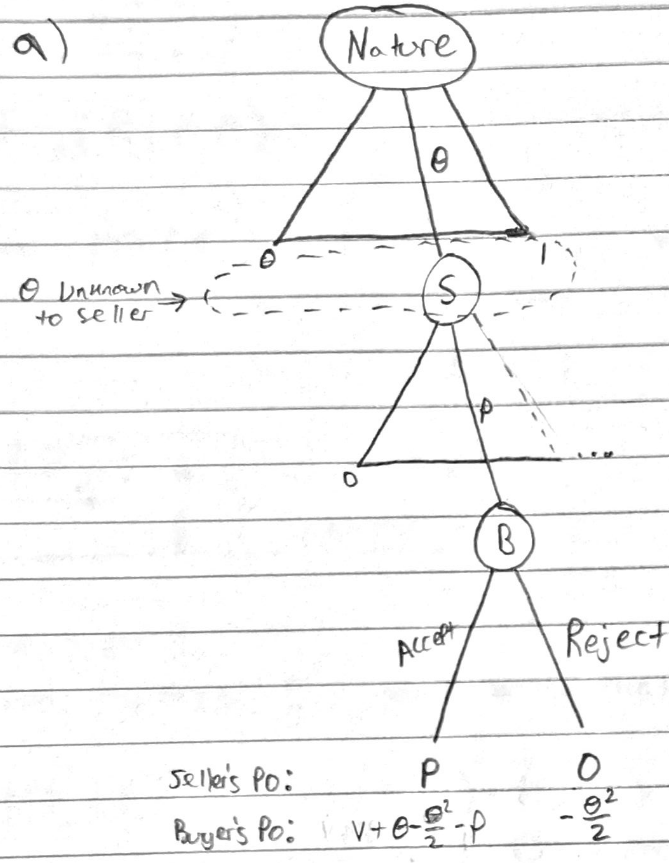
2018 Comp Review, Round I

Q4 a)



b) A Pure Strategy NE is a pair of strategies such that

- Players always make the same choice when faced with the same type/information (ie. there is no randomization over actions taken)
- Holding the strategy of the other player fixed, the actions chosen are best responses, given the info available (ie, they maximize Expected utility).

A PS for the buyer would be, when facing the same price, P, type θ, & parameter value V, the buyer would make the same choice everytime.

Eg. $\begin{cases} \text{Accept if} & \text{if } P < V + \theta \\ \text{Reject} & \text{otherwise} \end{cases}$

(spoilers. see next page)

c) **For Buyer:**

$$PO_B(R \mid P, \theta) = \frac{-\theta^2}{2}, \quad PO_B(A \mid P, \theta) = v + \theta - \frac{\theta^2}{2} - P$$

So $PO(A) \gtreqless P(R)$ when $v + \theta - \frac{\theta^2}{2} - P \gtreqless -\frac{\theta^2}{2}$

ie when $v + \theta - P \gtreqless 0$

ie when $\theta \gtreqless P - v$

$$\boxed{\begin{array}{lll} BR_B: & A & \text{if } \theta > P - v \\ & R & \text{otherwise} \end{array}}$$

For seller:

will maximize EU under belief that $\theta \sim U[0,1]$

$$\circledast \quad PO_S(P) = \text{Prob}(R \mid P) \cdot PO(R) + \text{Prob}(A \mid P) \cdot PO(A)$$
$$= \text{Prob}(R \mid P) \cdot 0 + \text{Prob}(A \mid P) \cdot P$$
$$= \text{Prob}(A \mid P) \cdot P$$

what is Prob $(A \mid P)$? Recall B plays A if $\theta > P - v$. So...

$$\text{Prob}(A \mid P) = \begin{cases} 1 & \text{if } P - v < 0 \quad (\text{since} \Rightarrow P - v < \theta \ \forall \theta \in [0,1]) \\ 1 - (P - v) & \text{if } P - v \in (0,1) \quad (\text{since} \Rightarrow P - v < \theta \ \forall \theta \in [P-v, 1]) \\ 0 & \text{otherwise, ie } P - v > 1 \quad (\text{since} \Rightarrow P - v \nless \theta \ \forall \theta \in [0,1]) \end{cases}$$

$$\Rightarrow PO_S(P) = \begin{cases} P & \text{if } P - v < 0\text{, ie } P < v \\ (1 - (P - v))P & \text{if } P - v \in (0,1)\text{, ie } P \in (v, v+1) \\ 0 & \text{otherwise, ie. } P - v > 1 \end{cases}$$

by substitution into $\circledast$

... So when is choosing $P < v$ better than choosing a $P \in (v, v+1)$?

formally

when is $PO(\tilde{P} < v) > PO(P \in (v, v+1))$?

