

- Derivative Markets and Instruments
- Basics of Derivative Pricing and valuation

## Derivative Markets and Instruments

### Derivative

- Derive its value from value or return of another asset or security

### Trade Place

- Exchange-traded derivative: standard and clearinghouse
  - Options and futures
- Dealers and OTC
  - Forwards and swaps

### Commitment

- **Forward** commitment
  - Legally binding promise to perform some action in the future
  - **Forward, futures, swaps**
  - Equities, indexes, bonds, foreign currencies, physical assets, interest rates
  - Premium: no
- **Contingent** claim
  - a claim that depends on a particular event
  - **options, credit derivative (default or ratings downgrade)**
  - takes **two** options to replicate the payoffs on a futures/forward contract
  - premium: long pay short

### Forward

- hedge
- at initiation: no payment, **zero** value
  - specify a forward price to buy
- future price rise, then buyer have positive value
- positions
  - long: buy the financial or physical asset
  - short: sell or deliver the asset
- deliver
  - physically deliver
  - cash-settled: forward – spot price at expiration
    - contracts for differences of non-deliverable forwards (NDFs)

### Futures

- standardized vs customized
- exchanged-traded vs private contracts that typically do not trade
- main features
  - active secondary market
  - greater **regulation**
  - backed by Clearinghouse (no CCP)
  - daily cash settlement of gains and losses

- minimum price fluctuation (tick size)
  - daily price move limit
  - settlement date
  - trading times
- settlement price
  - not closing price, **average** of prices during **last/closing** period
  - reduce price manipulation
  - used as settlement price and spot price of expired contract
- open interest
  - number of **outstanding** futures contracts of a specific kind
- speculator: gain exposure
- hedger: reduce exposure
- clearinghouse
  - split the trade, and act as the counterparty
  - can **reverse** the trade
  - remove counterparty risk
- margin 保证金
  - money deposited by both long and short
  - unlike **bond or stock**, no loan involved and no interest charge
  - mark-to-market: adjust margin balance based on gains and losses
- initial margin
  - posted before a trade.
  - Low, equals about one day's maximum price fluctuation
- maintenance margin
  - minimum margin must be maintained
  - if breached, must bring to **initial** margin
- price limits
  - settlement price change
  - limit move
    - price limit up 上限
    - price limit down 下限
  - locked limit
    - trade cannot take place and are locked in existing positions

## Swaps

- **exchange** a series of payments on **periodic** settlement dates over a certain time period until a **termination** date
  - at settlement date, payments are **netted**
  - length is the **tenor** of the swap
- most **participants** are **large** institutions (not individuals)
- **notional** principle: not exchanged
- plain vanilla interest rate swap
  - fixed-rate for floating-rate payment
  - fixed-rating payer: pay-fixed
  - floating-rate payer: pay-floating
- basis swap

- trading one **set of floating** rate payments for another
- replication
  - fixed payer
    - by being long the floating rate bond and being short the fixed-rate bond.
  - A series of off-the-market forwards

## Options

- the right but not the obligation
- exercise/strike price
- call option: buy, buyer (caller)
- put option: sell, seller (**writer**)
- option **premium**: price of option
- American options: exercised any time
- European options: exercised at expiration
  - At expire, they are the same
  - Before expiration, they differ

## Credit Derivative

- Provide the **bondholder (lender)** with protection against **downgrade** or **default** by the borrower
- Credit default swap (CDS) - default
  - insurance against **default**
  - Pay a series of cash and receive a payment once default
- Credit **spread** option - downgrade
  - A **call option** that is based on **bond's spread** relative to a benchmark
  - If bond's credit quality decrease, the yield **spread** will increase, and the bondholder will collect a payoff on the option.
- Pros and cons
  - cons: too risky, leverage
  - pros
    - provide **price** information
    - allow risk to be **managed and shifted** among the market participants
    - reduce **transaction** costs
- arbitrage
  - riskless, earn a return **greater** than **risk-free** rate
  - make a **profit** at **no risk** with **no capital** invested
  - exists when assets are **mispriced**
  - **eliminate misprice and improve market efficiency**
- Law of **once** price
  - two securities have identical cash flows in the future, regardless of future events should have the same price
  - Requires an investment

