

# Impact of half-wave plate systematics on the measurement of cosmic birefringence from CMB polarization

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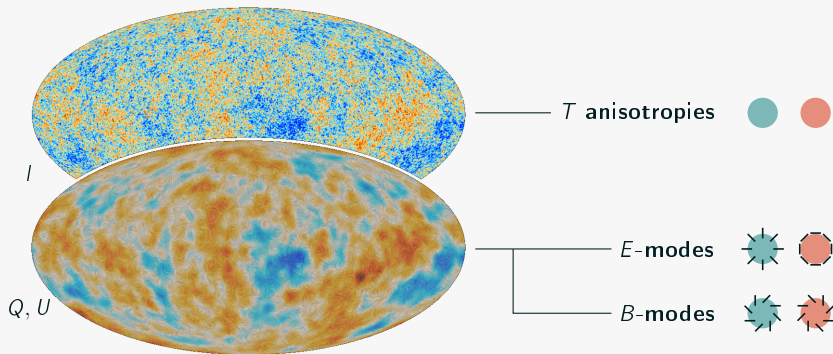
**Marta Monelli**

Max Planck Institut für Astrophysik  
Garching (Germany)

February 2nd, 2023

# CMB anisotropies

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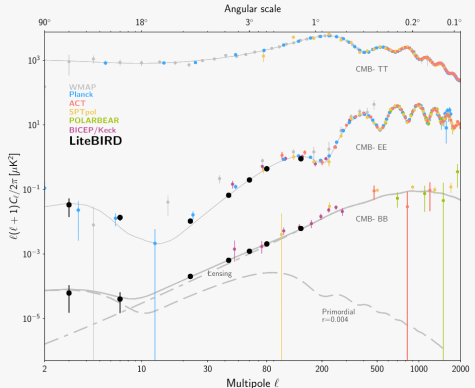


# searching $B$ -modes from inflation

*Expectation:* inflation-sourced perturbations leave traces on the CMB polarization.

Large scale  $B$ -modes can probe inflation.

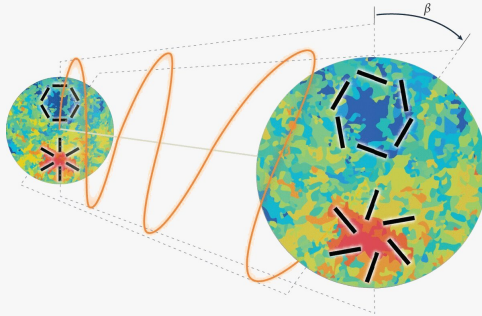
**Unprecedented sensitivity requirements!**



# a side effect: measuring cosmic birefringence

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CMB might also carry information about parity-violating new physics: **cosmic birefringence**.  
(time-dependent parity-violating pseudoscalar field)



mixing of  $E$  and  $B$  modes: 
$$\begin{cases} a_{\ell m, \text{obs}}^E = a_{\ell m}^E \cos 2\beta - a_{\ell m}^B \sin 2\beta, \\ a_{\ell m, \text{obs}}^B = a_{\ell m}^E \sin 2\beta + a_{\ell m}^B \cos 2\beta. \end{cases}$$


## trying to constrain $\beta$

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$$\begin{cases} C_{\ell,\text{obs}}^{TT} = C_{\ell}^{TT}, \\ C_{\ell,\text{obs}}^{EE} = \cos^2(2\beta)C_{\ell}^{EE} + \sin^2(2\beta)C_{\ell}^{BB} - \sin(4\beta)C_{\ell}^{EB}, \\ C_{\ell,\text{obs}}^{BB} = \cos^2(2\beta)C_{\ell}^{BB} + \sin^2(2\beta)C_{\ell}^{EE} + \sin(4\beta)C_{\ell}^{EB}, \\ C_{\ell,\text{obs}}^{TE} = \cos(2\beta)C_{\ell}^{TE} - \sin(2\beta)C_{\ell}^{TB}, \\ C_{\ell,\text{obs}}^{EB} = \sin(4\beta)(C_{\ell}^{EE} - C_{\ell}^{BB})/2 + \cos(4\beta)C_{\ell}^{EB}, \\ C_{\ell,\text{obs}}^{TB} = \sin(2\beta)C_{\ell}^{TE} + \cos(2\beta)C_{\ell}^{TB}. \end{cases}$$

$$C_{\ell,\text{obs}}^{EB} = \tan(4\beta)(C_{\ell,\text{obs}}^{EE} - C_{\ell,\text{obs}}^{BB})/2.$$

