Error in a Function

Definition: A function is said to be differentiable at point (x, y, z) of Δz is

$$\Delta f = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z$$

The Formula for the *relative error* and *Percentage error* is

$$Relative\ Error = \frac{|\Delta f|}{f} \qquad Percentage\ Error = \frac{|\Delta f|}{f}\times 100$$

Q1). Find the percentage error in the area of an ellipse when an error of 1 percent is made in measuring the semi major and minor axis?

Q2). Find the hypotenuse of right angle triangle whose sides are 6.03 units and 7.96 units also find the relative error?

Q3). A box is approximated by 3 by 4 by 5 feet, if each dimension is excess by 0.1 inch. Find the length of the diagonal, also find the excess volume.

Q4). The base of a box is a square of a side 5.005 cm and its depth is 7.998 cm, find the volume of a box also compute a relative and percentage error.

Solution of Non-Linear equation

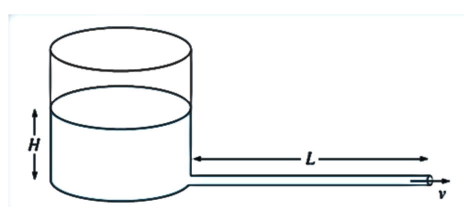
Q1). The velocity v of a falling parachutist is given by

$$v = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

Where $g = 9.81$ m/s^2. For a parachutist with a drag coefficient $c = 15$ kg/s. compute the mass m so that the velocity $v = 36$ m/s at time $t = 10$ sec. Use the Secant and Newton's Raphson methods to determine m to a level of $\varepsilon_s = 0.1\%$. (Exact value of m =59.95928)

Q2). The velocity of the water v (m/s) as depicted in the figure is discharged from a cylindrical tank through a long pipe can be computed as

$$v = \sqrt{2gH}\tanh\left[\frac{\sqrt{2gH}}{2L}t\right]$$



Where $g = 9.81$ m/s^2, H = initial head (m), L = pipe length (m), and t = elapsed time. Determine the head needed to achieve v = 5 m/s in 2.5 sec for a 4 m long pipe by using the Newton's Raphson methods, employ initial value of H = 1.0 with a stopping criteria ε_s = 1%. (Exact value of H =1.33033)

Q3). Water is flowing in a trapezoidal channel at a rate of Q = 20 m^3/s. The critical depth y for such a channel must satisfy the equation

$$0 = 1 - \frac{Q^2}{gA_c^3}B$$

Where $g = 9.81$ m/s^2, A_c = the cross-sectional area (m^2), and B = the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to depth y by

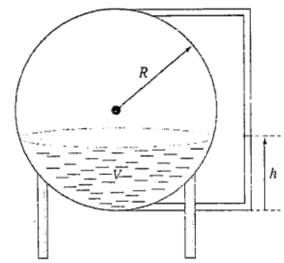
$$B = 3 + y \qquad \text{and} \qquad A_c = 3y + \frac{y^2}{2}$$

Solve for the critical depth using Secant method with initial values $y_0 = 0.5$ and $y_1 = 2.5$

Q4). You are required to design a spherical tank on hold water. The radius $R = 3.5\,m$. The volume of liquid it can hold can be calculated as,

$$V = \pi h^2 \frac{(4R - h)}{5}$$

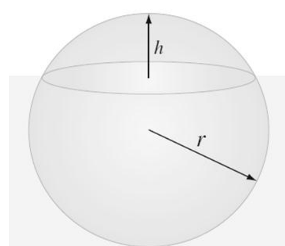
Where V=volume (m^3), h=depth of the water tank (m) and R is the radius of tank (m)



To what depth (h) must the tank be filled so that it holds 45 cubic meters? Approximate your result up to three decimal places.

