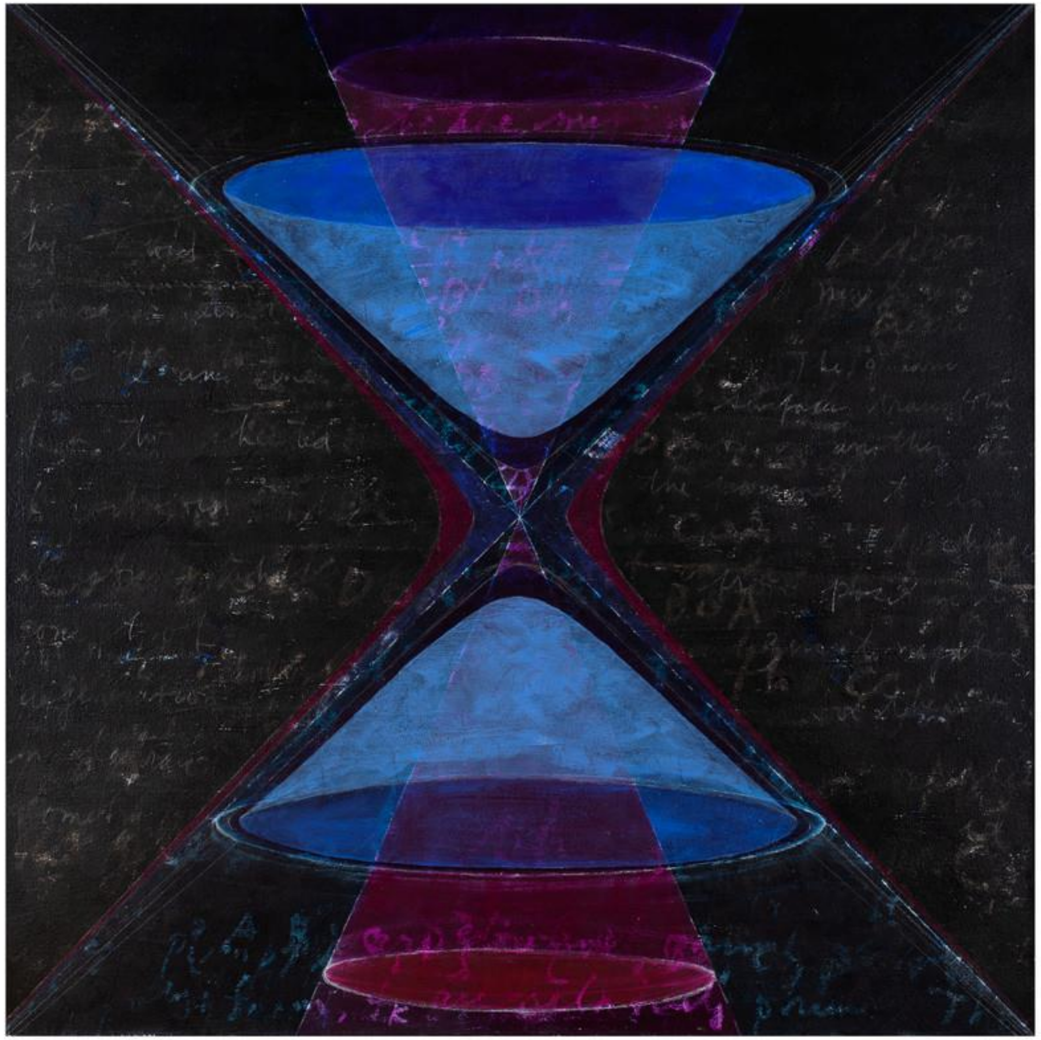
Stacking and balancing casual causality



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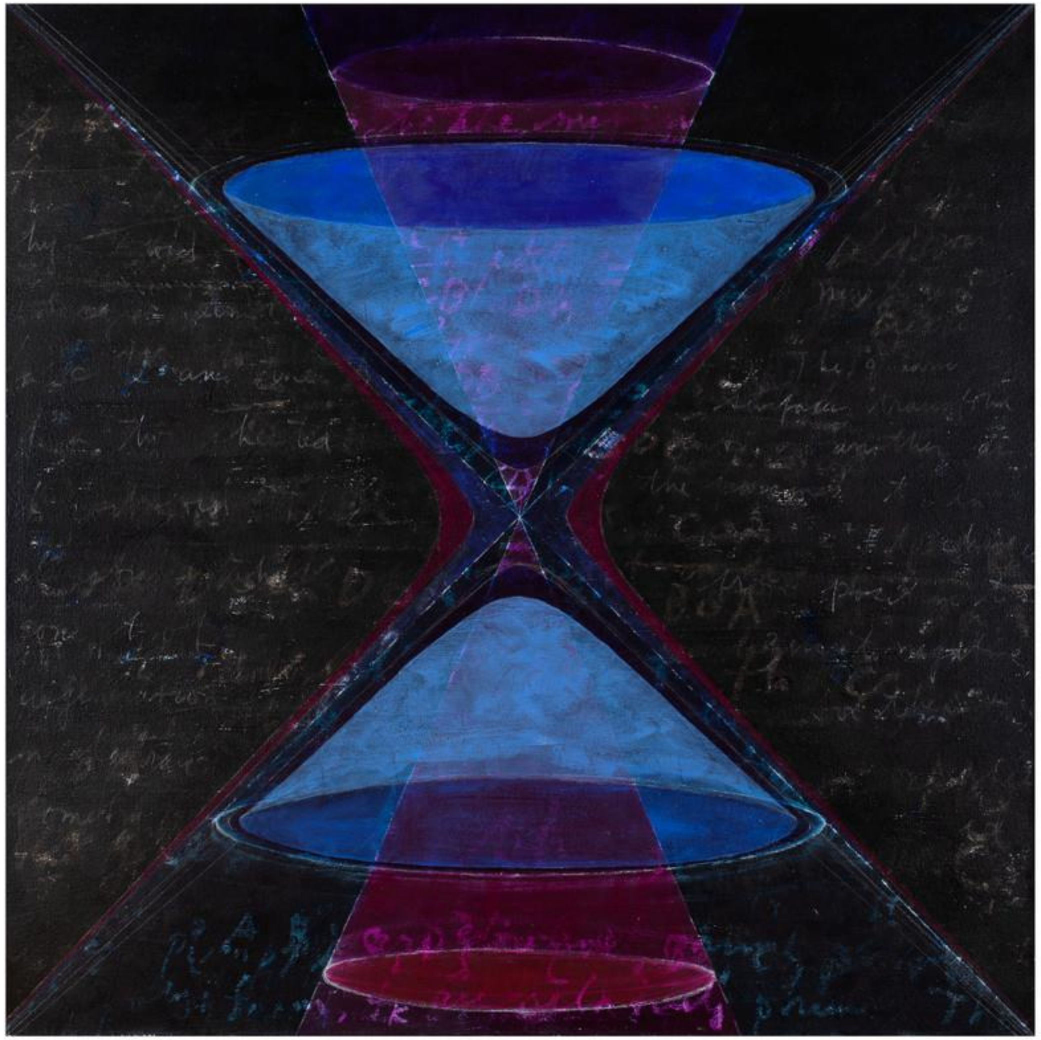


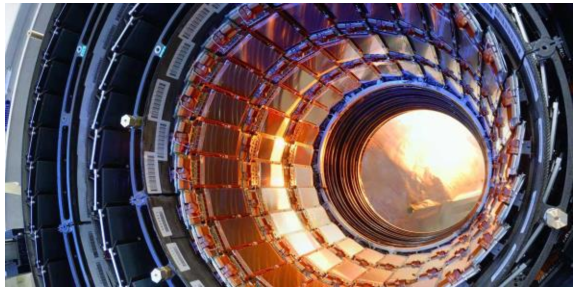
based on 2112.05031 & 2309.04534 in collaboration with C. de Rham, A. Margalit, A. J. Tolley

Motivation: EFTs of Gravity

Effective field theory of gravity

$$\mathcal{L}_{\rm IR} = \frac{M_{\rm Pl}^{D-2}}{2} R + \mathcal{L}_{\rm light}\left[g, \phi_-\right]$$

$$+ \ldots$$



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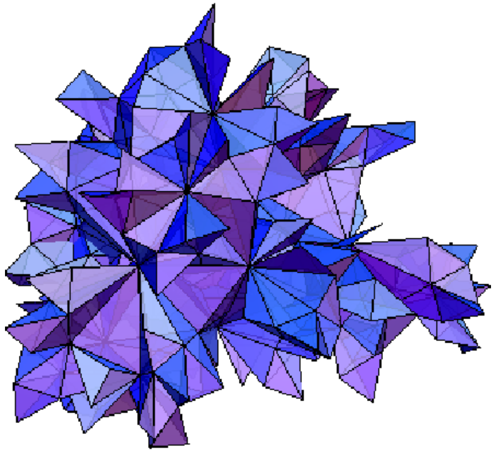
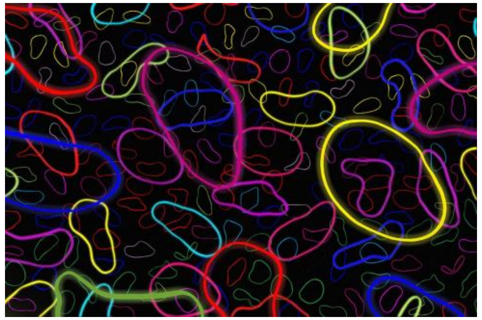




Energy/Size^{-1}

H_0 G_F α' $M_{\rm Pl}$

$$\mathcal{L}_{\rm UV} = \frac{M_{\rm Pl}^{D-2}}{2} R + \mathcal{L}_{\rm light}\left[g, \phi_-\right]$$

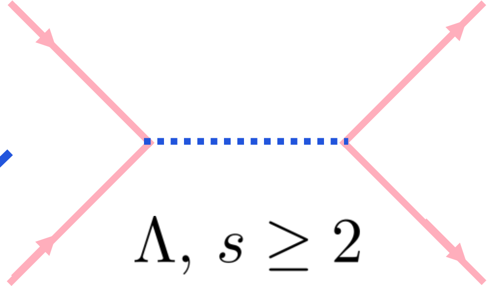
$$+ \mathcal{L}_{\rm heavy}\left[g, \phi_+\right]$$

?

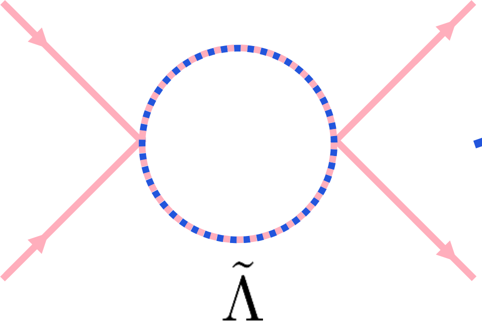
The **UV completion** of GR is unknown (please let me know if you do!), but we can write down a **generic effective action**.

