# Change point detection in Python

AMF seminar

## Charles Truong<sup>1</sup>

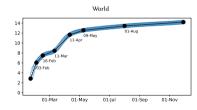
<sup>1</sup>Centre Borelli Université Paris-Saclay ENS Paris-Saclay, CNRS

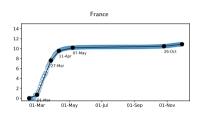
Wednesday 6th January

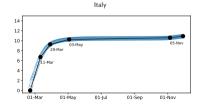




- ▶ Change point detection is a common task when dealing with non-stationary time series.
- ▶ Application example: study of COVID-19 infection curve [Jiang et al., 2020].
- Data from "Our World in Data" (ourworldindata.org).
- ► Cumulative reported deaths in log-scale.
- Piecewise linear trends (linear spline smoothing with optimal knot selection).
- ► The slope gives the growth rate ("log-return").



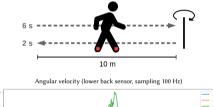




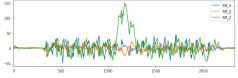
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- Application example: automatic diagnosis of neurologically impaired patients [Truong et al., 2019a].

Healthy and pathological subjects underwent a fixed protocol:

- standing still,
- walking 10m,
- turning around,
- walking back,
- standing still.



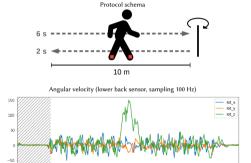
Protocol schema



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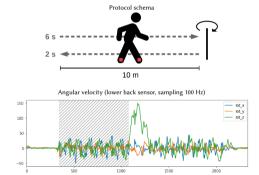
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2000

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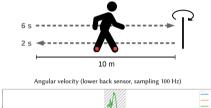
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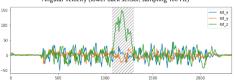
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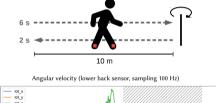
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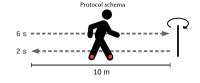
Can also be applied to finance, industrial monitoring, public health monitoring, etc. [Truong et al., 2020].

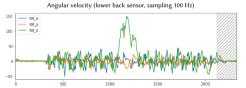
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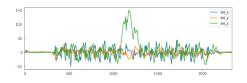
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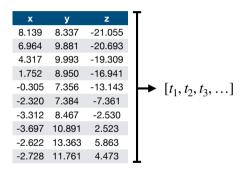




## What is change point detection?

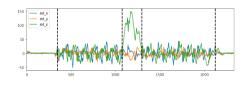
- ▶ Change point detection consists in finding the temporal boundaries between homogeneous time periods.
- ► Informally: "multivariate signal —> list of change point indexes"

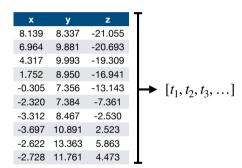




## What is change point detection?

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- 1. Introduction
- 2. What is change point detection
- 3. General principle of ruptures
- 4. Examples

Change in mean and variance (1-D)

Change in mean and variance (n-D

Change in distribution (parametric)

Change in distribution (non-parametric)

Gait analysis

- 5. Supervised change point detection
  - General principle

Learn the representation

6. Conclusion

# General principle

How to choose a segmentation?



$$\mathcal{T} = \{t_1, t_2, t_3\}$$

$$V(\mathcal{T}) = c(y_{0...t_1}) + c(y_{t_1...t_2}) + c(y_{t_2...t_3}) + c(y_{t_3...\mathcal{T}})$$

#### Problem 1.

Fixed number K of change points:

$$\widehat{\mathcal{T}} := \underset{\mathcal{T}}{\operatorname{arg \, min}} \ V(\mathcal{T}) \quad \text{s.t.} \ |\mathcal{T}| = K.$$

The "best segmentation" is the minimizer, denoted  $\widehat{\mathcal{T}}$ , of a criterion  $V(\mathcal{T})$ :

$$V(\mathcal{T}):=\sum_{k=0}^K c(y_{t_k\ldots t_{k+1}}).$$

Cost example:  $c(y) = \sum_t (y_t - \bar{y})^2$ .

#### Problem 2.

Unknown number of change points:

$$\widehat{\mathcal{T}} := \underset{\mathcal{T}}{\operatorname{arg \, min}} \ V(\mathcal{T}) \ + \ \operatorname{pen}(\mathcal{T})$$

where pen( $\mathcal{T}$ ) measures the complexity of a segmentation  $\mathcal{T}$ .

## General principle

Detection methods are the combination of three elements [Truong et al., 2020].

