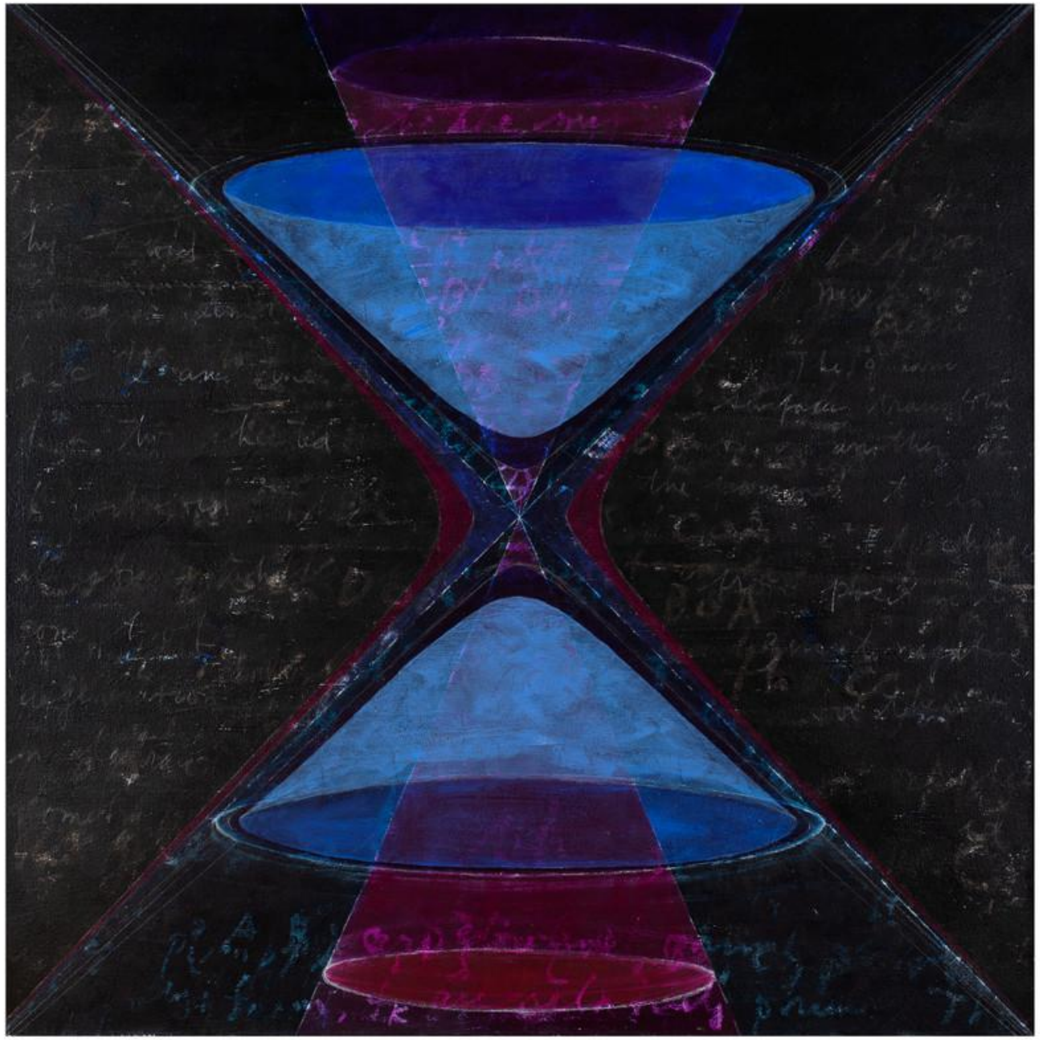
# Stacking and balancing casual causality



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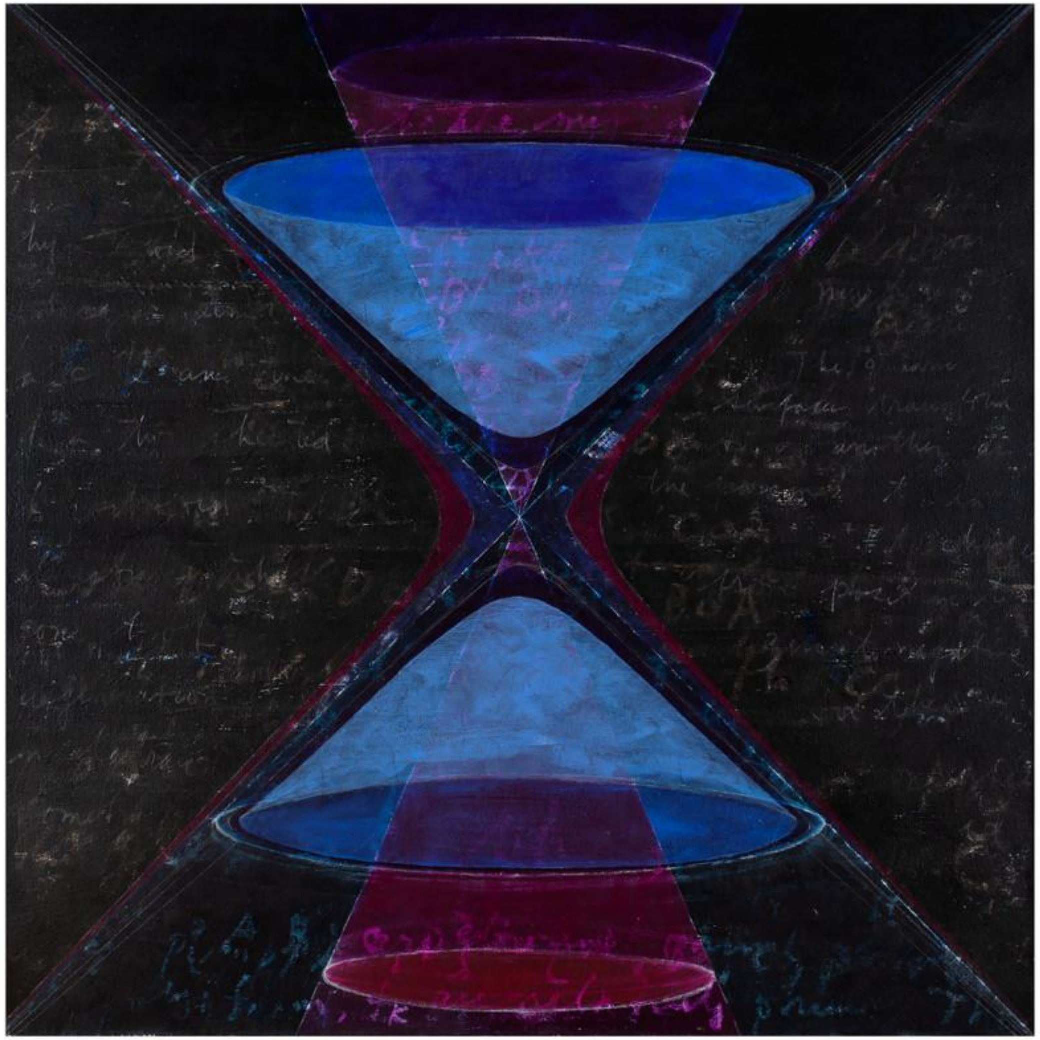
# Stacking and balancing casual causality



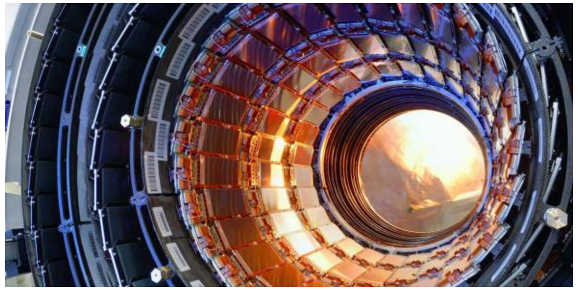
based on 2112.05031 & 2309.04534 in collaboration with C. de Rham, A. Margalit, A. J. Tolley

# Motivation: EFTs of Gravity

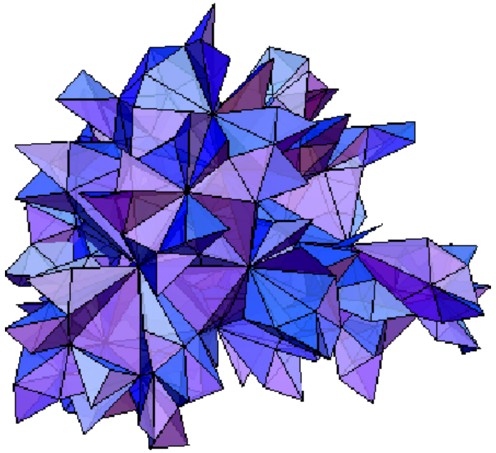
# Effective field theory of gravity

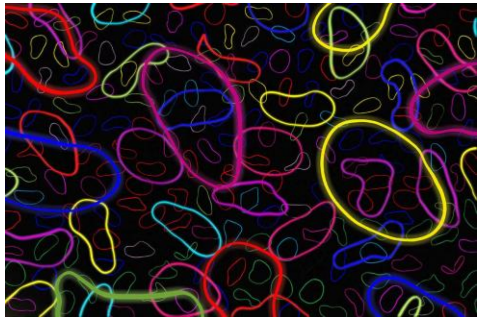
**Warning**: colour-coding!

$$\mathcal{L}_{\rm IR} = \frac{M_{\rm Pl}^{D-2}}{2} R + \mathcal{L}_{\rm light}\left[g, \phi_-\right] + \dots$$



$\Lambda$





Energy/Size$^{-1}$

$H_0$ $\quad G_F \quad \alpha' \quad M_{\rm Pl}$

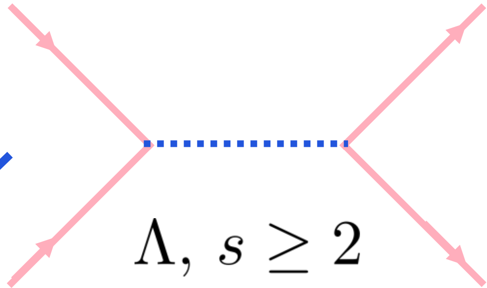
$$\mathcal{L}_{\rm UV} = \frac{M_{\rm Pl}^{D-2}}{2} R + \mathcal{L}_{\rm light}\left[g, \phi_-\right] + \mathcal{L}_{\rm heavy}\left[g, \phi_+\right]$$

?