Q5). According to *Archimedes principle*, the *buoyancy* force is equal to the weight of fluid displaced by the submerged portion of an object. For the sphere depicted in Fig. P5.19, use Newton Raphson method to determine the height h of the portion that is above water. Employ the following values for your computation, r = 1, ρ_s = density of sphere = 200 kg/m^3 and ρ_w = density of water = 1000 kg/m^3. Note that the volume of the above-water portion of the sphere can be computed with

$$V = \frac{\pi h^2}{3}(3r - h)$$



Solution

According to the question the given condition is

$$\rho_s V_s g = \rho_w V_w g$$

The volume of the above-water portion of the sphere is $V_w = \frac{4}{3}\pi r^3 - \frac{\pi h^2}{3}(3r - h)$

The volume of sphere is $V_s = \frac{4}{3}\pi r^3$

Q6). The Manning equation can be written for a rectangular open channel as

$$Q = \frac{\sqrt{S}\,(BH)^{5/3}}{n\,(B+2H)^{2/3}}$$

Where Q = flow (m^3/s), S = slope, H = depth, and n = Manning roughness coefficient. Formulate a Newton Raphson Scheme to solve the above equation for H. Given that Q = 5, S = 0.0002, B = 20, and n = 0.03.

Q7). The Redlich-Kwong equation of state is given by

$$p = \frac{RT}{v-b} - \frac{a}{v\,(v+b)\,\sqrt{T}}$$

Where T = absolute temperature (K) = $-40^0 C$, R = universal gas constant = 0.518 kJ/(kg K), p = absolute pressure (kPa) = 65,000 (kPa), and v = the volume of a kg of gas (m^3/kg), The parameters a and b are calculated by

$$a = 0.427\frac{R^2 T_c^{2.5}}{p_c} \text{ and } b = 0.0866R\frac{T_c}{p_c}$$

Where p_c = critical pressure = 4600 (kPa), and T_c = critical temperature = 191 (K). Employ iterative scheme to compute the volume of gas v.

Q8). The displacement of a structure is defined by the following equation for a damped oscillation

$$y = 8\,e^{-k\,t}\cos\omega\,t$$

Where k = 0.5 and $\omega = 3$. Use the Newton-Raphson method to determine the root to ε_s = 0.01%.

Q9). A uniform beam subject to a linearly increasing distributed load. The equation for the resulting elastic curve is

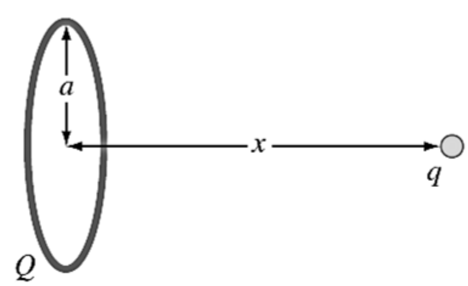
$$y = \frac{w_0}{120EIL}\left(-x^5 + 2L^2x^3 - L^4x\right)$$

By using any iterative method to determine the maximum deflection (that is, the value of x where dy/dx = 0). Then substitute this value into above equation to determine the value of maximum deflection. Use the following parameter values in your computation: L = 450 cm, E 5 50,000 kN/cm^2, I = 30,000 cm^4, and w_0 = 1.75 kN/cm.

Q10). A total charge Q is uniformly distributed around a ring-shaped conductor with radius a. A charge q is located at a distance x from the center of the ring as depicted in the figure. The force exerted on the charge by the ring is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQx}{(x^2+a^2)^{3/2}}$$

Where $\varepsilon_0 = 8.85\times10^{-12}$ $C^2/(N\text{-}m^2)$. Find the distance x where the force is 1N if q and Q are 2×10^{-5} C for a ring with a radius of 0.9 m.



