Stochastic Constraint Propagation for Mining Probabilistic Networks

originally presented at the 28th International Joint Conference on Artificial Intelligence (IJCAI), Macao 2019

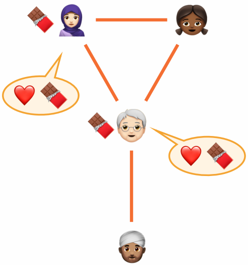
Anna Louise Latour, Behrouz Babaki, Siegfried Nijssen.

Universiteit Leiden
The Netherlands

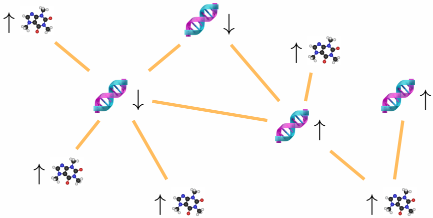
POLYTECHNIQUE MONTRÉAL

UCLouvain

Viral Marketing

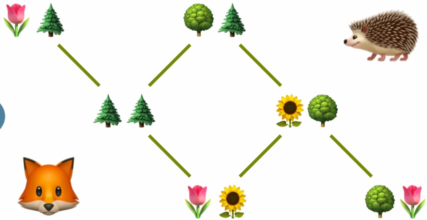


Signalling Regulatory Pathways

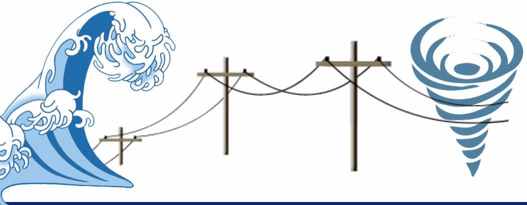


Stochastic
Constraint
Optimization
Problems

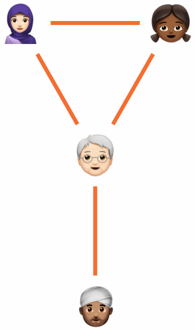
Landscape Connectivity



Powergrid Reliability



Example: Viral Marketing Problem



David Kempe, Jon Kleinberg, and Éva Tardos
Maximizing the spread of influence through a social network
KDD, 2003

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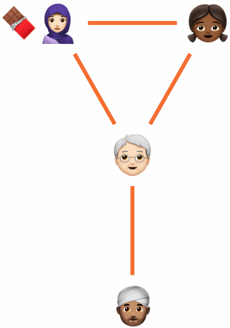

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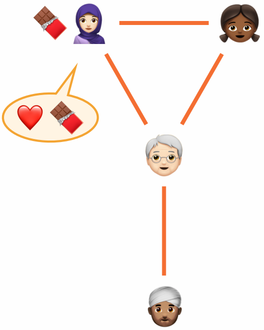

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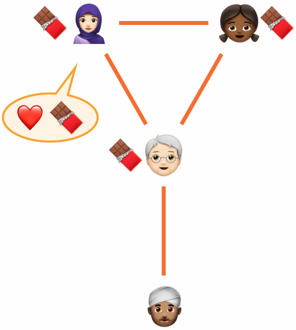

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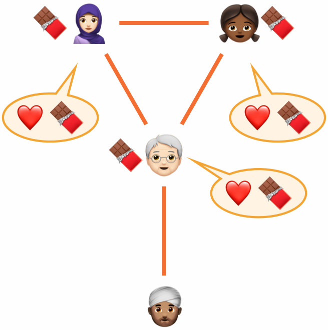

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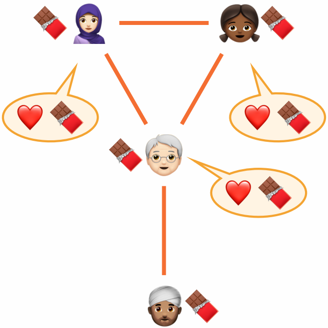

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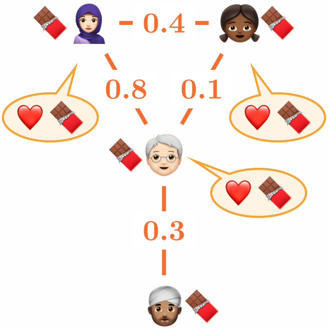

