Impact of half-wave plate systematics on the measurement of cosmic birefringence from CMB polarization

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May 23th, 2023

CMB anisotropies

Inhomogeneities at photon decoupling imprint anisotropies on the CMB.

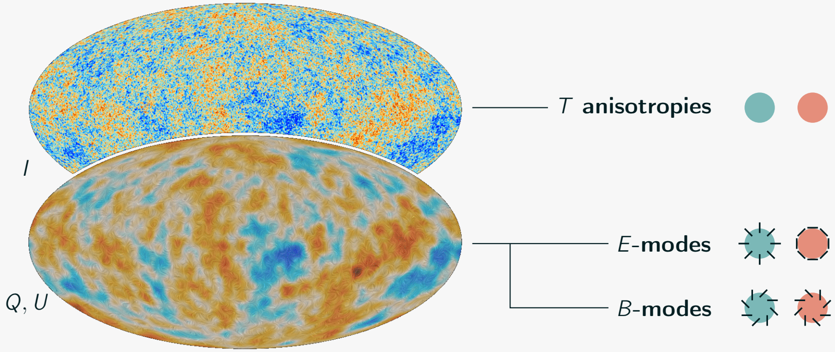

image credit: Jonathan Aumont

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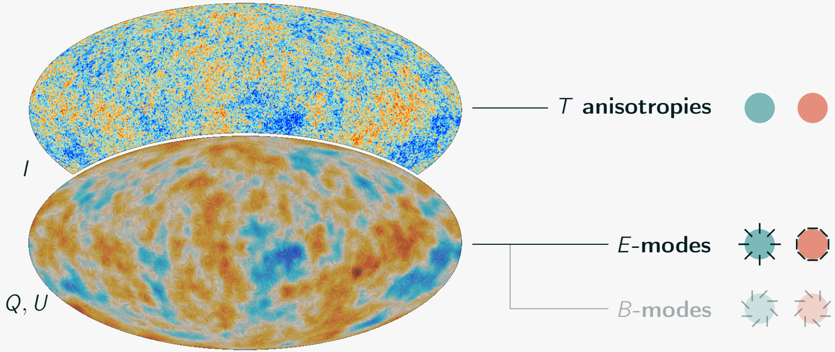

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new physics from CMB polarization

- **Inflation**-sourced tensor perturbations are expected to leave a distinctive signature (B-modes) on CMB polarization.

This is driving the development of a number of new missions:

- ☐ Simons Observatory,
- ☐ South Pole Observatory,
- ☐ CMB Stage-4,
- ☐ **LiteBIRD**.

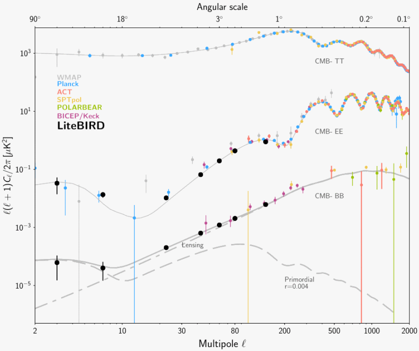

image credit: LiteBIRD Collaboration (2022) PTEP

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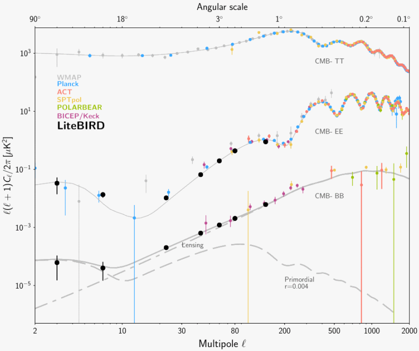

- **Parity-violating physics** could also imprint features on CMB polarization.

image credit: LiteBIRD Collaboration (2022) PTEP

signatures of parity violation

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Coupling a pseudoscalar χ to EM via a Chern-Simons term:

$$\mathcal{L}_{\mathrm{CS}} = -\frac{\alpha}{4f}\chi F_{\mu\nu}\widetilde{F}^{\mu\nu},$$

with $F_{\mu\nu} \propto \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, makes $+$ and $-$ photon helicity states propagate differently:

$$A''_{\pm} + \left(k^2 \mp \frac{k\alpha\chi'}{f}\right)A'_{\pm} = 0.$$

Difference in phase velocity
$\rightarrow$ rotation of the plane of linear polarization.

