Fixed Income

- Term Structure and Interest Rate Dynamics
- The Arbitrate-Free Valuation Framework
- Valuation and Analysis: Bonds with Embedded Options
- Credit Analysis Models
- Credit Default Swap

Term Structure and Interest Rate Dynamics

Summary

- **Benchmark Curve**
  - Spot rate curve
  - Forward rate curve (multiple)
  - YTM and return (single rate)
  - Par curve (PV=FV, coupon rate) **平价利率**
  - Swap rate curve (par bond, spot -> par)
  - Upward Curve: **forward > spot > par**
    - Spot rate is geometric mean of forward 几何平均
    - Par is fixed (some kind of average) of floating spot rate 平均去掉波动
  - Expected and **Realized** Return
    - Realized Return: use the actual rate
  - **Spot and Forward Evolvement**
    - Hold to maturity (coupon paying)
      - use short expected/real spot rate (f(n-1,1))
    - Hold for one year (zero coupon)
      - use expected/real spot rate (f(1,n-1)) at end of first year
      - Breakeven: use predicted f(1,n-1) at the beginning
- **Yield Spread**
  - Swap spread, I-spread, Z-spread, TED, LIBOR-OIS
- **Term Structure Theory**
  - Expectation, local expectation, liquidity
  - Segmented markets, preferred habitat
- **Interest Rate Models**
  - Equilibrium Models 平衡模型
    - Vasicek and CIR model
  - Arbitrage-Free Models 无套利
    - Ho-Lee Model
- **Yield Curve Risks**
  - level, steepness, curvature
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Spot Rates and Curve 即期利率

- $P_T = \frac{1}{(1+S_T)^T}$
  - S_T: spot rate
  - T: maturity
- Spot curve S_T $versus$ T

Forwards rates and Curve 远期利率

- Forward rates
- $S_T = \prod(1 + f_i)$

Yield-To-Maturity (YTM) 单一收益率

- A single rate for coupon bond

- $\sum_i \frac{CF_i}{(1+S_T)^i} = \sum_i \frac{CF_i}{(1+YTM)^i}$

Expected and Realized Return

- Assumptions
  - Bond is held to maturity
  - All payments are made on time and in full
    - Risk-free and no default
  - All coupons are **reinvested** at the **original** YTM
    - Least realistic assumption
- Violations
  - Interest rates are **volatile**
  - Yield curve is steeply sloped
  - Significant risk of **default**
  - Embedded **option** (put, call, conversion)
- Yield curve is not flat, coupon payment will not be reinvested at the YTM
  - Expected return will differ from yield
- Realized return
  - actual return
  - actual/expected spot rate, forward rate f(n-1,1) 迭代计算

Forward Pricing Model

- $P_{j+k} = P_j \times F_j^k$
- P_j is a discount factor

Forward Rate Model

- $\left(1+S_{j+k}\right)^{j+k} = \left(1+S_j\right)^j \left(1+f_j^k\right)^k$

Par Rate = coupon rate 平价利率

- YTM of bond trading at par (present value = face value)
- $PV = FV = \sum_i \frac{CF_i}{(1+YTM)^i}$

Bootstrapping (Par curve -> spot rate)

- Coupon = par rate * face value
- $R_1: par\ rate, F: face\ value, C: coupon = F \times R_1$
- $S_1 = R_1$
- $F = \frac{C}{1+S_1} + \frac{F+C}{(1+S_2)^2} \rightarrow S_2$
- $F = \sum_{i=1}^{n} \frac{C}{(1+S_i)^i} + \frac{F}{(1+S_n)^n} \rightarrow S_n$

Spot & forward Curve Shape

- Upward sloping
  - Risk premium
  - Inflation
- **Inverted** yield curve - downward
- Flat yield curve

- Humped yield curve
    - Intermediate-term interest are higher than short- and long-term rates

Spot and Forward Rates – Geometric Mean 几何平均

- Geometric mean
    - $(1+S_n)^\wedge n = (1+S_1)\times(1+f_1^1)\times(1+f_2^1)\times\cdots\times(1+f_{n-1}^1)$
- Relationship
    - Upward: forward curve > spot curve 上升不快
    - Downward: spot curve > forward curve 下降不快

Expected Spot rate and Forward rate

- Expected spot rate one year later = forward rate
    - $E(S_1^1) = f_1^1 = \frac{(1+S_2)^2}{1+S_1}$
        - one year later, expected spot rate is current forward rate
    - $E(S_1^2) = f_1^2 = \sqrt{\frac{(1+S_3)^3}{1+S_1}}$
        - one year later, expected two-year spot rate
    - $E(S_1^n) = f_1^n = \sqrt[n]{\frac{(1+S_{n+1})^{n+1}}{1+S_1}}$
- If expected **true** spot rates < forward rate predicted today: undervalue
    - Lower discount rate -> higher contract price -> buy the contract
    - 期望即期利率下降，低估，未来价格上升，买入
- If expected spot rate > forward rate: overvalue
    - Higher discount rate (may be credit risk is increased) -> price drop -> sell contract
    - 利率升高，情况变坏

Spot and Forward Evolvement

- $\mathrm{f(i,j)} \rightarrow (1+S_i)^i(1+f(i,j))^{j-i} = (1+S_j)^j$
- $\mathrm{f(n-1,1)}$: one-year forward rate 短期利率: 未来每年短期滚动利率
    - one-year forward rate from $\mathrm{n-1}$ to n
    - $\mathrm{f(n-1,1)} = \frac{(1+S_n)^n}{(1+S_{n-1})^{n-1}} - 1$
- $\mathrm{f(1,n-1)}$: n-1 year forward rate to maturity 长期利率
    - n-1 year forward rate from 1 to n
    - one year later, the forward rate to maturity
    - $\mathrm{f(1,n-1)} = \frac{(1+S_n)^n}{1+S_1} - 1$ predicted using original curve
    - The **breakeven rate** of purchasing a zero-coupon note either today or one year from today is equal to the one-year rate, n-1 forward
- **Hold to maturity (use f(n-1,1))**
    - Future forward rate increase, yield increase because coupons are reinvested at a higher rate
    - First year end, receive coupon c, $\mathrm{FV_1 = c}$
    - Second year end, receive coupon c, previous coupon reinvests
        - $\mathrm{FV_2 = C + C \times f(1,1)}$

- Third year end, receive coupon c, previous coupon reinvests
  - $\mathrm{FC_3} = \mathrm{C} + \mathrm{FV_2} \times \mathrm{f}(2,1)$
- $\mathrm{FC_n} = \mathrm{F} + \mathrm{C} + \mathrm{FV_{n-1}} \times \mathrm{f}(\mathrm{n}-1,1)$ 迭代计算公式
- $\mathrm{FC_n} = PV_{old} \times (1 + YTM_{new})^n \rightarrow YTM_{new}$

- **Hold before maturity** (one year)
  - spot rate $S_1, S_2, \cdots, S_n$ at the beginning
  - expected spot rate $ES_1, ES_2, \cdots, ES_{n-1}$ at the end of **first** year
  - $ES_{n-1} = f(1, n-1)$ one-year later, n-1 period forward rate
  - 1-year, …, N-year zero-coupon bond
  - $\mathrm{PV_0} = \frac{FV}{(1+S_n)^n} (1 \leq n \leq N)$
  - $\mathrm{PV_1} = \frac{FV}{(1+ES_{n-1})^{n-1}}$ (at the end of year 1, use expected spot rate)
  - $\mathrm{HPR} = \frac{(1+S_n)^n}{(1+ES_{n-1})^{n-1}} - 1$
  - If expected spot rate ES_{n-1} evolve as precited by original spot curve
    - $\mathrm{HPR} = \mathrm{S_1}$

EXHIBIT 1 SPOT AND FORWARD INTEREST RATES

Maturity (Years)	Spot Rates	Forward Rates (1 year) (*n* – 1 years forward)	Forward Rates (*n* – 1 year) (1 year forward)
(*n*)	*r*(*n*)	*f*(*n* – 1,1)	*f*(1,*n* – 1)
1	3.00%	3.00%	3.00%
2	4.00%	5.01%	5.01%
3	5.00%	7.03%	6.01%
4	6.00%	9.06%	7.02%
5	7.00%	11.10%	8.02%