Einstein-Hilbert +

Full **effective action** (redundantly parameterised):

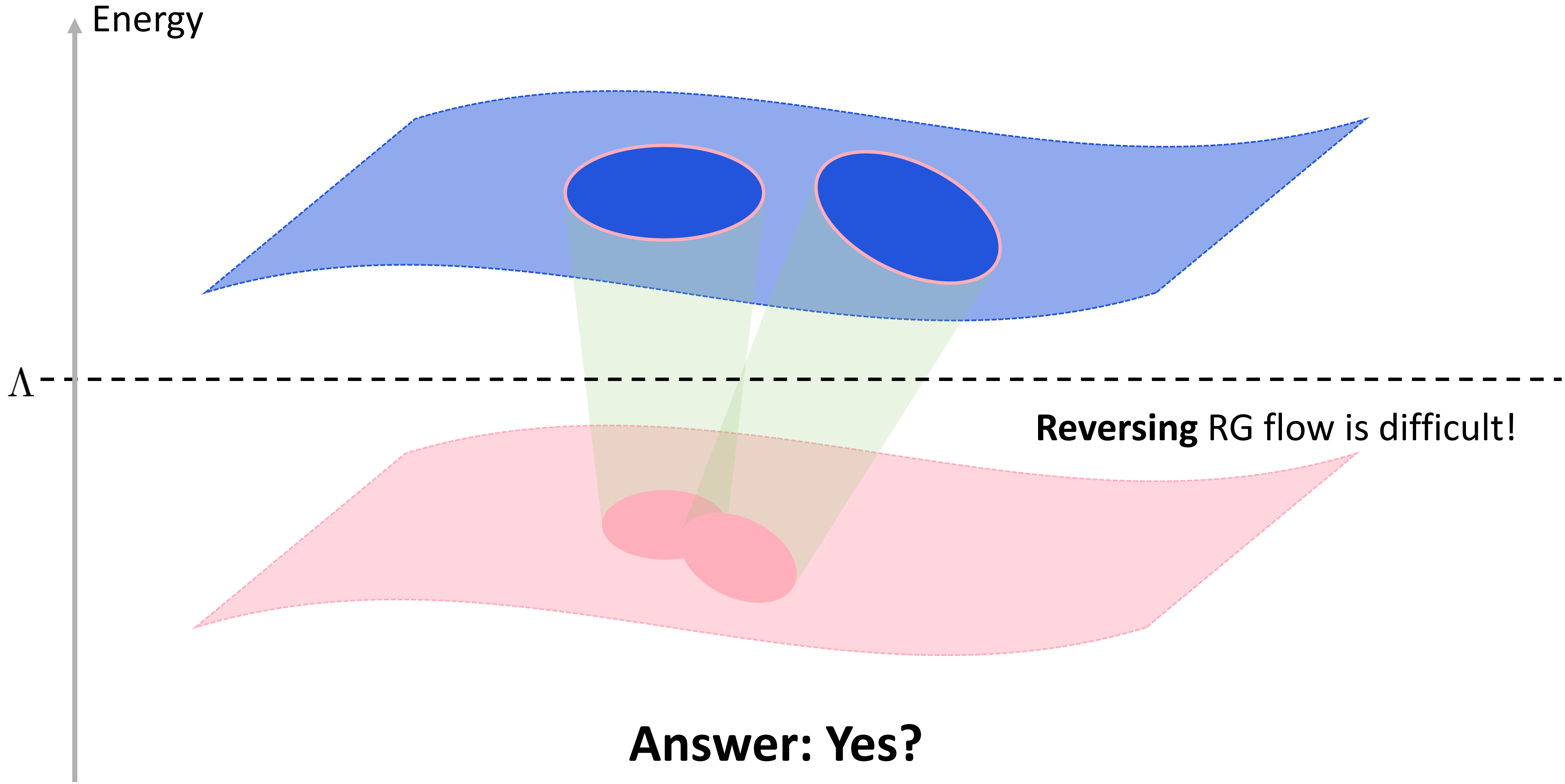


$$S_{\mathrm{EFT}} = \int d^D x \sqrt{-g} \Bigg[M_{\mathrm{Pl}}^{D-2} \left(\frac{1}{2} R + \Lambda^2 \sum_{m \geq 0, n \geq 2} c_{mn} \left(\frac{\nabla}{\Lambda} \right)^m \left(\frac{\mathrm{Riemann}}{\Lambda^2} \right)^n \right) + \tilde{\Lambda}^D \sum_{m \geq 0, n \geq 2} d_{mn} \left(\frac{\nabla}{\tilde{\Lambda}} \right)^m \left(\frac{\mathrm{Riemann}}{\tilde{\Lambda}^2} \right)^n \Bigg]$$



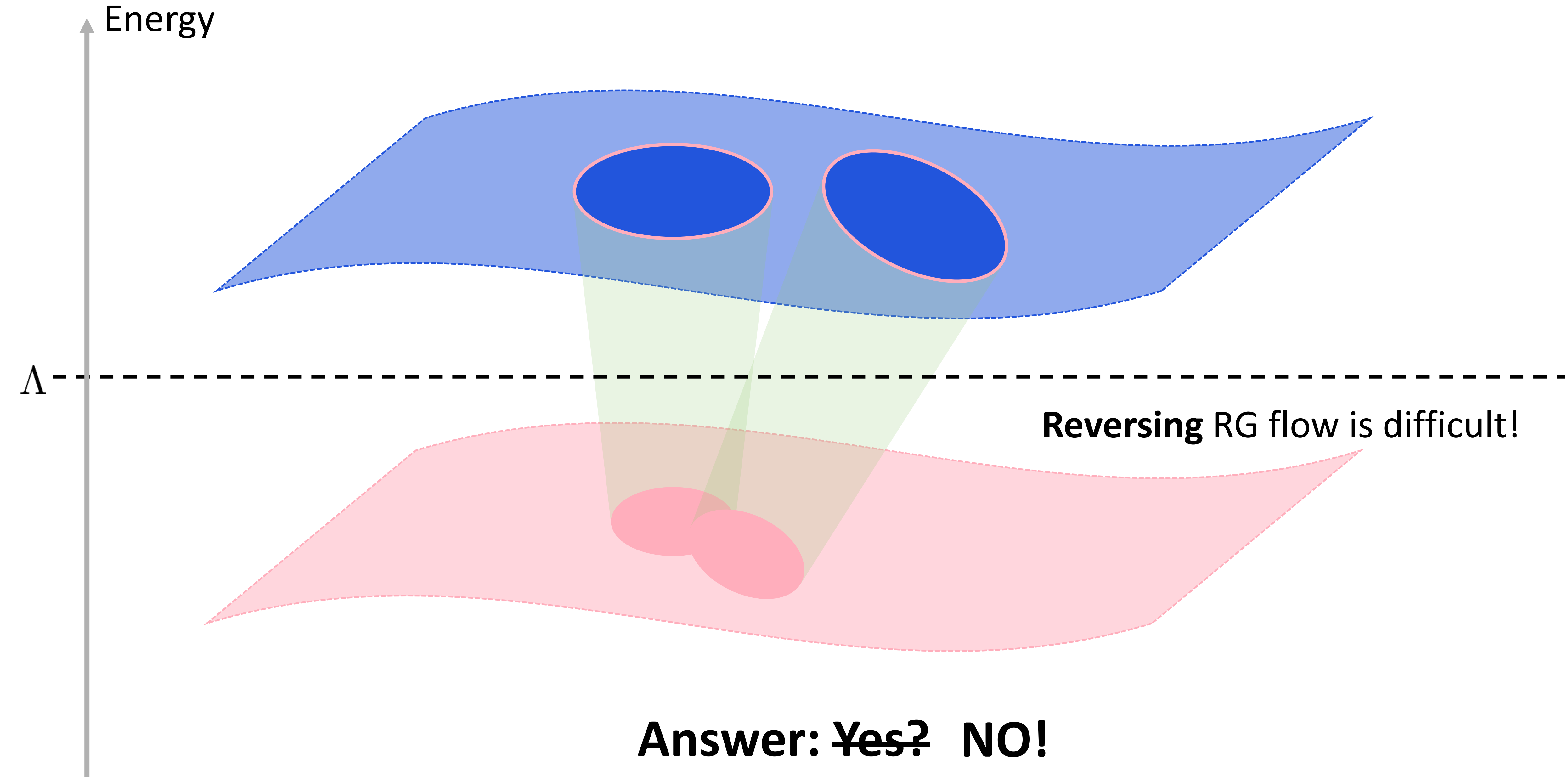
Question: Are all these terms physical?

RG flow



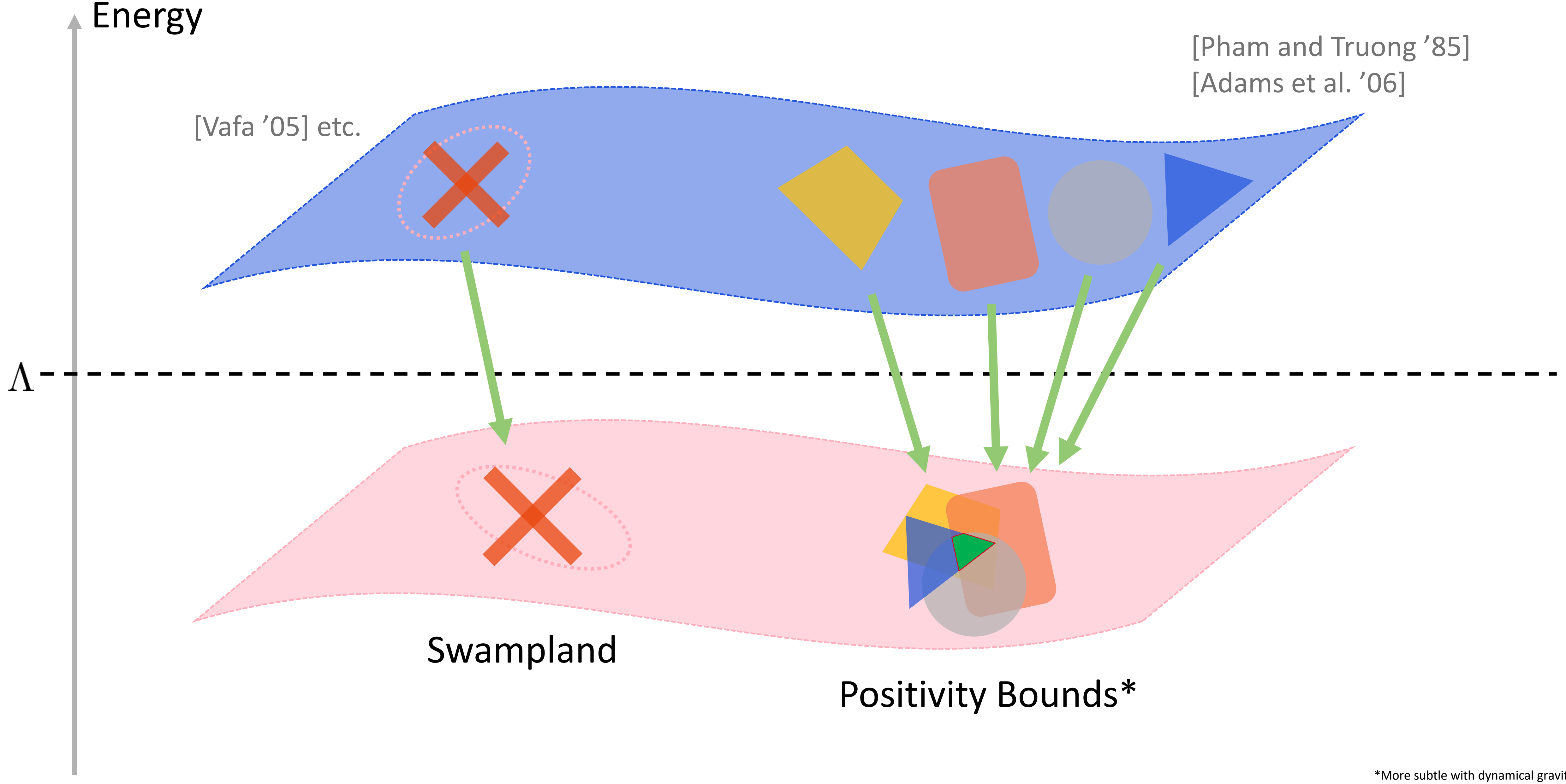
Answer: Yes?

