5%
  - Extract real return is 1.07/1.02-1=4.9%
  - $\mathbf{1 + nominal\ return = (1 + real\ return) \times (1 + inflation)}$
  - Increase in purchasing power
- Leveraged return
  - Multiple of return on the underlying asset
  - Percentage of cash investment
  - Derivative
  - Leveraged investment: deposit margin and borrow money
- Returns of major asset classes
  - Small-cap stocks: higher return and variance than large-cap stocks
  - Long-term bonds > long-term treasury bonds > Treasury bills
- Liquidity
  - Affect price
  - A major concern in emerging markets and infrequent traded securities

Variance

- Variance
  - Population and sample variance (n-1)
- Covariance
  - Sample covariance (n-1)
- Correlation
  - Zero: uncorrelated, not independent

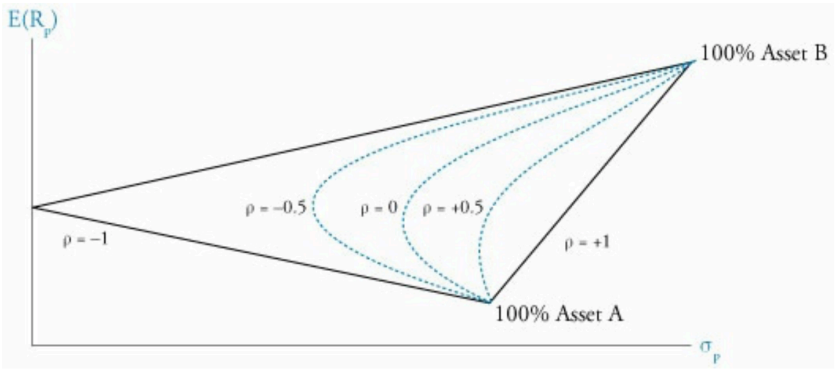
Risk preference

- Risk-averse: One dislikes risk, choose the one with less risk
  - Choose one with least risk
  - May select very risky portfolio if extra return can compensate the extra risk
- Risk-seeking (risk-loving): choose more risky investment
- Risk-neutral: no preference

Portfolio standard deviation

- $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$
- Minimize variance condition
  - $\frac{\partial\sigma_p}{\partial w_1} = \frac{w_1\sigma_1^2 + w_2\rho\sigma_1\sigma_2}{\sigma_p}$
  - $\frac{\partial\sigma_p}{\partial w_1} = \frac{\partial\sigma_p}{\partial w_2} \rightarrow w_1\sigma_1^2 + w_2\rho\sigma_1\sigma_2 = w_2\sigma_2^2 + w_1\rho\sigma_1\sigma_2$
  - $\rightarrow w_1(\sigma_1^2 - \rho\sigma_1\sigma_2) = w_2(\sigma_2^2 - \rho\sigma_1\sigma_2)$
  - $\rightarrow \frac{w_1}{w_2} = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - \rho\sigma_1\sigma_2}$
- Perfect positively correlation
  - $\sigma_p = w_1\sigma_1 + w_2\sigma_2$
  - Condition $w_1\sigma_1 = -w_2\sigma_2$
- Uncorrelated (zero correlation)
  - $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2}$
  - Condition $w_1\sigma_1^2 = w_2\sigma_2^2$
- Perfectly negative correlation
  - $\sigma_p = |w_1\sigma_1 - w_2\sigma_2|$
  - Condition $w_1\sigma_1 = w_2\sigma_2 \rightarrow \sigma_p = 0$
  - Zero variance can be constructed