- The first is a newly issued 7.00% coupon bond with a 5-year maturity issued at a price of $101.15 ($100.00 face value) with a yield to maturity of 6.72%. The second is newly issued zero-coupon bond with a 5-year maturity issued at a price of $71.30 ($100.00 face value) with a yield to maturity of 7.00%.
- **Coupon paying bond: realized return**
  - Coupon each year 7, reinvest start from the end of first year, use f(n-1,1)
  - $\mathrm{FV} = \Big(\big((7 \times 1.0501 + 7) \times 1.0703 + 7\big) \times 1.0906 + 7\Big) \times 1.111 + 107 = 141.87$
  - $(1 + \mathrm{YTM_{new}})^5 = \frac{141.87}{101.15} \rightarrow \mathrm{YTM_{new}} = 7\% > YTM_{old} = 6.72\%$
- **Zero-coupon bond: hold for one year**

- $S_1' = \frac{(1+S_5)^5}{(1+f(1,4))^4} - 1 = \frac{1.075^5}{\mathbf{1.0802^4}} - 1 = 3\%$

- **Zero-coupon bond: Break even for buy today or buy one year later**
    - $f(1,4) = \frac{(1+S_5)^5}{1+S_1} - 1 = \frac{1.075^5}{1.03} - 1 = 8.02\%$

Riding the yield curve – upward-sloping

- Rolling down the yield curve
    - Buy **longer** maturity than investment horizon
    - Both coupon yield and **capital** gain
- Conditions
    - **Upward** sloping yield curve
    - Interest rate curve **unchanged**
- Properties
    - Longer maturity -> more gain
    - Large difference between spot and future rates -> more gain
- Five year
    - Buy a five-year bond
        - Earn YTM
    - Buy a 30-year bond and sell a 25-year bond five years later
        - Earn YTM and capital gain

Swap Rate Curve – credit risk

- Swap fixed rate – **fixed rate**
    - One party pays a fixed rate
    - Counterparty make payments based on a floating rate
- Swap rate as **benchmark** is better than government bond yield
    - **Liquidity 流动性好**
        - **liquid** markets
        - Only exchange cash flow not principle
        - Easy to hedge for interest rate
    - **Credit Risk** 信贷风险
        - Reflect **credit risk** of commercial banks rather than governments
    - **Comparable** 可比较
        - more **comparable** across countries because they are not regulated by government
    - **Maturity 多期限**
        - Has yield quotes at many **maturities**
        - US T-bond on-the-run issues trading at only a small number of maturities
- Bank
    - Wholesale bank: swap curve
    - Retail bank: government bond
- $\boldsymbol{\sum_t \frac{SFR_T}{(1+S_t)^t} + \frac{1}{(1+S_T)^T} = 1}$
    - $V = V(\boldsymbol{fixed}) - V(floating) = 0$
    - $V(floating) = par\ value = FV = 1$

- $\sum_t SFR_T \times d_t + d_T = 1 \rightarrow \boldsymbol{SFR_T} = \frac{\mathbf{1} - \boldsymbol{d_T}}{\sum_i \boldsymbol{d_i}}$
    - $d_t = \frac{1}{(1+S_t)^t}$
- SFR can be thought as the **coupon** rate of a bond

Spread Measures

- Swap spread (default risk)
    - $\mathbf{swap\ spread_t} = \boldsymbol{swap\ rate_t} - \boldsymbol{on\ the\ run\ Treasury\ yield_t}$
    - difference at the same maturity
    - positive, lower credit risk of government bonds
    - LIBOR swap curve, **default risk** of commercial banks
- I-spread – credit and liquidity risk
    - Linear interpolation 线性插值
    - $\mathrm{spread_t} = S_i + (\mathrm{S_{i+1}} - S_i) \times \frac{t - T_i}{\mathrm{T_{i+1}} - T_i}$
        - $t \in [T_i, T_{i+1}]$
- Z-spread – corporate bond
    - A spread that's added to the entire spot rate curve (shift)
    - $PV_{market} = \sum_i \frac{CF_i}{(1+S_i+Z)^i}$
    - Zero volatility: assume zero interest rate volatility
    - Not good for bonds with embedded options
    - directly affected by the general **creditworthiness** of individual **debt** issuers
- TED spread (3-month LIBOR – T-bill) 交易对手风险
    - T: T-bill, ED: Eurodollar features contract
    - $\mathbf{TED\ spread = 3month\ LIBOR\ rate - 3\ month\ Tbill\ rate}$
    - Interbank **counterparty** risks
    - affected by banks' analysis of default of interbank loans, the TED spread is a measure of counterparty risk
- LIBOR-OIS spread – credit risk 货币市场、银行之间、流动风险
    - $\mathrm{TED\ spread = LIBOR\ rate - OIS\ rate}$
    - OIS: federal funds rate (minimal counterparty risk)
        - Interest rate swap rate 同行隔夜拆解利率
    - Credit risk and liquidity of money market securities
    - affected by bank's lending rates for **unsecured** overnight loans

Term Structure Theory 期限利率理论

- Unbiased expectations theory 期望理论：长期和短期一直
    - Investor's expectation changes the shape of interest rate
    - **Risk neutrality**: no risk premium for **maturity**
    - **Long-term** interest rates = mean of future **expected short-term** rates
    - If one-year spot rate is 5% and two-year spot rate is 7%
        - Then one-year forward rate in one year is 9%
- Local expectations theory 局部期望：短期一样，长期有差异
    - Risk-neutral only for **short** holding periods 短期一样
    - Risk premiums should exist for **long** term 长期有差别

- Short term: every bond (even long-maturity risky bond) earn the risk-free rate
- Not hold 不成立：只看持有期，不看 bond 本身的期限
  - Short HPR of long-maturity bond > short HPR of short-maturity bond

- Liquidity Preference Theory 流动性理论 (expectation + **liquidity**)
  - Forwards rates reflect expectations of future spot rate
  - Liquidity premium to compensate for exposure to interest rate risk
  - Liquidity premium is **positively** related to maturity
- Segmented Markets Theory 分割市场理论（每个期限独立由供需决定）
  - Rate is determined by **preferences** of borrowers and lenders
  - Drives the balances between supply of and demand for loans of different maturities
  - Yield at each maturity is determined **independently** of the yields at other maturities 不同期限，收益独立
- Preferred Habitat Theory 习惯性偏好（所有期限互相影响）
  - Forward rate = expected future spot rates + preferred maturity **premium （偏好转移溢价）**
  - Premium does not directly relate to maturity
  - **Imbalance** of supply and demand for funds in a maturity range will induce lenders and borrowers to shift their preferred habitats to one that has the **opposite** imbalance 期限互相平衡
  - Premiums are related to supply and demand for funds at various **maturities**

Interest Rate Models

- General short-term rate
  - $\Delta r = \lambda \Delta t + \sigma \Delta w$ (trend + random)
    - $\Delta w \sim N(0, \sqrt{\Delta t})$
- **Equilibrium Term Structure Models 平衡模型 - 推导**
  - Use of fundamental **economic** variables that drive interest rates
  - Sing-factor models: Vasicek and CIR model
- **Arbitrage-Free Models 无套利 – 拟合**
  - Assume market prices are correct, can match current market prices
  - Need to estimate parameters using historical data
  - Models: Ho-Lee model
- Ho-Lee model
  - $\Delta r = \lambda_t \Delta t + \sigma \Delta w$
  - λ_t: time-dependent drift
  - Symmetric/normal distribution of future rates
- Vasicek Model
  - $\Delta r = k(\theta - r)\Delta t + \sigma \Delta w$
    - k: mean-reversion speed
    - θ: long-run interest
  - interest rate can be negative
- CIR model
  - $\Delta r = k(\theta - r)\Delta t + \sigma\sqrt{r}\Delta w$
  - interest rate: mean-reverting to long-run value

