NOTE: hP Adaptivity: We can increase the accuracy of answer by decreasing h / increasing N.

Dated:

Simpson's 1/3 RD RULE OF INTEGRATION.

$$\int_a^b f(x)\,dx \simeq \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right]$$

$$\simeq \frac{1}{18}\left[0 + 4\left(\frac{1}{36} + \frac{9}{36} + \frac{25}{36}\right) + 2\left(\frac{4}{36} + \frac{16}{36}\right) + 1\right]$$

$$\Rightarrow \int_0^1 x^2\,dx \simeq \frac{1}{3} \quad \underline{\text{Ans}}$$

3. Simpson's 3/8 RULE OF INTEGRATION.

$$\int_a^b f(x)\,dx \simeq \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3) + y_6\right]$$

$$\simeq \frac{3}{48}\left[0 + 3\left(\frac{1}{36} + \frac{4}{36} + \frac{16}{36} + \frac{25}{36}\right) + 2\left(\frac{9}{36}\right) + 1\right]$$

$$\Rightarrow \int_0^1 x^2\,dx \simeq \frac{1}{3} \quad \underline{\text{Ans}}$$

Example 2: Consider the following data in which current (i) is noted against time (t). Find the total charges accumulated from $t = 0$ to $t = 1$.

$$i = \frac{dq}{dt} \Rightarrow dq = i\,dt$$

From $t = 0$ to $t = 1$: $dq = \int_0^1 i\,dt$.

Dated:

NUMBER SOLUTION OF O.D.Es

- ODE : Ordinary Differential equation.
- In differential equation we find dependant function.

$$\frac{dy}{dx} = xy \Rightarrow \int \frac{1}{y}\, dy = \int x\, dx$$

$$\ln y = x^2/2 + C$$

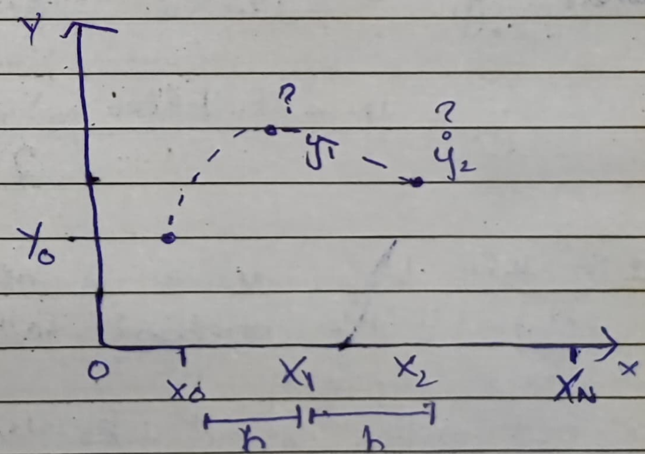
$$y = Ae^{x^2/2}$$

- $\frac{dy}{dx} = xy, \quad y(0) = 1 \Rightarrow$ Initial Value Problem. (I.V.P)

- In this, we know initial solution which is used to generate further solutions.

Method 1 : Euler's method: Consider an I.V.P :

$$\frac{dy}{dx} = f(x, y) \quad y(x_0) = y_0$$



- Slope at $(X_0, Y_0) = f(X_0, Y_0)$

$\Rightarrow$ If x_0 & x_1 form a linear line, we can assume slope $= \frac{y_1 - y_0}{x_1 - x_0} = \frac{y_1 - y_0}{h}$