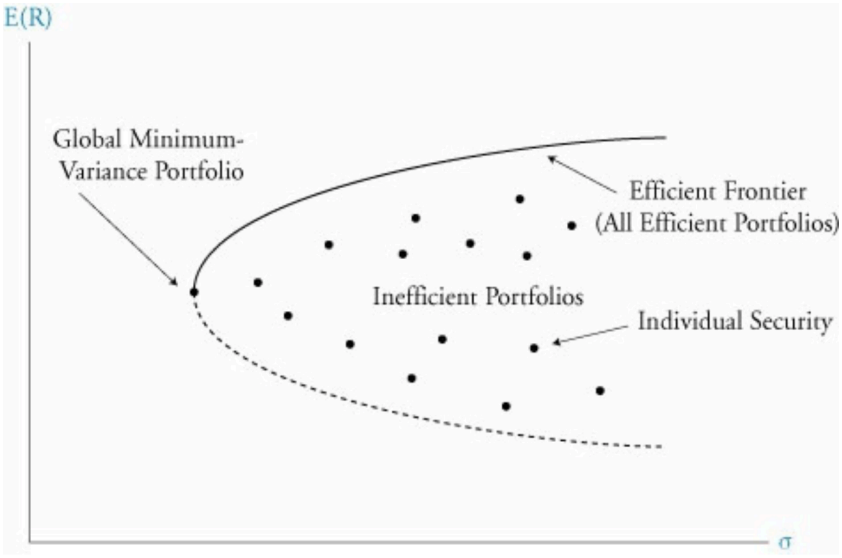
re 2: Risk and Return for Different Values of ρ



Minimum-variance

- Minimum-variance portfolio
  - Portfolio has minimum variance by varying asset weights
- Minimum-variance frontier
  - All the MVPs
- Efficient frontier
  - Assume **risk-averse**, higher return given the same risk
  - The curve that lies **above and to the right** of the minimum-variance frontier
- Global minimum-variance portfolio
  - The left point with **least** variance

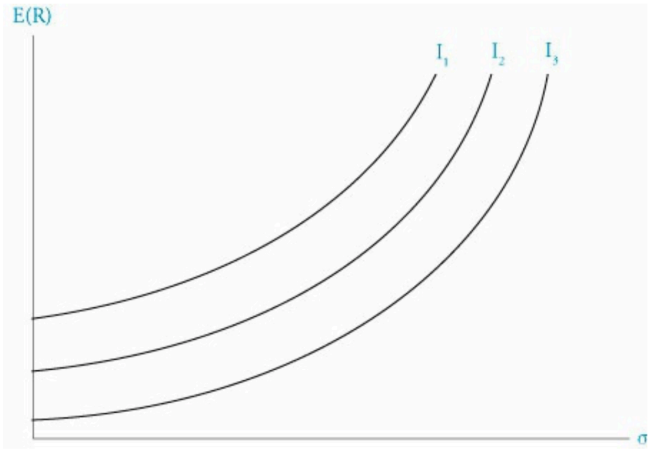
: Minimum-Variance and Efficient Frontiers



Investor utility

- Utility function
  - Investor preference in terms of risk and return
  - $\mathrm{U} = \mathrm{E}(\mathrm{r}) - \frac{1}{2}\gamma\sigma^2$
  - Risk preference γ
    - Marginal rewards for an additional risk
    - $\gamma > 0$ risk aversion (steeper)
    - $\gamma = 0$ risk neutral
    - $\gamma < 0$ risk seeking (flatter)
  - Risk aversion is best illustrated by a **positive** risk-return relationship
- Indifference curve
  - From economics, combination of risk and returns that is indifferent
  - Expected utility is the **same** for all points along a **single** indifference curve
  - Upward
  - More risk-averse: **steeper** curve