## trying to constrain $\beta$

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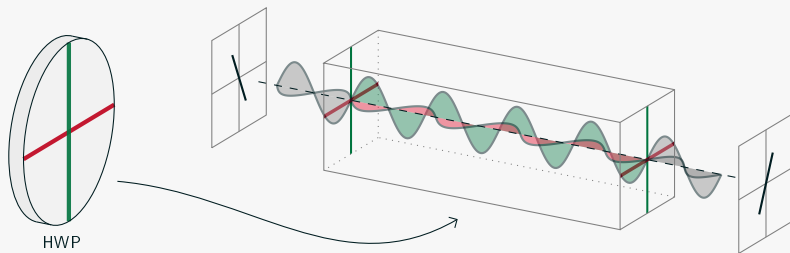
$$\begin{cases} C_{\ell,\text{obs}}^{TT} = C_{\ell}^{TT}, \\ C_{\ell,\text{obs}}^{EE} = \cos^2(2\beta)C_{\ell}^{EE} + \sin^2(2\beta)C_{\ell}^{BB} - \sin(4\beta)C_{\ell}^{EB}, \\ C_{\ell,\text{obs}}^{BB} = \cos^2(2\beta)C_{\ell}^{BB} + \sin^2(2\beta)C_{\ell}^{EE} + \sin(4\beta)C_{\ell}^{EB}, \\ C_{\ell,\text{obs}}^{TE} = \cos(2\beta)C_{\ell}^{TE} - \sin(2\beta)C_{\ell}^{TB}, \\ C_{\ell,\text{obs}}^{EB} = \sin(4\beta)(C_{\ell}^{EE} - C_{\ell}^{BB})/2 + \cos(4\beta)C_{\ell}^{EB}, \\ C_{\ell,\text{obs}}^{TB} = \sin(2\beta)C_{\ell}^{TE} + \cos(2\beta)C_{\ell}^{TB}. \end{cases}$$

$$C_{\ell,\text{obs}}^{EB} = \tan(4\beta)(C_{\ell,\text{obs}}^{EE} - C_{\ell,\text{obs}}^{BB})/2.$$

$$\beta = 0.35 \pm 0.14 \text{ (68\%CL)}$$

To extract this kind of information from CMB  
**systematics** have to be kept under control.

## the HWP: reducing systematics

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A **rotating** half-wave plate (HWP) as first optical element:

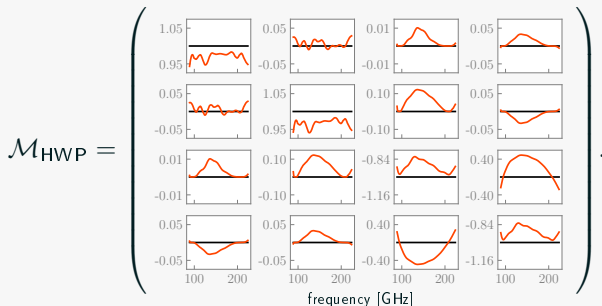
- ▶ modulates the signal to  $4f_{\text{HWP}}$ , allowing to “escape”  $1/f$  noise;
- ▶ makes possible for a single detector to measure polarization, reducing pair-differencing systematics.



# the HWP: inducing systematics

**Mueller calculus:** radiation described as  $S = (I, Q, U, V)$ , effect of polarization-altering devices parametrized by  $\mathcal{M}$ : so that  $S' = \mathcal{M} \cdot S$ .

For an ideal HWP,  $\mathcal{M}_{\text{ideal}} = \text{diag}(1, 1, -1, -1)$ , but let's look at a realistic case:



**how does this affect the observed maps?**

# steps we took in that direction

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- ▶ work on a **simulation pipeline** for a LiteBIRD-like mission;
- ▶ simulate observed maps in presence of non-ideal HWP;
- ▶ derive **analytical formulae** to interpret the output.

PREPARED FOR SUBMISSION TO JCAP

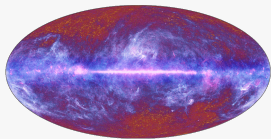
## Impact of half-wave plate systematics on the measurement of **cosmic birefringence** from CMB polarization

Marta Monelli,<sup>a</sup> Eiichiro Komatsu,<sup>a,b</sup> Alexandre Adler,<sup>c</sup> Matteo Billi,<sup>d,e,f</sup> Paolo Campeti,<sup>g,h</sup> Nadia Dachlythra,<sup>c</sup> Adriaan Duivenvoorden,<sup>h</sup> Jon Gudmundsson,<sup>c</sup> and Martin Reinecke.<sup>a</sup>

simulations

# what do we simulate

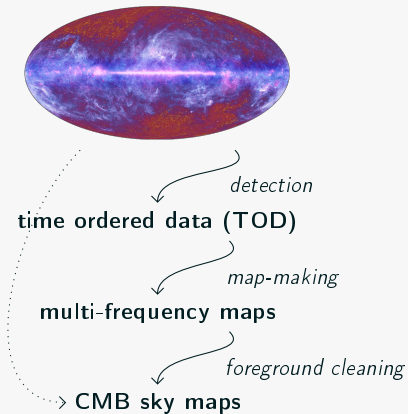
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→ CMB sky maps

