- Introduction to Fixed-Income Valuation

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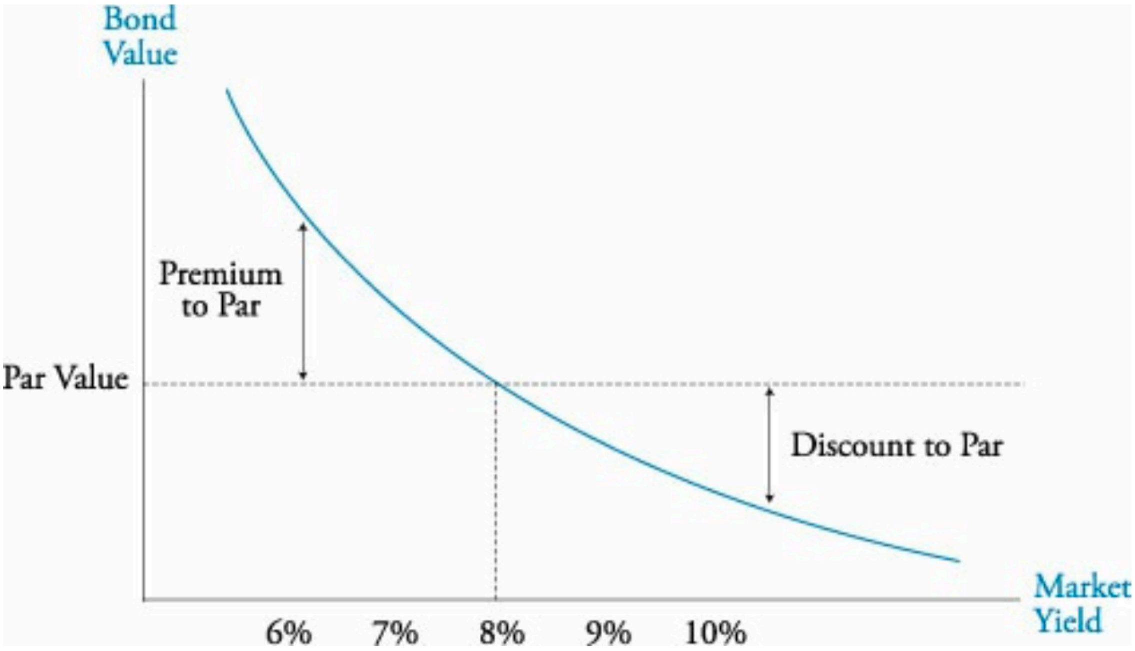
Bond Pricing

- Bond value
  - the summing of **present** values of all the cash flows
- **Yield-to-maturity** (YTM) or redemption yield
  - The market discount rate used to discount cash flows
- Define
  - Maturity Is $\mathbf{n}$ years, and face value is P
  - coupon at each year is C, paid annual
  - yield to maturity is Y
- Market Value / Present value (price)
  - $\mathrm{PV} = \sum_{t=1}^{n} \frac{C}{(1+Y)^t} + \frac{P}{(1+Y)^n}$
- **Reverse** Relation between yields and market value
  - When bond yields **increase** (decrease), its market value (present value) **decreases**
- **Discrete compounding**
  - Compounding $\mathbf{m}$ times a year
  - Equivalent to
    - $\mathrm{n} \times \mathrm{m}$ years, coupon is $\mathrm{c} = \frac{\mathrm{C}}{\mathrm{m}}$, interest is $\mathrm{y} = \frac{\mathrm{Y}}{\mathrm{m}}$
  - $\mathrm{PV} = \sum_{t=1}^{n\times m} \frac{\frac{C}{m}}{\left(1+\frac{Y}{m}\right)^{t\times m}} + \frac{P}{\left(1+\frac{Y}{m}\right)^{n\times m}} = \sum_{t=1}^{n\times m} \frac{c}{(1+y)^{t\times m}} + \frac{P}{(1+y)^{n\times m}}$

Convex Relationship

- Yield
  - Yield increase, bond price decrease
- Coupon rate
  - **Lower** coupon rate is more sensitive to change in yield
- Maturity
  - **Longer** maturity is more sensitive to change in yield
- Percentage change - Convex
  - Percentage decrease in value when yield increase by a given amount is **smaller** than the increase when yield increase by the same amount
- **Premium or discount**
  - coupon > yield -> premium
  - coupon = yield -> par
  - coupon < yield -> discount