4: Risk-Averse Investor's Indifference Curves



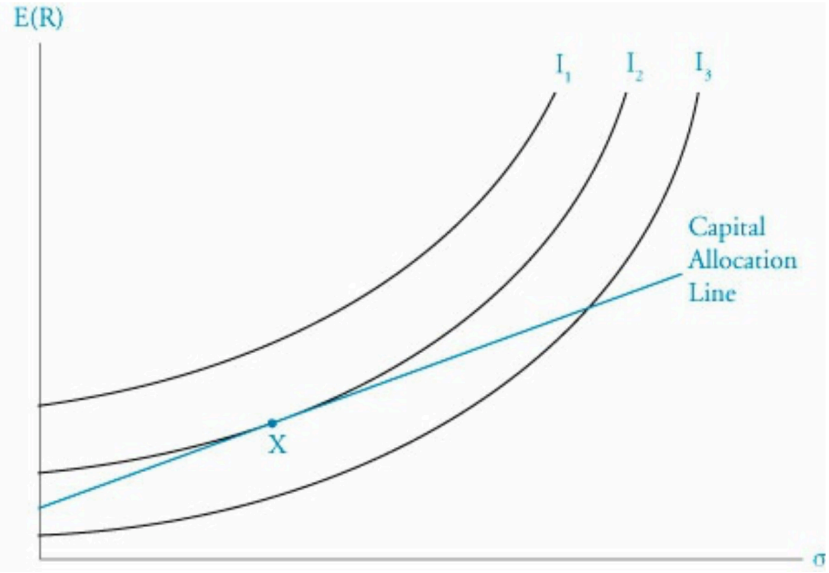
Two-fund separation theorem – capital allocation line

- Combine a risk-free asset and **optimal** risky portfolio
  - $R_p = w_f \times R_f + w_r \times R_r$
  - $\sigma_p^2 = w_r \times \sigma_r$
- Capital allocation line (CAL) – efficient portfolios
  - The line connects risk-free and risky portfolio
  - Different investors with different expectation have different CAL

Optimal Portfolio selection

- Intersection between capital allocation line and indifference curve (yield high return)

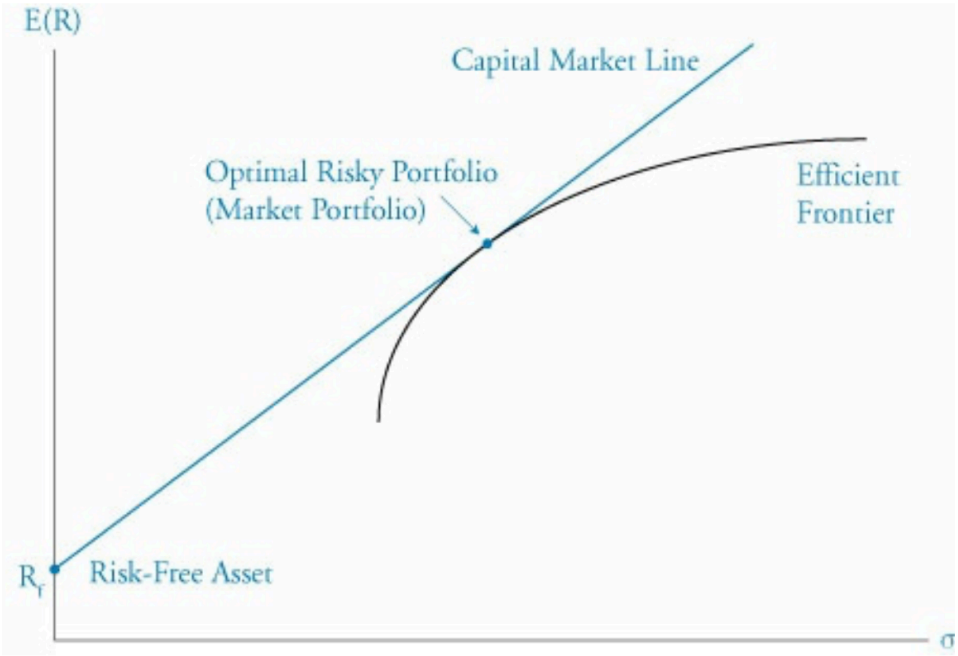
6: Risk-Averse Investor's Indifference Curves



Portfolio Risk and Return: Part II

Capital Market Line (CML)