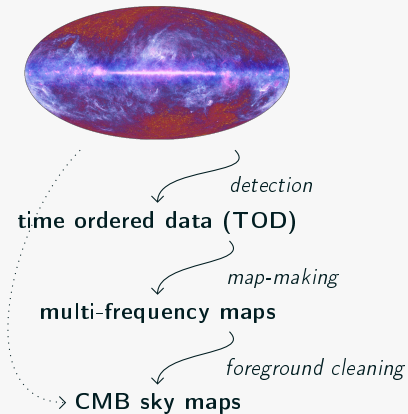
# what do we simulate

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# what do we simulate

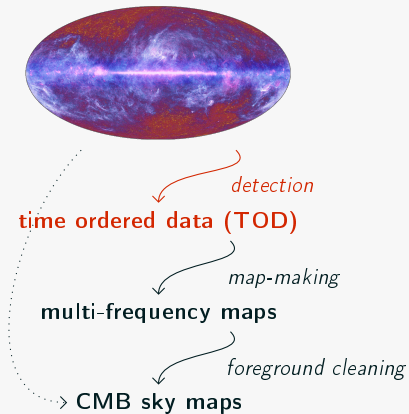
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**TOD:** collection of the signal detected by *each of the (4508) detectors* during the whole *(3-year) mission*.

# what do we simulate

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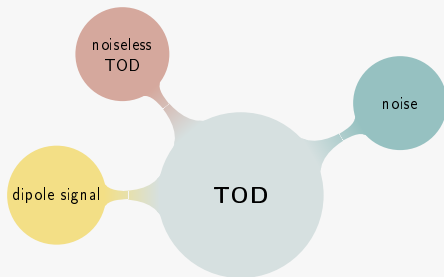


**TOD:** collection of the signal detected by *each of the (4508) detectors* during the whole *(3-year) mission*.

Simulating TOD is crucial in the planning of any CMB experiment: helps studying potential systematic effects.

# sketch of the pipeline

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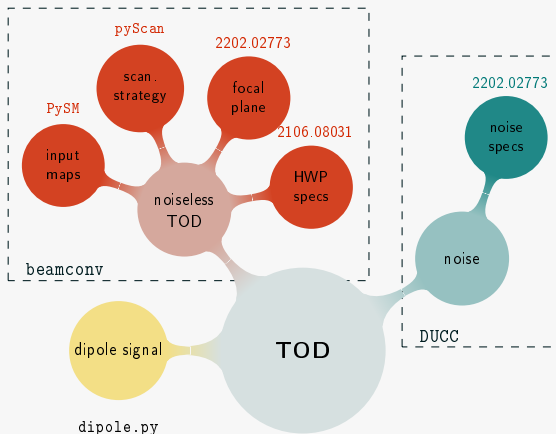




# sketch of the pipeline

beamconv: convolution code simulating TOD for CMB experiments with realistic polarized **beams**, scanning strategies and **HWP**.

DUCC: collection of basic programming tools for numerical computation: `fft`, `sht`, `healpix`, `totalconvolve`...



# working assumptions

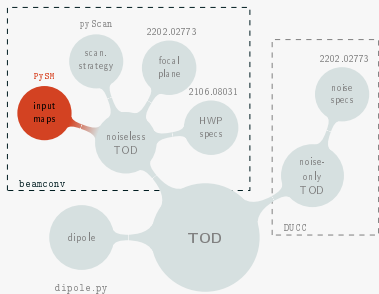
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To focus on the impact of **HWP non-idealities**, we consider a simplified problem:

- ▶ no noise,
- ▶ single frequency,
- ▶ CMB-only,
- ▶ simple beams,
- ▶ HWP aligned to the detector line of sight.

# input maps

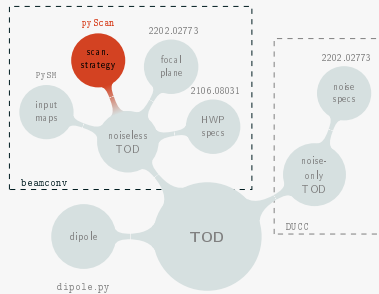
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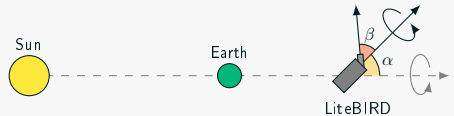
The pipeline can be fed with arbitrary input maps: **CMB**, **foregrounds**, or both.

