•
$$|G|=2$$
, $G=de,a$, $a\neq e$. $a^2=e$.
 $=\langle a\rangle$
 $G=\mathbb{Z}_2$

• If
$$0@=4$$
, $(a)=3e, a, a^2, a^3 \le G=(a)$.

•
$$a^2 = b^2 = c^2 = 0$$
.

$$\begin{array}{ccc}
\hline
ab = a & \Rightarrow & b = e & x \\
ab = b & \Rightarrow & a = e & x \\
\hline
ab = c & & & \\
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ab = c & \\
\hline
ab =$$

$$\{a,a,b,ab\}$$
, $a^2=b^2=a$ $ab=ba$

Klein four-group V4 or K4 (Vierergruppe)

Thm 6.1. Every group of order 4 is either 124, or the Va

Î	Q	a	b	c	
Q	e	a	b	c	\
a	a	e	C	b	
b	b	C	2	a	
C	C	b	a	Q	
					_

$$\frac{V_{\varphi}}{a^2 = \beta e, a, b, ab}$$

$$a^2 = b^2 = e, ab = ba$$

- V4 is abelian
 V4 is smallest non-cyclic group.
- all elements different from e have order 2

ix. Va can be seen as a subgroup of Sa

$$\begin{cases} 2 & (1 & 2 & 3 & 4) \\ 2 & 1 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$=$$
 $\{2, (12), (34), (12)(34)\}$ $\leq \mathcal{G}_{q}$

Éxample 6.1

$$a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad a = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$c = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Emercise

Enample 6.2 Va = 3e, a, b, ab)

§1 Direct product

Def 7.1 Let G, and G, be two groups. Then $G = G_1 \times G_2$ is a group, with the multiplication law $(g_1, g_2)(g_1', g_2') = (g_1g_1', g_2g_2')$.

G, × Gz is called the direct product of G, and G2.

$$(G_1 \times \{e_1\}) \leq G_1 \times G_2$$

Exercise: prove: Gix rest = Gi

· G, and G, are abelian, then G, ×G2 is abelian.

$$(g_{1}, g_{2}) (g_{1}', g_{2}') = (g_{1}g_{1}', g_{2}g_{2}')$$

$$= (g_{1}'g_{1}, g_{2}'g_{2})$$

$$= (g_{1}', g_{2}') (g_{1}, g_{2}).$$

 $|G_1 \times G_2| = |G_1||G_2|$

$$G_1 = G_2 = G_2$$

$$G_2 = G_2$$

$$(Z_2 \times Z_2) = \beta(e,e), (e,a), (a,e), (a,a)$$

$$(2,a)^2 = (2,a)(2,a) = (2,a^2) = (2,e).$$

$$\begin{array}{ccc}
\hline
Z_2 \times Z_2
\end{array} \simeq V_4$$

$$C_1 \times C_2$$

Lem 7.1 All groups of even order contain at least one-identity element whose order is two.

Pf.
$$|G| = 2n$$
, but has no element whose order is two.
 $G = 3e$, (g_1) , (g_2) ..., $(g_{2n-1})^3$.

Thm 7.1 A group of order 6 is either Z6 or S3.

$$|Pf|$$
 $|G| = 6$ 1.2,3,6.

$$\exists a, o(a) = 6.$$
 =) $G = (a).$ =) $G = Z_6.$

$$\exists a GG, o(a)=2.$$

$$(ab)=(ab)^{-1}=b^{1}a^{1}=(ba)=2.$$

$$(ab)=(ab)^{-1}=b^{1}a^{-1}=(ba)=2.$$

$$3e, a, a^2, b$$
 ... $3e, a, a^2, b$... $3e, a, a^2, b$... $3e, a, a^2, b$... $3e, a, b$

$$ab = ba^2$$

$$(ab=ba) \qquad (ab)^n = a^nb^n = e$$

$$n=b$$

$$(a^3=\alpha, b^2=\alpha, ab=ba^2) = 3$$

$$a = (123)$$
 $b = (12)$

$$|a| = (1, 2, 3), (6, 5), (7, 6), (7, 8), (7, 10)$$

$$(a,b)^n = (e,e)$$
.

(a,b)

$$\Rightarrow$$
 $(a^{n}, b^{n}) = (e, e)$

$$=) \qquad a^{n}=e \ , \qquad b^{n}=e$$

=)
$$n_{kp}$$
, n_{-lq} =) k_{-q} , l_{-p}

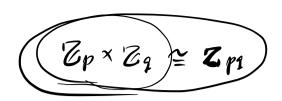
$$(u,v)^n$$
 (ℓ,ℓ)

$$=) \qquad n = (p) = (p) \qquad \Rightarrow \qquad (p) = (p') \qquad p = sp', \ g = sq'$$

$$\frac{k}{2} = \frac{2}{7} = \frac{2}{7}$$

$$\Rightarrow k=9', l=p' \frac{(3)p - p_1}{p'_1}$$





Exercise 2f a is abelian, o(a)=m, o(b)=n. prove

 $o(ab) = \frac{mn}{(m,n)}$