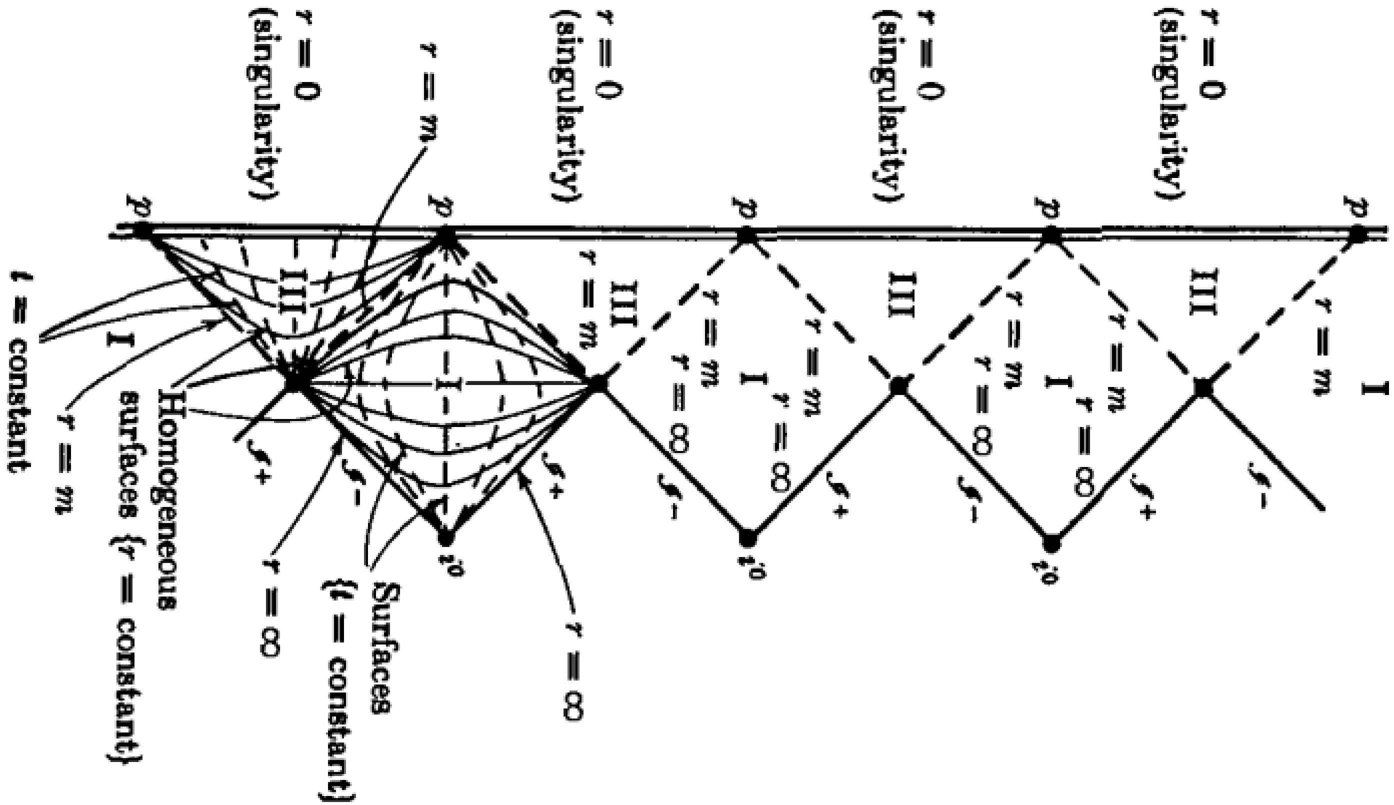
Quantum Gravity and Extremal Black Holes