**In the paper:**  $I$ ,  $Q$  and  $U$  input maps with  $n_{\text{side}} = 512$  from best-fit 2018 Planck power spectra;

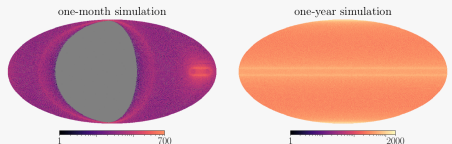
# scanning strategy



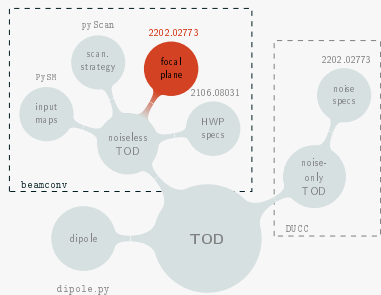
The pipeline can read or calculate pointings. We implemented some functionalities of pyScan in beamconv to deal with satellite missions.



**In the paper:** 1 year of LiteBIRD-like scanning strategy.



# focal plane specifics



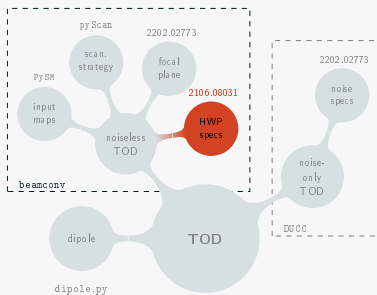
The pipeline can read from the Instrument Model Database (IMO):

```
{'name': 'M02_030_QA_140T',
 'wafer': 'M02',
 'pixel': 30,
 'pixtype': 'MP1',
 [...],
 'pol': 'T',
 'orient': 'Q',
 'quat': [1, 0, 0, 0]}
```

**In the paper:** 160 dets from M1-140.

specs.	values
$f_{\text{samp}}$	19 Hz
HWP rpm	39
FWHM	30.8 arcmin
offset quats.	[...]

# HWP specifics



In the paper: HWP is assumed to be ideal in the **first** simulation run (ideal TOD) and realistic in the **second** (non-ideal TOD).

Realistic HWP Mueller matrix elements as shown previously:

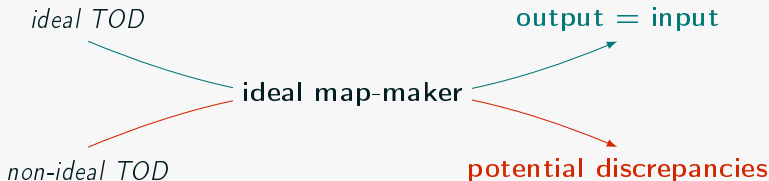
$$\mathcal{M}_{\text{HWP}} = \begin{pmatrix} \begin{matrix} 1.05 & 0.05 & 0.01 & 0.05 \\ 0.95 & -0.05 & -0.01 & -0.05 \end{matrix} \\ \begin{matrix} 0.05 & 1.05 & 0.10 & 0.05 \\ -0.05 & 0.95 & -0.10 & -0.05 \end{matrix} \\ \begin{matrix} 0.01 & 0.10 & -0.84 & 0.40 \\ -0.01 & -0.10 & -1.16 & -0.40 \end{matrix} \\ \begin{matrix} 0.05 & 0.05 & 0.40 & -0.84 \\ -0.05 & -0.40 & -1.16 & -1.16 \end{matrix} \end{pmatrix}.$$

frequency [GHz]

# what about maps?

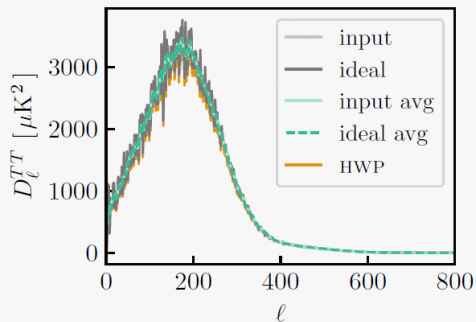
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Both ideal and non-ideal TOD processed by **ideal** bin-averaging map-maker.



# ideal vs non-ideal output spectra (1)

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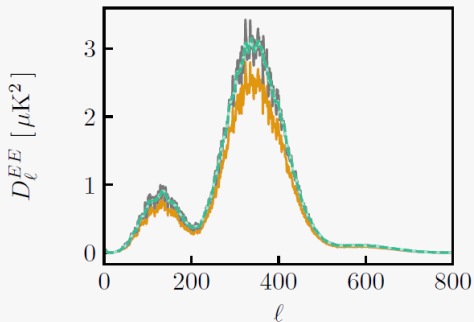
(beam transfer function not deconvolved)

►  $TT$  leaked a bit



# ideal vs non-ideal output spectra (1)

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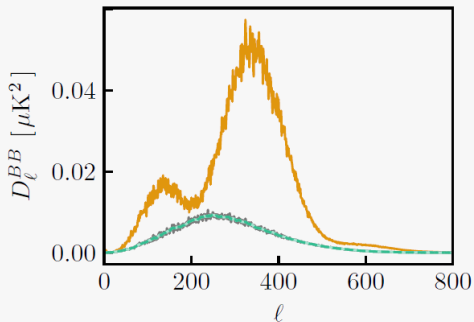
(beam transfer function not deconvolved)

▶  $TT$  leaked a bit

▶  $EE$  leaked a lot!