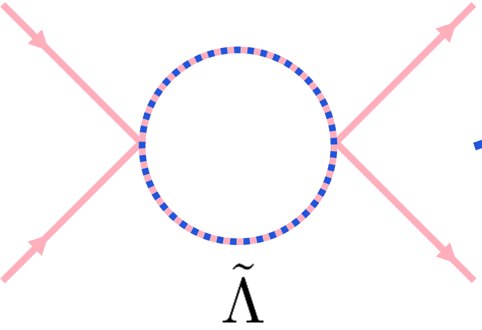
The **UV completion** of GR is unknown (please let me know if you do!), but we can write down a **generic effective action**.

# Einstein-Hilbert +

Full **effective action** (redundantly parameterised):

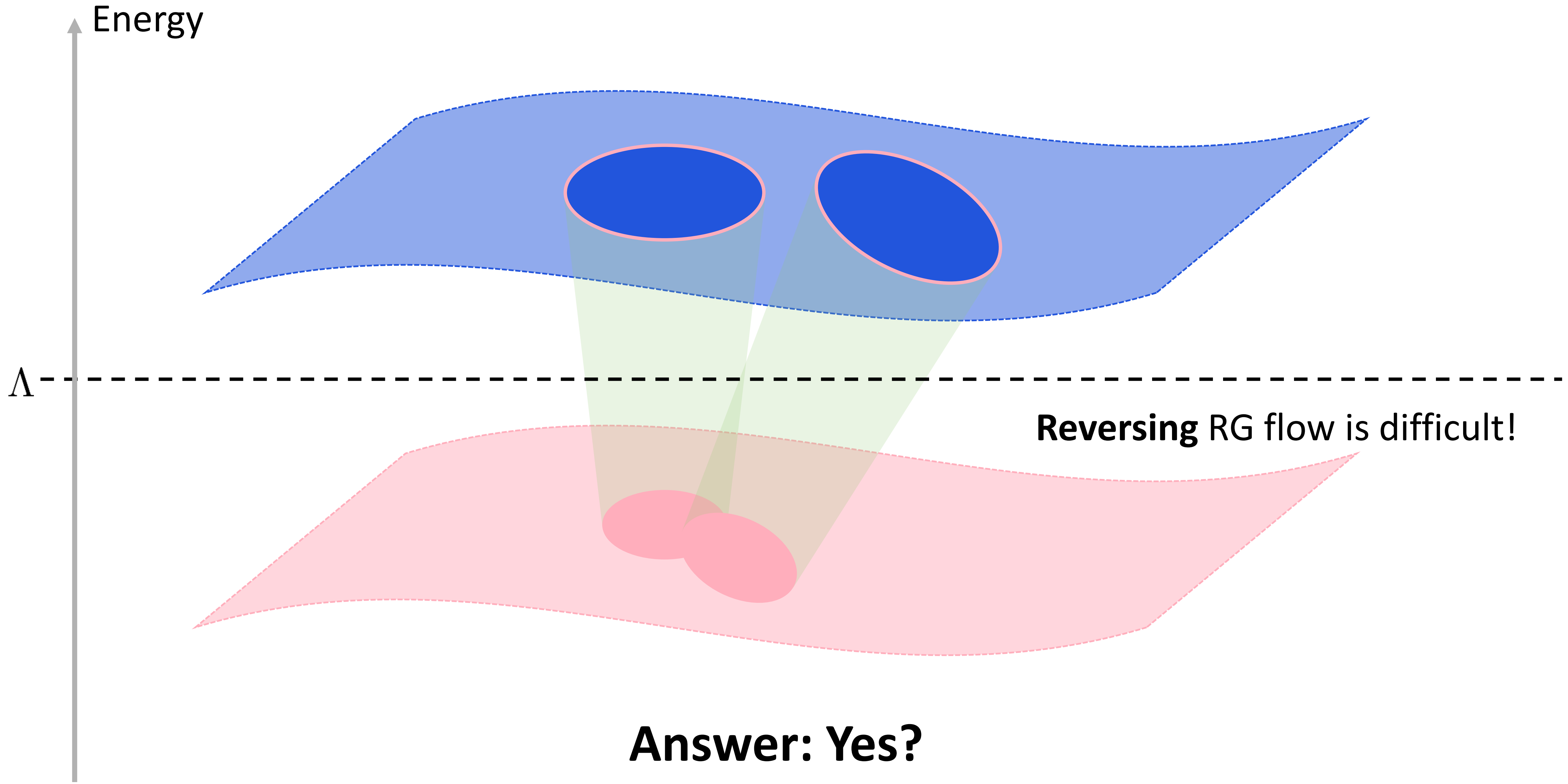



$$S_{\mathrm{EFT}} = \int d^D x \sqrt{-g} \Bigg[ M_{\mathrm{Pl}}^{D-2} \left( \frac{1}{2} R + \Lambda^2 \sum_{m \geq 0, n \geq 2} c_{mn} \left( \frac{\nabla}{\Lambda} \right)^m \left( \frac{\mathrm{Riemann}}{\Lambda^2} \right)^n \right) + \tilde{\Lambda}^D \sum_{m \geq 0, n \geq 2} d_{mn} \left( \frac{\nabla}{\tilde{\Lambda}} \right)^m \left( \frac{\mathrm{Riemann}}{\tilde{\Lambda}^2} \right)^n \Bigg]$$