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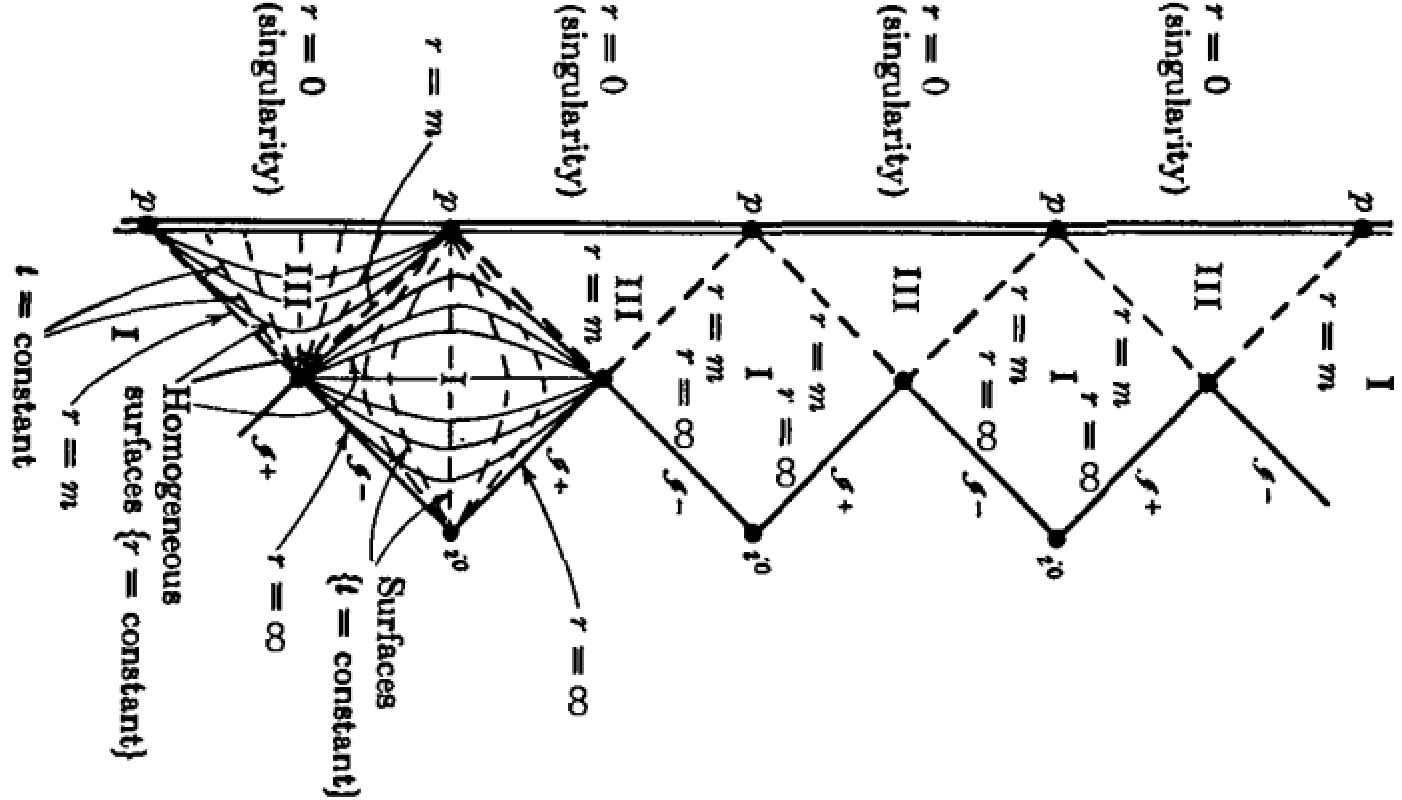
Quantum Gravity and Extremal Black Holes



based on 2407.XXXXX in collaboration with C. de Rham and A. J. Tolley

EFTs of Gravity

General relativity accurately describes gravity across various scales...

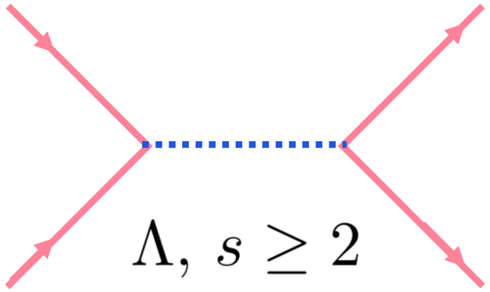
...but predicts its own breakdown: Need **UV completion**!