If optimal choice is $P \leq v$, then your $PO_S(P) = P$,

so $\boxed{P^* = v}$ (maximum w/o violating $P \leq v$).

If optimal choice is $P \in (v, v+1)$, then your $PO_S(P) = (1 + v - P)P$,

$= P + vP - P^2$. FOC_P: $1 + v - 2P^* = 0 \Rightarrow 2P^* = 1 + v \Rightarrow \boxed{P^* = \frac{1+v}{2}}$ \$

If optimal choice is $p - v > 1$, then to $\boxed{p^* = v+2}$ ... or really any p s.t. $p - v > 1$, & the player will get $PO = 0$

So our updated POs is...

$$PO_s = \begin{cases} v & \text{by choosing } p^* = v \quad \left(\begin{array}{c}\text{which satisfies} \\ p - v \leq 0\end{array}\right) \text{ (a)} \\ \left(1 + v - \frac{1+v}{2}\right)\left(\frac{1+v}{2}\right) = \left(\frac{1+v}{2}\right)^2 & \text{by choosing } p^* = \left(\frac{1+v}{2}\right) \quad \left(\begin{array}{c}\text{which satisfies} \\ p \in (v, v+1) \text{ when} \\ v \in (0,1)\end{array}\right) \text{ (b)} \\ 0 & \text{by choosing } p^* = v+2 \quad \left(\begin{array}{c}\text{which satisfies} \\ p - v > 1\end{array}\right) \text{ (c)} \end{cases}$$

So which p^* yields the highest payoff?

- when $v > 1$ ...

$$PO(p^* = v) > PO\left(p^* = \frac{1+v}{2}\right) > PO(p^* = v+2)$$

ie. $v > \left(\frac{1+v}{2}\right)^2 > 0$

(also note (b) no longer satisfies it's own condition... so not valid for $v > 1$)

$\Rightarrow p^* = v$ when $v > 1$

- when $v \in (0,1)$ ...

$$PO\left(p^* = \frac{1+v}{2}\right) > PO(p^* = v) > PO(p^* = v+2)$$

$\left(\frac{1+v}{2}\right)^2 > v > 0$

$\Rightarrow p^* = \left(\frac{1+v}{2}\right)^2$ when $v \in (0,1)$

- when $v \in (-1, 0)$ ...

$$PO\left(p^* = \frac{1+v}{2}\right) > PO(p^* = v+2) > PO(p^* = v)$$

ie. $\left(\frac{1+v}{2}\right)^2 > 0 > v$

$\Rightarrow p^* = \left(\frac{1+v}{2}\right)^2$ when $v \in (-1, 0)$

- when $v < -1$ ...

$$PO(p^* = v+2) > PO(p^* = v) \text{ \& } PO(p^* = v+2) > PO\left(p^* = \frac{1+v}{2}\right)$$

$0 > v$ & $0 > \left(\frac{1+v}{2}\right)^2$

$\Rightarrow p^* = v+2$ when $v < -1$

So $$\boxed{BR_s = \begin{cases} v & \text{if } v \geq 1 \\ \frac{v+1}{2} & \text{if } v \in (-1, 1) \\ v+2 & \text{otherwise} \end{cases}}$$

So our SPNE is...

$$\boxed{\begin{array}{ll} \text{Seller chooses } P = \begin{cases} V & \text{if } V \geq 1 \\ \frac{V+1}{2} & \text{if } V \in (-1,1) \\ V+2 & \text{otherwise} \end{cases} \\ \text{Buyer chooses } \begin{cases} \text{Accept} & \text{if } \Theta > P - V \\ \text{Reject} & \text{otherwise} \end{cases} \end{array}}$$