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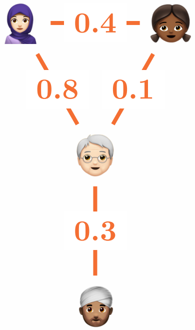

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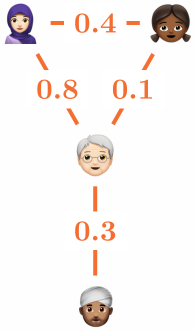

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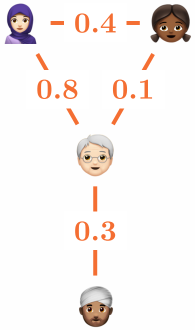

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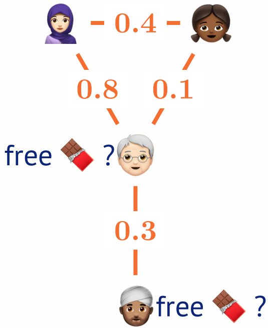

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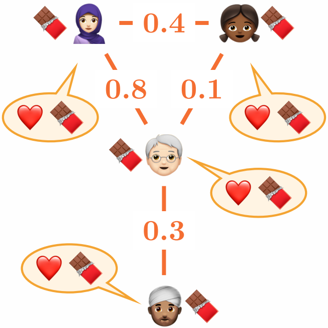

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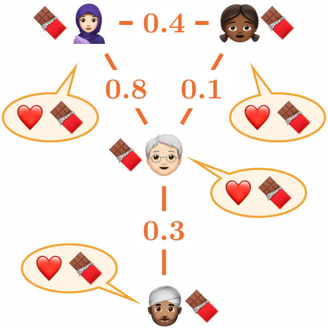

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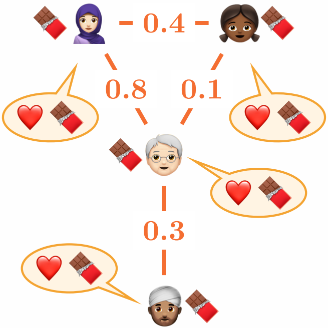

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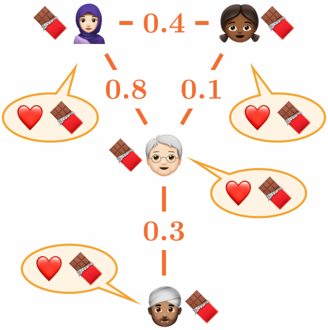

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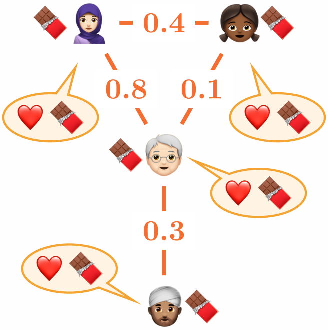

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CP technology
(search & propagation)

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OscaR: Scala in OR, 2012
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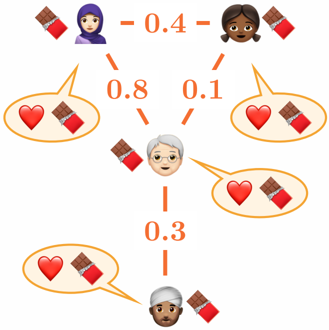

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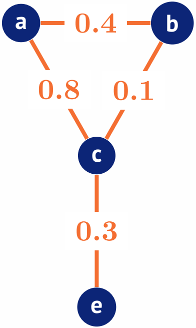

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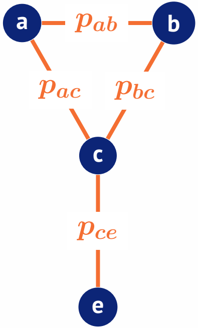

$$P(t_{xy} = 1) = p_{xy}$$
$$P(t_{xy} = 0) = (1 - p_{xy})$$

Boolean influence relationships are **independent**.