# ideal vs non-ideal output spectra (1)

---

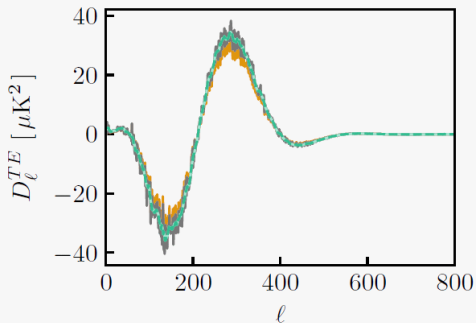


(beam transfer function not deconvolved)

- ▶ *TT* leaked a bit
- ▶ *EE* leaked a lot!
- ▶ *BB* larger (*EE* shape!)

# ideal vs non-ideal output spectra (1)

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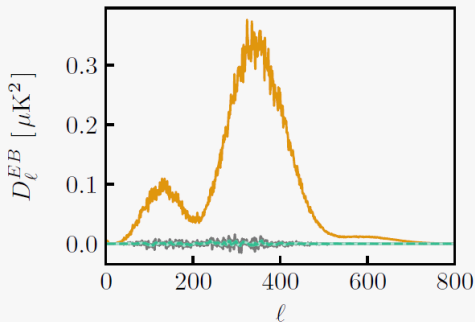


(beam transfer function not deconvolved)

- ▶ *TT* leaked a bit
- ▶ *EE* leaked a lot!
- ▶ *BB* larger (*EE* shape!)
- ▶ *TE* leaked a bit

# ideal vs non-ideal output spectra (1)

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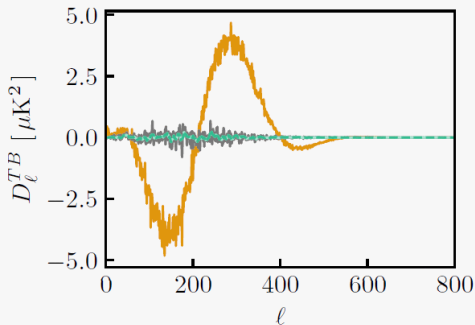


(beam transfer function not deconvolved)

- ▶ *TT* leaked a bit
- ▶ *EE* leaked a lot!
- ▶ *BB* larger (*EE* shape!)
- ▶ *TE* leaked a bit
- ▶ *EB* non-zero!

# ideal vs non-ideal output spectra (1)

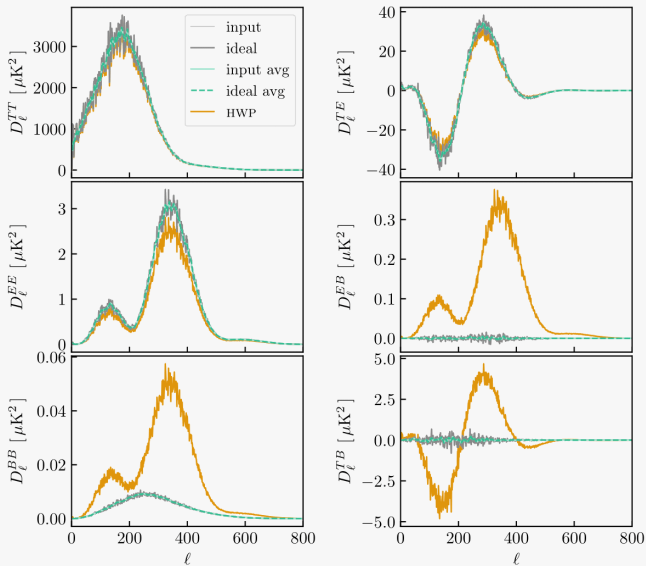
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(beam transfer function not deconvolved)

- ▶  $TT$  leaked a bit
- ▶  $EE$  leaked a lot!
- ▶  $BB$  larger ( $EE$  shape!)
- ▶  $TE$  leaked a bit
- ▶  $EB$  non-zero!
- ▶  $TB$  non-zero!

## ideal vs non-ideal output spectra (2)



how can we understand this?