- Market portfolio
  - Assume **homogeneous** expectation
  - The **tangency** portfolio
- Capital Market Line (CML)
  - $E(R_p) = R_f + \sigma_p \times \frac{E(R_m) - R_f}{\sigma_m}$
  - Market risk premium $E(R_m) - R_f$
  - Left: lending
  - Right: borrowing
- Passive investment
  - Informationally efficient
- Active portfolio
  - Models values are correct
  - Market prices are inaccurate



Risk Decomposition

- $\mathbf{total\ risk = systematic\ risk + unsystematic\ risk}$
- market portfolio: well-diversified
- diversification -> reduce unsystematic risk
- once use 30 securities, standard deviation remains constant
- increase number of stocks
  - systematic risk: can **increase or decrease**
  - unsystematic risk: decrease at a **decreasing** rate

Systematic risk

- assume diversification is **free**

- unsystematic risk is **not compensated** in equilibrium because it can be **eliminated** for free through diversification
- systematic risk is measured by contribution of a security to the risk of a **well-diversified** portfolio
- expected equilibrium **return** depends on its **systematic** risk

Return generating models

- factors
  - economic: GDP, inflation, consumer confidence
  - fundamental: earning, earnings growth, firm size, research expenditure
  - statistical: less likely to be included, they do not have theoretical basis
- multifactor model
  - $\mathrm{E}(\mathrm{R_p}) - \mathrm{R_f} = \beta_1 \times (E(R_1) - R_f) + \cdots + \beta_k \times (E(R_k) - R_f))$
  - Factor loading/sensitivity: β_k
  - The first factor is often the **market** factor $E(R_m) - R_f$
- **Fama and French**
  - Firm size (SMB): small cap – big cap
  - Firm (HML or VMG) - feedback
    - book value-to-market value, value stock – growth stock
    - Value stock: low market value
- Carhart
  - Momentum (UMD, WML): up-minus-down
  - Counter-feedback
- Single index model
  - $\mathrm{E}(\mathrm{R_p}) - \mathrm{R_f} = \beta \times (E(R_m) - R_f)$
- Market model
  - $\mathrm{R_p} = \alpha + \beta \times R_m + \epsilon$

Beta

- $\beta_k = \frac{cov(k,m)}{\sigma_m^2} = \rho \times \frac{\sigma_k}{\sigma_m}$
- Lower beta -> lower market risk
- **Security characteristic line (SCL)** – regression line
  - Excess return vs excess market return
  - Least square linear regression
  - Slope is the beta

Capital asset pricing model (CAPM)

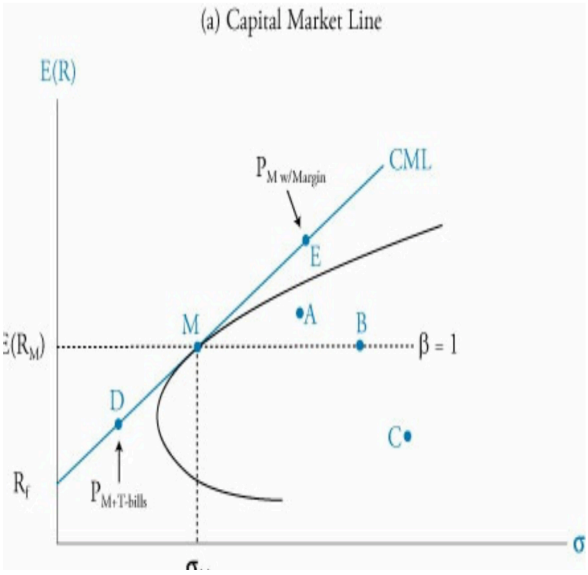
- **Security market line (SML)** - covariance
  - Return vs covariance between asset and market
  - $\mathrm{E}(\mathrm{R_i}) = \mathrm{R_f} + \frac{(E(R_m) - R_f)}{\sigma_m^2} \times \mathrm{cov(i, m)}$
- **Security market line (SML) - beta**
  - Return vs beta
  - $\mathbf{E(R_i) = R_f + \beta_i \times (E(R_m) - R_f)}$
- Assumptions
  - Risk **aversion**