RG flow



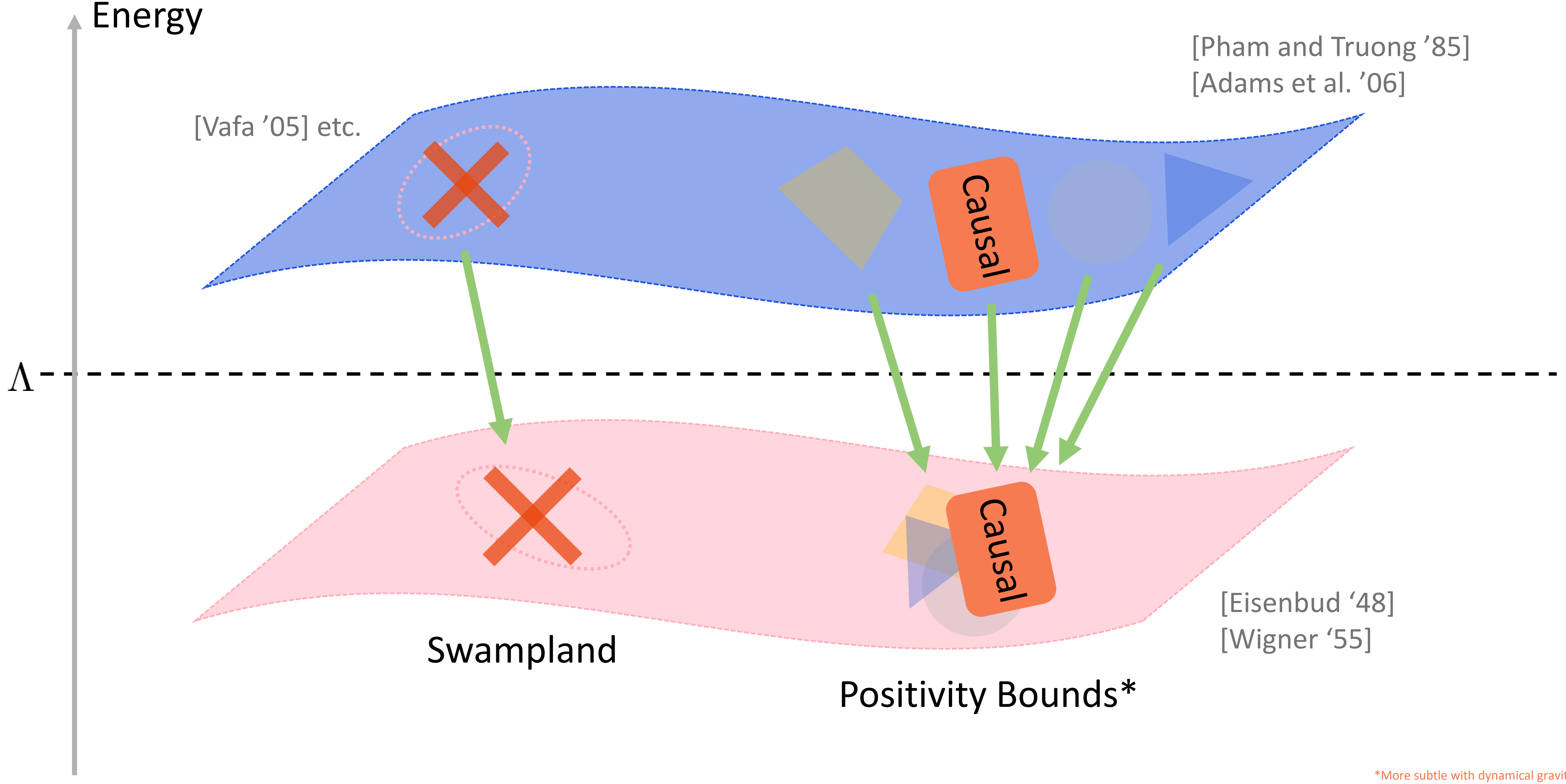
Answer: ~~Yes?~~ NO!

UV imprints on IR



*More subtle with dynamical gravity!

Causality



*More subtle with dynamical gravity!

Example: Consistency and Causality

Illustrative example on flat space: Goldstone

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + g(\partial\phi)^4 + \dots$$

In known UV completions, always find $g > 0$. Coincidence…?

…No! Propagation speed of perturbations about backgrounds $\bar{\phi} = c_\alpha x^\alpha$

$$v^2 = 1 - g\frac{4(c_\alpha p^\alpha)^2/|\mathbf{p}|^2}{1 - 2gc_\alpha u^\alpha}$$

So $g > 0$ directly linked to **subluminal** propagation speed of perturbations! [Adams et al. ’06]

→ Consistent with **positivity bounds**. Caveat: More subtle with **dynamical gravity** – technical and conceptual challenges !

[Cheung and Remmen ’17]
[Alberte, de Rham, Jaitly, and Tolley ’20]
[Tokuda, Aoki, and Hirano ’20]
etc.

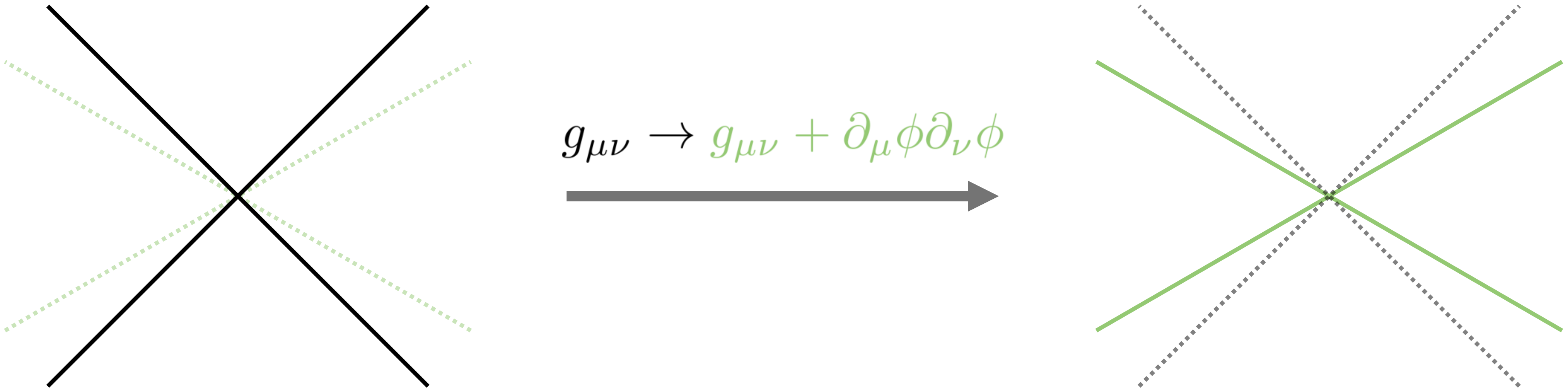
Causality

Goal: Use causality to identify consistent gravitational EFTs

Causality and Curved Spacetime

Causality and Time Delays

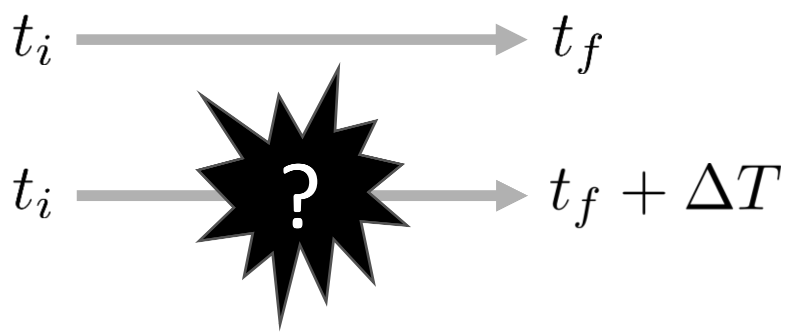
In gravitational EFTs, **field redefinitions** can change light cone structure



$$g_{\mu\nu} \to g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi$$

so propagation speeds are not invariant: (Sub-)luminal propagation **not meaningful** criterion.

→ Rephrase causality in terms of **time delay** ΔT: Assume spacetime is **asymptotically flat** and has causal **Killing vector** $k = \partial / \partial t$ associated with a conserved energy $E = -k \cdot u$



Eisenbud-Wigner Time Delay

Consider generic incoming wave packet and outgoing wave packet that differs by only by a **time delay**

$$|\mathrm{in}, g\rangle = \int_0^\infty \frac{dE}{2\pi} g(E) \hat{a}_E^{\mathrm{in}\dagger} |\mathrm{vac}\rangle , \quad |\mathrm{out}, g\rangle = e^{i\hat{P}_0 \Delta T} |\mathrm{in}, g\rangle$$

Given that

$$\langle \mathrm{vac}| \hat{a}_{E'}^{\mathrm{in}} \hat{S} \hat{a}_E^{\mathrm{in}\dagger} |\mathrm{vac}\rangle = 2\pi\delta(E - E') e^{2i\delta(E)}$$

then

$$\langle g, \mathrm{out}| \hat{S} |g, \mathrm{in}\rangle = \int_0^\infty \frac{dE}{2\pi} |g(E)|^2 e^{2i\delta(E) - iE\Delta T}$$

Take the profile $g(E)$ to be peaked around $\bar{E}$ with some width $\Delta E \ll \bar{E}$, so the **stationary phase approximation** gives

$$\Delta T = \left. \frac{2\partial\delta(E)}{\partial E} \right|_{E=\bar{E}} + \mathcal{O}(\Delta E^{-1})$$

→ **Eisenbud-Wigner** time delay, with intrinsic uncertainty!