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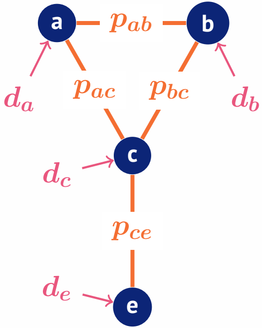

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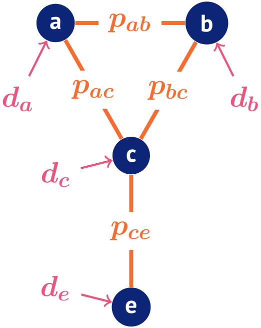

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Simplifying assumptions

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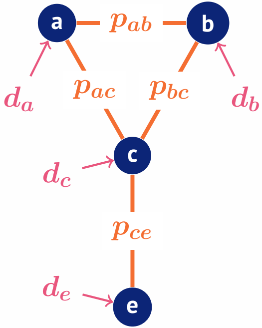

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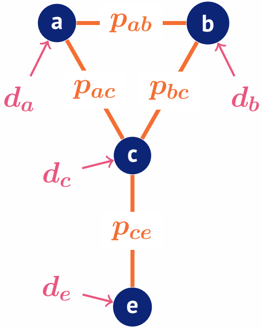

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- if person i gets a free sample ($d_i = 1$), they will buy it in the future;

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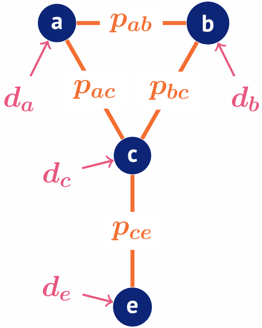

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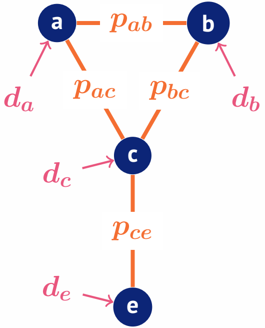
Boolean influence relationships are **independent**.

Simplifying assumptions

- influence relationships are symmetric;
- if person i gets a free sample ($d_i = 1$), they will buy it in the future;
- if person i buys chocolate and they have influence over j ($t_{ij} = 1$), then j will buy chocolate.

Example: Viral Marketing Problem



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Person e buys chocolate:

$$\phi_e =$$

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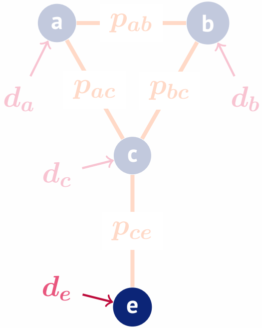

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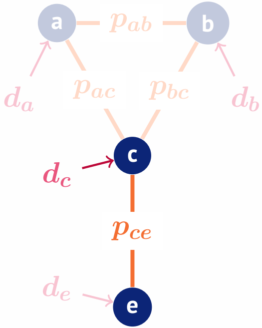

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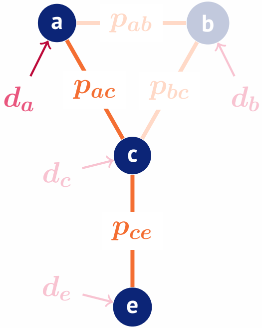

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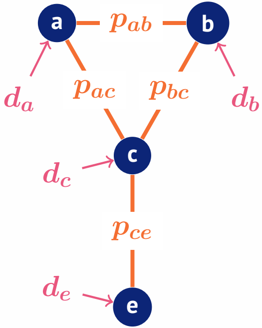

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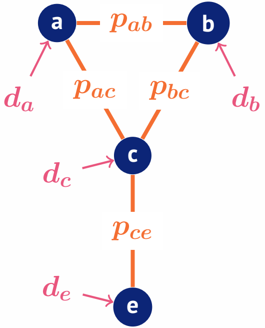

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$$\arg\max_{\sigma} \sum_{i \in \{a,b,c,e\}} P(\phi_i \mid \sigma)$$

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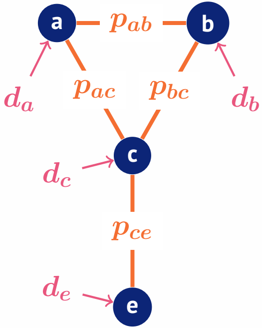

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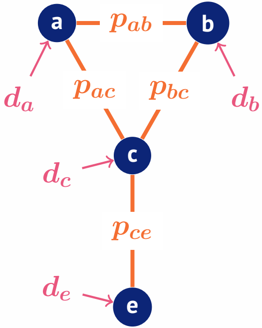