- volatility increase with interest rate

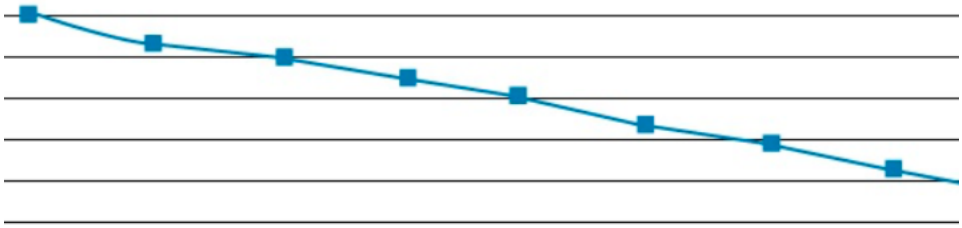
Yield Curve Risks

- Portfolio Duration
  - $\mathrm{D_p} = \sum_{\mathrm{i}} w_i D_i = \sum \mathrm{KRD_i}$
    - $\mathrm{KRD_i} = w_i \times D_i$
- Effective Duration 平行移动
  - Small **parallel** shifts in the yield curve 所有期限同时影响
  - shaping risks
    - change in yield curve
  - Cannot explain non-parallel shaping risks
  - Explains more than **75%** variation in return
- Key Rate Duration 非平行移动（期限）
  - Nonparallel yield curve shifts
  - One **maturity** at a time 一次影响一个期限
    - Sensitivity to changes in a single par rate, holding other spots rates constant
  - Change in bond value in response to 100bps change in corresponding key rate, holding other rates constant
  - A set of key duration
    - $\frac{\Delta \mathrm{P}}{\mathrm{P}} = -\sum_t D_t \times \Delta r_t$ 线性组合
    - D_t: key rate duration at maturity t
- LSC
  - Decompose changes to level, steepness and curvature changes
  - Level $\Delta \mathrm{X_L}$ 平移
    - parallel increase or decrease of interest rates
  - Steepness ($\Delta \mathrm{X}_S$) 坡度 - 长期上升，短期下降
    - long-term interest rates increase while short-term rates decrease
  - Curvature ($\Delta \mathrm{X}_C$) 曲度 - 中期不变，短期长期改变， 更加 convex
    - Increasing curvature, more **convex**
    - short- and long-term rates increase while intermediate rates do not change
  - $\frac{\Delta \mathrm{P}}{\mathrm{P}} = -D_L \times \Delta X_L - D_S \times \Delta X_S - D_C \times \Delta X_C$

Maturity Structure of Yield Curve Volatilities

- Interest rate volatility
  - Key concern for **fixed** income portfolio
  - Bonds with **embedded** options
- Term structure of interest rate **volatility**
  - Yield volatility versus maturity
  - Standard deviation of the change in bond yield
- More **volatile** in short-term interest rate 短期更加波动
- short-term interest rate volatility
  - monetary policy 货币政策
- long-term interest rate volatility

- real economy and inflation 经济和通货膨胀



The Arbitrate-Free Valuation Framework

Summaries

- **Arbitrage-free**
- **Interest rate tree generation**
    - volatility: historical or implied
    - probability $P_{up} = P_{down} = 0.5$
    - $\mathrm{R}_{\mathrm{up-and-then-down}} = R_{down-and-then-up}$
    - $R_{up} = R_{down} \times e^{2\sigma}$
    - $R_{middle} = f_{t,1}$**forward** rate <- spot rate
- Calibration
- Valuation
    - backward and pairwise
- MC: path dependency

Arbitrage-free

- law of one price
- arbitrage transaction
    - **no initial** cash outflow
    - a positive riskless profit in the future
- arbitrage conditions
    - **value additivity** 整体不等于部分
        - value of whole ~= sum of values of parts
        - stripping or reconstitution
    - **dominance** 优势
        - one asset trades at a lower price than another asset with identical characteristics
        - The price of any two risk-free securities with the same timing and the amount of payoffs must be the same.
- Risk-free securities
    - It only applies to risk-free securities and portfolios, not to risky ones

Binominal Interest Rate Tree - lognormal

- **Equal** probability of taking one of possible values
- **a lognormal random walk model**
    - non-negative interest rate
    - higher volatility at high rates
- **Properties**
    - $P_{up} = P_{down} = 0.5$
    - $\mathrm{R}_{\mathrm{up-and-then-down}} = R_{down-and-then-up}$
    - $R_{up} = R_{down} \times e^{2\sigma}$
- Rates – need forward
    - Par -> spot -> **forward**
- Volatility
    - Historical - most recent
    - Implied volatility – from derivative

Interest Rate Tree - Generation Rules

- 无套利拟合 Interest rate should be arbitrate-free values for the benchmark security
- 远期利率 The rate at each node is the **forward** rate for the entire period
- 2 倍标注差 Adjacent forward rates are **two standard deviation** apart ($e^{2\sigma}$)
    - $R_{up} = R_{down} \times e^{2\sigma\sqrt{t}}$ (t=1 yearly in most cases)
- The **middle** forward rate (or mid-point in case of even number of rates) in a period is approximately equal to the **implied** one-period **forward** rate for that period 中间节点的值是当期隐含远期利率
    - Par -> spot -> forward
    - $1 + S_2 = (1 + S_1) \times (1 + f_{1,1}) \rightarrow \mathbf{R_{middle}} = \mathbf{f_{1,1}}$
    - $R_{up} = R_{middle} \times e^{\sigma}$
    - $R_{down} = R_{middle} \times e^{-\sigma}$

Interest Rate Tree

- Valuation
    - generating the cash flows that are interest rate dependent
    - supplying the **interest rates** used to determine the present value of the cash flows
- Assumptions
    - interest rate model such as a lognormal model of interest rates
    - volatility of interest rates
- Volatility can be measured relative to the current level of rates
    - Use a lognormal distribution, interest rate movements are proportional to the level of rates and are bounded at the low end by zero.

Valuation – backward induction

- Interest rate is defined at the beginning, t=0, 1, 2
- $V_0 = \frac{1}{1+r_0} \times (p_u \times (CF_1 + V_1^u) + p_d \times (CF_1 + V_1^d))$
- $V_0 = \frac{1}{1+r_0} \times (\boldsymbol{CF_1} + p_u \times V_1^u + p_d \times V_1^d)$

Calibration – Forward Generation and Backward Valuation

- An iterative process
- Interest rate bounds
    - ensures that the **upper and lower** rates are consistent with the volatility assumption, the interest rate model
- **value** equals to the observed **market** value
    - The cash flows of the bond are discounted using the interest rate tree, and if this doesn't produce the correct price, another pair of forward rates is selected and the process is repeated.
- Arbitrage free – same value using spot rate curve or binomial tree
    - Any option-free bond should have the same value whether using the spot rate curve or the binomial tree.
- Implementation - spreadsheet

    - Can be done using **spreadsheet** software (e.g., the solver function in Microsoft Excel) and is thus relatively easy to do without the knowledge of **special** programming techniques and without **great expense**

Valuation – Pairwise

- A n-period tree has 2^n pairwise paths, because the first year is fixed
    - For a one-year tree, the choices from Time 0 are u or d, where u indicates interest rates going up and d indicates interest rates going down.
    - For a two-year tree, interest rates can increase or decrease from where they are after one year, so there are 2 × 2 = 4 paths, or uu, ud, du, and dd.
- A n-year bond 2^{n-1} pairwise paths 第一年固定
    - Interest rate is fixed in the first year, after that each year, each node will create two paths
- $V = \frac{1}{2^n}\sum_{paths} V_{path}$
- $V_{path} = \sum_t \frac{CF_t}{(1+S_t)^t} = \sum_{t=1}^{n} \frac{C}{(1+S_t)^t} + \frac{F}{(1+S_n)^n}$
- $1 + S_t = (1 + S_0) \times \cdots \times (1 + f_{n-1,1})$ 利率树上的是 forward

Monte Carlo

- Path dependency
    - RMBS: Payment risk
    - Mortgage-backed securities
- Forward-rate simulation
    - Interest rate term structure model
- **Calibration**: add **drift**
    - To be consistent with market prices
    - Add a **constant** to **all interest** rates on all paths if estimated value is higher
    - The benchmark bond is option-free, so its value should not be affected by interest rate volatility
    - Won't help
        - adjust the volatility assumption
        - increase the number of simulations
- number of paths
    - Monte Carlo requires simulation of a **limited** (though large) number of interest rate paths out of all possible paths
    - binomial tree valuation considers **all** the interest rate paths
- mean reversion
    - **Upper and lower** bounds on interest rates
- Increase number of paths
    - Increase accuracy in a statistical sense 统计精确
    - Does not closer to true fundamental value 不代表接近真实值