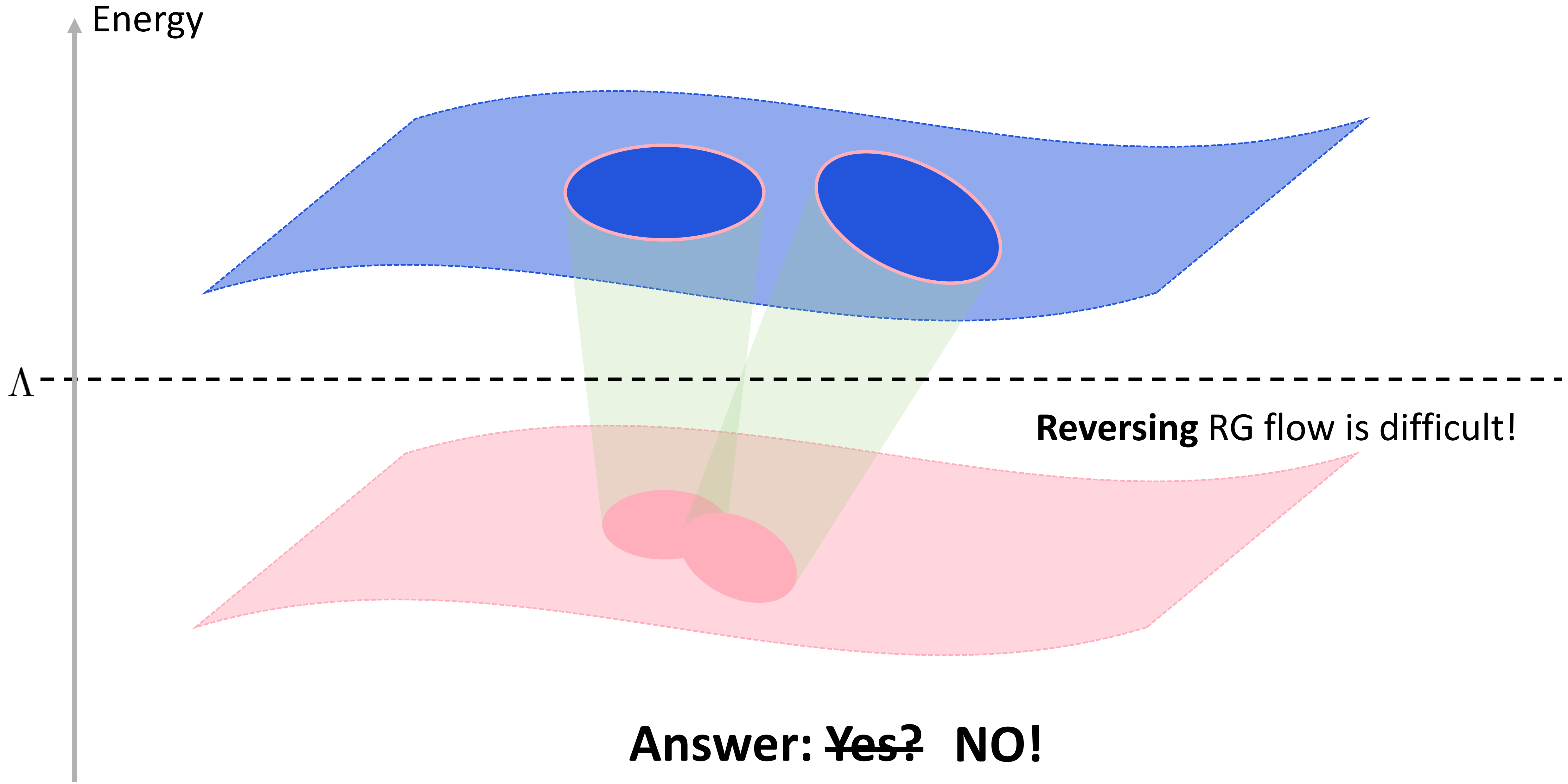

**Question: Are all these terms physical?**

# RG flow




**Answer: Yes?**

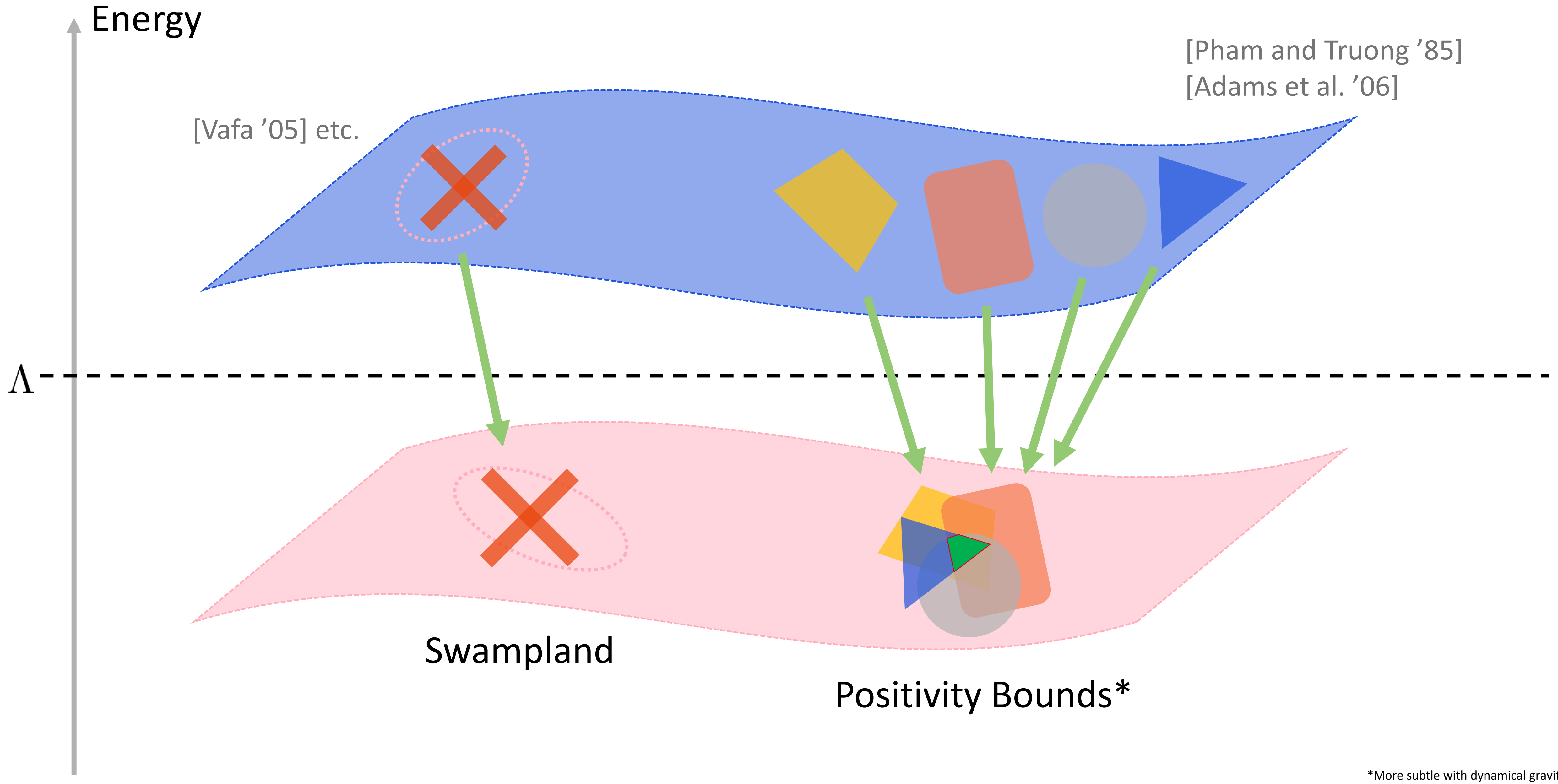
# RG flow




**Reversing** RG flow is difficult!

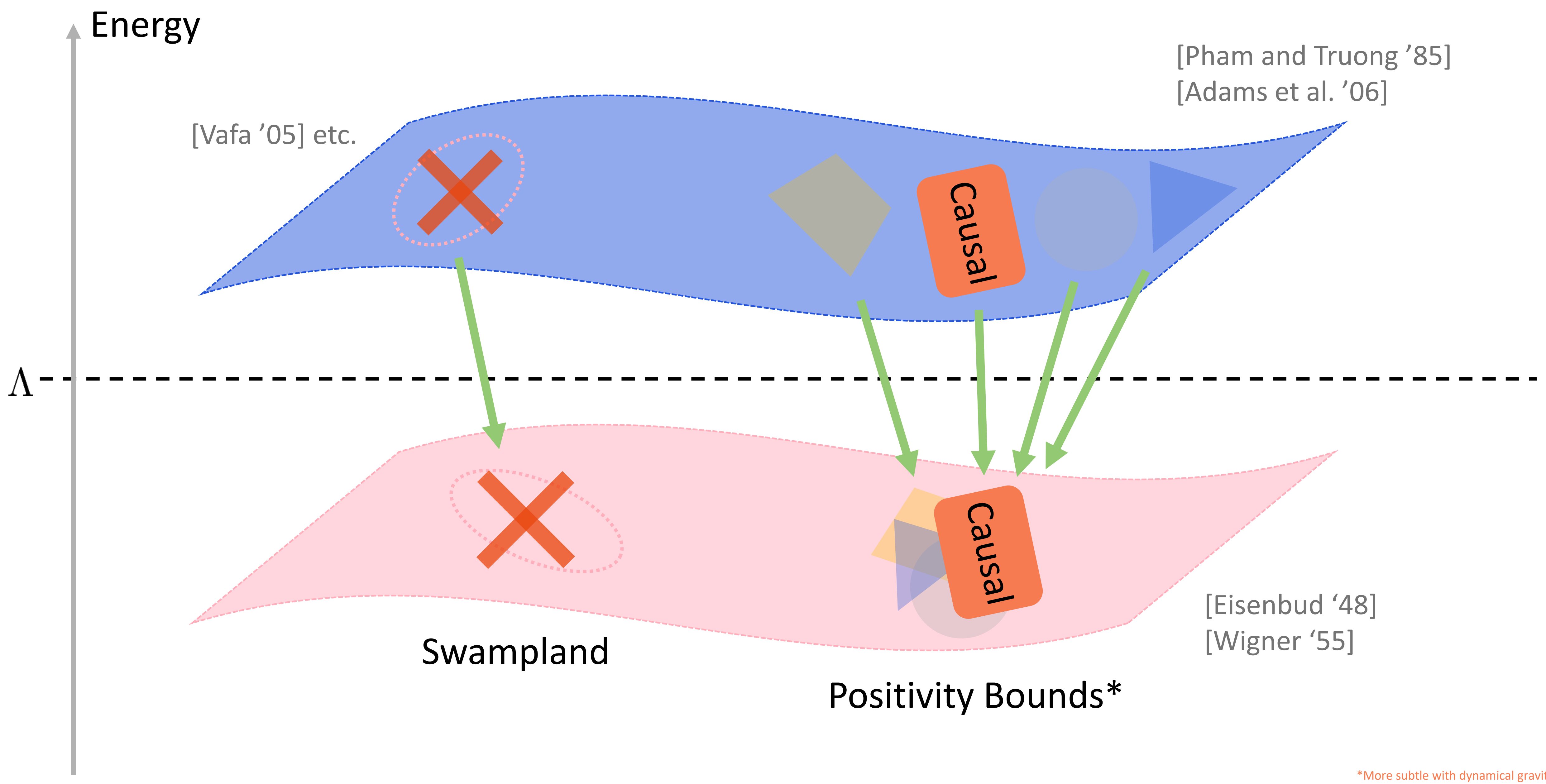
**Answer: ~~Yes?~~ NO!**

# UV imprints on IR




*More subtle with dynamical gravity!

# Causality




*More subtle with dynamical gravity!

# Example: Consistency and Causality

Illustrative example on flat space: Goldstone

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + g(\partial\phi)^4 + \ldots$$

In known UV completions, always find $g > 0$. Coincidence…?

…No! Propagation speed of perturbations about backgrounds $\bar{\phi} = c_\alpha x^\alpha$

$$v^2 = 1 - g\frac{8(c_\alpha p^\alpha)^2/|\mathbf{p}|^2}{1 - 4gc_\alpha c^\alpha}$$

So $g > 0$ directly linked to **subluminal** propagation speed of perturbations! [Adams et al. '06]

→ Consistent with **positivity bounds**. Caveat: More subtle with **dynamical gravity** – technical and conceptual challenges !

[Cheung and Remmen '17]
[Alberte, de Rham, Jaitly, and Tolley '20]
[Tokuda, Aoki, and Hirano '20]
etc.