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repeatedly solve:

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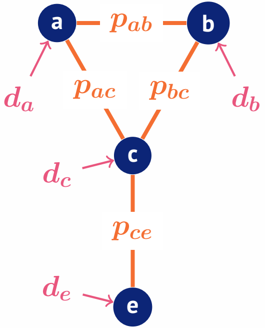

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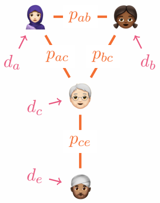
Existing (generic) method



A.L.D. Latour, B. Babaki, A. Dries, A. Kimmig, G. Van den Broeck, S. Nijssen. *Combining Stochastic Constraint Optimization and Probabilistic Programming: From Knowledge Compilation to Constraint Solving.* CP, 2017

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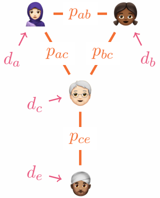

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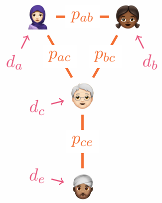
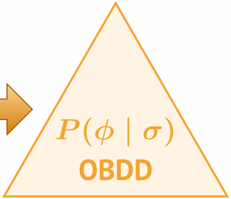


$$\begin{aligned}\phi_e = & d_e \vee (d_c \wedge t_{ce}) \vee \\ & (d_a \wedge t_{ac} \wedge t_{ce}) \vee \\ & (d_b \wedge t_{bc} \wedge t_{ce}) \vee \\ & (d_a \wedge t_{ab} \wedge t_{bc} \wedge t_{ce}) \vee \\ & (d_b \wedge t_{ab} \wedge t_{ac} \wedge t_{ce})\end{aligned}$$



A.L.D. Latour, B. Babaki, A. Dries, A. Kimmig, G. Van den Broeck, S. Nijssen.
Combining Stochastic Constraint Optimization and Probabilistic Programming: From Knowledge Compilation to Constraint Solving. CP, 2017

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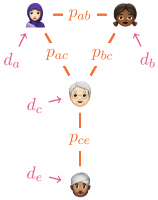

$$
\begin{aligned}
\phi_e = & \, d_e \vee (d_c \wedge t_{ce}) \vee \\
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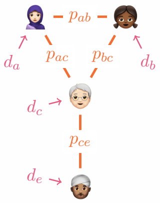

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\phi_e = d_e \lor (d_c \land t_{ce}) \lor \\
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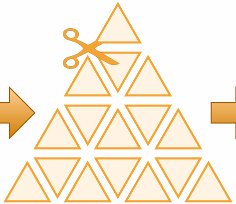

CP
or
MIP

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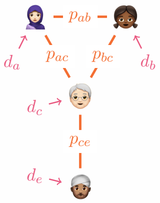

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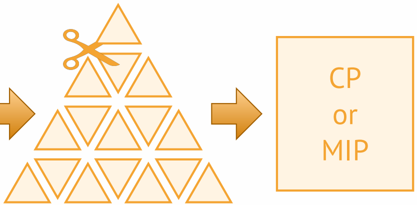
$$

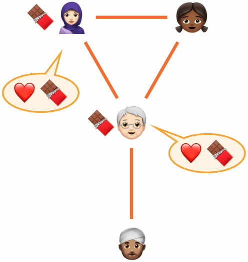


Decomposition method.

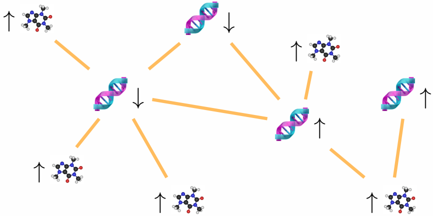
A.L.D. Latour, B. Babaki, A. Dries, A. Kimmig, G. Van den Broeck, S. Nijssen. *Combining Stochastic Constraint Optimization and Probabilistic Programming: From Knowledge Compilation to Constraint Solving.* CP, 2017

Observation 1: existing method does **not guarantee** Generalized Arc Consistency (**GAC**) $\longrightarrow$ **inefficient**;