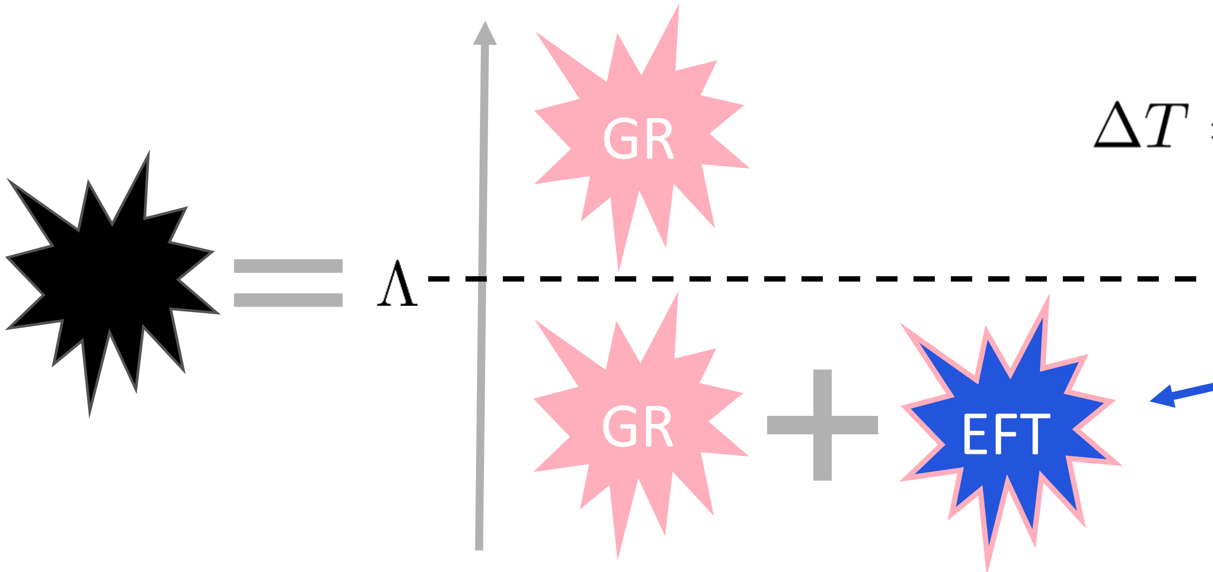
Is causality just $\Delta T > 0$?

Subtlety 1: Uncertainty principle puts limit on “observations” via **resolvability**

→ Waves with frequency ω cannot measure time delays ΔT with

$$|\Delta T| \lesssim \omega^{-1}$$

Subtlety 2: Need to distinguish effect of **background** geometry from EFT correction



$$\Delta T = \Delta T^{\mathrm{GR}} + \Delta T^{\mathrm{EFT}}$$

can isolate in LIF

Background effect due to GR should set **reference**

→ To determine **causality of EFT**, study EFT contribution.

Infrared Causality

Putting this together:

Infrared Causality
<u>Violation</u>

$\Updownarrow$

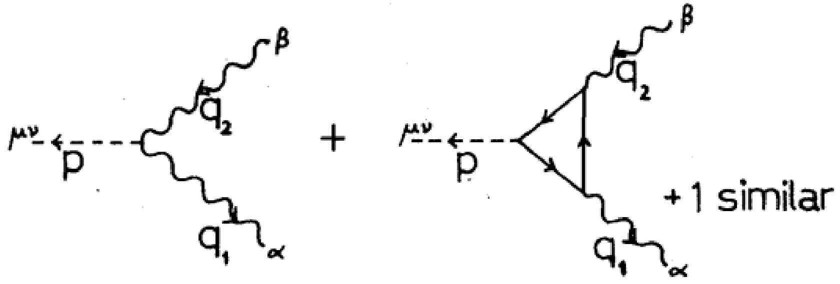
$$\boxed{\begin{gathered}\Delta T^{\mathrm{EFT}} < 0 \\ \textbf{AND} \\ \left|\Delta T^{\mathrm{EFT}}\right| \gtrsim \omega^{-1}\end{gathered}} \quad \Bigg\} \quad \Delta T^{\mathrm{EFT}} \lesssim -\omega^{-1}$$

Let's try this!

Example: QED on Curved Spacetime

QED on fixed curved background

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}(i\gamma^\mu D_\mu-m_e)\psi\right]$$



Integrating out the electron [Drummond and Hathrell ’80]

$$W=\int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{320\pi}\frac{\alpha}{m_e^2}R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}+\mathcal{O}\left(\frac{\alpha}{m_e^{2n}}\right)\right]$$

E.g. on Schwarzschild (with Schwarzschild radius r_g): **Gravitational birefringence**

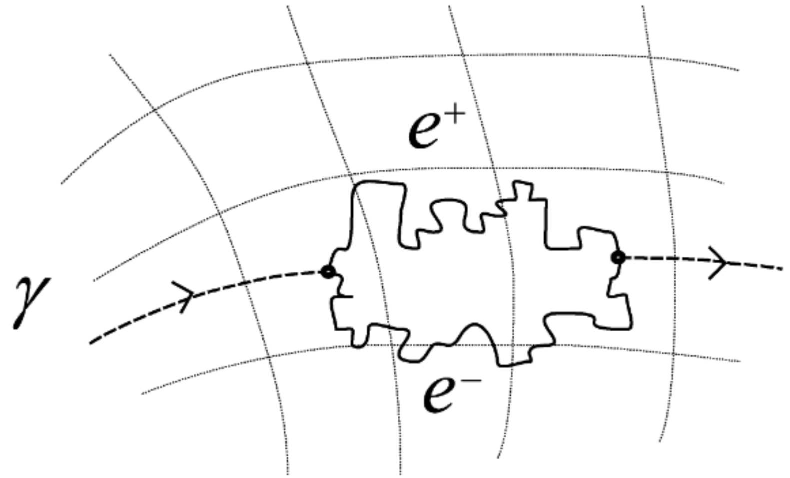
$$c_s^2-1\sim\pm\frac{1}{m_e^2}\frac{r_g}{r^3}\quad\longrightarrow\quad\Delta T^{\mathrm{EFT}}\sim\pm\frac{2r_g}{b^2m_e^2}$$

Signals causality violation, but resolved within (partial) UV completion itself!

[Hollowood and Shore ’07]

→ Causality at low energies violated by integrating out electron...?

Example: QED on Curved Spacetime



Lesson 1: Naïve trustworthiness of truncation $\Lambda \overset{?}{=} m_e/\sqrt{\alpha}$ is not true (Lorentz invariant) EFT **cut-off.** Need to think of asymptotic expansion

$$\Lambda = \lim_{n\to\infty} \left(\frac{m_e^{2n}}{\alpha}\right)^{1/2n} = m_e$$

Lesson 2: IR causality can be diagnosed purely **within EFT!** Within regime of validity

[de Rham and Tolley '20]

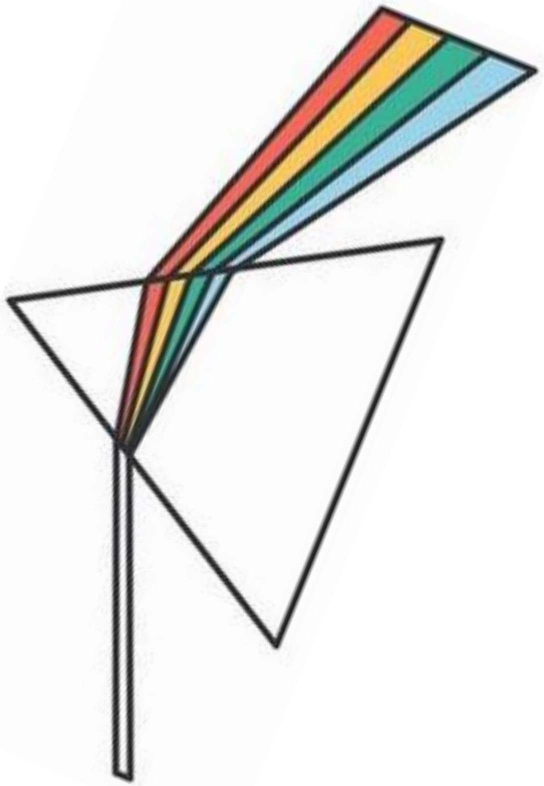
$$\left|\Delta T^{\mathrm{EFT}}\right| \ll \omega^{-1}$$

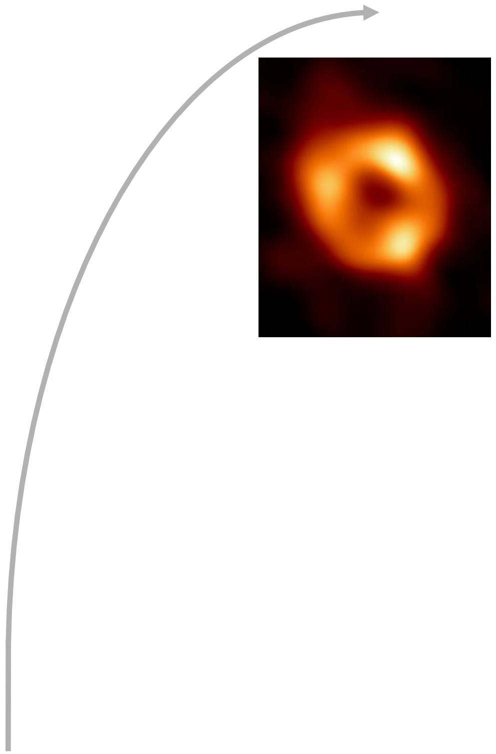
→ unresolvable!

EFTs on pp-waves

Testing Ground: Black Holes

Like to smash things into each other to study them: Scatter gravitons off **black hole**!





Technically challenging: Gauge invariant basis variables remain same, but master variables receive EFT corrections [Kodama & Ishibashi ’03]

→ IR causality consistent with (gravitational) **positivity bounds** [CYRC, de Rham, Margalit, and Tolley ‘21]

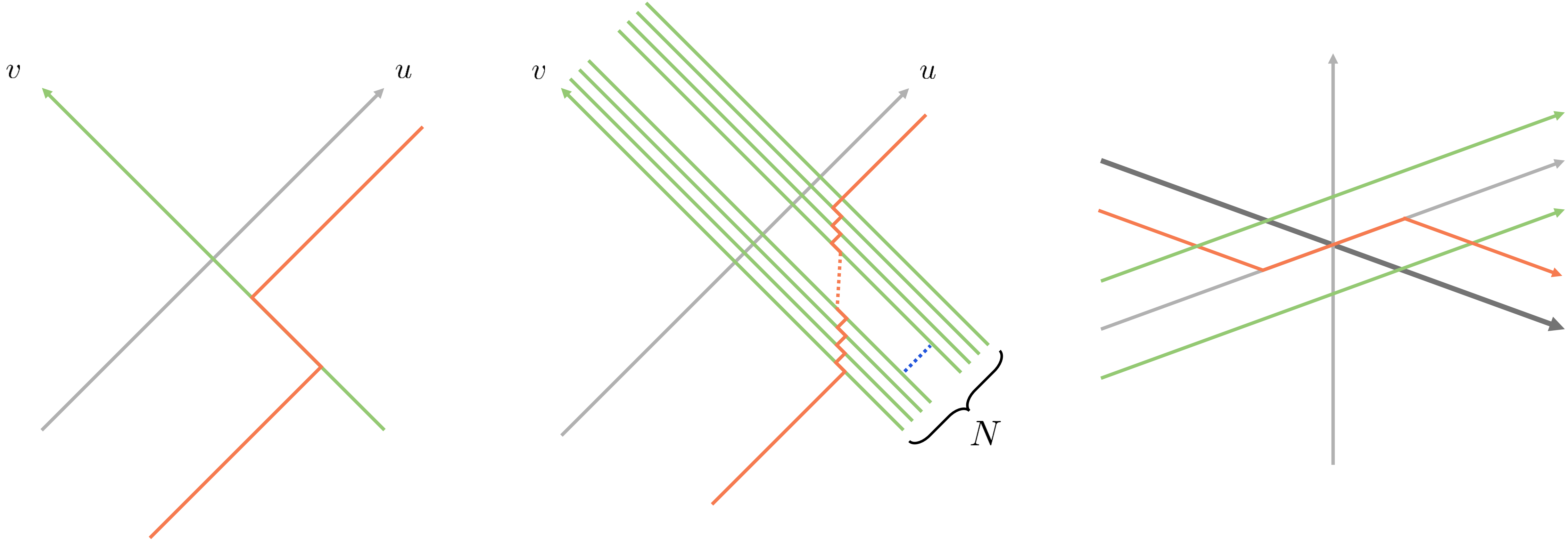
Aichelburg-Sexl Boost: Shockwaves

Instead, take Aichelburg-Sexl boost to **shockwave** spacetime

Spoiler: Same conclusion for single shockwave and black hole, but more interesting configurations with shockwaves! [Camanho, Edelstein, Maldacena, and Zhiboedov '14]