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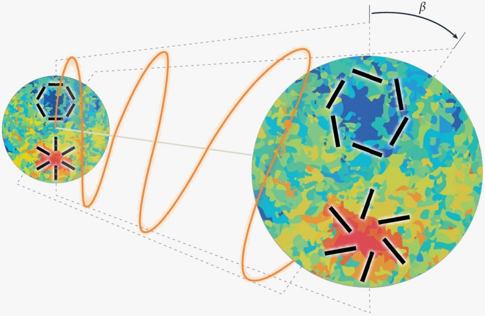
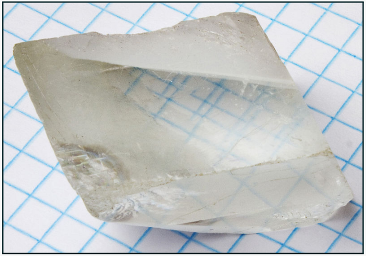


Difference in phase velocity
$\rightarrow$ rotation of the plane of linear polarization.

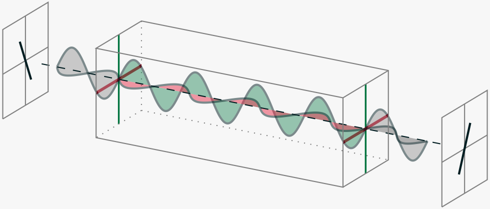
image credit: Yuto Minami

why "cosmic birefringence"?

Birefringence: property of a material whose refractive index depends on the polarization and propagation direction of light.



Thinner slabs, normal incidence:
no double refraction, only retardance.



Both optical and cosmic birefringence rotate polarization vectors.

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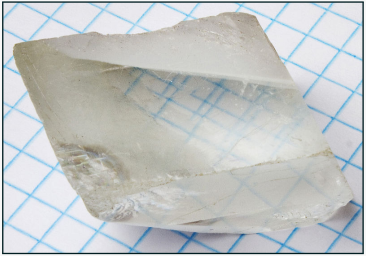

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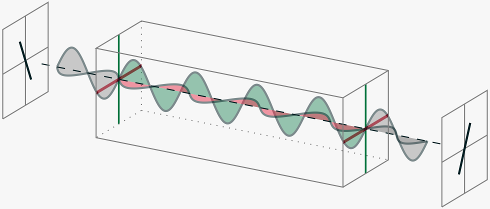
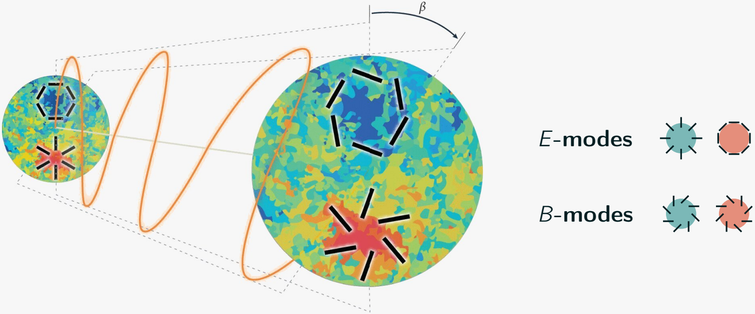


(this is a half-wave plate, by the way)

Both optical and cosmic birefringence rotate polarization vectors.

effect in harmonic space



Mixing of E and B modes: $\begin{cases} a^E_{\ell m,\mathrm{obs}} = a^E_{\ell m}\cos 2\beta - a^B_{\ell m}\sin 2\beta, \\ a^B_{\ell m,\mathrm{obs}} = a^E_{\ell m}\sin 2\beta + a^B_{\ell m}\cos 2\beta. \end{cases}$

image credit: Yuto Minami

hints of cosmic birefringence

$$\begin{cases} C_{\ell,\mathrm{obs}}^{TT} = C_\ell^{TT}, \\ C_{\ell,\mathrm{obs}}^{EE} = \cos^2(2\beta) C_\ell^{EE} + \sin^2(2\beta) C_\ell^{BB} - \sin(4\beta) C_\ell^{EB}, \\ C_{\ell,\mathrm{obs}}^{BB} = \cos^2(2\beta) C_\ell^{BB} + \sin^2(2\beta) C_\ell^{EE} + \sin(4\beta) C_\ell^{EB}, \\ C_{\ell,\mathrm{obs}}^{TE} = \cos(2\beta) C_\ell^{TE} - \sin(2\beta) C_\ell^{TB}, \\ C_{\ell,\mathrm{obs}}^{EB} = \sin(4\beta)(C_\ell^{EE} - C_\ell^{BB})/2 + \cos(4\beta) C_\ell^{EB}, \\ C_{\ell,\mathrm{obs}}^{TB} = \sin(2\beta) C_\ell^{TE} + \cos(2\beta) C_\ell^{TB}. \end{cases}$$