Viral Marketing

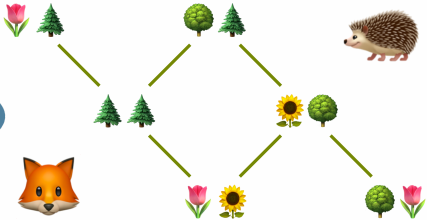


Signalling Regulatory Pathways

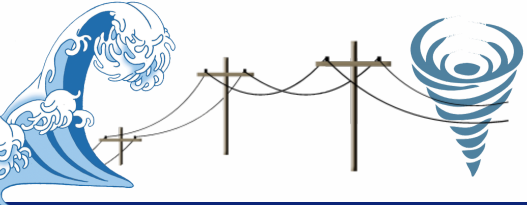


monotonic distributions

Landscape Connectivity



Powergrid Reliability



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Recall: optimization is repeated **constraint solving**:

$$\text{solve } \sum_{\phi \in \mathbf{\Phi}} P(\phi \mid \sigma) > \theta \text{ for increasing } \theta;$$

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Recall: optimization is repeated **constraint solving**:

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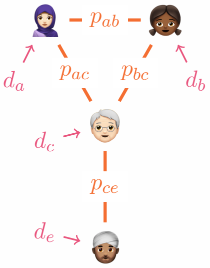
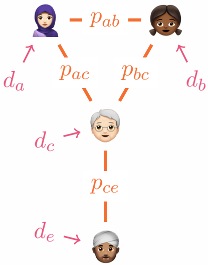
GOAL: create constraint propagation algorithm for Stochastic Constraints on Monotonic Distributions (SCMDs), which guarantees GAC.


p_{ab}
p_{ac}
p_{bc}
d_a
d_b
d_c
p_{ce}
d_e



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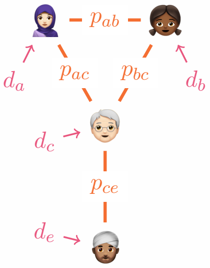
$$P\left(\phi_e \mid \sigma = \{d_a, d_c\}\right) \geq \theta$$



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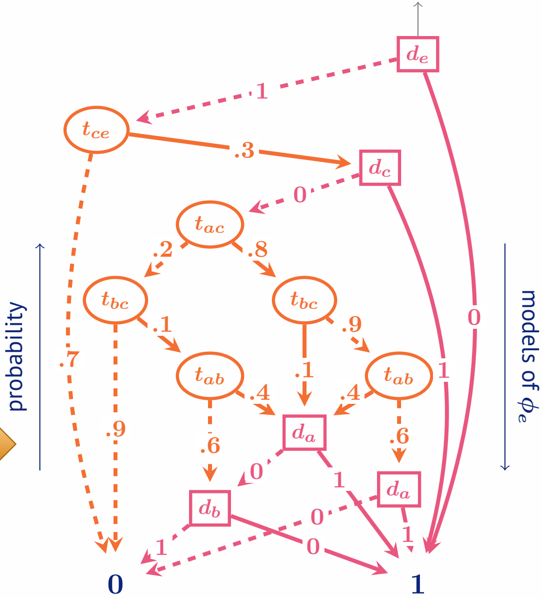
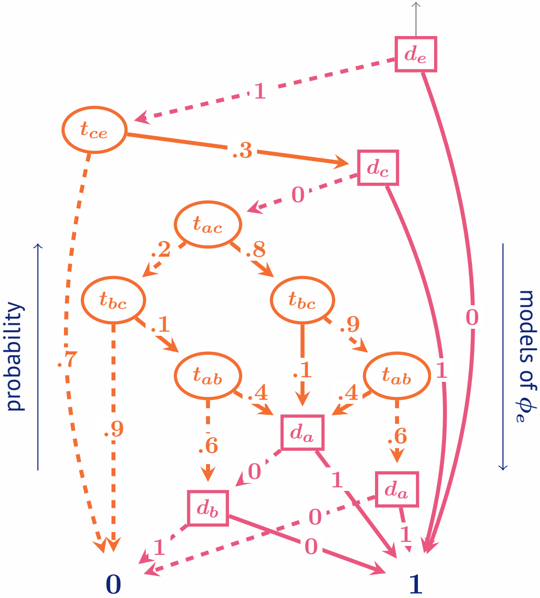


Ordered Binary Decision Diagram (**OBDD**) encodes **probability distribution**

$$P(\phi_e \mid \sigma = \{d_a, d_c\}) \geq \theta$$



Ordered Binary Decision Diagram (**OBDD**) encodes **probability distribution** (*not* the solutions to the constraint).