Effective Field Theory perspective: Use most general **local** action

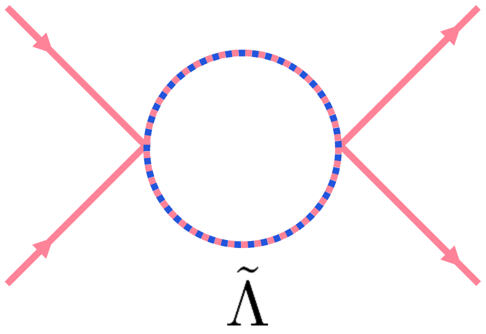
1) consistent with **symmetries**,

2) organised in **derivative expansion**, and

3) with **coefficients** fixed by dimensional analysis.



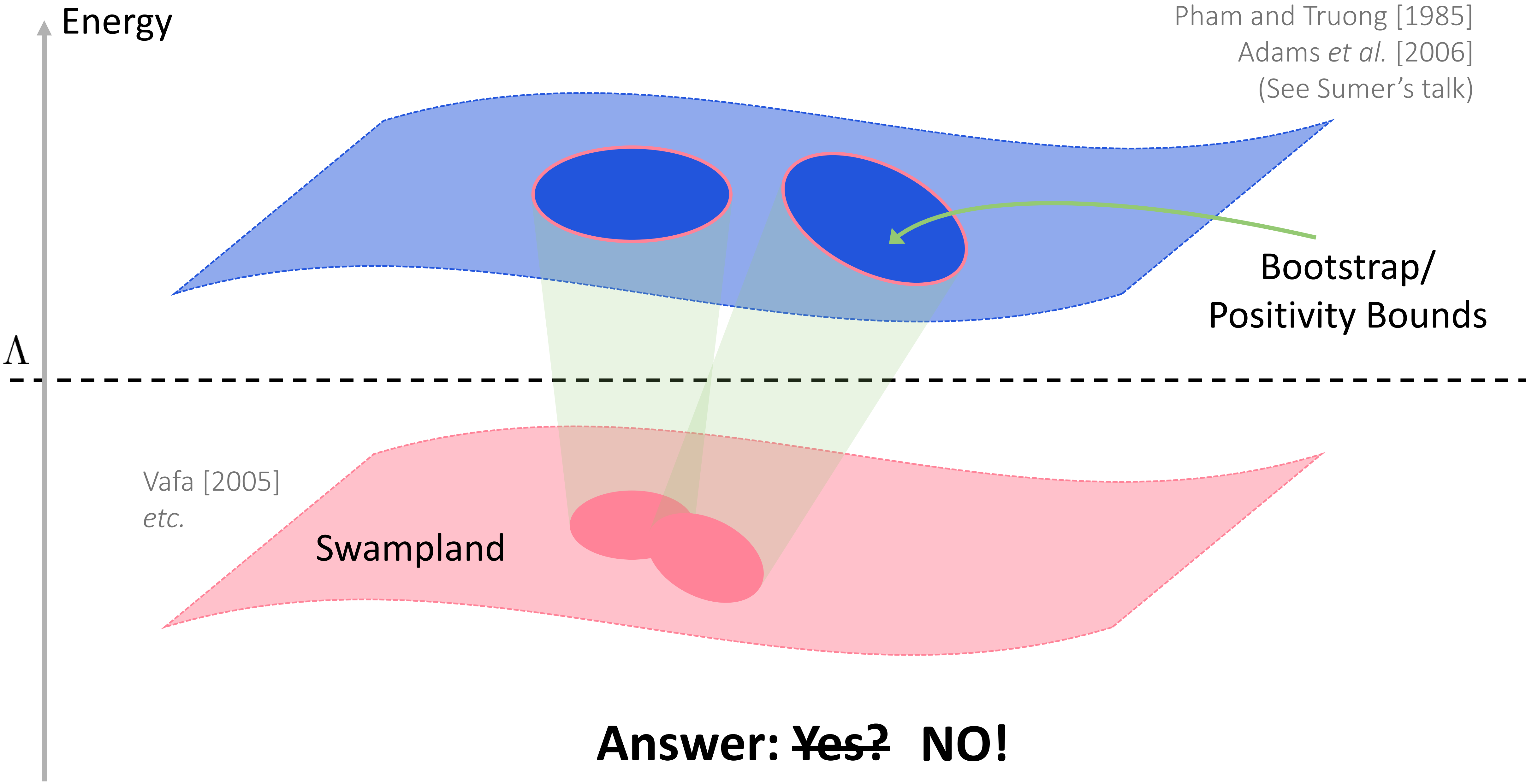
$$S_{\mathrm{EFT}} = M_{\mathrm{Pl}}^{D-2} \int d^D x \sqrt{-g} \left[\frac{1}{2} R + \Lambda^2 \sum_{m \geq 0, n \geq 2} c_{mn} \left(\frac{\nabla}{\Lambda} \right)^m \left(\frac{\mathrm{Riemann}}{\Lambda^2} \right)^n \right]$$