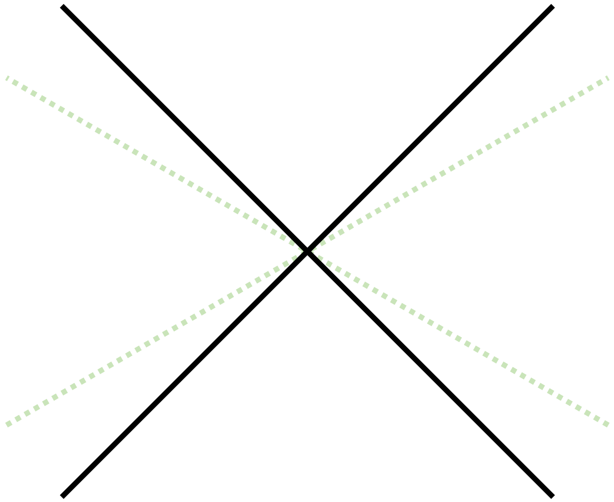
# Causality

**Goal: Use causality to identify consistent gravitational EFTs**

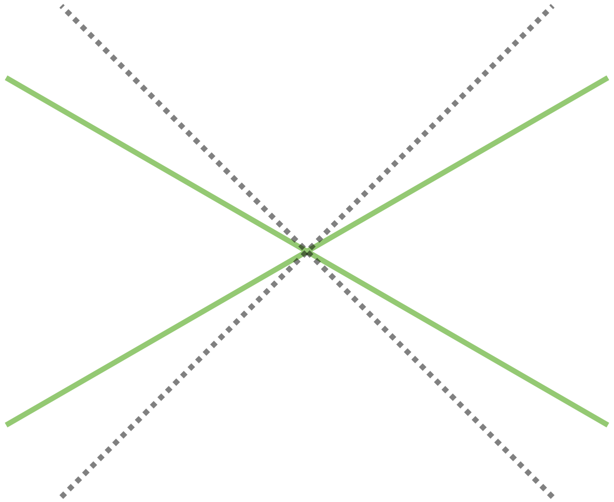
# Causality and Curved Spacetime

# Causality and Time Delays

In gravitational EFTs, **field redefinitions** can change light cone structure

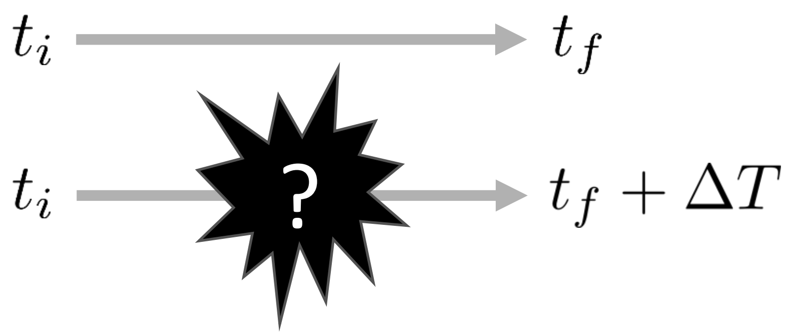


$$g_{\mu\nu} \to g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi$$



so propagation speeds are not invariant: (Sub-)luminal propagation **not meaningful** criterion.

→ Rephrase causality in terms of **time delay** $\Delta T$: Assume spacetime is **asymptotically flat** and has causal **Killing vector** $k = \partial/\partial t$ associated with a conserved energy $E = -k \cdot u$



# Eisenbud-Wigner Time Delay

Consider generic incoming wave packet and outgoing wave packet that differs by only by a **time delay**

$$|\mathrm{in}, g\rangle = \int_0^\infty \frac{dE}{2\pi} g(E) \hat{a}_E^{\mathrm{in}\dagger} |\mathrm{vac}\rangle \,, \quad |\mathrm{out}, g\rangle = e^{i\hat{P}_0 \Delta T} |\mathrm{in}, g\rangle$$

Given that

$$\langle \mathrm{vac}| \hat{a}_{E'}^{\mathrm{in}} \hat{S} \hat{a}_E^{\mathrm{in}\dagger} |\mathrm{vac}\rangle = 2\pi\delta(E - E')e^{2i\delta(E)}$$

then

$$\langle g, \mathrm{out}| \hat{S} |g, \mathrm{in}\rangle = \int_0^\infty \frac{dE}{2\pi} |g(E)|^2 e^{2i\delta(E) - iE\Delta T}$$

Take the profile $g(E)$ to be peaked around $\bar{E}$ with some width $\Delta E \ll \bar{E}$, so the **stationary phase approximation** gives

$$\Delta T = \left. \frac{2\partial\delta(E)}{\partial E} \right|_{E=\bar{E}} + \mathcal{O}(\Delta E^{-1})$$

→ **Eisenbud-Wigner** time delay, with intrinsic uncertainty!

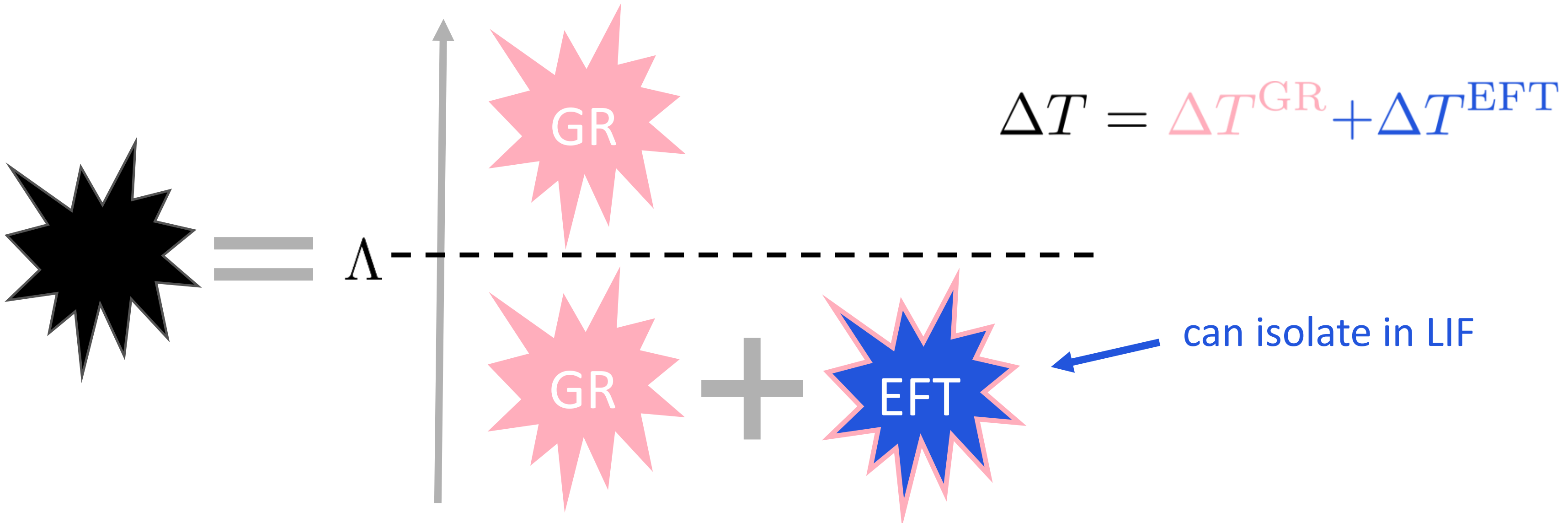
# Is causality just $\Delta T > 0$?

**Subtlety 1**: Uncertainty principle puts limit on “observations” via **resolvability**

→ Waves with energy $E$ cannot measure time delays $\Delta T$ with

$$|\Delta T| \lesssim E^{-1}$$

**Subtlety 2**: Need to distinguish effect of **background** geometry from EFT correction




Background effect due to GR should set **reference**

→ To determine **causality of EFT**, study EFT contribution.

# Infrared Causality

Putting this together:

**Infrared Causality**
**<u>Violation</u>**

$\Updownarrow$

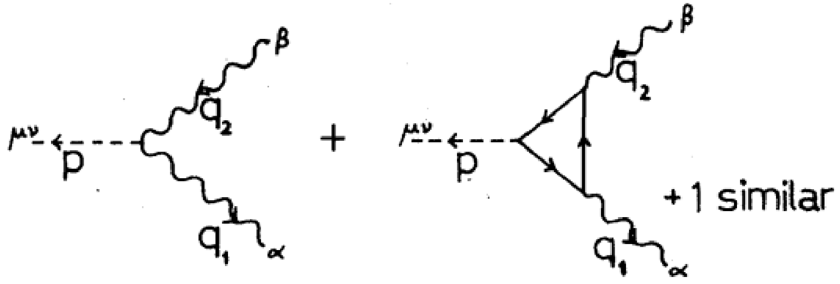
$$\boxed{\begin{gathered}\Delta T^{\mathrm{EFT}} < 0 \\ \textbf{AND} \\ \left|\Delta T^{\mathrm{EFT}}\right| \gtrsim E^{-1}\end{gathered}} \Bigg\} \quad \Delta T^{\mathrm{EFT}} \lesssim -E^{-1}$$

Let's try this!

# Example: QED on Curved Spacetime

QED on fixed curved background

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi \right]$$




Integrating out the electron [Drummond and Hathrell '80]

$$W = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{320\pi} \frac{\alpha}{m_e^2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \mathcal{O}\left( \frac{\alpha}{m_e^{2n}} \right) \right]$$

E.g. on Schwarzschild (with Schwarzschild radius $r_g$): **Gravitational birefringence**

$$c_s^2 - 1 \sim \pm \frac{1}{m_e^2} \frac{r_g}{r^3} \quad \longrightarrow \quad \Delta T^{\mathrm{EFT}} \sim \pm \frac{2 r_g}{b^2 m_e^2}$$

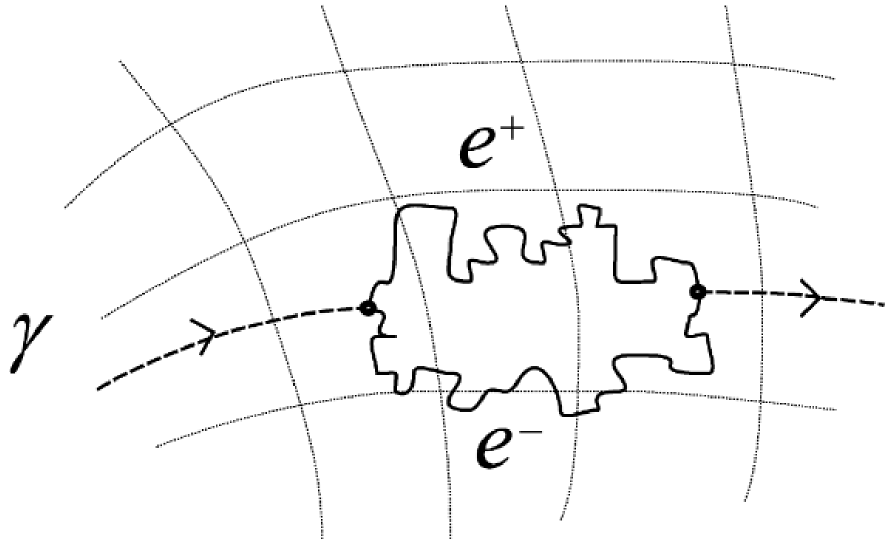
Signals causality violation even though UV completion is causal!