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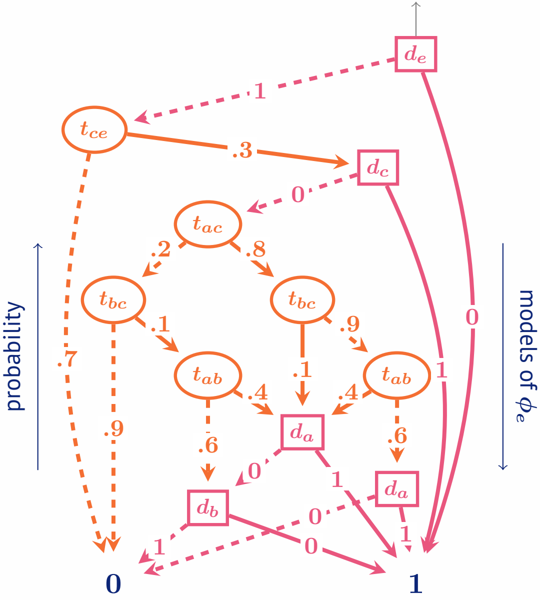

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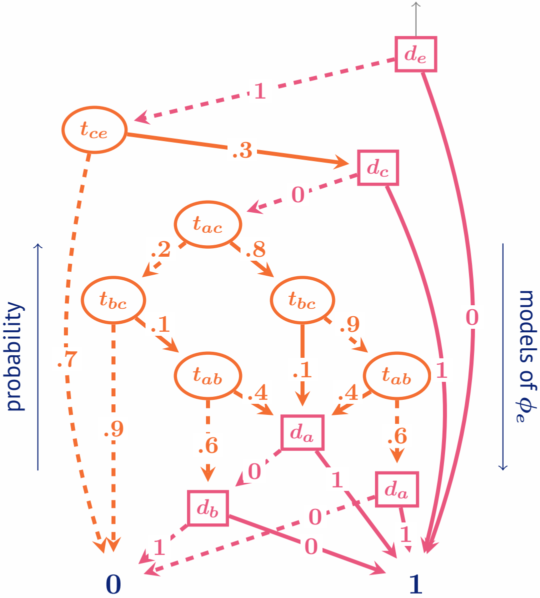

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Use OBDD to evaluate **strategy σ**. **Complexity** of one sweep: $O(m)$, with $m = |\text{OBDD}|$.

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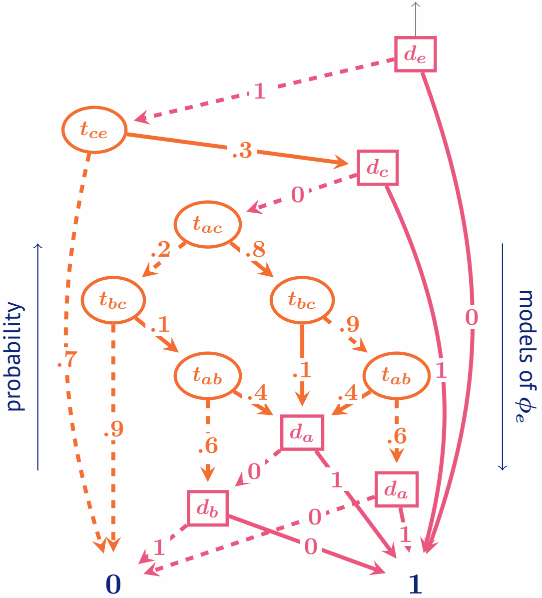