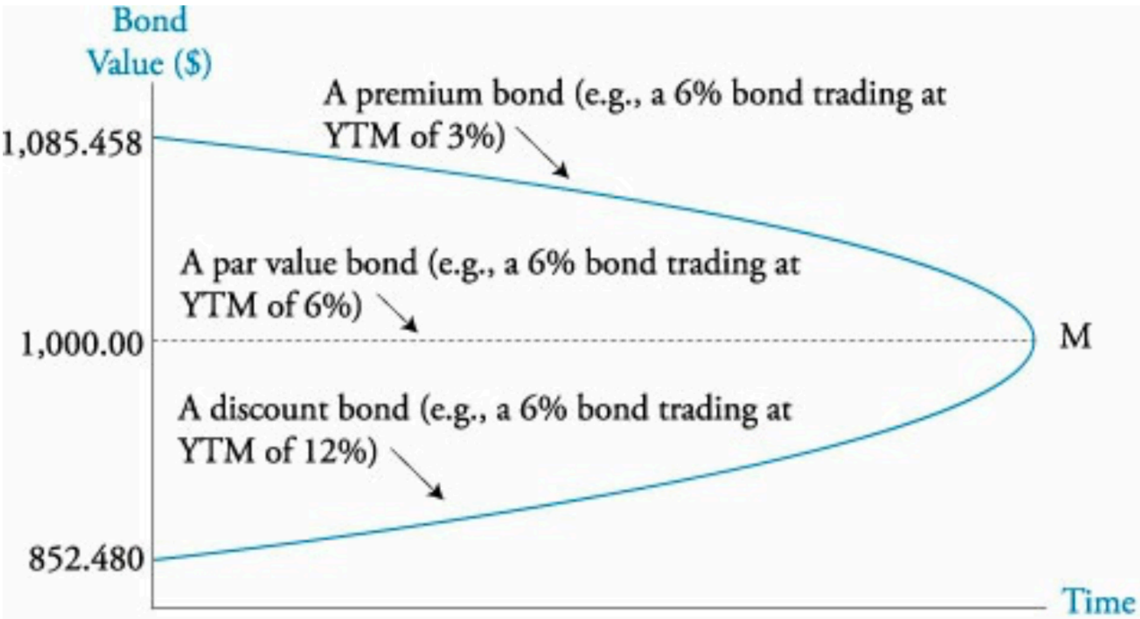
Figure 1: Market Yield vs. Bond Value for an 8% Coupon Bond



Price and Maturity – pull-to-par

- regardless of yield, price will **converge** to par value as maturity approaches
- constant-yield price trajectory

Figure 3: Premium, Par, and Discount Bonds



Spot Rate

- yield-to-maturity uses the same discount rate for all bond cash flow
- in reality, discount rate depends on the time period of the cash flow
- **spot rate / zero rate** / zero-coupon rate
    - market discount rate for a single payment (zero-coupon bond) in the future

- Assume spot rate at time t is S_t
- Bond price is
  - $\mathrm{PV} = \sum_{t=1}^{n} \frac{C}{(1+S_t)^t} + \frac{P}{(1+S_n)^n}$
- It is also called **no-arbitrate** price
- The bond value is slighter greater than its par value

Full Price and Accrued interest 全价和应计利息

- Previous price assume settlement date is on a coupon payment date
- Full/dirty/invoice price 多一次复利
  - When settle between coupon payment dates, the full price is
  - $\text{Full price} = \text{ price on last coupon date} \times \left(1 + \frac{\mathrm{Y}}{\mathrm{m}}\right)^{\frac{d}{D}}$
  - Price on last coupon date can be calculated
  - $\text{d is the days} = \text{settlemnt date} - \text{last coupon payment date}$
  - $\text{D is the days between two coupon payment dates}$
  - BA II 计算器时间格式：MM.DDYY
- Accrued interest 线性
  - $\text{accrued interst} = \text{coupon } \times \frac{\mathrm{d}}{\mathrm{D}}$
- flat/clean price
  - $\text{Flat price} = \text{full price} - \text{accured interest}$
  - Flat price is **different** from price on last coupon date
  - Bond dealer often quote the **flat** price, more stable
- Days
  - Actual/actual: government
  - 30/360: corporate
- Range
  - $[\text{start date, end date}) \rightarrow days = end\ date - start\ date$
  - Aug 15 to Aug 21, days is 21-15=6 (Aug 21 is excluded)