acausality
acausality
acausality
acausality

Stacking and Balancing Causality



(More Precise) Goal: Constrain EFT operators using IR causality

Review: Pp-waves

In Brinkmann coordinates (u, v, x^i)

$$ds^2 = 2du\,dv + F(u, x^i)du^2 + \delta_{ij}dx^i dx^j$$

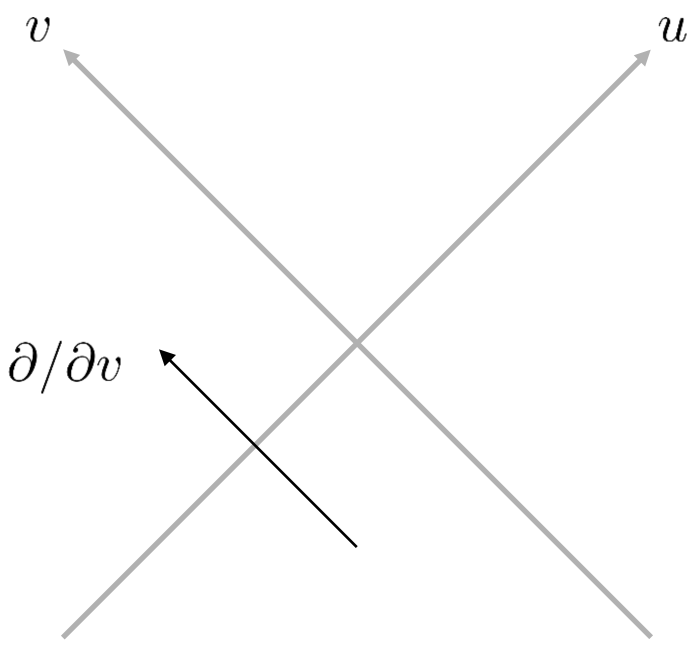
Only non-vanishing component of Riemann tensor

$$R_{uiuj} = -\frac{1}{2}\partial_i\partial_j F$$

Vacuum Einstein's equations impose

$$R_{\mu\nu} = 0 \longrightarrow \partial_i\partial^i F = 0$$

→ Harmonic $F(u, x^i)$!



Within this class of solutions: **Rank-0 and -2 contractions** of Riemann tensors and covariant derivatives e.g.

$$R_{\mu\nu}, \quad (R^m)^\lambda{}_{\mu\lambda\nu}, \quad \nabla_\alpha\nabla_\beta(R^n)^\alpha{}_\mu{}^\beta{}_\nu, \quad \ldots$$

vanish.

Surfin' on pp-waves

Pp-waves satisfying vacuum Einstein equation are background solutions **at all orders** in EFT

$$\text{Background eq.} \sim \left.\frac{\delta S_{EFT}}{\delta g^{\mu\nu}}\right|_{\text{pp-wave}}$$

$$= 0$$

However, equations for perturbations $h_{\mu\nu}$ on pp-wave background

$$\text{Perturbation eq.} \sim \left.\frac{\delta^2 S_{\text{EFT}}}{\delta g^{\mu\nu}\delta g^{\rho\sigma}}\right|_{\text{pp-wave}} h^{\rho\sigma}$$

$$+ \text{perm.}$$

$$\neq 0$$

not trivially satisfied!

→ **EFT corrections non-zero**!

Regime of Validity

EFT breaks down when probed…

1) at too small length scales or high energies → **background** (trivial for pp-waves)
2) by particles with too high energies → **perturbations** (non-trivial for pp-waves!)

Find parameter controlling asymptotic expansion using **Lorentz scalars** towards infinity (see QED). Crucially:

$$R_{\mu\nu\alpha\beta}\delta R^{\mu\nu\alpha\beta} \neq 0$$

Fourier transform perturbations $\nabla h \to ikh$, then constraints take schematic form

$$\text{“}\lim_{a,b,c\to\infty}\left(\frac{\nabla}{\Lambda}\right)^a\left(\frac{\text{Riemann}}{\Lambda^2}\right)^b\left(\frac{k}{\Lambda}\right)^{2c+b} \ll 1\text{”}$$

→ **EFT constraints**:

conserved quantity for $\partial/\partial v$

$$\partial_r \ll \Lambda, \quad k^\mu\partial_\mu \ll \Lambda^2, \quad \frac{\partial_r F}{r}k_v^2 \ll \Lambda^4$$

“Shockwaves are not solutions in the EFT of gravity”

Shockwaves are pp-waves with

$$F(u,r) = \frac{4\Gamma\left(\frac{D-4}{2}\right)}{\pi^{(D-4)/2}}\delta(u)\frac{G|P_u|}{r^{D-4}}$$

→ Solutions to Einstein’s equations with **ultra-relativistic** (delta function) source

$$T_{uu} = -P_u\delta(u)\delta^{(D-2)}(\mathbf{x})$$

(also obtained via **Aichelburg-Sexl boost** from Schwarzschild black hole).

However:

$$\frac{\partial_r F}{r}k_v^2 = -\frac{4(D-4)\Gamma\left(\frac{D-4}{2}\right)}{\pi^{(D-4)/2}}\delta(u)\frac{G|P_u|k_v^2}{r^{D-6}} \to \infty \not< \Lambda^4$$

so shockwaves are outside EFT regime of validity → need to **regulate** e.g. as Gaussian

$$\delta(u) \to \frac{1}{\sqrt{2\pi}L}e^{-u^2/2L^2}, \quad L \gg k_v/\Lambda^2$$

Leading-order EFT: Gauss-Bonnet Gravity

Leading-order EFT in $D \geq 5$

$$S_{\rm EFT} = \int d^D x \sqrt{-g} M_{\rm Pl}^{D-2} \left(\frac{1}{2} R + \frac{c_{\rm GB}}{\Lambda^2} R_{\rm GB}^2 + \mathcal{O}\left(\Lambda^{-4}\right) \right)$$

$$R_{\rm GB}^2 = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

→ **Einstein-Gauss-Bonnet gravity!**

Equations for perturbations (in **light cone gauge** $h_{v\mu} = 0$):

$$\bar{g}^{\mu\nu} \partial_\mu \partial_\nu h_{ij} - 8 \frac{c_{\rm GB}}{\Lambda^2} \partial_v^2 X_{ij} = 0, \quad X_{ij} = (\partial_m \partial_{(i} F) h_{j)}{}^m - \frac{\bar{g}_{ij}}{D-2} (\partial_m \partial_n F) h^{mn}$$

Decompose $x^i \to (r, x^\alpha)$ and assume **spherical symmetry** to decouple modes

$$\bar{g}^{\mu\nu} \partial_\mu \partial_\nu \Phi_M + a_M \frac{c_{\rm GB}}{\Lambda^2} \frac{\partial_r F}{r} \partial_v^2 \Phi_M = 0, \quad a_M = \left(8(D-4), 4(D-4), -8, -8\right)$$

$$\Phi_M = \left(h_{rr}, h_{r\alpha}, h_{\alpha\beta}, g^{33} h_{33} - g^{\alpha\alpha} h_{\alpha\alpha}\right)$$

Stacking Causality

JWKB Approximation

Fourier transform of perturbation equations $\partial_v \to ik_v$ is a **Schrödinger-like equation**

$$i\frac{\partial\Phi_M}{\partial u} = -\frac{1}{2k_v}\nabla^2\Phi_M + V\Phi_M, \quad u \to \text{“time”}, \quad k_v \to \text{“mass”}$$

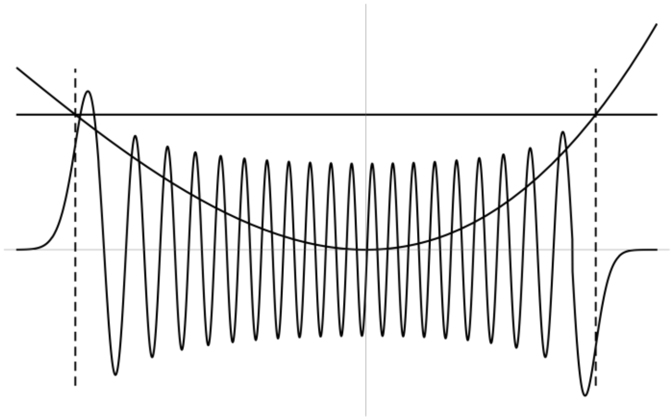
$$V(u,r) = -\frac{k_v}{2}F(u,r) + a_M k_v \frac{c_{\mathrm{GB}}}{\Lambda^2}\frac{\partial_r F(u,r)}{r}$$

Solve this using **JWKB Ansatz** and treat Laplacian perturbatively:

$$\Phi_M(u,r) = \Phi_0 \exp[i\delta_M(u,r)],$$

$$\delta_M(u,r) = \delta_M^{(0)}(u,r) + \delta_M^{(1)}(u,r) + \ldots$$

The approximation **valid** as long as $|\delta^{(0)}(u,r)| \gg |\delta^{(1)}(u,r)|$, i.e. until $u = u_{\max}$ defined by



$$\left|\int_0^{u_{\max}} du \boldsymbol{\nabla} V(u,r)\right| \sim V(u_{\max},r)$$

→ Can’t accumulate time delay indefinitely!