$$C_{\ell,\mathrm{obs}}^{EB} = \tan(4\beta)(C_{\ell,\mathrm{obs}}^{EE} - C_{\ell,\mathrm{obs}}^{BB})/2.$$

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$$C_{\ell,\mathrm{obs}}^{EB} = \tan(4\beta)(C_{\ell,\mathrm{obs}}^{EE} - C_{\ell,\mathrm{obs}}^{BB})/2.$$

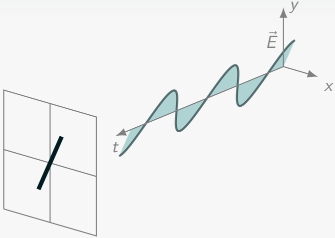
$$\beta = 0.35 \pm 0.14 \ (68\%\mathrm{CL})$$

To be confirmed (or not) by future polarization observations!

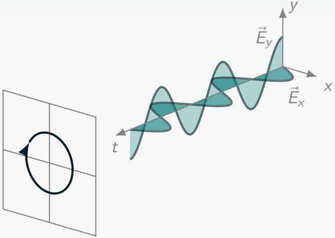
Minami and Komatsu (2020) Phys. Rev. Lett. 125

measuring polarization

describing polarization: Stokes vectors



linearly polarized



circularly polarized

$$\text{Stokes vector } \vec{S} \equiv \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \equiv \begin{pmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2\mathrm{Re}(E_x E_y^*) \\ -2\mathrm{Im}(E_x E_y^*) \end{pmatrix}$$

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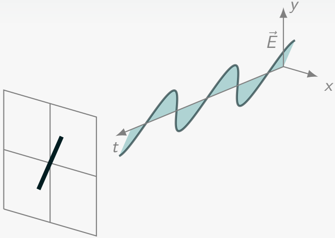

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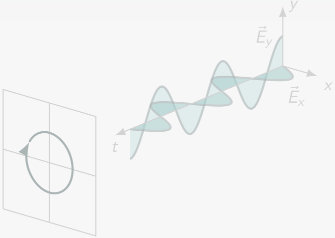

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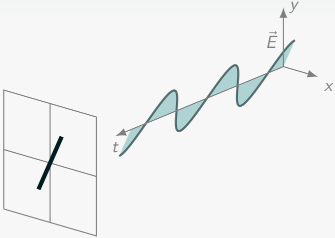

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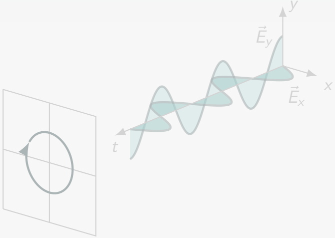

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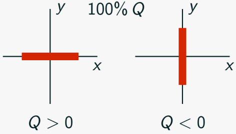

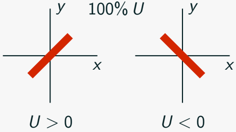

matrix methods for computing polarization

Mueller calculus: radiation described as $S = (I, Q, U)$, effect of polarization-altering devices parametrized by $\mathcal{M}$ so that $\mathsf{S}' = \mathcal{M} \cdot \mathsf{S}$.

$$\mathcal{M}_{\mathsf{pol}} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{M}_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) \\ 0 & -\sin(2\theta) & \cos(2\theta) \end{pmatrix}, \quad \ldots$$

Given two optical elements in series with $\mathcal{M}_1$ and $\mathcal{M}_2$,
their combined effect can be described by $\mathcal{M}_2\mathcal{M}_1$.