- If $h \to 0 \Rightarrow \frac{y_1 - y_0}{h} = f(x_0, y_0)$

$$\boxed{y_1 = y_0 + h f(X_0, Y_0) \text{ at } X = X_1}$$

$\therefore h = \frac{X_N - X_0}{N}$

Similarly, $y_2 = y_1 + hf(x_1, y_1)$ at $x = x_2$

$\vdots$

$y_N = y_{N-1} + hf(x_{N-1}, y_{N-1})$ at $x = x_N$

So, this means $y_{i+1} = y_i + hf(x_i, y_i)$ at $x = x_{i+1}$

$\forall i = 0, 1, 2 \ldots N-1, N$

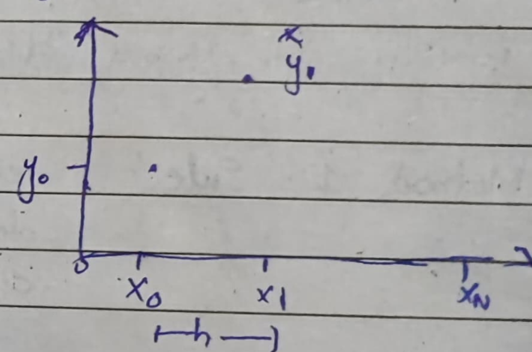
Dated:

$\leftarrow h \rightarrow$

- Above formula is ~~Eul~~ Euler method for Ordinary D.E.
- You need 2 of these 3 to solve:
  1. h
  2. N
  3. X_N

2. MODIFIED EULER'S METHOD:

$$\frac{dy}{dx} = f(x,y) \quad ; \quad y(x_0) = y_0.$$



In modified rules:

$$y_1^* = y_0 + h f(x_0, y_0) \text{ at } x_1$$

where y_1^* = predicted solution

$$y_1 = y_0 + \frac{h\,[f(x_0,y_0) + f(x_1, y_1^*)]}{2}$$

- In this logic, answer is not dependent only on h but also on slopes. This method takes 2 slopes and take their average.
- Best option is to take slopes of extreme points

✓ Trapezia

Similarly,
$$y_2^* = y_1 + h f(x_1, y_1) \text{ at } x_2$$
$$y_2 = y_1 + \frac{h\,[f(x_1,y_1) + f(x_2, y_2^*)]}{2}$$

So,
$$y_{i+1}^* = y_i + h f(x_i, y_i) \text{ at } x_{i+1}$$
$$y_{i+1} = y_i + \frac{h\,[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^*)]}{2}$$

System
contour
bazaar
Venture dive
26 m
Folio 3
10 pearl

Dated: 3 June '2024.

3. Range - Kutta method of Order 4 (RK - method of order 4).

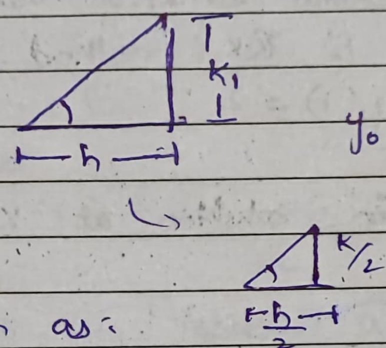
$$\frac{dy}{dx} = f(x, y) \quad , \quad y(x_0) = y_0.$$

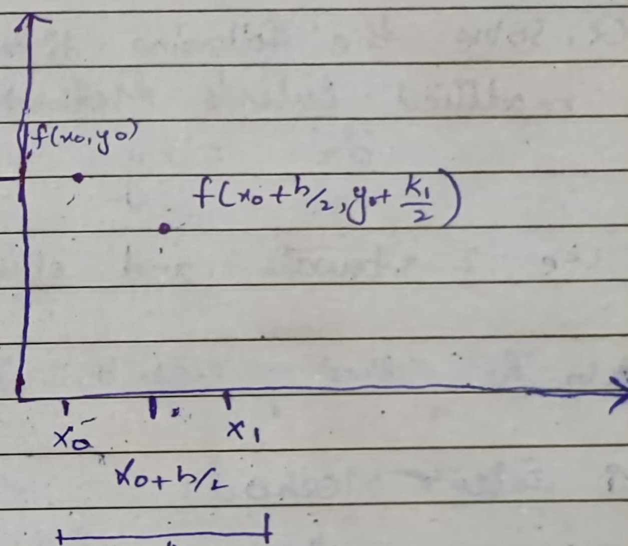
- In this method, we take slope at 4 different points & then we take weighted average of these points

$\Rightarrow \tan\theta = \frac{K_1}{h}$

$\therefore \tan\theta = \text{slope}$

$\boxed{K_1 = h \times \text{slope}}$





so K_1 can be written as:

$$K_1 = hf(x_0, y_0).$$

Similarly, $K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$K_4 = hf(\overbrace{x_0 + h}^{x_1}, y_0 + K_3)$$

* Proportion of K and h remains same so if it is $(x_0 + 3h/4$ then $y_0 \Rightarrow y_0 + 3K_1/4$.