Suppressed by loops

→ Treat as **standard QFT** (careful about breakdown!)

Question: Are all these terms physical?



Answer: ~~Yes?~~ NO!

Deforming Extremal Charged Black Holes

Charged Black Holes in AdS

- Gravity + Maxwell in D dimensions

$$S_{\mathrm{EM}} = \int \mathrm{d}^D x \sqrt{-g} \left[\frac{1}{2\kappa} \left(R + \frac{(D-2)(D-1)}{L^2} \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

→ Asymptotically flat limit when $L \to \infty$.

- Spherically symmetric and static **background solutions**

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r)} + r^2 \mathrm{d}\Omega_d^2, \qquad F = \Psi'(r)\mathrm{d}t \wedge \mathrm{d}r$$

$d = D - 2$

→ Anti-de Sitter Reissner-Nordström (AdS RN)

$Q^2 = \frac{D-3}{D-2} \kappa q^2$

$$A(r) = B(r) = f(r) := 1 + \frac{r^2}{L^2} - \frac{M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}, \qquad \Psi(r) = \frac{q}{r^{D-3}}$$

The Extremal Limit

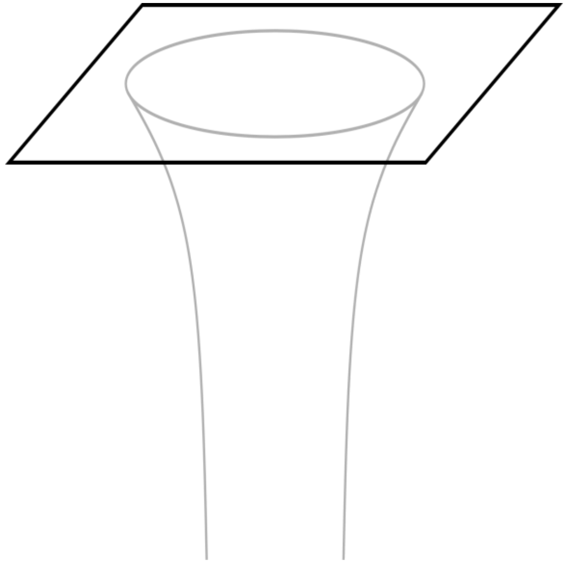
- Solution possesses two **horizons**

$$r_\pm = \frac{M}{2} - \sqrt{\left(\frac{M}{2}\right)^2 - Q^2}$$

→ Degenerate to extremal horizon $r_H := r_+ = r_-$ in **extremal limit**.

$\rho = r - r_+$

- **Extremal** **near-horizon geometry** is leading-order in ρ / r_H :



$$\mathrm{d}s^2 = \frac{2}{f''(r_H)}\left[-\rho^2\left(\frac{f''(r_H)}{2}\mathrm{d}t\right)^2 + \frac{\mathrm{d}\rho^2}{\rho^2}\right] + r_H^2 \mathrm{d}\Omega_d^2$$

i.e. $\mathrm{AdS}_2 \times S^d$!

→ **Generic** to extremal black holes!

Kunduri, Lucietti, and Reall [2007]

Deformations

What about solutions with less symmetry?

→ Interested in *stationary* deformations to *near-horizon* regions of *extremal* black holes!

Horowitz, Kolanowski, and Santos [2022, 2023]
Gralla & Zimmermann [2018]

- From full analysis of gravitational perturbations: Deformations h

$$h = c_- \rho^{\gamma_-} + c_+ \rho^{\gamma_+}, \qquad \gamma_\pm = \frac{1}{2}\left(-1 \pm \sqrt{1 + 4U} \right)$$

Effective Potential

→ Finite B.C. @ $\rho = 0$ require $c_- = 0$.