Matrix methods are extremely convenient to manipulate polarization, since one does not work with the electromagnetic field itself.

an example: pair-differencing systematics

Polarization can be measured by comparing the readings of pairs of (orthogonal) detectors:

$$
\begin{aligned}
d_1 &= a \cdot \mathcal{M}_{\mathrm{pol}} \cdot S = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \end{pmatrix} = I + Q \,, \\
d_2 &= a \cdot \mathcal{M}_{\mathrm{pol}} \mathcal{M}_{\pi/2} \cdot S \qquad\qquad = I - Q \,.
\end{aligned}
$$

This method can lead to detection of spurious polarization.

the path forward

How will next generation CMB experiments deal with this?

- LiteBIRD,
- Simons Observatory,
- South Pole Observatory,
- CMB Stage-4.

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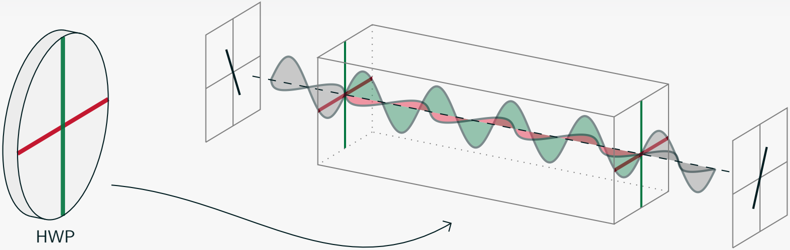
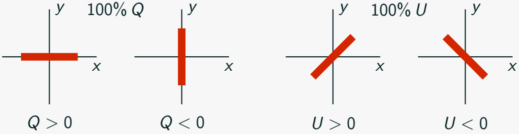
They **all** plan to employ rotating **half-wave plates (HWPs)** as polarization modulators.

the HWP: reducing systematics

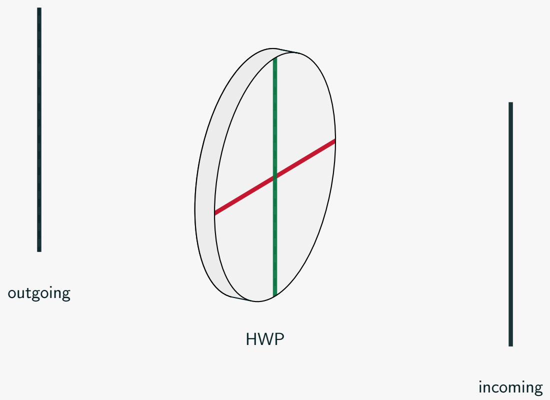




$$\mathcal{M}_{\text{HWP}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

A **rotating** half-wave plate (HWP) as first optical element can help to control systematics.

ideal rotating HWP



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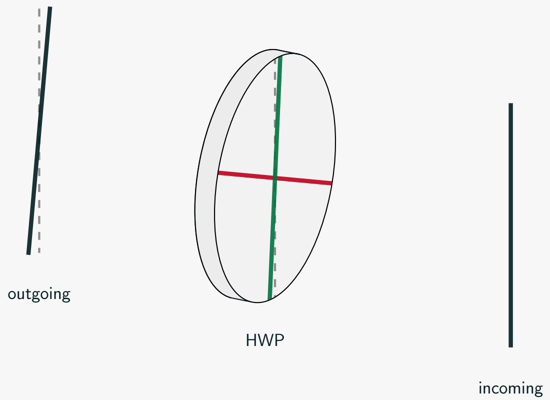

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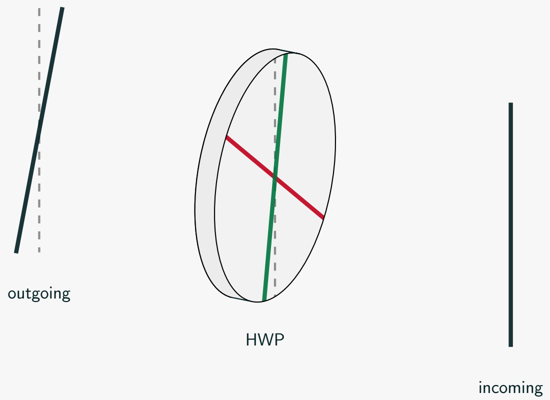

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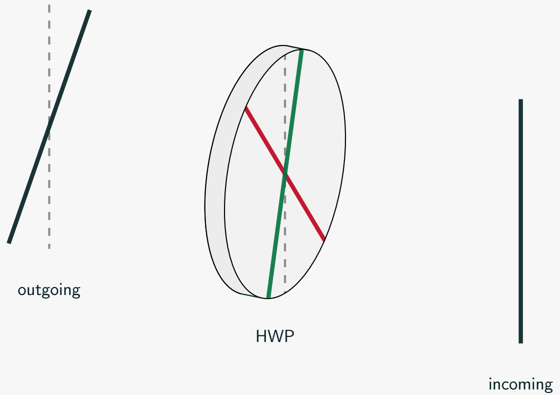