→ Causality at low energies violated by integrating out electron...?

# Example: QED on Curved Spacetime

**Perspective 1:** Causality restored as new degrees of freedom are restored. [Hollowood and Shore '07]

→ That's new physics!




Whether we like it or not *we are all low-energy physicists*.

**Lesson 2:** IR causality can be diagnosed purely **within EFT!** Within regime of validity

[de Rham and Tolley '20]

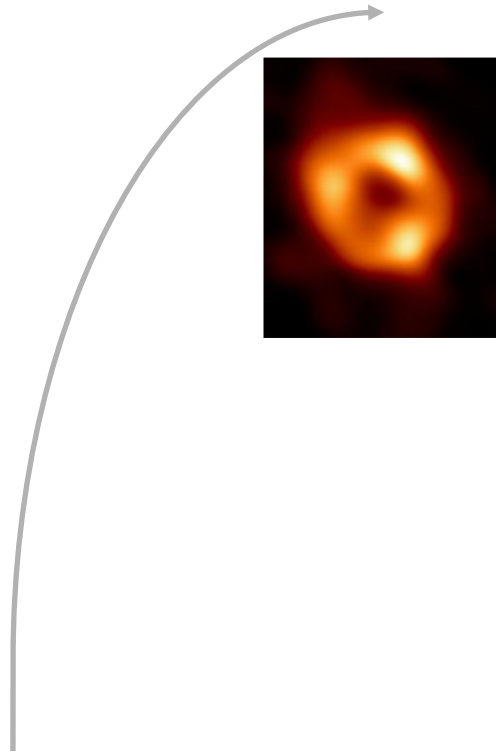
$$\left|\Delta T^{\mathrm{EFT}}\right| \ll E^{-1}$$

→ unresolvable!

# EFTs on pp-waves

# Testing Ground: Black Holes

Like to smash things into each other to study them: Scatter gravitons off **black hole**!



**Technically challenging**: Gauge invariant basis variables remain same, but master variables receive EFT corrections [Kodama & Ishibashi ’03]

→ IR causality consistent with **gravitational positivity bounds** [CYRC, de Rham, Margalit, and Tolley ‘21]

# Aichelburg-Sexl Boost: Shockwaves

Instead, take Aichelburg-Sexl boost to **shockwave** spacetime


Spoiler: Same conclusion for single shockwave and black hole, but **more interesting configurations** with shockwaves! [Camanho, Edelstein, Maldacena, and Zhiboedov '14]

# Stacking and Balancing Causality

Suppose that $\Delta T^{\mathrm{EFT}} < 0$ is possible. Then

$$|\Delta T^{\mathrm{EFT}}| \sim |c_{\mathrm{EFT}}| T \leq T_0$$

A (relatively) larger time advance would lead to

$$|\Delta T^{\mathrm{EFT}}| \sim |c_{\mathrm{EFT}}| \alpha T \leq \tilde{T}$$

so

$$|c_{\mathrm{EFT}}| \lesssim \frac{1}{\alpha} \frac{T_0}{T}$$

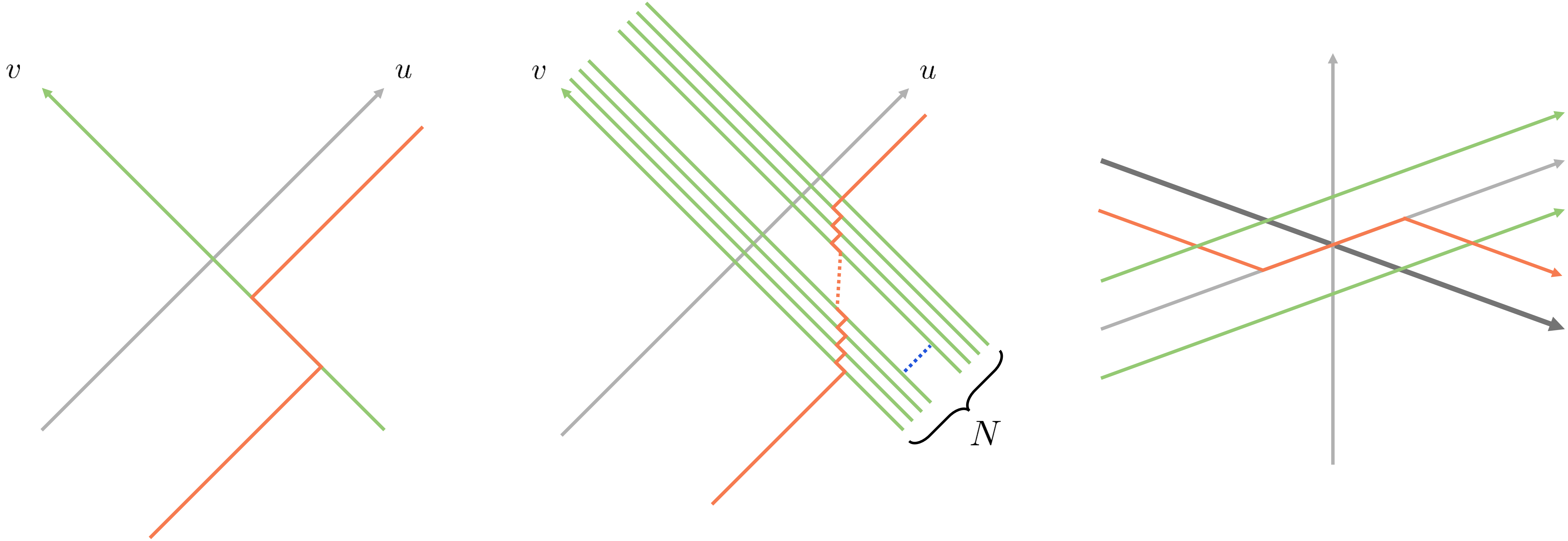
[CEMZ '14]
[Question at "Gravity at YITP 2022"]

In fact: As $\alpha \to \infty$ , we will find

$$c_{\mathrm{EFT}} = 0$$

→ Essentially avoids the **resolvability criterion**!

# Preview: Stacking and Balancing Causality

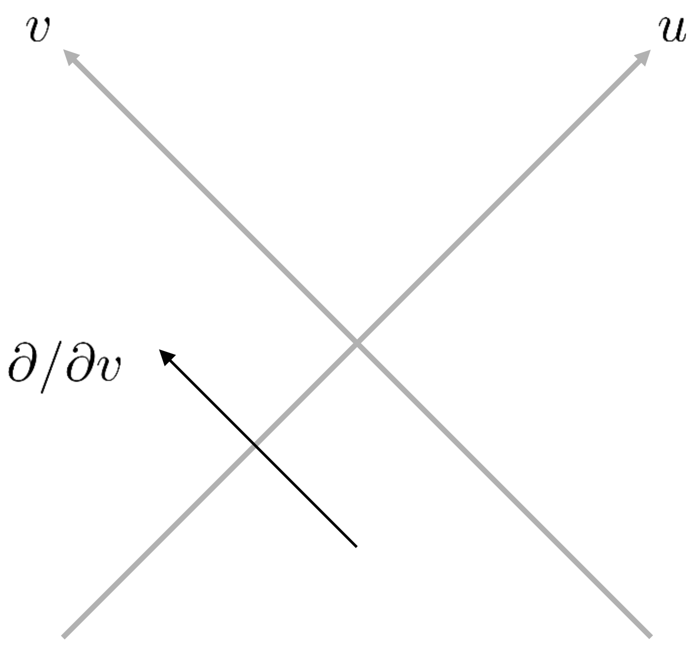



**(More Precise) Goal: Constrain EFT operators using IR causality**

# Review: Pp-waves

In Brinkmann coordinates $(u, v, x^i)$

$$ds^2 = 2du\,dv + F(u, x^i)du^2 + \delta_{ij}dx^i dx^j$$




Only non-vanishing component of Riemann tensor

$$R_{uiuj} = -\frac{1}{2}\partial_i\partial_j F$$

Vacuum Einstein's equations impose

$$R_{\mu\nu} = 0 \longrightarrow \partial_i\partial^i F = 0$$

→ Harmonic $F(u, x^i)$ !

Within this class of solutions: **Rank-0 and -2 contractions** of Riemann tensors and covariant derivatives e.g.

$$R_{\mu\nu}, \quad (R^m)^\lambda{}_{\mu\lambda\nu}, \quad \nabla_\alpha\nabla_\beta(R^n)^\alpha{}_\mu{}^\beta{}_\nu, \quad \ldots$$

vanish.