- Weighted Average: $y_1 = y_0 + \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$ at $x = x_1$

- General Equation: $K_1 = hf(x_i, y_i)$

$K_2 = hf(x_i + h/2, y_i + K_1/2)$

$K_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{K_2}{2}\right)$

• RK method of order 2 : Modified Euler.

Dated:

$K_4 = h f(x_i + h, y_i + K)$.

$\therefore \quad y_{i+1} = y_i + \frac{(K_1 + 2K_2 + 2K_3 + K_4)}{6}$ at $X = X_i$.

Q. Solve the following differential equation using Euler's method, modified Euler's Method & RK method of order 4.

$\frac{dn}{dy} = xy, \quad y(1) = 2$

Use 2 intervals and obtain solution at $X = 1.5$

* in RK' Method of order 4, error depends upon h^4.

i. Euler's Method:

$y_{i+1} = y_i + hf(x_i, y_i) \quad \forall\ i = 0, 1$

$h = \frac{1.5 - 1}{2} = 0.25$

$\therefore \quad x_0 = 1, \quad y_0 = 2 \quad f(x, y) = xy.$

$i = 0 \Rightarrow y_1 = y_0 + hf(x_0, y_0)$

$y_1 = 2 + (0.25)[x_0, y_0]$

$y_1 = 2 + 0.25 [1, 2] \Rightarrow y_1 = 2.5$ at $X_1 = 1.25$

$i = 1 \quad y_2 = y_1 + hf(x_1, y_1)$

$y_2 = 2.5 + 0.25 [x_1, y_1]$

$y_2 = 3.28125$ at $X_2 = 1.5$.

Dated:

2. Modified Euler:

- $y^*_{i+1} = y_i + hf(x_i, y_i) \quad \forall \; x = x_{i+1}$

- $y_{i+1} = y_i + h\left[\frac{f(x_i, y_i) + f(x_{i+1}, y^*_{i+1})}{2}\right]$

$i = 0 \Rightarrow$

$y_1 = y_0 + h f(x_0, y_0)$

$y_1 = 2 + 0.25 \, [x_0, y_0]$

$y_1^* = 2.5$

$y_1 = y_0 + h\left[\frac{f(x_0, y_0) + f(x_1, y_1)}{2}\right]$

$y_1 = 2 + 0.25 \left[\frac{f(1, 2) + f(1.25, 2.5)}{2}\right]$

$x_1 = 1.25, \quad y_1 = 2.6466.$

$i = 1 \Rightarrow$

$y_2^* = y_1 + h f(x_1, y_1)$

$y_2^* = 2.6406 + 0.25 \, (1.25, 2.6406)$

$y_2^* = 3.4658$

$y_2 = 2.6406 + h\left[\frac{f(1.5, 3.4658) + f(1.25, 2.6406)}{2}\right]$

$x_2 = 1.5, \quad y_2 = 3.7030.$

3. R.K Method:

$K_1 = h f(x_0, y_0)$

$K_1 = 0.25 \, f(1, 2)$

$K_1 = 0.5$

$K_2 = h f\left(x_i + \frac{h}{2}, y_i + \frac{K_1}{2}\right)$

Dated:

$$K_2 = 0.25 \times f\left(1 + \frac{0.25}{2}, 2 + \frac{0.05}{2}\right)$$

$$K_2 = 0.6328$$

$$K_3 = hf\left(x_i + h/2, y_i + K_2/2\right)$$
$$K_3 = 0.25 \times f\left(1 + 0.25/2, 2 + 0.6328/2\right)$$
$$K_3 = 0.6514$$