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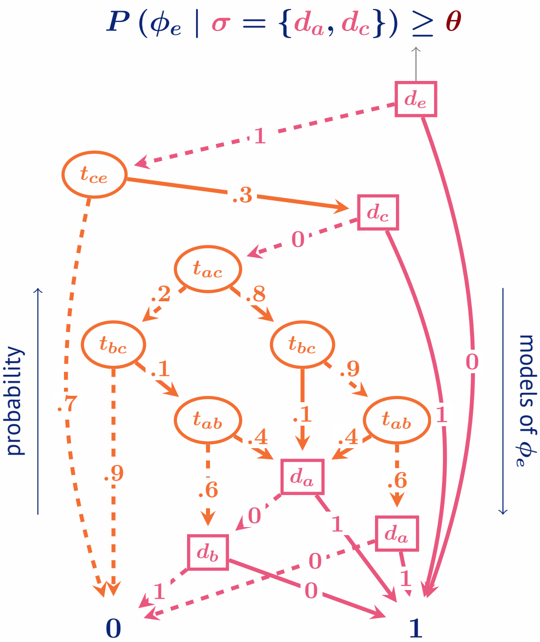
Naïve method has complexity $O(m \cdot n)$, where n is the number of unbound variables



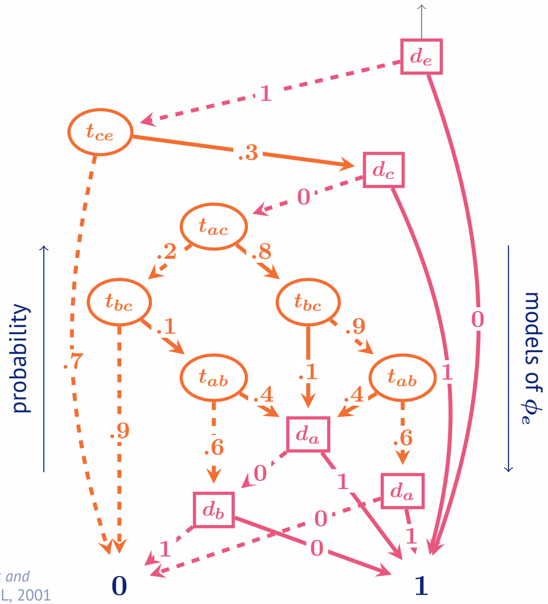
Ordered Binary Decision Diagram (**OBDD**) encodes **probability distribution** (*not* the solutions to the constraint).

solve $\sum_{\phi \in \Phi} P(\phi \mid \sigma) \geq \theta$

Use OBDD to evaluate **strategy σ**. **Complexity** of one sweep: $O(m)$, with $m = |\text{OBDD}|$.

Smart, incremental (full sweep) method has complexity $O(m+n)$, using **derivatives**.

$P(\phi_e \mid \sigma = \{d_a, d_c\}) \geq \theta$



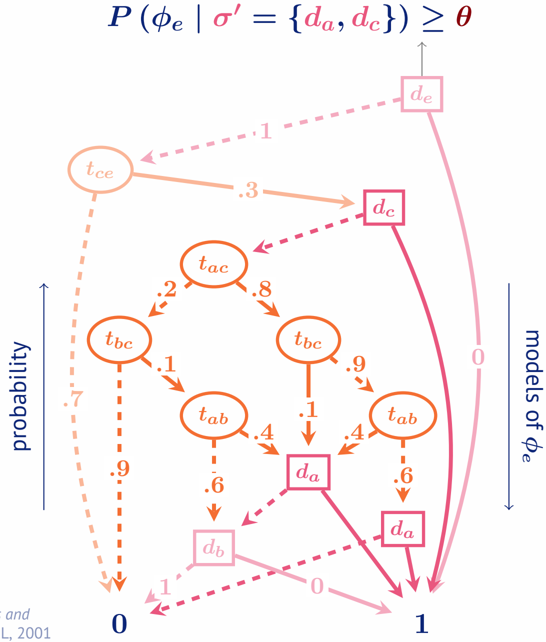
Adnan Darwiche. *On the tractable counting of theory models and its application to belief revision and truth maintenance.* JANCL, 2001

Ordered Binary Decision Diagram (**OBDD**) encodes **probability distribution** (*not* the solutions to the constraint).

$$\text{solve} \sum_{\phi \in \Phi} P(\phi \mid \sigma) \geq \theta$$

Use OBDD to evaluate **strategy σ**. **Complexity** of one sweep: $O(m)$, with $m = |\text{OBDD}|$.

Smart, incremental (partial sweep) method has complexity $O(m + n)$, using **derivatives**.