Solution of System of Linear Equations

Q1). An electronics company produces transistors, resistors, and computer chips. Each transistor requires four units of copper, one unit of zinc, and two units of glass. Each resistor requires three, three, and one units of the three materials, respectively, and each computer chip requires two, one, and three units of these materials, respectively. Putting this information into table form, we get:

Component	Copper	Zinc	Glass
Transistors (x)	4	1	2
Resistors (y)	3	3	1
Computer chips (z)	2	1	3
Total	960	510	610

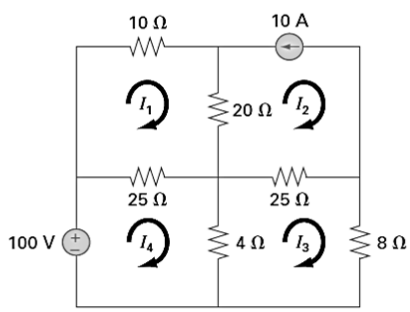
Supplies of these materials vary from week to week, so the company needs to determine a different production run each week. For example, one week the total amounts of materials available are 960 units of copper, 510 units of zinc, and 610 units of glass. Set up the system of equations modeling and apply triangulation method to solve for the number of transistors, resistors, and computer chips to be manufactured this week.

Q2). An electrical engineer supervises the production of three types of electrical components. Three kinds of material—metal, plastic, and rubber—are required for production. The amounts needed to produce each component are

Component	Metal	plastic	Rubber
1 (x)	4	1	2
2 (y)	3	3	1
3 (z)	2	1	3
Total	3.89	0.095	0.282

Where metal, plastic and rubber are in gram per component. If totals of 3.89, 0.095, and 0.282 *kg* of metal, plastic, and rubber, respectively, are available each day, how many components can be produced per day? Apply any technique to calculate the no. of components that can be manufactured per day.

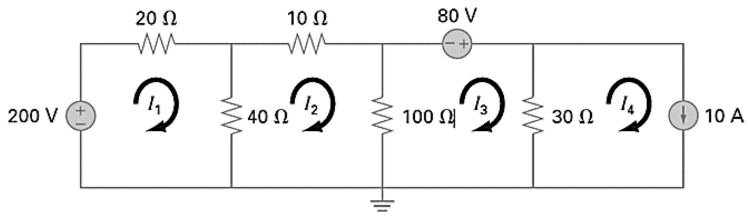
Q3). The following system of equations was generated by employing the mesh current law to the circuit as depicted in the figure



$$55I_1 - 25I_4 = -200$$
$$-37I_3 - 4I_4 = -250$$
$$-25I_1 - 4I_3 + 29I_4 = 100$$

Find the currents I_1, I_3, and I_4 by using the Gauss-Seidel method

Q4). The following system of equations was generated by employing the mesh current law to the circuit as depicted in the figure



$$60I_1 - 40I_2 = 200$$
$$-40I_1 + 150I_2 - 100I_3 = 0$$
$$-100I_2 + 130I_3 = 230$$

Find the currents I_1, I_2, and I_3.

Q5). Idealized spring-mass systems have numerous applications throughout engineering. The arrangement of four springs in series being depressed with a force of 2000 *N*. At equilibrium, force-balance equations can be developed defining the interrelationships between the springs,

$$k_2(x_2 - x_1) = k_1x_1$$
$$k_3(x_3 - x_2) = k_2(x_2 - x_1)$$
$$k_4(x_4 - x_3) = k_3(x_3 - x_2)$$
$$F = k_4(x_4 - x_3)$$

Where the *k*'s are spring constants. If k_1 through k_4 are 50, 50, 75, and 225 N / m, respectively, calculate the *x*'s.

Q6). Three blocks are connected by a weightless cord and rest on an inclined plane Applying method to the one used in the analysis of the falling parachutists produces the following set of simultaneous equations

$$100\,a + T = 519.72$$
$$50\,a - T + R = 216.55$$
$$25\,a - R = 108.28$$

Solve for acceleration *a* and the tensions *T* and *R* in the two ropes.