Valuation and Analysis: Bonds with Embedded Options

Summaries

- **Callable/puttable bond**
  - $\mathrm{V_{callable}} = \mathrm{V_{straight}} - V_{call}$
  - $\mathrm{V_{puttable}} = \mathrm{V_{straight}} + V_{put}$
- Value embedded bond with binomial tree
  - Option type, when can exercise?
- OAS
  - $\mathrm{V_{callable}} = \mathrm{V_{straight}} - V_{call} \rightarrow \mathrm{OAS_{call}} = ZS - \mathrm{V_{call}}$
  - $\mathrm{V_{puttable}} = \mathrm{V_{straight}} + V_{put} \rightarrow \mathrm{OAS_{put}} = ZS + V_{put}$
- **Valuation Factors**
  - **Straight** bond value is **negative** related to interest rate **level**
  - **Option** value is **positively** related to interact rate **volatility**
  - Interest Rate Level -> affect straight bond
  - Interest Rate Volatility -> affect option -> effect on embedded bond and OAS
  - Yield Curve Shape
- **Effective Duration**
  - Embedded < option free
  - Zero coupon = maturity
  - Floater = next time to rest
- One-Sided duration
  - 两边小，中间大
- Key rate duration
  - Shaping risk
  - interest 和 coupon 反比
- **Effective Convexity**
  - Puttable: always positive
  - Callable: negative convexity when interest rate decrease
  - Embedded bond convexity <=option-free bond convexity
- Caps and floors – coupon
- **Convertible** bond
  - Bond vs stock return characteristics
  - Conversion ratio N: 1 bond can exchange for N stocks
  - Market stock price S
  - Bond issue value 发行价格
    - Conversion price = **issue** price / N 转换价格=发行价值
  - Straight value – do not convert 持有价格（不行权价格）
  - Conversion value – convert 转换/**行权**价格（行权后价格）
    - Conversion value = N *S
  - Minimal value 最低价格
    - Minimal value = Max (straight value, conversion value)
  - Market value – market selling price 市场买卖价格
    - market conversion price K = market value / N 市场转换价格
    - market premium = K – S
    - market premium ratio = (K-S)/S

        - premium over straight = (K-K1)/K1 = market value / straight value -1

Embedded options
- Manage interest rate risk
- Issue bonds at an attractive coupon rate

Option Style
- European-style
    - exercised on a single day **immediately** after lockout period
- American-style
    - Exercised at **any** time after the lockout period
- Bermudan-style
    - Can be exercised at **fixed** dates after the lockout period

Simple Options
- Callable bond 可赎回
    - Issuer can buy the bond back
- Puttable bond 可卖回
    - Investor can sell the bond back
- Extendible bond 可展期
    - Investor extend the maturity
    - A puttable bond with a longer maturity
    - 等价于一个期限更长的 puttable bond，可以中途归还

Complex options
- Convertible bond
    - Convert debt to common stock
- Estate put 死亡 put
    - Allows the **hers** of an investor to put the bond back to the issuer upon the **death** of the investor
    - Inversely related to life expectancy
- Sinking fund bonds (sinkers)
    - Issuer to set aside funds periodically to retire the bond
    - Reduce the credit risk of the bond
    - Related issuer options: call provision, acceleration provisions, delivery options

Value Relation
- $\mathrm{V_{callable}} = \mathrm{V_{straight}} - V_{call}$
- $\mathrm{V_{puttable}} = \mathrm{V_{straight}} + V_{put}$
- **Straight** bond value is **negative** related to interest rate **level**
- **Option** value is **positively** related to interact rate **volatility**

Valuation
- Callable bond
    - Lower of price and call price

    - Call for low rate
- Puttable bond
    - Higher of price and put price
    - Put for high rate

Interest Rate Volatility 利率波动

- Option values are **positively** related to volatility 期权价值增加
- Straight bond is affected by **level** of interest rates but unaffected by changes in volatility 债券价值不变

Interest Rate Level 利率水平

- Interest rate level -> straight bond 债券价格 -> moneyness of option 期权价值
- Option value
    - Put is **positively** related to interest rate level
    - Call is **negative** related to interest rate level
- Interest rate **increase**
    - Straight bond **decrease**
    - Put option: more likely exercise the option, value **increase**
    - Puttable bond: decreases less rapidly
    - Call option: not affected too much
    - Callable bond: not affected too much
- Interest rate decrease
    - Straight bond **increase**
    - Put option: not affected too much
    - Puttable bond: not affected too much
    - Call option: more likely exercise the option, value **increase**
    - Callable bond: increase less rapidly

Shape of Yield Curve

- Upward sloping -> interest rate increase
    - Call option value decrease (move to out-of-money)
    - Put option value increase (high probability in-the-money)
- Upward sloping flattens (mean interest rate decrease)
    - Call option value increases (move to in-the-money)
    - Put option value decreases (move to out-of-money)

Option-Adjusted Spread (OAS) – risky bond

- A **constant** spread that's added to **all** one-period rates such that the calculated value equals the market price of the risky bond
- $\mathrm{V_{callable}} = \mathrm{V_{straight}} - V_{call} \rightarrow \mathrm{OAS_{call}} = ZS - \mathrm{V_{call}}$
- $\mathrm{V_{puttable}} = \mathrm{V_{straight}} + V_{put} \rightarrow \mathrm{OAS_{put}} = ZS + V_{put}$
- OAS is added to the tree after adjustment for embedded option
- OAS is calculated after the **option risk** has been **removed 考虑了期权风险**
- OAS reflect the **credit risk** over benchmark since option risk is removed
- If bond OAS > OSA benchmark bonds, then the bond is likely **underpriced**.
- Callable bond

- Z-spread = credit risk + liquidity risk + **option** risk
- OAS = credit risk + liquidity risk

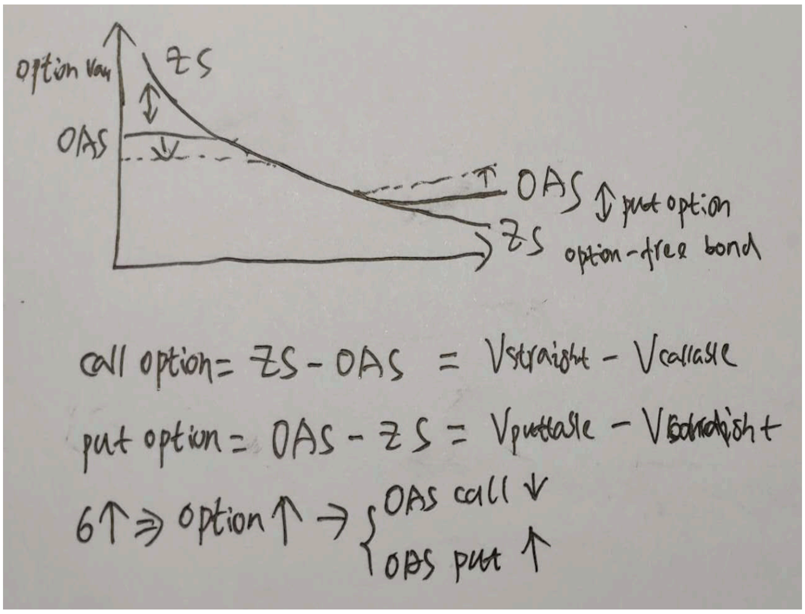
- Similar credit risk should have the same OAS
  - OAS is higher than peer OAS -> **undervalued** with low price
  - OAS is lower than peer OAS -> overvalued with high price
  - OAS is close to peer OAS -> fairly priced

	Treasury benchmark	Bond sector benchmark	Issuer-specific benchmark
G-spread Z-spread	Credit risk	Credit risk	
	Liquidity risk	Liquidity risk	Liquidity risk
	Option risk	Option risk	Option risk
OAS	Credit risk	Credit risk	
	Liquidity risk	Liquidity risk	Liquidity risk
OAS > 0	Over if actual OAS < required OAS; Undervalued if OAS > required OAS	Over if actual OAS < required OAS; Undervalued if OAS > required OAS	Undervalued
OAS = 0	Over	Over	Fairly priced
OAS < 0	Over	Over	Over