## Basics of Derivative Pricing and valuation

### Costs and Benefits

- Costs of owning asset
  - storage, insurance, opportunity cost
- Benefits
  - monetary (dividend, interest)
  - non-monetary (**convenience** yield)
    - significant for commodities
    - if it is hard to sell short, owning it has convenience

### Cost of carry

- Net cost of holding an asset
- $S_0 = \frac{E(S_T)}{(1+R_f+risk\ premium)^T} + PV(benefits) - PV(cost)$
- Risk averse: require **risk premium**
- Risk neutral: only risk-free rate

### No-arbitrage condition/risk-neutral pricing

- Portfolio, discount at **risk-free** rate
- Structure
  - Long asset, short derivative
  - $PV(asset + short\ forward) \times (1 + R_f)^T = FV(asset + short\ forward)$
- If cost of buying asset and selling forward < PV
  - Borrow money, buy asset, sell forward, earn **excessive** risk-free return

### Replication

- **Risky asset + derivative = risk-free asset**
- Example
  - Long stock, short derivative at 50 on the stock, six months
  - $S - F(50) = \frac{50}{(1+R_f)^{0.5}}$
  - $F(50) = S - \frac{50}{(1+R_f)^{0.5}}$ 
    - Buy stock, borrow cash
  - $-F(50) = -S + \frac{50}{(1+R_f)^{0.5}}$ 
    - Sell stock, lend money

### Forward

- Forward - Bond
  - **Risky bond+ credit protection = risk-free bond**
  - Portfolio
    - Buy stock, sell call option at 40, buy a put at 40, will receive 40
- value and price
  - value 合同价值
    - value of forward and futures are **zero** at initiation
    - change due to price of underlying **asset**
  - forward price 合同价格 (买资产的价格)
    - specified in contract, remain **constant**
    - the price to buy the **underlying** asset

- $F_0(T) = S_0 \times (1 + R_F)^T$
- Value – from longer side (spend F to buy and get asset S)
  - $V_0(T) = S_0 - \frac{F_0(T)}{(1+R_F)^T}$
  - $V_t(T) = S_t - \frac{F_0(T)}{(1+R_F)^{T-t}}$  (long side value, pay F in the future and own S)
- Value - Cost and benefits
  - $V_0(T) = S_t + PV_t(cost) - PV_t(benefit) - \frac{F_0(T)}{(1+R_f + risk\ premium)^{T-t}}$
  - Current asset + costs – benefit – future contract price

### Forward rate agreement (FRA)

- LIBOR is the underlying rate
  - US dollar libor, Eurodollar time deposit, interbank US dollars in London
- **Long**
  - Pay **fixed**, receive **floating**
- Hedge borrowing and lending
- Intend to borrow fund -> take long pay fixed, receive floating
  - Receive a payment if borrowing cost increase
- Intend to lend money -> take short
  - Rate decline -> receive a payment
  - Rate increase -> make a payment
- Synthetic FRA
  - Borrow 30\*90 forward -> borrow **120** and lend 30

### Forward and futures differ

- Differ in standardisation, liquid, counterparty risk
- Futures
  - Gains and losses are settled daily
  - Margin balance is adjusted daily
  - Excess margin can be withdrawn
  - Insufficient can be deposited
- The same
  - Interest rates are known
  - Interest rates are **constant**, or simply **uncorrected** with future prices
  - They are equal, ignoring the time value of money (interest rate)

### Correlation between interest rate and asset price ( $F = S \cdot (1+r)^T$ )

- **Positive**
  - Prefer long in future (excess cash realized earlier)
  - Long side -> earn more if interest rate is high
    - Future price > forward price
  - Long side -> loss more if rate is low
- Zero
  - No preference
- Negative
  - Prefer long in **forward**
  - Future: rising price -> more profits -> lower interest rate -> more reinvest risk