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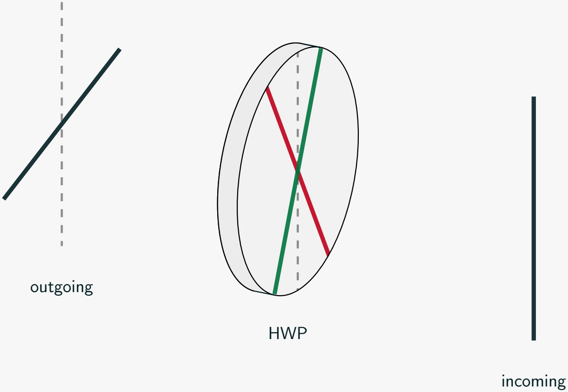

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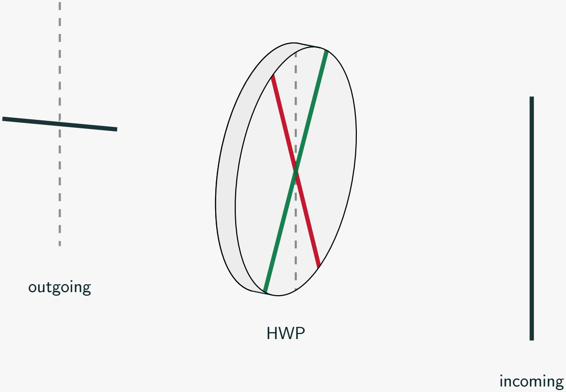

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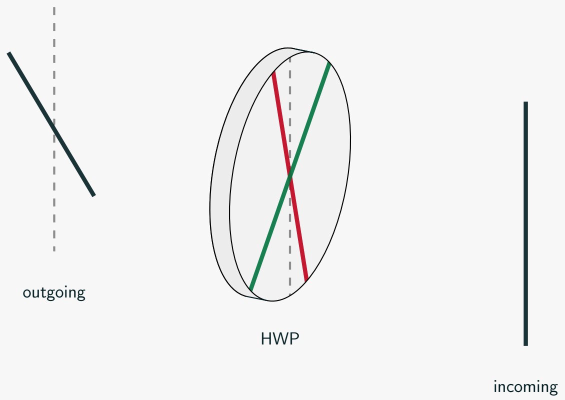

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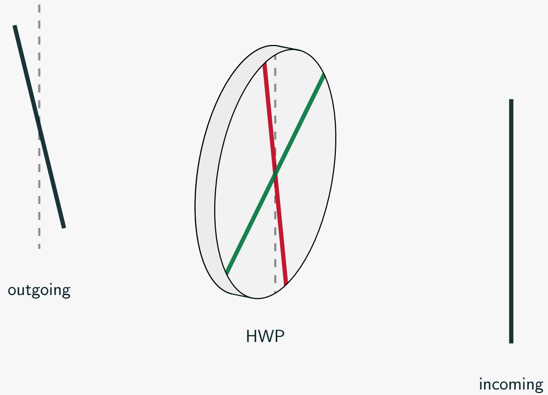

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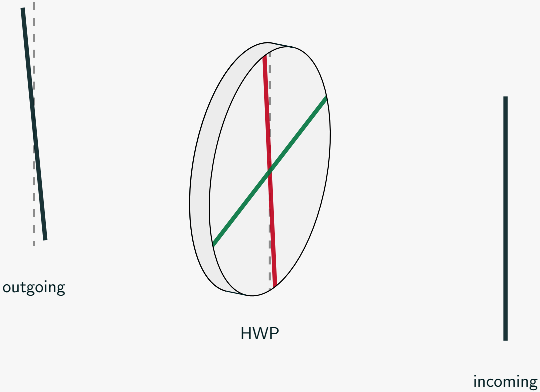

