- Understanding fixed-income risk and return

## Understanding fixed-income risk and return

**Sources of return fixed-rate bond**

- Coupon and principal payments
- Reinvest coupon
- Capital gain or loss is sold prior to maturity
    - Bond held to maturity has no such gain or loss

**Assumption**

- Interest rate earned on reinvested coupon equals to YTM

**Conclusions**

- YTM1: YTM of the bond when it is purchased
- Largest gain of a zero-coupon bond held to maturity is interest income
- Hold to maturity -> return = YTM1
- Sell bond prior to maturity and YTM is unchanged -> return = YTM1
- Market YTM for the bond, assumed reinvestment rate, **increases** after the bond is purchased but before the first coupon date （YTM 增加）
    - Hold to maturity (longer) -> return > YTM1 (more reinvested) 持有长回报高
    - Hold for **short** time -> return < YTM1 (less reinvested) 短期持有回报小
- Market YTM for the bond, assumed reinvestment rate, **decreases** after the bond is purchased but before the first coupon date
    - Hold to maturity (longer) -> return < YTM1 (less reinvested) 持有到期回报少
    - Hold for **short** time -> return > YTM1 (more capital gain) 短期持有回报高

**Carrying value 市值**

- Value of a bond between its purchase and sale
- Capital gain or loss is measured relative to carrying value

At the **middle** of holding period $0 < t < T$

- Carrying value (backward) 未来值的折现
    - N= $\mathbf{T} - \mathbf{t}$ (TTM), PMT=C, I/Y=YTM, FV=100-> PV
- Coupon interest
    - $t \times C$
- Coupon interest and reinvested interest (forward) coupon 的复利和未来值
    - $N = t$, PMT=C, I/Y=YTM, PV=**0** -> FV
    - Reinvested income: $FV - t \times C$

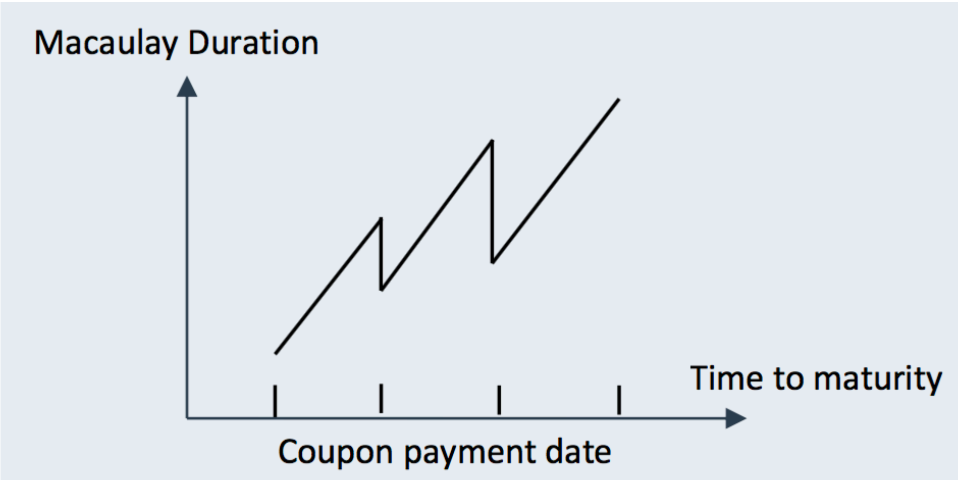
**Risks**

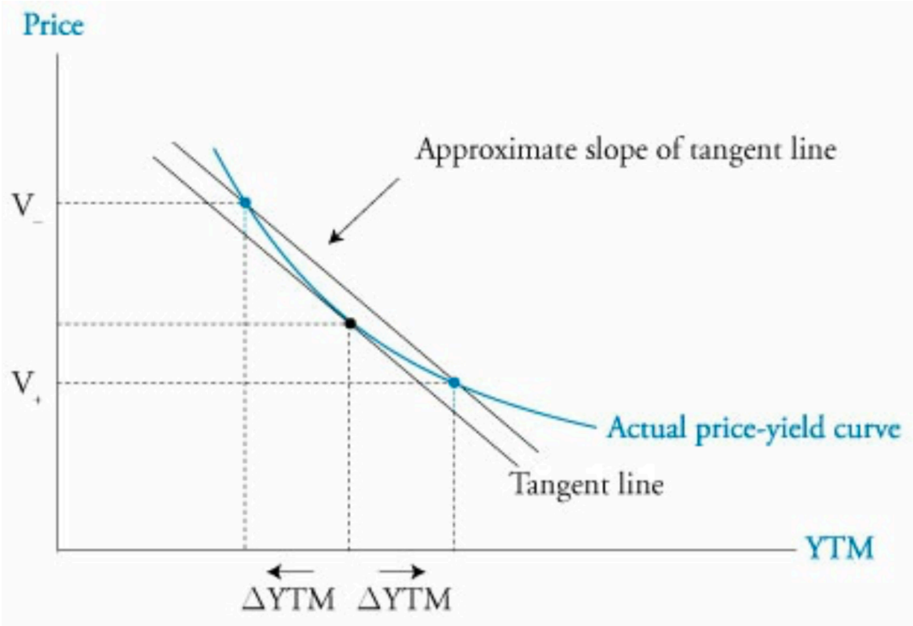
- **Market price risk 市场价格风险**
    - Uncertainty about price due to YTM
- **Reinvestment risk 再投资风险**