# Surfin' on pp-waves

Pp-waves satisfying vacuum Einstein equation are background solutions **at all orders** in EFT

$$\text{Background eq.} \sim \left.\frac{\delta S_{EFT}}{\delta g^{\mu\nu}}\right|_{\text{pp-wave}} = 0$$

However, equations for perturbations $h_{\mu\nu}$ on pp-wave background

$$\text{Perturbation eq.} \sim \left.\frac{\delta^2 S_{\text{EFT}}}{\delta g^{\mu\nu}\delta g^{\rho\sigma}}\right|_{\text{pp-wave}} h^{\rho\sigma} + \text{perm.} \neq 0$$

not trivially satisfied!

→ **EFT corrections non-zero**!

# Regime of Validity

**EFT breaks down** when probed…

1) at too small length scales or high energies → **background** (trivial for pp-waves)
2) by particles with too high energies → **perturbations** (non-trivial for pp-waves!)

Find parameter controlling asymptotic expansion using **Lorentz scalars** towards infinity (see QED). Crucially:

$$R_{\mu\nu\alpha\beta}\delta R^{\mu\nu\alpha\beta} \neq 0$$

Fourier transform perturbations $\nabla h \to ikh$, then constraints take schematic form

$$\text{“}\lim_{a,b,c\to\infty}\left(\frac{\nabla}{\Lambda}\right)^a\left(\frac{\text{Riemann}}{\Lambda^2}\right)^b\left(\frac{k}{\Lambda}\right)^{2c+b} \ll 1\text{”}$$

→ **EFT constraints**:

conserved quantity
for $\partial/\partial v$

$$\partial_r \ll \Lambda, \quad k^\mu\partial_\mu \ll \Lambda^2, \quad \frac{\partial_r F}{r}k_v^2 \ll \Lambda^4$$

# "Shockwaves are not solutions in the EFT of gravity"

**Shockwaves** are pp-waves with

$$F(u,r) = \frac{4\Gamma\left(\frac{D-4}{2}\right)}{\pi^{(D-4)/2}}\delta(u)\frac{G|P_u|}{r^{D-4}}$$

→ Solutions to Einstein's equations with **ultra-relativistic** (delta function) source

$$T_{uu} = -P_u \delta(u)\delta^{(D-2)}(\mathbf{x})$$

(also obtained via **Aichelburg-Sexl boost** from Schwarzschild black hole).

However:

$$\frac{\partial_r F}{r}k_v^2 = -\frac{4(D-4)\Gamma\left(\frac{D-4}{2}\right)}{\pi^{(D-4)/2}}\delta(u)\frac{G|P_u|k_v^2}{r^{D-6}} \to \infty \not< \Lambda^4$$

so shockwaves are outside EFT regime of validity → need to **regulate** e.g. as Gaussian

$$\delta(u) \to \frac{1}{\sqrt{2\pi}L}e^{-u^2/2L^2}, \quad L \gg k_v/\Lambda^2$$

# Leading-order EFT: Gauss-Bonnet Gravity

Leading-order EFT in $D \geq 5$

$$S_{\rm EFT} = \int d^D x \sqrt{-g} M_{\rm Pl}^{D-2} \left( \frac{1}{2} R + \frac{c_{\rm GB}}{\Lambda^2} R_{\rm GB}^2 + \mathcal{O}\left(\Lambda^{-4}\right) \right)$$

$$R_{\rm GB}^2 = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

→ **Einstein-Gauss-Bonnet gravity!**

Equations for perturbations (in **light cone gauge** $h_{v\mu} = 0$ ):

$$\bar{g}^{\mu\nu} \partial_\mu \partial_\nu h_{ij} - 8 \frac{c_{\rm GB}}{\Lambda^2} \partial_v^2 X_{ij} = 0, \quad X_{ij} = (\partial_m \partial_{(i} F) h_{j)}{}^m - \frac{\bar{g}_{ij}}{D-2} (\partial_m \partial_n F) h^{mn}$$

Decompose $x^i \to (r, x^\alpha)$ and assume **spherical symmetry** to decouple modes

$$\bar{g}^{\mu\nu} \partial_\mu \partial_\nu \Phi_M + a_M \frac{c_{\rm GB}}{\Lambda^2} \frac{\partial_r F}{r} \partial_v^2 \Phi_M = 0, \quad a_M = (8(D-4), 4(D-4), -8, -8)$$

$$\Phi_M = \left( h_{rr}, h_{r\alpha}, h_{\alpha\beta}, g^{33} h_{33} - g^{\alpha\alpha} h_{\alpha\alpha} \right)$$

# Stacking Causality

# JWKB Approximation

Fourier transform of perturbation equations $\partial_v \to ik_v$ is a **Schrödinger-like equation**

$$i\frac{\partial \Phi_M}{\partial u} = -\frac{1}{2k_v}\nabla^2\Phi_M + V\Phi_M, \quad u \to \text{"time"}, \quad k_v \to \text{"mass"}$$

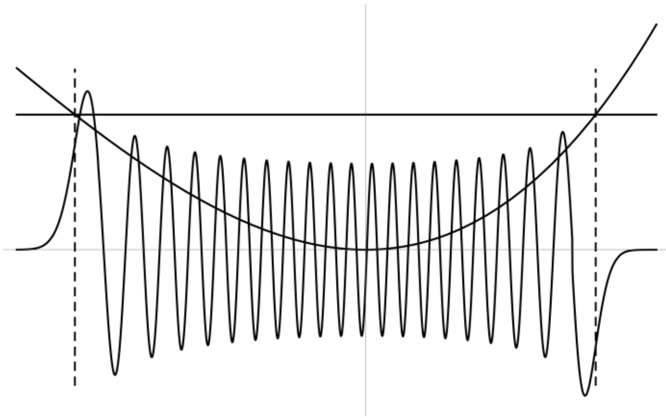
$$V(u,r) = -\frac{k_v}{2}F(u,r) + a_M k_v \frac{c_{\rm GB}}{\Lambda^2}\frac{\partial_r F(u,r)}{r}$$

Solve this using **JWKB Ansatz** and treat Laplacian perturbatively:

$$\Phi_M(u,r) = \Phi_0 \exp[i\delta_M(u,r)],$$

$$\delta_M(u,r) = \delta_M^{(0)}(u,r) + \delta_M^{(1)}(u,r) + \ldots$$

The approximation **valid** as long as $|\delta^{(0)}(u,r)| \gg |\delta^{(1)}(u,r)|$, i.e. until $u = u_{\rm max}$ defined by



$$\left|\int_0^{u_{\rm max}} du \boldsymbol{\nabla} V(u,r)\right| \sim V(u_{\rm max}, r)$$

→ Can't accumulate time delay indefinitely!

# Eikonal Time Delay

Leading-order JWKB phase shift reproduces the **eikonal** phase shift. **Cumulative time delay** for particle localised at impact parameter $r = b$,

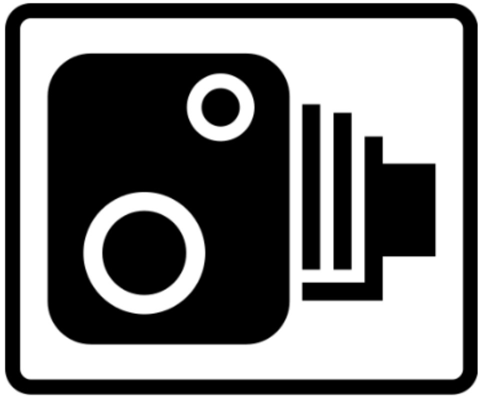
$$\Delta T(u) = 2\,\frac{\partial \delta_0(u,r)}{\partial k_v}\bigg|_{r=b} = \left( \int_0^u F(u,r)du' - 4a_M \frac{c_{\rm GB}}{\Lambda^2} \int_0^u \frac{\partial_r F(u,r)}{r} \right)\bigg|_{r=b}$$

Therefore:

$$\Delta T^{\rm EFT}(u) = -4a_M \frac{c_{\rm GB}}{\Lambda^2} \int_0^u \frac{\partial_r F}{r}\bigg|_{r=b}, \qquad a_M = \big(+8(D-4), +4(D-4), -8, -8\big)$$

→ No definite sign! Causality violation for any non-zero $c_{\rm GB}$ …?

Hmm let's see…

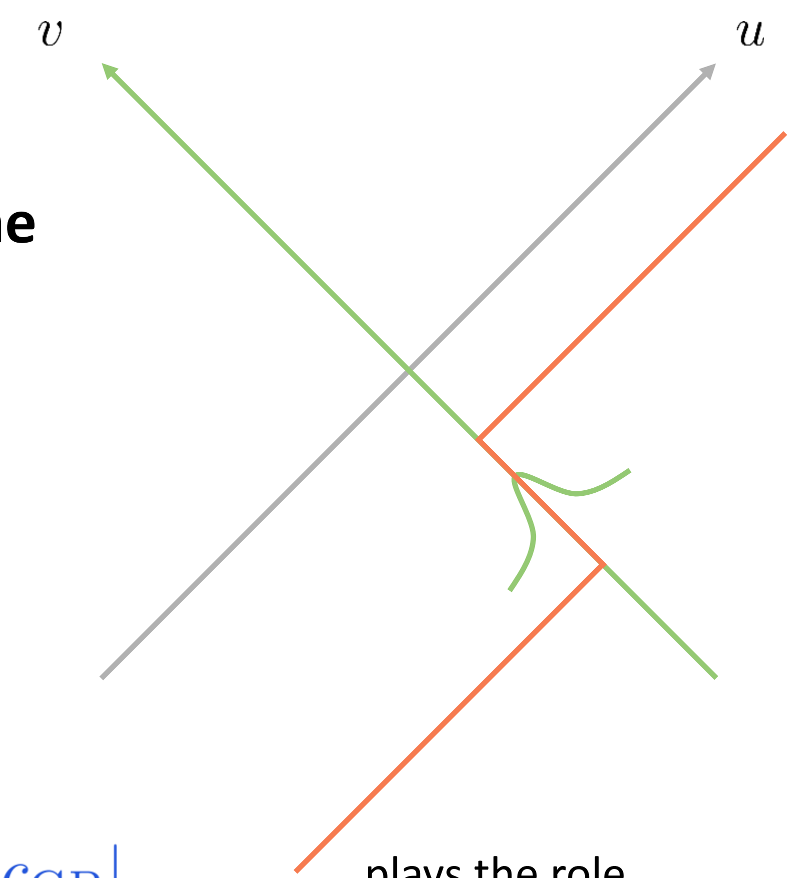



Am I going too fast?