Eikonal Time Delay

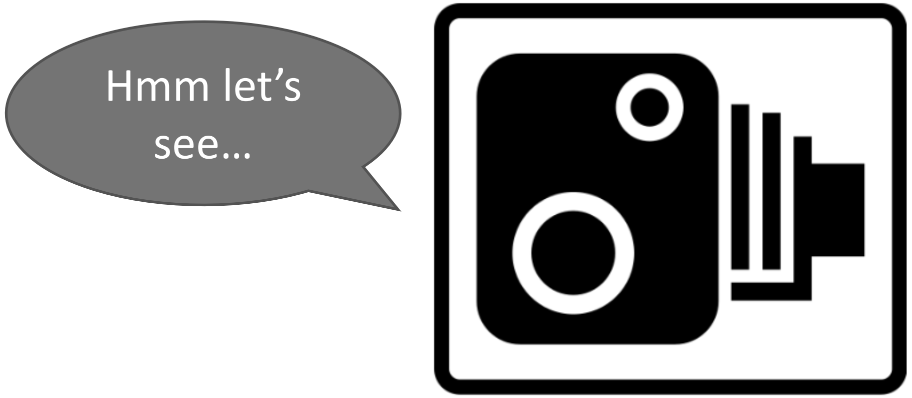
Leading-order JWKB phase shift reproduces the **eikonal** phase shift. **Cumulative time delay** for particle localised at impact parameter $r = b$,

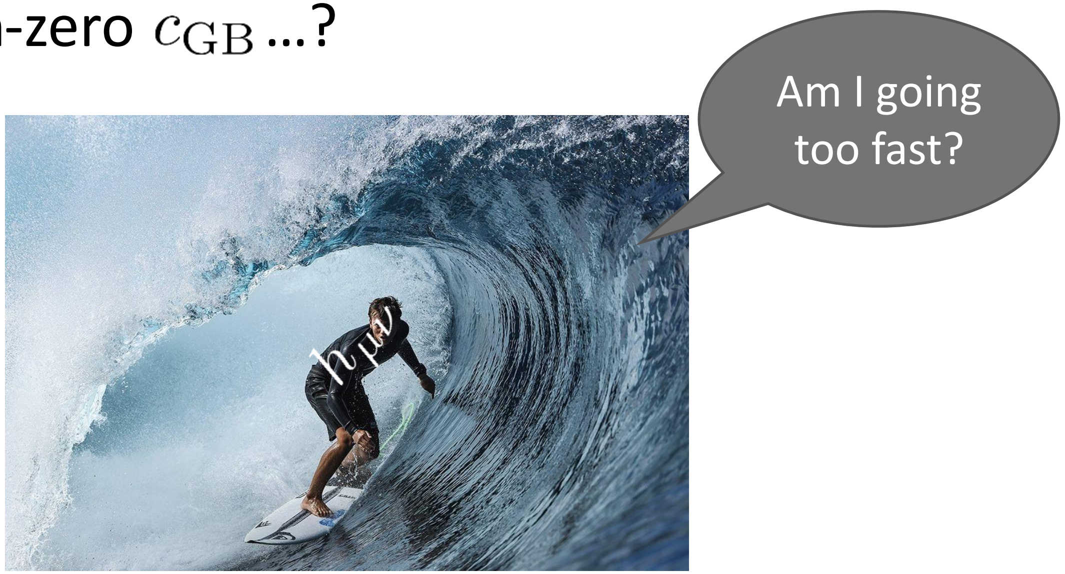
$$\Delta T(u) = 2\,\frac{\partial \delta_0(u,r)}{\partial k_v}\bigg|_{r=b} = \left(\int_0^u F(u,r)du' - 4a_M \frac{c_{\rm GB}}{\Lambda^2}\int_0^u \frac{\partial_r F(u,r)}{r}\right)\bigg|_{r=b}$$

Therefore:

$$\Delta T^{\rm EFT}(u) = -4a_M \frac{c_{\rm GB}}{\Lambda^2}\int_0^u \frac{\partial_r F}{r}\bigg|_{r=b}, \qquad a_M = \big(+8(D-4), +4(D-4), -8, -8\big)$$

→ No definite sign! Causality violation for any non-zero $c_{\rm GB}$ …?

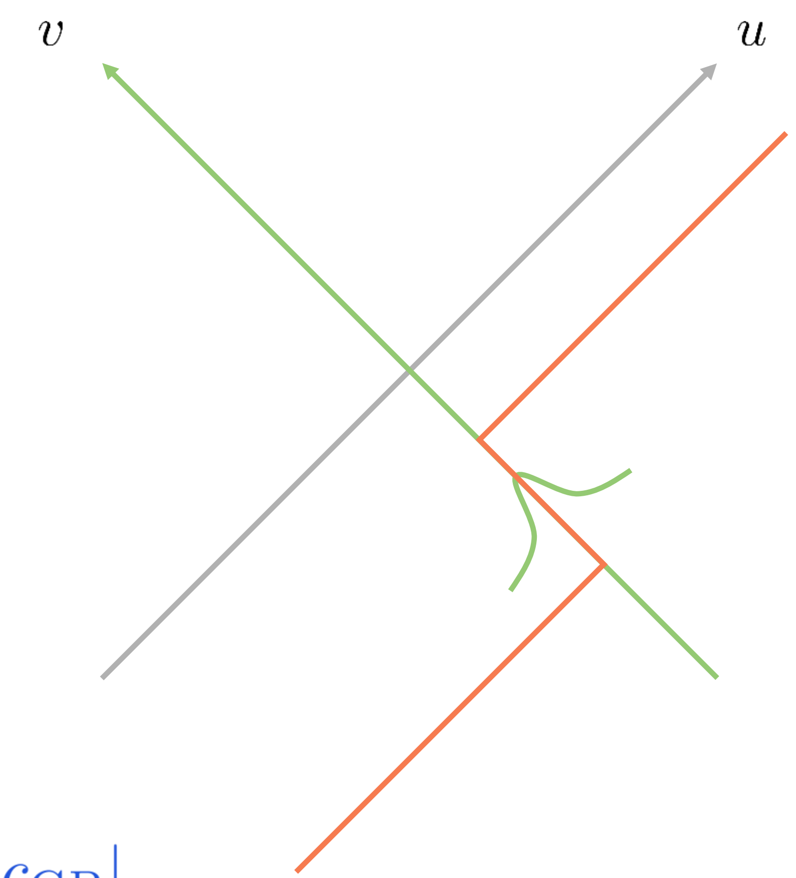




Localised Source

For sources with arbitrary profile $f = f(u)$ in time **localised** at $r = 0$:

$$F(u,r) = \frac{f(u)}{r^{D-4}}$$



1) Validity of **eikonal** approximation imposes

$$\int_0^{u_{\max}} du \frac{f(u)}{b^{D-2}} \sim \sqrt{\frac{f(u_{\max})}{b^{D-2}}}$$

2) **EFT** regime of validity

$$\frac{f(u)}{b^{D-2}} k_v^2 \ll \Lambda^4$$

so time delay:

$$|\Delta T^{\mathrm{EFT}}| \sim \frac{|c_{\mathrm{GB}}|}{\Lambda^2} \int_0^{u_{\max}} du \frac{f(u)}{b^{D-2}} \sim \frac{|c_{\mathrm{GB}}|}{\Lambda^2} \sqrt{\frac{f(u_{\max})}{b^{D-2}}} \ll \frac{|c_{\mathrm{GB}}|}{k_v}$$

→ Same as with spherical symmetry: **IR causality** does not require $c_{\mathrm{GB}} = 0$!

Special Case: N Stacked Shockwaves

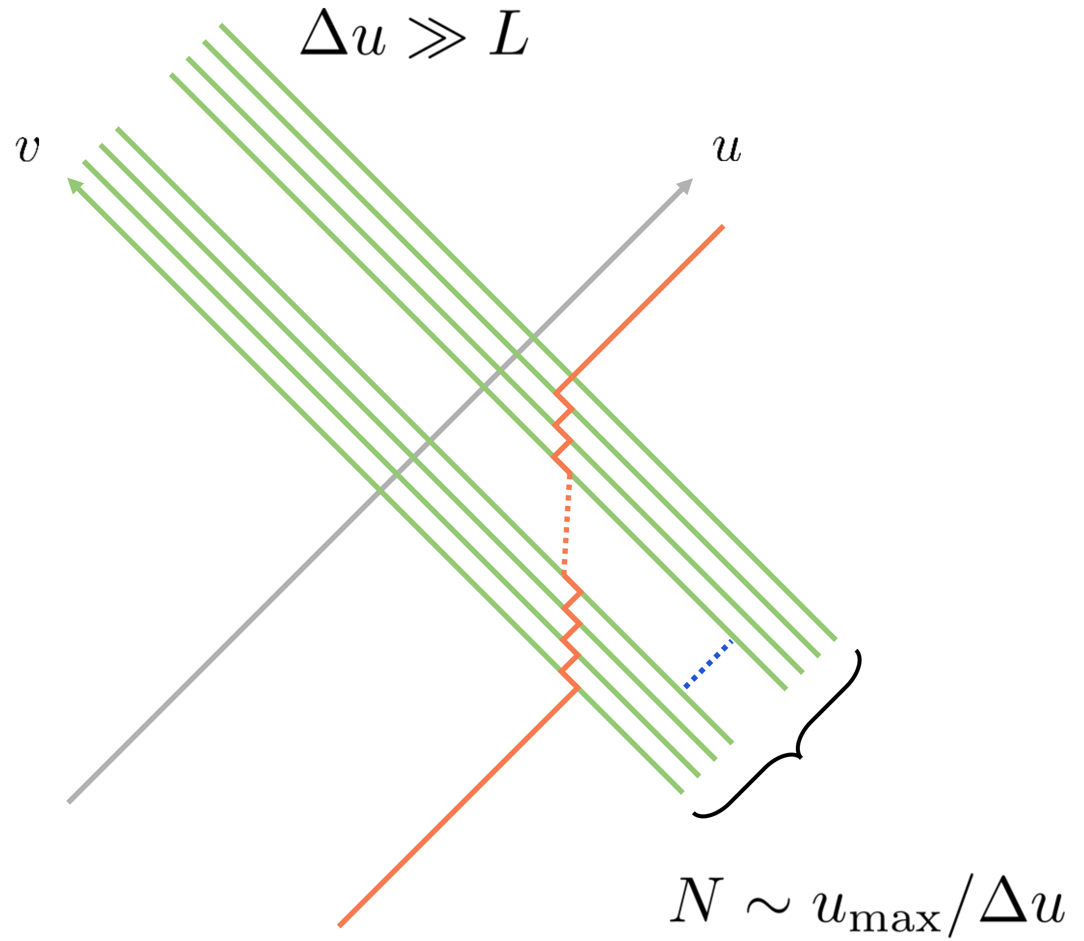
Stack N regulated shockwaves with width L and separated by Δu

$$f(u) = \frac{1}{\sqrt{2\pi}L}\frac{4\Gamma\left(\frac{D-4}{2}\right)}{\pi^{(D-4)/2}}G|P_u|\sum_{n=1}^{N} e^{-(u-n\Delta u)^2/2L^2}$$

[Camanho, Edelstein, Maldacena, and Zhiboedov '14]

When shocks sufficiently separated:

$$\left|\Delta T^{\mathrm{EFT}}_{(N)}\right| \sim N\left|\Delta T^{\mathrm{EFT}}_{(1)}\right|$$



To **maximise** causality violation: Want N as large as possible!

However validity of JWKB sets $u_{\max}$ and validity of EFT bounds Δu above

$$\Delta u \gg L \gg \Lambda^2/k_v$$

→ **Cannot** make N arbitrarily large!