# modeling the TOD

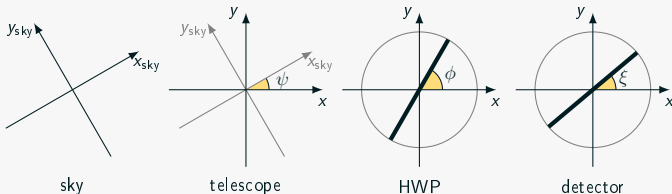
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How beamconv computes the TOD:

$$d_t = \sum_{s\ell m} \left[ B_{\ell s}^I a_{\ell m}^I + \frac{1}{2} (-2B_{\ell s}^P a_{\ell m}^P + 2B_{\ell s}^P - 2a_{\ell m}^P) + B_{\ell s}^V a_{\ell m}^V \right] \sqrt{\frac{4\pi}{2\ell+1}} e^{-is\psi_t} {}_sY_{\ell m}(\theta_t, \phi_t),$$

beam coefficients (*or combinations of them if HWP non-ideal*).

**In the paper:**  $d = (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{\xi-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \cdot S.$





# modeling the observed maps

---

**(minimal) TOD:** signal detected by 4 detectors.

**map-maker:** bin-averaging assuming ideal HWP.

**estimated output maps:** linear combination of  $\{I, Q, U\}_{\text{in}}$ .

$$\begin{pmatrix} d^{(0)} \\ d^{(90)} \\ d^{(45)} \\ d^{(135)} \end{pmatrix} = \begin{pmatrix} (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{0-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{90-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{45-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \\ (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{135-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \end{pmatrix} \cdot \begin{pmatrix} I_{\text{in}} \\ Q_{\text{in}} \\ U_{\text{in}} \end{pmatrix}$$

Being *ideal*, map-making amounts to apply  $(\hat{A}^T \hat{A})^{-1} \hat{A}^T$  to the TOD:

$$\hat{S} = (\hat{A}^T \hat{A})^{-1} \hat{A}^T A \cdot S.$$

## estimated output maps

---

$$\hat{I} = m_{ij} l_{in} + (m_{iq} Q_{in} + m_{iu} U_{in}) \cos(2\alpha) + (m_{iq} U_{in} - m_{iu} Q_{in}) \sin(2\alpha),$$

$$\begin{aligned} \hat{Q} = \frac{1}{2} \big\{ & (m_{qq} - m_{uu}) Q_{in} + (m_{qu} + m_{uq}) U_{in} + 2 m_{qi} l_{in} \cos(2\alpha) + 2 m_{ui} l_{in} \sin(2\alpha) \\ & + [(m_{qq} + m_{uu}) Q_{in} + (m_{qu} - m_{uq}) U_{in}] \cos(4\alpha) \\ & + [-(m_{qu} - m_{uq}) Q_{in} + (m_{qq} + m_{uu}) U_{in}] \sin(4\alpha) \big\}, \end{aligned}$$

$$\begin{aligned} \hat{U} = \frac{1}{2} \big\{ & (m_{qq} - m_{uu}) U_{in} - (m_{qu} + m_{uq}) Q_{in} - 2 m_{ui} l_{in} \cos(2\alpha) + 2 m_{qi} l_{in} \sin(2\alpha) \\ & + [-(m_{qq} + m_{uu}) U_{in} + (m_{qu} - m_{uq}) Q_{in}] \cos(4\alpha) \\ & + [(m_{qu} - m_{uq}) U_{in} + (m_{qq} + m_{uu}) Q_{in}] \sin(4\alpha) \big\}, \end{aligned}$$

where  $\alpha = \phi + \psi$ . For **good** coverage and **rapidly spinning** HWP:

$$\hat{\mathbf{S}} \simeq \begin{pmatrix} m_{ij} l_{in} \\ [(m_{qq} - m_{uu}) Q_{in} + (m_{qu} + m_{uq}) U_{in}]/2 \\ [-(m_{qu} + m_{uq}) Q_{in} + (m_{qq} - m_{uu}) U_{in}]/2 \end{pmatrix}.$$

# equations for the $\hat{C}_\ell$ s

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Expanding  $\hat{S}$  in spherical harmonics:

$$\hat{C}_\ell^{TT} \simeq m_{ii}^2 C_{\ell,\text{in}}^{TT},$$

$$\hat{C}_\ell^{EE} \simeq \frac{(m_{qq} - m_{uu})^2}{4} C_{\ell,\text{in}}^{EE} + \frac{(m_{qu} + m_{uq})^2}{4} C_{\ell,\text{in}}^{BB} + \frac{(m_{qq} - m_{uu})(m_{qu} + m_{uq})}{2} C_{\ell,\text{in}}^{EB},$$