# Localised Source

For sources with arbitrary profile $f = f(u)$ in time **localised** at $r = 0$ :

$$F(u,r) = \frac{f(u)}{r^{D-4}}$$




1) Eikonal approximation valid up to **maximum scattering time**

$$\int_0^{u_{\max}} du \frac{f(u)}{b^{D-2}} \sim \sqrt{\frac{f(u_{\max})}{b^{D-2}}}$$

2) **EFT** regime of validity

$$\frac{f(u)}{b^{D-2}} k_v^2 \ll \Lambda^4$$

so time delay:

$$|\Delta T^{\mathrm{EFT}}| \sim \frac{|c_{\mathrm{GB}}|}{\Lambda^2} \int_0^{u_{\max}} du \frac{f(u)}{b^{D-2}} \sim \frac{|c_{\mathrm{GB}}|}{\Lambda^2} \sqrt{\frac{f(u_{\max})}{b^{D-2}}} \ll \frac{|c_{\mathrm{GB}}|}{k_v}$$

plays the role of $E^{-1}$

→ Same as in spherical symmetry: **IR causality** consistent with $|c_{\mathrm{GB}}| \lesssim 1$!

# Special Case: $N$ Stacked Shockwaves

**Stack** $N$ regulated shockwaves with width $L$ and separated by $\Delta u$

$$f(u) = \frac{1}{\sqrt{2\pi}L} \frac{4\Gamma\left(\frac{D-4}{2}\right)}{\pi^{(D-4)/2}} G|P_u| \sum_{n=1}^{N} e^{-(u-n\Delta u)^2/2L^2}$$

[Camanho, Edelstein, Maldacena, and Zhiboedov '14]

When shocks sufficiently separated:

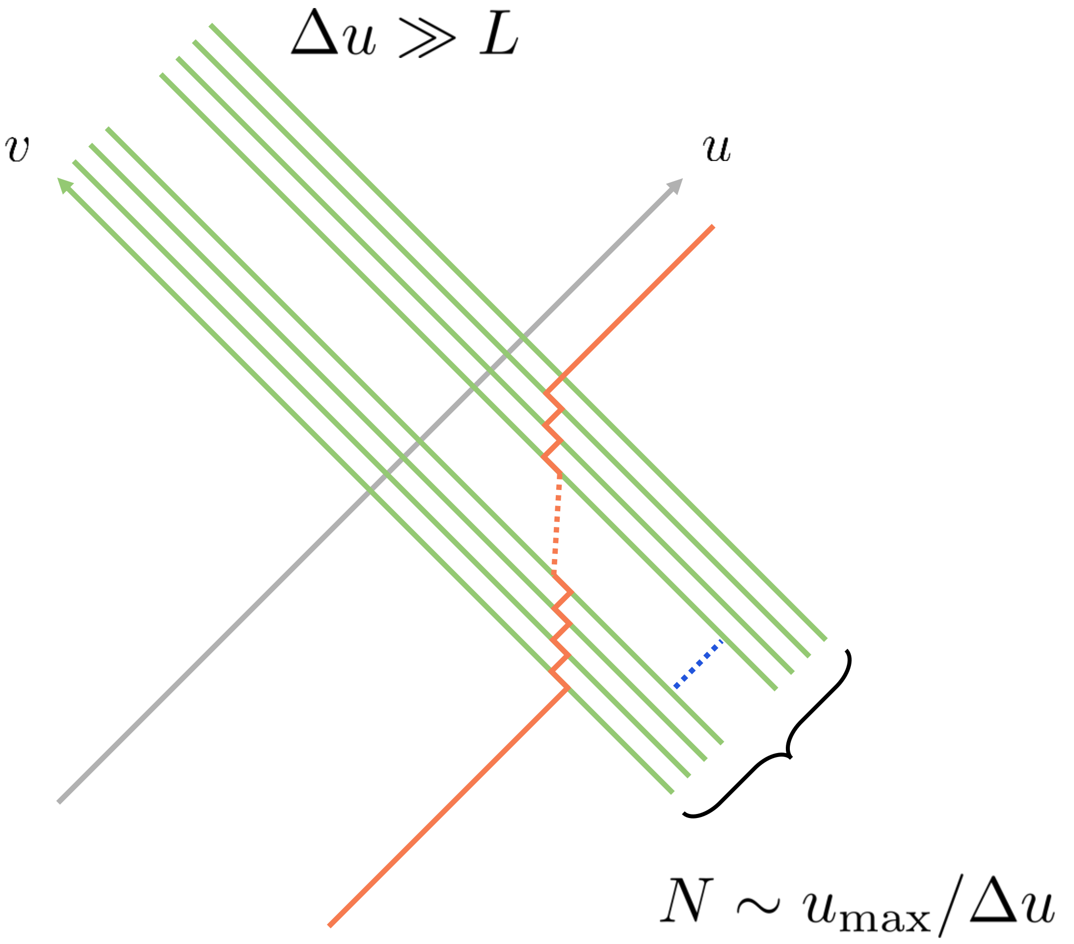
$$\left|\Delta T^{\mathrm{EFT}}_{(N)}\right| \sim N \left|\Delta T^{\mathrm{EFT}}_{(1)}\right|$$

To **maximise** causality violation: Want $N$ as large as possible!

However validity of JWKB sets $u_{\mathrm{max}}$ and validity of EFT bounds $\Delta u$ below

$$\Delta u \gg L \gg \Lambda^2/k_v$$

→ **Cannot** make $N$ arbitrarily large!



# Stacked Shockwaves: Classical Perspective

JWKB approximation at leading order

$$k_v \frac{d^2\mathbf{x}}{du^2} = -\boldsymbol{\nabla} V(u,\mathbf{x})$$

→ **Newton's equation**! Transverse displacement estimate:

$$\Delta r(u) \sim -\frac{1}{k_v}\int_0^u du' \int_0^{u'} du'' \partial_r V(u,r)\bigg|_{r=b} = -\int_0^u du' \int_0^{u'} du'' \partial_r F(u,r)\bigg|_{r=b}$$

Approximation only valid until this is small relative to impact parameter. This sets $u_{\max}$

$$\Delta r(u_{\max}) \sim b \quad \longrightarrow \quad \int_0^{u_{\max}} du \int_0^u du' \frac{f(u')}{b^{D-2}} \sim 1$$

and the EFT contribution to the time delay is not resolvable:

$$|\Delta T_{\mathrm{EFT}}(u_{\max})| \ll \frac{|c_{\mathrm{GB}}|}{k_v} \int_0^{u_{\max}} du \int_0^u du' \frac{f(u')}{b^{D-2}} \sim \frac{|c_{\mathrm{GB}}|}{k_v}$$

→ Validity of JWKB equivalent to **negligibility of scattering**

# Stacked Shockwaves: Quantum Perspective

Can separate interaction picture time-evolution operator for $N$ **isolated** scattering events,

$$\hat{U}(t_N, t_0) = \mathcal{T} \prod_{n=1}^{N} \hat{U}(t_n, t_{n-1})$$

For sufficiently **long time intervals**

$$\hat{S}_{\text{total}} \approx \mathcal{T} \prod_{n=1}^{N} \hat{S}_n \approx (\hat{S}_1)^N \to \Delta T_{\text{total}} = N \Delta T_1$$

→ Too quick!

Example: $N$ identical impulses $\hat{K}$

$$\hat{H}_{\text{int}}(t) = \sum_{n=1}^{N} \delta[t - (t_{n-1} + a_n)] \hat{K}, \quad 0 < a_n < t_n - t_{n-1}$$

S-matrix for individual scattering events **not identical** (for generic interaction)

$$\hat{S}_n = e^{i\hat{H}_0(t_{n-1}+a_n)} e^{-i\hat{K}} e^{-i\hat{H}_0(t_n+a)}$$

→ Effect of $\hat{H}_0$ is **diffusion**!