Stacked Shockwaves: Classical Perspective

JWKB approximation at leading order

$$k_v \frac{d^2\mathbf{x}}{du^2} = -\boldsymbol{\nabla} V(u, \mathbf{x})$$

→ **Newton's equation**! Transverse displacement estimate:

$$\Delta r(u) \sim -\frac{1}{k_v}\int_0^u du' \int_0^{u'} du'' \partial_r V(u,r)\bigg|_{r=b} = -\int_0^u du' \int_0^{u'} du'' \partial_r F(u,r)\bigg|_{r=b}$$

Approximation only valid until this is small relative to impact parameter. This sets $u_{\max}$

$$\Delta r(u_{\max}) \sim b \quad \longrightarrow \quad \int_0^{u_{\max}} du \int_0^u du' \frac{f(u')}{b^{D-2}} \sim 1$$

and the EFT contribution to the time delay is not resolvable:

$$|\Delta T_{\mathrm{EFT}}(u_{\max})| \ll \frac{|c_{\mathrm{GB}}|}{k_v} \int_0^{u_{\max}} du \int_0^u du' \frac{f(u')}{b^{D-2}} \sim \frac{|c_{\mathrm{GB}}|}{k_v}$$

→ Validity of JWKB equivalent to **negligibility of scattering**

Stacked Shockwaves: Quantum Perspective

Can separate interaction picture time-evolution operator for N **isolated** scattering events,

$$\hat{U}(t_N, t_0) = \mathcal{T} \prod_{n=1}^{N} \hat{U}(t_n, t_{n-1})$$

For sufficiently **long time intervals**

$$\hat{S}_{\text{total}} \approx \mathcal{T} \prod_{n=1}^{N} \hat{S}_n \approx (\hat{S}_1)^N \to \Delta T_{\text{total}} = N \Delta T_1$$

→ Too quick!

Example: N identical impulses $\hat{K}$

$$\hat{H}_{\text{int}}(t) = \sum_{n=1}^{N} \delta[t - (t_{n-1} + a_n)] \hat{K}, \quad 0 < a_n < t_n - t_{n-1}$$

S-matrix for individual scattering events **not identical** (for generic interaction)

$$\hat{S}_n = e^{i\hat{H}_0(t_{n-1}+a_n)} e^{-i\hat{K}} e^{-i\hat{H}_0(t_n+a)}$$

→ Effect of $\hat{H}_0$ is **diffusion**!

Balancing Causality

Scatter No More

Scattering in transverse direction crucial to see bound on time delay!

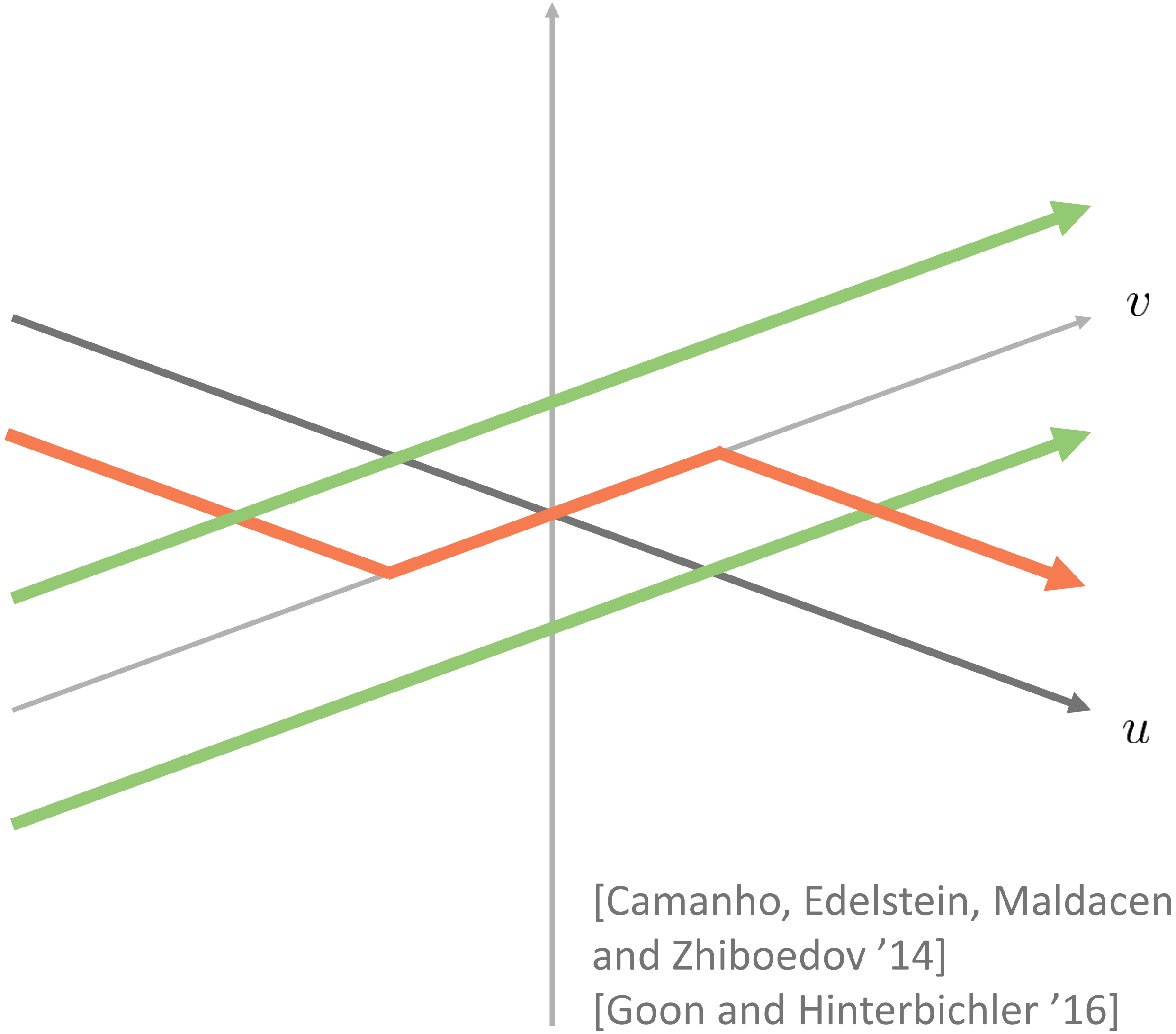
Propagate **between balancing** sources

$$F(u,\mathbf{x}) = f(u)\left(\frac{1}{|\mathbf{x}-\mathbf{b}|^{D-4}} + \frac{1}{|\mathbf{x}+\mathbf{b}|^{D-4}}\right)$$

By **symmetry**, no scattering in the transverse directions!

Accumulate time delay indefinitely to maximise causality violation…?

→ **No**, this is unstable!



[Camanho, Edelstein, Maldacena, and Zhiboedov ’14]
[Goon and Hinterbichler ’16]

Instability Timescale

Choose $\mathbf{b} = b\hat{\mathbf{z}}$. Classical equations of motion near origin

$$k_v \frac{d^2 z}{du^2} = -\frac{\partial V}{\partial z} \sim k_v \Omega^2 z, \quad \Omega^2 \sim \frac{1}{k_v} \frac{\partial^2 V}{\partial z^2}\bigg|_{\mathbf{x}=\mathbf{0}} < 0$$

JWKB Ansatz solution

$$z(u) \sim \frac{1}{\Omega(u)^{1/2}} \exp\left[\pm i \int_0^u du' \Omega(u')\right]$$

Instability becomes relevant at $u = u_{\text{inst}}$ defined by

$$\left| \int_0^{u_{\text{inst}}} du\, \Omega(u) \right| \sim \int_0^{u_{\text{inst}}} du \sqrt{\frac{f(u)}{b^{D-2}}} \sim 1$$

In fact, **uncertainty** of time delay operator in semiclassical approximation

$$\delta T \gtrsim 2^{-3/2} \left| \int^{u_{\text{inst}}} du [1 - 2u\Omega(u)]\, \Omega(u) \exp\left(2 \int_0^u du' \Omega(u')\right) \right|$$

→ To avoid scattering, need localised wavepackets: Far from S-matrix eigenstates!

Unbalanced Shockwaves

Either way, $u_{\rm inst}$ acts as $u_{\rm max}$, placing bound on time delay:

$$k_v|\Delta T_{\rm EFT}(u_{\rm max})| \sim k_v \frac{|c_{\rm GB}|}{\Lambda^2}\int_0^{u_{\rm max}} du \frac{f(u)}{b^{D-2}} \ll |c_{\rm GB}| \int_0^{u_{\rm max}} du \sqrt{\frac{f(u)}{b^{D-2}}} \sim |c_{\rm GB}|$$

Once again: **IR causality** not sufficient to rule out GB operator (despite lack of scattering classically!)

Gravity is unstable, so this holds for **generic configurations**: Sum of squared “frequencies” is non-positive

$$\sum_{n=1}^{D-2} \omega_n^2 = (\Omega^2)^i{}_i = \frac{1}{k_v}\frac{\partial^2 V}{\partial x^i \partial x_i}\bigg|_{\mathbf{x}=\mathbf{x}_0} = -\frac{1}{2}\partial_i\partial^i F(\mathbf{x}=\mathbf{x}_0) \leq 0$$

so at least one unstable direction.

→ In **Born approximation** (cf. paper), can reproduce lack of scattering classical limit etc.
Perturbation theory out of control when EFT contribution large!

Conclusion

IR Causality of Gauss-Bonnet Gravity

For scattering off single black hole and shockwave, multiple shock waves, and between shockwaves, always:

$$k_v \left|\Delta T^{\mathrm{EFT}}\right| \ll \left|c_{\mathrm{GB}}\right|$$

Perspective 1: IR causality imposes

$$|c_{\mathrm{GB}}| \lesssim 1$$

[Camanho, Edelstein, Maldacena, and Zhiboedov ’14]
[Reall, Tanahashi, and Way ‘14]

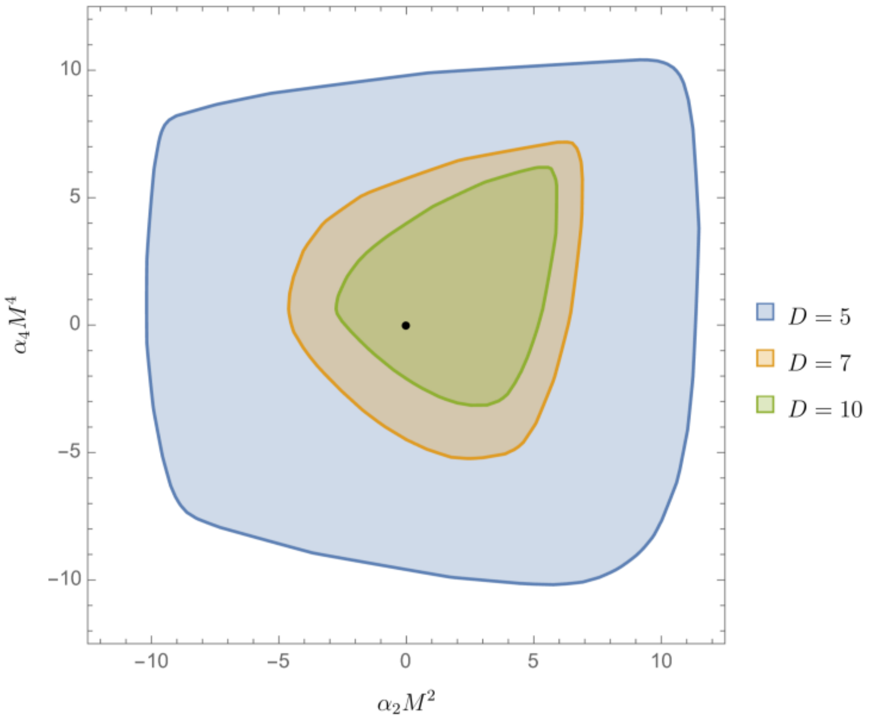
In contrast to earlier claims that causality requires $c_{\mathrm{GB}} = 0$.