- Uncertainty about total coupon payments and invested income due to uncertainty about future reinvestment rates
- Long time horizon (reinvestment risk > market price risk)
  - Price converges to par, so no interest rate risk
  - More reinvestment risks
- Short investment horizon (market price risk > reinvestment risk)
  - Interest rate risk increase, reinvestment risk decreases
  - Yield decrease -> price increase -> more gain
  - Yield increase -> price decrease -> less return

**Duration (衡量利率风险)**

- Measure bond's **interest rate** risk or sensitivity of its **full** price to its change in yield
- For option-free bonds
- Macaulay Duration
  - Weighted average of the number of years until each of the bond's
  - $\mathrm{D_{macaulay}} = \frac{\sum_t t \times PV_t}{\sum_t PV_t}$
  - Semi-annual bond: weighted semi-annual periods
  - Not best estimation of interest rate sensitivity
  - Nonconvertible perpetual bond
    - $\mathrm{macaulay} = \frac{1+\mathrm{Y}}{\mathrm{Y}}$
- Modified Duration
  - $\mathrm{D_{modified}} = \frac{\mathrm{D_{macaulay}}}{1+\frac{Y}{m}}$
    - Discount one more period
  - Percentage change in bond's price
    - $\frac{\mathbf{\Delta P/P}}{\mathbf{\Delta Y}} = -\mathbf{D_{modified}} \rightarrow \frac{\Delta P}{P} = -D_{modified} \times \Delta Y$
- Approximate Modified Duration
  - $\mathrm{D_{modified}} = \frac{V_- - V_+}{2 \times \Delta Y \times V_0}$
    - $\frac{\Delta \mathrm{P}}{\Delta \mathrm{Y}} = \frac{V_+ - V_-}{2 \times \Delta Y} \rightarrow \mathrm{D_{modified}} = -\frac{\Delta P}{\Delta Y} \times \frac{1}{\mathrm{V_0}}$
  - Linear estimation, good estimate for small change






**Effective Duration (curve parallel shift 曲线的平移, embedded bonds)**

- Bonds with embedded options
  - Callable bond, puttable bond
  - mortgage-backed bond, a prepayment option, similar to a call option
- bond price
  - based on **path** of interest rates
  - should use **effective duration** to estimate interest risk
  - it is based on bond price from a pricing model
  - bond price based on YTM cannot be used because **uncertain** future cash flow
- effective duration is used to measure interest rate sensitivity
  - change not the YTM, but the benchmark **yield curve 基于 curve 的平移**
- $\mathrm{D}_{\text{effective}} = \frac{V_{-} - V_{+}}{2 \times \Delta curve \times V_0}$
- Separate
  - Changes in **benchmark yields**
  - changes in **yield spread** for credit and liquidity risk
- modified duration
  - no distinction between changes in benchmark yield and changes in spread
- effective duration
  - reflects only the sensitivity of bond's value to changes in benchmark yield curve
  - credit duration: changes in credit spread
- effective duration does not necessarily provide **better** estimate for smaller changes in yield, maybe large change in yield produce more predictable prepayments or calls than small changes

**Key Rate Duration (non-parallel shift)**

- Key Rate Duration / partial duration

    - Measure impact of nonparallel shift
- Key rate durations 一次改变一个 maturity
    - A key rate duration is the sensitivity of bond value changes to spot rate for a specific **maturity**, holding other spot rates **constant**
    - A key rate duration can be calculated for each maturity on the spot rate curve
    - For a portfolio, **sum** the individual effects to get overall effects

