PV of Cash Flow

- Time t_i (number of years = months/12)
  - Time t is 0.5 for six months
  - Time t is 1 for 12 months
- Future cash flow at time t_i is C_i, which can be coupon and/or principle value
- t_i period spot rate is z_i, which is annualized
- Present value at time t_i （用复利法）

$$PV_i = \frac{C_i}{(1+z_i)^{t_i}}$$

哪个是正确的？

- $PV_i = \frac{C_i}{1+z_i \times t_i}$（错误，单利）
- $PV_i = \frac{C_i}{(1+z_i)^{t_i}}$ **（正确，复利）**

假如是半年的话

- $PV = \frac{C}{\sqrt{1+z}}$ 和 $PV_i = \frac{C}{1+\frac{z}{2}}$很接近
  - 因为当$x \to 0$时，$\sqrt{1+x} \cong 1+\frac{x}{2}$

假如是 3 年时，$PV = \frac{C}{(1+z)^3}$明显比$PV = \frac{C}{1+z\times 3}$对。

Figure 4: Duration Calculation

Year	CF for 5-Year Bond	CF for 1-Year Bond	Spot Rate	PV(CF)	t × PV(CF)
1	\$5	\$103.5	3.50%	\$104.83	\$104.83
2	\$5	\$0	3.90%	\$4.63	\$9.26
3	\$5	\$0	4.19%	\$4.42	\$13.26
4	\$5	\$0	4.21%	\$4.24	\$16.96
5	\$105	\$0	5.10%	\$81.88	\$409.38
				\$200.00	\$553.69

验证：第 3 年的利率是 4.19%, FV 是 5， $PV = \frac{C}{(1+z)^3} = \frac{5}{(1+0.0419)^3} = 4.42$

例题

44. An analyst is using the delta-normal method to determine the VaR of a fixed income portfolio. The portfolio contains a long position in 1-year bonds with a $1 million face value and a 6% coupon that is paid semi-annually. The interest rates on six-month and twelve-month maturity zero-coupon bonds are, respectively, 2% and 2.5%. Mapping the long position to standard positions in the six-month and twelve-month zeros, respectively, provides which of the following mapped positions?

C

A. $30,000 and 1,030,000

B. $29,500 and 975,610

C. $29,703 and 1,004,878

D. $30,300 and 1,035,000

Answer: C

The long position is mapped into a combination of market values of the zero-coupon bonds that provide the same cash flows:

$$\Delta P \approx -D^{*} \times P \times \Delta y + \frac{1}{2} \times C \times P \times (\Delta y)^2$$

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1-year bond, face value 1m, 6% coupon, semi-annually

Time (months)	Bond + Coupon (million)	Spot rate	PV (million)
6	0.03m	2%	$\frac{\mathbf{0.03}}{\mathbf{(1+0.02)^{0.5}}} = \mathbf{0.029704}$
12	1+0.03=1.03m	2.5%	$\frac{1.03}{1+0.025} = 1.004878$

问题：在计算半年的折现时哪个是对的？为什么

- $\frac{0.03}{1+\frac{0.02}{2}} = 0.029703$
- $\frac{0.03}{(1+0.02)^{0.5}} = 0.029704$ （这个是正确的）

参考：

https://www.investopedia.com/terms/s/spot_rate_yield_curve.asp

For example, suppose that a two-year 10% coupon bond with par value of $100 is being priced using Treasury spot rates. The Treasury spot rates for the subsequent four periods (each year is composed of two periods) are 8%, 8.05%, 8.1% and 8.12%, and the four corresponding cash flows are $5 (calculated as 10%/2 x $100), $5, $5, $105 (coupon payment plus principal value at maturity). When the spot rates are plotted against the maturities, we get the spot rate or the zero curve.

Using the bootstrap method, the number of periods will be designated as 0.5, 1, 1.5, and 2, where 0.5 is the first 6-month period, 1 is the cumulative second 6-month period, and so on.

The present value for each respective cash flow will be:

$= \$5/1.08^{0.5} + \$5/1.0805^{1} + \$5/1.081^{1.5} + \$105/\$1.0812^{2}$

$= \$4.81 + \$4.63 + \$4.45 + \$89.82$

$= \$103.71$