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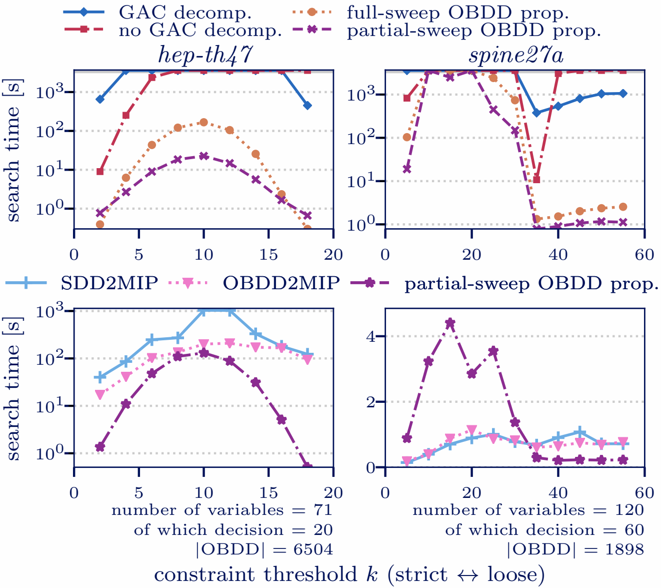
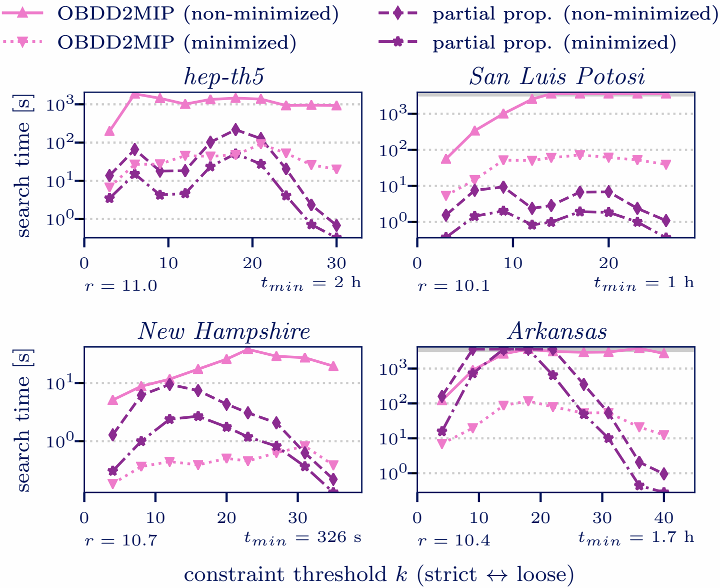
SCMD propagator vs existing methods



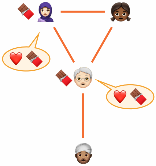
Scalability of SCMD propagator vs MIP

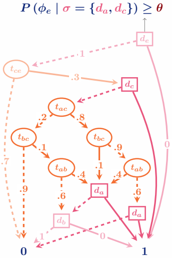


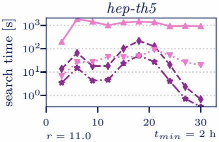
Main contribution

A new global constraint propagator for Stochastic Constraints on Monotonic Distributions (SCMDs) which:

- guarantees GAC;
- has linear complexity;
- outperforms existing CP-based methods and complements MIP-based methods;
- scales better with OBDD size than existing MIP-based methods.







contact: `a.l.d.latour@liacs.leidenuniv.nl`

code & more **results**: `github.com/latower/SCMD`

new work: D. Fokkinga, A.L.D. Latour, M. Anastacio, S. Nijssen, H. Hoos. *Programming a Stochastic Constraint Optimisation Algorithm, by Optimisation*. IJCAI Data Science meets Optimization workshop, 2019. `ada.liacs.nl/papers/FokEtAl19.pdf`

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Theme by Joost Schalken. Updated by Pepijn van Heiningen & Anna Louise Latour.

Acknowledgements

We thank Hélène Verhaeghe for her input and suggestions. This work was supported by the Netherlands Organisation for Scientific Research (NWO). Behrouz Babaki is supported by a postdoctoral scholarship from IVADO through the Canada First Research Excellence Fund (CFREF) grant.