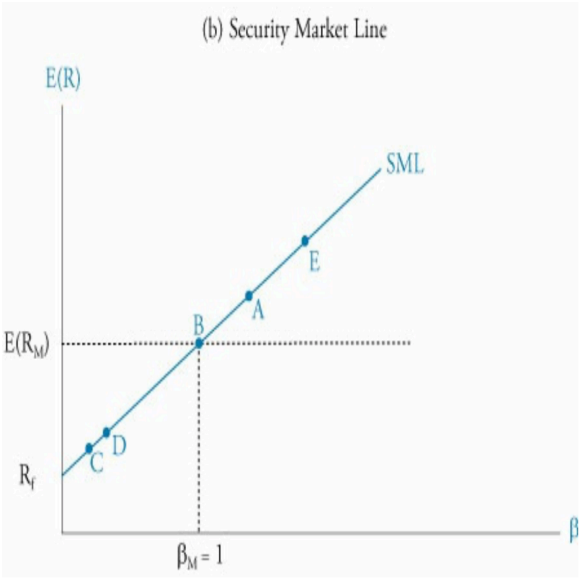
- Utility maximizing investor
- Frictionless market: no tax, transaction cost
- One-period horizon
- Homogeneous expectation: return, variance, correlation
- Divisible asset: infinitely divisible
- Competitive market: all price **takers**

- Required rate of return
  - Expected return and its required return (by investor) are equal
  - Use **CAPM** to estimate required rate of return
- **Misprice** - Undervalue or overvalue – not on the SML line
  - Forecast return (真实回报) and required return (理论回报)
  - **Overvalue**: forecast return < required return 回报低于预期，高估
  - **Undervalue**: forecast return > required return 回报高于预期，低估
  - Properly valued: forecast return = required return
  - 左低右高：在图里画，线的左边是低估，线的右边是高估

CML and SML

- CML
  - Only **efficient portfolio** on the CML
- SML
  - All **properly priced** securities and portfolios





Risk-adjusted Return

- **Sharpe ratio – CAL slope**
  - $\mathrm{SR} = \frac{\mathrm{R_p} - R_f}{\sigma_p}$
- **M-Squared – CAL (same rank with Sharpe ratio)**
  - $\mathrm{M}^2 = R_f + \frac{R_p - R_f}{\sigma_p} \times \sigma_\mathrm{m} - R_m = \frac{R_p - R_f}{\sigma_p} \times \sigma_\mathrm{m} - (R_m - R_f)$
  - $M^2 = SR \times \sigma_m - (R_m - R_f)$
  - $P \rightarrow P^* \ (with\ market\ risk) \rightarrow P^* - M$

- **Treynor ratio – SML slope**
  - $\mathrm{TR} = \frac{\mathrm{R_p} - R_f}{\beta_p}$
- **Jensen's alpha (same rank with Treynor ratio)**
  - $\alpha = \mathrm{R_p} - E(E_p) = R_p - \left(R_f + \beta \times (R_m - R_f)\right)$
  - $\rightarrow \frac{\alpha}{\beta} = \frac{R_p - R_f}{\beta} - (\mathrm{R_m} - \mathrm{R_f}) = \mathrm{TR} - (\mathrm{R_m} - \mathrm{R_f})$
- **total risk or systematic risk**
  - total risk: Sharpe ratio and M-Squared
    - a single manager
  - systematic risk: Treynor and Jensen's alpha
    - multiple managers, overall fund is diversified

