Report 1.10

Taylor Grimm

June 2023

Work problem 1.12, pg. 30 of Agresti
A researcher routinely tests using a nominal $P(\text{type I error}) = 0.05$, rejecting H_0 if the P-value ≤ 0.05. An exact test using test statistic T has null distribution $P(T = 0) = 0.30, P(T = 1) = 0.62$, and $P(T = 2) = 0.08$, where a higher T provides more evidence against the null.

a. With the usual P-value, show that the actual $P(\text{type I error}) = 0$.

   Under the null distribution given above, the lowest probability is $P(T = 2) = 0.08$. So, even if we observe the most extreme possible test statistic of $T = 2$, we will not reject H_0 since $0.08 > 0.05$, so $P(\text{type I error}) = 0$.

b. With the mid P-value, show that the actual $P(\text{type I error}) = 0.08$.

   The mid P-value is computed as $\frac{1}{2}P(T = 2) + P(T > 2) = \frac{1}{2}.08 + 0 = 0.04$, so we reject H_0 since $.04 < 0.05$. This results in an actual $P(\text{type I error}) = 0.08$ since $P(T = 2) = 0.08$.

c. Find $P(\text{type I error})$ in parts (a) and (b) when $P(T = 0) = 0.30$, $P(T = 1) = 0.66$, $P(T = 2) = 0.04$. Note that the test with mid P-value can be conservative or liberal. The exact test with ordinary P-value cannot be liberal.

   With the usual P-value, we would reject if we observe $T = 2$, which has probability of 0.04 under the null distribution, so $P(\text{type I error}) = 0.04$.

   With the mid P-value, observing $T = 2$ yields a mid P-value of $\frac{1}{2}P(T = 2) + P(T > 2) = \frac{1}{2}.04 + 0 = 0.02 < 0.05$, which causes us to reject H_0. In this case, the $P(\text{type I error}) = 0.04$.

d. In part (a), a randomized-decision test generates a uniform random variable U from $[0, 1]$ and rejects H_0 if both $T = 2$ and $U \leq \frac{5}{8}$. Show the actual $P(\text{type I error}) = 0.05$. Is this a sensible test?

   Since the uniform random variable is independent of the value of T, the probability of $T = 2$ and $U \leq \frac{5}{8}$ is $P(T = 2) \times P(U \leq \frac{5}{8}) = 0.08 \times \frac{5}{8} = 0.05$. Since we reject H_0 if the P-value ≤ 0.05, we reject H_0 here.

The actual P(type I error) is the probability, under the null, of $T = 2$ and $U \leq \frac{5}{8}$, which is 0.05.

This test is not sensible.