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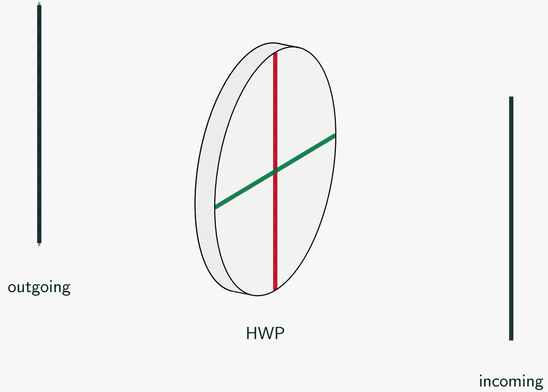

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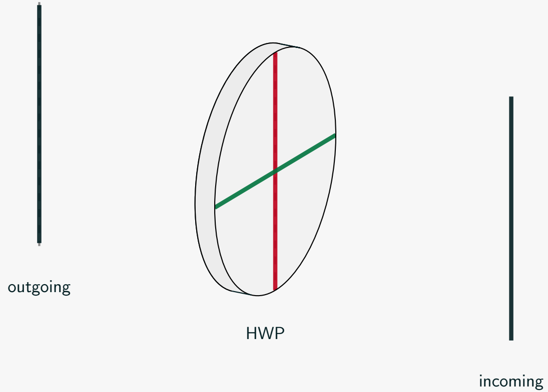
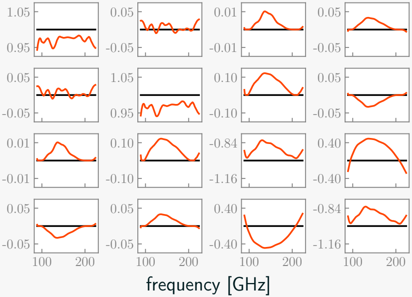


- The intrinsic signal is modulated to $4f_{\mathbf{HWP}}$ and can be distinguished from spurious signal (no/different modulation).

the HWP Mueller matrix

For an ideal HWP, $\mathcal{M}_{\text{ideal}} = \text{diag}(1, 1, -1, -1)$, but let's look at a realistic case:

$$\mathcal{M}_{\text{HWP}} = $$

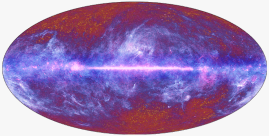


How does this affect the observed maps?

Giardiello et al. (2022) A&A 658

modeling the HWP effect

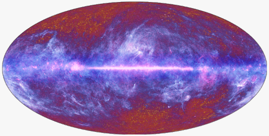
how to propagate systematics



CMB sky maps

image credit: *Planck* collaboration

how to propagate systematics



detection

time ordered data (TOD)

map-making

multi-frequency maps

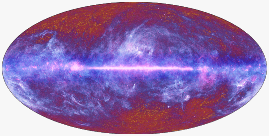
foreground cleaning

CMB sky maps

TOD: collection of the signal detected by *each of the (4508) detectors* during *the whole (3-year) mission*.

image credit: *Planck* collaboration

how to propagate systematics



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Simulating and **modeling** TOD is crucial in the planning of any CMB experiment: it helps studying potential systematic effects.

image credit: *Planck* collaboration

HWP impact on CB: working assumptions

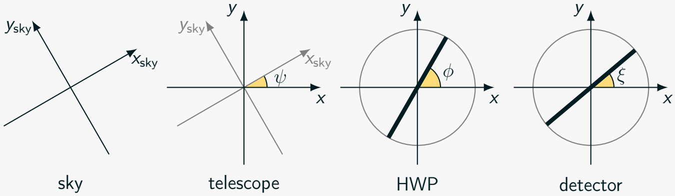
To focus on the impact of **HWP non-idealities** we consider a simplified problem:

- no noise,
- single frequency,
- CMB-only,
- simple beams,
- HWP aligned to the detector line of sight.

modeling the TOD

(minimal) TOD: signal detected by 4 detectors.

$$\begin{pmatrix} d^{(0)} \\ d^{(90)} \\ d^{(45)} \\ d^{(135)} \end{pmatrix} = \begin{pmatrix} (1\ 1\ 0) \cdot \mathcal{R}_{0-\phi} \mathcal{M}_{\mathrm{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{90-\phi} \mathcal{M}_{\mathrm{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{45-\phi} \mathcal{M}_{\mathrm{HWP}} \mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{135-\phi} \mathcal{M}_{\mathrm{HWP}} \mathcal{R}_{\phi+\psi} \end{pmatrix} \cdot \begin{pmatrix} I_{\mathrm{in}} \\ Q_{\mathrm{in}} \\ U_{\mathrm{in}} \end{pmatrix}$$