$$K_4 = hf(x_i + h, y_i + K_3)$$
$$= 0.25 \times f(1 + 0.25, 2 + 0.6514) \Rightarrow 0.8285$$

$$y_1 = y_0 + \frac{(K_1 + 2K_2 + 2K_3 + K_4)}{6} = 2 + \frac{(0.5 + 0.6328 \times 2 + 2 \times 0.6514 + 0.8285)}{6}$$

$$y_1 = 2.6494 \quad \text{at} \quad x_1 = 1.25.$$

At $(x_1, y_1) \Rightarrow (1.25, 2.6494)$

$$K_1 = h f(x_0, y_0) \Rightarrow 0.25 \times f(1.25, 2.6494)$$
$$K_1 = 0.8279$$

$$K_2 = hf(x_0 + h/2, x_1 + K_1/2) \Rightarrow 0.25 \times f\left(1.25 + 0.25/2, 2.6494 + \frac{0.8279}{2}\right)$$
$$K_2 = 1.0530$$

$$K_3 = hf(x_0 + h/2, x_1 + K_2/2) \Rightarrow 0.25 \times f\left(1.25 + 0.25/2, 2.6494 + \frac{1.0530}{2}\right)$$
$$K_3 = 1.0917$$

$$K_4 = hf(x_i + h, y_i + K_3) = 0.25 \times f\left(1.25 + \frac{0.25}{2}, 2.6494 + 1.0917\right)$$
$$K_4 = 1.4029$$

$$y_2 = y_1 + \frac{(K_1 + 2K_2 + 2K_3 + K_4)}{6} = 2.6494 + \frac{(0.8279 + 2 \times 1.053 + 2 \times 1.0917 + 1.4029)}{6}$$

$$\boxed{y_2 = 3.7361 \quad \text{at} \quad x = 1.5}$$

Dated:

At $x = 1.5$:

$y_{Euler} = 3.28125$, $y_{mod.Euler} = 3.7030$, $y_{R.K} = 3.7360$

Exact Solution

$\frac{dy}{dx} = xy \Rightarrow \frac{1}{y} dy = x\,dx$

$\ln y = x^2/2 + C$

$\ln 2 = \frac{1^2}{2} + C$

$C = \ln 2 - 1/2$

$\Rightarrow \ln y = \frac{x^2}{2} + \ln 2 - \frac{1}{2}$

$y = e^{\frac{x^2-1}{2}} + \ln(2)$

At $x = 1.5$

$y = e^{\frac{1.5^2-1}{2}} + \ln(2)$

$y = 3.73649$

* We will compare exact solution with every numerical solution.

Conclusion : Standard R.K ~~Euler~~ Method is most accurate followed by Modified Euler & then Euler.

* Root Mean Square Error (RMSE) : $\sqrt{\frac{\Sigma (y_{exact} - y_{R.K})^2}{N}}$

Dated:

Solution Of A System Of Linear Equations.

- Direct Method
  1. LU - Decomposition Method
- Indirect Method.
  2. Jacobi Iterative Method.
  3. Gauss - Seidle Iterative Method.

1. LU - Decomposition Method: Consider a system of equations of size $n \times n$ i.e. $A\underline{x} = \underline{b}$ → (1)

* $n \times n$: No. of unknowns × No. of Equations.