Cost function

Search method

Constraint

Criterion 
$$V(\mathcal{T})$$
 to minimize:  $V(\mathcal{T}) := \sum_{k=0}^{K} c(y_{t_k..t_{k+1}})$ .

#### Problem 1.

Fixed number *K* of change points:

$$\widehat{\mathcal{T}} := \begin{bmatrix} \arg \min_{\mathcal{T}} & V(\mathcal{T}) \\ \end{bmatrix}$$
 s.t.  $|\mathcal{T}| = K$ 

#### Problem 2.

Unknown number of change points:

$$\widehat{\mathcal{T}} := \begin{array}{c|c} \operatorname{arg\ min} & V(\mathcal{T}) + \end{array} \begin{array}{c|c} \operatorname{pen}(\mathcal{T}) \end{array}$$

where pen( $\mathcal{T}$ ) measures the complexity of a segmentation  $\mathcal{T}$ .

# General principle

A modular architecture.

```
First import and data loading.
[319]:
         1 import ruptures as rpt
           signal = get signal(...) # user defined
[329]: 1 # cost function
                                                                             Choosing the cost function
         2 c = rpt.costs.CostL2()
                                                                             Here, c(y) = \sum_t (y_t - \bar{y})^2.
                                                                             Choosing the search method
         1 # search method
f3301:
         2 algo = rpt.Binseq(jump=5, min size=10, custom cost=c)
                                                                             Here, binary segmentation.
                                                                             Fitting the algorithm.
[331]: 1 # fit algo
         2 algo.fit(signal)
                                                                             Choosing the constraint
[332]: 1 # predict change points
         2 # fixed number of changes
                                                                             Then detecting the change points ("predict").
         3 bkps = algo.predict(n bkps=10)
         4 # or penalized detection
         5 bkps = algo.predict(pen=50)
                                                                             Measuring the detection accuracy.
        1 from ruptures.metrics import hausdorff
[333]:
         3 error = hausdorff(true bkps, bkps)
```

## A discrete optimization problem

Minimize the sum of cost over all segmentations:

$$\min_{t_1,t_2,...,t_K} \sum_{k=0}^K c(y_{t_k..t_{k+1}}).$$

or

$$\min_{t_1,t_2,...,t_K} \sum_{k=0}^K c(y_{t_k..t_{k+1}}) + \beta K.$$

- A naive implementation is prohibitive  $\binom{T}{K}$  segmentations).
- The problem is solved recursively using Bellman's dynamic programming.
- ▶ For most cost functions, the complexity is  $\mathcal{O}(T^2)$  in operations and  $\mathcal{O}(T)$ .
- ▶ Heuristics to approximately solve this problem exist: binary segmentation (with variants) and window-sliding. Complexity in  $\mathcal{O}(T)$ .

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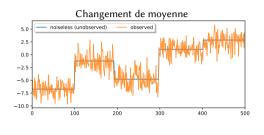
Change in mean and variance (n-D)

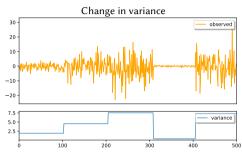
Change in distribution (parametric)

Change in distribution (non-parametric)

- Supervised change point detection General principle Learn the representation
- Conclusion

## Change in mean and variance (1-D)





Cost function:

$$c(y_{a...b}) = \sum_{t=a}^{b-1} (y_t - \bar{y}_{a...b})^2$$

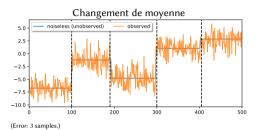
where  $\bar{y}_{a..b}$  is the empirical mean of  $y_{a..b}$ .

Cost function:

$$c(y_{a..b}) = (b - a) \log(\hat{\sigma}_{a..b})$$

where  $\hat{\sigma}_{a..b}$  the empirical standard-deviation  $y_{a..b}$ .

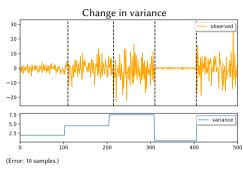
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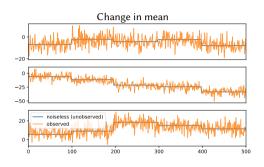


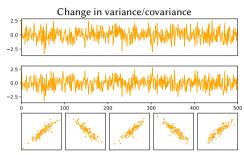
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## Change in mean and variance (n-D)





Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} ||y_t - \bar{y}_{a..b}||^2$$

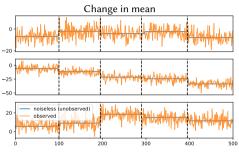
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$$c(y_{a...b}) = (b-a) \log \det \hat{\Sigma}_{a...b}$$

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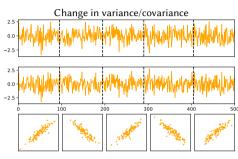


(Error: 7 samples.)

#### Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} ||y_t - \bar{y}_{a..b}||^2$$

where  $\bar{y}_{a..b}$  is the empirical mean of  $y_{a..b}$ .



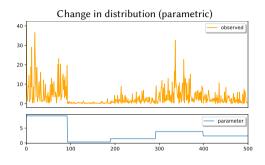
(Error: 3 samples.)

Cost function:

$$c(y_{a...b}) = (b-a) \log \det \hat{\Sigma}_{a...b}$$

where  $\hat{\sigma}_{a..b}$  is the empirical covariance matrix of  $y_{a..b}$ .

# Change in distribution (parametric)



Cost function:

$$c(y_{a..b}) = -\max_{\theta} \log f_{\theta}(y_{a..b})$$

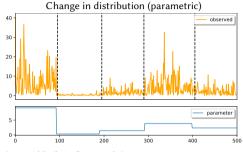
where  $f_{\theta}$  is the density of the chosen distribution, parametrized by  $\theta.$ 

**Important fact.** The estimated change points converge to the true changes.

[Lavielle, 1999, Detection of multiples changes in a sequence of dependant variables. Stochastic Processes and Their Applications, 83(1), 79–102.]

- Not necessarily i.i.d. observations.
- Can be strongly dependant (but stationary).
- ▶ ...

# Change in distribution (parametric)



(exponential distribution, Error: 6 samples.)

Cost function:

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- Not necessarily i.i.d. observations.
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- **...**

# Change in distribution (non-parametric)

When the underlying distribution is unknown:

- ► [Arlot et al., 2019, A kernel multiple change-point algorithm via model selection. Journal of Machine Learning Research, 20(162), 1–56.]
- [Matteson and James, 2014, A nonparametric approach for multiple change point analysis of multivariate data. Journal of the American Statistical Association, 109(505), 334–345.]
- [Ross and Adams, 2012, Two nonparametric control charts for detecting arbitrary distribution changes. Journal of Quality Technology, 44(2), 102–117.]

The kernel approach is particularly interesting because it can deal with non-numerical data: symbolic signals, texts, functional time series,...

### General principle:

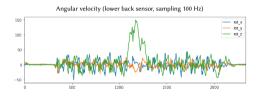
- The signal is mapped to a high-dimensional space:  $y_t \longrightarrow \phi(y_t)$ .
- **Detection** of change in the mean of the  $\phi(y_t)$ .

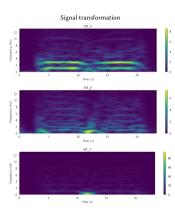
#### Cost function:

$$c(y_{a..b}) = \sum_{t=a}^{b-1} \|\phi(y_t) - \bar{\mu}\|_{\mathcal{H}}^2$$

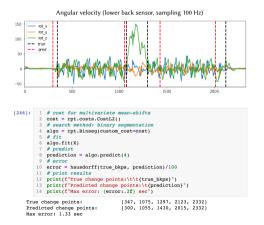
where  $\bar{\mu}$  is the empirical mean of  $\{\phi(y_t)\}_{a...b}$ . (Computed using the kernel trick.)

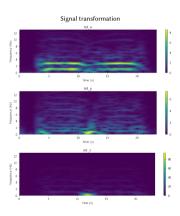
- ▶ To simplify the detection task, the signal is transformed (here, short-term Fourier transform).
- ► Then mean-shifts are detected.





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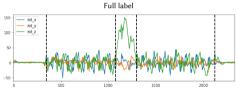
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- 5. Supervised change point detection General principle Learn the representation
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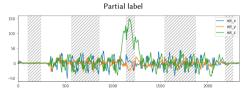
## Supervised change point detection

General principle

- ▶ How to integrate expert knowledge to calibrate the change point detection? [Truong et al., 2019b]
- ► The expert provides the target segmentation: either full or partial label.
- ► Labels are hard to collect. The easier for the clinicians, the better.



The exact change point locations are provided.



Only homogeneous periods (hatched areas) are provided (weakly supervised).

Labels are transformed into constraints. Intuitively, the problem is:

Learn a transformation  $\Psi$  such that  $d(\Psi(x_t), \Psi(x_s)) \le u$  if  $x_t$  and  $x_s$  similar  $d(\Psi(x_t), \Psi(x_s)) \ge l$  if  $x_t$  and  $x_s$  dissimilar (u > 0) and (u > 0)

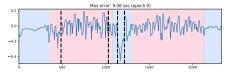
Two samples are *similar* if they belong to the same regime.

Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).

Epoch by epoch (epoch 0)



True segmentation: alternating colors.

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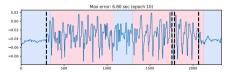
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Epoch by epoch (epoch 10)



True segmentation: alternating colors.

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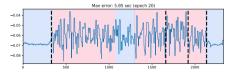
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Epoch by epoch (epoch 20)



True segmentation: alternating colors.

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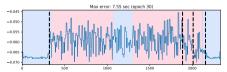
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Epoch by epoch (epoch 30)



True segmentation: alternating colors.

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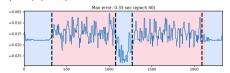
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Epoch by epoch (epoch 40)



True segmentation: alternating colors.

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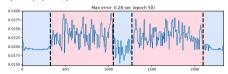
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This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and max-pooling (with tensorflow).

Epoch by epoch (epoch 50)



True segmentation: alternating colors.

Labels are transformed into constraints. Intuitively, the problem is:

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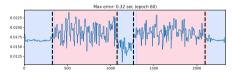
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This setting can be used to learn a deep representation.

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Epoch by epoch (epoch 60)



True segmentation: alternating colors.

Labels are transformed into constraints. Intuitively, the problem is:

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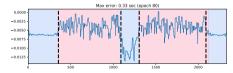
Two samples are *similar* if they belong to the same regime.

Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).

Epoch by epoch (epoch 80)



True segmentation: alternating colors.

Labels are transformed into constraints. Intuitively, the problem is:

Learn a transformation  $\Psi$  such that  $d(\Psi(x_t), \Psi(x_s)) \le u$  if  $x_t$  and  $x_s$  similar  $d(\Psi(x_t), \Psi(x_s)) \ge l$  if  $x_t$  and  $x_s$  dissimilar (u > 0 and l > 0)

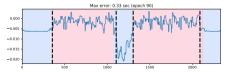
Two samples are *similar* if they belong to the same regime.

Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).

Epoch by epoch (epoch 90)



True segmentation: alternating colors.

Labels are transformed into constraints. Intuitively, the problem is:

Learn a transformation  $\Psi$  such that  $d(\Psi(x_t), \Psi(x_s)) \le u$  if  $x_t$  and  $x_s$  similar  $d(\Psi(x_t), \Psi(x_s)) \ge l$  if  $x_t$  and  $x_s$  dissimilar (u > 0) and l > 0)

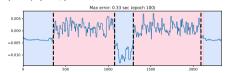
Two samples are *similar* if they belong to the same regime.

Two samples are *dissimilar* if they belong to consecutive regimes.

This setting can be used to learn a deep representation.

Here, two layers of temporal separable convolutions and maxpooling (with tensorflow).

Epoch by epoch (epoch 100)



True segmentation: alternating colors.

### Conclusion

- Code for those experiments will be available on my GitHub github.com/deepcharles.
- New methods are frequently implemented in ruptures.
- Extensions to graph/network data soon.
- ▶ Differentiable dynamic programming for end-to-end unsupervised representation learning.

### References



Arlot, S., Celisse, A., and Harchaoui, Z. (2019).

A kernel multiple change-point algorithm via model selection.

Journal of Machine Learning Research, 20(162):1-56.



Jiang, F., Zhao, Z., and Shao, X. (2020).

Time series analysis of COVID-19 infection curve: a change-point perspective.

Journal of Econometrics.



Lavielle, M. (1999).

Detection of multiples changes in a sequence of dependant variables.

Stochastic Processes and their Applications, 83(1):79-102.



Matteson, D. S. and James, N. A. (2014).

A nonparametric approach for multiple change point analysis of multivariate data. *Journal of the American Statistical Association*, 109(505):334–345.



Ross, G. J. and Adams, N. M. (2012).

Two nonparametric control charts for detecting arbitrary distribution changes.

Journal of Quality Technology, 44(2):102–117.

### References



Truong, C., Barrois-Müller, R., Moreau, T., Provost, C., Vienne-Jumeau, A., Moreau, A., Vidal, P.-P., Vayatis, N., Buffat, S., Yelnik, A., Ricard, D., and Oudre, L. (2019a).

 $\boldsymbol{A}$  data set for the study of human locomotion with inertial measurements units.

Image Processing On Line, 9.



Truong, C., Oudre, L., and Vayatis, N. (2019b).

Supervised kernel change point detection with partial annotations.

In Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pages 1–5, Brighton, UK.



Truong, C., Oudre, L., and Vayatis, N. (2020).

 $Selective\ review\ of\ off line\ change\ point\ detection\ methods.$ 

Signal Processing, 167.