'e 11: M-squared for a Portfolio

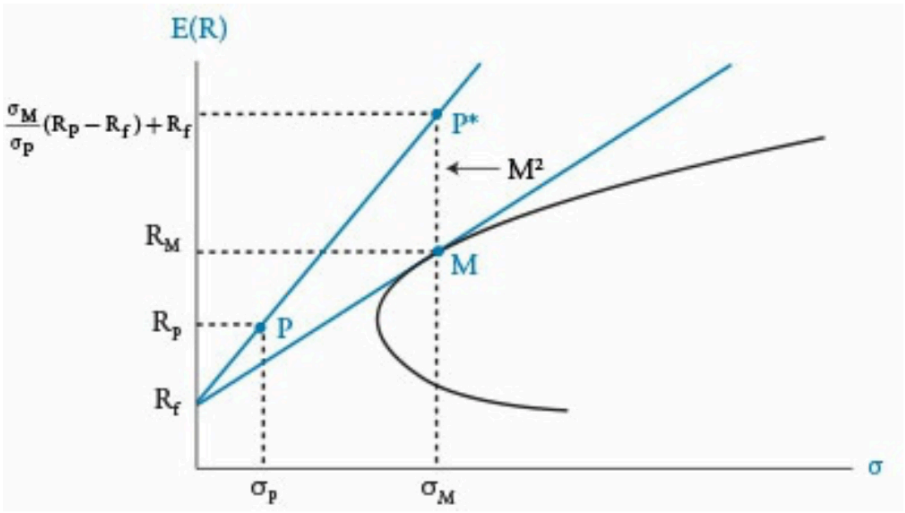
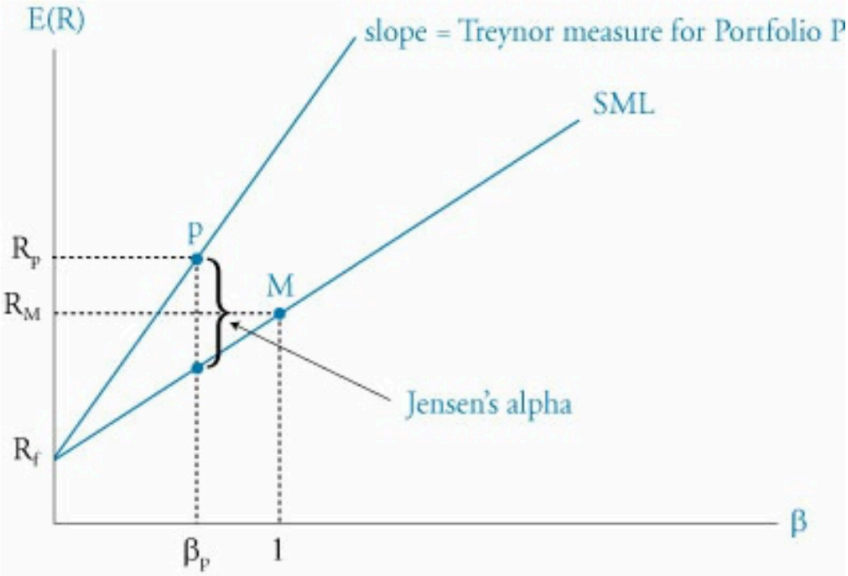


Figure 12: Treynor Measure and Jensen's Alpha



Lines (capital: standard deviation/total risk, security: beta/systematic risk)

- capital **allocation** line (CAL) – return vs standard deviation

    - a combination of a risk-free and **a risky** portfolio
- capital **market** line (CML) - return vs standard deviation
    - a combination of a risk-free and **the optimal market/tangency risky** portfolio
    - all investors hold the same **market** portfolio
- security **characteristic** line – excessive return vs excess market return
    - linear regression line used to find beta
- security **market** line (SML) - return vs beta (or covariance)
    - CAPM

Basic of portfolio planning and construction

Investment policy statement (IPS)