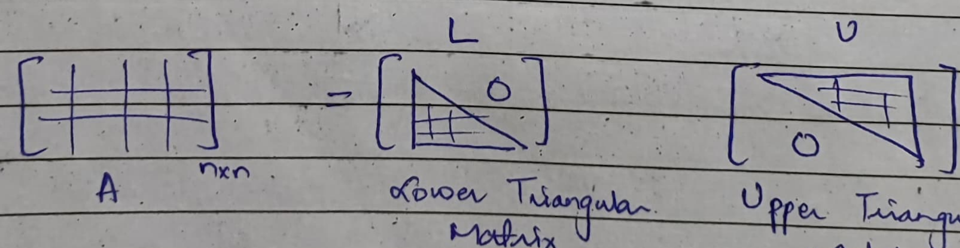
- Let $A = LU$. —— (2)

$\Rightarrow LU\underline{x} = \underline{b}$

Let $U\underline{x} = \underline{y}$ —— (3)

$\Rightarrow L\underline{y} = \underline{b}$ —— (4)

- Eq (1) :



Eq (4)

$$\underbrace{\begin{bmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ \vdots & & \ddots & \\ & & & 6 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{\underline{y}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}_{\underline{b}}$$

$L_{11} y_1 = b_1$

$y_1 = \dfrac{b_1}{L_{11}}$

- This method is called forward swapping / substitution.

Dated:

Put this y values in eq (iii)

$$\begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ 0 & U_{22} & \cdots & \\ 0 & 0 & & U_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

- $U_{nn} x_n = y_n$

$$\boxed{x_n = \frac{y_n}{U_{nn}}}$$

- This method is called Backward Subsitution

Example:

Eq (2) :

$$\underset{A}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}} = \underset{L}{\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}_{3\times3}} \underset{U}{\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}_{3\times3}}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} = \begin{bmatrix} L_{11}U_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21}U_{11} & L_{21}U_{12}+L_{22}U_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{11}+L_{32}U_{12}+L_{33}U_{33} \end{bmatrix}_{3\times3}$$

- Number of unknowns $= 12$ & No. of Equations $= 9$

$\Rightarrow$ we want Unknowns = Equations, so we assume 3 unknown values.

$\Rightarrow$ E.g we assume L_{11}, L_{21} & $L_{31} = 1$

$\Rightarrow$ we solve this left to right & top to bottom.

$$\begin{bmatrix} L_{11}U_{11} \xrightarrow{1} & L_{11}U_{12} & L_{11}U_{13} \\ '' \xrightarrow{2} & '' & '' \\ '' & '' & '' \end{bmatrix}$$

Dated:

Example: Solve the following system of a linear equation using LU - Decomposition Method.

$$2x + 3y + 4z = 10$$
$$4x + 8y + 9z = 20$$
$$8x + 12y + 10z = 30$$

Ans. $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 8 & 9 \\ 8 & 12 & 10 \end{bmatrix}$, $\underline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $b = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$

$$AX = b \quad \text{——— } (1)$$

Let $A = LU$

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 8 & 9 \\ 8 & 12 & 10 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 8 & 9 \\ 8 & 12 & 10 \end{bmatrix} = \begin{bmatrix} L_{11}U_{11} & L_{11}V_{12} & L_{11}V_{13} \\ L_{21}U_{11} & L_{21}U_{12}+L_{22}U_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}V_{13}+L_{32}U_{23}+L_{33}V_{33} \end{bmatrix}$$

Let $L_{11} = L_{22} = L_{33} = 1$

$L_{11}U_{11} = 2 \Rightarrow U_{11} = 2$, $U_{12} = 3$, $U_{13} = 4$

$L_{21}U_{11} = 4 \Rightarrow L_{21} = 2$

$L_{21}U_{12} + L_{22}U_{22} = 8$

$(2)(3) + 1\,U_{22} = 8$; $V_{22} = 2$

$L_{21}U_{13} + L_{22}U_{23} = 9$

$(2)(4) + (1)U_{23} = 9$

$U_{23} = 1$

$L_{31} = 4$

$L_{32} = 0$

$U_{33} = -6.$

Dated:

This implies :

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & -6 \end{bmatrix}$$

- $L U \underline{x} = \underline{b} \longrightarrow (2)$

Let $U \underline{x} = \underline{y} \longrightarrow (3)$

$\Rightarrow L \underline{y} = \underline{b} \quad - (4)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

* By forward substitution :