**Interest rate risk**

- Increase maturity -> increase risk
    - Payment **further** are more sensitive to changes in discount rate
    - Special
        - A discount coupon bond, increase maturity -> decrease Macaulay duration
        - Duration first increase with longer maturity, and then decreases over a range of relatively long maturities until it approaches the duration of a perpetuity, which is (1+Y)/Y
- Increase coupon rate -> decrease risk
    - More bond value will be paid sooner
    - Duration of a zero-coupon bond > duration of a coupon bond
- Increase YTM -> decrease risk
    - Use slope as proxy for risk, higher yield, slope is flatter
- Call or put provision -> decrease risk (effective duration)
    - Call option
        - Yield fall -> value increase
        - Callable bond price = straight bond – call option
    - Put option

**Special Bond durations**

- Perpetual bond
    - $\text{Macaulay Duration} = \frac{1+\text{Y}}{\text{Y}} = 1 + \frac{1}{Y}$
    - $\text{Modified Duration} = \frac{1}{\text{Y}}$
- Zero-coupon bond
    - $\text{Macaulay Duration} = \text{TTM}$
    - $\text{Effective Duration} = \frac{\text{TTM}}{1+\text{Y}}$

**Portfolio Duration**

- Cash flow approach
    - Weighted average number of periods until portfolio's cash flow will be received
    - **Theoretically** correct
    - The yield is cash flow yield, the IRR of bond portfolio
    - **Inconsistent** with duration capturing the relationship between YTM and price
    - **Not work** for portfolio with **embedded** bonds since cash flow are unknown
- Weighted average of duration
    - Weighted average of durations of individual bonds

    - $\mathrm{D_{portfolio}} = \sum_i w_i \times D_i$
        - $w_i = \frac{bond\ full\ price}{portfolio\ full\ price}$
        - Use market value
  - Practical approximation, often used in **practice**
  - Work with bonds with **embedded** options using effective durations
  - Limitation
    - YTM of every bond should change by the same amount to create a **parallel** shift
  - The same duration when yield curve is flat
  - less accurate when there is greater **variation** in yields

**Money Duration**

- money duration is duration in currency
  - $\text{money duration} = \text{annual modified duration} \times \text{bond full price}$
  - $\frac{\Delta P/P}{\Delta Y} = -D \rightarrow \frac{\Delta P}{\Delta Y} = -(D \times P) = -D_{\text{money}}$
- money duration of 100 par value (每 100 平价对应的 money duration)
  - $\text{money duration per 100 of par value} = \text{annual modified duration} \times \text{bond } \textbf{full} \text{ price per 100 of par value}$
- change in bond value
  - $\Delta\text{bond value} = \text{money duration} \times \Delta\text{YTM}$
  - $\frac{\Delta P/P}{\Delta Y} = -D \rightarrow \Delta P = -(D \times P) \times \Delta Y = -D_{\text{money}} \times \Delta Y = \frac{V_- - V_+}{2}$