Matrix Pricing

- Estimate the **yield** 收益率
  - for **not traded or infrequently** traded bonds
  - Use the YTMs of traded bonds that have very close **credit quality** and are similar in **maturity and coupon**
  - Use linear interpolation on maturity
- Estimate the **spread 利差**
  - Treasury bond is used as benchmark yield for US corporate bonds
  - linear interpolation on spread

Yield Measures Fixed-Rate Bonds

- effective yield 有效利率 (remove effects of compounding 去掉复利次数的影响)
  - the compound return that depends on how many coupon payments
  - the frequency of coupon payments is **periodicity** of annual rate
  - Given quoted yield Y

        - $EAF = \left(1+\frac{Y}{m}\right)^m - 1$
    - Can adjust it for different periodicity based on the same EAF
        - $\left(1+\frac{Y_1}{m_1}\right)^{m_1} = \left(1+\frac{Y_2}{m_2}\right)^{m_2}$
- Street convention yield or true yield
    - Street convention yield 名义收益
        - on **stated** coupon payment dates
    - true yield 真实收益
        - based on **actual** payment date
        - some coupon dates fall on weekends and holidays will be made the next business day
    - street convention yield > true yield
- current yield 只是 **coupon** 收益（无资本利得和再投资）
    - only annual interest income (no capital gains & losses, no reinvestment income)
    - $\text{current yield} = \frac{\textbf{annual}\text{ cash coupon}}{bond\ price} (bond\ price = flat\ price)$
- simple yield (coupon 收益，溢价或者折价的线性折旧)
    - take premium or discount into account
    - assume discount or premium **declines evenly** over the remaining years to maturity
    - $\text{simple yield} = \frac{\textbf{annual}\text{ cash coupon}}{bond\ price} + \frac{\text{strainght line amoritization}}{bond\ price}$
        - $bond\ price = flat\ price$
- yield-to-maturity
    - Hold until mature
- Yield-to-call
    - It can be calculated for each possible date and price
    - Yield-to-first call, yield-to-first par call
    - PV is current value, FV is the call price, N is the time from now to call date
- Yield-to-worst
    - The **lowest** yield-to-maturity and yield-to-call
- Option-adjusted yield (remove the effects of option 去掉期权的影响)
    - Callable bond price = option-free bond – call option
        - Option-free=add the value of call option to the bond's flat price
    - Option-adjusted yield < yield-to-maturity
        - Callable bonds have higher yield to compensate the call option
    - Compare embedded option bonds to similar option-free bonds

Floating Note Yields

- Value of FRN are **more stable** than fixed-rated debt of similar maturity because coupon interest rates are **reset periodically** based on reference rate
- Arrear
    - Interest rate is set at the **beginning** of a period
    - Payment is made at the **end** of a period
- Interest rate = reference rate + margin
    - Margin reflect credit risk

- The liquidity and tax treatment can also affect margin
- **Quoted margin** 报价利差(用于计算 coupon)
  - The margin used to **calculate coupon** payment
  - Coupon rate = LIBOR + quoted margin
- **Required/discount** margin 必要利差 (用于折现，平价时的利差)
  - The margin required to return the FRN to its **par** value
  - Yield = LIBOR + required/discount margin
- Credit quality
  - When credit quality is unchanged - par
    - **quoted margin = required margin**
    - FRN returns to its par value at each reset date when next coupon payment is reset to the current market rate (plus or minus appropriate margin)
  - When credit quality decrease - discount
    - **Required margin** increase > quoted margin, so FRN sells at a discount
  - When credit quality increase – premium
    - Required margin decrease < quoted margin, so FRN sells at a premium
- Value
  - Used **quoted** margin to estimate future cash flow
  - Use **required** margin to discount future cash flow into present value

Money Market Yields

- Can be discount from face value or add-on yield, can be 360-day or 365-day
  - Both discount and add-on yield are **simple** yield
- US treasury bill: discount, 360-day
- LIBOR and bank CD: add-on-yield
- **Bond equivalent yield**: 365-day add-on yield

Yield curve 收益率曲线

- Shows yield by maturity
- Term structure of interest rate 期限结构
  - The yields at different maturities (terms) for like securities or interest rates

Spot curve/Spot rate yield curve (即期利率曲线 , zero-coupon bond 零息债券)

- Yield curve for single payments in the future, such as zero-coupon bonds or stripped Treasury bonds
- The term structure of **spot rate (即期利率)**
- Also named **zero** curve (**zero-spread**) or stirp **curve** (stripped Treasuries)
- Usually quoted on a semi-annual bond basis