- **Starting point** of portfolio management process
- **Establish a target asset allocation strategy**
- Reason
  - Goals in terms of risk and return
  - Should be compatible with risk tolerance
- Major components
  - Description of client: circumstances, situation, objective
  - Statement of purpose
  - Duties and responsibilities of manager, custodian of asset, client
  - Procedures to update IPS
  - Investment objectives
    - Return and Risk tolerance
  - Investment constraints
    - Time, liquidity, tax, legal and regulatory, unique needs and preferences
  - Investment **guidelines**
    - policy execution, asset types permitted, **leverage**
  - Evaluation of performance: benchmark portfolio
  - **Appendices** – **allocation/deviation/rebalance**
    - Strategic (baseline) asset allocation
    - Permitted deviations from policy portfolio allocations
    - How and when to balance portfolio allocation

Risk Objective

- Absolute risk
  - **Probability** of specific results, **percentage or dollar** loss
  - Not strict limits on results
  - Nominal or real return
    - At least 6% per annum or 3% more than annual **inflation** rates
- Relative risk
  - Specific to **benchmark** and can be **strict or probability**
  - Bank: cost of fund (deposit rate)
  - Returns on peer portfolios: top quartile
    - Not investable portfolio
- Make sure risk and return are compatible

Willingness and Ability

- Ability
  - Financial circumstances
- Willingness
  - Attitudes and beliefs
  - Subjective and based on questionnaire
- Overall risk tolerance: the lower of ability and willingness
- Relation
  - High willingness but low ability -> low risk

- Low willingness but high ability -> educate

Situation		Risk tolerance
Willingness > Ability		Ability (Education)
Willingness < Ability	Return Objective = Willingness	Willingness (Reevaluation)
	Return Objective = Ability	Ability (Education)

Investment constraints

- Liquidity
  - Turn asset into spendable cash in a short time without having to make significant price concessions
  - Liquid: bank, insurance, mutual fund
  - Illiquid: hedge fund, private equity
- Time horizon
  - Longer time -> more risk and less liquidity
- Tax situation
  - Different tax treatment
  - Different accounts: retirement, personal, trust
  - Prefer Tax-free bonds
  - Prefer equities that produce capital gains – lower rate than other types of income
  - Focus on **after-tax** return
- Legal and regulatory
  - Restriction
    - Certain types of securities and assets
    - Percentage allocations to specific types of investments
- Unique circumstances
  - Specific preferences
  - Ethical preference: tobacco or firearms
  - Human rights abuses
  - Religious preference
  - Diversification needs

Strategic Asset allocation

- **Allocation**
  - specific the **percentage** allocations to the included asset classes
- **Correlation**
  - **High** correlation within an asset class
  - **Low** correlation between asset classes
- **Classes**
  - Traditional: cash, bond, equities, real estate

    - Alternative: private equity, managed or passively, artwork, intellectual property
- Equities
    - Domestic and foreign
    - Large and small
    - Developed and emerging markets
- Bonds
    - Maturities: short and long
    - Issuer: Government or corporate
    - Rating: investment grade or speculative (high yield)
- Data
    - Returns, standard deviations, correlations

Active Portfolio Construction

- **strategic** asset allocation **- long-term 长期**
    - **Efficient frontier**
    - Identify the optimal portfolio then find the **strategic** asset allocation
- **Tactical** asset allocation – **short-term** opportunities 短期
    - Vary the **asset allocation weight** to take advantage of short-term opportunities
    - Depend on existence of **short-term** opportunities and manager's **ability**
- **Security selection – inefficiencies – higher than expected return 利用市场无效**
    - Deviation from **index** weights on individual securities within the asset class
    - Depend on manager's skill and the opportunities (**mispricing** or inefficiencies)
- Risk budgeting
    - Set overall risk limit for the portfolio

Multiple managers

- Issues
    - Offsetting active positions
        - Some may overweight while other may underweight
        - Risk budget is **underutilized**
    - Excessive trading
        - All managers actively manage portfolio relative to an index -> trading may be **excessive**
        - Negative consequence: higher capital gains taxes
- Core-satellite
    - Majority/**core** in **passively** managed indexes
    - Smaller/**satellite** in **active** strategies
    - Reduce likelihood of **excessive** trading and **offsetting** active positions