Lagrange Interpolation

Q1). Use the Lagrange interpolation formula calculate $F(3.1)$ from the following table

x	0	1	2	3	4	5
***F*(*x*)**	2	7	9	12	15	21

Q2). Find the unique polynomial $P(x)$ of degree 4 or less such that $P(1) = 1$, $P(2) = 9$, $P(3) = 24$, $P(4) = 37$, and $P(5) = 54$ by using the Lagrange interpolation formula also compute $P(3.6)$

Q3). Using the Lagrange interpolation formula find the value of *y* when *x* = 9.5 from the following table

x	5	7	9	11	13
y	7	10	12	14	19

Newton's Forward and Backward Interpolation

Q1). Calculate the value of $e^{1.76}$ from the following given data

x	1.7	1.8	1.9	2.0	2.1
y = e^x^	5.47	6.05	6.68	7.39	8.17

Q2). The following data given the melting point of an alloy of Lead and Zinc. Find the melting point of the alloy containing 86 percent of lead.

Percentage of Lead (p)	50	60	70	80	90
Temperature (Q ^{0}C)	210	235	285	315	376

Q3). The population of a certain town is given in the following table. Estimate the population in the years 1923 and 1959

Year	1920	1930	1940	1950	1960
Population (in Thousands)	15.87	20.65	24.12	28.34	33.78

Curve Fitting

Q1). A chemical company is working to examine the effect of extraction time on the efficiency of an extraction operation is obtained from the following data

x (Extraction time in minutes)	3	4	9	7	6	5
y (Efficiency)	11	12	17	14	19	20

Fit the following curves on the above data set

(i) Linear Curve

(ii) Quadratic Curve

(iii) Exponential Curve

Q2). Fit the linear and second degree curve on the following data

x	2	4	6	8	10
y	4.0496	5.0391	7.1698	10.2181	12.3194

Numerical Integration

Q1). Evaluate the following integral

$$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\, dx$$

(a) Analytically (b) single application of the trapezoidal rule (c) composite trapezoidal rule (d) Simpson one third and three-eighth rules (e) Compute Percentage Error.

Q2). Suppose that the upward force of air resistance on a falling object is proportional to the square of the velocity. For this case, the velocity can be computed as

$$\int_0^1 v(t)\,dt.\; v(t)=\sqrt{\frac{gm}{c_d}}\tanh\left[\sqrt{\frac{gc_d}{m}}\,t\right]$$

Where c_d = a second-order drag coefficient, g = 9.81 ms^{-2}, m = 68.1 kg, and c_d = 0.25 kgm^{-1}

(a) Apply analytical integration to determine how far the object falls in 10 sec.

(b) Make the same evaluation, but compute the integral by using the Simpson rule when no. of sub-intervals equal to 6. $t \in [0,1]$

Q3). An 11-m beam is subjected to a load, and the shear force follows the equation

$$V(x)=5+0.25x^2$$

Where V is the shear force and x is length in distance along the beam. We know that $V = dM/dx$, and M is the bending moment. Integration gives the relationship

$$M = M_0 + \int_0^x V\,dx$$

If M_o is zero and x = 11, calculate M using (a) analytical integration, (b) trapezoidal rule and Simpson rules (one-third, and three-eighth) when no. of intervals equal to 7.

Q4). The work produced by a constant temperature, pressure volume thermodynamic process can be computed as

$$W = \int p\,dV$$

Where W is work, p is pressure, and V is volume. Using a combination of the trapezoidal rule, Simpson rules (one-third, and three-eighth), use the following data to compute the work in kJ (kJ = kN - m)

Volume (m^3)	0.25	0.5	0.75	1.0	1.25	1.50	1.75	2.0
Pressure (kPa)	336	294.4	266.4	260.8	260.5	249.6	193.6	165.6

Q5). A projectile is fired from the ground level, the acceleration of the projectile is described the integral $\int_0^1 f(x)\,dx$ where $f(x)=\sqrt{2x+1}$. Find the velocity of the projectile by using the ***Simpson's rule*** when interval is divided into seven sub-intervals and also find the total distance covered by the projectile after 25 seconds and also compute the percentage error?