Interest Rate Volatility

- Replacement (option bond -> OAS, bond -> Z)
  - If the option is not executed -> ZS
  - If the option is executed -> OAS
  - $\mathrm{V_{callable}} = \mathrm{V_{straight}} - V_{call} \rightarrow \mathrm{OAS_{call}} = ZS - \mathrm{V_{call}}$
  - $\mathrm{V_{puttable}} = \mathrm{V_{straight}} + V_{put} \rightarrow \mathrm{OAS_{put}} = ZS + V_{put}$
- Volatility Increase - Callable
  - Increase, ZS unchanged, option increase
  - callable: $\sigma \uparrow \rightarrow \mathrm{V_{call}} \uparrow \rightarrow OAS_{call} \downarrow$
  - Calculated value decrease, **closer** to market value, need **less** OAS
- Volatility Increase - Puttable
  - puttable: $\sigma \uparrow \rightarrow \mathrm{V_{put}} \uparrow \rightarrow OAS_{put} \uparrow$
  - Calculated value increase, **further** to market value, need **more** OAS

Assumed Level of Volatility	Value				OAS_{CALL}	OAS_{PUT}
	Calls	Puts	Callable	Putable		
High	High	High	Low	High	Low	High
Low	Low	Low	High	Low	High	Low



Option-free bond - Modified

- **Modified** duration and convexity
  - Assume cash flow is not affected
  - $\frac{\Delta \mathrm{P/P}}{\Delta y} = -D$

Embedded Bond - Effective Duration

- Embedded bond: **effective** duration and effective convexity
- $Effecive\ duration = \frac{BV_{-y} - BV_{+y}}{2 \times y \times BV_0}$
- $effective\ convexity = \frac{BV_{-y} + BV_{+y} - 2 \times BV_0}{y^2 \times BV_0}$
- Steps
  - Benchmark interest rates, interest rate volatility, OAS
  - Add a positive parallel shift
  - Build a new binomial interest rate tree using the new yield curve
  - Add OAS to the forward rate at each node
  - Compute value
  - Repeat for a negative parallel shift

Effective Duration Comparisons

- $ED(embedded\ call\ or\ put\ option) \le ED(straight)$ 因为行权
- $ED(zero\ coupon) \approx maturity$
- $ED(coupon\ coupon) < maturity$ 有些现金提前支付
- $ED(floater) \approx time\ (in\ years)\ to\ next\ reset$
  - Floater = par value, full floater -> duration =0
  - If there is a time to reset, each range as a **zero-coupon** bond
- Puttable bond
  - Interest rate increase -> ED decrease
- Callable bond
  - Interest rate decrease -> ED decrease

One-Sided duration 单边/不对称/上升或者下降

- One-sided up-duration and one-sided down-duration
- 两边低，中间高
- Callable (at the money) 靠近左边
    - Duration(down) < Duration(up) 下降有极限
- Puttable (at the money) 靠近右边
    - Duration(down) > Duration(up) 上升有极限

Key Rate Duration / Partial Duration - Shaping Risk 不同期限的利率

- Interest rate sensitivity of changes in yields of specific **maturities**
- **Shaping** risk: capture the changes in the shape of yield curve
- Each time, shift the yield at on maturity, holding others constant
- Can be negative
    - Maturity points that are shorter than the bond maturity if the bond is a zero-coupon bond or has a very low coupon

Key Rate Duration Analysis - Coupon

- Option-free
    - Issue at par
        - Partial duration at maturity is most **important**
        - The rest is zero
    - Issue at a discount – low coupon 低分红，低价值，高 Duration 风险
        - Partial duration at maturity is **higher** than duration at par
        - The rest is zero or **negative**
    - Issue at a premium – high coupon 高 coupon 高溢价
        - Partial duration at maturity is **lower** than duration at par
        - The rest is zero or **positive**
- Callable
    - Low coupon 低分红，低价值，不容易被 call
        - Less likely to be called
        - Highest effect: maturity-matched rate
    - High coupon 高分红，高价值，容易被 call
        - More likely to be called because high price
        - Highest effect: **time-to-exercise** rate
- Puttable
    - Low coupon 低分红，低价值，容易被 put
        - More likely to be put because low price
        - Highest effect: **time-to-exercise** rate
    - High coupon
        - Less likely to be called
        - Highest effect: maturity-matched rate
- 技巧
    - High coupon , high value, more cash earlier 高分红，价值高
        - Call 在价值高时容易被 call

        - Put 在价值低时容易被 put

Effective Convexity

- Straight: positive convexity
- Puttable: positive convexity
- Callable
    - positive when interest rates are high
    - **Negative** convexity when interest rates are low
- Embedded bond convexity <=option-free bond convexity
    - When interest rate too small or too large, Embedded bond convexity goes to **zero**
- Interest rate decrease or yield curve further flatten
    - Callable and puttable bonds will rise but **less** rapidly than the straight bond

Capped and Floored Floaters - Coupon

- Floater
    - **Coupon** adjusted every period based on a reference rate
    - Paid in arrears 开始决定，结束付钱
        - Determined at the beginning of the period
        - Paid at the end of the period
        - present value is always equal to FV
- Capped floater coupon 有上限
    - A cap to the coupon rate
    - Value = straight floater – **embedded cap (similar to callable option)**
- Floored floater coupon 有下限
    - A floor to the coupon rate
    - Value = straight floater + embedded floor (similar to put option)
- Backward induction
    - Coupon is changed but interest rate is not changed 改变分红，不改变利息
- 技巧
    - Coupon 和 interest 相反

Convertible Bond

- Convert to a fixed number of common shares during a specific timeframe (conversion period) and at a fixed amount of money (conversion price)
- Conversion ratio **N** 转换比率，一个债券换多少股票
    - 1 bond can exchange for N stocks
- Market stock price S
- Bond issue value 发行价格
    - Conversion price = **issue** price / N 转换价格=发行价值
- Straight value – do not convert 持有价格（不行权价格）
- Conversion value – convert 转换/**行权**价格（行权后价格）
    - Conversion value = N * S
- Minimal value 最低价格
    - Minimal value = Max (straight value, conversion value)

- Market value – market selling price 市场买卖价格
  - market conversion price K = market value / N 市场转换价格
  - market premium = K – S
  - market premium ratio = (K-S)/S
  - premium over straight = (K-K1)/K1 = market value / straight value -1
- Return
  - stock price increase (conversion value): S/K – 1
  - stock price decrease (straight value): K1/K – 1

Conversion Price

- affected by stock splits/dividends
- change of control: put options
  - conversion price is **lower**
  - hard puts: redeemable for cash
  - soft puts: issuer decides whether to redeem the bond for cash, stock, subordinated debentures, or a combination of the three
- Cash Dividend
  - Pay cash dividend -> price drop -> conversion value drop -> conversion price drop or conversion ratio increase
  - The **cash** dividend is higher than the **threshold** dividend, which will typically result in a **reduction** in the conversion price, which results in an increase in the conversion ratio

Convertible Bond Value

- call option on stock
  - $V_{convertible} = V_{straight} + V_{call-option-on-stock}$
- Properties
  - **Lower** coupon on convertible bonds
  - Value increase when the common stock price goes up

Convertible Bond Risk-Return Characteristics

- Stock price decreases, bond will **overperform** because there is a floor straight value
  - Fixed-income equivalent (**busted** convertible) 当成债券，没有去执行期权
- stock price rises, bond will **underperform** because of the conversion **premium**
  - common stock equity
- stock price stable, bond will overperform because of the **coupon**
  - most of the time