Other branch always arises from **regular** sub-extremal geometry.
Hence **physical even if singular**!

Gubser [2000]

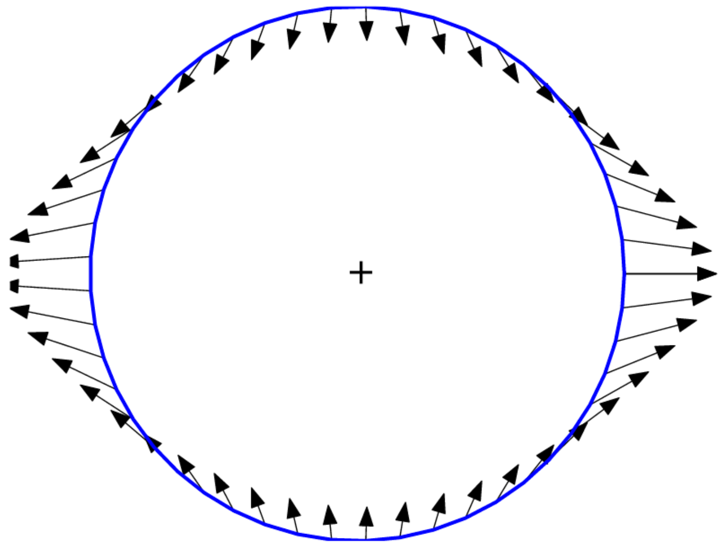
Singularities

Scaling in terms of original metric perturbations is $h_{..} \sim \rho^{\gamma}$.

- **Scalar invariants** on deformed geometry scale as:

$$S \sim \rho^{n\gamma}, \quad n \in \mathbb{N}^{+}$$

→ Scalar polynomial **singularity** when $\gamma < 0$.



- Perturbations to the **Weyl tensor** scale as:

$$\delta C_{....} \sim \rho^{\gamma-2}$$

→ Parallel-propagated **singularity** when $\gamma < 2$.

Tidal force on particles travelling along geodesics of deformed background.
Artefact of **geodesic approximation**/breakdown of wordline EFT.

Horowitz, Kolanowski, Remmen, and Santos [2024]

EFT Corrections

Deformations in the EFT of Gravity

Parameterise corrections from UV with higher-derivative **EFT corrections**

$$S = S_{\mathrm{EM}} + S_{\mathrm{EFT}}$$

- Due to rigidity of near-horizon geometry:

$$h_{..} \sim \rho^{\gamma}, \qquad \gamma = \gamma_{\mathrm{GR}} + \gamma_{\mathrm{EFT}}$$

→ EFT correction **resums** into exponent

Specifically:

$$S_{\mathrm{EFT}} = \int \mathrm{d}^D x \sqrt{-g} \sum_{\mathcal{O}} \frac{c_{\mathcal{O}}}{\Lambda^{[\mathcal{O}]-D}} \mathcal{O} \quad \longrightarrow \quad \gamma_{\mathrm{EFT}} = \sum_{\mathcal{O}} c_{\mathcal{O}} \gamma_{\mathcal{O}}$$

- **Marginal case**: When $\gamma_{\mathrm{GR}} = 0$, singularity of horizon sensitive to sign of

$$\hat{\gamma} := \gamma_{\mathrm{EFT}}\big|_{\gamma_{\mathrm{GR}}=0}$$

Example: EFT Correction

Specific yet generic EFT correction

$$S = S_{\mathrm{EM}} + \frac{\kappa c}{\Lambda^2} \int \mathrm{d}^5x \sqrt{-g}(F_{\mu\nu}F^{\mu\nu})^2.$$

- Static and spherically symmetric **background** solutions with

$$A(r) = B(r) = f(r) - \frac{c}{\Lambda^2}\frac{12Q^4}{r^{10}}, \qquad \Psi(r) = \frac{q}{r^2} - 16c\frac{\kappa}{\Lambda^5}\frac{q^3}{r^8}$$

→ Modifies extremality bound.