$y_1 = 10 \qquad 2y_1 + y_2 = 20 \qquad \Rightarrow \underline{y} = \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix}$

$y_2 = 0, \qquad y_3 = -10.$

Eq (3) : Let $U x = \underline{y} \rightarrow (3)$

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ \cancel{x_2} \\ Z \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix}$$

By backward substitution:

$-6z = -10$

$z = \frac{10}{6} = \frac{5}{3}$

$$\underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 35/12 \\ -5/6 \\ 5/35 \end{bmatrix}$$

* This method is not suitable for large problems because it give exact answer

Dated:

- How to determine solution does not exist:

① . $A = LU$

$\det(A) = \det(LU)$

$= \text{determinant}(L) \times \det(U).$

- If any diagonal value of L or U is '0', then determinant will be 0. Hence, $\det(A) = 0$ & solution won't exist.

② .

2. JACOBI ITERATIVE METHOD: Consider a $n \times n$ system:

$$A\underline{x} = \underline{b} \quad \text{—— ①.}$$

Let $A = L + D + U$ —— ②.

$\Rightarrow (L + D + U)\underline{x} = \underline{b}$

$\Rightarrow (L+U)\underline{x} + D\underline{x} = \underline{b}$

- $D\underline{x} = \underline{b} - (L+U)\underline{x}$

$\underline{x}^{i+1} = D^{-1}(\underline{b} - (L+U)\underline{x}^{i}) \rightarrow ③$

- We would have used inverse of A in eq ① but we used eq 3 because diagonal's inverse is easy to find (reciprocate all diagonal values).

So, we write this algo as $\underline{x}^{i+1} = D^{-1}(\underline{b} - (L+U)\underline{x}^{i})$ (x^0 as input)

Dated:

- Above algorithm is iterative in nature. We start with initial seed vector X^0. Above algorithm will converge to exact solution, provided that A is a diagonally ~~matrix~~. dominant matrix.

- Diagonally dominant matrix: It is a matrix whose all diagonal's absolute values are greater than sum of all other values in the row of that particular diagonal.

$$\begin{bmatrix} 10 & 3 & 2 \\ 4 & 9 & -2 \\ 8 & 10 & 20 \end{bmatrix} \quad \begin{matrix} |10| > |3| + |2| \; ✓ \\ |9| \geq |4| + |-2| \; ✓ \\ |20| \geq |8| + |10| \; ✓ \end{matrix}$$

x ——— x

$$X^{i+1} = D^{-1}(b - (L+U)X^i) \quad \forall \, i = 0, 1, 2, 3$$

Consider 3×3:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\Rightarrow x_1 = \frac{b_1}{a_{11}} - \frac{(a_{12}x_2 + a_{13}x_3)}{a_{11}}, \quad x_2 = \frac{b_2}{a_{22}} - \frac{(a_{21}x_1 + a_{23}x_3)}{a_{22}}$$

$$\Rightarrow x_3 = \frac{b_3}{a_{33}} - \frac{(a_{31}x_1 + a_{32}x_2)}{a_{33}}.$$

Assume $X^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{bmatrix}$

$$x_1^1 = \frac{b_1}{a_{11}} - \frac{(a_{12}x_2^0 + a_{13}x_3)}{a_{11}}, \quad x_2^1 = \frac{b_2}{a_{22}} - \frac{(a_{21}x_1^0 + a_{23}x_3)}{a_{12}}$$

$$x_3^1 = \frac{b_3}{a_{33}} - \frac{(a_{31}x_1^0 + a_{32}x_2)}{a_{31}}$$

Dated:

- We put this value of $x_0, x_1, x_2 = 1$ again to get $x_0, x_1, x_2 = 2$ & so, on till we get final answer.