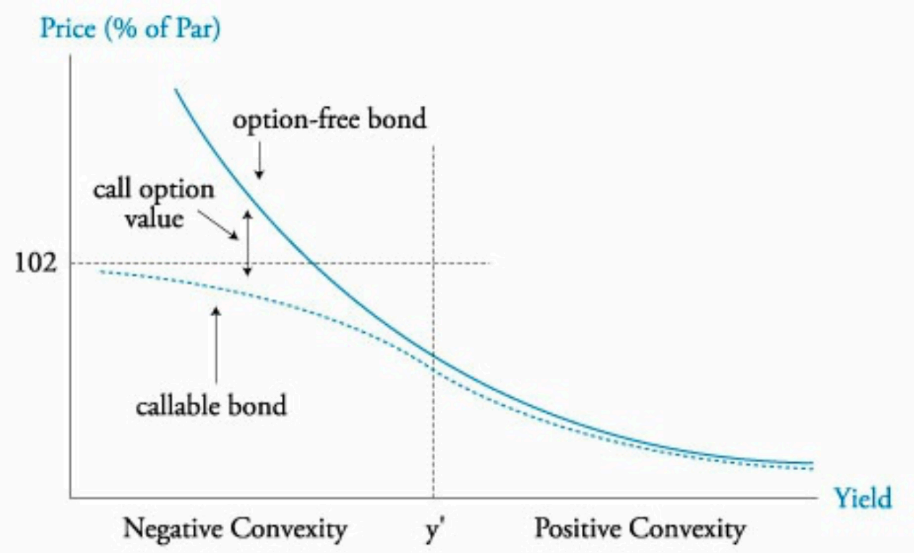
**Per value of a basis point (PVBP)**

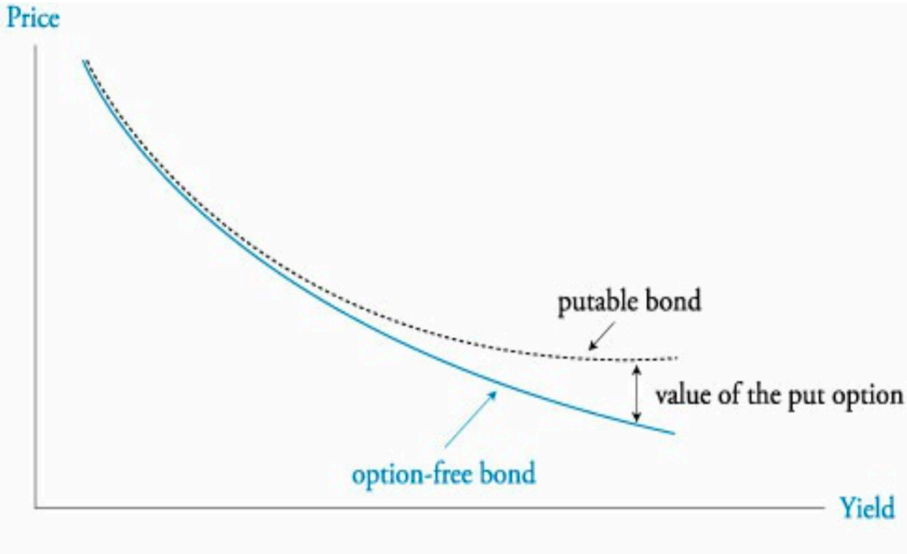
- Money change in full price of a bond when its YTM changes by one basis point (0.1%)
- $\text{PVBP} = \Delta P = -D_{\text{modified}} \times P \times \Delta Y = \frac{V_- - V_+}{2}$
  - $\Delta Y = 0.01\%$
- If paid semi-annual, the BP is still 0.01%
  - $y_{\pm} = 0.5 \times (y \pm 0.01\%)$

**Convexity**

- Linear estimation underestimate price
- Convexity is percentage change in secondary derivative
  - $\textbf{Convexity} = \frac{\Delta^2 P}{\Delta Y^2 \times P} = \frac{V_- + V_+ - 2V_0}{(\Delta Y)^2 \times V_0}$
- Effective convexity (bonds with embedded options)
  - $\text{Effective Convexity} = \frac{\Delta^2 P}{\Delta \text{Curve}^2 \times P} = \frac{V_- + V_+ - 2V_0}{(\Delta Curve)^2 \times V_0}$
- Factors (similar to duration)
  - Longer maturity, lower coupon rate, lower yield -> increase convexity
  - Two bonds with equal duration, more dispersed cash flow -> high convexity
- Convexity
  - Option-free: positive
  - Callable: negative when yield is small
  - Puttable: greater convexity when yield is large

**Figure 3: Price-Yield Function of a Callable vs. an Option-Free Bond**




Percentage change in Price

- $\frac{\Delta \text{P}}{\text{P}} = -D \times \Delta Y + \frac{1}{2} \times C \times (\Delta Y)^2$

**Term Structure of yield volatility**

- Relation between volatility of bond yields and their times to maturity
- Volatility of bond's price
  - Sensitivity of bond's price to a given change in yield
  - **Volatility** of bond's yield
- Short-term more volatility than long-term
  - Short-term could have more price volatility than a longer-term

**Duration – HPR and investment horizon**

- If investment horizon and Macaulay duration are matched
  - A parallel shift in yield curve prior to first coupon payment will not affect HPR
  - **Market** price risk and **reinvestment** risk **offset** each other

- Increase in yield
  - Short-term: less capital gain and small increase in reinvestment income
  - **Long-term**: more reinvestment income
- Decrease in yield
  - **Short-term**: more **capital gain** and small decrease in reinvestment income
  - Long-term: less **reinvestment** income
- **Duration gap**
  - **Gap = Macaulay duration – investment horizon**
  - Positive gap
    - Macaulay duration > investment horizon
    - More market price risk from increasing interest rate
    - 利息上升导致价格下降
  - Negative gap
    - Macaulay duration < investment horizon
    - More reinvestment risk from decreasing interest rate
    - 利率下降导致再投资下降

**Credit and liquidity spread**

- Benchmark interest = real rate of return + expected inflation
- Bond's spread = credit premium + liquidity premium
- Change in a spread
  - $\frac{\Delta \mathrm{P}}{\mathrm{P}} = -D \times \Delta spread + \frac{1}{2} \times C \times (\Delta spread)^2$