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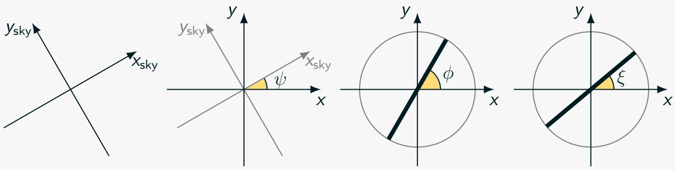
response matrix A



sky telescope HWP detector

modeling the observed maps

map-maker: bin-averaging $\widehat{S} = (\widehat{A}^T\widehat{A})^{-1}\widehat{A}^T A \cdot S$ assuming ideal HWP.

$$\widehat{A} = \begin{pmatrix} (1\ 1\ 0) \cdot \mathcal{R}_{0-\phi}\mathcal{M}_{\text{ideal}}\mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{90-\phi}\mathcal{M}_{\text{ideal}}\mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{45-\phi}\mathcal{M}_{\text{ideal}}\mathcal{R}_{\phi+\psi} \\ (1\ 1\ 0) \cdot \mathcal{R}_{135-\phi}\mathcal{M}_{\text{ideal}}\mathcal{R}_{\phi+\psi} \end{pmatrix}$$

ideal TOD

non-ideal TOD

ideal map-maker

output = input

potential discrepancies

estimated ouput maps

$$
\begin{aligned}
\widehat{I} &= m_{ii} I_{\rm in} + (m_{iq} Q_{\rm in} + m_{iu} U_{\rm in}) \cos(2\alpha) + (m_{iq} U_{\rm in} - m_{iu} Q_{\rm in}) \sin(2\alpha)\,, \\
\widehat{Q} &= \frac{1}{2}\Big\{(m_{qq} - m_{uu}) Q_{\rm in} + (m_{qu} + m_{uq}) U_{\rm in} + 2 m_{qi} I_{\rm in} \cos(2\alpha) + 2 m_{ui} I_{\rm in} \sin(2\alpha) \\
&\qquad + \big[(m_{qq} + m_{uu}) Q_{\rm in} + (m_{qu} - m_{uq}) U_{\rm in}\big] \cos(4\alpha) \\
&\qquad + \big[-(m_{qu} - m_{uq}) Q_{\rm in} + (m_{qq} + m_{uu}) U_{\rm in}\big] \sin(4\alpha)\Big\}\,, \\
\widehat{U} &= \frac{1}{2}\Big\{(m_{qq} - m_{uu}) U_{\rm in} - (m_{qu} + m_{uq}) Q_{\rm in} - 2 m_{ui} I_{\rm in} \cos(2\alpha) + 2 m_{qi} I_{\rm in} \sin(2\alpha) \\
&\qquad + \big[-(m_{qq} + m_{uu}) U_{\rm in} + (m_{qu} - m_{uq}) Q_{\rm in}\big] \cos(4\alpha) \\
&\qquad + \big[(m_{qu} - m_{uq}) U_{\rm in} + (m_{qq} + m_{uu}) Q_{\rm in}\big] \sin(4\alpha)\Big\}\,,
\end{aligned}
$$

where $\alpha = \phi + \psi$. For **good** coverage and **rapidly spinning** HWP:

$$
\widehat{\mathsf{S}} \simeq \begin{pmatrix} m_{ii} I_{\rm in} \\ [(m_{qq} - m_{uu}) Q_{\rm in} + (m_{qu} + m_{uq}) U_{\rm in}]/2 \\ [(m_{qq} - m_{uu}) U_{\rm in} - (m_{qu} + m_{uq}) Q_{\rm in}]/2 \end{pmatrix}.
$$