- Scaling exponents obtained by decoupling and solving perturbations equations.
  - Marginal deformation for every harmonic ℓ:

$$k_S^2 = \ell(\ell+2)$$

$$\hat{\gamma} = -\frac{c}{\Lambda^2 r_H^2}\frac{72k_S^2(k_S^2-4)^2}{15k_S^4 - 128k_S^2 + 256}$$

→ Negative for sign $c > 0$ (expected from positivity bounds and WGC).

Breakdown of Breakdowns

Sign of $\hat{\gamma}$ and hence presence of curvature singularities on horizon **UV sensitive!**

- Suggests breakdown of EFT near horizon, but...
  - **EFT expansion** under control when

$$r_H \Lambda \gg 1, \quad h \sim h_0 \rho^\gamma \ll (\Lambda r_H)^4$$

  - **Metric perturbation theory** under control when

$$h \sim h_0 \rho^\gamma \ll 1$$

→ Metric perturbation theory out of control before EFT breaks down!

- This is supported by the following example...

Example: UV Avatar

Einstein-Maxwell-Dilaton system

$$S_{\mathrm{EMD}} = \int \mathrm{d}^D x \sqrt{-g} \left[\frac{1}{2\kappa}(R - 2\Lambda) + \sum_{i=1}^{N} \left(-\frac{1}{4N} e^{\alpha \phi_i} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_\mu \phi_i \nabla^\mu \phi_i - \frac{1}{2} m_i^2 \phi_i^2 \right) \right]$$

Useful to define

$$\frac{1}{m_{\mathrm{eff}}^2} = \sum_{i=1}^{N} \frac{1}{m_i^2}$$

- Near-horizon geometry in previous example are full solution.

- Find **scaling exponents** of deformations from full analysis of gravitational perturbations.

  In marginal case:

$$\hat{\gamma} = -\frac{\alpha^2}{N^2 r_H^2 m_{\mathrm{eff}}^2 \kappa} \frac{3k_S^2 (k_S^2 - 4)^2}{4\left(15k_S^4 - 128k_S^2 + 256\right)} \left(\sum_{i=1}^{N} \frac{4 r_H^2 m_{\mathrm{eff}}^2}{r_H^2 m_i^2 + k_S^2} - 1 \right)$$

Example: UV Avatar

Interesting features...!

- Good **UV behaviour**: Two-derivative theory

- When $m_i^2 r_H^2 \gg 1$, tree-level effective action includes F^{2n}-terms ($n > 1$). At leading order, reproduce F^4-correction from previous example with

$$c = \frac{1}{32}, \qquad \Lambda^2 = \frac{N^2 \kappa m_{\text{eff}}^2}{\alpha^2}$$

→ Presents **partial UV completion!**

- Scaling exponents at leading-order manifestly negative:

$$\hat{\gamma} = -\frac{\alpha^2}{N^2 r_H^2 m_{\text{eff}}^2 \kappa} \frac{9k_S^2(k_S^2 - 4)^2}{4\,(15k_S^4 - 128k_S^2 + 256)} + \dots$$

→ Singularity already present in the UV.

A Conjecture

Leading EFT

EFT corrections in $D=5$ up to four-derivatives

$$S_{\mathrm{EFT}} = \int \mathrm{d}^5x\sqrt{-g}\bigg[\frac{c_1}{\kappa\Lambda^2}R^2 + \frac{c_2}{\kappa\Lambda^2}R_{\mu\nu}R^{\mu\nu} + \frac{c_3}{\kappa\Lambda^2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2}RF^2$$
$$+ \frac{c_5}{\Lambda^2}R_{\mu\nu}F^{\mu}{}_{\lambda}F^{\nu\lambda} + \frac{c_6}{\Lambda^2}R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \frac{\kappa c_7}{\Lambda^2}(F^2)^2 + \frac{\kappa c_8}{\Lambda^2}F_{\mu}{}^{\nu}F_{\nu}{}^{\rho}F_{\rho}{}^{\sigma}F_{\sigma}{}^{\mu}\bigg]$$

- Field-redefinition **invariant** combinations are c_3, c_6,

$$c_0 = \frac{1}{2}\left[c_1 + 11c_2 + 31c_3 + 6c_4 + 12\left(c_5 + c_6\right) + 18\left(2c_7 + c_8\right)\right]$$
$$c_9 = c_2 + c_5 + c_8$$

- Perturbatively admits static and spherically symmetric **background solutions.** Determine **scaling exponents** from deformations.