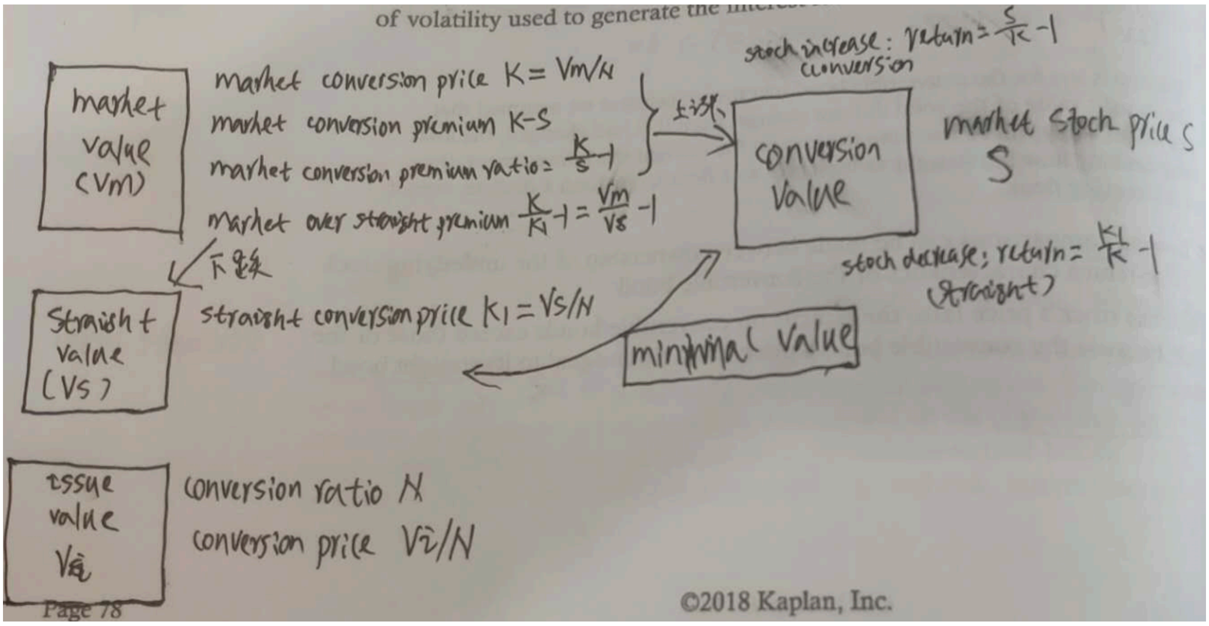
Convertible Embedded Bond

- call option - forced conversion
  - most convertible bond has a **call** option that can be exercised if the common stock price rises to more than the conversion price, resulting in a forced conversion (bondholders must either convert and accept shares or accept a fixed call price).
  - Forced conversion prevents bondholders from continuing to receive coupon payments at the expense of current shareholders.

- protecting shareholders from dilution

Convertible and Callable Bond

- Convertible bonds are more **difficult** to value than callable bonds because the analyst must consider all of the factors that affect the value of a callable bond, including interest rates and interest rate volatility, plus all of the factors that affect the issuer's common stock price and options on its common stock, such as its price volatility.
- An interest rate tree is used to value both a callable and a convertible bond.



Convertible Bond

- Conversion ratio **N** 转换比率，一个债券换多少股票
  - The number of common shares per bond
- **Conversion price** (price of a common share) 一个股票的买入价格
  - conversion price $= \frac{\text{bond issue price}}{conversion\ ratio}$ 执行价格
  - **stated** conversion price 合约里规定的价格
- **market** price of stock **S** 股票市场价格（用现金买入）
- **Conversion** value 转换价值/股票的市值
  - Value of common stock into which the bond can be converted
  - conversion value $=$ conversion ratio $\times$ **market price of stock**
- straight/investment value V_{straight} 债券本身价值
  - market value of the bond if it were not convertible
- minimum value of a convertible bond 转换债最小价值
  - minimum value of a convertible bond $=$ max (straight value, conversion value)
- market price of convertible bond 转换债市场价值
- market conversion price 市场转换价格**K**（用债券买入股票的价格）
  - **market conversion price** $= \frac{\textbf{market price of convertible bond}}{conversion\ ratio}$

- market conversion premium 转换溢价（债券买入价格高，out-of-money）
  - $\textbf{market converion premium} = \text{market conversion price} - \text{market price} = K - S$ 债券买入价格 - 现金买入价格（多花的钱）
- market conversion premium ratio 转换溢价比率
  - $\text{market conversion premium ratio} = \frac{\text{market conversion price}}{\textit{market price per share}} - 1 = \frac{K}{S} - 1$
- downside risk – premium over straight value
  - $\text{premium over straight} = \frac{\textit{market price of convertible bond}}{\textit{straight value}} - 1$
- return from investing in the **convertible** bond
  - $K \to \max\left(S, \frac{V_{\text{straight}}}{N}\right) = \frac{\text{minimal value}}{N}$
  - $\text{return} = \frac{\textit{minimal value}}{\textit{market price}} - 1$
- return from investing directly in stock
  - $\text{return} = \frac{\text{market price}}{\textit{old market price}} - 1$

债券	**Bond value** **债券价值**	**Conversion ratio** **转换比率**	**Stock price** **股票价格**
Stated 发型时 合约规定	Issue value 发行价	Stated conversion ratio	Stated conversion price 转换时买入成本价格
Not convert 不转换	**Straight** value $\mathbf{V}_{\textit{straight}}$ 未来现金流的现值		
Convert 转换成股票	Conversion value $\mathbf{S} \times \mathbf{N}$ 转换后股票的价值	conversion ratio	Market price of stock S 股票市场价格（现金买）
Convertible bond 最小值 **回报**	Minimum value = max (straight value, conversion value)	conversion ratio	
Market price 市场价值 **成本**	Market price of $\mathbf{K} \times \mathbf{N}$ convertible bond	conversion ratio	Market conversion price K 市场转换价格（用债券买）
	Premium over straight = market price / straight value – 1		Market conversion premium = K-S
	Return = minimum value / market price - 1		Market conversion premium ratio = K/S-1

Bond Analytics

- **call-put parity**
  - $\text{C} - \text{P} = \text{PV(forward price of the bond on the exercise date)} - PV(\textit{exercise price})$
- Option free bonds does not depend on interest rate volatility
- Check interest rate volatility: short > long

Credit Analysis Models

Summaries

- **Expected loss**
  - Exposure
  - loss severity, recovery rate (percentage)
  - loss given default, recovery (amount)
  - probability of default:
    - hazard rate
    - risk-neutral probability
- **Credit Value Adjustment (CVA)**
  - CVA = risk bond price – risk-free bond price
- Credit Scores/Rating/Migrations
- Credit Models
  - Structural Models - Option Analogy
  - Reduced Form Models – Default Intensity
- Credit Spreads (CVA, VND)

Exposure (E)

- Exposure at a time t = current cash flow and present value at t of future cash flow
- 最后时刻的 exposure 是 face value
  - $\mathrm{E_T} = FV$
- 反向递推 backward induction
  - $\mathrm{E_t} = \frac{E_{t-1}}{1+r} + C_t$ (value + dividend)
- Example
  - 3-year, 5% coupon, 100 par value, rate 2.5%
  - $\mathrm{E_3} = 100 + 5 = 105$
  - $\mathrm{E_2} = \frac{E_3}{1+r} + 5 = \frac{105}{1.025} + 5 = 107.44$
  - $\mathrm{E_1} = \frac{E_2}{1+r} + 5 = \frac{107.44}{1.025} + 5 = 109.82$

Loss and Recovery Rate

- Loss severity (LS)
- Recovery rate (RR)
  - $\mathrm{RR} + \mathrm{LS} = 1$

Loss and Recovery Amount

- Recovery (R)
  - Recovery is the recovery amount $\mathrm{R} = \mathrm{E} \times \mathrm{RR}$
- Loss given default (LGD)
  - $\mathbf{LGD} = \mathbf{E} \times (\mathbf{1} - \mathbf{RR})$
  - $\mathbf{LGD} = \mathbf{E} - \mathbf{R} = \mathbf{exposure} - \mathbf{recovery}$
- $\mathrm{Recovery} + \mathrm{LGD} = \mathrm{E}$

Probability of default (PD)

- Initial probability of default is **hazard** rate, also conditional probability
- $PS_t = (1 - \mathrm{hazard})^{\mathrm{t}}$

- $\mathrm{PD}_t = PS_{t-1} \times hazard = (1 - \mathrm{hazard})^{\mathrm{t}-1} \times hazard$

Expected loss (EL)

- $\mathbf{EL = PD \times LGD = PD \times E \times RR}$

Present Value of Expected Loss (PVEL)

- $\mathrm{PVEL = risk\ free\ bond - risky\ bond}$
- $\mathrm{PVEL_t} = \frac{\mathrm{EL_t}}{(1+r)^t} = EL_t \times DF_t$ discount factor
- Maximum amount that pay to an insurer to bear the risk
- Time value adjustment 时间价值
    - Present value
- Risk-neutral probability 风险中心定价

Credit Spread

- $\mathrm{spread} = \mathbf{YTM}\ \mathrm{of\ risky\ bond - YTM\ of\ risk\ free\ bond}$

Credit Value Adjustment (CVA)