→ Consistent with bootstrap and **positivity bounds**!

Can understand mild violation of positivity bounds from resolvability criterion $\Delta T^{\mathrm{EFT}} \gtrsim -\omega^{-1}$

Perspective 2: For EFTs $|c_{\mathrm{GB}}| \lesssim 1$ natural

→ GB gravity does not violate **IR causality**



[Caron-Huot and Li ‘22]

Summary

Conclusion

- In curved spacetime, correct notion to learn about EFTs is **IR causality**
  - To make statements about EFTs, need to properly identify **regime of validity** of EFT and approximations used.

- EGB gravity not ruled out by IR causality
  - **consistent** with gravitational positivity bounds!
  - Resolvability gives complementary understanding of **mild violation of positivity.**

Outlook

- Use infrared causality on less symmetric **backgrounds** to get more bounds on different EFT operators? [Carrillo González, de Rham, Jaitly, Pozsgay, Tokareva, and Tolley ‘22 & ’23]

- More physically: **de Sitter**? [Bittermann, McLoughlin, and Rosen ‘22]
  - IR causality is more local than asymptotic causality!
  - Extend using notion of de Sitter S-Matrix [Melville and Pimentel ‘23]

Thanks for your attention!
Questions?

Bonus Slides

Time Delay in Field Theory

Given spectral decomposition of **full S-matrix**,

$$\hat{\mathbb{S}} = \sum_{I,J} \int_0^\infty dE \, |E, I\rangle \, \hat{\mathbb{S}}_{IJ} \, \langle E, J|$$

time delay operator on full Fock space is

$$\Delta\hat{\mathbb{T}} = -i \sum_I \int_0^\infty dE \frac{\partial}{\partial \epsilon} \left(\hat{\mathbb{S}}^\dagger \, |E - \frac{\epsilon}{2}, I\rangle \, \langle E + \frac{\epsilon}{2}, I| \hat{\mathbb{S}} \right) \Bigg|_{\epsilon=0}$$

Recover **Wigner-Smith** operator when projected onto single-particle S-matrix $\hat{S}$:

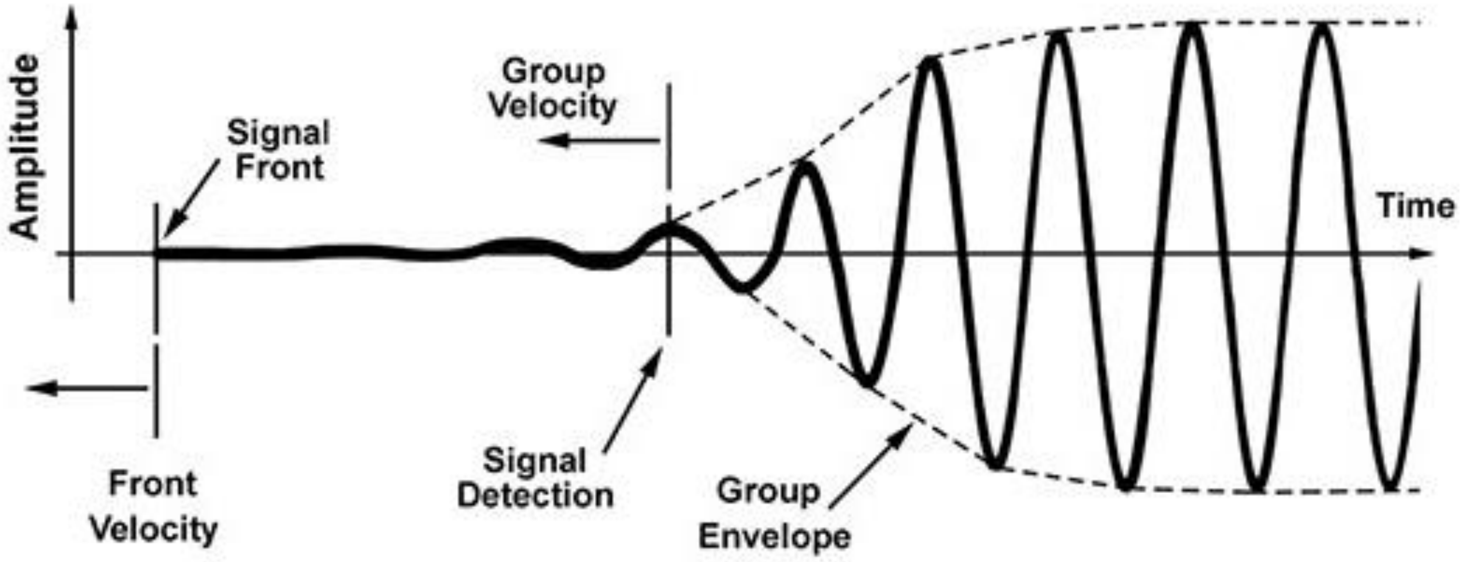
$$\Delta\hat{T} = -\frac{i}{2} S^\dagger \frac{\partial \hat{S}}{\partial E} + \frac{i}{2} \frac{\partial \hat{S}^\dagger}{\partial E} S$$

In elastic region, recover **Eisenbud-Wigner** time delay when evaluated on eigenstates of the S-matrix :

$$\hat{S} \, |\delta\rangle = e^{2i\hat{\delta}} \, |\delta\rangle \rightarrow \langle \delta| \, \Delta\hat{T} \, |\delta\rangle = 2 \frac{\partial \delta}{\partial E}$$

→ Key point: Known expressions use **various approximations!**

Infrared Causality and Front Velocities



Front velocity sets causality

$$v_{\text{front}} = \lim_{\omega \to \infty} v_{\text{phase}}(\omega)$$

precisely correspond to **high-frequency modes**.

Regime of Validity

To estimate regime of validity: Bound **Lorentz scalars** at asymptotic infinity. Schematically:

$$\left(\frac{\nabla}{\Lambda}\right)^a \left(\frac{A}{\Lambda^4}\right)^b \left(\frac{k}{\Lambda}\right)^c \ll 1, \quad A_{\mu\nu} = R_{\mu\alpha\nu\beta}k^\alpha k^\beta$$

For $a \to \infty$

$$\left(\frac{\Box}{\Lambda^2}\right)^{a/2} \left(\frac{S}{\Lambda^{[S]}}\right)^p \ll 1 \longrightarrow \frac{|\boldsymbol{\nabla}|}{\Lambda} \sim \frac{\partial_r}{\Lambda} \ll 1$$

For $b \to \infty$

$$\mathrm{Tr}(A^b) = \underbrace{A^{\alpha_b}{}_{\alpha_1} A^{\alpha_1}{}_{\alpha_2} \ldots A^{\alpha_{b-1}}{}_{\alpha_b}}_{b \text{ times}} \ll \Lambda^{4b} \longrightarrow A \sim \frac{\partial_r F}{r} k_v^2 \ll \Lambda^4$$

For $p \to \infty, q \to \infty$ for $p + q = a$

$$[(k^\mu \nabla_\mu)^p A_{\alpha\beta}][(k^\nu \nabla_\nu)^q A^{\alpha\beta}] \ll \Lambda^{8+2a} \longrightarrow k^\mu \partial_\mu \ll \Lambda^2$$

Regime of Validity: Sanity Check

To check bounds on Lorentz scalars will be realised: Compute **higher-order EFT** correction due to

$$S_{\text{eff}} = \int d^D x \sqrt{-g}\, M_{\text{Pl}}^{D-2} \left(\frac{1}{2} R + \frac{c_{\text{GB}}}{\Lambda^2} R_{\text{GB}}^2 + \frac{c_{R^3}}{\Lambda^4} (R^3) + \frac{c_{R^4}}{\Lambda^6} (R^4) + \dots \right)$$

E.g. equations of motion for (transverse) tensor perturbations

$$\begin{aligned}
&\Box \Phi_T - 8 \frac{c_{\text{GB}}}{\Lambda^2} \frac{\partial_r F}{r} \partial_v^2 \Phi_T \\
&+ 24 \frac{c_{R^3}}{\Lambda^4} \left[\frac{\partial_u \partial_r F}{r} \partial_v^3 \Phi_T - (D-2) \frac{\partial_r F}{r^2} \partial_r \partial_v^2 \Phi_T - 4(D-2) \frac{\partial_r F}{r^3} \partial_v^2 \Phi_T \right] \\
&+ 16 \frac{c_{R^4}}{\Lambda^6} \left(\frac{\partial_r F}{r} \right)^2 \partial_v^4 \Phi_T + 192 \frac{c_{\text{GB}}\, c_{R^3}}{\Lambda^6} \left(\frac{\partial_r F}{r} \right)^2 \partial_v^4 \Phi_T = 0
\end{aligned}$$

Leading-order theory not trustworthy when corrections dominate:

$$\partial_r \ll \Lambda, \quad k^\mu \partial_\mu \ll \Lambda^2, \quad \frac{\partial_r F}{r} k_v^2 \ll \Lambda^4$$

→ Reproduces EFT regime of validity.

Example: Consistency and Infrared Causality

Illustrative example on curved spacetime: **Goldstone**

$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 + \frac{g}{\Lambda^D} (\nabla \phi)^4 + \dots \right]$$

With spherical symmetry for scalar

$$\bar{\phi}'(r) = \frac{\alpha}{r^{D-2} C(r)} + \mathcal{O}\left(\Lambda^{-D}\right)$$

Matter sources **backreaction** to geometry via Einstein's equation

$$\Box \left(h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right) = -\frac{2}{M_{\mathrm{Pl}}^{D-2}} T_{\mu\nu}$$

→ Total time delay:

$$\Delta T \sim \left[\left(\frac{r_g}{b} \right)^{D-3} + \frac{\alpha^2}{M_{\mathrm{Pl}}^{D-2} b^{2D-6}} \right] b + \frac{g}{\Lambda^D} \frac{\alpha^2}{b^{2D-3}}$$

Example: Consistency and Infrared Causality

On flat space (without dynamical gravity), causality and positivity bounds imposed

$$g > 0$$

With gravity:

Asymptotic causality: Extremising for tightest bounds

$$\Delta T \gtrsim -\omega^{-1} \longrightarrow g \gtrsim -\left(\frac{\Lambda}{M_{\rm Pl}}\right)^{(D-2)/2}$$

→ Not natural (analytic), and weaker than gravitational positivity bounds!

Infrared causality:

$$\Delta T^{\rm EFT} \gtrsim -\omega^{-1} \longrightarrow g \gtrsim -\left(\frac{\Lambda}{M_{\rm Pl}}\right)^{D-2}$$

→ Agrees with gravitational positivity bounds!