- Future suffer loss earlier

## Swaps

- Equivalence: **bonds**
  - Issue a fixed-rate bond and use the proceed to buy a floating-rate bond
- Equivalence: a **series** of forward rate contracts
  - Each with a **forward** rate = swap **fixed** rate
  - A series of forward contracts with **swap** price
- Price and value
  - Value is zero at initialization and obtained through **replication**
  - Price is **not zero** and is determined by **replication** and is **fixed**
- One contract settles each quarter
  - One known payment and **three unknown** payments
  - Long size S-F
- Forward contracts
  - Zero value forward contract for each of future dates
- **Off-market forward**
  - With a **contract** rate that gives it a **non-zero** value at **initialization**
  - Swap has **zero** at initiation
  - Must consists of some off-market forwards with **positive** and some **negative** present values, so that the sum is zero
- Currency swaps
  - The underlying principal is denominated in different currencies and is typically exchanged at the **start and end** of the swap

## Option

- Moneyness
  - At, in, out
- Intrinsic/exercise value:  $\max(S-X, 0)$
- Time value/speculative value
- *option premium = intrinsic value + time value*

## Option factors

- **valuation**
  - $c = S \times N(d_1) - K \times e^{-r \times T} N(d_2)$
  - $p = K \times e^{-r \times T} N(-d_2) - S \times N(-d_1)$
  - $d_{1,2} = \frac{\ln S - \ln K + r \times T}{\sigma \sqrt{t}} \pm \frac{1}{2} \sigma^2 \times T$
- Underlying price
  - Call, higher price, high value
  - Put: reverse
- Exercise price
  - Call, higher price, lower value
  - Put: reverse
- Risk-free rate of interest
  - Call, higher -> higher value
- **Volatility** of underlying (both increase)

- Make options valuable, increase make call and put increase
  - If volatility is zero, time value is zero
- Time to expire
  - Call
    - Longer time increase volatility -> increase value
  - Most put
    - Longer time increase option values
  - **Deep-in-the-money** European put
    - higher risk-free rate, longer time -> **reduce value**
- Costs and benefits of holding the asset ( $S \rightarrow S + \text{cost} - \text{benefit}$ )
  - $\text{call} - \text{put} = (\text{stock} + \text{PV}(\text{costs}) - \text{PV}(\text{benefits})) - \frac{\text{strike price}}{(1+R_F)^t}$
  - If there are benefits, call values are decreased, put are increased
  - If there are **storage** costs, make the call option more valuable

### Option Call-put parity

- $\text{call} - \text{put} = \text{stock} - \frac{\text{strike price}}{(1+R_F)^t}$
- **fiduciary** call (bond + call)
  - a call with strike price X and a riskless bond that pay X
  - if it is out of money, it is X
  - if it is in the money, it is  $X + (S-X) = S$
- **protective** put (stock + put)
  - a share of stock + put on the stock
  - if put is in the money, it is  $S+(X-S) = X$
  - if the put is out of money, it is S

### call-put-forward-parity

- $\text{call} - \text{put} = \frac{F_0(T)}{(1+R_F)^t} - \frac{\text{strike price}}{(1+R_F)^t}$
- the asset must be purchase at the expiration in forward price

### one-period binomial model

- $U$  size of up move
- $D = \frac{1}{U}$  size of down move
- $\pi_u = \frac{1+R_f-D}{U-D}$  **risk-neutral** probability of up move
- $\pi_d = 1 - \pi_u$  probability of down move
- $S_u = S_0 \times U$
- $S_d = S_0 \times D = \frac{S_0}{U}$
- Procedure
  - Compute stock price
  - Payoff of the option at maturity
  - Expected value of option
  - Discounting it back using **risk-free** rate
- Used for hedging

### European and American Options

- At expiration -> same
  - they have the same value
- Before expiration
  - they are equal unless right of exercise has **positive** value
- Call option **without cash flow** -> same
  - no advantage to exercise early, they are the same
- Call option **with cash flow (valuable)**
  - **American options > European options**
  - If there is a dividend, the price will drop on the ex-dividend day
  - Better to sell it before ex-dividend price or hold to receive the dividend
- Put option **with cash flow** (not valuable except **deep in the money put**)
  - Early exercise **not valuable** because price drop is good for put option
    - Cash flow -> decrease price -> put more valuable
  - **Deep in the money (American options > European options)**
    - 20 drops to 0
    - No further space to drop, but can increase

Type	Sub-type	At Expiration	Before Expiration	
			No cash flow	Has cash flow
Call	All	Equal	Equal	American > European
Put	Deep in the money		Equal	Equal
	Other			American > European