- $\mathrm{CVA} = \sum_i^n PV(EL_i)$
    - **Sum** of **present value** of **expected loss** for each period
- **Steps**
    - exposure $\mathrm{E_t} = \frac{E_{t-1}}{1+r} + C_t$ (value + coupon)
    - $\mathrm{LGD_t = E_t \times (1 - RR)}$
    - $\mathrm{PD}_t = (1 - \mathrm{hazard})^{\mathrm{t}-1} \times hazard$
    - $EL_t = \mathrm{PD}_t \times LGD_t$
    - $PVEL_t = \frac{EL_t}{(1+r)^t}$
    - $\mathrm{CVA} = \sum_i^n PV(EL_i)$

Credit Value Adjustment (CVA) – bond price

- $\boldsymbol{CVA = risk\ free\ bond\ price - risky\ bond\ price}$
    - 债券价格的折扣
- $risk\ free\ bond\ price = PV(\mathrm{N, R_f, PMT, FV})$
- $risky\ bond\ price = PV(\mathrm{N, YTM, PMT, FV})$

Risk Neutral Probability of Default

- Risk-neutral - implied in the current market
- Risk free approach
    - $\mathrm{PV_{free}} = \frac{FV}{1+R_f}$
- Risky approach 1
    - $\mathrm{PV_{risky}} = \frac{FV}{1+YTM}$
- Risky approach 2 推导思路
    - $\mathrm{PV_{risky}} = \frac{PD \times FV \times RR + (1-PD) \times FV}{1+R_f}$
        - $= \frac{FV - FV \times PD \times (1-RR)}{1+R_f} = \frac{FV - EL}{1+R_f}$
- $YTM - R_f \approx PD \times LS = EL' = spread$

- $\frac{FV}{1+y} = \frac{FV-EL}{1+R_f} \rightarrow \frac{1+R_f}{1+y} = 1 - \frac{EL}{FV}$
- $\rightarrow \frac{y-R_f}{1+y} = \frac{\mathrm{EL}}{\mathrm{FV}} = PD \times LS$

- Given the market price
  - The PD and RR are **positively** related

Internal Rate of Return (IRR)

- PV = present value of **risky** bond = risk free bond - CVA
- If bond default
  - PV: - risky bond price
  - FV: **recovery** at time t

Credit Scores

- Higher score for better quality

Credit Ratings

- Issued for debt, securities, government and quasi-government debt
- **Ordinal** ratings
- Account for expected loss
- Agencies: Moody, S&P, Fitch
- Notching
  - Lower of the rating by one or more levels for more **subordinated** debt
  - Account for **LGD** differences
- Outlook
  - Positive, negative, stable
- High related bonds have lower spreads

Credit Migration

- Change in rating, change in credit risk
- Percentage Change in the price
- Change to a certain grade
  - $\frac{\Delta \mathrm{P}}{\mathrm{P}} = -D_{modified} \times \boldsymbol{\Delta spread}$
  - Modified duration
- Expected change
  - $\frac{\Delta \mathrm{P}}{\mathrm{P}} = -\sum_i p_i \times D_{modified} \times (\mathbf{spread_i} - \mathbf{spread})$
- The probabilities of change are **not symmetrically** distributed around the current rating.
- They are skewed toward a **downgrade** rather than toward an upgrade.
- Increases in credit spreads for **downgrades** are usually larger than decreases in spreads for upgrades.

Credit Models

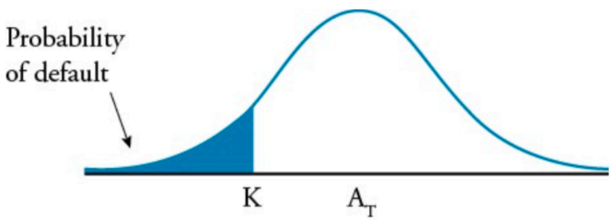
- Structural Models
- Reduced Models

Structural Models - Option Analogy

- Based on balance sheet and option pricing theory (lognormal distribution)
- $A_T = \mathrm{V_d} + V_e$
- **Shareholder: buy option, debt holder: sell option**
- **Call option - shareholder**
    - A **call option** on asset with strike price equal to debt
        - $\mathrm{V_c} = \max(0, \mathrm{A_T} - \mathrm{K})$
        - $\mathrm{V_c} = \mathrm{A_t} N(d_1) - Ke^{-rT} N(d_2)$
    - $\mathrm{V_e} = \mathrm{V_c} = \max(0, \mathrm{A_T} - \mathrm{K})$
        - Long a call option 持有期权
    - $\mathrm{V_d} = A_T - V_c = \min(K, A_T)$
        - Long company asset 持有公司资产
        - Short a call option
- **Put option - debtholder**
    - A **call option** on asset with strike price equal to debt
        - $\mathrm{V_p} = \max(0, K - A_T)$ put option
    - $\mathrm{V_d} = \min(K, A_T) = K - \max(0, K - A_T) = K - V_p$
        - Long the risk-free debt 持有债务
        - Short a put option 卖出期权
    - $\mathrm{V_e} = \mathrm{A_T} - V_d = (A_T - K) + \mathrm{V_p}$
        - Long net asset $A_T - K$ 持有净值产
        - Buy a put option 买入期权

Structural Models - Probability of Default

- Probability of default is **endogenous**
- If asset value falls below default barrier K, the company defaults
- $\mathrm{PD} = \mathrm{P}(\mathrm{A_T} < \mathrm{K}) = \mathrm{N}(-\mathrm{d_1})$



Structural Models

- Advantages
    - **endogenous**
    - Economic rationale for default and explains why default occurs
    - Utilize option pricing models to value risky debts
- Disadvantages
    - Assume a simple balance sheet structure, complex ones cannot be modeled
    - Existence of **off-balance** sheet debt makes default barrier inaccurate
    - Assets of the company are **traded** in the market
    - Do not consider business cycle
    - Assume a fixed risk-free rate

Reduced Form Models

- Assume some **debt** is traded
- **Statistically** model when default occurs
- Default is randomly occurring exogenous variable
- Use **historical** data
- **default intensity -> PD**
  - the probability of default over the next time period
  - Modeled using regression models
- Advantages
  - **Exogenous**
  - No **assumption** about assets of company
  - Default intensity is allowed to **vary** as company fundamental changes as well as when the state of the economy changes
  - Reflect business cycle
- Disadvantages
  - Do not explain why default occurs
  - Default is a **random** event (surprise)

Credit Spread Analytics

- $\text{credit spread} = \text{YTM of risky bond} - \text{YTM of benchmark}$
- bootstrap method can be used
- $\mathbf{CVA = VND - V_{bond}}$
- Value given no default $\text{VND} = \text{PV}(\text{risky bond}|\mathbf{risk\ free}\ \text{rate})$
- $\text{V}_{\text{bond}} = \text{PV}(\text{risky bond}|\text{YTM of risky bond})$

CVA Calculation 利率波动

- **Zero Volatility – interest rate - deterministic cash flow 确定现金流**
  - $\text{VND} = \text{PV}(\text{N}, \mathbf{R_F}, \text{PMT}, \text{FV})$ 无风险利率
  - $\text{V}_{\text{bond}} = \text{PV}(\text{N}, \mathbf{R_F} + \boldsymbol{spread}, \text{PMT}, \text{FV})$ 债券利率
  - $\mathbf{CVA = VND - V_{bond}}$ **简化公式**
- **Non-zero volatility – Interest rate tree 期望现金流**
  - Binominal tree of **benchmark** rate
  - Probabilistic -> deterministic cash flow 平均风险敞口
  - **Backward induction** – calculate value at each node
    - $\text{VND} = \text{V}_0$ first node is the **VND**
  - **Expected exposure** at each **period** 每期平均现金流
    - $\text{EE}_\text{t} = \sum_i p_{t,i} \times CF_{t,i} = \sum_i p_{t,i} \times V_{t,i} + Coupon$
  - **Expected loss** at each period
    - $\text{EL}_\text{t} = EE_t \times PD_t \times (1 - RR)$
  - Present value of expected loss
    - $PVEL_t = PV(EL_t) = \frac{EL_t}{(1+R_f)^t}$
  - $CVA = \sum_t PV(EL_t)$
  - $V_{bond} = VND + CVA$