- Generalized Jacobi Iterative Form:

$$x_1^{i+1} = \frac{b_1}{a_{11}} - \frac{(a_{12}x_2^{i} + a_{13}x_3^{i})}{a_{11}} \quad ; \quad x_2^{i+1} = \frac{b_2}{a_{22}} - \frac{(a_{21}x_1^{i} + a_{23}x_3^{i})}{a_{22}}$$

$$x_3^{i+1} = \frac{b_3}{a_{33}} - \frac{(a_{31}x_1^{i} + a_{32}x_2^{i})}{a_{33}}$$

- Above scheme is Jacobi Iterative method in generalized form.

3. Gauss - Seidel Iterative Method.

$$x_1^{i+1} = \frac{b_1}{a_{11}} - \frac{(a_{12}x_2^{i} + a_{13}x_3^{i})}{a_{11}}$$

$$x_2^{i+1} = \frac{b_2}{a_{22}} - \frac{(a_{12}x_1^{i+1} + a_{23}x_3^{i})}{a_{22}}$$

$$x_3^{i+1} = \frac{b_3}{a_{33}} - \frac{(a_{31}x_1^{i+1} + a_{32}x_2^{i+1})}{a_{33}}$$

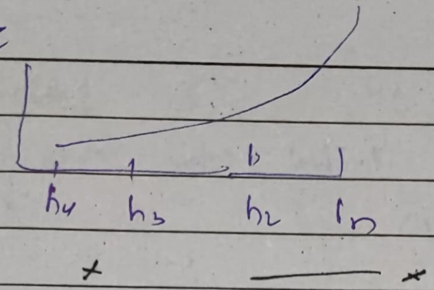
- Above scheme is called Gauss-Seidel Iterative Method and it converges for a diagonally dominant matrix.

- It is more effective & have less iteration.

$$x^{i+1} = (D+L)^{-1}Ux^{i} + (D+L)^{-1}b$$

Dated: RMSE

Graph :



- 12 Coefficients to be calculated
  6 obvious — 6 to be found.

Q. Solve the following system of linear equations using:
1. Jacobi Iterative Method.
2. Gauss - Seidel iterative Method.

Use starting guess $x^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = [0 \; 0 \; 0]^t$

such that $|Ax - b| < 10^{-2}$.

$$4x + 3y + 9z = 10$$
$$3x + 8y + 2z = 20$$
$$10x + 3y + 4z = 40.$$

Solution : For making the system diagonally dominant, we swap R_1 & R_3

$$10x + 3y + 4z = 40$$
$$3x + 8y + 2z = 20$$
$$4x + 3y + 9z = 10.$$

$\Rightarrow x = \frac{40}{20} - \frac{(3y + 4z)}{10}$ $\quad y = \frac{20}{8} - \frac{(3x+2z)}{8}$

$z = \frac{10}{9} - \frac{(4x + 3y)}{9}$

1. For Jacobi Method: $x^{i+1} = \frac{40}{10} - \frac{(3y^i + 4z^i)}{10}$

$y^{i+1} = \frac{20}{8} - \frac{(3x^i + 2z^i)}{8}$

Dated:

$$z^{i+1} = \frac{10}{9} - \frac{(4x^i + 3y^i)}{9}$$

for $\forall$ $i = 0, 1, 2, \ldots$

initial vector values $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \to x^0 \\ \to y^0 \\ \to z^0 \end{matrix}$

For $i = 0$

$$x^1 = 4 - \frac{(3y^0 + 4z^0)}{10} \quad , \quad y^1 = \frac{20}{8} - \frac{(3x^0 + 2z^0)}{8}$$

$$x^1 = 4 \qquad\qquad y^1 = 5/2$$

$$z^1 = \frac{10}{9} - \frac{(4x^0 + 3y^0)}{9}$$

$$z^1 = 10/9 \qquad \text{so, } \underline{X^1} = \begin{bmatrix} 4 \\ 5/2 \\ 10/9 \end{bmatrix}$$

index, not power

For $i = 1$

$$x^{2} = 4 - \frac{(3y^1 + 4z^1)}{10} = 4 - \frac{(3 \times 5/2 + 4 \times 10/9)}{10}$$

$$x^2 = 101/36$$

$$y^2 = \frac{20}{8} - \frac{(3x^1 + 2z^1)}{8} = \frac{13}{18}$$

$$z^2 = \frac{10}{9} - \frac{(4x^1 + 3y^1)}{9} = -\frac{3}{2}$$

$$\underline{X^2} = \begin{bmatrix} 101/36 \\ 13/18 \\ -3/2 \end{bmatrix}$$

* ~~Such~~ $|Ax - B| < 10^{-2}$ ~~said~~ stopping condition will not be in exam due to long calculation
* instead in exam, it will be mentioned "perform 2 iteration" etc.