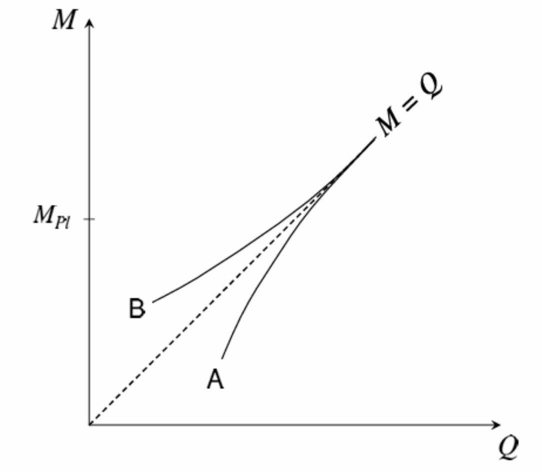
→ Subtle: EFT changes asymptotics.

Weak Gravity Conjecture

When $L \to \infty$, this is constrained by the **Weak Gravity Conjecture (WGC)**.

- **Super-extremal** black holes in tension with **Weak Cosmic Censorship.**
  - **WGC**: UV physics should allow super-extremal states to allow for decay without super-extremality:

$$\frac{M/2}{|Q|} < 1$$



- For **small** black holes: Extremal charge-to-mass ratio should **decrease** with decreasing mass
  - At leading order (four derivatives), WGC implies

$$\frac{M/2}{|Q|} = 1 - \frac{1}{3\Lambda^2 r_H^2} c_0 \longrightarrow c_0 > 0$$

Arkani-Hamed, Motl, Nicolis, and Vafa [2006]
Kats, Motl, and Padi [2006]

Near-Horizon Negativity

Examples (Einstein-Maxwell-Dilaton and scalar toy model) suggest following **speculative conjecture**...

- **Near-Horizon Negativity**: EFTs **consistent** with UV completions have

$$\hat{\gamma} \lesssim 0$$

- For four-derivative corrections to Einstein-Maxwell, **marginal scaling exponent** is

$$\hat{\gamma} = -\frac{512 k_S^2}{\left(16 - 3k_S^2\right)^2 \left(5k_S^2 - 16\right) \Lambda^2 r_H^2} \tilde{c}_0$$

with

$$\begin{aligned} 1024\tilde{c}_0 =& 2c_0 \left(k_S^2 - 4\right)^2 \left(11k_S^2 - 56\right) \\ & - \left(k_S^2 - 8\right) \left[c_3 \left(69k_S^4 - 544k_S^2 + 960\right) - 16c_6 \left(3k_S^4 - 28k_S^2 + 64\right)\right] \end{aligned}$$

Near-Horizon Negativity

→ **Near-Horizon Negativity** implies

$$\tilde{c}_0(\ell) > 0, \quad \forall \ell$$

- For $L \to \infty$, marginal mode is $\ell = 2$

$$\tilde{c}_0(\ell = 2) = c_0 > 0$$

→ Reproduces (and hence strictly stronger than) **AF WGC**.

Kats, Motl, and Padi [2007]
Horowitz, Kolanowski, Remmen, and Santos [2023]

- **Other bounds** obtained from $\ell > 2$.
  - For instance, as $\ell \to \infty$

$$\tilde{c}_\infty = \lim_{k_S \to \infty} \frac{\tilde{c}_0}{k_S^6} = \frac{22c_0 - 69c_3 + 48c_6}{1024} > 0$$

Conclusion

Summary

- **UV sensitivity** of extremal black holes
  - Deformations to near-horizon geometry are UV sensitive, but no **breakdown of EFT**!

    → Generalisation to **extremal black branes**! [WIP] with A. Kovacs

  → Better understanding of relation to **Aretakis instability** and **Love Numbers**.

- **Constraints** on EFTs
  - **Near-horizon negativity** (speculative): Generalisation of bound from **WGC** for leading EFT.

    → Physical intuition – Holography, energy conditions?

Thanks for your attention!
Questions?