Credit Spread Components

- $\text{risk free nominal rate} = \text{real rate} + \text{expected inflation}$
- $\text{spread} = \text{risk free rate} + \text{credit risk} + \text{liquidity risk} + \cdots$

Credit Spread Term Structure

- spread change at different maturity
- to create the curve
  - all bonds should have **similar** credit characteristics
- spread is **positively** related to probability of default
- spread is **negatively** related to recovery rate
- most are upward sloping
- inverted sloping
  - **cyclical**: high-yield issuers in **cyclical** industries at the **bottom** of a cycle where investors are looking past the bottom of the cycle
  - bonds that have a **very high** likelihood of default where a primary pricing mechanism is based on the recovery amount in default
  - **refinance**: an issuer refinance a near-dated bond with a longer-term bond
    - newly issued bonds are generally more liquid

Credit Spread Term Structure – Factors

- **credit quality**
  - higher quality, lower spread
- **Financial Conditions**
  - expansion -> narrow spread
  - downturn -> widen spread
- market demand and supply – liquidity
  - less liquid -> widen spread
- equity market volatility
  - volatility -> widen spreads

Securitized Debt

- SPE
  - Issuer: higher leverage and lower cost
  - Investor: diversification, more stable cash flow
- credit exposure
  - collateral pool quality
  - origination and servicing quality
- collateral pool
  - homogeniety
    - **similarity** of assets
    - homogenous: credit card ABC
    - heterogeneous
  - granular
    - **transparency** of assets
    - **highly granular**: hundreds of clearly defined loans
    - **more discrete**: a few loans would warrant examination of each obligation separately

            - leveraged loan CLO (bank loan)
    - short-term granular and homogenous -> statistical-based
        - trade receivables
    - medium-term granular and homogenous -> portfolio-based
    - discrete and non-granular -> individual loan level
- servicer quality
    - manage the pool and service the pool
    - **operating** and **counterparty** risk
- credit enhancement
    - internal
        - tranche: waterfall
        - overcollateralization
        - excess servicing spread: primary form of protection
    - external
        - bank, insurance, loan originators
- covered bond - **recourse**
    - senior secured bonds backed by a collateral pool as well as by the issuer
    - because of the additional collateral protection, usually have higher credit ratings than the issuing institutions have

Credit Default Swap

Summaries

- CDS
- Spread: calculated, fixed, and difference as upfront fee
- Types: Single-name (CTD) and Index CDS
- Credit Event
- Settlement: cash and physical
- Upfront premium% = (spread - coupon) * duration of CDS
- CDS price ($100) = 100 – upfront premium%
- CDS
    - Monetizing, naked, long/short, curve trade

CDS

- insurance contract
    - if a credit event occurs, the credit protection buyer gets compensated by the credit protection seller
- notional principal
    - a face value of protection
- Reference asset
- Credit Risk
    - protection **seller**: **long** credit risk
    - protection **buyer**: **short** credit risk
    - CDS buyer = short CDS
- CDS Spread 基于信用风险
    - The premium paid by the buyer
    - Should be based on credit risk
- CDS - **Standardization** – coupon rate 标准化，支付固定
    - A fixed coupon on CDS
    - 1% for investment-grade securities
    - 5% for high-yield securities
- CDS Spread Difference 差异
    - Buyer pay a fixed coupon rate 每期支付固定
    - Buyer/seller pays the difference **upfront** 期初解决差异
        - Diff * notional principal
- Protection buyer - **Put** option
- ISDA
    - Standardized contracts

Settlement

- Cash settlement 买卖
    - Buyer: give reference obligation to seller
    - Seller: pay the **full** amount
- Physical settlement 补全
    - Seller: pays the par value – market value
        - Principle * payout
        - Payout = 1 - recovery rate

Single-Name CDS

- Reference obligation
  - Fixed-income security, usually a **senior unsecured**, senior CDS
- Reference Entity
  - The issuer of the reference obligation
- Credit event 违约或者平级或者更高级违约
  - Reference entity defaults on the reference obligation
  - Or defaults on any other issue that is ranked pari passu (same rank) or higher
- Cheapest-to-deliver 同等级别里最便宜的
  - Market value of the cheapest-to-deliver (CTD) bond that has the same **seniority** as the reference obligation

Index CDS

- Reference obligation
  - Cover multiple issuers simultaneously
  - Protection for each issuer is equal
- notional principal 保额平均分配
  - total principal is the sum of protection on all issuers
- Pricing
  - Depends on correlation of default
  - High correlation -> high spread

Credit Events

- Bankruptcy
- Failure to pay
  - Issuer misses a scheduled coupon or principal payment
- Restructuring
  - Issuer forces its creditors to accept terms that are different than the specified in the original issue.
  - US do not consider it 美国不考虑重组

Determinations Committee (DC)

- 15-member group of ISDA declares when a credit event has occurred
- A supermajority vote (>=13) is required for a credit event to be declared

CDS Pricing

- PD
  - Multiple-year horizon, increase over time
  - Hazard rate: conditional probability
- LGD
  - LGD = E * (1 - recovery rate)
- Spread ~= EL

Upfront Payment - PV

- **premium** leg – buyer pay - coupon

    - buyer pays to seller
- **protection** leg – seller - **spread**
    - contingent payment seller pays to buyer in case of a default
- **upfront** payment (from **buyer** side)
- buyer pay: upfront + coupon, seller: spread
- $\mathbf{upfront\ payment = PV(protection\ leg|spread) - PV(premium\ leg|coupon)}$

Upfront Payment – CDS Duration
- $\mathbf{upfront\ payment\ \% \approx (spread - coupon) \times duration_{cds}}$
- $\text{Spread} = \text{coupon} + \frac{\text{upfront premium\%}}{duration_{cds}}$

CDS Price
- $\text{PV(premium leg|coupon)} = \text{PV(protection leg|spread)} - \text{upfront payment}$
- $\mathbf{CDS\ price\ (per\ 100\ notional)} \approx \mathbf{100 - \textit{upfront premium}\%}$

Valuation after Inception of CDS
- Spread decrease, seller profit; spread increase, buyer profit
- $\frac{\Delta P}{P} = D \times \Delta\text{spread} \rightarrow$
    - Spread increase, gain for buyer
- $\text{profit} = \Delta\text{spred } \times \text{Duration } \times \text{notional principal}$

Monetizing CDS - Offset
- **monetizing** the gain
    - capture value from an in-the-money CDS exposure
- enter **offset** transaction to unwind exposure
- protection buyer if spread increase
    - sell the original CDS with its remaining maturity
- profit
    - difference between upfront premium received and paid

Naked CDS 不买债券，直接买 CDS，以后卖出
- buyer has no underlying exposure

Long/Short Trade
- purchase protection on **one** reference entity while selling protection on **another** related reference entity

Credit Curve – Term Structure
- upward sloping if longer maturity bonds have a higher spread
- constant hazard rate -> flat credit curve

Curve Trade – buy and sell
- a type of long/short trade
- buy and sell the **same** entity with different maturity

- upward-sloping credit curve
    - sell short-term CDS 卖掉短期
    - buy long-term CDS 买入长期
- curve flatten trade: upward-sloping -> flatten
    - buy short-term CDS
    - sell long-term CDS

Basis Trade – buy (sell) bond and buy (sell) CDS

- difference in credit spreads between **bond** markets and **CDS** market
- mispricing will be temporary and the disparity should eventually disappear after it is recognized
- **bond** spread 4%, CDS spread 3%
    - buy the bond and buy CDS
- bond spread 3%, CDS spread 4%
    - sell CDS, sell bond

Leveraged Buyout (LBO) – buy stock and buy CDS (short)

- issue debt to repurchase equity
- debt increase spread
- A acquire B with a premium issue a 5-and 10-year debt
    - A CDS spread increase, stock price decrease
    - Solution: buy B's share, **short** CDS (buyer)
- **purchase** stock and purchase CDS protection
- **purchase CDS = short CDS**

Index CDS 整体和部分

- Constituents is priced different than index CDS spread

CDO – buy and sell

- **Cash CDO** (**debt**)
    - Portfolio of debt securities
- **Synthetic** CDO (**CDS**)
    - Similar credit risk exposure to that of a cash CDO
    - But is assembled using CDS rather than debt securities
    - **Risky bond = risk-free bond - CDS**
- If synthetic CDO cost < cash CDO cost
    - Buy synthetic CDO and sell cash CDO