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angular power spectra

Expanding $\widehat{S}$ in spherical harmonics:

$$\widehat{C}_\ell^{TT} \simeq m_{ii}^2 C_{\ell,\text{in}}^{TT},$$
$$\widehat{C}_\ell^{EE} \simeq \frac{(m_{qq}-m_{uu})^2}{4} C_{\ell,\text{in}}^{EE} + \frac{(m_{qu}+m_{uq})^2}{4} C_{\ell,\text{in}}^{BB} + \frac{(m_{qq}-m_{uu})(m_{qu}+m_{uq})}{2} C_{\ell,\text{in}}^{EB},$$
$$\widehat{C}_\ell^{BB} \simeq \frac{(m_{qq}-m_{uu})^2}{4} C_{\ell,\text{in}}^{BB} + \frac{(m_{qu}+m_{uq})^2}{4} C_{\ell,\text{in}}^{EE} - \frac{(m_{qq}-m_{uu})(m_{qu}+m_{uq})}{2} C_{\ell,\text{in}}^{EB},$$
$$\widehat{C}_\ell^{TE} \simeq \frac{m_{ii}(m_{qq}-m_{uu})}{2} C_{\ell,\text{in}}^{TE} + \frac{m_{ii}(m_{qu}+m_{uq})}{2} C_{\ell,\text{in}}^{TB},$$
$$\widehat{C}_\ell^{EB} \simeq \frac{(m_{qq}-m_{uu})^2-(m_{qu}+m_{uq})^2}{4} C_{\ell,\text{in}}^{EB} - \frac{(m_{qq}-m_{uu})(m_{qu}+m_{uq})}{2} (C_{\ell,\text{in}}^{EE} - C_{\ell,\text{in}}^{BB}),$$
$$\widehat{C}_\ell^{TB} \simeq \frac{m_{ii}(m_{qq}-m_{uu})}{2} C_{\ell,\text{in}}^{TB} - \frac{m_{ii}(m_{qu}+m_{uq})}{2} C_{\ell,\text{in}}^{TE}.$$

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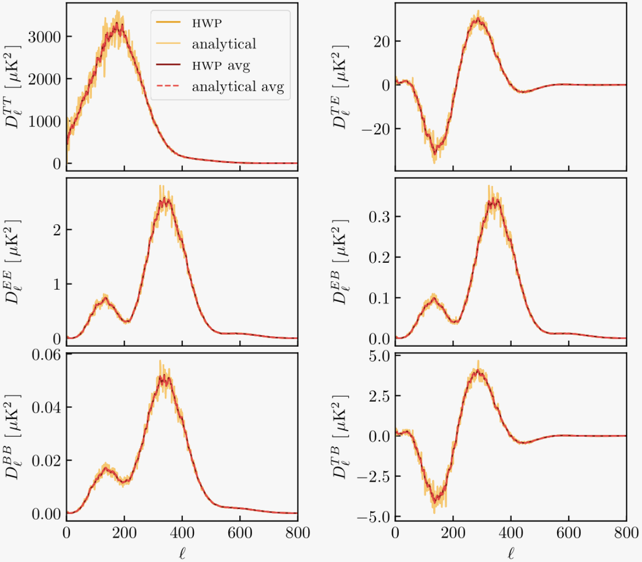
analytical vs simulated output spectra



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impact on cosmic birefringence

HWP-induced miscalibration

Analytic $\widehat{C}_\ell$s satisfy the relations:

$$\begin{cases} \widehat{C}_\ell^{EB} \simeq \tan(4\widehat{\theta}) \left[\widehat{C}_\ell^{EE} - \widehat{C}_\ell^{BB}\right]/2 \\ \widehat{C}_\ell^{TB} \simeq \tan(2\widehat{\theta})\widehat{C}_\ell^{TE} \end{cases}$$

The HWP induces an additional miscalibration, **degenerate** with cosmic birefringence and polarization angle miscalibration!

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our formulae suggest

$$\widehat{\theta} \equiv -\frac{1}{2}\arctan\frac{m_{qu}+m_{uq}}{m_{qq}-m_{uu}} \sim 3.8^\circ,$$

compatibly with simulations.

The HWP induces an additional miscalibration, **degenerate** with cosmic birefringence and polarization angle miscalibration!

This doesn't mean that the HWP will keep us from measuring β, but it shows how important it is to carefully calibrate $\mathcal{M}_{\mathrm{HWP}}$.

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conclusions and outlook

- much information is still hidden in CMB polarization (for instance, **cosmic birefringence** as a signature of parity-violating physics),
- new physics can be probed only if systematics are well under control,
- a rotating HWP can help, but it induces additional systematics which should be accounted for (**HWP-induced miscalibration**),

- we are now provided with an analytical model and a simulation pipeline that can be used to study the impact of the HWP in more realistic scenarios. this is key for the planning of the next generation of CMB experiments.