# Balancing Causality

# Scatter No More

**Scattering** in transverse direction crucial to see bound on time delay!

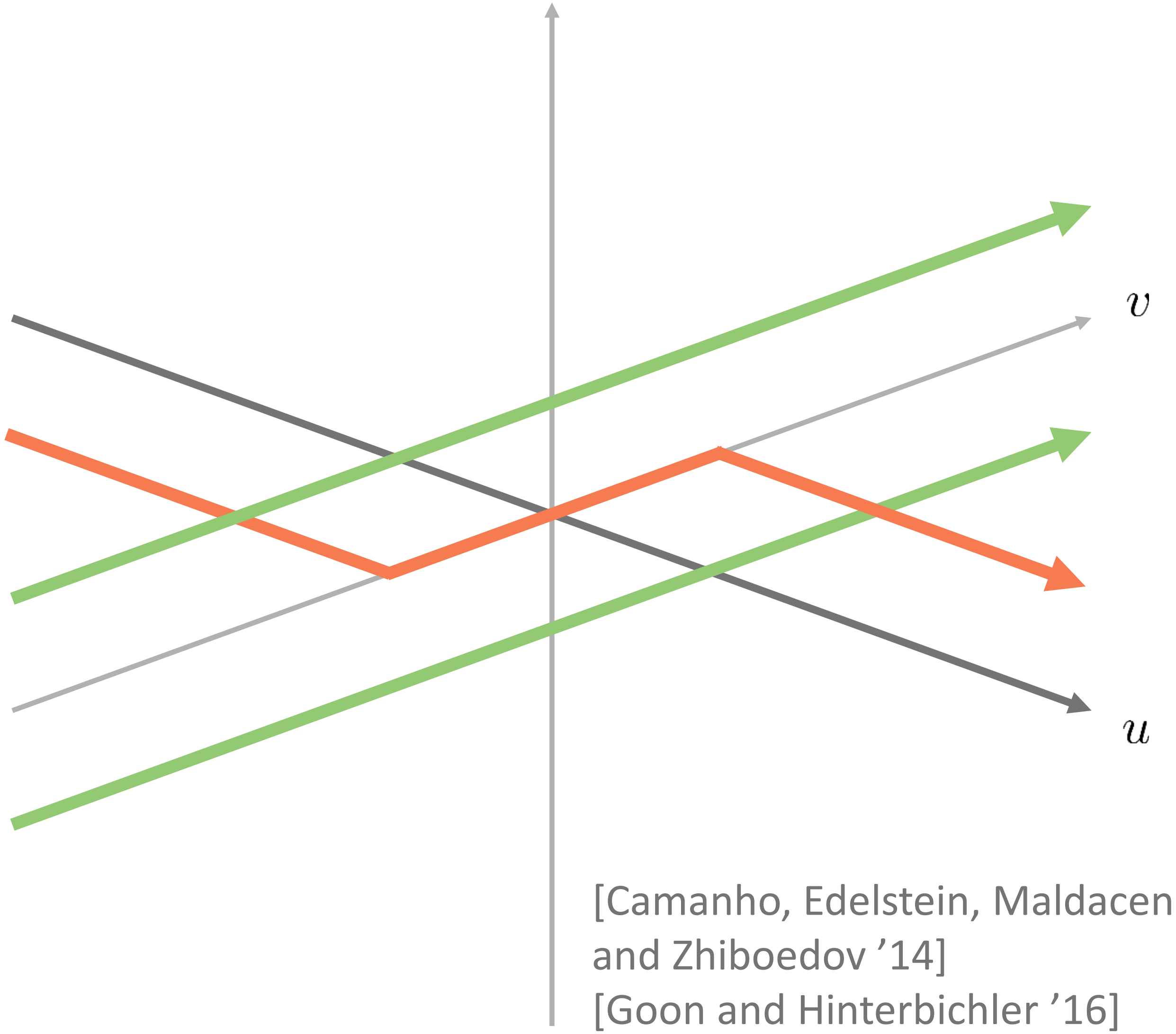
Propagate **between balancing** sources

$$F(u,\mathbf{x}) = f(u)\left(\frac{1}{|\mathbf{x}-\mathbf{b}|^{D-4}} + \frac{1}{|\mathbf{x}+\mathbf{b}|^{D-4}}\right)$$

By **symmetry**, no scattering in the transverse directions!

Accumulate time delay indefinitely to maximise causality violation…?

→ **No**, this is unstable!




[Camanho, Edelstein, Maldacena, and Zhiboedov ’14]
[Goon and Hinterbichler ’16]

# Instability Timescale

Choose $\mathbf{b} = b\hat{\mathbf{z}}$. Classical equations of motion near origin

$$k_v \frac{d^2 z}{du^2} = -\frac{\partial V}{\partial z} \sim k_v \Omega^2 z, \quad \Omega^2 \sim \frac{1}{k_v} \frac{\partial^2 V}{\partial z^2}\bigg|_{\mathbf{x}=\mathbf{0}} < 0$$

JWKB Ansatz solution

$$z(u) \sim \frac{1}{\Omega(u)^{1/2}} \exp\left[\pm i \int_0^u du' \Omega(u')\right]$$

**Instability** becomes relevant at $u = u_{\text{inst}}$ defined by

$$\left| \int_0^{u_{\text{inst}}} du\, \Omega(u) \right| \sim \int_0^{u_{\text{inst}}} du \sqrt{\frac{f(u)}{b^{D-2}}} \sim 1$$

In fact, **uncertainty** of time delay operator in semiclassical approximation

$$\delta T \gtrsim 2^{-3/2} \left| \int^{u_{\text{inst}}} du [1 - 2u\Omega(u)]\, \Omega(u) \exp\left(2 \int_0^u du' \Omega(u')\right) \right|$$

→ To avoid scattering, need localised wavepackets: Far from S-matrix eigenstates!

# Unbalanced Shockwaves

Either way, $u_{\text{inst}}$ acts as $u_{\max}$, placing bound on time delay:

$$k_v |\Delta T_{\text{EFT}}(u_{\max})| \sim k_v \frac{|c_{\text{GB}}|}{\Lambda^2} \int_0^{u_{\max}} du \frac{f(u)}{b^{D-2}} \ll |c_{\text{GB}}| \int_0^{u_{\max}} du \sqrt{\frac{f(u)}{b^{D-2}}} \sim |c_{\text{GB}}|$$

Once again: **IR causality** consistent with $|c_{\text{GB}}| \lesssim 1$!

Gravity is unstable, so this holds for **generic configurations**: Sum of squared "frequencies" is non-positive

$$\sum_{n=1}^{D-2} \omega_n^2 = (\Omega^2)^i{}_i = \frac{1}{k_v} \frac{\partial^2 V}{\partial x^i \partial x_i}\bigg|_{\mathbf{x}=\mathbf{x}_0} = -\frac{1}{2} \partial_i \partial^i F(\mathbf{x} = \mathbf{x}_0) \leq 0$$

so at least one unstable direction.

→ See paper for more details: Come to the same conclusion in **Born approximation** (can reproduce lack of scattering classical limit etc.) and **Perturbation theory** (smaller regime of validity)!

# Conclusion

# IR Causality of Gauss-Bonnet Gravity

For scattering off single black hole and shockwave, multiple shock waves, and between shockwaves, always:

$$k_v \left|\Delta T^{\mathrm{EFT}}\right| \ll |c_{\mathrm{GB}}|$$

**Perspective 1:** IR causality imposes

$$|c_{\mathrm{GB}}| \lesssim 1$$

[Camanho, Edelstein, Maldacena, and Zhiboedov '14]
[Reall, Tanahashi, and Way '14]

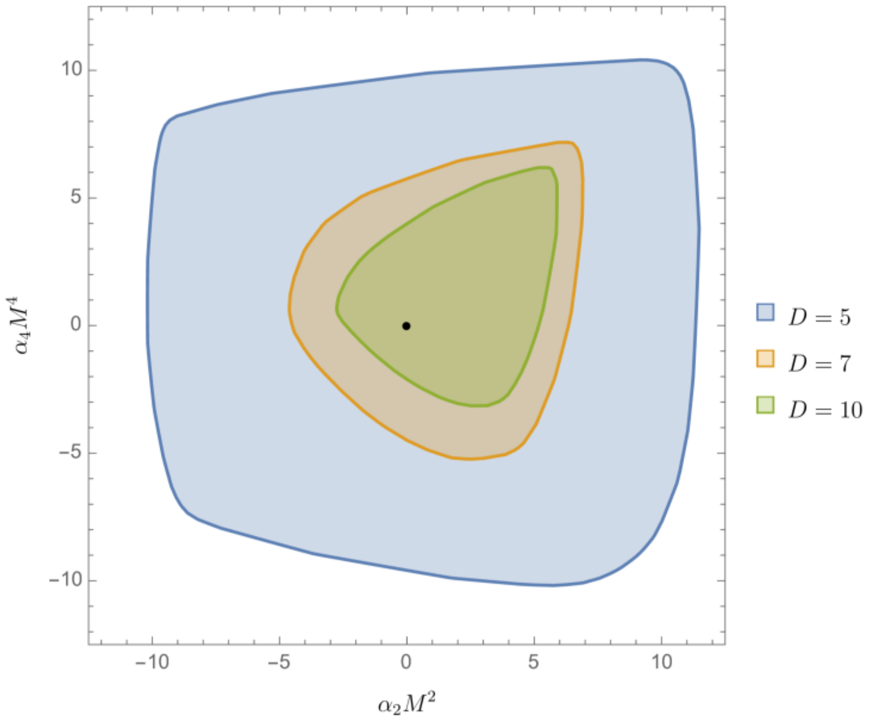
(In contrast to earlier claims that causality requires $c_{\mathrm{GB}} = 0$ )

→ Consistent with bootstrap and **positivity bounds**!

Can understand mild violation of positivity bounds from resolvability criterion $\Delta T^{\mathrm{EFT}} \gtrsim -\omega^{-1}$

**Perspective 2:** For EFTs $|c_{\mathrm{GB}}| \lesssim 1$ natural

→ GB gravity does not violate **IR causality**




[Caron-Huot and Li '22]

# Summary

**Conclusion**

- In curved spacetime, correct notion to learn about EFTs is **IR causality**
  - To make statements about EFTs, need to properly identify **regime of validity** of EFT and approximations used.

- EGB gravity not ruled out by IR causality
  - **consistent** with gravitational positivity bounds!
  - Resolvability gives complementary understanding of **mild violation of positivity.**

**Outlook**

- Use infrared causality on less symmetric **backgrounds** to get more bounds on different EFT operators? [Carrillo González, de Rham, Jaitly, Pozsgay, Tokareva, and Tolley ‘22 & ’23]

- More physically: **de Sitter**? [Bittermann, McLoughlin, and Rosen ‘22]
  - IR causality is more local than asymptotic causality!
  - Extend using notion of de Sitter S-Matrix [Melville and Pimentel ‘23]

Thanks for your attention!
Questions?