Yield curve for coupon bonds (收益率曲线， 有息债券, 完全基于 spot curve)

- Shows YTM for coupon bonds at various maturities
- Calculated yields for specific maturities and use linear interpolation for estimation
- Equation
  - $\sum_{t=1}^{n} \frac{C}{(1+Y)^t} + \frac{F}{(1+Y)^n} = \sum_{t=1}^{n} \frac{C}{(1+S_t)^t} + \frac{F}{(1+S_n)^n} \to Y$

Par curve/Par bond yield curve (平价利率曲线，平价债券，完全基于 spot curve)

- Not yield on actual bond, but the yield that is constructed from **spot** curve
- YTM of a par theoretical bond at each maturity
- Not directly observed yields
- Use spot rates and let PV = FV=100, YMT=coupon rate
  - $100 = \sum_{t=1}^{n} \frac{C}{(1+S_t)^t} + \frac{100}{(1+S_n)^n} \rightarrow \mathrm{YTM} = \frac{C}{100}$

Forward Curve 远期利率曲线 (fully depends on spot curve, 完全基于 spot curve)

- Forward rates are yield for future periods
- **Forward curve**
  - Usually shows the yield of a 1-year securities for each future year, quoted on a semi-annual bond basis
- Notation **ayby or $\mathbf{a_y b_y}$**
  - **ayby** means b-year forward rate a-year from now on
  - 2y1y means 1-year forward rate 2 years from now on
- Spot and Forward Relation
  - $(1 + S_n)^n = (1 + S_1) \times (1 + 1y1y) \times (1 + 2y1y) \times \cdots \times (1 + (n-1)_y 1_y)$
  - $(1 + S_n)^n = (1 + S_x)^x \times \left(1 + (x-1)_y 1_y\right) \times \cdots \times (1 + (n-1)_y 1_y)$
- Forward rate based on spot rate
  - $(1 + S_{n+1})^{n+1} = (1 + S_n)^n \times (1 + ny1y)$
  - $\rightarrow ny1y = \frac{(1+S_n)^n}{(1+S_{n+1})^{n+1}}$
- Semi-annual
  - $\left(1 + \frac{S_1}{2}\right)^2 = \left(1 + \frac{S_{0.5}}{2}\right) \times \left(1 + \frac{0.5y0.5y}{2}\right)$

Yield Spread

- Yield spread
  - Difference between yields of two bonds
- Benchmark spread
  - A yield spread relative to a benchmark bond
- G-spread (benchmark is government bond)
  - Fixed-income securities, **on-the-run government** bonds are used as benchmarks
  - Maturity must match
- I-spread (interpolated, benchmark is a swap rate)
  - Use rates for interest rate swaps in the same current and with the same tenor as a bond
  - Yield spreads relative to swap rates are known as interpolated spreads
- **Libor**
  - **Floating-rate securities** typically use **Libor** as benchmark rate
- Factors Analysis
  - Macroeconomic factors
    - Affect all bonds, the yields increase but spreads are constant
  - Microeconomic (Firm-specific/industry-specific)

        - The yields increase and the spread increase
        - Credit risk or issuer's liquidity
- Disadvantages
    - G-spread and I-spreads assume spot yield curve is **flat**, but normally it is upward-sloping

Zero-volatility and Option-Adjusted Spreads – based on spread curve

- Z-spread / zero-volatility spread (parallel shift in benchmark spot curve)
    - The spread added to each spot rate to discount the bond to its market value
    - a **parallel** shift in spot curve
    - $\mathrm{PV} = \sum_{\mathrm{t=1}}^{\mathrm{n}} \frac{C}{(1+S_t+Z)^t} + \frac{F}{(1+S_n+Z)^n}$
- Option-adjusted spread
    - Used for bonds with embedded options
    - It is the spread to the government spot rate curve that the bond would have if it were **option-free**
        - $\mathrm{PV} = \sum_{\mathrm{t=1}}^{\mathrm{n}} \frac{C}{(1+S_t+OAS)^t} + \frac{F+opton\ value}{(1+S_n+OAS)^n}$
    - OAS < Z spread
- Callable bond
    - $\text{callable bond price} = \text{option free bond price} - \text{call option}$
    - $\rightarrow\ \text{Z spread} = \text{OAS} + \ \text{option value}$
- Puttable bond
    - $\text{puttable bond price} = \text{option free bond price} + \text{put option}$
    - $\rightarrow\ \text{Z spread} = \text{OAS} - \ \text{option value}$