Dated:

2. For Gauss Seidel: $x^{i+1} = \frac{40}{10} - \frac{(3y^i + 4z^i)}{10} = 4$

For $i=0$

$y^{i+1} = \frac{20}{8} - \frac{(3x^{i+1} + 2z^i)}{8} = \frac{20}{8} - \frac{(3\times4+0)}{8} = 1$

$z^{i+1} = \frac{10}{9} - \frac{(4x^{i+1} + 3y^{i+1})}{9} = \frac{10}{9} - \frac{(4\times4+3\times1)}{9} = \frac{-9}{9} = -1$

$\underline{X_1} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$

For $i=1$: $x^2 = 4 - \frac{(3y^1 + 4z^1)}{10} = 4 - \frac{(3\times1 + 4\times -1)}{10}$

$x^2 = 41/10$

$y^2 = \frac{20}{8} - \frac{(3x^2 + 2z^1)}{8} = \frac{20}{8} - \frac{(3\times 41/10 + 2\times -1)}{8}$

$\therefore y^2 = 97/80$

$z^2 = \frac{10}{9} - \frac{4x^2 + 3y^2}{9} = \frac{10}{9} - \frac{4\times 41/10 + 3\times 97/80}{9} = \frac{-805}{720}$

Dated:

INTERPOLATION and CURVE FITTING

- **Interpolation:** To obtain a reading / find missing data from known dataset using mathematical, statistical or graphical technique.

⇒ Interpolation is either done for "equally spaced data" e.g $x = 1, 2, 3, 4$ or "unequally spaced data" e.g $x = 1, 1.5, 3, 4$

Interpolation

⇒ Equally Spaced

1. Newton's Forward Difference Formula (NFDF)
2. Newton's Backward Difference Formula (NBDF)

⇒ Unequally Spaced

3. Langrange Interpolation Formula

- **Forward / Backward Differences:** Consider $(n+1)$ equally spaced data set. i.e (x_i, y_i) ∀ $i = 0, 1, 2 \dots n$.

	x	y	Δy	$\Delta^2 y$		$\Delta^n y$
h	x_0	y_0	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	- - -	
	x_1	y_1	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		
	x_2	y_2	⋮	⋮	- - -	
	⋮	⋮	⋮	⋮		$\Delta^n y_0$
	⋮	⋮	⋮	⋮	- - -	
	⋮	⋮	⋮	$\Delta^2 y_{n-2}$		
	x_{n-1}	y_{n-1}	$\Delta y_{n-1} = y_n - y_{n-1}$			
	x_n	y_n				

- For $n+1$ values, n differences will be calculated.

Forward Operation: $\Delta y_i = y_{i+1} - y_i$

- Backward: $\nabla y_i = y_i - y_{i-1}$

Dated:

Forward Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	1 y_0	1 Δy_0	6 $\Delta^2 y_0$	6 $\Delta^3 y_0$	0 $\Delta^4 y_0$	0 $\Delta^5 y_0$	0 $\Delta^6 y_0$
1	2	7	12	6	0	0	
2	9	19	18	6	0		
3	28	37	24	6			
4	65	61	30				
5	126	91					
6	217						

Inference: At $\Delta^3 y$ (Third Order), our diffeence becomes constant. This means our data follows the trend of "cubic polynomial".

$\Rightarrow$ At the degree where difference becomes constant, our data follows that trend.

- If the n^{th} difference is constant, then the data is following n^{th} degree polynomial.