Q6). The cross-sectional area of a channel can be computed as

$$A_c = \int_0^B H(y)\,dy$$

Where B is the total channel width (m), H is the depth (m), and y is the distance from the bank (m). In a similar fashion, the average flow Q ($m^3\,s^{-1}$) can be computed as

$$Q = \int_0^B U(y)\,H(y)\,dy$$

Where U is the water velocity (m/s). Use these relationships and a numerical method to determine A_c and Q for the following data

x, m	**0**	**3**	**6**	**9**	**12**	**15**
H, m	0.5	1.3	1.25	1.7	1	0.25
U, m/s	0.03	0.06	0.05	0.12	0.11	0.02

Q8). If a capacitor initially holds no charge, the voltage across it as a function of time can be computed as

$$V(t) = \frac{1}{C}\int_0^t i\,(t)\,dt$$

If C = 10^{-5} farad, use the following current data to develop a plot of voltage versus time

t, sec	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**
i, 10^{-3} A	0.2	0.3683	0.3819	0.2282	0.0486	0.0082	0.1441

Q9). A river is 80 meter wide. The depth d in feet at a distance x ft. from one bank is given from the following table.

x	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**
y	0	4	7	9	12	16	14	7	4

Find approximately the area of cross-section. The area of cross-section can be computed as

$$A = \int_0^x f(x)\,dx$$

Q10). A rocket is launched from the ground level, its acceleration is registered during the first 45 seconds is described by the integral $\int_0^{45} f(x)\,dx$ where $f(x) = \sqrt{x\,e^{-x}}$, by applying the Simpson compute the velocity of the rocket at 45 seconds when no. of interval is divided in to seven sub-intervals..

Q11). A solid of revolution is formed by rotating about x axis, the area between the x axis, the lines $x = 0$ and $x = 1$, and the curve passing through the points with the following coordinates

x	**0**	**2.5**	**5.0**	**7.5**	**10.0**	**12.5**	**15.0**
y	5	5.5	6.0	6.75	6.25	5.5	4.0

Estimate the volume of the solid is generated. The volume of the solid of revolutions is computed as

$$V = \int_0^{15} \pi\, y^2\,dx$$

ODE's Numerical Solution

Q1). For the given differential equation $\frac{dy}{dx} = 2x + y,\ y(0) = 1$. Compute y (1.0) by suing

(i) Euler Method.
(ii) Modified Euler Method.
(iii) Runge-Kutta method of Order 4. (Taking step size $h = 0.25$)

Q2). Solve the differential equation $\frac{dy}{dt} = \frac{t+y}{t},\ y(2) = 2.$ Compute $y(2.4)$ by using

(i) Euler Method.
(ii) Modified Euler Method.
(iii) Runge-Kutta method of Order 4. (Taking step size $h = 0.2.$)

Also compare the results with actual solution. The actual solution is $y(t) = t\left(\ln(0.5t) + 1\right)$

Q3). Find the actual length of the steel tower $y(t)$, if after an earthquake it started to deform with time *t*, modelled by the differential equation $\frac{dy}{dt} = t^2 - 2yt + y^2,\ y(0) = 1,\ h = 0.2$ at $y(0.6)$ by using

(i) Euler Method.
(ii) Runge-Kutta method of Order 2.
(iii) Runge-Kutta method of Order 4.

Q4). An inductance of 0.2 Henneries and at a resistance of 0.5 ohm are connected in series with a E.M.F E volts is modelled by the differential equation

$$\frac{dI}{dt} = \frac{1}{L}\left(E - RI\right)$$

Initially the current in the circuit at time t = 0 is zero. Calculate the current at the end of 7 seconds when E = 3 volts by using

(i) Euler Method.
(ii) Runge-Kutta method of Order 2.
(iii) Runge-Kutta method of Order 4. (Taking step size $h = 1.75$)

Also compare the results with actual solution. The actual solution is $I(t) = 5\left(1 - e^{-10t}\right)$

Note that the ***Runge –Kutta method of second order*** is ***the modified Euler method***. The formula for the R-K method of order 2 is,

$$k_1 = h\,f\left(x_n, y_n\right)$$
$$k_2 = h\,f\left(x_n + h, y_n + k_1\right)$$
$$k = 0.5\left(k_1 + k_2\right)$$
$$y_{i+1} = y_i + k$$