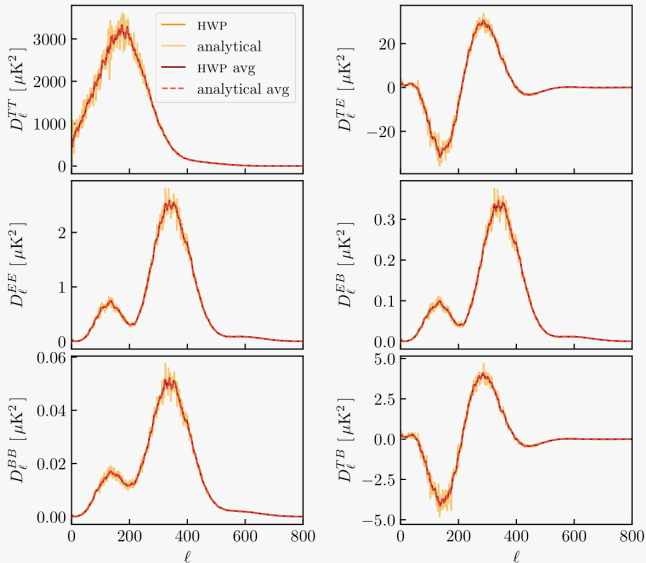
$$\hat{C}_\ell^{BB} \simeq \frac{(m_{qq} - m_{uu})^2}{4} C_{\ell,\text{in}}^{BB} + \frac{(m_{qu} + m_{uq})^2}{4} C_{\ell,\text{in}}^{EE} - \frac{(m_{qq} - m_{uu})(m_{qu} + m_{uq})}{2} C_{\ell,\text{in}}^{EB},$$

$$\hat{C}_\ell^{TE} \simeq \frac{m_{ji}(m_{qq} - m_{uu})}{2} C_{\ell,\text{in}}^{TE} + \frac{m_{ji}(m_{qu} + m_{uq})}{2} C_{\ell,\text{in}}^{TB},$$

$$\hat{C}_\ell^{EB} \simeq \frac{(m_{qq} - m_{uu})^2 - (m_{qu} + m_{uq})^2}{4} C_{\ell,\text{in}}^{EB} - \frac{(m_{qq} - m_{uu})(m_{qu} + m_{uq})}{2} (C_{\ell,\text{in}}^{EE} - C_{\ell,\text{in}}^{BB}),$$

$$\hat{C}_\ell^{TB} \simeq \frac{m_{ji}(m_{qq} - m_{uu})}{2} C_{\ell,\text{in}}^{TB} - \frac{m_{ji}(m_{qu} + m_{uq})}{2} C_{\ell,\text{in}}^{TE}.$$

# analytical vs non-ideal output spectra



impact on cosmic birefringence

# HWP-induced miscalibration

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Analytic  $\hat{C}_\ell$ s satisfy the relations:

$$\begin{cases} \hat{C}_\ell^{EB} \simeq \tan(4\hat{\theta}) [\hat{C}_\ell^{EE} - \hat{C}_\ell^{BB}] / 2 \\ \hat{C}_\ell^{TB} \simeq \tan(2\hat{\theta}) \hat{C}_\ell^{TE} \end{cases}$$

The HWP induces an additional miscalibration,  
**degenerate** with cosmic birefringence and polarization angle  
miscalibration!

# HWP-induced miscalibration

---

Analytic  $\hat{C}_\ell$ s satisfy the relations:

$$\begin{cases} \hat{C}_\ell^{EB} \simeq \tan(4\hat{\theta}) [\hat{C}_\ell^{EE} - \hat{C}_\ell^{BB}] / 2 \\ \hat{C}_\ell^{TB} \simeq \tan(2\hat{\theta}) \hat{C}_\ell^{TE} \end{cases}$$

our formulae suggest

$$\hat{\theta} \equiv -\frac{1}{2} \arctan \frac{m_{qu} + m_{uq}}{m_{qq} - m_{uu}} \sim 3.8^\circ,$$

compatibly with simulations.

The HWP induces an additional miscalibration,  
**degenerate** with cosmic birefringence and polarization angle  
miscalibration!

This doesn't mean that the HWP will keep us from measuring  $\beta$ ,  
but it shows how important it is to carefully calibrate  $\mathcal{M}_{\text{HWP}}$ .

simple generalizations



## including frequency dependence

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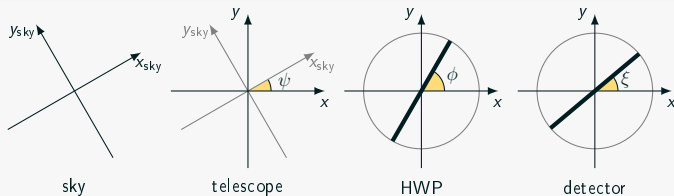
How does  $d = (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{\xi-\phi} \mathcal{M}_{\text{HWP}} \mathcal{R}_{\phi+\psi} \cdot S$  change when the **frequency dependence** of  $\mathcal{M}_{\text{HWP}}$  and signal is taken into account?

$$d = (1 \ 0 \ 0) \cdot \mathcal{M}_{\text{det}} \mathcal{R}_{\xi-\phi} \int d\nu \mathcal{M}_{\text{HWP}}(\nu) \mathcal{R}_{\phi+\psi} \cdot S(\nu).$$

Assuming an ideal map-maker and retracing the same steps as before:

$$\hat{\theta} = -\frac{1}{2} \arctan \frac{\langle m_{qu} + m_{uq} \rangle}{\langle m_{qq} - m_{uu} \rangle}, \quad \text{where } \langle \cdot \rangle = \int d\nu \cdot (\nu) S(\nu).$$

# instrument miscalibration



So far, we assumed  $\begin{cases} \hat{\psi} \equiv \psi, \\ \hat{\phi} \equiv \phi, \\ \hat{\xi} \equiv \xi, \end{cases}$  but more generally  $\begin{cases} \hat{\psi} \equiv \psi + \delta\phi, \\ \hat{\phi} \equiv \phi + \delta\psi, \\ \hat{\xi} \equiv \xi + \delta\xi. \end{cases}$

Taking such (frequency-independent) deviations into account:

$$\hat{\theta} = -\frac{1}{2} \arctan \frac{\langle m_{qu} + m_{uq} \rangle}{\langle m_{qq} - m_{uu} \rangle} + \delta\theta, \quad \text{where } \delta\theta \equiv \delta\xi - \delta\psi - 2\delta\phi.$$

# steps forward

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Even *more general* generalizations worth exploring:

- ▶ including a realistic band pass,
- ▶ allowing for miscalibrations to depend on  $\nu$ .

**For how long can we push the analytical formulae?**

the importance of calibration

## how does the map-model change

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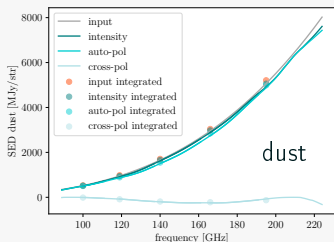
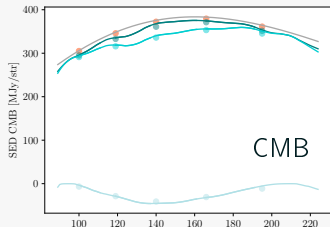
**Without HWP:** 
$$\begin{pmatrix} I_j \\ Q_j \\ U_j \end{pmatrix} = \sum_{\lambda} g_{\lambda} \begin{pmatrix} I_{\lambda} \\ Q_{\lambda} \\ U_{\lambda} \end{pmatrix} + n,$$

**With HWP:** 
$$\begin{pmatrix} I_j \\ Q_j \\ U_j \end{pmatrix} = \sum_{\lambda} \begin{pmatrix} g_{\lambda}^{ii} & 0 & 0 \\ 0 & g_{\lambda}^{qq-uu} & g_{\lambda}^{qu+uq} \\ 0 & g_{\lambda}^{qu+uq} & g_{\lambda}^{qq-uu} \end{pmatrix} \begin{pmatrix} I_{\lambda} \\ Q_{\lambda} \\ U_{\lambda} \end{pmatrix} + n,$$

where  $g_{\lambda} = \frac{\int d\nu G(\nu) S_{\lambda}(\nu)}{\int d\nu G(\nu)}, \quad g_{\lambda}^{ii} = \frac{\int d\nu G(\nu) m_{ii}(\nu) S_{\lambda}(\nu)}{\int d\nu G(\nu)},$  and so on.

HWP non-idealities contribute to **gain**, **polarization-efficiency** and **cross-polarization leakage**.

# effective SEDs



$$\sum_{\lambda} \begin{pmatrix} g_{\lambda}^{ii} & 0 & 0 \\ 0 & g_{\lambda}^{qq-uu} & g_{\lambda}^{qu+uq} \\ 0 & g_{\lambda}^{qu+uq} & g_{\lambda}^{qq-uu} \end{pmatrix} \begin{pmatrix} I_{\lambda} \\ Q_{\lambda} \\ U_{\lambda} \end{pmatrix} + n,$$

- ▶ Since all these effects are frequency dependent, they affect each component differently,
- ▶ An imprecise calibration of  $\mathcal{M}_{\text{HWP}}$  can lead to complications in the component separation step.

- ▶ we are now provided with a **simulation pipeline** that can be easily adapted to study more complex problems (adding noise, more realistic beams...);
- ▶ the **analytical formulae** represent an alternative tool to study the same problems more effectively (but approximately);
- ▶ obvious application: exploiting the analytical formulae to derive **calibration requirements** for the HWP Mueller matrix elements, so that they don't prevent us from detecting ***B*-modes**, measuring **cosmic birefringence**, nor spoil the **foreground cleaning** procedure.